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Equilibrium Concepts in Exhaustible Resource Economics

by

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ABSTRACT

This thesis considers the appropriate equilibrium concept in the dynamic games when industry faces a natural resource that is exhaustible.

The first model of this thesis considers the effect that a capacity constraint has on a pipeline's ability to extract rents from shippers. When binding, a capacity constraint prevents shippers from substituting shipments across time. This raises the profits to the pipeline in the subgame perfect equilibrium by reducing rent-dissipation due to the Coase Conjecture. We show that this effect is most pronounced when the pipeline has a low discount rate, but that it may also happen with a high discount rate. We also show that using a capacity constraint alone that the pipeline cannot increase its profits to the full commitment level.

In the second model of this thesis we develop a theory of 'oil'igopolistic oil exploration in which strategic exploration and production are derived jointly in a three period subgame perfect equilibrium. We predict that producers with smaller proven reserves will do more exploration than producers with larger proven reserves. The rationale for this result is that producers gain no commitment power from exploration if their proven reserves are sufficiently large. Smaller producers, in contrast, can alter the path of production by their rivals by increasing the rate of exploration early in the game. These predictions are consistent with country-level production and reserve data in the post-World War II era.

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DEDICATION

To Eduard and Samuel:

Venujem vám túto prácu a celé svoje štúdium. Ďakujem vám za vašu podporu, ktorou ste ma počas týchto rokov povzbudzovali.

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1. Introduction

The most common equilibrium concept in industrial organization is the Nash equilibrium. In the Nash equilibrium, the agent takes the strategies of other agents as given and does not consider the possibility of influencing them (Nash 1950). In dynamic games, the agent chooses his actions after observing actions of his opponents and agents confront one another repeatedly. In these games, the Nash equilibrium concept must be refined to consider strategic behavior of agents.

The theory of dynamic games generally distinguishes between the open loop Nash equilibrium and the subgame perfect Nash equilibrium (Selten 1975). The open loop strategy is a function of time and the initial state. Thus this strategy does not account for what would happen if some agent were to deviate or if the state is changed by other events in the future. The subgame perfect strategy does reflect the observed state in the action and hence is a function of state and time. Thus the subgame perfect Nash equilibrium is dynamically consistent in the sense that at each point in time agents choose actions that maximize their returns in the subgame beginning at that node.

This thesis discusses and analyzes the appropriate equilibrium concept in dynamic games in markets for a natural resource that is exhaustible. This analysis is done for two different market scenarios.

The first model of this thesis considers a dynamic game between two sides of the market, in particular, the monopsony provider of transportation services of a nonrenewable natural resource, i.e. pipeline, and the competitive sellers of that resource. It is well known (e.g., Bulow 1982) that the open loop equilibrium concept yields higher profits for the monopsonist, yet this equilibrium is implausible since it is not dynamically consistent. Indeed, pipelines are built to serve for several decades, yet servicing contracts are written for much shorter time periods. However short contracts are not favorable to pipelines because reserve depletion raises the price the monopsonist has to pay to sellers. Even though sellers act non-strategically, sellers anticipate that the monopsonist has an incentive to pay a higher price in the later period, and hence they will not sell in the current period. Indeed, the Coase conjecture suggests that if the periods are very short or the cost of waiting is very small the sellers can force the monopsonist to buy the competitive quantity of natural resource today at the competitive price: the monopsony power is destroyed by the ability of the sellers to substitute intertemporally.

We consider the effect that a capacity constraint has on a pipeline's ability to gain monopsony power in the subgame perfect Nash equilibrium. We show that a capacity constraint can prevent shippers from substituting their shipments across time, which can raise the profits to the pipeline compared with the unconstrained subgame perfect equilibrium profits, and reduce Coase Conjecture rent-dissipation. We show that this effect is most pronounced when the pipeline has a low discount rate, but that it may even happen with a very high discount rate.

The second model of this thesis discusses how oligopoly firms, which differ in their stocks of proven and unproven reserves, compete over production and the level of exploration of the nonrenewable resource. The 'oil'igopoly theory of oil production, when the game does not consider the exploration phase, yields similar solutions to the open loop and the subgame perfect Nash equilibria (Loury 1986, Polasky 1992). However, when exploration is added to this game, producers may strategically invest in exploration to increase their production capacity and the open loop Nash equilibrium is no longer an appropriate equilibrium concept. Thus this chapter develops a theory of 'oil'igopolistic oil exploration in which strategic exploration and production are derived jointly in a three period subgame perfect equilibrium. The 'oil'igopoly theory of oil exploration predicts that producers with smaller proven reserves will do more exploration than producers with larger proven reserves. The intuition for this result is that producers gain no commitment power from exploration if their proven reserves are sufficiently large. Smaller producers, in contrast, can alter the path of production by their rivals by increasing the rate of exploration early in the game.

Both models of this thesis analyze problems in exhaustible resource economics. Both models are solved in a discrete time framework with a finite number of periods, where backwards induction methods of the subgame perfect equilibrium can be utilized. In the monopsony pipeline model, only two time periods are necessary to show that the open loop and the subgame perfect Nash equilibria differ. In the 'oil'igopoly theory of oil exploration three time periods are required to obtain a difference in the equilibrium concepts. The correct equilibrium concept in each of these models is the subgame perfect Nash equilibrium. However, in both models, the open loop and the subgame perfect equilibrium differ only under certain conditions. These conditions ensure that actions taken today influence one's rival's future actions and those changes in their future actions affect the agent's own profits.

Even though both models use the same equilibrium concept (the subgame perfect Nash equilibrium), because of different settings of the models, different techniques are used, and different questions are asked in the two models. In the 'oil'igopoly theory of oil exploration, we solve for the Nash equilibrium among producers within each period, where producers have control over extraction and exploration activities. In the monopsony pipeline model, we solve for the subgame perfect equilibrium, where the buyer has the first-mover advantage and thus chooses prices (knowing the desired quantities that maximize his profit), and suppliers are Stackelberg followers with control over production quantities within periods.

In the monopsony pipeline problem, there is only one type of reserves – proven. The question is how much of the proven reserves to use and how this quantity affects the expectations on the subgame perfect Nash equilibrium. In the 'oil'igopoly theory of oil exploration, producers produce from two types of reserves – proven and unproven, that are all used up by the end of period 3. The question is when to explore the unproved reserves to turn them into the proven reserves.

In the monopsony pipeline problem, strict convexity of production cost is required to get the open lop and subgame perfect Nash equilibria that differ in price-production contracts. In the 'oil'igopoly theory of oil exploration, strict convexity of exploration cost is required to get the open loop and subgame perfect Nash equilibria that differ in exploration and extraction activities.

Finally, we obtained surprising results in both problems. In the monopsony pipeline problem, we assume concavity of the buyer's utility function $u(q_t)$. We find that this condition is not the same as a capacity constraint (Kahn 1986). In the 'oil'igopoly theory of oil exploration we find that the medium-sized firms in terms of proven reserve holdings have a strategic incentive to increase exploration activity in subgame perfect comparing with the open loop Nash equilibrium. Both large and medium sized firms have incentive to decrease equilibrium subgame perfect production comparing with the open loop equilibrium.

This thesis proceeds as follows: Chapter 2 illustrates the motivation and literature review for the monopsony pipeline problem. Chapter 3 presents the complete monopoly pipeline model. Chapter 4 discusses the motivation and literature review for the theory of 'oil'igopoly oil exploration, where the theory is derived in chapter 5.

2. Motivation and Literature Review: the Monopoly Pipeline Problem

Pipelines are the primary method of transporting oil and gas across land. In the United States, there currently are over two hundred natural gas pipelines with over 297,000 miles of natural gas transmission lines, of which about 20,000 miles are major trunk lines with the balance being gathering lines. An additional 1.8 million miles of pipelines lines distribute natural gas to consumers.[1] There are also over 200,000 miles of oil and refined products transmission pipelines, of which almost 55,000 miles are "trunk" lines, 8-24 inches in diameter.[2] Pipelines are not just a North American phenomenon. In Western Europe there were over 36,000 kilometers of oil pipelines transporting over 800 million

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[1] "Changes in U.S. Natural Gas Transportation Infrastructure in 2004," Energy Information Agency, U.S. Department of Energy, 2005.
[2] Association of Oil Pipelines, http://www.aopl.org/go/site/888/.

cubic meters of oil in 2003.[3]

Pipelines in the United States presently account for around 17% of the volume of goods transported, but account for only about two percent of the total costs of transportation.[4] Pipelines are perfectly designed to take advantage of economies of scale, since most of the costs of providing pipeline services are the fixed costs of putting the pipeline in place. In the United States, pipeline ownership is disassociated from the shippers. This has important consequences for the distribution of economic rents between the pipeline, which is often a monopoly, and the shippers, who are often small price-taking firms.

The following chapter examines how the dynamics of oil and gas field extraction affect the ability of a pipeline to extract rents from shippers. While most of the economics literature regarding pipelines has focused on its natural monopoly aspects, we focus on the issue of how the choice of capacity size in the pipeline affects the competitive environment in which the pipeline operates. In particular, we view pipeline capacity as a means in which a pipeline may overcome the problem it faces in its relations to shippers – that of being incapable of committing itself to a long-term pricing scheme.

Formally, the problem faced by a pipeline is similar to two other problems that have received much attention in economics. In particular, the problems faced by a durable goods monopolist and the problem faced by an importer of an exhaustible resource.

Coase (1972) noted that a durable goods monopolist faces the problem that his customers know that he has an incentive to ignore their capital losses from future sales.

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[3] "Performance of European Cross-Country Oil Pipelines," Report 3/05, CONCAWE, Brussels, 2005, www.concawe.org.
[4] Association of Oil Pipelines, http://www.aopl.org/go/site/888/.

With a constant marginal cost, Coase conjectured that since future sales are a perfect substitute for current sales, that the monopolist will produce the competitive quantity "in the twinkling of an eye". This happens because consumers of the durable good realize that while the durable goods monopolist has an incentive to set the price high in the beginning, it will later wish to lower that price to attract further customers. This conjecture was found to be true by Stokey (1981) and Gul et al (1986). However, in an important extension, Kahn (1986) found that if the marginal cost of production is increasing in the output level, that the monopolist has an incentive to spread production out over time, and that this unravels the Coase conjecture.

Coase also conjectured that the monopolist might regain his monopoly power through leasing, rather than sales, so that his customers know that the monopolist will internalize the capital losses from additions to the stock of durable goods. Bulow (1982) formalized this part of Coase's conjecture, and found that so long as the durable goods monopolist has a credible commitment device that his profits will be larger than if he does not have such a commitment device. The question then is what constitutes a credible commitment device?

The same story as the "Coase conjecture" occurs with an exhaustible resource importer who tries to extract economic rents from resource producers by setting an import tariff. Bergstrom (1982) showed that when extraction costs are constant, even in the dynamic model, all rent can be captured by importer. When the marginal cost of production is stock-dependent and rises as reserves deplete, producers will realize that the tariff will decline in the future, and will shift supply across periods (Karp 1984, Newbery and Maskin 1993, Hörner and Kamien 2004).

Our contribution to this literature is to show the conditions under which a capacity constraint by the pipeline serves as a credible commitment device for the pipeline. The model we consider is that of a monopsonistic buyer who faces a number of sellers. Following Bulow (1982), we consider only a simple two-period model, which means that profits to the monopsonist are not driven to zero even when the monopsonist has no other commitment device. Analogous to Kahn (1986), we assume that the buyer's marginal utility is decreasing in the output level. This gives the monopsonist an incentive to spread production across both periods. Second, he producers who use the pipeline face costs which are increasing as the remaining reserves are depleted. This has the effect of ensuring that the marginal cost curves of the producers are continuous, so that we avoid the "pacman" solution of Bagnoli et al. (1989).

We find that the discount rate has a huge effect upon whether or not the pipeline is able to use pipeline capacity as a commitment device with shippers. This is surprising because in the full commitment equilibrium, the quantity the pipeline ships declines over time for any discount rate. However, in the no commitment possible equilibrium, the quantity the pipeline ships may increase over time. This is most likely to occur when the value of a dollar earned in the future is high – which is to say the discount rate is low.

It is precisely this case, where production is increasing over time in the unconstrained subgame perfect equilibrium, in which the capacity constraint is useful to the pipeline. The reason is quite intuitive. The strategic effect comes from the fact that the monopsonist can affect second period output levels with first period choices. If capacity constrains the second period output level, then the strategic effect disappears for the monopsonist.

Capacity constraints of various forms have been considered in the literature. Kahn (1986) showed that an increasing marginal cost of production – of which a capacity constraint is a limiting case – causes the Coase conjecture to partially unravel. Our model shows that even with constant marginal profit from production – which is equivalent to constant marginal cost of production in Kahn's model – that we obtain the result that a capacity constraint can increase profits to the pipeline. Thus capacity effects are separate from curvature of the cost function. Denicolo and Garella (1999) show that restricting sales to a portion of buyers willing to pay the first period price (given their expectations of the second period price) increases the monopolists' profits. In our model, the capacity constraint allows the monopolist to raise the price in the first period, which means he leaves less surplus on the table relative to the rationing equilibrium. Finally, MacAfee and Wiseman (2006) consider a model in which the monopolist can expand capacity at a low but positive cost in every period. In contrast, we have capacity as a choice made at the beginning of the game which cannot be altered later. Nevertheless, the lesson drawn from both papers is quite similar: capacity constraints raise profits to the pipeline.

The model presented in the next chapter is organized as follows. Section 3.1 identifies the assumptions on costs and pipeline profits. Section 3.2 derives the full commitment open loop Nash equilibrium. Section 3.3 derives the unconstrained subgame perfect equilibrium. Section 3.4 adds the capacity constraint to the subgame perfect equilibrium. Section 3.5 derives the optimal capacity level. Section 3.6 concludes.

3. Capacity Constraints as a Commitment Device in Pipeline Rent Extraction

3.1. Assumptions

We assume that pipeline services are essential to bringing the production of non-renewable resource to market. The suppliers of the resource are assumed to be price taking, and hence non-strategic, but they are forward looking rational actors. They face a single monopsony pipeline who buys output q_t from the competitive producers, paying price p_t per unit in periods t = 1,2.[5] We assume that the pipeline is fully depreciated after two periods. Both the suppliers and the buyer discount second period profits at the common factor δ, where $0 < \delta < 1$. The initial stock is R and the stock decreases at production rate q_1 resulting in stock $R - q_1$ at the beginning of period $t = 2$ and in stock $R - q_1 - q_2$, which must be non-negative, at the end of period two.

The pipeline's net-of-acquisition-costs profits in each period are denoted as $u(q_t)$, and depend only on the quantity produced, q_t. Suppliers' extraction costs are denoted as $C(q_t, R-q_{t-1})$, and depend both on the quantity produced and the initial reserves. We assume that the cost and profit functions obey the following conditions:

A.1: $C(q_t, R-q_{t-1}) = \int_{q_{t-1}}^{q_{t-1}+q_t} c(R-q)\,dq$, where $q_0 \equiv 0$, $c'(.) < 0$ and $c''(.) \geq 0$;

A.2: $u'(q_t) > 0$ and $u''(q_t) \leq 0$, $t = 1, 2$;

[5] The problem of the pipeline where monopoly supplier offers processing and transmission services is formally equivalent to monopsony buyer of oil or natural gas. Let ρ_t denote the price paid to producers by consumers and let τ_t denote the tariff chosen by the pipeline. Then the producer receives price $p_t = \rho_t - \tau_t$.

A.3: $u'(0) > c(0)$;

A.4: $u'(0) < c(0) - Rc'(0)$.

Assumption A.1 defines the extraction cost for suppliers of the resource such that the unit cost of extraction, $c(R - q_t)$, rises at an increasing rate as reserves are exhausted.[6] In the pipeline considered here, the marginal factor cost of *cumulative* production q starting with stock R is

$$m(q) = c(R - q) - qc'(R - q). \tag{3.1}$$

An implication of A.1 is that the marginal factor cost is greater than the average factor cost, i.e., $m(q) > c(R - q) > 0$, and that the marginal factor cost is increasing in q, i.e., $m'(q) > 0$.[7]

Assumption A.2 defines the buyer's profits in each period as an increasing concave function of the quantity purchased in that period.[8,9]

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[6] In the durable goods monopoly problem, the assumption that $c'(.) < 0$ in A.1 is equivalent to the assumption that marginal revenue is less than average revenue to the durable goods monopolist.

[7] Equation (3.1) is written with the initial stock as R. Thus, in period one there is no problem with the interpretation of (3.1) as the marginal factor cost. However, when $q_1 > 0$, the marginal factor cost in period two is $c(R-q_1-q_2) - q_2c'(R-q_1-q_2) < m(q_1+q_2)$. However, $m(q_1+q_2) = c(R-q_1-q_2) - (q_1+q_2)c'(R-q_1-q_2)$ is then the marginal factor cost of *cumulative* production.

[8] To get a concave profit function, it must either be that the demand curve, $D(q_t)$, for the output q_t is downwards sloping or that the cost of transporting the resource product, $M(q_t)$, is rising at an increasing rate in q_t. Then $u(q_t) = D(q_t)q_t - M(q_t)$, and $u'(q_t) > 0$ for $D(q_t) + D'(q_t)q_t > M'(q_t)$ and $u''(q_t) < 0$ for $2D'(q_t) + D''(q_t)q_t < M'(q_t)$.

[9] In the durable goods monopoly case, $u'(q_t) > 0$ is equivalent to positive marginal cost of production, and $u''(q_t) < 0$ is equivalent to increasing marginal costs. In Bulow (1982), marginal cost was constant. Kahn (1986) was the first to examine the durable goods monopoly case where marginal cost was increasing.

Assumptions A.3 and A.4 make the game dynamic. Assumption A.3 guarantees that the buyer is capable of purchasing the last unit of reserves, so that if positive reserves exist in any period, there will be positive level of production. Assumption A.4 ensures that the buyer does not wish to consume all of the reserves in a single period.

We turn now to the characterization of the full-commitment equilibrium.

3.2. Open Loop Nash Equilibrium

We begin by deriving the open loop Nash equilibrium. In the Nash equilibrium both buyer and sellers have perfect commitment power. While unrealistic, this equilibrium serves as a benchmark from which to compare both the unconstrained no-commitment equilibrium and the capacity constrained equilibrium below, as it produces the highest possible profits for the pipeline.[10] In the open loop equilibrium the buyer commits to the price p_t in each of the two periods at time $t = 0$ and the sellers commit to the quantity supplied q_t in each of the two periods at time $t = 0$. We assume throughout that the buyer acts as a Stackelberg leader and the sellers act as Stackelberg followers, although we will refer to the solution where all choices are made at time $t = 0$ as the Nash equilibrium.

Taking the prices p_1 and p_2 as given, the sellers choose q_1 and q_2 to maximize

[10] Bulow (1982) provides the simplest example of why perfect commitment power of each side of the market in the Nash equilibrium is unrealistic. In his example, a durable goods monopolist faces a linear demand for the good and has constant (zero) marginal cost of production – corresponding to $u''(q_t) = 0$ in our model. In the Nash equilibrium, the durable goods monopolist produces only in period one, and hence behaves exactly as a non-durable goods monopolist, producing to the quantity where marginal revenue is equal to zero. In the second period, both consumers and the durable goods monopolist benefit from additional production since the intercept of the residual demand curve is greater than marginal cost. See also Karp (1984).

$$\pi^S = p_1 q_1 - \int_0^{q_1} c(R-q)dq + \delta\left(p_2 q_2 - \int_{q_1}^{q_1+q_2} c(R-q)dq\right) \tag{3.2}$$

subject to non-negativity constraints on production and the resource stock constraint that $q_1 + q_2 \le R$. Let λ denote the Lagrange multiplier associated with the resource stock constraint in the sellers' objective function. In equilibrium, λ will be the scarcity rental value of the *in situ* resource stock. The sellers' first-order necessary conditions are given by (3.3)-(3.5):[11]

$$\frac{\partial L^S}{\partial q_1} = p_1 - (1-\delta)c(R-q_1) - \delta c(R-q_1-q_2) - \lambda \le 0, \tag{3.3}$$

$$\frac{\partial L^S}{\partial q_2} = \delta\left[p_2 - c(R-q_1-q_2)\right] - \lambda \le 0, \tag{3.4}$$

$$\frac{\partial L^S}{\partial \lambda} = (R-q_1-q_2) \ge 0,\ \lambda \ge 0 \text{ and } \lambda\left[R-q_1-q_2\right] = 0. \tag{3.5}$$

We shall refer to the open loop Nash equilibrium values of the choice variables with a subscript 'OL'. The inequalities in (3.3) and (3.4) reflect that a corner solution is possible in the q_t. Equations (3.3) and (3.4) together imply a Hotelling intertemporal price arbitrage condition when positive quantities of resource are supplied in both periods, i.e.,

$$p_1^{OL} - c\left(R-q_1^{OL}\right) = \delta\left[p_2^{OL} - c\left(R-q_1^{OL}\right)\right] \tag{3.6}$$

[11] Throughout the chapter, we shall indicate that the non-negativity constraint on production binds by writing the first-order condition in production as a strict inequality. However, we shall explicitly include a multiplier (denoted as θ or θ_t) for the non-negativity constraint on the suppliers' scarcity rent, λ or λ_t.

when $q_1 > 0$ and $q_2 > 0$.

The buyer's discounted stream of net profits is given by:

$$\pi^B = u(q_1) - p_1 q_1 + \delta\left[u(q_2) - p_2 q_2\right]. \tag{3.7}$$

The buyer chooses q_1, q_2, p_1, p_2 and λ to maximize the discounted flow of net profits subject to satisfying the sellers' first-order conditions, (3.3)-(3.5). The Lagrangian for the buyer's problem can be stated as:

$$L^B = u(q_1) - p_1 q_1 + \delta\left[u(q_2) - p_2 q_2\right] + \mu_2\left[\delta\left[p_2 - c(R - q_1 - q_2)\right] - \lambda\right]$$

$$+\ \mu_1\left[p_1 - (1-\delta)c(R - q_1) - \delta c(R - q_1 - q_2) - \lambda\right] + \phi\left[R - q_1 - q_2\right] + \theta\lambda.$$

The multipliers μ_1 and μ_2 are for the constraints (3.3) and (3.4), and the multipliers ϕ and θ are for the resource supply constraint and the non-negativity constraint on scarcity rent in (3.5), respectively. The buyer is unconstrained in his choice of the prices to be offered. Thus the first-order necessary conditions in the prices satisfy

$$\frac{\partial L^B}{\partial p_1} = -q_1 + \mu_1 = 0, \tag{3.8}$$

$$\frac{\partial L^B}{\partial p_2} = -\delta q_2 + \delta \mu_2 = 0. \tag{3.9}$$

From (3.8) and (3.9), we get that $\mu_1^{OL} = q_1^{OL}$ and $\mu_2^{OL} = q_2^{OL}$, respectively. Therefore, we may write the buyer's first-order necessary conditions in the choice of the quantities produced in each period and in the value of the scarcity rent as

$$\frac{\partial L^B}{\partial q_1} = u'(q_1) - p_1 + (q_1 + q_2)\ \delta c'(R - q_1 - q_2)$$

$$+ q_1 (1-\delta) c'(R - q_1) - \phi \ \leq 0, \tag{3.10}$$

$$\frac{\partial L^B}{\partial q_2} = \delta\left[u'(q_2) - p_2\right] + (q_1 + q_2)\ \delta c'(R - q_1 - q_2) - \phi \ \leq 0, \tag{3.11}$$

$$\frac{\partial L^B}{\partial \lambda} = -q_1 - q_2 + \theta = 0, \tag{3.12}$$

$$\theta \geq 0,\ \ \lambda \geq 0, \text{ and } \theta\lambda = 0, \tag{3.13}$$

$$\phi \geq 0,\ \ R - q_1 - q_2 \geq 0, \text{ and } \phi\left[R - q_1 - q_2\right] = 0, \tag{3.14}$$

where the inequalities in (3.10) and (3.11) correspond to the non-negativity constraints in production. Note that combining (3.3) and (3.10), and combining (3.4) and (3.11) yields

$$\delta\left[u'\left(q_2^{OL}\right) - m\left(q_1^{OL} + q_2^{OL}\right)\right] \leq \phi + \lambda, \tag{3.15}$$

$$u'\left(q_1^{OL}\right) - (1-\delta)\, m\left(q_1^{OL}\right) - \delta m\left(q_1^{OL} + q_2^{OL}\right) \leq \phi + \lambda. \tag{3.16}$$

Thus, if $q_1 > 0$ and $q_2 > 0$, then

$$u'\left(q_1^{OL}\right) - \delta u'\left(q_2^{OL}\right) = (1-\delta)\, m\left(q_1^{OL}\right)\ . \tag{3.17}$$

Comparing (3.6) and (3.17), we see that while discounted prices rise at rate $(1-\delta)c(R-q_1)$, discounted marginal profits to the buyer rise at rate $(1-\delta)m(q_1)$ > $(1-\delta)c(R-q_1)$. This is because the buyer internalizes the cost increase to infra-marginal production whereas producers only require that price rise to cover the difference in cost at the margin.

We may now state the main result of this section, characterizing the open loop equilibrium:

Theorem 3.1: Under assumptions A.1-A.4, the open loop Nash equilibrium satisfies

$$q_1^{OL} > q_2^{OL} > 0, \tag{3.18}$$

$$q_1^{OL} + q_2^{OL} < R, \tag{3.19}$$

$$p_2^{OL} \leq c\left(R - q_1^{OL} - q_2^{OL}\right), \tag{3.20}$$

$$\begin{aligned} u'\left(q_2^{OL}\right) &\leq c\left(R - q_1^{OL} - q_2^{OL}\right) - \left(q_1^{OL} + q_2^{OL}\right)c'\left(R - q_1^{OL} - q_2^{OL}\right) \\ &= m\left(q_1^{OL} + q_2^{OL}\right), \end{aligned} \tag{3.21}$$

$$p_1^{OL} - \delta p_2^{OL} \geq (1-\delta)c\left(R - q_1^{OL}\right), \tag{3.22}$$

$$u'\left(q_1^{OL}\right) - \delta u'\left(q_2^{OL}\right) \geq (1-\delta)m\left(q_1^{Ol}\right). \tag{3.23}$$

Condition (3.18) implies that the two-period Nash equilibrium for a monopsonistic pipeline has higher level of production in period one than in period two. Indeed, period two production may be zero. Condition (3.19) implies that in the Nash equilibrium, the monoposonistic pipeline leaves some resource stock un-exploited, even though it is economically feasible to produce. This drives the scarcity rental value of the resource stock to zero for both buyer and sellers. The weak inequalities in equations (3.20)-(3.23) hold with equality when $q_2^{OL} > 0$. Thus, when production is positive in period two, the price sellers receive equals the marginal cost of extraction and the quantity the pipeline transports is where marginal revenue product is equal to the marginal factor cost. Condition (3.22) gives the rate at which the price must increase over time in order for suppliers to be indifferent between selling in either period (equality) or to be willing to only supply in period one (strict inequality), given that sellers' extraction costs rise as reserves are depleted. Likewise, (3.23) gives the condition under which the buyer is indifferent between purchasing positive quantities in each period (equality) or willing to only purchase a positive quantity in period one (strict inequality), taking into account the amount by which the price must rise in period two given extraction in period one.

We prove Theorem 3.1 by the following three lemmas. The first lemma shows that the buyer drives the resource rental price to zero, implying that the buyer extracts all of the rents.[12]

Lemma 3.1: Under Assumptions A.1-A.4, in the Nash equilibrium if production is positive in at least one period, then the scarcity rental value to suppliers is driven to zero.

[12] Because the periods are of fixed length, the monopsonist cannot capture infra-marginal rents due to the upwards sloping extraction cost function. Thus, suppliers of the input earn positive rents in equilibrium.

Proof: Suppose that $\lambda^{OL} > 0$ when $q_1^{OL} + q_2^{OL} > 0$. $\lambda^{OL} > 0$ implies that $\theta^{OL} = 0$. However, (3.12) implies that $q_1^{OL} + q_2^{OL} = \theta^{OL} = 0$, which is a contradiction. *Q.E.D.*

Next we show that the resource is not fully exhausted in the Nash equilibrium.

Lemma 3.2: Under Assumptions A.1-A.4, in the Nash equilibrium the resource is not fully exhausted.

Proof: Suppose that $\phi^{OL} > 0$, which implies that $q_1^{OL} + q_2^{OL} = R$. Thus, for $R > 0$, production must occur in at least one of two periods. There are two cases to consider.

(*i*) Suppose that $q_2^{OL} > 0$. Then from (3.15) and assumption A.2 we get

$$u'(0) - m(R) > u'\left(q_2^{OL}\right) - m(R) = \phi^{OL}/\delta.$$

However the expression on the left is negative by assumption A.4, which is a contradiction.

(*ii*) Next, suppose that $q_1^{OL} = R > 0$, which implies that $q_2^{OL} = 0$. Then assumption A.2 implies that (3.15) can be written as

$$u'(0) - m(R) > u'(R) - m(R) = \phi^{OL} > 0.$$

Again, this contradicts assumption A.4. *Q.E.D.*

The final lemma of this section shows that production in period one is always larger than production in period two in the Nash equilibrium.

Lemma 3.3: Under Assumptions A.1-A.4, in the Nash equilibrium the buyer always offers a price contract that results in positive production in both periods with production in period one larger than production in period two.

Proof: First, assume that both $q_1^{OL} = q_2^{OL} = 0$. Then by assumption A.1, (3.15) or (3.16) can be written as

$$u'(0) \leq c(R) \leq c(0),$$

which contradicts assumption A.3. Next, assume that $q_1^{OL} > 0$ and $q_2^{OL} > 0$. Then (3.15) and (3.16) together with assumption A.1 and Lemma 3.1, imply that

$$u'(q_1^{OL}) - u'(q_2^{OL}) = (1-\delta)\left[m(q_1^{OL}) - m(q_1^{OL} + q_2^{OL})\right] < 0$$

Thus, if both periods have positive production, period one production is larger. Next, suppose that $q_1^{OL} > q_2^{OL} = 0$. Then (3.15) and A.1 implies

$$u'(0) < c(R - q_1^{OL}) < c(0),$$

which contradicts assumption A.3. Finally, assume that $q_1^{OL} = 0$ and $q_2^{OL} > 0$. The first-order necessary conditions (3.15) and (3.16), together with assumption A.1, imply that

$$u'(0) - u'(q_2^{OL}) \leq (1-\delta)\left[m(0) - m(0 + q_2^{OL})\right] < 0.$$

This implies that $u'(0) < u'(q_2^{OL})$, which contradicts assumption A.2. *Q.E.D.*

Lemmas 3.1 and 3.3 together imply that the first period equilibrium quantity is strictly positive, which proves condition (3.18) of Theorem 3.1. Condition (3.19) follows from Lemmas 3.2 and 3.3. Conditions (3.20)-(3.23) follow from condition (3.18).

Next, we turn to the equilibrium in which firms are unable to commit to future actions.

3.3. Closed Loop Subgame Perfect Nash Equilibrium

In the closed loop, or subgame perfect Nash equilibrium, no one can commit at $t = 0$ to do something at times $t = 1$ or $t = 2$ that is not in their best interest at that time. Within each period, we continue to assume that the buyer is the Stackelberg leader and the sellers are Stackelberg followers. We solve for this equilibrium by backwards induction and refer to the equilibrium as the subgame perfect equilibrium.

3.3.A. Subgame Perfect Equilibrium in Period Two

The sellers' problem at the beginning of the second period consists of finding q_2 that maximizes their profit in the second period, taking as given the price p_2 offered by buyer and the quantity remaining from the first period, $R - q_1$. We assume for now that $R - q_1 >$ 0, otherwise there is no period two choice to be made. We shall consider the validity of this assumption below. The sellers' (undiscounted) second period profits are

$$\pi_2^S = p_2 q_2 - \int_{q_1}^{q_1+q_2} c(R-q)\,dq. \tag{3.24}$$

The sellers' choice of q_2 must be non-negative, i.e., $q_2 \geq 0$, and it must satisfy the resource stock constraint that $q_2 \leq R - q_1$. The corresponding sellers' Lagrangian function is:

$$L_2^S = p_2 q_2 - \int_{q_1}^{q_1+q_2} c(R-q)\,dq + \lambda_2 [R - q_1 - q_2].$$

The sellers' first-order necessary conditions are thus:

$$\frac{\partial L_2^S}{\partial q_2} = p_2 - c(R - q_1 - q_2) - \lambda_2 \leq 0, \tag{3.25}$$

$$\frac{\partial L_2^S}{d\lambda} = R - q_1 - q_2 \geq 0, \lambda \geq 0 \text{ and } \lambda_2 [R - q_1 - q_2] = 0. \tag{3.26}$$

Condition (3.25) says that sellers produce only when the price covers the marginal cost of resource extraction, $c\left(R - q_1 - q_2^{SP}\right)$, plus the scarcity rent, λ_2. The Kuhn-Tucker conditions for the second period production constraint are given by (3.26).

Since the buyer is the Stackelberg leader, the buyer's problem in period two can be thought of as choosing p_2, q_2, and λ_2 to maximize

$$\pi_2^B = u(q_2) - p_2 q_2. \tag{3.27}$$

subject to the constraints (3.25) and (3.26). Thus the buyer's Lagrangian is

$$L_2^B = u(q_2) - p_2 q_2 + \mu_2 \left[p_2 - c(R - q_1 - q_2) - \lambda\right] + \phi_2 [R - q_1 - q_2] + \theta_2 \lambda,$$

where θ_2 is the multiplier on the non-negativity constraint for the scarcity rental value, ϕ_2

is the multiplier on the resource stock constraint given in (3.26), and μ_2 is the multiplier on the price constraint (3.25). The first-order necessary condition for the buyer's choice of the price p_2 is

$$\frac{\partial L_2^B}{\partial p_2} = -q_2 + \mu_2 = 0. \tag{3.28}$$

Thus (3.28) implies $\mu_2^{SP} = q_2^{SP}$. Using (3.28), the first-order necessary conditions for the choices of q_2, λ_2, ϕ_2 and θ_2 must satisfy

$$\frac{\partial L_2^B}{\partial q_2} = u'(q_2) - p_2 + q_2 c'(R - q_1 - q_2) - \phi_2 \leq 0, \tag{3.29}$$

$$\frac{\partial L_2^B}{\partial \lambda} = -q_2 + \theta_2 = 0, \tag{3.30}$$

$$\phi_2 \geq 0,\ R - q_1 - q_2 \geq 0, \text{ and } \phi_2 [R - q_1 - q_2] = 0, \tag{3.31}$$

$$\theta_2 \geq 0,\ \lambda_2 \geq 0, \text{ and } \theta_2 \lambda_2 = 0. \tag{3.32}$$

There are two differences between the second period subgame perfect equilibrium given by (3.25)-(3.26) and (3.28)-(3.32) and the second period Nash equilibrium given by (3.4), (3.9) and (3.11). A trivial difference is that λ and ϕ in the Nash equilibrium are replaced by λ_2/δ and ϕ_2/δ in the subgame perfect equilibrium. The significant difference is that only q_2 appears as a coefficient on $c'(R - q_1 - q_2)$ in the subgame perfect equilibrium condition (3.29), while q_1+q_2 appears as the coefficient on $c'(R-q_1-q_2)$ in the Nash equilibrium condition (3.11). This difference prohibits us from making an explicit statement about the upper bound on the magnitude of q_2.

We summarize the second period equilibrium as follows:

Theorem 3.2: The second period equilibrium satisfies

$$0<q_2^{SP}\leq R-q_1, \tag{3.33}$$

$$p_2^{SP}=c\left(R-q_1-q_2^{SP}\right), \tag{3.34}$$

$$u'\left(q_2^{SP}\right) \leq c\left(R-q_1-q_2^{SP}\right)-q_2^{SP}c'\left(R-q_1-q_2^{SP}\right). \tag{3.35}$$

The upper bound on q_2^{SP} in (3.33) is a weak inequality. This is unlike the Nash equilibrium, where it was possible to show that $q_2^{OL}<R-q_1^{OL}$, and it occurs because only q_2^{SP} appears as a coefficient to $c'\left(R-q_1-q_2^{SP}\right)$ in (3.29). Condition (3.34) implies that the buyer drives the scarcity rental value to zero. The weak inequality in (3.35) reflects the fact that the scarcity rental value to the buyer may not be zero, if the entire stock is exhausted.

We prove the strict inequality in (3.33) and the strict equality in (3.34) in Theorem 3.2 by the following two lemmas:

Lemma 3.4: Under assumptions A.1-A.4, in the subgame perfect equilibrium, if there is positive stock at the beginning of the second period, then the second period quantity is positive.

Proof: Suppose that $R-q_1>0$ and that $q_2^{SP}=0$. Then $\lambda_2^{SP}=\phi_2^{SP}=0$ by (3.26) and (3.31), respectively. Combining (3.25) and (3.29) yields

$$u'(0)<c(R-q_1)<c(0),$$

where the first inequality is necessary to drive $q_2^{SP}=0$ and the second inequality uses A.1. However, this contradicts A.3. *Q.E.D.*

Next, we show that the buyer drives the scarcity rental value to sellers to zero.

Lemma 3.5: Under assumptions A.1-A.4, in the subgame perfect equilibrium, the buyer drives the scarcity rental value for sellers to zero.

Proof: Suppose that $\lambda_2^{SP}>0$. By (3.32), this implies that $\theta_2^{SP}=0$. However, by (3.30), $q_2^{SP}=\theta_2^{SP}=0$, which is a contradiction to Lemma 3.4. *Q.E.D.*

The weak inequalities occur because we cannot determine whether or not q_2^{SP} reaches its upper bound of $R-q_1$.

Next, we derive the relationship between q_2^{SP} and q_1. This relationship is crucial to understanding how the subgame perfect equilibrium differs from the open loop Nash equilibrium.

Proposition 3.1: Under assumptions A.1-A.3, if $R-q_1>0$, then (*i*) $-1\leq \partial q_2^{SP}/\partial q_1<0$, (*ii*) $\partial p_2^{SP}/\partial q_1\geq 0$, and (*iii*) $\partial q_2^{SP}/\partial p_1=\partial p_2^{SP}/\partial p_1=0$.

Proof: There are two cases to consider, depending upon whether or not q_2^{SP} less than or equal to $R-q_1$. First, suppose that $q_2^{SP} < R-q_1$. Then by Lemma 3.4, (3.26) and (3.31), we have that $\lambda_2^{SP} = \phi_2^{SP} = 0$. Thus (3.35) implies

$$u'\left(q_2^{SP}\right)= c\left(R-q_1-q_2^{SP}\right) - q_2^{SP}c'\left(R-q_1-q_2^{SP}\right). \tag{3.36}$$

Implicitly differentiating (3.36) yields

$$\frac{dq_2^{SP}}{dq_1} = -\frac{c'\left(R-q_1-q_2^{SP}\right)-q_2^{SP}c''\left(R-q_1-q_2^{SP}\right)}{u''\left(q_2^{SP}\right)+2c'\left(R-q_1-q_2^{SP}\right)-q_2^{SP}c''\left(R-q_1-q_2^{SP}\right)}. \tag{3.37}$$

By A.1 and A.2, each term in the numerator and denominator of (3.37) is negative in sign. The denominator contains two additional negative terms relative to the numerator. This proves the strict equality parts of (*i*). To get the effect of q_1 on p_2^{SP}, we differentiate (3.34) with respect to q_1 using (3.37) to yield

$$\frac{\partial p_2^{SP}}{\partial q_1} = -c'\left(R-q_1-q_2^{SP}\right)\left[1+\frac{\partial q_2^{SP}}{\partial q_1}\right] > 0. \tag{3.38}$$

This proves the strict inequality part of (*ii*).

Second, suppose that $\phi_2^{SP} > 0$, so that $q_2^{SP} = R-q_1$. Then, $\partial q_2^{SP}/\partial q_1 = -1$. This proves the lower-bound equality part of condition (*i*). Furthermore, if $q_2^{SP} = R-q_1 > 0$, then $p_2^{SP} = c(0)$, so $\partial p_2^{SP}/\partial q_1 = 0$. This proves the equality part of condition (*ii*).

Part (*iii*) follows by noting that p_1 does not appear in (3.33)-(3.35). *Q.E.D.*

Note that the comparative statics in Proposition 3.1 depend upon the properties of the function $c(R-q)$ in A.1 If $c(R-q)$ were a constant, so that $c'(.)=c''(.)=0$, then the second period equilibrium is unaffected by changes in q_1 and p_1.

3.3.B. Subgame Perfect Equilibrium in Period One

We now turn to the period one equilibrium. Let the optimized value of second period profits to sellers be denoted as π_2^{S*}. Then in period one, sellers choose q_1 to maximize

$$\pi_1^S = p_1 q_1 - \int_0^{q_1} c(R-q)\,dq + \delta\pi_2^{S*}, \tag{3.39}$$

subject to a non-negativity constraint on q_1 and the resource stock constraint that $q_1 \leq R$. We let λ_1 denote the Lagrange multiplier resource stock constraint in period one. Then the Lagrangian is

$$L_1^S = p_1 q_1 - \int_0^{q_1} c(R-q)\,dq +$$

$$\delta\left\{p_2^{SP} q_2^{SP} - \int_{q_1}^{q_1+q_2^{SP}} c(R-q)\,dq + \lambda_2^{SP}\left[R-q_1-q_2^{SP}\right]\right\} + \lambda_1\left[R-q_1\right].$$

Because sellers are assumed to be price takers, we set $\partial p_2^{SP}/\partial q_1 = 0$ in the sellers' first order conditions below.[13] Thus by the envelope theorem, the sellers' first-order conditions are

$$\frac{\partial L_1^S}{\partial q_1} = p_1 - (1-\delta)\, c(R-q_1) - \delta c\left(R-q_1-q_2^{SP}\right) - \delta\lambda_2^{SP} - \lambda_1 \le 0, \tag{3.40}$$

$$R-q_1 \ge 0,\ \ \lambda_1 \ge 0, \text{ and } \lambda_1\left[R-q_1\right]=0. \tag{3.41}$$

From (3.33) we see that condition (3.40) imposes a constraint on the difference in prices across the two periods that must be satisfied in order for production in period one to be positive. Note that the Lagrange multiplier for the resource constraint in period one is zero by Theorem 3.2. Conditions (3.41) are the Kuhn-Tucker conditions on the resource stock constraint in period one.

Turning to the buyer, the buyer in period one chooses q_1 and p_1 to maximize

$$\pi_1^B = u(q_1) - p_1 q_1 + \delta\pi_2^{B*}. \tag{3.42}$$

subject to the constraints (3.40) and (3.41). Let ϕ_1 denote the multiplier on the resource stock constraint and θ_1 denote the multiplier on the non-negativity constraint on λ_1. Then the buyer's Lagrangian can be written as

$$L_1^B = u(q_1) - p_1 q_1 + \delta\pi_2^{B*} + \mu_1\left[p_1 - (1-\delta)c(R-q_1) - \delta c\left(R-q_1-q_2^{SP}\right)\right]$$

$$- \mu_1\left[\delta\lambda_2^{SP} + \lambda_1\right] + \phi_1\left[R-q_1\right] + \theta_1\lambda_1.$$

[13] This follows the practice in the literature from Bulow (1982) forward.

Using the envelope theorem, the buyer's first-order necessary condition of the price is:

$$\frac{\partial L_1^B}{\partial p_1} = -q_1 + \mu_1 = 0. \tag{3.43}$$

Thus, (3.43) implies that $\mu_1^{SP} = q_1^{SP}$. Given this, the remaining first-order conditions are

$$\frac{\partial L_1^B}{\partial q_1} = u'(q_1) - p_1 + (1-\delta) q_1 c'(R - q_1) + \delta\left(q_1 + q_2^{SP}\right) c'\left(R - q_1 - q_2^{SP}\right)$$

$$+ \delta q_1 c'\left(R - q_1 - q_2^{SP}\right)\frac{\partial q_2^{SP}}{\partial q_1} - \delta\phi_2^{SP} - \phi_1 \leq 0, \tag{3.44}$$

$$\frac{\partial L_1^B}{\partial \lambda_1} = \theta_1 - \mu_1 = 0, \tag{3.45}$$

$$\theta_1 \geq 0,\ \lambda_1 \geq 0, \text{ and } \theta_1\lambda_1 = 0, \tag{3.46}$$

$$\phi_1 \geq 0,\ R - q_1 \geq 0, \text{ and } \phi_1[R - q_1] = 0. \tag{3.47}$$

Condition (3.44) shows that, like (3.40), both Lagrange multipliers for the resource stock constraints appear. (3.44) also contains the strategic effect term, $\partial q_2^{SP}/\partial q_1$. Equations (3.45)-(3.47) are the Kuhn-Tucker conditions on the scarcity rent and resource stock constraints.

Next, we prove that it is optimal for buyer to offer such a contract that yields a positive quantity of resource being produced in the first period if the quantity produced is positive in the second period.

Lemma 3.6: Under assumptions A.1-A.4, in the subgame perfect equilibrium, production in period one is positive.

Proof: Suppose that (3.40) and (3.44) hold with strict inequality, so that $q_1^{SP} = 0$. Then by Lemma 3.4, $q_2^{SP} > 0$ and $\lambda_1^{SP} = \phi_1^{SP} = 0$ since $R > 0$, and $\lambda_2^{SP} = 0$ by Lemma 3.5. Then (3.40) and (3.44) imply

$$u'(0) - (1-\delta)m(0) - \delta m\left(q_2^{SP}\right) < \delta\phi_2^{SP}, \tag{3.48}$$

And (3.25) and (3.29) imply

$$\delta\left[u'\left(q_2^{SP}\right) - m\left(q_2^{SP}\right)\right] = \delta\phi_2^{SP}. \tag{3.49}$$

Subtracting (3.49) from (3.48) yields

$$u'(0) - \delta u'\left(q_2^{SP}\right) - (1-\delta)m(0) \leq (\delta - 1)\phi_2^{SP} < 0. \tag{3.50}$$

Thus, because δ < 1, (3.50) can be rearranged to yield

$$u'(0) - m(0) < \delta\left[u'\left(q_2^{SP}\right) - m(0)\right] < u'\left(q_2^{SP}\right) - m(0). \tag{3.51}$$

However, this violates assumption A.2. *Q.E.D.*

Lemma 3.7: Under assumptions A.1-A.4, in the subgame perfect equilibrium, the buyer drives the seller's scarcity rental value, λ_1^{SP}, to zero.

Proof: By (3.45) and Lemma 3.6, $\theta_1^{SP} = q_1^{SP} > 0$. Hence, by (3.41), $\lambda_1^{SP} = 0$.

Next, we show that $q_1^{SP} < R$. This will imply that $\phi_1^{SP} = 0$.

Lemma 3.8: Under assumptions A.1-A.4, in the subgame perfect equilibrium, some reserves remain at the end of period one.

Proof: Suppose that $\phi_1^{SP} > 0$, which implies that $q_1^{SP} = R$. Then (3.40), (3.44), and A.2 imply that

$$u'(0) - m(R) > u'(R) - m(R) \geq \phi_1^{SP} > 0, \tag{3.52}$$

where the weak inequality is because ϕ_2^{SP} could be positive as well. However, the first inequality violates assumption A.4. *Q.E.D.*

Lemmas 3.6-3.8 imply that the first period subgame perfect equilibrium satisfies the following:

Theorem 3.3: Under assumptions A.1-A.4, the first period subgame perfect equilibrium satisfies

$$0 < q_1^{SP} < R, \tag{3.53}$$

$$p_1^{SP} - (1-\delta)\, c\left(R - q_1^{SP}\right) - \delta c\left(R - q_1^{SP} - q_2^{SP}\right) = 0, \tag{3.54}$$

$$u'\left(q_1^{SP}\right) - (1-\delta) m\left(q_1^{SP}\right) - \delta m\left(q_1^{SP} + q_2^{SP}\right)$$

$$+ \delta q_1^{SP} c'\left(R - q_1^{SP} - q_2^{SP}\right) \frac{\partial q_2^{SP}}{\partial q_1} \leq 0. \tag{3.55}$$

Notice that there are two differences between the first period subgame perfect equilibrium conditions given by (3.53)-(3.55) and the equivalent conditions for the Nash equilibrium. Both concern (3.55). First, the last expression contains the effect changes in q_1^{SP} have upon q_2^{SP}. This is the Coasean effect, absent from the Nash equilibrium condition, and which is positive in sign by A.1 and Proposition 3.1. The second difference is that (3.55) holds as an inequality, since we cannot rule out that in the second period we consume all of the resource stock.

We have one final task in this section, which is to compare the subgame perfect equilibrium values of q_1^{SP} and q_2^{SP}. We do this in two steps. First, we show that we get a different relationship between q_1^{SP} and q_2^{SP} at the limiting values of δ. Then we find the conditions under which q_1^{SP} and q_2^{SP} are monotonic in δ.

Proposition 3.2: Under assumptions A.1-A.4, in the subgame perfect equilibrium, (*i*) when $\delta = 0$ then $q_1^{SP} > q_2^{SP}$, and (*ii*) when $\delta = 1$ then $q_1^{SP} < q_2^{SP}$.

Proof: (*i*) Assume first that $\delta = 1$. Then (3.55) and (3.35) can be written as

$$u'\left(q_1^{SP}\right) - c\left(R - q_1^{SP} - q_2^{SP}\right) + q_1^{SP} c'\left(R - q_1^{SP} - q_2^{SP}\right) + q_2^{SP} c'\left(R - q_1^{SP} - q_2^{SP}\right)$$

$$+ \; q_1^{SP} c'\left(R - q_1^{SP} - q_2^{SP}\right)\frac{\partial q_2^{SP}}{\partial q_1} \;=\; \phi_2^{SP}, \tag{3.56}$$

$$u'\left(q_2^{SP}\right) - c\left(R - q_1^{SP} - q_2^{SP}\right) + q_2^{SP} c'\left(R - q_1^{SP} - q_2^{SP}\right) \;=\; \phi_2^{SP}. \tag{3.57}$$

Subtracting (3.57) from (3.56) and rearranging using Proposition 3.1 yields

$$u'\left(q_2^{SP}\right) - u'\left(q_1^{SP}\right) = q_1^{SP} c'\left(R - q_1^{SP} - q_2^{SP}\right)\left[1 + \frac{\partial q_2^{SP}}{\partial q_1}\right] < 0. \tag{3.58}$$

Thus, when $\delta = 1$, $u'\left(q_2^{SP}\right) < u'\left(q_1^{SP}\right)$ and $q_2^{SP} > q_1^{SP}$.

(*ii*) Assume now that $\delta = 0$. Then (3.55) and (3.35) can be written as

$$u'\left(q_2^{SP}\right) - c\left(R - q_1^{SP} - q_2^{SP}\right) + q_2^{SP} c'\left(R - q_1^{SP} - q_2^{SP}\right) = \phi_2^{SP} > 0, \tag{3.59}$$

$$u'\left(q_1^{SP}\right) - c\left(R - q_1^{SP}\right) + q_1^{SP} c'\left(R - q_1^{SP}\right) = 0. \tag{3.60}$$

Now, we subtract (3.60) from (3.59) to get $u'\left(q_2^{SP}\right) - u'\left(q_1^{SP}\right) > -q_2^{SP} c'\left(R - q_1^{SP} - q_2^{SP}\right) + q_1^{SP} c'\left(R - q_1^{SP}\right) + \left[c\left(R - q_1^{SP} - q_2^{SP}\right) - c\left(R - q_1^{SP}\right)\right]$. For $q_2^{SP} > q_1^{SP}$, the left-hand side of this expression is negative and right-hand side is positive, which contradicts. Thus, $q_1^{SP} > q_2^{SP}$.

Q.E.D.

An implication of Proposition 3.2 is that there exists a $\hat{\delta}$ such that $0 < \hat{\delta} < 1$, for which $q_2^{SP} = q_1^{SP} = \hat{q}^{SP}$. From Theorem 3.2 $\hat{q}^{SP}$ solves:

$$u'\left(\hat{q}^{SP}\right) - c\left(R - 2\hat{q}^{SP}\right) + \hat{q}^{SP} c'\left(R - 2\hat{q}^{SP}\right) = 0. \tag{3.61}$$

Then by (3.61) and Theorem 3.3 $\hat{\delta}$ solves:

$$\hat{\delta} = \frac{c\left(R-\hat{q}^{SP}\right)-c\left(R-2\hat{q}^{SP}\right)-\hat{q}^{SP}\left[c'\left(R-\hat{q}^{SP}\right)-c'\left(R-2\hat{q}^{SP}\right)\right]}{c\left(R-\hat{q}^{SP}\right)-c\left(R-2\hat{q}^{SP}\right)-\hat{q}^{SP}\left[c'\left(R-\hat{q}^{SP}\right)-c'\left(R-2\hat{q}^{SP}\right)\left[2+\frac{\partial q_2^{SP}}{\partial q_1}\right]\right]}. \quad (3.62)$$

The question that remains is whether or not $\hat{\delta}$ is unique. The next proposition makes clear the condition that must hold in order for $\hat{\delta}$ to be unique:

Proposition 3.3: Under assumptions A.1-A.4, in the subgame perfect equilibrium, $\partial q_1^{SP}/\partial\delta < 0$ if

A.5 $\quad m\left(q_1^{SP}\right)-m\left(q_1^{SP}+q_2^{SP}\right)+q_1^{SP}c'\left(R-q_1^{SP}-q_2^{SP}\right)\frac{\partial q_2^{SP}}{\partial q_1}<0, \qquad$ for all δ.

Proof: Totally differentiating the first-order condition (3.55) using Proposition 3.1 yields

$$\left\{u''\left(q_1^{SP}\right)-(1-\delta)m'\left(q_1^{SP}\right)-\delta m'\left(q_1^{SP}+q_2^{SP}\right)\left(1+\frac{\partial q_2^{SP}}{\partial q_1}\right)+\delta c'\left(R-q_1^{SP}-q_2^{SP}\right)\frac{\partial q_2^{SP}}{\partial q_1}\right.$$

$$\left.-\delta q_1^{SP}c''\left(R-q_1^{SP}-q_2^{SP}\right)\frac{\partial q_2^{SP}}{\partial q_1}\left(1+\frac{\partial q_2^{SP}}{\partial q_1}\right)dq_1^{SP}\right\}\, dq_1^{SP}$$

$$= -\left(m(q_1^{SP})-m(q_1^{SP}+q_2^{SP})+q_1^{SP}c'\left(R-q_1^{SP}-q_2^{SP}\right)\frac{\partial q_2^{SP}}{\partial q_1}\right)d\delta. \quad (3.63)$$

The coefficient on dq_1^{SP} is negative by second-order conditions, and the coefficient on $d\delta$ is the expression in A.5. *Q.E.D.*

So long as condition A.5 holds, $\partial q_1^{SP}/\partial\delta$ < 0, and Proposition 3.1 ensures that

$\partial q_2^{SP}/\partial\delta > 0$. This condition, taken together with the results of Proposition 3.2, are sufficient to ensure that a unique value of $\hat{\delta}$ exists such that for $\delta < \hat{\delta}$, $q_1^{SP} > q_2^{SP}$, and for $\delta \geq \hat{\delta}$ then $q_1^{SP} \leq q_2^{SP}$. Notice that this condition simply says that the strategic effect in the last term of A.5 does not dominate the differences between the marginal factor cost of producing quantity q_1^{SP} and the marginal factor cost of producing quantity $q_1^{SP} + q_2^{SP}$ all in one period.

3.4. The Effect of a Capacity Constraint when Capacity is Costless

A capacity constraint restricts how much can be bought or sold in each period. If it restricts how much can be sold in the second period, it also eliminates the strategic effect of changes in the period one quantity. The question is whether the buyer can gain monopsony power by limiting his ability to purchase in the second period, i.e. by restricting his capacity. If so, then capacity serves as a credible commitment device for the buyer. In section 3.4.A, we highlight the effect a capacity constraint has on the buyer's subgame perfect profit for the case where $\delta > \hat{\delta}$. This is the case in which a capacity constraint is most likely to have a positive effect, because of the presence of the strategic term in (3.55). Section 3.4.B shows the local effects on profits for the case where $\delta \leq \hat{\delta}$. Section 3.4.C shows that profits are unambiguously lowered by a capacity constraint in the open loop equilibrium. We leave to section 3.5 the case where capacity is costly.

3.4.A. The Effect of a Capacity Constraint on Subgame Perfect Profits when $\delta > \hat{\delta}$

We shall denote the pipeline capacity as Q. The capacity constraint implies that $q_1 \leq Q$ and $q_2 \leq Q$. To illustrate the effect of the capacity constraint, we consider the case where the constraint binds in period two, but not in period one. This occurs when $\delta > \hat{\delta}$. Let $Q^A = q_2^{SP}$ denote the value of Q, such that the constraint just binds.

Let us consider the case where $Q < Q^A$, so that in equilibrium $q_2^{CSP} = Q$, but $q_1^{CSP} < Q$. The sellers' second period Lagrangian in this case is

$$L_2^S = p_2 q_2 - \int_{q_1}^{q_1+q_2} c(R-q)\,dq + \eta_2 [Q - q_2],$$

where η_2 is the multiplier on the capacity constraint. The sellers' period two first-order necessary conditions are:

$$\frac{\partial L_2^S}{\partial q_2} = p_2 - c(R - q_1 - q_2) - \eta_2 = 0, \tag{3.64}$$

$$\frac{\partial L_2^S}{\partial \eta_2} = Q - q_2 \geq 0,\ \eta_2 \geq 0, \text{ and } \eta_2 [Q - q_2] = 0. \tag{3.65}$$

Thus (3.64) and (3.65) form constraints to the buyer's second period problem. The second period Lagrangian for the buyer can be written as

$$L_2^B = u(q_2) - p_2 q_2 + \mu_2 [p_2 - c(R - q_1 - q_2) - \eta_2] + \chi_2 \eta_2 + \kappa_2 [Q - q_2],$$

where κ_2 is the multiplier on the capacity constraint, χ_2 is the multiplier on $\eta_2 \geq 0$; and, as before, μ_2 is the multiplier on (3.64). Given that the capacity constraint binds, it follows that $\chi_2^{CSP} = \mu_2^{CSP} = q_2^{CSP} = Q > 0$, so that $\eta_2^{CSP} = 0$ and the first-order condition for the buyer in second period quantity choice can be written as

$$\frac{\partial L_2^B}{\partial q_2} = u'(Q) - p_2^{CSP} + Qc'(R - q_1 - Q) - \kappa_2^{CSP} = 0. \tag{3.66}$$

Given that $\eta_2^{CSP} = 0$, the second period solutions for p_2^{CSP} and κ_2^{CSP} are given jointly by (3.64) and (3.66). From (3.64), we see that an increase in q_1 raises p_2^{CSP}:

$$\partial p_2^{CSP} / \partial q_1 = -c'(R - q_1 - Q) > 0. \tag{3.67}$$

Moving to the first period, the sellers' first period Lagrangian is

$$L_1^S = p_1 q_1 - \int_0^{q_1} c(R-q)\,dq +$$

$$\delta\left\{ p_2^{CSP} q_2^{CSP} - \int_{q_1}^{q_1+q_2^{CSP}} c(R-q)\,dq + \eta_2^{CSP}\left[Q - q_2^{CSP}\right]\right\} + \eta_1\left[Q - q_1\right].$$

We assume again that sellers ignore the price effect of changes in period one quantity, since they are price takers. Thus by the envelope theorem, the sellers' first-order conditions when $q_1 > 0$ are

$$\frac{\partial L_1^S}{\partial q_1} = p_1 - (1-\delta)\, c(R-q_1) - \delta c\left(R - q_1 - q_2^{CSP}\right) - \eta_1 = 0, \tag{3.68}$$

$$Q - q_1 \geq 0,\ \ \eta_1 \geq 0 \ \text{ and } \ \eta_1 [Q - q_1] = 0. \tag{3.69}$$

The buyer treats (3.68) and (3.69) as constraints. Given the second period equilibrium, the buyer's first period Lagrangian can be written as:

$$L_1^B = u(q_1) - p_1 q_1 + \delta\left[u(Q) - Qc(R - q_1 - Q)\right] + \kappa_1 [Q - q_1] + \chi_1 \eta_1$$

$$+ \mu_1 \left[p_1 - (1-\delta)c(R-q_1) - \delta c(R - q_1 - Q) - \eta_1\right].$$

Again, it follows that $\mu_1^{CSP} = \chi_1^{CSP} = q_1^{CSP}$, so that when $0 < q_1^{CSP} < Q$, the buyer's first-order condition in first period quantity is

$$\frac{\partial L_1^B}{\partial q_1} = u'(q_1) - p_1 + (1-\delta) q_1 c'(R - q_1) + \delta(q_1 + Q) c'(R - q_1 - Q) = 0. \tag{3.70}$$

We may now write the equilibrium values of q_1, p_1 and p_2 satisfying (3.64), (3.68) and (3.70) as $q_1(Q)$, $p_1(Q)$, $p_2(Q)$, respectively. Writing the solution as an indirect profit function for the buyer in the capacity choice Q, after cancelling out the terms involving the prices, yields

$$\pi^B\left(Q \mid Q^B < Q < Q^A\right) = u\left(q_1(Q)\right) + \delta u(Q) - q_1(Q) c\left(R - q_1(Q)\right)(1-\delta)$$

$$- \left(q_1(Q) + Q\right) c\left(R - q_1(Q) - Q\right)\delta. \tag{3.71}$$

Differentiating $\pi^B\left(Q \mid Q^B < Q < Q^A\right)$ with respect to Q using the envelope theorem yields the effect of the capacity constraint:

$$\frac{\partial \pi^B\left(Q \mid Q^B < Q < Q^A\right)}{\partial Q} = \delta\left[\kappa_2^{CSP} + q_1 c'\left(R - q_1 - Q\right)\right]. \qquad (3.72)$$

Equation (3.72) contains two effects. The $\delta\kappa_2^{CSP}$ term is the typical Lagrange multiplier effect of a constraint. This effect reduces profits as the constraint is tightened, since $\kappa_2^{CSP} >$ 0. The second effect, however, is negative. Intuitively, the second effect occurs because the constraint eliminates the strategic effect term, $q_1 c'\left(R - q_1 - Q\right)\delta$, from the subgame perfect equilibrium condition (3.55), since $q_2^{CSP} = Q$ implies that $\partial q_2^{CSP} / \partial q_1 = 0$. The elimination of the strategic effect means that a tightening of the capacity constraint increases profits, since $q_1 c'\left(R - q_1 - Q\right)\delta < 0$.

Next, we find the lower bound of the interval where profits are defined by (3.71). Implicitly differentiating (3.70) yields the effect of the capacity constraint on the first period equilibrium quantity:

$$\frac{\partial q_1^{CSP}}{\partial Q} = \frac{\delta m'\left(q_1 + Q\right)}{u''\left(q_1\right) - \left(1 - \delta\right) m'\left(q_1\right) - \delta m'\left(q_1 + Q\right)} \subset (-1,0). \qquad (3.73)$$

Thus, tightening the capacity constraint raises q_1^{CSP}. This implies that as Q reduces in size, both constraints must eventually bind, so that $q_1^{CSP} = q_2^{CSP} = Q^B$, for a Q^B such that $0 < Q^B < Q^A$ that solves:

$$u'\left(Q^B\right)-(1-\delta)\, m\left(Q^B\right)-\delta m\left(2Q^B\right)=0. \tag{3.74}$$

For now, note only that by A.1 and A.2, Q^B is decreasing in δ. We next show that in the interval (Q^B, Q^A] the buyer's profits are decreasing in Q.

Proposition 3.4: Under assumptions A.1-A.4, when $\delta > \hat{\delta}$, profits are decreasing in Q over the interval (Q^B, Q^A].

Proof: Suppose that $Q^B < Q \leq Q^A$ and that $\delta > \hat{\delta}$, so that $q_2^{CSP} = Q > q_1^{CSP}$. Then the rate of change in profits is given by (3.72). Suppose that the right-hand-side of (3.72) is positive in sign, so that decreases in Q decrease profits. Then, by (3.66), we may write (3.72) as

$$\frac{\partial \pi^B\left(Q \mid Q^B < Q < Q^A\right)}{\partial Q} = \delta\left[u'(Q)-m\left(q_1^{CSP}+Q\right)\right] > 0. \tag{3.75}$$

Thus, the term in square brackets must be positive. If we subtract (3.70) from (3.75) we get

$$u'(Q) - u'\left(q_1^{CSP}\right) - (1-\delta)\left[m\left(q_1^{CSP}+Q\right)-m\left(q_1^{CSP}\right)\right] > 0. \tag{3.76}$$

However, $u''(.) < 0$ by A.2 and $m'(.) > 0$ by A.1, so that the inequality in (3.76) cannot hold for $q_1^{CSP} < Q$. *Q.E.D.*

Proposition 3.4 shows that the buyer's profits are increasing as capacity is constrained throughout the region (Q^B,Q^A). We now show that the buyer's profits jump at Q^A. At Q^A, $q_2^{SP} = q_2^{CSP} = Q^A$, and the first period quantity satisfies (3.70) in the capacity constrained case and (3.55) in the case where the capacity is not constrained. As the strategic effect from (3.55) disappears in (3.70), q_1 discontinuously drops at Q^A.[14] (See Fig. 3.1.) This drop in the constrained first period quantity at Q^A lowers the price the buyer must pay in each period. Since the second period quantity can not adjust because of the constraint, the buyer's profits discontinuously jump up at Q^A (see Fig. 3.2):

$$\partial\pi^B\left(Q^A\right)/\partial q_1^{SP} = -\delta q_1^{SP} c'\left(R - q_1^{SP} - Q^A\right)\partial q_2^{SP}/\partial q_1 < 0.$$

Therefore, in region $(Q^B,Q^A]$, the buyer's constrained profits are strictly larger than the profits he could earn in the unconstrained subgame perfect equilibrium.

Since profits are strictly increasing as Q decreases in the region $(Q^B,Q^A]$, to find the capacity, Q^*, that maximizes the buyer's profits we must consider the equilibrium in which the constraint binds in both periods. If the capacity constraint holds in both periods, then the second period equilibrium is given by (3.64)-(3.66), and again it follows that $\eta_2^{CSP} = 0$ and that the strategic effect vanishes, since $q_2^{CSP} = Q$. Since the strategic effect is absent on both sides of Q^B, it follows that the buyer's profits are continuous at Q^B. As the constraint also binds in period one, we may write the seller's necessary conditions as (3.68) and (3.69), but the buyer's first-order-condition (3.70) is now written as

$$\frac{\partial L_1^B}{\partial q_1} = u'(Q) - p_1 + (1-\delta)\,Qc'(R-Q) + \delta 2Qc'(R-2Q) - \kappa_1^{CSP} = 0. \tag{3.77}$$

Thus, by (3.64)-(3.69) and (3.77), equilibrium profits to the buyer can be written as

[14] The term $u'(q_1) - (1-\delta)m(q_1) - \delta m(q_1 + Q^A)$ is a decreasing function of q_1. In (3.70), this is set equal to zero. In (3.55), it is set equal to $-\delta q_1 c'(R - q_1 - Q^A)\partial q_2/\partial q_1 < 0$. Thus, $\lim_{Q\to Q^A} q_1^{CSP}(Q) < q_1^{SP}$.

$$\pi^B\left(Q \mid Q<Q^B\right) = u(Q)(1+\delta) - Qc(R-Q)(1-\delta) - 2Qc(R-2Q)\delta. \tag{3.78}$$

Differentiating the buyer's profits with respect to Q using (3.64)-(3.69) and (3.77) yields

$$\frac{\partial \pi^B\left(Q \mid Q<Q^B\right)}{\partial Q} = \kappa_1^{CSP} + \delta\kappa_2^{CSP} + \delta Qc'(R-2Q). \tag{3.79}$$

Note that (3.79) like (3.72), has both a negative term and a positive term.

We now show that a unique value of Q^* that maximizes $\pi^B\left(Q \mid Q<Q^B\right)$ exists in the interval $[0,Q^B)$.

Proposition 3.5: Under assumptions A.1-A.4, when $\delta > \hat{\delta}$, a unique capacity level, Q^*, that maximizes the buyer's subgame perfect equilibrium profits, exists in the interval $0 < Q^* < Q^B$.

Proof: Existence. When $Q < Q^B$, we may use (3.64)-(3.69) and (3.77) to show that (3.79) may be written as

$$\frac{\partial \pi^B\left(Q \mid Q<Q^B\right)}{\partial Q} = u'(Q)(1+\delta) - (1-\delta)\, m(Q) - 2\delta m(2Q). \tag{3.80}$$

Taking the limit of (3.80) as $Q \to 0$ yields

$$\lim_{Q\to 0} \frac{\partial \pi^B\left(Q \mid Q<Q^B\right)}{\partial Q} = \left[u'(0)-m(0)\right](1+\delta) > 0. \tag{3.81}$$

The inequality follows from A.3. Next, taking the limit as $Q \to Q^B$ using (3.68) and (3.77) yields

$$\lim_{Q \to Q^B} \frac{\partial \pi^B (Q \mid Q < Q^B)}{\partial Q} = \left[u'(Q^B) - m(2Q^B)\right]\delta =$$

$$(1-\delta)\delta\left[m(Q^B) - m(2Q^B)\right] < 0. \qquad (3.82)$$

The second inequality in (3.82) follows from subtracting (3.77), which is zero at Q^B, from the expression on the right-hand side of the first equality in (3.82). The inequality follows from assumption A.1. This proves that there exists a value of Q^* such that (3.80) vanishes in the interval $(0,Q^B)$.

Uniqueness: To prove uniqueness, differentiate (3.80) with respect to Q:

$$\frac{\partial^2 \pi^B (Q \mid Q < Q^B)}{\partial Q^2} = u''(Q)(1+\delta) - (1-\delta)\, m'(Q) - 2\delta m'(2Q) < 0. \qquad (3.83)$$

This is negative by A.1 and A.2. Therefore, a unique value of Q^* occurs where the right-hand-side of (3.80) vanishes. *Q.E.D.*

Propositions 3.4 and 3.5 together imply that when $\delta > \hat{\delta}$, the buyer's subgame perfect equilibrium profits are maximized at Q^*, where $Q^* < Q^B$. Figs. 3.1 and 3.2 illustrate this result. Fig. 3.1 shows that at Q^A, both q_1 and κ_2 discontinuously jump. This translates into a jump in profits, as shown in Fig. 3.2. However, there are no jumps at Q^B.

Figure 3.1: Equilibrium Production with a Capacity Constraint, $\delta > \hat{\delta}$

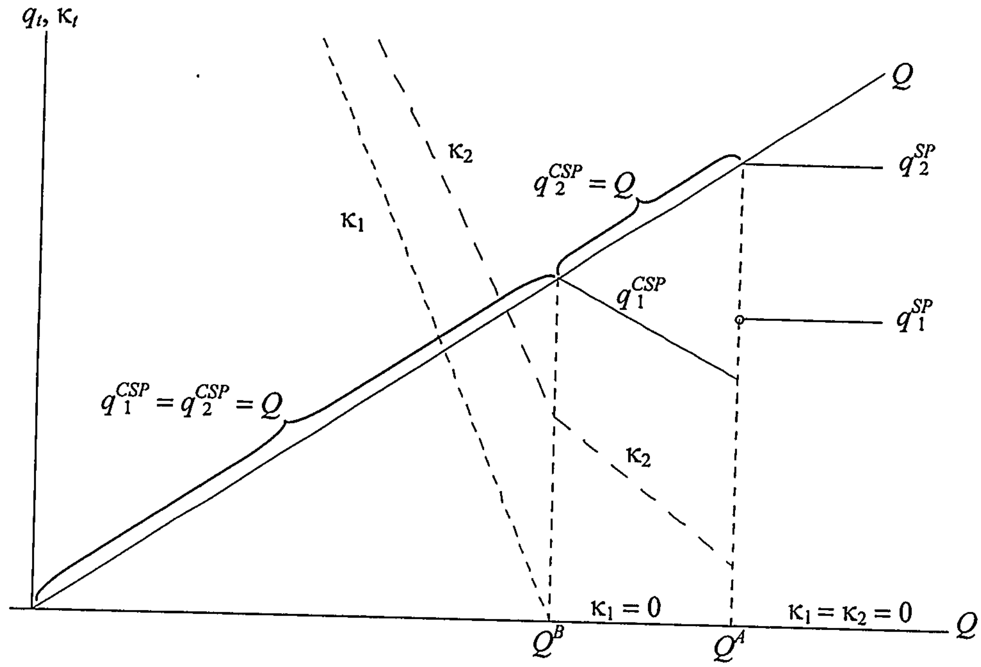
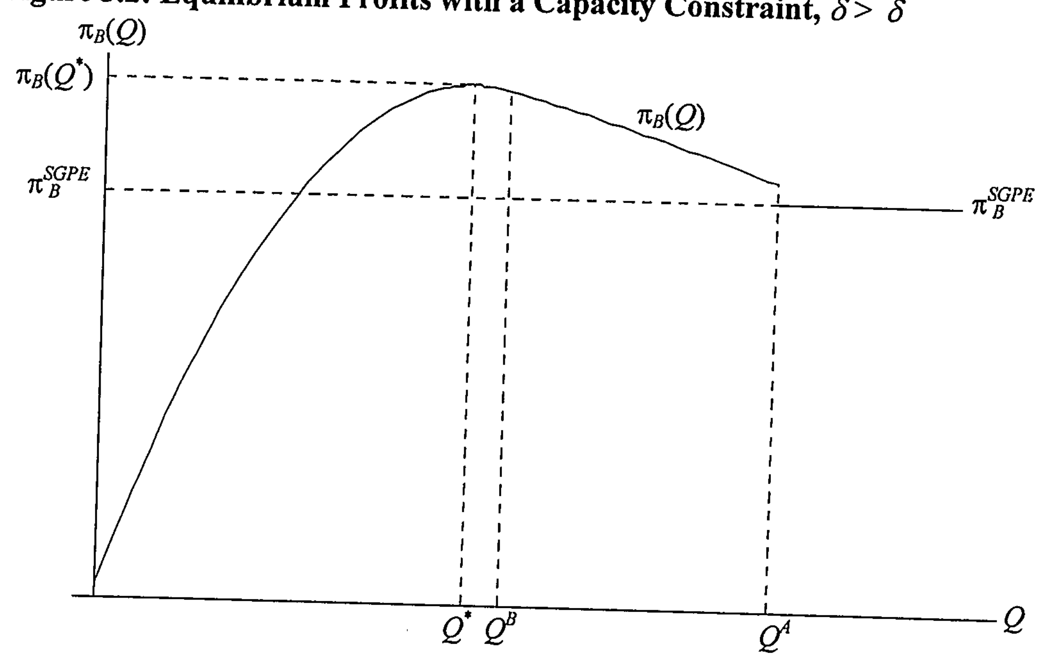


Figure 3.2: Equilibrium Profits with a Capacity Constraint, $\delta > \hat{\delta}$



3.4.B. The Effect of a Capacity Constraint on Subgame Perfect Profits when $\delta < \hat{\delta}$

Next, we turn to the case where $\delta \leq \hat{\delta}$. In this case, the unconstrained subgame perfect equilibrium is characterized by $q_1^{SP} > q_2^{SP}$. Thus, a capacity constraint affects the period one quantity, rather than the period two quantity, which means that if there is a strategic effect, it shall remain in the constrained first order conditions. Let Q^C denote the value of Q such that $q_1^{SP} = Q^C$ just binds. Then the case we consider has $Q < Q^C$, so that $q_1^{CSP} = Q$, but $q_2^{CSP} < Q$.

Given that $q_2^{CSP} < Q$, the period two equilibrium is identical to that analyzed in Section 3.3. Again, we shall ignore the case where the stock is fully exhausted. It follows that there exists a strategic effect from Proposition 3.1. The first period sellers' Lagrangian can be written as

$$L_1^S = p_1 q_1 - \int_0^{q_1} c(R-q)\,dq +$$

$$\delta\left\{p_2^{CSP} q_2^{CSP} - \int_{q_1}^{q_1+q_2^{CSP}} c(R-q)\,dq + \eta_2^{CSP}\left[Q - q_2^{CSP}\right]\right\} + \eta_1\left[R - q_1\right].$$

Thus by the envelope theorem and the assumption that sellers act as price takers, the sellers' first-order conditions when $q_1^{CSP} = Q$ binds are

$$\frac{\partial L_1^S}{\partial q_1} = p_1 - (1-\delta)\,c(R-Q) - \delta c\left(R - Q - q_2^{CSP}\right) - \eta_1 = 0, \tag{3.84}$$

$$Q - q_1 \geq 0,\ \eta_1 \geq 0 \text{ and } \eta_1 \left[Q - q_1\right] = 0. \tag{3.85}$$

Thus (3.84) and (3.85) form the constraints for the buyer's problem. Using the notation developed above, the buyer's Lagrangian is

$$L_1^B = u(q_1) - p_1 q_1 + \delta\left[u\left(q_2^{CSP}\right) - p_2^{CSP} q_2^{CSP}\right] + \chi_1 \eta_1 + \kappa_1\left[Q - q_1\right]$$

$$+ \mu_1\left[p_1 - (1-\delta)c(R-Q) - \delta c\left(R - Q - q_2^{CSP}\right) - \eta_1\right].$$

It follows that if $\kappa_1^{CSP} > 0$, then $q_1^{CSP} = Q$. This in turn implies that $\chi_1^{CSP} > 0$ and $\eta_1^{CSP} = 0$. Therefore, the buyer's quantity first-order condition can be written as

$$\frac{\partial L_1^B}{\partial q_1} = u'(Q) - p_1 + Qc'(R-Q)\,(1-\delta) + \left(Q + q_2^{CSP}\right) c'\left(R - Q - q_2^{CSP}\right)\delta$$

$$+ Qc'\left(R - Q - q_2^{CSP}\right)\delta\frac{\partial q_2^{CSP}}{\partial q_1} - \kappa_1^{CSP} = 0. \tag{3.86}$$

When the capacity constraint binds in period one but not in period two, the system given by (3.34), (3.35) (which holds as an equality) and (3.84) implicitly define $q_2(Q)$, $p_2(Q)$, and $p_1(Q)$.

Given (3.34) and (3.35) implicitly define the second period equilibrium as a function of Q, we may again use Proposition 3.1 show that $\partial q_2^{CSP}/\partial Q \subset (-1,0]$. This means that as Q is reduced, q_2^{CSP} increases, which means that there exists some value Q^D, where $0 < Q^D < Q^C$, such that for $Q \leq Q^D$, $q_1^{CSP} = q_2^{CSP} = Q$. Note that Q^D solves:

$$u'(Q^D)-c(R-2Q^D)+Q^Dc'(R-2Q^D)=0 \tag{3.87}$$

Thus, unlike the points Q^A, Q^B, and Q^C, the point Q^D is independent of the discount factor.[15] To find the effect of the constraint on the buyer's profits in the interval $(Q^D,Q^C]$, define the present value stream of profits as

$$\pi^B(Q \mid Q^D < Q \leq Q^C) = u(Q) + \delta u(q_2(Q)) - Qc(R-Q)(1-\delta)$$

$$- [Q+q_2(Q)]\, c(R-Q-q_2(Q))\, \delta. \tag{3.88}$$

Then it follows from (3.35) and (3.86) that the effect of a change in Q on the buyer's profits is

$$\frac{\partial \pi^B(Q \mid Q^D < Q \leq Q^C)}{\partial Q} = K_1^{CSP}. \tag{3.89}$$

Hence, we may state the following result:

Proposition 3.6: When assumptions A.1-A.4 hold and $\delta < \hat{\delta}$, then in the interval $(Q^D,Q^C]$, tightening the capacity constraint *reduces* the buyer's profits.

The reason for this result is that the capacity constraint does not eliminate the strategic term – compare (3.44) with (3.86). All that the capacity constraint does is prohibit the buyer from choosing the unconstrained first period quantity, q_1^{CSP}, so this reduces his

[15] In the appendix A, we use this to show that Q^D is the limiting value of Q^A, Q^B, and Q^C as δ approaches critical values.

profits. The profit function is continuous at Q^C.

However, it is possible that there exists a local maxima in the interval $(0,Q^D]$. In the appendix A, we show the conditions under which a local maxima exists. In general, it is not possible to say whether this local maxima yields profits that exceed those of the unconstrained subgame perfect equilibrium, as occurred when $\delta > \hat{\delta}$, as in section 3.4.A.[16] However, in a linear example, we have found that profits are improved for a much larger range of δ than we could prove generally.[17]

3.4.C. The Effect of a Capacity Constraint on Open Loop Profits

We have seen that a capacity constraint can increase the buyer's profits when the buyer cannot commit to future policies without the constraint for the case where $\delta > \hat{\delta}$. We now ask if the same can be said of a buyer who already possesses commitment power through some other means. The answer, of course, is no – the monopsonist does not need the commitment device of the capacity constraint when he already poses a credible commitment device. Nevertheless, it is instructive to see why this is the case.

Recall that in Theorem 3.1, we found that $q_1^{OL} > q_2^{OL} \geq 0$. Thus, in the constrained open loop equilibrium, we shall either have that $q_1^{COL} = Q > q_2^{COL}$ or $q_1^{COL} = q_2^{COL} = Q$. If the first

[16] The comparison is between $\pi^B(Q^*) = u(Q^*)(1+\delta) - (1-\delta)Q^*c(R - Q^*) - \delta 2Q^*c(R - 2Q^*)$ and $\pi^B(Q^C, q_2^{SP}) = u(Q^C) + \delta u(q_2^{SP}) - (1 - \delta)Q^C c(R - Q^C) - \delta(Q^C + q_2^{SP})c(R - Q^C - q_2^{SP})$. It follows that $Q^* < q_2^{SP} < Q^C$. From this, we may deduce that $u(Q^*)(1+\delta) < u(Q^C) + \delta u(q_2^{SP})$, but that $(1-\delta)Q^*c(R - Q^*) + \delta 2Q^*c(R - 2Q^*) < (1 - \delta)Q^C c(R - Q^C) - \delta(Q^C + q_2^{SP})c(R - Q^C - q_2^{SP})$. Thus, it is not possible to tell whether the capacity constrained profits are greater or less than the unconstrained profits.

[17] The linear example assumes that $c(R - q) = \sigma - \gamma(R - q)$ and that $u(q) = \omega q$. In this case, $\hat{\delta} = 2/3$. However, profits are improved by restricting capacity down to the value of $\delta = ½$.

case occurs, it is possible to have $q_2^{COL} = 0$, although as the constraint continues to tighten, it will happen that the quantity in period two increases. Also, as we also showed in Theorem 3.1 that the resource constraint $q_1 + q_2 \leq R$ does not bind in the unconstrained open loop equilibrium, we shall ignore that constraint. When both period quantities are positive, under assumptions A.1-A.4, the open loop constrained equilibrium can be shown to satisfy:

$$p_2^{COL} = c\left(R - q_1^{COL} - q_2^{COL}\right), \tag{3.90}$$

$$p_1^{COL} = (1-\delta)c\left(R - q_1^{COL}\right) + \delta c\left(R - q_1^{COL} - q_2^{COL}\right), \tag{3.91}$$

$$\delta\left[u'\left(q_2^{COL}\right) - m\left(q_1^{COL} + q_2^{COL}\right)\right] - \kappa_2^{COL} = 0, \tag{3.92}$$

$$u'\left(q_1^{COL}\right) - (1-\delta)\, m\left(q_1^{COL}\right) - \delta m\left(q_1^{COL} + q_2^{COL}\right) - \kappa_1^{COL} = 0, \tag{3.93}$$

$$q_1^{COL} \leq Q,\ \kappa_1^{COL} \geq 0,\ \kappa_1^{COL}\left(Q - q_1^{COL}\right) = 0,\ q_2^{COL} \leq Q,\ \kappa_2^{COL} \geq 0,$$

$$\text{and } \kappa_2^{COL}\left(Q - q_2^{COL}\right) = 0. \tag{3.94}$$

Equations (3.90) and (3.91) represent the intertemporal pricing conditions that make the sellers indifferent between selling in each period. These are identical to the conditions (3.3) and (3.4) in the unconstrained problem, given that the buyer always drives to zero the scarcity rental value for sellers. Equations (3.92) and (3.93) are the conditions that make the buyer indifferent between purchasing in each period. These differ from the unconstrained case only when the capacity constraint binds.

When the capacity constraint binds in period one (i.e., when $\kappa_1^{COL} > 0$) but not in period two ($q_2^{COL} < Q$), the buyer's open loop equilibrium indirect profits in terms of the capacity constraint are given by

$$\pi^B\left(Q \mid Q^G < Q < Q^F\right) = u(Q) + \delta u\left(q_2^{COL}\right) - Qc(R-Q)(1-\delta)$$

$$- \left(Q + q_2^{COL}\right) c\left(R - Q - q_2^{COL}\right) \delta . \tag{3.95}$$

Differentiating these profits with respect to Q using (3.90)-(3.94) yields

$$\frac{\partial \pi_B^{COL}}{\partial Q} = \kappa_1^{COL} > 0. \tag{3.96}$$

Note that when the first period constraint binds but the second period constraint does not, (3.92) shows that how q_2^{COL} changes with Q:

$$\frac{\partial q_2^{COL}}{\partial Q} = \frac{m'\left(Q + q_2^{COL}\right)}{u''\left(q_2^{COL}\right) - m'\left(Q + q_2^{COL}\right)} \subset (-1,0). \tag{3.97}$$

Thus there exists some value Q^G, where $0 < Q^G < Q^F$, such that for $Q \leq Q^G$, the capacity constraint binds in both periods.

When both constraints bind ($\kappa_1^{COL} > 0$ and $\kappa_2^{COL} > 0$), then the buyer's constrained open loop profits are given by

$$\pi^{B}\left(Q \mid Q<Q^{G}\right)=u(Q)(1+\delta)-Qc(R-Q)(1-\delta)+2Qc(R-2Q)\delta, \tag{3.98}$$

which is identical to (3.78) except for the domain over which it holds. Differentiating the profits in (3.98) with respect to Q using (3.90)-(3.94) yields

$$\frac{\partial \pi_{B}^{COL}}{\partial Q}=\kappa_{1}^{COL}+\kappa_{2}^{COL}>0. \tag{3.99}$$

Thus, we have proved the following:

Proposition 3.7: When assumptions A.1-A.4 hold, the open loop equilibrium profits of the buyer are never improved by restricting the capacity below Q^{F}.

Next, we compare the constrained open loop with the constrained subgame perfect Nash equilibrium profits. At point $Q = 0$, profits under both the constrained open loop and constrained subgame perfect equilibrium are zero. Following (3.99), with increases in Q open loop profit increases at the rate of $\kappa_{1}^{COL}+\kappa_{2}^{COL}$. Using (3.92)-(3.93) when $Q < Q^{G}$, (3.99) may be written as $u'(Q)(1+\delta)-(1-\delta)m(Q)-2\delta m(2Q)$ that is identical with (3.80). Proposition 3.5 implies that (3.78) increases up to the point Q^{*} and then it decreases. Proposition 3.7 implies that profits given by (3.98) strictly increase in domain $(0,Q^{G})$. Thus it must be true that $Q^{*} > Q^{G}$. Thus the constrained open loop profits given by (3.98) and the constrained subgame perfect profits given by (3.78) are identical when $Q < Q^{G}$.

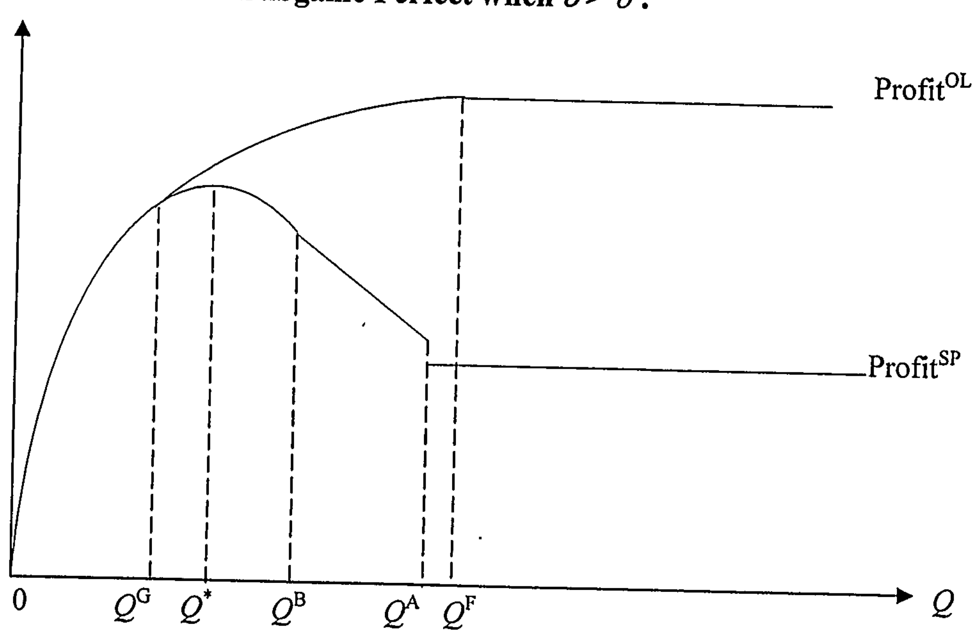
For Q in the domain (Q^{G}, Q^{B}), $q_{1}^{CSP}=q_{2}^{CSP}=Q$ and $q_{2}^{COL}<q_{1}^{COL}=Q$. Then, the rate at which the constrained open loop profit increases given by (3.96) and the rate at which the

constrained subgame perfect profit changes given by (3.80). To see that the constrained open loop profit increases at the greater rate than the constrained subgame perfect profit, we subtract (3.80) from (3.96) and get:

$$\delta\left[u'\left(q_2^{COL}\right)-u'(Q)\right]-\delta\left[m\left(Q+q_2^{COL}\right)-m(2Q)\right]>0.$$

Both expressions on the left-hand side of inequality are zero when $Q = Q^G$. For $Q^G < Q < Q^B$ the open loop profits are higher than the subgame perfect profits. Since $Q^G < Q^* < Q^B$ and Proposition 3.5 indicates Q^* to be a unique capacity level that maximizes the buyer's subgame perfect profits, we have proved the following general result:

Figure 3.3: Comparison of the Buyer's Equilibrium Profits: Constrained Open Loop and Constrained Subgame Perfect when $\delta > \hat{\delta}$.



Proposition 3.8: When assumptions A.1-A.4 hold, despite the outcome of Proposition 3.5, the open loop equilibrium profits of the buyer are always greater than the subgame perfect equilibrium profits when $Q > Q^G$.

3.5. Endogenous Capacity

In this section, we assume that capacity is costly and we solve for optimal the capacity, Q^{**}, that maximizes the buyer's subgame perfect equilibrium profits. Let the cost of capacity be denoted as $v(Q)$, where $v'(Q) \geq 0$ and $\partial^2\pi^B(Q)/\partial Q^2 - v''(Q) < 0$. Thus the monopsonist chooses the size of pipeline that maximizes his discounted present value of profit, $\Pi^B(Q) = \pi^B(Q) - v(Q)$. The first-order necessary condition is:

$$\frac{\partial\pi^B(Q^{**})}{\partial Q} - v'(Q^{**}) = 0 \tag{3.100}$$

where partial derivative is defined by whichever is appropriate from (3.72) or (3.79). Equation (3.100) gives the solution for optimal capacity of pipeline. Equation (3.100) indicates two things. First, if the marginal cost of pipeline is zero, $v'(Q) = 0$, then $\partial\pi^B(Q^{**})/\partial Q$ must equal zero as well. In this case, $Q^{**} = Q^*$. As Proposition 3.5 indicates, if Q^* exists, then Q^* maximizes the constrained subgame perfect Nash equilibrium profit. Thus, Q^* would be the best monopsonist's choice when the cost of pipeline is zero. Second, if the marginal cost of pipeline is nonzero, i.e., if $v'(Q^{**}) > 0$, then derivative of profits in (3.100) must be positive as well, which puts $Q^{**} < Q^{*}$. Thus the monopsonist's optimal choice of pipeline size will be always lower than Q^{*}. This occurs because when the monopsonist faces positive costs of capacity, he gains two things from restricting capacity: lower capacity costs and the elimination of his incentive to defect from the Nash equilibrium in future periods.

3.6. Conclusions

This chapter shows that a pipeline has an incentive to restrict the size of the pipeline in order to prevent the Coase Conjecture result that its profits be dissipated by producers shifting production across time.

An empirical implication of our model is that pipeline capacity should be smaller when discount factors are relatively large. Thus, we hypothesize that pipelines in countries with high levels of political unrest will tend to be larger, as the strategic effect of constraining the pipeline size is more likely to be offset by the incentive to rapidly extract the resource.

4. Motivation and Literature Review: an 'Oil'igopoly Theory of Exploration

The theory of 'oil'igopoly, developed by Salant (1976) and extended by Loury (1986) and Polasky (1992),[18] has the simple yet elegant prediction that producers holding

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[18] Salant (1981, 1982), Lewis and Schmalensee (1979) and Ulph and Folie (1980) have also used a Nash strategies to model the world oil market. See Mason and Polasky (2005) and Benchakround, Gaudet, and van Long (2004) for recent extensions to the Nash model. See Gilbert (1978), Newbery (1981), Ulph (1982), Groot, Withagen and de Zeeuw (1992, 2003) for Stackelberg cartel-fringe models. Karp (1984), Maskin and

larger reserves of oil tend to produce larger quantities of oil but a smaller proportion of their reserves in each period. Polasky found support for this prediction using a cross-section of oil producing nations. However, a key limitation of the theory of 'oil'igopoly is that they solved only for the Nash equilibrium. It is well known that the Nash equilibrium to dynamic games is not dynamically consistent. However, Eswaran and Lewis (1986) showed that when producers possess well defined property rights, the Nash Equilibrium differs only slightly from the dynamically consistent subgame perfect equilibrium.[19]

When exploration is added to the game, it is no longer clear that the Nash equilibrium will yield results that are qualitatively similar to the dynamically consistent subgame perfect equilibrium.[20] This paper shows that the Nash equilibrium produces a much different result than the dynamically consistent subgame perfect equilibrium. The reason for this difference can be seen as follows. If one views exploration as the costly process of moving reserves from the "unproven" to the "proven" state, then it becomes clear that exploration may have strategic implications.[21] This occurs because once the exploration has occurred, the exploration costs become sunk. As exploration costs are on the order of hundreds of thousands of dollars for a well drilled on land to tens of millions of dollars for a well drilled at sea, sinking the discovery cost results in a substantial lowering of the

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Newbery (1990), Karp and Newbery (1993) consider Stackelberg models in which governments extract rents from exhaustible resource industries over time. These models also focus on the difference between open loop (Nash) and dynamically consistent (subgame perfect) equilbria.

[19] This result differs from those in Levhari and Mirman (1980) and Reignanum and Stokey (1985), who found substantial economic differences between the equilibrium concepts when the resource stocks are commonly owned.

[20] Hotelling (1931) was the first model of competitive industry extraction. Competitive models of exploration appear in Pindyk (1978), Arrow and Chang (1982), and Swierzbinski and Mendelsohn (1989).

[21] Proven reserves are those reserves for which exploration has already demonstrated the existence of an economically viable deposit. Unproven reserves are those reserves that the geologic indicators suggest exist, but which have not yet been discovered, or transformed into proven reserves, through exploration.

marginal cost of production.[22] Having lowered its marginal costs of future production, a producer has a credible threat to its rivals that it will produce a larger quantity in the next period. Hence, this threat induces one's rivals to tilt their production profile towards the present, which raises the present value of future production to the producer.

The strategic advantage conveyed by exploration is similar to that obtained from an increase in plant capacity, or R&D research to lower production costs in the industrial organization literature (e.g., Dixit 1980, 1986, Fudenberg and Tirole 1994, Bulow, Geanakopolis and Klemperer 1985).[23] This strategic aspect of exploration leads us to model the game using subgame perfection as the equilibrium concept. Thus, the game is solved by backwards induction. Given that an exhaustible resource market exhausts the resource in the final period of the game, there can be no strategic effects in a two period game. This means that the game must be at least three periods long in order to see the strategic effects. Hence, we solve for the dynamically consistent equilibrium in a three period game in which producers compete not only in the output market, but also in the process of exploration.

Like the theory of 'oil'igopoly, we find that producers holding larger proven reserves extract a larger quantity but a smaller proportion of their reserves in each period prior to exhaustion. We find that this relationship also holds as well for unproven reserves. The reason for these results is similar to the logic in the theory of 'oil'igopoly. Larger producers produce a smaller proportion of their reserves because an increase in the output, which

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[22] Average drilling costs in the United States were approximately seven hundred thousand dollars for an onshore well and over twelve million dollars for an offshore well in 2002. Source: *Basic Petroleum Databook*, American Petroleum Institute 2006.

[23] The literature on strategic investments is surveyed in Tirole (1990, pp. 314-336).

depresses the price, has a greater effect on their revenues than for a smaller producer. Here, this effect is amplified by the strategic advantages of holding larger reserves.

The second result, which is novel to this paper, is that producers with smaller proven reserves will do more exploration than producers holding larger reserves, all else constant. This occurs because producers holding small levels of proven reserves are more likely to run out of those reserves in the next period. Therefore, for these producers, the benefit of additional proven reserves is very high. In contrast, producers holding proven reserves in sufficient quantity to produce from these reserves in the next period already have a credible commitment device to signal to rivals that they will produce a larger quantity in subsequent periods, so for them the benefit from holding larger quantities of proven reserves is small. For example, Saudi Arabia's Aramco, holds proven reserves that will last between seventy and eighty years at its current production levels.[24] Thus, exploration is the primary instrument for gaining a strategic advantage for small producers, while constraints on production are the primary instrument for gaining a strategic advantage for larger producers. This is unlike other models of strategic investment, where firms with an initial cost advantage tend to exploit that advantage by investing at higher levels than their rivals.[25] This occurs because exhaustible resource producers face an intertemporal constraint that production in any period must be less than the sum of proven reserves and reserve additions in that period.

[24] There have been only about 2000 wells drilled in the Gulf region, compared to over a million drilled in the United States (see "Really Big Oil," *The Economist*, August 10, 2006).
[25] In an asymmetric strategic investment model, such as in Dixit (1980), a firm with an initial cost advantage makes higher levels of investment, all else constant. An example is provided in the conclusions section, where this issue is discussed more fully.

An important limitation of the theory of 'oil'igopoly, and to the model that we present, is that producers face no uncertainty over their reserve holdings. We also follow the theory of 'oil'igopoly assumption that producers do not face competition over their own reserves, whether proven or unproven.[26] An important implication of these assumptions is that we can abstract from informational issues associated with exploration.[27]

This paper is closest in spirit to earlier papers by Bulow and Geanakopolis (1983) and Hartwick and Sadorsky (1990). These papers were also interested in the strategic effects from exploration from higher-cost stocks due to exploration's role as a commitment device. However, these papers make an important assumption that limits the generality of their results. In both papers, producers produce in only two periods. If producers were to exhaust their entire reserves in the second period, then there can be no strategic effect from exploration, as we show below. To circumvent this problem, both sets of authors make assumptions which effectively leave some reserves unexploited. In Hartwick and Sadorsky, producers in the first period choose both the level of exploration and production, but in the second period producers only produce from their remaining proven reserves – they do no further exploration. In Bulow and Geanakopolis, producers in each period extract from lower cost reserves and from a higher cost backstop technology. Depletion of reserves raises the future marginal costs of extraction from those reserves. However, the lower cost

[26] This assumption follows from evidence that most of the significant players in the world oil market are state-owned producers, which face little or no competition for access to the resource stocks within their own countries. Sixteen of the top twenty oil producers by reserve holdings are state owned producers. See "Really Big Oil," *The Economist*, August 10, 2006).

[27] See Mason (1986), Isaac (1987), Polasky (1996), and Hendricks and Porter (1996) for models of information transmission in exploration. These models implicitly assume that mineral rights are not secure. These models have focused on whether there is too little or too much exploration from an information gathering perspective and whether the timing of exploration has strategic information effects.

reserves were not exhausted in their model. In each of these models, current production affects subsequent profits, but it cannot affect subsequent behaviour. Thus in each of these models there exists a subgame in which the excess reserves are exploited, but the effect of this subgame on the remainder of the game is ignored.

In contrast, we solve for the entire production and exploration path, including the endgame in which reserves are completely exhausted. Thus, we offer a complete characterization of the dynamics of the game. Like Hartwick and Sadorsky and Bulow and Geanakopolis, we find that producers behave strategically by over- exploring relative to an open loop benchmark. However, unlike these authors, we also find that producers postpone some exploration to the final period in which they are active, as the proven reserves have lower marginal costs of extraction (e.g., Hartwick 1977).

The next chapter is organized as follows. In Section 5.1, we present the basic equilibrium results of the model, beginning with what happens when there is only one period left before oil is exhausted, and then working back to two periods before exhaustion. Section 5.2 derives the main results regarding strategic exploration by moving back one more period and asking how producers behave at that point, given the effects on rivals' subsequent behaviour. It is in this section that we show why smaller producers are the ones most likely to be doing the most exploration. Section 5.3 includes a simple empirical test of the hypothesis that smaller producers do more exploration using country-level data over the post World War II era. Section 5.4 concludes by discussing the results of this paper in relation to other strategic investment models.

5. An 'Oil'igopoly Theory of Exploration

5.1. 'Oil'igopoly Exploration and Production

The game we consider is restricted to thee periods. In this section, we describe the behaviour of producers in the last two periods in the game. In section 5.2, we show how these results affect strategic exploration.

5.1.A. Notation and Assumptions

At the beginning of each period t, let n_t producers hold reserves of one type or the other. Proven reserves held by the i^{th} producer at the beginning of period t are denoted as R_{it}; unproven reserves are denoted as S_{it}. Thus initial reserves held by producer i are denoted as R_{i1} and S_{i1}, respectively. We let $\boldsymbol{R}_t = \{R_{1t},R_{2t},\ldots,R_{n_t t}\}$ and $\boldsymbol{S}_t = \{S_{1t},S_{2t},\ldots,S_{n_t t}\}$ denote the vectors of stocks held at the beginning of period t by all n_t producers active in that period; $R_t = \Sigma_{i=1}^{nt} R_{it}$ and $S_t = \Sigma_{i=1}^{nt} S_{it}$ denote the stocks held at the beginning of period t by all n_t producers; and $R_{-it} = \Sigma_{j\neq i}^{nt} R_{jt}$ and $S_{-it} = \Sigma_{j\neq i}^{nt} S_{jt}$ denote the sum of reserves held by all producers other than producer i at the beginning of period t. We assume that the stocks of proven and unproven reserves for all producers are common knowledge.

We let n_t denote the number of producers holding one stock or the other and m_t denote the number of producers who exhaust their total reserves in period t. Thus, the number of producers evolves according to $n_{t+1} = n_t - m_t$, $t = 1, 2, 3$. Since the game ends in three periods, all producers exhaust their reserves by period three (i.e., $n_4 = 0$). Therefore, $\Sigma_{t=1}^{3} m_t = n_1$. For a given allocation of reserves of each type, the number of producers exhausting each type is endogenous. However, rather than deriving the equilibrium number of producers that exhaust in each period, without loss of generality, we shall fix the number of producers exhausting in each period, i.e., the $\{m_t\}_{t=1,2,3}$, and derive the conditions on the reserve holdings that have to be satisfied in equilibrium in order for this number of producers to rationally exhaust in each period.

In each period, producer i chooses a level of output, q_{it}, and a level of reserve additions, w_{it}, $t = 1,2,3$, $i = 1,\ldots,n_t$. The model is deterministic, so each unit of exploration yields a fixed quantity of reserve additions. Given the production and reserve additions choices made by producers in period t, the stocks of proven and unproven reserves held by the i^{th} producer evolve according to

$$R_{it+1} = R_{it} + w_{it} - q_{it}, \qquad i = 1,\ldots,n_t,\ t = 1,2,3, \tag{5.1}$$

$$S_{it+1} = S_{it} - w_{it}, \qquad i = 1,\ldots,n_t,\ t = 1,2,3. \tag{5.2}$$

The price at time t is denoted by $P_t = P(Q_t)$, where $Q_t = \Sigma_{i=1}^{nt} q_{it}$, and where the demand function $P(Q_t)$ is positive valued and decreasing in aggregate output. The extraction and discovery costs are denoted by $c_i(q_{it})$ and $d_i(w_{it})$, respectively. These are assumed to be positive-valued, homogeneous of degree $r \geq 2$ for discovery cost function and $r \geq 1$ for extraction cost function, increasing convex functions in the level of production and the level of exploration, respectively.[28] Homogeneity of degree r implies $g_i'(jx) = j^{r-1} g_i'(x)$. Thus for all $j > 0$ and $r \geq 2$, when $x = 0$ it must be true that $g_i'(0) = 0$. Thus by homogeneity we

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[28] It will become clear below that strict convexity of the function $d_i(w_{it})$ is required to obtain effects. See Proposition 5.6.

set $d_i'(0)$ to be zero. For simplicity, we assume that the cost of extraction and discovery are independent of reserves.[29]

As we wish to restrict the model to three periods so that we can use backwards induction, we assume that the demand and cost functions satisfy:

Assumption A.1: $c_i'(0) + d_i'(0) < P(0) < \infty.$

The first inequality ensures that producers wish to ultimately exhaust their reserves, as the marginal revenue exceeds the cost of extraction for the last unit of reserves. The second inequality implies that the resource is not essential to production in the economy. This assumption ensures that exhaustion occurs in finite time.

In addition, we make two regularity assumptions which, when taken together, ensure that the best-response functions are stable. This is necessary in order for strategic effects to take place. These assumptions are:

Assumption A.2: $P'(Q_t) + q_{it}P''(Q_t) < 0.$

Assumption A.3: $c_i''(q_{it}) - P'(Q_t) > 0.$

Assumption A.2 implies that producer i's marginal profit is lowered by an increase in the output of any other producer. This implies that the goods are strategic substitutes and occurs because the choice variable is the output (Bulow, Geanakopolis, and Klemperer 1985). Assumption A.3 implies that the demand function intersects the marginal cost

[29] Thus the only grade differential in the stocks is the difference between proven and unproven reserves. See Swierzbinskin and Mendelsohn (1989), *inter alia*, for a model of grade differentials under competitive extraction and exploration.

function from above. Taken together, assumptions A.2 and A.3 imply that second order conditions are satisfied for each producer, and these assumptions together are also sufficient conditions to yield the existence of a unique and stable equilibrium (Vives 1999, Theorem 2.7).

We also assume that producer i's total profits, not just marginal profits, are decreasing in the output level of other producers (*cf.* Tirole 1990, p. 326). In a single period game, this automatically holds, since the effect on producer i's profits of an increase in the output of other producers, Q_{-it}, is simply $P'(Q_t)q_{it} < 0$. However, when we move to the second period in a game in which all producers exhaust their stock by the third period, so that $q_{i3} = R_{i2} + S_{i2} - q_{i2}$ for all producers that produce in period three, then the assumption that profits are decreasing in the second period output of other producers can be written as

Assumption 4: $$q_{i2}P'(Q_2) - \beta q_{i3}P'(Q_3) < 0, \qquad i = 1,2,\ldots,n_2.$$

Finally, one dollar earned or spent one period in the future is discounted at the common rate $\beta \in (0,1)$.

Now, we turn to the analysis of the game, which we begin in period three, the final period in which any producers produce.

5.1.B. The Period Three Equilibrium.

In the final period of the game, there are n_3 producers with R_{i3} and S_{i3} reserve holdings. Then the problem faced by producer i at the beginning of period three is to choose q_{i3} and w_{i3}, taking the actions of the other producers fixed, to maximize

$$V_{i3} = P(Q_3)q_{i3} - c_i(q_{i3}) - d_i(w_{i3}), \qquad i = 1,\ldots,n_3. \qquad \textbf{P3}$$

subject to the constraints that

$$R_{i3} + w_{i3} - q_{i3} \geq 0, \qquad i = 1,\ldots,n_3, \tag{5.3}$$

$$S_{i3} - w_{i3} \geq 0, \qquad i = 1,\ldots,n_3, \tag{5.4}$$

Given that all producers are assumed to exhaust in period three, the constraints (5.3) and (5.4) are each binding, so that in equilibrium, $w_{i3} = S_{i3}$ and $q_{i3} = R_{i3} + S_{i3}$. Let λ_i and μ_i denote the Kuhn-Tucker multipliers for the constraints (5.3) and (5.4), respectively. Then the first-order necessary conditions for each producer can be written as

$$\frac{\partial V_{i3}}{\partial q_{i3}} = P(R_3 + S_3) + (R_{i3} + S_{i3})P'(R_3 + S_3) - c_i'(R_{i3} + S_{i3}) - \lambda_i = 0, \; i = 1,\ldots,n_3, \tag{5.5}$$

$$\frac{\partial V_{i3}}{\partial w_{i3}} = -d_i'(S_{i3}) + \lambda_i - \mu_i = 0, \qquad i = 1,\ldots,n_3. \tag{5.6}$$

Condition (5.5) says that each producer equates marginal revenue with marginal extraction costs plus scarcity rent. Condition (5.6) implies that marginal discovery costs are equated with the net scarcity rent. Note that assumption A.2 implies that λ_i is decreasing in the aggregate reserves held by all other producers at the beginning of period three.

Next, we turn to the condition which is both necessary and sufficient for all producers to exhaust in period three. Suppose that producer i enters period three with positive values of both stocks. In order for producer i to exhaust in period three, the marginal profits from period three must exceed those from waiting another period, taking the actions of all other producers as fixed. The discounted marginal profits to such producer who waits until period four are $\beta[P(0) - c_i'(0) - d_i'(0)]$, since all other producers are assumed to have exhausted in period three. Combing (5.5) and (5.6) by eliminating λ_i and comparing the marginal profits in period three and four yields the condition that must hold if producer i is to exhaust in period three, given that he holds both types of reserves:

$$P(R_3 + S_3) + (R_{i3} + S_{i3})P'(R_3 + S_3) - c_i'(R_{i3} + S_{i3}) - d_i'(S_{i3})$$

$$= \mu_i > \beta\left[P(0) - c_i'(0)\right], \qquad \text{if } S_{it} > 0,\ i = 1,\ldots,n_3. \tag{5.7}$$

For a producer that holds only unproven reserves the condition (5.7) is unchanged, except that $R_{i3} = 0$. But the condition for a producer that holds only proven reserves would not contain the $-d_i'(S_{i3})$ term, and μ_i would be replaced by λ_i (see (5.5)):

$$P(R_3 + S_3) + \ R_{i3}P'(R_3 + S_3) - c_i'(R_{i3}\ )$$

$$= \lambda_i > \beta\left[P(0) - c_i'(0)\right], \qquad \text{if } S_{i3} = 0,\ i = 1,\ldots,n_3. \tag{5.8}$$

These conditions say that in order to be satisfied by exhausting in period three, the rents earned by producer i in period three must be greater than the present value of the rents earned by waiting one period, taking as given the actions of the other producers. The Nash equilibrium is that all producers exhaust if, and only if, the inequalities in (5.7) and (5.8) holds for all n_3 producers. The equilibrium condition for a particular producer is illustrated graphically in Fig. 5.1.

Under assumptions A.2 and A.3, the marginal profits to the producer in each period are downward sloping functions. The solid lines in Fig. 5.1 are the marginal profits for a producer holding only proven reserves, while the dashed lines in Fig. 5.1 are the marginal profits for a producer who holds unproven reserves.[30] The marginal profits for a producer holding unproven reserves are lower than the marginal profits for a producer holding only proven reserves because of the marginal discovery costs.

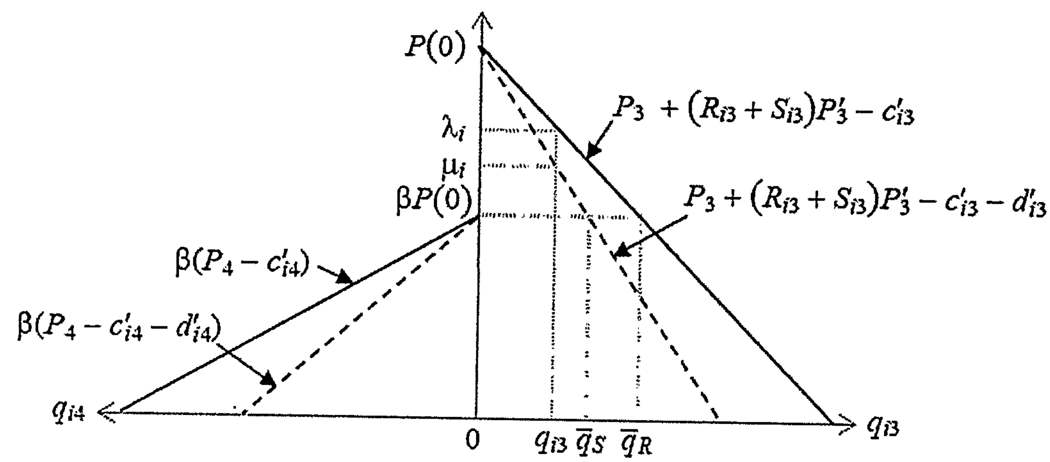
The quantities $\overline{q}_S$ and $\overline{q}_R$ are the maximum total reserves that can be held by a producer holding both proven and unproven reserves or just proven reserves, respectively, given the holdings of all other producers, such that producer i will rationally exhaust in period three. Thus if producer i holds reserves $R_{i3} + S_{i3} < \overline{q}_S$, then producer i maximizes his profits by exhausting both types of reserves in period three, and if producer i holds only proven reserves $R_{i3} < \overline{q}_R$, then producer i maximizes his profits by exhausting his proven reserves in period three. If this condition holds for all producers, then period three is the equilibrium time of exhaustion.

Note that $\overline{q}_S < \overline{q}_R$ in Fig. 5.1. This occurs because unproven reserves face the additional cost of discovery, $d_i'(S_{i3})$, relative to proven reserves. We can see that a producer carrying only proven reserves into period three has a credible threat to produce a larger quantity in period three. Nevertheless, this threat does not affect the *behaviour* of the other producers,

[30] For brevity, we shall use time subscripts for the prices and costs when the functional arguments are suppressed. Thus, $c_{it} \equiv c_i(q_{it})$, $c_{it}' \equiv c_i'(q_{it})$, and $c_{it}'' \equiv c_i''(q_{it})$; and similarly for $d_i(w_{it})$ and $P(Q_t)$.

so long as the other producers exhaust in period three.[31] Rather, this threat only affects the other producers' *profits*, but it does not affect q_{i3}.[32] Thus, we may state:

Figure. 5.1: Rational Exhaustion in Period Three by Producer *i*.



Lemma 5.1: Under assumptions A.1-A.4, if all producers exhaust no later than period three, then a producer who carries a larger stock into period three affects the profits of other producers, but not the behaviour of other producers. Therefore, the subgame perfect equilibrium and the Nash equilibrium for the subgame beginning at period two are identical.

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[31] If, by carrying additional reserves into period three, producer j were to cause producer i to alter *in which period* producer i exhausted, there would be a change in the *behavior* of producer i. In Fig. 5.2, this corresponds to lowering the marginal profit function sufficiently in period three so that $R_{i3} + S_{i3} > \bar{q}_S$. We do not consider this strategic effect in this paper.

[32] The effect on marginal profits is $\partial\lambda_i/\partial Q_{-i3} = \partial\mu_i/\partial Q_{-i3} = P'(R_3 + S_3) + (R_{i3} + S_{i3})P''(R_3 + S_3) < 0$ by A.2. The effect on profits is also negative: $\partial V_{i3}/\partial Q_{-i3} = (R_{i3} + S_{i3})P'(R_3 + S_3) < 0$.

Therefore, in period two, without loss of generality, we may solve for the equilibrium by using an open loop solution, since the open loop and subgame perfect solutions are identical.

5.1.C. Period Two Subgame Perfect Equilibrium

We turn now to the problem faced by producer i in period two, given that it holds reserves R_{i2} and S_{i2}, where $R_{i2} + S_{i2} > 0$, but either R_{i2} or S_{i2} could be zero, and given that at least $m_3 > 0$ producers rationally exhaust in period three, and no producers exhaust beyond period three.

By Lemma 5.1, producer i's problem in period two is to choose exploration and production $\{q_{i2}, q_{i3}, w_{i2}, w_{i3}\}$, taking the choices of all other producers as fixed, to maximize[33]

$$V_{i2} = P(Q_2)q_{i2} - c_i(q_{i2}) - d_i(w_{i2}) + \beta[P(Q_3)q_{i3} - c_i(q_{i3}) - d_i(w_{i3})], \qquad \mathbf{P2}$$

subject to the following constraints (the Kuhn-Tucker multipliers associated with each constraint are written in parentheses):[34]

$$R_{i2} + w_{i2} + w_{i3} - q_{i2} - q_{i3} \geq 0, \qquad (\lambda_i) \qquad i = 1,\ldots,n_2, \qquad (5.9)$$

[33] Since producer i's reserves are exhausted in period three, we could write this as a backwards induction:

$$\mathbf{P2'} \qquad V_{i2} = P(Q_2)q_{i2} - c_{i2}(q_{i2}) - d_{i2}(w_{i2}) + \beta V_{i3}^*(\mathbf{R}_3,\mathbf{S}_3),$$

where $V_{i3}^*(\mathbf{R}_3,\mathbf{S}_3)$ is the solution to problem **P3**. However, since producer j cannot affect producer i's period three production, problems **P2′** and **P2** are equivalent.

[34] The multipliers λ_i and μ_i are now written as the present value of the resource stocks R_{it} and S_{it} in *period two*. The values of λ_i and μ_i in section 5.1.B, which are the present value in period three, equal λ_i/β and μ_i/β in period two, respectively.

$$S_{i2} - w_{i2} - w_{i3} \geq 0, \qquad (\mu_i) \qquad i = 1,\ldots,n_2, \qquad (5.10)$$

$$R_{i2} + w_{i2} - q_{i2} \geq 0, \qquad (\phi_i) \qquad i = 1,\ldots,n_2. \qquad (5.11)$$

Constraint (5.9) is a feasibility constraint on production due to the exhaustible nature of the resource. Constraint (5.10) is a feasibility constraint on exploration. Constraint (5.11) ensures that extraction in period two is feasible given the beginning proven reserves and the reserve additions in period two.

The first-order necessary conditions for maximization of **P2** include (5.9)-(5.11) and

$$\frac{\partial V_{i2}}{\partial q_{i2}} = P(Q_2) + P'(Q_2)q_{i2} - c_i'(q_{i2}) - \lambda_i - \phi_i \leq 0, \quad i = 1,\ldots,n_2, \qquad (5.12)$$

$$\frac{\partial V_{i2}}{\partial q_{i3}} = \beta[P(Q_3) + P'(Q_3)q_{i3} - c_i'(q_{i3})] - \lambda_i \leq 0, \qquad i = 1,\ldots,n_3, \qquad (5.13)$$

$$\frac{\partial V_{i2}}{\partial w_{i2}} = -d_i'(w_{i2}) + \lambda_i - \mu_i + \phi_i \leq 0, \qquad i = 1,\ldots,n_2, \qquad (5.14)$$

$$\frac{\partial V_{i2}}{\partial w_{i3}} = -\beta d_i'(w_{i3}) + \lambda_i - \mu_i \leq 0, \qquad i = 1,\ldots,n_3. \qquad (5.15)$$

Each of these holds as an equality when the choice variable is non-negative. The marginal value of the proven reserves, R_{i2}, to producer i at the beginning of period two is $\lambda_i + \phi_i$ and the marginal value of unproven reserves to producer i at the beginning of period two is μ_i. The conditions (5.12) and (5.13) have the usual interpretation that the marginal profit from extraction in each period is equal to the marginal value of the remaining resource stock. Equations (5.14) and (5.15) reveal that a similar dynamic is at work with unproven reserves.

Given that a producer with positive stocks will produce at least in period two, we may use (5.13) to eliminate the shadow value of proven reserves, λ_i, from (5.12) to yield an intertemporal arbitrage rule in terms of marginal profits from production:

$$\pi_1^{i2}(q_{i2}^*, Q_{-i2}) - \beta\pi_1^{i3}(R_{i2} + S_{i2} - q_{i2}^*, Q_{-i3}) - \phi_i \geq 0 \quad i = 1,\ldots,n_2, \tag{5.16}$$

where $\pi_1^{it}(q_{it}^*, Q_{-it}) \equiv P(Q_t) + P'(Q_t)q_{it}^* - c_{it}'(q_{it}^*)$ is the equilibrium marginal profit from extraction in period t, given that discovery costs are sunk and holding the output of all other producers, Q_{-it}, constant.[35] When production is positive in both periods and the constraint (5.11) does not bind, so that ϕ_i is zero, then (5.16) holds as a strict equality. This means that the marginal profits from production are equal in present value, which is Hotelling's rule for an oligopolist (e.g., Salant 1976, Loury 1986, or Polasky 1992).

When $\phi_i > 0$ and production is positive in both periods, then the Hotelling condition (5.16) reflects the increase in extraction costs due to having to extract only from unproven reserves in period three, so that $\pi_1^{i2}(q_{i2}^*, Q_{-i2}) > \beta\pi_1^{i3}(R_{i2} + S_{i2} - q_{i2}^*, Q_{-i3})$. This means that the value of the marginal reserves declines in present value when (5.11) is binding.

Similarly, when the producer chooses not to produce in period three, (5.16) implies that $\pi_1^{i2}(R_{i2} + S_{i2}, Q_{-i2}) > \beta\pi_1^{i3}(0, Q_{-i3})$, which is the condition under which the producer does better by exhausting in period two than by taking some reserves into period three.

A similar expression can be obtained for marginal discovery costs, substituting in from

[35] Subscripts denote partial derivatives: $\pi_1^{it} \equiv \partial\pi^{it}/\partial q_{it}$, $\pi_2^{it} \equiv \partial\pi^{it}/\partial Q_{-it}$, $\pi_{11}^{it} \equiv \partial^2\pi^{it}/\partial q_{it}^2$, and $\pi_{12}^{it} \equiv \partial^2\pi^{it}/\partial q_{it}\partial Q_{-it}$.

(5.11) we can write (5.14) and (5.15) in terms of period two exploration to provide an intertemporal optimization condition for reserve additions:

$$d_i'(w_{i2}^*) - \beta d_i'(S_{i2} - w_{i2}^*) \gtreqless \phi_i,\ i = 1,\ldots,n_2. \tag{5.17}$$

However, this inequality can go in either direction, depending upon which period(s) exploration occurs. Our first result is that exploration is positive in any period in which production occurs, which implies that (5.17) holds with equality:

Proposition 5.1: Under assumptions A.1-A.4, if producer i has a positive quantity of unproven reserves at the beginning of period two (i.e., $S_{i2} > 0$) and rationally exhausts in period two or three, then producer i explores in each period in which he produces.

Proof: This proposition is proven by the following lemmas.

Lemma 5.2: With positive quantities of the unproven reserves and production in both periods two and three, if there is zero exploration in some period, it will be in period two, not period three.

Proof: Suppose not. Suppose that $w_{i2} > 0$ and that $\lambda_i - \mu_i < \beta d_i'(0)$. Then, $d_i'(S_{i2}) - \phi_i = \lambda_i - \mu_i$ and $w_{i3} = 0$. Then we obtain that

$$d_i'(S_{i2}) - \phi_i = \lambda_i - \mu_i < \beta d_i'(0). \tag{5.18}$$

Since the producer is assumed to extract in the third period, it is not possible that $w_{i2} = S_{i2} > 0$ and $\phi_i > 0$ both occur. Thus, let $\phi_i = 0$. Then this equation implies that $d_i'(S_{i2}) < \beta d_i'(0) =$

0, which is a contradiction, since $d_i' \geq 0$.

Lemma 5.3: If producer i extracts all of his proven reserves by period two, so that the constraint (5.11) binds, then he also explores in period two.

Proof: Suppose not. Suppose that $\phi_i > 0$, that $q_{i2} = R_{i2}$, and that $d_i'(0) - \phi_i > \lambda_i - \mu_i$. Then $d_i'(0) - \phi_i > \lambda_i - \mu_i$ implies that $w_{i2} = 0$, which means that $w_{i3} = S_{i2}$, so that $d_i'(0) - \beta d_i'(S_{i2}) > \phi_i$. Since $d_i'(0) = 0$, we get that $-\beta d_i'(S_{i2}) > \phi_i > 0$ that contradicts.

Lemma 5.4: If producer i produces in both periods and the constraint (5.11) does not bind, then producer i will have positive level of exploration level in period two.

Proof: Suppose not. Suppose that $q_{i2} < R_{i2}$, that production occurs in both periods, and that $w_{i2} = 0$. Then $q_{i2} < R_{i2}$ implies that $\phi_i = 0$ and $d_i'(0) > \lambda_i - \mu_i$ implies that $w_{i2} = 0$. Since $w_{i2} = 0$, $w_{i3} = S_{i2} > 0$. Therefore, $d_i'(0) > \beta d_i'(S_{i2})$. Since $d_i'(0) = 0$, $\beta d_i'(S_{i2}) < 0$ contradicts.

The only other possibility is that producer i exhausts in period two but does not explore in period two. Assumption A.1 implies that each producer exhausts all stocks, so it cannot be equilibrium behaviour for producer i to shut down before exhausting his unproven reserves.

This completes the proof of Proposition 5.1.

Proposition 5.1 shows that because unproven reserves have higher marginal costs to extract than proven reserves, each producer will produce from the high cost reserves in the final period in which it operates, even given the strategic advantages of early transformation into proven reserves. This was assumed *not* to occur in Hartwick and Sadorsky (1990), but Proposition 5.1 shows that if the producer is given the choice of exploring in the last period, he will do so. The reason for this result is that producers still have an incentive to produce from the lowest cost reserves first as in Hartwick (1977). Given that there are no strategic effects between periods two and three by Lemma 5.1, this effect dominates in the final two periods of the game.

Proposition 5.1 implies that (5.17) can be written as

$$d_i'(w_{i2}^*) - \beta d_i'(S_{i2} - w_{i2}^*) = \phi_i, \qquad i = 1,\ldots,n_2. \tag{5.19}$$

Thus, marginal exploration costs are constant in present value when the constraint (5.11) is not binding, and fall in present value when the constraint (5.11) is binding. In what follows, we use the fact that when the constraint (5.11) is not binding, (5.19) implies that there exists a value of $w_{i2}^* = w_{i2}(S_{i2})$ such that

$$d_i'\left(w_{i2}(S_{i2})\right) \equiv \beta d_i'\left(S_{i2} - w_{i2}(S_{i2})\right), \qquad \text{for } S_{i2} > 0. \tag{5.20}$$

It follows from (5.20) that $0 < w_{i2}'(S_{i2}) < 1$.[36] Note that $w_{i2}(0) = 0$, since the quantity of unproven reserves is known with certainty.

While proposition 5.1 eliminates all equilibria with zero exploration in either period in which production is positive, there remain three possible outcomes for a producer that produces in one or both the two remaining periods, depending upon whether or not the constraint (5.11) binds when production occurs in period three, and on whether or not the

[36] For example, if d_i'' is a positive constant, then $w_{i2}'(S_{i2}) = \beta/(1+\beta)$.

producer produces in period three:

Case A: producer i explores and extracts in periods two and three and the constraint (5.11) does not bind, i.e., $\{w_{i2}^{*} = w_{i2}(S_{i2}) \text{ and } q_{i2}^{*} < R_{i2} + w_{i2}(S_{i2})\}$.

Case B: producer i explores and extracts in periods two and three but the constraint (5.11) binds, i.e., $\{S_{i2} > w_{i2}^{*} > w_{i2}(S_{i2}) \text{ and } q_{i2}^{*} = R_{i2} + w_{i2}^{*}\}$.

Case C: producer i exhausts in period two, i.e., $\{w_{i2}^{*} = S_{i2} \text{ and } q_{i2}^{*} = R_{i2} + S_{i2}\}$.

To characterize the equilibrium choices made by producer i when taking as given the actions of all other producers, we define the following terms:[37]

$$\psi_i(q_{i2} \mid Q_{-i2}) \equiv \pi_1^{i2}(q_{i2}, Q_{-i2}) - d_i'(q_{i2} - R_{i2})$$

$$- \beta[\pi_1^{i3}(R_{i2} + S_{i2} - q_{i2}, Q_{-i3}) - d_i'(R_{i2} + S_{i2} - q_{i2})], \tag{5.21}$$

where $w_{i2} = q_{i2} - R_{i2}$, and $q_{i3} = w_{i3} = R_{i2} + S_{i2} - q_{i2}$ from (5.11) and (5.13). Thus, from (5.12) -(5.15), $\psi_i(q_{i2}^{*} \mid Q_{-i2}) = 0$ when (5.11) is binding. Similarly, let:

$$\eta_i(q_{i2} \mid Q_{-i2}) \equiv \pi_1^{i2}(q_{i2}, Q_{-i2}) - d_i'\big(w_{i2}(S_{i2})\big)$$

$$- \beta[\pi_1^{i3}(R_{i2} + S_{i2} - q_{i2}, Q_{-i3}) - d_i'\big(S_{i2} - w_{i2}(S_{i2})\big)], \tag{5.22}$$

where $w_{i2} = w_{i2}(S_{i2})$ is given by (5.20), and $q_{i3} = R_{i2} + S_{i2} - q_{i2}$ from (5.11). Thus,

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[37] Given that all producers exhaust in three periods, $Q_{-i3} = R_{-i3} + S_{-i3} - Q_{-i3}$, so implicit in (5.21) and (5.22) are the stocks held by other producers.

$\eta_i(q_{i2}^* \mid Q_{-i2}) = 0$ when (5.11) is not binding. The difference between (5.21) and (5.22) is the value of w_{i2}. When $\phi_i = 0$, $w_{i2}^* = w_{i2}(S_{i2})$, but when the constraint (5.11) is binding, $\phi_i > 0$ implies that $w_{i2}^* > w_{i2}(S_{i2})$, since (5.19) implies that w_{i2}^* is increasing in ϕ_i. When $q_{i2}^* = R_{i2} + w_{i2}(S_{i2})$, $\psi_i(q_{i2}^* \mid Q_{-i2})$ is identical to $\eta_i(q_{i2}^* \mid Q_{-i2})$.

We proceed by first deriving and interpreting the conditions that hold for a particular producer that produces in periods two and three to have a unique best response to what the remaining industry is doing. Then we find a set of conditions on the demand and cost function that ensures that the best-reply mappings of all producers contract to a unique equilibrium.

Proposition 5.2: Under assumptions A.1-A.4, holding constant the actions of all other producers, if producer i produces in period two or in periods two and three, then producer i's unique choice of extraction and exploration exists satisfying $\psi_i(q_{i2}^*) = 0$ and (5.19) when (5.11) binds, if and only if,

$$\psi_i(R_{i2} + S_{i2} \mid Q_{-i2}) < 0 \qquad \text{and} \qquad \psi_i(R_{i2} + w_{i2}(S_{i2}) \mid Q_{-i2}) > 0; \tag{5.23}$$

and satisfying $\eta_i(q_{i2}^*) = 0$ and (5.20) when (5.11) does not bind, if and only if,

$$\eta_i(R_{i2} + w_{i2}(S_{i2}) \mid Q_{-i2}) < 0 \quad \text{and} \quad \eta_i(0 \mid Q_{-i2}) > 0. \tag{5.24}$$

Proof: (*i*) Uniqueness: Since uniqueness is easiest to prove, we begin with it. When (5.11) binds, so that (5.21) and (5.19) define the equilibrium, we see from (5.21) that

$$\psi_i'(q_{i2}) = \pi_{11}^{i2}(q_{i2}, Q_{-i2}) \; - \; d_{i2}''(q_{i2} - R_{i2})$$

$$+ \beta[\pi_{11}^{i3}(R_{i2} + S_{i2} - q_{i2}, Q_{-i3}) \; - d_{i3}''(R_{i2} + S_{i2} - q_{i2})] \; < 0, \tag{5.25}$$

where $\pi_{11}^{it}(q_{it}, Q_{-it}) \equiv 2P_t' + q_{it}P_t'' - c_{it}'' < 0$ in order for second order conditions to hold. Similarly, we see that the left-hand-side of (5.19) is strictly increasing in q_{i2}:

$$d_{i2}''(q_{i2} - R_{i2}) + \beta d_{i3}''(R_{i2} + S_{i2} - q_{i2}) \; > 0. \tag{5.26}$$

Thus, if $\psi_i(q_{i2}^*) = 0$ for some feasible q_{i2}^* and (5.19) holds, then q_{i2}^* is unique.

The proof for (5.22) proceeds similarly. Differentiating (5.22) with respect to q_{i2} yields

$$\eta_i'(q_{i2}) = \; \pi_{11}^{i2}(q_{i2}, Q_{-i2}) \; + \beta\pi_{11}^{i3}(R_{i2} + S_{i2} - q_{i2}, Q_{-i3}) < 0. \tag{5.27}$$

Thus, if $\eta_i(q_{i2}^*) = 0$ for some feasible q_{i2}^* and (5.20) holds, then q_{i2}^* is unique.

(*ii*) Existence (sufficiency): In the case where (5.11) is binding, (5.21) and (5.19) describe the equilibrium. Feasibility requires that

$$R_{i2} + S_{i2} > q_{i2}^* = R_{i2} + w_{i2}^* \; > \; R_{i2} + w_{i2}(S_{i2}). \tag{5.28}$$

Combining (5.28) with the monotonicity assumption A.2, we see that assumptions A.2 and A.3 are sufficient to prove the existence of an equilibrium when (5.11) binds.

When (5.11) does not bind, the corresponding feasibility condition is

$$0 < q_{i2}^* < R_{i2} + w_{i2}(S_{i2}). \tag{5.29}$$

Thus, by the monotonicity assumption A.2, we obtain (5.24) as the sufficient conditions to ensure a unique equilibrium.

(*iii*) Existence (necessity): To prove that the conditions in (5.23) are necessary to obtain an equilibrium when the constraint (5.11) binds, suppose one of the conditions is not binding. Suppose that $\psi_i(R_{i2} + S_{i2}) > \psi_i(R_{i2} + w_{i2}(S_{i2})) > 0$. Then by (5.25), no feasible value of q_{i2}^* exists that satisfies $\psi_i(q_{i2}^*) = 0$. Similarly, when (5.11) does not bind, if $\eta_i(0) > \eta_i(R_{i2} + w_{i2}(S_{i2})) > 0$, no feasible value of q_{i2}^* exists such that $\eta_i(q_{i2}^*) = 0$. This completes the proof.

The economic interpretation of the conditions (5.23) and (5.24) are straightforward. Let us consider (5.23) first. The condition $\psi_i\left(R_{i2} + S_{i2} \mid Q_{-i2}\right) < 0$ can be written as

$$\pi_1^{i2}(R_{i2} + S_{i2}, Q_{-i2}) - d_i'(S_{i2}) < \beta[\pi_1^{i3}(0, Q_{-i3}) - d_i'(0)]. \tag{5.30}$$

This means that it is profitable for producer i to carry some of its production forward to period three. Since this condition forms the boundary between the cases where production ends in period two and continues to period three, we summarize it as follows:

Proposition 5.3: Under assumptions A.1-A.4, and taking the actions of all other producers as fixed, producer i will produce in period three, rather than ending production in period two if, and only if, $\psi_i\left(R_{i2} + S_{i2} \mid Q_{-i2}\right) < 0$.

Proof: (*i*) Sufficiency: If the inequality in (5.30) holds, then producer i will hold some of its reserves for production in period three.

(*ii*) Necessity: If the inequality in (5.30) is reversed, then producer i prefers to exhaust in period two, rather than holding some reserves to period three. This completes the proof.

Note that the boundary created by $\psi_i(R_{i2} + S_{i2} \mid Q_{-i2}) = 0$ intersects the boundary created by $\psi_i(R_{i2} + w_{i2}(S_{i2}) \mid Q_{-i2}) = 0$ at $S_{i2} = 0$. This occurs because when $S_{i2} = 0$, $w_{i2}(0) = 0$.

The condition $\psi_i(R_{i2} + w_{i2}(S_{i2}) \mid Q_{-i2}) > 0$ in (5.23) can be written as

$$\pi_1^{i2}(R_{i2} + w_{i2}(S_{i2}), Q_{-i2}) \; > \; \beta\pi_1^{i3}(S_{i2} - w_{i2}(S_{i2}), Q_{-i3}). \tag{5.31}$$

The interpretation of (5.31) is that marginal profits in period two exceed those in period three in present value when second period production equals $q_{i2}^{*} = R_{i2} + w_{i2}(S_{i2})$. It is this condition that causes the producer to increase its production from unproven reserves.

When the constraint (5.11) is not binding ($\phi_i = 0$), the relevant conditions is (5.24) . The condition $\eta_i(R_{i2} + w_{i2}(S_{i2}) \mid Q_{-i2}) < 0$ is the reverse of the inequality in (5.31). If the present value of marginal profits when period two production equals $q_{i2}^{*} = R_{i2} + w_{i2}(S_{i2})$ is less than the present value of marginal profits in period three given remaining reserves of $S_{i2} - w_{i2}(S_{i2})$, the producer wishes to keep some of these reserve additions for use in period three.

Finally, the condition $\eta_i(0) > 0$ can be written as

$$\pi_1^{i2}(0, Q_{-i2}) \; > \; \beta\pi_1^{i3}(R_{i2} + S_{i2}, Q_{-i3}). \tag{5.32}$$

This condition says that the producer wishes to have some production in period two, rather than holding all production until period three.

The condition $\psi_i(R_{i2} + w_{i2}(S_{i2}) \mid Q_{-i2}) = \eta_i(R_{i2} + w_{i2}(S_{i2}) \mid Q_{-i2}) = 0$ serves to form a boundary separating the cases where (5.11) is and is not binding in R_{i2} and S_{i2} space. Totally differentiating this condition and solving for the slope of this locus yields

$$\left.\frac{\partial R_{i2}}{\partial S_{i2}}\right|_{\psi_i(R_{i2}+w_{i2}(S_{i2}))=0} = \frac{w'_{i2}\pi_{11}^{i2}-(1-w'_{i2})\beta\pi_{11}^{i3}}{-\pi_{11}^{i2}},$$

where $\pi_{11}^{it} < 0$ by assumptions A.2 and A.3. In general, this expression is ambiguous in sign. However, in the special case of linear demand, constant marginal extraction cost, and quadratic exploration costs, the slope of the $\psi_i(R_{i2} + w_{i2}(S_{i2}) \mid Q_{-i2}) = 0$ locus is zero. This locus is shown in Figure 5.2 as the boundary where $q_{i2}^* \leq R_{i2} + w_{i2}(S_{i2})$. Since $\partial\psi_i(R_{i2} + w_{i2}(S_{i2}) \mid Q_{-i2})/\partial R_{i2} = \pi_{11}^{i2} < 0$, an increase in R_{i2} causes the constraint (5.11) not to bind.

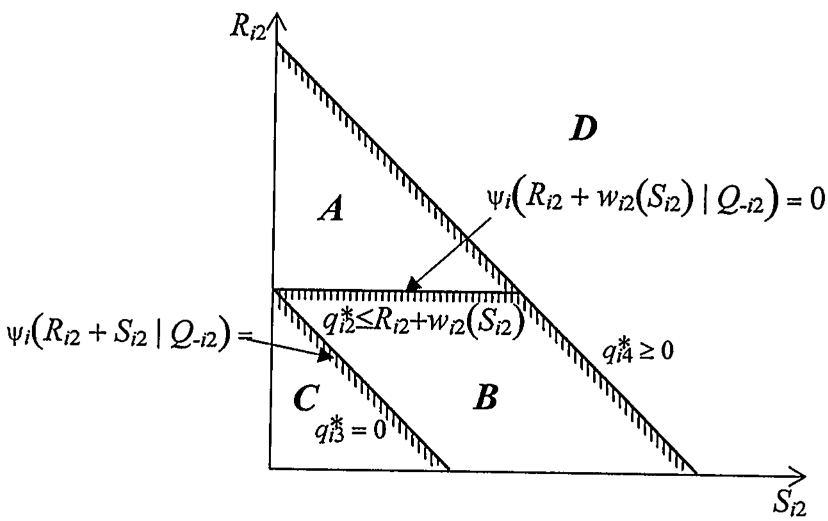
The next result derives the boundaries for where $q_{i4}^* < 0$.

Proposition 5.4: Under assumptions A.1-A.4, and taking the actions of all other producers fixed, producer i exhausts in period three for $\{R_{i2}, S_{i2}\}$ satisfying (5.23) and (5.7) if (5.11) binds or by (5.24) and (5.7), if (5.11) does not bind.

Proof: See appendix B.

Propositions 5.1-5.4 establish the conditions under producer i has a unique equilibrium response in which production and exploration are non-negative in periods 2 and possibly period three, given the stocks $\boldsymbol{R}_2$ and $\boldsymbol{S}_2$, and the equilibrium actions of other producers. The next proposition shows the conditions under which a Nash equilibrium among the set of active producers exists and is unique.

Figure 5.2: Equilibrium Exploration and Exhaustion for Producer *i*.



Notes—The areas A and B correspond to the areas where the constraint (5.11) is not binding and is binding, respectively, and where producer i produces in both periods two and three. Producers with reserves in area C rationally exhaust in period two (or earlier). A producer holding reserves in area D would prefer to exhaust in period four (or later).

Proposition 5.5: Under assumptions A.1-A.4, there exists a unique Nash equilibrium beginning in period two which is characterized by (5.23) and (5.24) holding for all m_3 producer that exhaust in period three and characterized by (5.31) for all m_2 producers that exhaust in period two.

Proof: See appendix B.

5.1.D. Properties of the Second Period Value Function

Let $V_{i2}^*(\boldsymbol{R}_2,\boldsymbol{S}_2)$ denote the maximized value of problem **P2** for producer i. To obtain strategic effects in exploration, it is necessary for the equilibrium values q_{i2}^*, q_{i3}^*, w_{i2}^*, and w_{i3}^* to depend upon the initial reserves of the other producers. That is, if $q_{it}^* = q_{it}^*(R_{i2}, S_{i2})$ and $w_{it}^* = w_{it}^*(R_{i2}, S_{i2})$ *only* (i.e., the solutions to the maximization problem **P2** depend only own reserves), then there is no strategic effect from production and exploration in period one, since q_{it}^* and w_{it}^* are not affected by the resource stocks of the other producer (Eswaran and Lewis 1986).

We begin by showing the effect on producer i's stream of future profits beginning in period two of an increase in R_{i2} and S_{i2} when that producer produces in both periods two and three. The stream of profits for a producer producing in both periods two and three can be written as

$$V_{i2}^*(\boldsymbol{R}_2,\boldsymbol{S}_2) = P(Q_2)q_{i2}^* - c_i(q_{i2}^*) - d_i(w_{i2}^*) + \beta[P(Q_3)q_{i3}^* - c_i(q_{i3}^*) - d_i(w_{i3}^*)], \qquad (5.33)$$

where $q_{i3}^* = R_{i2} + S_{i2} - q_{i2}^*$ for all producers who produce in period three. For producers for whom (5.11) does not bind, $w_{i2}^* = w_{i2}(S_{i2})$, while for producers for whom (5.11) binds, $w_{i2}^* = q_{i2}^* - R_{i2} > w_{i2}(S_{i2})$.

Using the envelope theorem, differentiating second period profits with respect to R_{i2} when (5.11) does not bind yields

$$\text{(case } A\text{)} \qquad \frac{\partial V_{i2}^*}{\partial R_{i2}} = \beta(P_3 + P_3' q_{i3}^* - c_{i3}') + (P_2' q_{i2}^* - \beta P_3' q_{i3}^*)\sum_{j\neq i} \frac{\partial q_{j2}^*}{\partial R_{i2}}, \qquad (5.34)$$

And when (5.11) does bind, differentiating second period profits with respect to R_{i2} yields

$$\text{(case } B\text{)} \qquad \frac{\partial V_{i2}^*}{\partial R_{i2}} = d'_{i2} + \beta(P_3 + P'_3 q_{i3}^* - c'_{i3} - d'_{i3}) + (P'_2 q_{i2}^* - \beta P'_3 q_{i3}^*)\sum_{j \neq i} \frac{\partial q_{j2}^*}{\partial R_{i2}} \tag{5.35}$$

Differentiating the second period profit function with respect to S_{i2} yields

$$\text{(cases } A \text{ \& } B\text{)} \quad \frac{\partial V_{i2}^*}{\partial S_{i2}} = \beta(P_3 + P'_3 q_{i3}^* - c'_{i3} - d'_{i3}) + (P'_2 q_{i2}^* - \beta P'_3 q_{i3}^*)\sum_{j \neq i} \frac{\partial q_{j2}^*}{\partial S_{i2}}, \tag{5.36}$$

for both the case where (5.11) binds and where it does not bind.

The first set of terms on the right-hand-side of (5.34)-(5.36) are the direct effects to the producer of having more of that type of stock in period two. By the definitions of the Nash equilibrium given by (5.23) and (5.24), these terms are each positive in sign in equilibrium. The term $(P'_2 q_{i2}^* - \beta P'_3 q_{i3}^*)$ is negative in sign by A.4, since this term corresponds to the derivative of producer i's profits with respect to output by producer. We shall turn to the terms involving $\partial q_{j2}^*/\partial R_{i2}$ and $\partial q_{j2}^*/\partial S_{i2}$ after we write down the conditions for a producer who exhausts in period two.

The value function for a producer who ends production in period two is simply

$$V_{i2}^*(\boldsymbol{R}_2, \boldsymbol{S}_2) = P(Q_2)(R_{i2} + S_{i2}) - c_i(R_{i2} + S_{i2}) - d_i(S_{i2}). \tag{5.37}$$

Differentiating (5.37) with respect to R_{i2} and S_{i2} yields:

$$\text{(case } C\text{)} \qquad \frac{\partial V_{i2}^*}{\partial R_{i2}} = P_2 + P'_2 q_{i2}^* - c'_{i2} + P'_2 q_{i2}^* \sum_{j \neq i} \frac{\partial q_{j2}^*}{\partial R_{i2}}, \tag{5.38}$$

(case *C*) $$\frac{\partial V_{i2}^{*}}{\partial S_{i2}} = P_2 + P_2' q_{i2}^{*} - c_{i2}' - d_{i2}' + P_2' q_{i2}^{*} \sum_{j \neq i} \frac{\partial q_{j2}^{*}}{\partial S_{i2}} \tag{5.39}$$

Now we turn to the terms involving the summations of $\partial q_{j2}^{*}/\partial R_{i2}$ and $\partial q_{j2}^{*}/\partial S_{i2}$. Following Tirole (1990, p. 326), we may use the chain rule to write these sums as

$$\sum_{j \neq i} \frac{\partial q_{j2}^{*}}{\partial R_{i2}} = \left(\frac{\partial q_{i2}^{*}}{\partial R_{i2}}\right) \sum_{j \neq i} \frac{\partial q_{j2}^{*}}{\partial q_{i2}} \quad \text{and} \quad \sum_{j \neq i} \frac{\partial q_{j2}^{*}}{\partial S_{i2}} = \left(\frac{\partial q_{i2}^{*}}{\partial S_{i2}}\right) \sum_{j \neq i} \frac{\partial q_{j2}^{*}}{\partial q_{i2}}. \tag{5.40}$$

The sign of these expressions depend on the slopes of the best-response functions of all other producers to producer *i*'s output level. Given assumptions A.2 – A.4, the goods are strategic substitutes, so that the slopes of the best-response functions are negative. However, we need these summations to be negative in net given the interactions among the set of all other producers.[38] Lemma 5.2 shows that this is so.

Lemma 5.5 (Dixit 1986)**:** Under assumptions A.1-A.4, the sum of the $\partial q_{j2}^{*}/\partial q_{i2}$ in (5.40) are negative for all producers who produce in period two and three, and zero for producers who end production in period two.

Proof: See appendix B.

Next, we show the effect of own proven and unproven reserve holdings on the second

[38] The best response functions $q_{i2}^{*} = \rho_i(Q_{-i2})$ describe how producer *i* responds to changes in the output of all other producers. To see how all other producers simultaneously respond to a change in producer *i*'s output, we need to solve the system of equations $H_{-i}\boldsymbol{dq}_{-i2} = bdq_{i2}$ to obtain $\boldsymbol{dq}_{-i2} = H_{-i}^{-1} bdq_{i2}$, where $\boldsymbol{dq}_{-i2} = \{dq_{12},\ldots,dq_{i-12},dq_{i+12},\ldots,dq_{n2}\}$ is the vector of dq_{j2} for $j \neq i$, H_{-i} is the Jacobian matrix for the first-order conditions for all producers other than producer *i* with diagonal elements a_j equal to the second order conditions on q_{j2} and off diagonal elements b_j in row *j* (where these are defined in the text below), and $b = \{b_1,\ldots,b_{i-1},b_{i+1},\ldots,b_{n2}\}$ is the vector of the cross-effects on marginal profits.

period output of each type of producer.

Proposition 5.6: Under assumptions A.1-A.4, (*i*) $\partial q_{i2}^*/\partial R_{i2} > 0$ and $\partial q_{i2}^*/\partial S_{i2} > 0$, and (*ii*) $\partial q_{i2}^*/\partial R_{i2} = \partial q_{i2}^*/\partial S_{i2}$ when $d_{it}''(w_{it}) = 0$ or constraint (5.11) does not bind, and (*iii*) $\partial q_{i2}^*/\partial R_{i2} > \partial q_{i2}^*/\partial S_{i2}$ when $d_{it}''(w_{it}) > 0$ and when constraint (5.11) binds.

Proof: (*i*) Write the total differential of a producer that produces in both periods two and three first-order condition in its own output q_{i2}^* as

$$a_i dq_{i2}^* + b_i \Sigma_{j\neq i}\, dq_{j2}^* \;=\; -e_i dR_{i2} - f_i dS_{i2}, \tag{5.41}$$

where a_i and b_i are defined as above, and where $e_i = f_i \equiv -\beta(2P_3' + P_3''q_{i3}^* - c_{i3}'') > 0$ for producers for whom (5.11) is not binding, and for a producer for whom (5.11) binds $e_i = d_{i2}''$ $- \beta(2P_3' + P_3''q_{i3}^* - c_{i3}'' - d_{i3}'') > 0$ and $f_i = -\beta(2P_3' + P_3''q_{i3}^* - c_{i3}'' - d_{i3}'') > 0$. For the case where R_{i2} changes, (5.41) implies

$$\frac{\partial q_{i2}^*}{\partial R_{i2}} = \frac{-e_i}{\left(a_i + b_i \sum_{j\neq i} \frac{\partial q_{j2}^*}{\partial q_{i2}}\right)} = \left(\frac{e_i}{-(a_i - b_i)}\right)\left(\frac{1 + \sum_{j\neq i} \frac{b_j}{a_j - b_j}}{\frac{a_i}{a_i - b_i} + \sum_{j\neq i} \frac{b_j}{a_j - b_j}}\right) \equiv e_i\Gamma_i > 0, \tag{5.42}$$

where Γ_i is $-(a_i - b_i)^{-1}$ times the second expression in brackets in the second equality. Γ_i is positive since $a_i - b_i < 0$ and both $a_i/(a_i - b_i) > 0$ and $b_i/(a_i - b_i) > 0$ for all i. Thus, $\partial q_{i2}^*/\partial R_{i2}$ $= e_i\Gamma_i > 0$ and by an equivalent process, it can be shown that $\partial q_{i2}^*/\partial S_{i2} = f_i\Gamma_i > 0$. For producers that produce in only period two, $q_{i2}^* = R_{i2} + S_{i2}$, so that $\partial q_{i2}^*/\partial R_{i2} = \partial q_{i2}^*/\partial S_{i2} = 1$.

(*ii*) When the constraint (5.11) does not bind or $d''_{i2} = 0$, $e_i = f_i$, so that $\partial q^*_{i2}/\partial R_{i2} - \partial q^*_{i2}/\partial S_{i2} = 0$.

(*iii*) When $d''_{i2} > 0$, $\partial q^*_{i2}/\partial R_{i2} - \partial q^*_{i2}/\partial S_{i2} = d''_{i2}\Gamma_i > 0$. This completes the proof.

Note that when (5.11) is not binding, that $\partial q^*_{i2}/\partial R_{i2} = \partial q^*_{i2}/\partial S_{i2}$, and when (5.11) is binding, $\partial q^*_{i2}/\partial R_{i2} - \partial q^*_{i2}/\partial S_{i2} = d''_{i2}\Gamma_i > 0$ for $d''_{i2} > 0$, where $\Gamma_i > 0$ is defined in (5.42). These results affect whether or not exploration gives the producer a strategic advantage, since an increase in first period exploration, w_{i1}, increases R_{i2} and decreases S_{i2} at the same rate. It should be clear, therefore, that when (5.11) is not binding, we shall find *no strategic effect from exploration.* This foreshadows the main result in the next section, which is that producers with large quantities of proven reserves do not have a strategic incentive to explore. Furthermore, note that the expression $\partial q^*_{i2}/\partial R_{i2} - \partial q^*_{i2}/\partial S_{i2} = d''_{i2}\Gamma_i > 0$ shows that strict convexity of the $d_i(w_{it})$ functions is necessary to obtain strategic effects from exploration.

Next, we state a sufficient condition for the 'oil'igopoly theory of production result that $\partial q^*_{i2}/\partial R_{i2} < 1$, which also extends to unproven stocks: $\partial q^*_{i2}/\partial S_{i2} < 1$. This condition works for producers who produce in both periods two and three:

Corollary to Proposition 5.6: Under assumptions A.1-A.4, for producers that produce in both periods two and three, a sufficient condition for $\partial q^*_{i2}/\partial R_{i2} < 1$ is that

$$\frac{d''_{i2} + \beta d''_{i3} - (P'_2 + q^*_{i2}P''_2 - c''_{i2})}{d''_{i2} + \beta d''_{i3} - P'_2 - \beta(P'_3 + q^*_{i3}P''_3)} < \sum_{j \neq i} \frac{P'_2 + q^*_{i2}P''_2 + \beta(P'_3 + q^*_{i3}P''_3)}{P'_2 - c''_{i2} + \beta(P'_3 - c''_{i3})} \qquad (5.43)$$

Proof: See appendix B.

While the condition (5.43) is not very intuitive, when $P''_t = c''_{it} = d''_{it} = 0$, this condition collapses to $2/(N-1) < 1 + \beta$, which holds for *all* $N \geq 3$. Thus, like the theory of 'oil'igopoly, this model also has the property that output is increasing at a decreasing rate in proven reserves.[39]

5.2. Strategic Exploration and Extraction

Now we are prepared to ask the central question of this paper. In this section we derive the strategic effects from exploration and from production, and show which types of producers will alter their behaviour relative to the open loop Nash equilibrium based on those incentives.

5.2.A. Exploration and Production in Period One.

The problem faced by producer i in period one is to choose output q_{i1} and exploration w_{i1} to maximize

$$\max_{\{q_{i1}\, w_{i1}\}} \quad V_{i1} = P(Q_1)q_{i1} - c_i(q_{i1}) - d_i(w_{i1}) + \beta V^*_{i2}(\boldsymbol{R}_2, \boldsymbol{S}_2), \quad i = 1, \ldots, n_1, \qquad \textbf{P1}$$

where the value function $V^*_{i2}(\boldsymbol{R}_2, \boldsymbol{S}_2)$ is given by (5.33) or (5.37), depending on whether or

[39] A similar condition can be obtained for the effect of an increase in unproven reserves. Indeed, when (11) is not binding, we know that the effects are identical, and when (11) is binding, we need only substitute $f_i = e_i + d''_{it}$ for e_i in the condition (B.13).

not the producer produces in period three. Producer i's choices are subject to the constraints

$$w_{i1} \leq S_{i1}, \tag{5.44}$$

$$q_{i1} \leq R_{i1} + w_{i1}, \tag{5.45}$$

which are analogous to the constraints (5.10) and (5.11), respectively.

Let γ_i and κ_i denote the value of Lagrange multiplier on the constraints (5.44) and (5.45), respectively. By the envelope theorem, the solution to **P1** for a producer that produces in periods two and three when (5.11) is not binding must satisfy (5.44), (5.45), and the following:

(case A)
$$\frac{\partial V_{i1}}{\partial q_{i1}} = P(Q_1) + P'(Q_1)q_{i1}^* - c_i'(q_{i1}^*) - \kappa_i - \beta\frac{\partial V_{i2}^*}{\partial R_{i2}}$$

$$= P(Q_1) + P'(Q_1)q_{i1}^* - c_i'(q_{i1}^*) - \kappa_i - \beta^2[P_3 + P_3'q_{i3}^* - c_{i3}']$$

$$- \beta[q_{i2}^*P_2' - \beta\, q_{i3}^*P_3']\left(\frac{\partial q_{i2}^*}{\partial R_{i2}}\right)\sum_{j\neq i}\frac{\partial q_{j2}^*}{\partial q_{i2}} \leq 0, \quad i = 1,\ldots,n_1, \tag{5.46}$$

(case A)
$$\frac{\partial V_{i1}}{\partial w_{i1}} = -d_i'(w_{i1}^*) + \kappa_i - \gamma_i + \beta\left(\frac{\partial V_{i2}^*}{\partial R_{i2}} - \frac{\partial V_{i2}^*}{\partial S_{i2}}\right)$$

$$= -d_i'(w_{i1}^*) + \kappa_i - \gamma_i + \beta d_i'(w_{i2}^*)$$

$$+ \beta[q_{i2}^*P_2' - \beta\, q_{i3}^*P_3']\left(\frac{\partial q_{i2}^*}{\partial R_{i2}} - \frac{\partial q_{i2}^*}{\partial S_{i2}}\right)\sum_{j\neq i}\frac{\partial q_{j2}^*}{\partial q_{i2}} \leq 0, \; i = 1,\ldots,n_1. \tag{5.47}$$

5.2.B. Strategic Exploration and Production

The strategic effects appear in the terms of the third lines in both (5.46) and (5.47), which have been written using (5.34)-(5.36). These are strategic effects because producer i chooses the stocks it takes into period two, knowing the effect this has upon the exploration and output choices that they will make in the next period, the $\partial q_{i2}^*/\partial R_{i2}$ and $\partial q_{i2}^*/\partial R_{i2} - \partial q_{i2}^* /\partial S_{i2}$ terms in (5.46) and (5.47), respectively, and how this affects the choices made by the other producers, through the $\Sigma_{j\neq i}\partial q_{j2}^*/\partial q_{i2}$ terms. By assumptions A.4 and Proposition 5.6, these effects are non-negative on exploration and non-positive on production, relative to the open loop (Nash) equilibrium. Absent these effects, the equilibrium is identical to the Nash equilibrium, in which only the expressions on the second lines of (5.46) and (5.47) appear in the first-order conditions.

An immediate result, which follows from Proposition 5.6, is the following:

Proposition 5.7: Under assumptions A.1-A.4, producers that hold proven reserves in sufficient quantities that they will produce in both periods two and three and for whom the constraint (5.11) is *not binding* (i.e., case A producers) have a strategic incentive to restrict output, but do not have a strategic incentive to increase exploration.

Proof: When (5.11) is not binding, (5.42) implies that $\partial q_{i2}^*/\partial R_{i2} = \partial q_{i2}^*/\partial S_{i2}$, so the strategic effect vanishes. Thus, there is no strategic effect from exploration. To see that the strategic effect decreases first period production, rewrite (5.46) as

$$P(Q_1) + P'(Q_1)q_{i1}^* - c_{i1}'(q_{i1}^*) - \kappa_i = \beta[P(Q_2) + P'(Q_2)q_{i2}^* - c_{i2}'(q_{i2}^*)]$$

$$+\beta\left[P'(Q_2)q_{i2}^{*}-\beta P'(Q_3)q_{i3}^{*}\right]\left(\sum_{j\neq i}\frac{\partial q_{j2}^{*}}{\partial q_{i2}}\right)\left(\frac{\partial q_{i2}^{*}}{\partial R_{i2}}\right). \tag{5.48}$$

The expression on the left-hand-side is the marginal profit from period one production. The expression on the right hand side is the marginal profit from second period production plus (the term on the second line) the strategic effect of holding higher reserves in period three. As the strategic effect is in net positive in sign, the producer has a greater incentive to withhold production in the first period relative to the open loop equilibrium. This completes the proof.

The result that producers for whom (5.11) is not binding do not have a strategic incentive for exploration follows from the fact that when (5.11) is not binding, Proposition 5.6 implies that $\partial q_{i2}^{*}/\partial R_{i2} = \partial q_{i2}^{*}/\partial S_{i2}$ and that first period exploration increases R_{i2} and decreases S_{i2} at the same rate. There does exist a strategic effect from production, even though $\partial q_{i2}^{*}/\partial R_{i2} = \partial q_{i2}^{*}/\partial S_{i2}$, since R_{i2} is affected by first period production, but S_{i2} is not affected by first period production.

Proposition 5.7 is counter-intuitive, because it suggests that having more market power does not necessarily give one an incentive to act strategically. As we mentioned in the introduction, the reason for this is that a producer with reserves lasting well beyond the next period gains nothing from having converted more reserves to the proven state. This occurs because when (5.11) is not binding, that producer already has a credible commitment to produce a large quantity in both periods two and three.

Next, consider the equivalent conditions for a producer for whom the constraint (5.11) *is* binding along the equilibrium path. By definition, this is a producer for who proven reserves are insufficient for period two production. The equivalent first-order conditions to (5.46) and (5.47) are

$$\text{(case } B\text{)} \qquad P(Q_1) + P'(Q_1)q_{i1}^* - c'_{i1} - \kappa_i \le \beta d'_{i3} + \beta\left[P_2 + P'_2 q_{i2}^* - c'_{i2} - d'_{i2}\right]$$

$$+ \beta\left[q_{i2}^* P'_2 - \beta\, q_{i3}^* P'_3\right]\left(\frac{\partial q_{i2}^*}{\partial R_{i2}}\right)\sum_{j \ne i} \frac{\partial q_{j2}^*}{\partial q_{i2}}, \qquad i = 1,\ldots,n_1, \qquad (5.49)$$

$$\text{(case } B\text{)} \qquad d'_{i1}(w_{i1}^*) - \kappa_i + \gamma_i \;\le\; \beta d'_{i2} + \beta\left[q_{i2}^* P'_2 - \beta\, q_{i3}^* P'_3\right]\left(\frac{\partial q_{i2}^*}{\partial R_{i2}} - \frac{\partial q_{i2}^*}{\partial S_{i2}}\right)\sum_{j \ne i} \frac{\partial q_{j2}^*}{\partial q_{i2}},$$

$$i = 1,\ldots,n_1. \qquad (5.50)$$

The next proposition summarizes the strategic effects for this type of producer:

Proposition 5.8: Under assumptions A.1-A.4, producers that hold proven and unproven reserves in sufficient quantities that they will produce in both periods two and three, but for whom the proven reserve holdings are insufficient to produce in period three, so that the constraint (5.11) is binding (case *B* producers), have both a strategic reason to restrict output and a strategic reason to increase exploration.

Proof: The strategic interaction terms appear on the second lines of (5.49) and (5.50). Both are positive in sign by Proposition 5.6. This completes the proof.

Thus, producers whose holdings of reserves are sufficient to get to period three but whose proven reserves are insufficient to last until period three are able to exert strategic pressure on those producers with large enough reserves that they still have proven reserves at the end of period three. This occurs because when (5.11) is binding for a producer, then $\partial q_{i2}^{*}/\partial R_{i2} - \partial q_{i2}^{*}/\partial S_{i2} > 0$, so that exploration gives the producer a credible commitment to produce a larger quantity in period two. Thus exploration gives these producers a strategic incentive for exploration that does not occur when (5.11) is not binding, since in that case an increase in first period exploration, w_{i1}, increases R_{i2} and decreases S_{i2} at the same rate. These producers also have a strategic incentive to restrict production for the same reason as the type A producers.

Finally, consider a producer who rationally exhausts his stocks in period two. For this type of producer, the equivalent conditions for maximizing **P1** are

$$\text{(case } C\text{)} \qquad P(Q_1) + P'(Q_1)q_{i1}^{*} - c_{i1}'(q_{i1}^{*}) - \kappa_i \leq \beta[P_2 + P_2'q_{i2}^{*} - c_{i2}']$$

$$+ \beta P'(Q_2)q_{i2}^{*}\left(\frac{\partial q_{i2}^{*}}{\partial R_{i2}}\right)\sum_{j\neq i}\frac{\partial q_{j2}^{*}}{\partial q_{i2}}, \qquad (5.51)$$

$$\text{(case } C\text{)} \qquad d_{i1}'(w_{i1}^{*}) - \kappa_i + \gamma_i \leq \beta d_{i2}' + \beta P'(Q_2)q_{i2}^{*}\left(\frac{\partial q_{i2}^{*}}{\partial R_{i2}} - \frac{\partial q_{i2}^{*}}{\partial S_{i2}}\right)\sum_{j\neq i}\frac{\partial q_{j2}^{*}}{\partial q_{i2}} \qquad (5.52)$$

The next proposition shows that these producers do not have a strategic reason to explore in period one:

Proposition 5.9: Under assumptions A.1-A.4, a producer with sufficient reserves to produce in period two, but insufficient reserves to produce in period three, gains a strategic advantage from withholding production, but does not gain a strategic advantage from increasing exploration.

Proof: Recall that $\partial q_{i2}^{*}/\partial R_{i2} = \partial q_{i2}^{*}/\partial S_{i2} = 1$ for producer that exhausts in period two, since $q_{i2}^{*} = R_{i2} + S_{i2}$. Thus, the strategic effect vanishes in the exploration equation, but remains in the production equation. This completes the proof.

The reason these producers do not gain from exploration is because second period output for these producers equals $q_{i2}^{*} = R_{i2} + S_{i2}$, so that the term in brackets in (5.52) equals zero, since an increase in exploration increases R_{i2} at the same rate as it decreases S_{i2}.

Propositions 5.7-5.9 suggest that only producers that exhaust their proven reserves in period two, but have sufficient unproven reserves to continue production in period three, have a strategic incentive to explore for oil. Producers with sufficient reserves to have proven reserves at the beginning of period three already have sufficient reserves to have a credible commitment that they will produce a large quantity in period three. Thus, they do not gain anything by exploring beyond the level that would occur in the open loop equilibrium, although they do gain from restricting production. Producers with aggregate reserves sufficient only to exhaust in period two also do not gain a strategic advantage from exploration in period one, although they too gain a strategic advantage from restricting output in period one. This suggests that the strategic incentive to explore is highly non-linear in the size of proven reserves. Those with very small and those with very large proven reserves have no strategic incentive to over explore relative to the open loop equilibrium, but those with intermediate level reserves have an incentive to over explore relative to the open loop equilibrium.

5.2.C. Characterization of Period One Exploration

Now that we have seen the strategic effects, there remains but one task. That is to characterize the equilibrium in period one. We begin our analysis of the equilibrium in period one by ignoring the strategic effects and assuming that neither of the constraints (5.44) or (5.45) binds. Then the choice of production given by (5.46), (5.49) or (5.51), and says that marginal profits from extraction in period one are equated with the discounted value of additional proven reserves in period three. Thus, this is again a simple Hotelling result which implies that discounted marginal profits are equated across periods. Equations (5.47), (5.50) and (5.52) give a similar Hotelling result that discounted marginal costs of exploration are equal across periods. Obviously, the strategic effects alter the interpretation of these results in the same way as in Proposition 5.2, as will having either (5.44) or (5.45) bind.

Next, we show that if the constraint (5.45) is binding, so that producer i's proven reserves in period two are zero, then producer i will not have positive proven reserves at the end of any subsequent period.

Proposition 5.10: Under assumptions A.1-A.4, if a producer extracts all of its proven reserves in period one, he will not subsequently hold positive quantities of proven reserves.

Proof: See appendix B.

These propositions eliminate all but three possible combinations of exploration activities for the m_3 producers that produce in all three periods.[40] We can conclude that so long as initial unproven reserves are positive, the producer explores in every subsequent period. Furthermore, for each period in which producer i takes some proven reserves into the next period, it extracts only from the lower cost proven reserves. Lastly, if proven reserves are exhausted prior to unproven reserves, then the producer will not rebuild these proven reserves in any subsequent period.

5.3. Empirical Evidence of Strategic Exploration

In this section, we present a simple test of the hypothesis that strategic incentives matter in exploration. When the proven reserve holdings of a cross-section of oil producing countries is examined (see Figure 5.3), it becomes clear that producers with smaller reserves at the beginning of the 1950s tended to have higher rates of growth of their proven reserves. As we observe no countries that have exhausted their reserves, we interpret this evidence as support for the hypothesis that producers with smaller proven reserves are likely to engage in strategic exploration, while producers holding larger proven reserves are not likely to engage in strategic exploration.

We present a test of whether countries with smaller reserves do more exploration by regressing the rate of reserves growth on initial reserves using data from the 99 countries

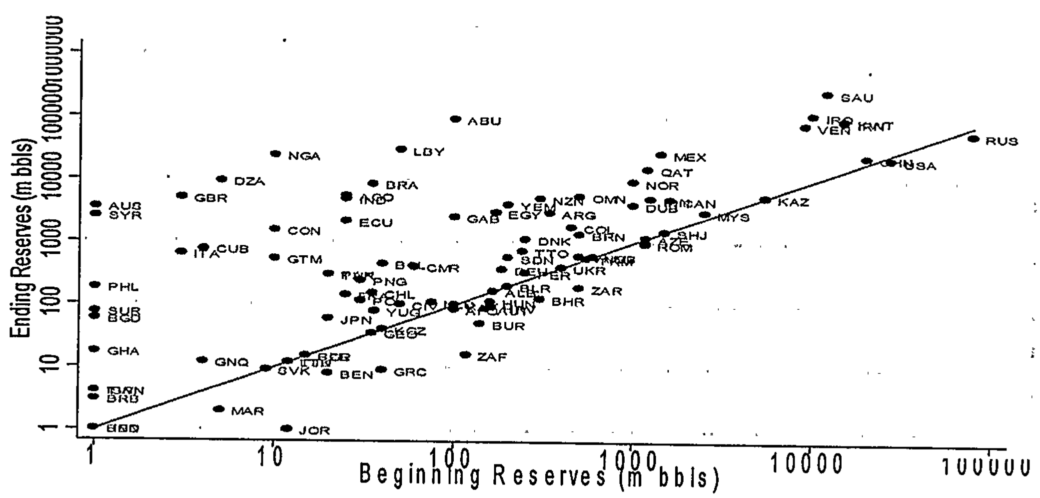
[40] There are also m_2 producers which only hold sufficient reserves to produce in period two (those who take reserves equivalent to the area C in Figure 5.2), and there are m_1 producers whose reserve holdings are insufficient to even produce in period two. These producers' choices cannot be influenced by the actions of the remaining producers, but those who take reserves into period two can affect the behavior of producers who continue to produce into period three.

holding oil reserves in the post World War II era that is depicted in Fig. 5.3. The regression result is:

$$\frac{ln(R_{T,i},R_{0,i})}{T_i} = \underset{(0.51)}{4.09} - \underset{(0.24)}{1.11}\, ln(R_{0,i}),\ \text{adjusted } R^2 = 0.18,\ N = 99.$$

(Standard errors are in parentheses.) T_i is the number of years each country is observed in the data, and $ln(R_{T,i})$ and $ln(R_{0,i})$ are the natural log of ending and beginning reserves, respectively.

Figure 5.3: Reserves Growth and Initial Reserves, 1952-2002



Notes:—Reserves data is by country and is in log scale. The number of years between initial reserves and ending reserves differs across countries. Countries above the 45° line exhibit reserve growth. Source: *Oil & Gas Journal.*

Fig. 5.3, and the regression results show that countries with smaller initial reserves tended to have higher rates of growth in their proven reserves over the period 1952-2002. While there is greater variation in the countries with smaller reserves, the percentage changes in reserves is highest for countries that initially had smaller reserves. This regression supports the hypothesis that smaller countries do indeed explore more relative to the larger countries: a one percent increase in initial reserves results in a 1.11 percent reduction in reserves growth.

These results are not a perfect test of the hypothesis, as some producing countries have multiple firms doing the production. In addition, the largest oil producers tend to be dominated by state-owned-firms. While it is possible that state owned firms may respond to economic incentives differently than privately owned firms, it would be an interesting coincidence if these firms tended to behave as we suggest forward looking strategic firms would behave. Finally, there may be other reasons – political unrest, higher levels of risk, etc. (e.g., Bohn and Deacon 2001) – that would cause producers in parts of the world with larger reserve bases to behave differently. We have not conditioned for this in our regressions. Nevertheless, our results, simple as they are, provide tantalizing evidence that the world oil market behaves as suggested by the 'oil'igopoly theory of exploration.

5.4. Conclusions

This chapter has developed a three-period model of 'oil'igopolistic exploration and production. We have solved for the dynamically consistent (subgame perfect) equilibrium in a model in which producers compete both by production in the output market and by converting unproven reserves into proven reserves.

The most interesting conclusion from this study is that smaller producers – or at least those with smaller proven reserves – are most likely to engage in strategic exploration, all else equal. As we argued above, the intuition behind this result is that producers with large proven reserves already have a credible commitment to produce large quantities in the future periods.

This result, however, appears to be unique to an exhaustible resource model. In the strategic investment model of Dixit (1980), both large and small firms have an incentive to invest strategically. However, in that model, the firm with an initial cost advantage – which translates into higher production levels – tends to make higher levels of strategic investment. This can be seen in a simple example, where the profits to two duopolists are given by

$$\pi_i = (a - q_1 - q_2 - c_i + k_i)q_i - \tfrac{1}{2}dk_i^2, \quad i = 1,2. \tag{5.53}$$

Where q_i is output, price is given by $p = a - q_1 - q_2$, c_i is initial marginal cost of production, and k_i is the reduction in marginal costs attained by investing at cost $\tfrac{1}{2}dk_i^2$. We let $a = 12$, $d = 4$, $c_1 = 4$, and $c_2 = 2$. Then it follows that the equilibrium values of k_i and $c_i - k_i$ are as in Table 5.1.

From Table 5.1, it is clear that both firms over invest in the subgame perfect equilibrium relative to the Nash equilibrium. The percentage change in costs is higher in the subgame perfect equilibrium for both firms. However, the most interesting thing to note form Table 5.1 is that Firm 2, which has an initial cost advantage of 100%, makes much higher levels of investment both in absolute terms and in percentage reduction terms. This suggests that the reason smaller producers strategically explore has more to do with the constraint (5.11) binding.

Table 5.1: Dixit's (1980) Strategic Investment in R&D with Asymmetric Firms

	k_1	k_2	$c_1 - k_1$	$c_2 - k_2$	$\%\Delta c_1$	$\%\Delta c_2$
SGPE	0.625	1.625	3.375	0.375	-.156	-.812
NE	0.485	1.15	3.51	0.848	-.122	-.576

Producers for whom proven reserves shall be exhausted in the next period, however, do gain a strategic advantage from having lowered their costs of production by sinking the cost of exploration.

Also, because these producers are large producers they gain less from a reduction in costs because a large producer faces a larger reduction in price when it expands its output (e.g., Nordhaus 1969). Thus, smaller producers are those who do the most exploration. Furthermore, a producer with large proven reserves already has sufficient low cost reserves on hand to credibly increase its output.

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Mathematical Appendix

Appendix A: The constrained subgame perfect equilibrium when $\delta \leq \hat{\delta}$.

In section 3.4.A, we derived the constrained subgame perfect equilibrium profits for the case where $\delta > \hat{\delta}$. In section 3.4.B, we showed that the constrained subgame perfect equilibrium profits are decreasing with tightening the capacity constraint in the interval (Q^D, Q^C) where $\delta < \hat{\delta}$. Next, we show what happens when $Q \leq Q^D$ where $\delta \leq \hat{\delta}$.

Results of Propositions 3.1 and 3.2 ensure that there exists a unique value of $\hat{\delta}$ such that $q_1^{SP} = q_2^{SP} = \hat{q}^{SP}$ where $\hat{q}^{SP}$ is given by (3.61) and is a unique value since (3.61) is strictly decreasing in q. Thus, a capacity constraint will affect both period quantities where $\delta = \hat{\delta}$. This implies that $Q^A = Q^C = \hat{q}^{SP}$. Comparing (3.61) with (3.87) we see that $\hat{q}^{SP} = Q^D$. Thus the limiting value for both Q^A and Q^C is Q^D. To see if Q^D is also limiting value for Q^B, given by (3.74), we find next the discount factor for which $Q^D = Q^B$ and $q_1^{CSP} = q_2^{CSP} = \hat{q}^{SP}$. Notice that Q^D is discount factor invariant and Q^B decreases with discount factor. Thus, there exists a unique value $\tilde{\delta}$ such that for $\delta < \tilde{\delta}$, $Q^B > Q^D$, and for $\delta \geq \tilde{\delta}$ then $Q^B \leq Q^D$ and $\tilde{\delta}$ solves:

$$\tilde{\delta} = \frac{c\left(R - \hat{q}^{SP}\right) - c\left(R - 2\hat{q}^{SP}\right) - \hat{q}^{SP}\left[c'\left(R - \hat{q}^{SP}\right) - c'\left(R - 2\hat{q}^{SP}\right)\right]}{c\left(R - \hat{q}^{SP}\right) - c\left(R - 2\hat{q}^{SP}\right) - \hat{q}^{SP}\left[c'\left(R - \hat{q}^{SP}\right) - 2c'\left(R - 2\hat{q}^{SP}\right)\right]}. \tag{A.1}$$

Expressing $\tilde{\delta}$ by means of $\hat{q}^{SP} = Q^D$ enables us directly to compare magnitudes of discount factor and state: $0 < \tilde{\delta} < \hat{\delta}$.

This disparity in the discount factor bears further complication into the constrained problem. We have to divide our analysis when $Q \leq Q^D$ over two intervals: $(0, \tilde{\delta}]$ and $(\tilde{\delta}, \hat{\delta}]$.

A.1. The Effect of a Capacity Constraint on Subgame Perfect Profits when $\delta < \tilde{\delta}$

First, we consider the case when $\delta < \tilde{\delta}$ and thus $Q^D < Q^B$. If $Q < Q^D$, then the capacity constraint binds in both periods. We saw in (3.79) the effect that a change in the capacity constraint has upon the subgame equilibrium profits of the buyer. It is clear from Proposition 3.5 that by assumption A.3, marginal profits are decreasing in Q in the domain $(0, Q^B)$ and that a value of unique Q^* exits. Since $Q^D < Q^B$, the existence of Q^* is questionable.

The local maximum exists if $Q^* < Q^D$. Since Q^D is discount factor invariant and Q^* decreases with discount factor[41], there exists a unique value $\bar{\delta}$ such that for $\delta < \bar{\delta}$, $Q^* > Q^D$, and for $\delta \geq \bar{\delta}$ then $Q^* \leq Q^D$ and $\bar{\delta}$ solves:

$$\bar{\delta} = \frac{c\left(R-\hat{q}^{SP}\right)-c\left(R-2\hat{q}^{SP}\right)-\hat{q}^{SP}\left[c'\left(R-\hat{q}^{SP}\right)-c'\left(R-2\hat{q}^{SP}\right)\right]}{c\left(R-\hat{q}^{SP}\right)-c\left(R-2\hat{q}^{SP}\right)-\hat{q}^{SP}\left[c'\left(R-\hat{q}^{SP}\right)-3c'\left(R-2\hat{q}^{SP}\right)\right]}. \tag{A.2}$$

Expressing $\bar{\delta}$ by means of $\hat{q}^{SP} = Q^D$ enables us directly to compare magnitudes of discount

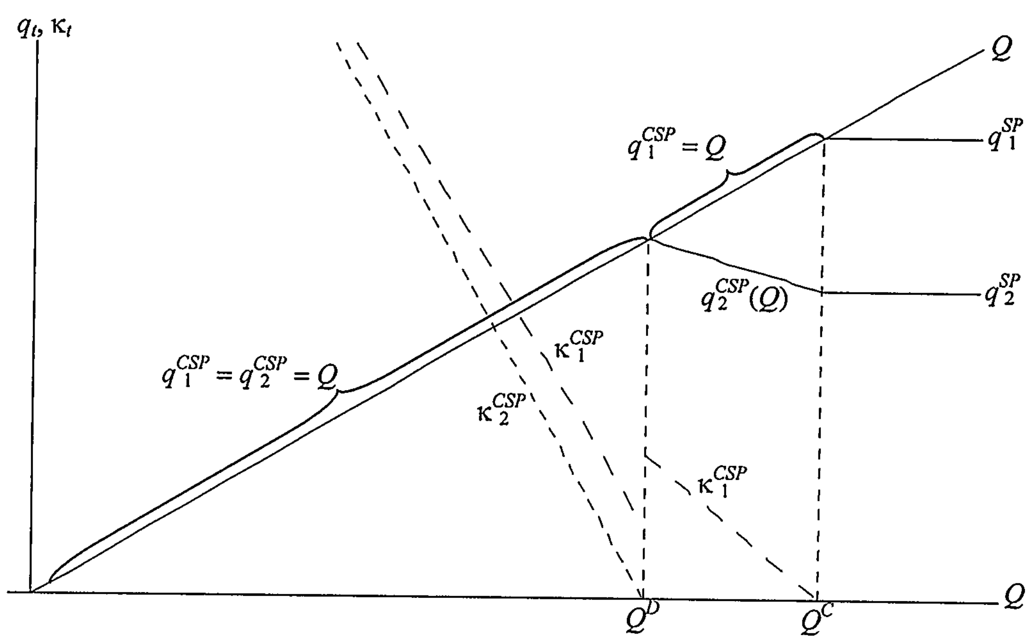
[41] $\partial Q^*/\partial\delta = -\left[u'(Q^*)-m(2Q^*)\right]-\left[m(Q^*)-m(2Q^*)\right]/\left[(1+\delta)u''(Q^*)-(1-\delta)m'(Q^*)-4\delta m'(Q^*)\right]<0.$

factor and state: $0 < \bar{\delta} < \tilde{\delta} < \hat{\delta}$. This allows us to state the following proposition:

Proposition A.1: When assumptions A.1-A.4 hold and $\bar{\delta} < \delta < \tilde{\delta}$, then in the interval $(0,Q^D]$, there exists a local maxima, Q^*, for the buyer's profits.

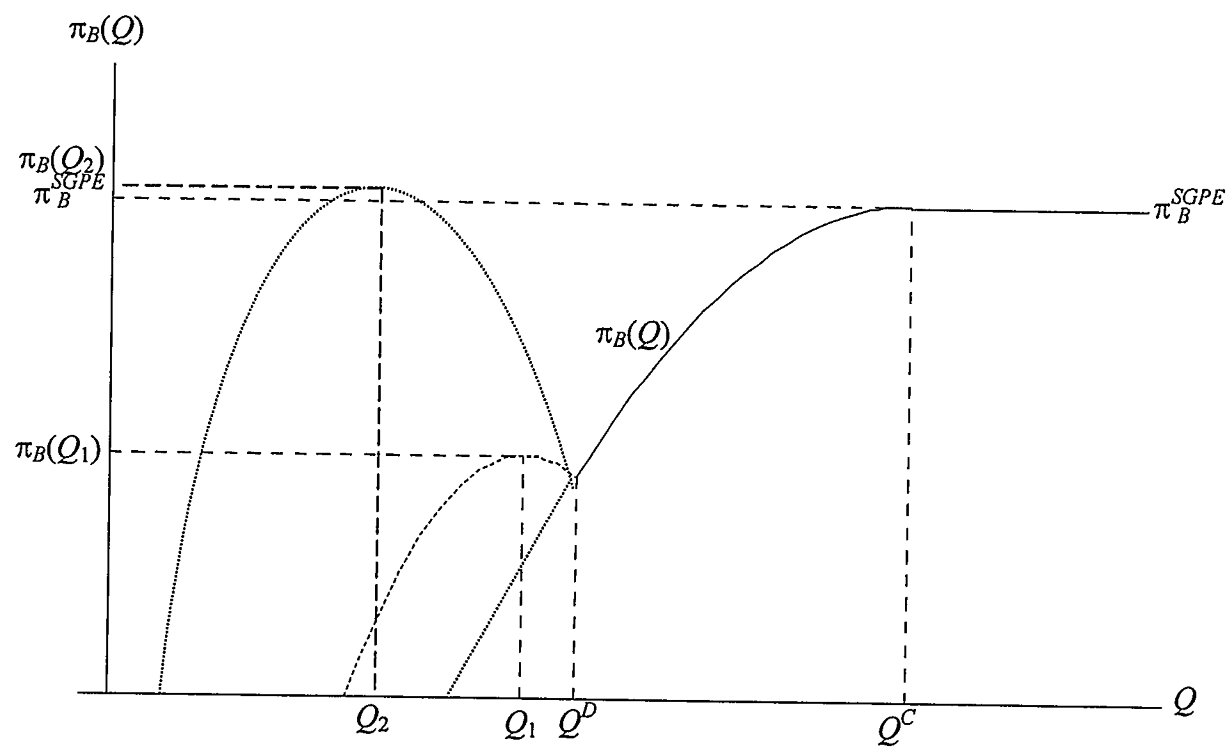
Proof: The rate at which the buyer's profits change as Q increases is given by (3.79). As $Q \to 0$, $\partial \pi^B \left(Q | Q < Q^D\right)/\partial Q > 0$ and $\partial^2 \pi^B (Q \mid Q < Q^D)/\partial Q^2 < 0$. Then since $\bar{\delta} < \delta$, $Q^* < Q^D$. Thus Q^* exists in the domain $(0,Q^D]$ and is a local maximum.

Figure A.1: Equilibrium Production with a Capacity Constraint, $\delta < \tilde{\delta} < \hat{\delta}$



Two examples of the local maxima when $\delta < \hat{\delta}$ and one example when the local maximum does not exist within interval $(0, Q^D]$ are illustrated in Figure A.2. Figure A.1 illustrate that there are no jumps in production in Q^D and Q^C and thus profits are continuous. In general it is not possible to say whether the local maxima profits are greater or less than the unconstrained profits when $\bar{\delta} < \delta$.[42] What Proposition A.1 shows is that there might exist local maxima in the domain $(0,Q^D]$. It does not necessarily imply that the buyer profits from restricting capacity, although this is possible.

Figure A.2: Equilibrium Profits with a Capacity Constraint, $\delta < \tilde{\delta} < \hat{\delta}$



A.2. The Effect of a Capacity Constraint on Subgame Perfect Profits when $\tilde{\delta} < \delta < \hat{\delta}$

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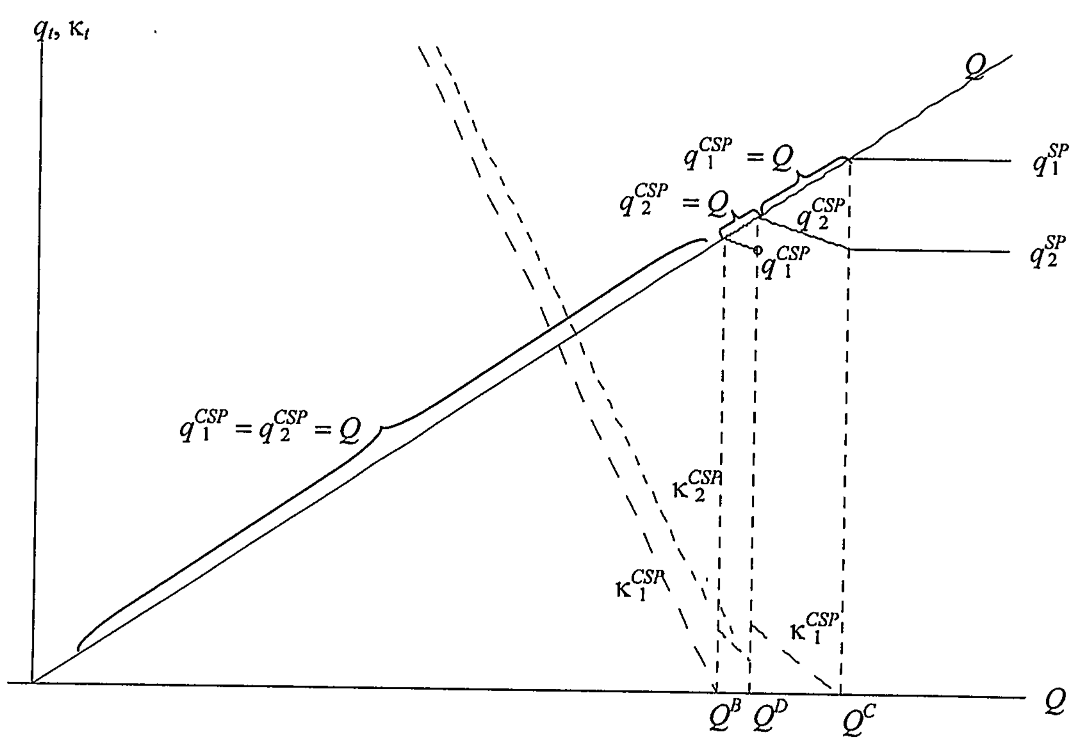
[42] As stated in footnote 15, in the linear example $\hat{\delta} = 2/3$, $\tilde{\delta} = ½$, and $\bar{\delta} = 1/3$. Constrained profits are improved by restricting capacity for $\delta > ½$. Thus, in Figure 5, Q_2 and $\pi_B(Q_2)$ are not applicable.

We now consider the case when $\tilde{\delta} < \delta < \hat{\delta}$ and $Q^B < Q^D$. Thus to find constrained subgame perfect equilibrium profits over the domain $(0, Q^D)$, we spread analysis over two intervals: $(0, Q^B]$ and $(Q^B, Q^D]$.

At Q^D, $q_1^{CSP} = q_2^{CSP} = Q$ and strategic effect from the subgame perfect equilibrium vanishes. Since $Q^B < Q^D$, and Q^B refers to the capacity size which clears for vanished strategic term, then it must be true that at Q^D there is the effect of missing strategic term. Thus q_1^{CSP} discontinuously jumps that translates in the positive jump in the profit at Q^D as shown in Figs A.3 and A.4.

Figure A.3: Equilibrium Production with a Capacity Constraint, $\tilde{\delta} < \delta < \hat{\delta}$

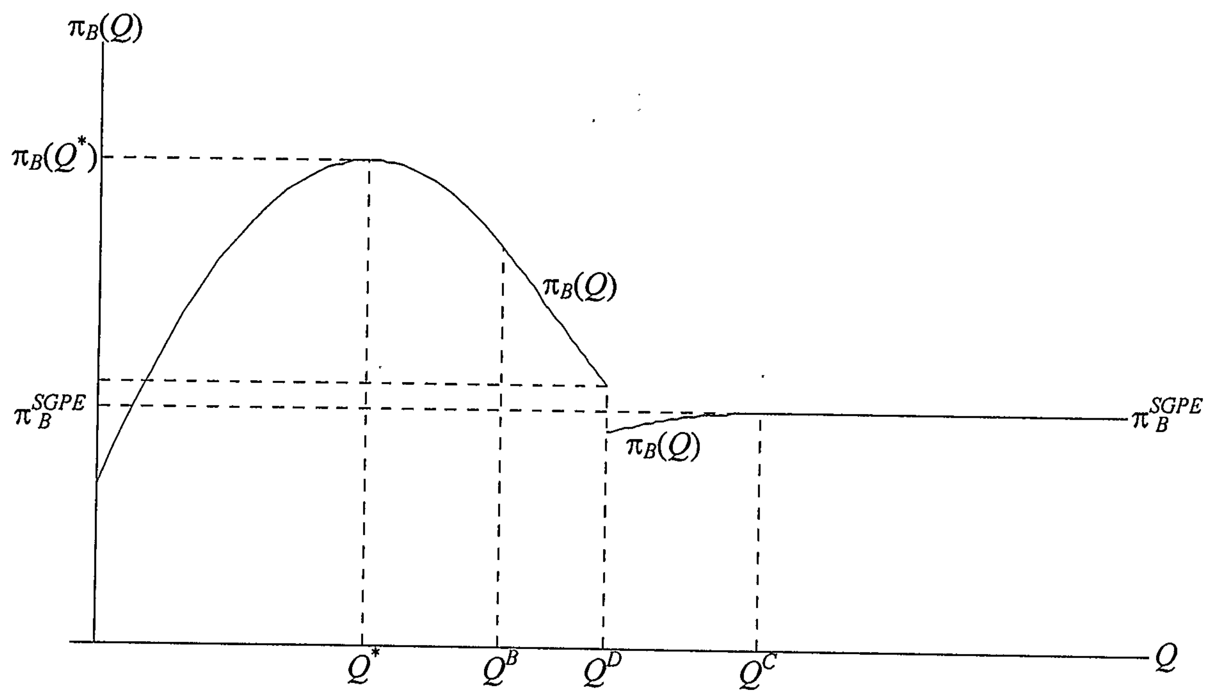


If $Q^B < Q^D$, then from section 3.4.A. profits are described by (3.71), $q_1^{CSP} < q_2^{CSP} = Q$, and the rate of change is determined by (3.80). Proposition 3.4 states that profits increase with shrinking size of capacity.

If $Q < Q^B$ then the capacity constraint binds in both periods and we apply the outcomes of Proposition 3.5. Thus there exists the local maximum Q^* in the domain $(0, Q^B]$.

Again, in general it is not possible to say whether the local maxima profits are greater or less than the unconstrained profits[43]. We can say with certainty that for critical value of $\delta = \hat{\delta}$, constrained profits are grater than the unconstrained.

Figure A.4: Equilibrium Profits with a Capacity Constraint, $\tilde{\delta} < \delta < \hat{\delta}$



[43] See footnote 17. Thus, where $\tilde{\delta} < \delta < \hat{\delta}$, constrained equilibrium profits are always improved.

Appendix B: Proofs of Propositions and Theorems in the Chapter 5

Proof of Proposition 5.4

Producer i rationally ends production in period three only if

$$\tau_i(R_{i2}, S_{i2}) \;=\; \pi_1^{i3}(q_{i3}^*, Q_{-i2}) - d'_{i3}(w_{i3}^*) - \beta\, \pi_1^{i4}(0, 0) \;\geq\; 0. \tag{B.1}$$

(*i*) Suppose that $q_{i2}^* < R_{i2} + w_{i2}^*$. Then $q_{i3}^* = R_{i2} + S_{i2} - q_{i2}^*$, and $w_{i3}^* = S_{i2} - w_{i2}^*$. Thus, let

$$\tau_{i1}(R_{i2}, S_{i2}) \;\equiv\; \pi_1^{i3}(R_{i2} + S_{i2} - q_{i2}^*, Q_{-i2}) - d'_{i3}(S_{i2} - w_{i2}^*) - \beta\pi_1^{i4}(0, 0). \tag{B.2}$$

When $S_{i2} = 0$, the value of $R_{i2} = \hat{R}$ such that $\tau_{i1}(\hat{R}, 0) = 0$ must satisfy $\pi_1^{i3}(\hat{R} - q_{i3}^*, Q_{-i3}) = \beta\pi_1^{i4}$ (0, 0). It can be shown (*cf.* (5.42)) that $0 < \partial q_{i2}^*/\partial R_{i2} < 1$. Thus, $\hat{R} - q_{i2}^*$ lies between zero and $\hat{R}$. Let $\bar{R}$ solve $\pi_1^{i2}(\bar{R}, Q_{-i2}) \equiv \beta\pi_1^{i3}(0, Q_{-i3})$, which is the boundary given in Proposition 5.3 for ending production in period two. Since $\hat{R} - q_{i2}^*$ is strictly positive, it follows that $\hat{R} > \bar{R}$. Thus, in the region where $S_{i2} = 0$, there exists a set of values of R_{i2} such that producer i wishes to exhaust in period three and follow the strategy outlined in Proposition 5.2. It can also be shown that along the locus of points where $\tau_{i1}(R_{i2}, S_{i2}) = 0$, that

$$\left.\frac{dR_{i2}}{dS_{i2}}\right|_{\tau_{i1}(R_{i2}, S_{i2}) = 0} = -\frac{(1 - \partial q_{i2}^*/\partial S_{i2})\pi_1^{i3} - (1 - \partial w_{i2}^*/\partial S_{i2})d''_{i3}}{(1 - \partial q_{i2}^*/\partial R_{i2})\pi_1^{i3}} < 0, \tag{B.3}$$

Since $0 < \partial w_{i2}^*/\partial S_{i2} < 1$, and $0 < \partial q_{i2}^*/\partial S_{i2} = \partial q_{i2}^*/\partial R_{i2} < 1$.

(*ii*) Next, consider the case where $q_{i2}^* = R_{i2} + w_{i2}^*$. Then $w_{i3}^* = q_{i3}^* = R_{i2} + S_{i2} - q_{i2}^*$. In this case, let

$$\tau_{i2}(R_{i2}, S_{i2}) \;\equiv\; \pi_1^{i3}(R_{i2} + S_{i2} - q_{i2}^*, Q_{-i3}) - d_{i3}'(R_{i2} + S_{i2} - q_{i2}^*) - \beta\pi_1^{i4}(0, 0), \qquad \text{(B.4)}$$

where q_{i2}^* solves $\psi_i(q_{i2}^*) = 0$. In this case, the $\tau_{i2}(R_{i2}, S_{i2})$ loci is again downward sloping:

$$\left.\frac{dR_{i2}}{dS_{i2}}\right|_{\tau_{i2}(R_{i2},S_{i2}) = 0} = -\frac{1 - \partial q_{i2}^*/\partial S_{i2}}{1 - \partial q_{i2}^*/\partial R_{i2}} < 0, \qquad \text{(B.5)}$$

since $0 < \partial q_{i2}^*/\partial S_{i2} < \partial q_{i2}^*/\partial R_{i2} < 1$.

This implies that there exist values $\{R_{i2}, S_{i2}\}$ such that producer i wishes to produce in period three but not in period four. When $R_{i2} = 0$, the corresponding value of $S_{i2} = \hat{S}$ such that $\tau_{i2}(0, \hat{S}) = 0$ must satisfy

$$\pi_1^{i3}(\hat{S} - q_{i2}^*, Q_{-i3}) - d_{i3}'(\hat{S} - q_{i2}^*) \;=\; \beta\pi_1^{i4}(0, 0). \qquad \text{(B.6)}$$

Let $\bar{S}$ solve $\pi_1^{i2}(\bar{S}, Q_{-i2}) \equiv \beta\pi_1^{i3}(0,Q_{-i3})$, which is the boundary given in Proposition 5.3 for ending production in period two, Since $0 < \partial q_{i2}^*/\partial S_{i2} < 1$, $\hat{S} - q_{i2}^*$ is strictly positive. This implies that in the region where $R_{i2} = 0$, that $\hat{S} > \bar{S}$, which completes the proof.

Proof of Proposition 5.5

(i) Existence (Vives 1999, theorem 2.7). To prove existence, it is necessary to prove that the best-reply functions are strongly decreasing in the output of the other producers. Assumptions A.2, A.3 and A.4 ensure that the slope of the best-reply functions $\rho_{i2}(Q_{-i2})$ are strongly decreasing:

$$\rho_{i2}'(Q_{-i2}) \;=\; -\left(\frac{P_2' + q_{i2}P_2'' + \beta(P_3' + q_{i3}P_3'')}{P_2' + q_{i2}P_2'' + \beta(P_3' + q_{i3}P_3'') - (c_{i2}'' - P_2') - \beta(c_{i3}'' - P_3')}\right). \qquad \text{(B.7)}$$

Both the numerator and the denominator of the term in brackets are negative, so the whole expression is negative. Therefore, under assumptions A.2 and A.3, the best-response functions are strictly decreasing. Given this, Vives (1999, theorem 2.7) implies that an equilibrium exists.

(*ii*) Uniqueness. To prove uniqueness, it is necessary to also show that the best-response map $\rho(\cdot) \equiv \{\rho_{12}(Q_{-1}),\ldots,\rho_{n_2 2}(Q_{-n_2})\}$ is a contraction. Vives (1999, theorem 2.8) proves that if the slopes of the best-reply functions are strongly decreasing in the output of the other producers and greater than -1 in value, then a unique equilibrium exists. Note that assumptions A.2 and A.3 imply that

$$0 > P_2' + q_{i2}P_2'' + \beta(P_3' + q_{i3}P_3'')$$

$$> P_2' + q_{i2}P_2'' + \beta(P_3' + q_{i3}P_3'') - (c_{i2}'' - P_2') - \beta(c_{i3}'' - P_3') \qquad \text{(B.8)}$$

Dividing through by -1 times the right-hand-side reveals that $\rho_{i2}'(Q_{-i2}) > -1$. Thus, the condition on the best-response functions is met. This completes the proof.

Proof of Lemma 5.5

This proof follows Dixit (1986). Write the total differential of the j^{th} producer's first order condition on the choice of q_{j2}^* as

$$a_j dq_{j2}^* + b_j \Sigma_{k\neq i,j}\, dq_{k2}^* \;=\; -b_j dq_{i2}, \qquad j \neq i, \qquad \text{(B.9)}$$

where, by A.2 and A.3, $a_j \equiv 2P_2' + P_2''q_{j2}^* - c_{j2}'' + \beta(2P_3' + P_3''q_{j3}^* - c_{j3}'') < 0$ for a producer for whom (5.11) is not binding and $a_j \equiv 2P_2' + P_2''q_{j2}^* - c_{j2}'' - d_{j2}'' + \beta(2P_3' + P_3''q_{j3}^* - c_{j3}'' - d_{j3}'') < 0$ for a producer for whom (5.11) is binding, and $b_j = P_2' + P_2''q_{j2}^* + \beta(P_3' + P_3''q_{j3}^*) < 0$ for all producers that continue to produce in period three. We can rewrite (B.9) as

$$dq_{j2}^* + \left(\frac{b_j}{a_j - b_j}\right) dQ_{-i2} = -\left(\frac{b_j}{a_j - b_j}\right) dq_{i2}. \tag{B.10}$$

Summing over all $j \neq i$ and solving for how the aggregate output by other producers changes as q_{i2} increases yields

$$\frac{dQ_{-i2}}{dq_{i2}} = -\left(1 + \sum_{j \neq i} \frac{b_j}{a_j - b_j}\right)^{-1} \sum_{j \neq i} \frac{b_j}{a_j - b_j}. \tag{B.11}$$

Thus

$$\frac{\partial q_{j2}^*}{\partial q_{i2}} = -\left(\frac{b_j}{a_j - b_j}\right)\left(1 + \sum_{k \neq i} \frac{b_k}{a_k - b_k}\right)^{-1} < 0, \tag{B.12}$$

since $b_j/(a_j - b_j) > 0$ for all j. This completes the proof for those producers that produce into period three. For producers that end production in period two, $q_{j2}^* = R_{j2} + S_{j2}$. Thus, these producers do not respond at all to changes in q_{i2}. This completes the proof.

Proof of Corollary to Proposition 5.6

The necessary condition for (5.42) to be less than one can be written as

$$\frac{e_i - a_i}{e_i - a_i + b_i} < \sum_{j \neq i} \frac{b_j}{a_j - b_j}. \tag{B.13}$$

This expression can be rearranged to yield (5.43). This completes the proof.

Proof of Proposition 5.10

By assumption, $R_{i2} = 0$. Now, suppose that the conclusion does not follow. Then it must be that $q_{i2}^* < w_{i2}^*$, if the producer continues to produce to period three. (If the producer does not continue to produce in period three, then all reserves are exhausted in period two, which proves the proposition.) Thus $\phi_i = 0$, since (5.11) is not binding. Since the feasibility constraint (5.11) must bind, it requires that $q_{i3}^* > w_{i3}^*$. However, $\phi_i = 0$ implies that w_{i2}^* solves (5.20), so that $w_{i3}^* > w_{i2}^*$. Therefore $q_{i2}^* < w_{i2}^* < w_{i3}^* < q_{i3}^*$. However $\phi_i = 0$ also implies that q_{i2}^* solves (5.22), so that $q_{i2}^* > q_{i3}^*$. This is a contradiction, which completes the proof.