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PRODUCTION SCHEDULING AND INVENTORY CONTROL FOR A BREWERY

by

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The undersigned certify that they have read, and recommend to the Faculty of Graduate Studies for acceptance, a thesis entitled "Production Scheduling and Inventory Control for a Brewery" submitted by Eric Richard Mortell in partial fulfillment of the requirements for the degree of Master of Arts.

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ABSTRACT

The objective of this study is to schedule production for a brewery over a twelve month period such that production and inventory costs are minimized. This is a linear programming problem with integer constraints and, because of the problem size, a computer package called "Ophelie II" was employed.

In setting up the model several difficulties were encountered in trying to keep the problem size and thereby computing costs to a minimum and yet still retain assumptions that were realistic in terms of the actual operation. For the most part, these difficulties were overcome but in retrospect there are several improvements that could have been made.

Solutions were obtained for both the continuous problem and the integer problem. These solutions were not truly optimal but they were sufficiently close to enable comparisons to be made with the actual operating results. At the outset of the study, it was anticipated that substantial savings would result. However this was not the case.

ACKNOWLEDGEMENTS

I would like to sincerely thank Dr. Van de Panne, my supervisor, for the assistance and guidance he provided and for his patience, as this thesis took far longer to write than it should have. Also, I am indebted to Mr. K. Stuart of Computer Research Associates Ltd. for his assistance in setting up the model and arranging for computer time and to Molson's Western Breweries Limited and Molson's Brewery (Ontario) Limited who provided both technical and financial aid. Finally, I would like to thank my wife Shannon, as without her understanding this thesis might never have been completed.

TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

Chapter 1

INTRODUCTION

The brewing of beer is an age old art that has developed over the centuries. In Chapter 2, the history of beer is discussed, followed by a fairly detailed description of the raw materials used in the production of beer. Finally, the actual brewing process is set out in all its stages from the inputing of the raw materials to when the finished product is finally packaged about a month later.

Chapter 3 defines the problem at hand. At the outset of this study, the objective was to minimize all production costs for a 12-month period under the constraint that demand must always be met. However, it was soon recognized that this would be an almost impossible task so that steps had to be taken to reduce the problem size.

For the particular brewery under consideration, it was discovered that the bottleshop and the warehouse were the bottlenecks of the production process. This, together with the facts that raw material costs are irrelevant and the major production costs are incurred in the bottleshop permitted the assumption that the only relevant part of the production process is the bottleshop. The second step taken to reduce the problem size was to assume that the three products that were marketed by the brewery were available in only one package size as opposed to the actual number of 3. Finally, the daily demand and productive capacities were combined to form weekly demands and capacities resulting in 52 time periods instead of 312 (6 days x 52 weeks).

The problem as briefly described above is a linear programming problem with integer constraints. There are four approaches to solving this type of problem which can be defined as implicit enumeration, the cutting plane method, Benders' algorithm and the branch and bound method.

1.1. Implicit Enumeration

This method was stimulated to a great extent by E. Balas.[1] With this method computing times tend to increase exponentially with the number of integer variables and therefore it has not been developed to any great extent. However, there has been some success in using this method to solve smaller problems of a special structure.[2]

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[1] E. Balas, An Additive Algorithm for Solving Linear Programs With Zero-One Variables, Operations Research, Vol. 15, No. 5 (September-October, 1967), pp. 915-957.

[2] For a bibliography of problems using this method see A.M. Geoffrion and R.E. Marsten, Integer Programming Algorithms: A Framework and State-of-the-Art Survey, Management Science, Vol. 18, No. 9, (May 1972), p. 472.

1.2. The Cutting Plane Method

This approach was developed by Gomory and was the first method developed for solving integer programming problems.[3] Although this method works well in practice its success has been almost exclusively with very small problems. However, an exception to this statement is that network, travelling salesman and crew scheduling problems of a large size have been successfully solved by a Glen Martin of Control Data Corporation,[4] although much of his work is unpublished.

1.3. Benders' Algorithm

In the method developed by Benders',[5] the problem is decomposed into a continuous problem and an all-integer problem. This method works better than the cutting plane method but is still not suitable for large problems. The most successful computational experience with this method has been with

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[3] For detailed information on this method see M.L. Balinski and K. Spielberg, Methods for Integer Programming: Algebraic, Combinatorial and Enumerative, in J.S. Afronsky (ed.), Progress in Operations Research, Vol. III, Wiley, New York, 1969.

[4] For a bibliography of computational experience using this method see A.M. Geoffrion and R.E. Marsten, op. cit., p. 484.

[5] J.F. Benders, Partitioning Procedures for Solving Mixed Programming Problems, Numerische Mathematik, Vol. 4 (1962), pp. 238-252.

refinery design models.[6] As well, investment planning problems have been solved using this method.[7]

1.4. Branch and Bound Method

The branch and bound method was developed by Land and Doig[8] and, according to present knowledge, appears to be the most effective general approach to mixed integer problems. Unlike the three methods previously discussed, there are commercially available computer packages to solve integer programming problems using the branch and bound method. Specifically, these packages are "Ophelie II" through Control Data Corporation, "MPSX" through I.B.M. and "Umpire," available on the Univac 1108.[9]

Being that it is an extremely difficult and time consuming procedure to write a program to solve an integer programming problem, it was decided to employ one of the commercially available packages. At that time, The University

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[6] J.P. Childress, Five Petrochemical Industry Applications of Mixed Integer Programming, Bonner & Moore Associates, Inc., Houston, March 1969.

[7] For a bibliography of these computational experiences see A.M. Geoffrion and R.E. Marsten, op. cit., p. 482.

[8] A.H. Land and A.G. Doig, An Automatic Method of Solving Discrete Programming Problems, Econometrica, Vol. 28 (1960), pp. 497-520.

[9] For a description of the algorithms used in each of these packages and an extensive bibliography of computational experience, see A.M. Geoffrion and R.E. Marsten, op. cit., pp. 472-480.

of Calgary computers supported the I.B.M. "Mathematical Programming System". However, it only supported the linear programming part of the system and because the problem had integer constraints the M.P.S. system and hence the University computers could not be used.

At this point in time consideration was given to abandoning the project but it was discovered through a local computer research firm that Control Data Canada had available on the C.D.C. 6600 the "Ophelie II" package. As Control Data had just installed a computer at The University of Calgary and as this package was not supported by the University arrangements were made to utilize their facilities.

In arriving at a solution to a problem with integer constraints, the Ophelie II system first generates the solution to the general linear programming problem and then generates the integer solution. Chapter 4 analyses the continuous solution. However, it is only analyzed for a 4-week period as, in an attempt to keep computing costs to a minimum, the problem size was again reduced by running the problem thirteen times in groups of four weeks. Unfortunately, what was achieved were optimal solutions for 13 groups of 4 weeks rather than an optimal solution for the 52 weeks together. Because commercial computer facilities were being used as opposed to the University computers, this error could not be corrected.

The integer solution is discussed in Chapter 5. In this case, it was decided to examine the solutions for the entire group of 13 four week periods as it was felt that while the solution was not truly optimal, it was fairly close.

Finally, Chapter 6 comments on the model and on the conclusions resulting from the study.

Chapter 2

THE PRODUCTION PROCESS

2.1. History of Beer

Beer is the oldest beverage made by man and, according to archaeologists, was first produced at about the same time as bread. Its popularity continued to grow from these early times into the ancient Greek and Roman cultures, but it was in the Christian era that beer really came into its own. This was largely through the influence of the monasteries which brewed and improved the beer. In fact, the monks really pioneered the hotel industry by providing hospitality to pilgrims and other travelers.[1]

The introduction of beer to Canada was by the early French explorers. The first commercial brewery in Canada was founded by the Intendant Jean Talon. However, the industry as we know it today is usually considered to have its beginnings with the establishment of a brewery on the banks of the St. Lawrence by John Molson in 1786. Today, Molson Breweries Limited are still operating a brewery at this location.

Since the early beginning, production techniques have changed substantially. But nevertheless, the product today is still produced through natural fermentation as it was in the beginning.

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[1] Brewers Association of Canada, Brewing in Canada (Montreal, Quebec: Ronalds Federated, 1965), pp. 1-2.

2.2. Raw Materials

2.2.1. Malt

Before proceeding into a discussion of the various stages of the production process, it would be advantageous to first of all examine the basic raw materials which are used in the production of beer. Specifically, they are malt, cereal adjuncts, water, hops and yeast.

The basic ingredient of beer is malt, which is barley that has been allowed to germinate to a limited extent and then kiln-dried to halt further growth. There are three distinctive types of barley that are suitable for malting:[2]

1. The two-rowed types which are mainly European.
2. The six-rowed Manchurian types which are typical of American brewing barleys.
3. The six-rowed barleys of the Mediterranean, California, Chile and Australia.

In Canada, the barleys used for brewing purposes are of the six-rowed Manchurian type[3] and to a limited extent the two-rowed European variaties of Hannchen and Betzes.

After the farmer has harvested his barley crop, he will send a sample to one of the malting plants for analysis. If it meets the high standards required for malting purposes, the malting company will pay a premium price for it.

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[2]W. J. Bradley, lecture notes from course in Brewing, University of Saskatchewan.

[3]The names of the types of barley grown in Canada are Paragon, Conquest, Olli, Parkland, Montcalm and O.A.C. 21.

Between the time that the barley is harvested and the commencement of malting, a rest period of from two to three months is necessary in order to ensure even growth. For this purpose the barley is dried at approximately $110^{o}F$ for twelve hours to reduce the moisture content to between 10% and 12%, which will minimize the danger of heating and mold development.

Following this, the barley is now ready for the malting operation, the purpose of which is to bring about sufficient growth of the barley embryo to enable it to secrete the enzyme activity that later on will be needed to degrade the starch content of the mash.[4]

Basically, the malting process is designed to supply to the barley those conditions of temperature and moisture that would normally be supplied by nature at the start of the growing season. Then when the enzyme activity is at the proper stage, heat is applied to remove the moisture and therefore stop growth. The malting process can be broken into three phases:[5]

1. Steeping
2. Germination
3. Drying

2.2.1.1. Steeping

In order to promote germination, adequate moisture is required. Therefore the barley is put through a steeping operation

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[4]Ibid

[5]Ibid

which takes approximately 40 - 60 hours and raises the moisture content of the kernels from approximately 12% to 42%. Also, the size of the kernel will increase from 20% - 45%. The temperature during this process should be from 60^{o} - 70^{o}F.

2.2.1.2. Germination

After the steeping process is completed, germination can begin. This process takes from 5 - 6 days at a temperature between 55^{o} and 70^{o}F. During this period, two things happen:[6]

a. Enzyme systems are developed which are utilized for the conversion of starches to sugar.

b. The barley is modified so that the starch is readily available for enzyme utilization.

2.2.1.3. Drying

When the malt has been modified to the desired degree, the barley moves into the final stage known as drying or kilning. There are two objectives of this process:

a. to arrest germination and modification.

b. to give the malt the characteristic flavour for the type of beer which is to be brewed from it.

To arrest the germination process, the moisture content has to be reduced to 5% or less which is simply a matter of drying the grain as economically as possible. However, obtaining the desired flavour is a far more complex problem as different temperatures and time periods produce different flavours. These flavours come from coloured aromatic bodies called melanoidins which are

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[6] Ibid

formed by the combination of simple proteins with simple sugars under the influence of high temperatures and moisture.

The kilning process takes approximately 48 hours at temperatures ranging from 120°F to 180°F. At the conclusion, the malt is usually stored from 1 - 4 weeks prior to shipment and use in the brewery so as to let the moisture content in the bin balance.

Most of the malt used in the brewery is a blend of several different varieties. This blending is done by the malster either prior to storage or as the shipment is being loaded.

2.2.2. Cereal Adjunct

An adjunct is a non-malted material which has a high starch or sugar content and which may be used in the production of alcohol. The reason for their use is that during the expensive malting process there is a loss of extract yielding materials amounting to 7% to 12% of the dry weight of the original grain. Therefore the adjuncts are used to replace a portion of the malt required in a brew.[7]

The most widely used adjuncts in North America are corn and corn products i.e. corn grits, refined grits and corn flakes. Corn has a very high starch content and the large starch granules are easily liquidified and converted to sugar. Also, corn has a low protein content which is a definite advantage as protein content can result in instability and a lack of brilliance in the final product.

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[7] Ibid.

2.2.3. Water

The water that is used for brewing must be:[9]

1. Free from bacteria,
2. Low in carbonate content,
3. Have moderate amounts of calcium and magnesium salts,
4. Low in nitrogen content,
5. Have a slightly acid pH[10]
6. Be free of all off-odors and flavours.

Many of the breweries use city water which often does not meet the requirements for brewing. As a result this water has to be treated prior to its use by part or all of the following steps:[11]

1. De-chlorination
2. Filtering through a sand and gravel filter
3. Filtering through activated carbon filters
4. Demineralization

If all of the above procedures were carried out the water would be very much like distilled water due to the removal of the salts in the demineralizer. Therefore, the water has to be retreated with salts to obtain the desired brewing level. This is usually done in the brewhouse with the proper amount being added for a single brew.

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[9] Ibid.

[10] pH is a term used to determine the alkalinity or acidity in a product. It is broken into 14 points, 7 is neutral, above 7 is alkali, below 7 is acid.

[11] Ibid.

2.2.4. Hops

Hops are the female flowers of a hop vine and are used to impart the characteristic flavour to beer and give it its delicate aroma. Immediately after they are picked, they are dried, reducing the moisture content from approximately 80% to around 9% - 12%. Normally, the air used for drying the hops is heated to 145^{o} - $150^{o}F$, drying the hops in about 10 hours.[12]

After the drying process, the hops are baled in jute sacking and then put into storage at temperatures of 38^{o} - $40^{o}F$ and a humidity of 50%.

2.2.5. Yeast

Yeast is a single celled organism which is capable of carrying out all of the functions of life i.e. digesting absorbed foods, using them for reproduction and excreting waste materials. It is one of the most important ingredients of beer as it converts the sugars into alcohol and carbon dioxide as well as giving off intermediate products which impart characteristic flavours to the final product.[13]

There are two main types of yeast used in brewing, namely top fermenting and bottom fermenting. Top fermenting yeast is used for ale production and rises to the top during fermentation while bottom fermenting yeast is used in lager production and settles to the bottom of the vessel.

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[12] Ibid.

[13] Ibid.

2.3. The Brewing Process

The brewing process can be divided into five distict phases, the first being that of malt and adjunct (cereal) storage and preparation.

2.3.1. Malt and Adjunct Storage and Preparation

The malt and adjunct i.e. usually corn, arrive at the brewery in rail car quantities from where they are unloaded into large storage bins which can be as large as 12' high. The adjunct or cereal does not require any further processing and will therefore go directly from storage to the cereal weigh hopper where it is held until it is ready to be used.

This is not the case with the malt. From the storage bin, the malt goes to the cleaning reel where the chaff and any other large foreign particles are removed. From there, it goes to the scale hopper where each batch is weighed to the same amount. These weighed batches are then passed on to the malt mill where the kernels are crushed and loosened from the husks. The husks, however, are not damaged or broken as they are required for filtering later in the process. Finally, the ground malt is removed into the gound malt weigh hopper where it is also held until it is ready for use.[14]

In modern breweries, all of the above operations of elevating, cleaning, weighing and grinding are carried out automatically from a central control panel.

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[14] Ibid.

2.3.2. Brewing

2.3.2.1. Mashing

From the cereal weigh hopper, the adjunct is transferred to the cereal cooker. There it is mixed with water and, through a series of time-temperature steps, it is brought up to the boiling point where it is held for 20 - 30 minutes. Usually a small amount of malt is added in with the adjunct to prevent the adjunct from forming a gelatinous mass which cannot be pumped and also to start the enzyme reactions. In the cereal cooker the mash is heated with steam jackets surrounding the vessel. This is done to ensure that there is no carry-over of off-odors or flavours from any live steam.

At the same time, the malt is transferred from the ground malt weigh hopper to the mash cooker where it is also mixed with water and, through a time-temperature sequence, its temperature is also increased. At the appropriate time, the boiling mash from the cereal cooker is transferred to the mash cooker. This assists in raising the temperature of the main mash up to the conversion temperature i.e. 154°F - 167°F. It is in this temperature range that the enzymes that convert the starches to sugars are most active. The mash is held at this temperature until all the starches have been converted. At this point, the temperature is increased to between 167°F - 176°F which stops all the enzyme action. Finally, the mash is pumped into the "Lauter Tun" approximately 2½ - 3 hours from the start of the mashing process.

2.3.2.2. Lauter Tun

The lauter tun is a false bottomed vessel with small slits in it. This allows the liquid to pass through but holds the husks from the grain. These husks form a filter bed which helps to retain the remaining portion of the kernels but still allows the liquid to pass through. This liquid is known as wort (pronounced wert) and is composed of maltose, dextrins, simple proteins, complex proteins and various other materials.[15]

When the liquid level in the lauter tun is reduced to approximately the height of the grain bed, an operation called "sparging" takes place. This consists of spraying a carefully treated and measured amount of hot water over the grain bed so as to wash out any remaining fermentable extract.[16]

Before going into the kettle, the wort passes through the grant where it is checked for clarity and quality. From start to finish, the lautering time is approximately two hours.

2.3.2.3. Boiling and Cooling

In the brew kettle, the wort is brought to a boil and held for 1½ - 2 hours to:[17]

1. Concentrate the fermentable extract by boiling off excess water.

2. Destroy any latent enzyme activity.

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[15]Ibid.

[16]The spent grains that are left are dumped into a separate hopper and then sold as cattle feed.

[17]Ibid.

3. Precipitate protein material.

4. Sterilize the wort.

5. Extract the hop characteristics desired from the hops.

During the later part of the boiling operation, hops are added. When boiling is complete they are immediately removed from the wort in the hop separator, a vessel which allows the liquid to pass through but retains the hops.[18]

After the liquid passes through the hop separator, it goes into the hot wort tank for a period of time to allow the protein etc. (called trub) to settle to the bottom. At the end of the settling period, the wort is passed through a cooler and is cooled to the fermenting temperature i.e. approximately $50^{o}F$ for lager and $60^{o}F$ for ale. The time required for the wort to pass from the brew kettle to the fermenting tanks is approximately two hours.

As a matter of interest, the annual capacity of the brewing process for the particular brewery under consideration was at that time 1,056,000 barrels.

2.3.3. Fermenting

As the wort is going into the fermenting vessel, clean sterile air is forced into the line to provide the oxygen necessary for the yeast to start a good vigorous fermentation. Also, the yeast is pumped into the fermenter while it is being filled with

[18] The spent hops are disposed of as they are of no further value.

wort to ensure that the yeast is evenly distributed.[19]

In many breweries, starter fermenting tanks are used, as during the cooling of the wort there is a coagulation of protein which will settle out and an accumulation of resins and tannins which will rise to the top. Therefore, in order to eliminate these impurities in the final yeast crop, the fermentation is begun in a starting tank and is pumped to a fermenting vessel when the trub has settled out.[20]

During fermentation, the sugars are converted to alcohol and carbon dioxide plus numerous intermediate products. As the fermentation progresses the yeast changes from an aerobic organism (one that requires oxygen) to an anaerobic one (no oxygen required). The carbon dioxide produced gradually purges the beer of all the air and is taken from the top of the fermenter and expelled to the atmosphere. However, if the carbon dioxide that is liberated is pure it is collected and stored for future use i.e. carbonating the final product.

The fermentation process takes 7 to 9 days. At the completion, the beer is cooled down to the desired temperature (usually 30°F) and the yeast crop is allowed to settle. At the completion of the settling and cooling the beer is transferred from the fermenter to the primary storage or ruh storage tanks.

The annual capacity of the fermenting process for the

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[19] On average, 6,000,000 yeast cells are added to every cubic centimeter of wort.

[20] ibid.

brewery under consideration was at that time 589,600 barrels.

2.3.4. Primary and Secondary Storage

Primary storage is used for the proper aging and mellowing of the beer. The time period varies anywhere from 4-14 days but generally averages about 10.

At the end of the primary storage period the beer is pre-filtered through diatomaceous earth to remove all suspended particles and thus eliminate the haze which is in the beer at this point. During filtering, CO_2 is added to the beer to eliminate any oxygen that may have been picked up in transferring from one vessel to another and also to put some of the desired CO_2 into the beer.

Following this, beer is put into secondary or final storage tanks where it reaches its full maturity in an average of 7 days. In these tanks, the beer is held at a temperature of 31^{o} - $33^{o}F$ so that the compounds which will form chill haze develop. These compounds are then removed in the final filteration so that there is no danger of a haze forming when the consumer chills the beer in his refrigerator. During this filteration the CO_2 level in the beer is increased to the proper level on its way to the bottling and racking tanks where it may remain 1-2 days prior to use.

The combined annual capacity of the primary and secondary storage was 498,666 barrels.

2.3.5. Racking

Racking involves the filling of kegs with draught beer and, since these kegs are reusable great care is taken in washing them as follows:

1. Water sprays on the outside of the keg.
2. Interior of the keg rinsed with water.
3. Outside of keg brushed and washed.
4. Inside of keg washed with hot water.
5. Inside of keg washed with solution
6. Cold water sprayed on inside of keg.
7. Keg drained and dischared to inspector. If it is clean a cork is placed in the tap hole and it is ready for filling.

To fill the keg, a machine known as a racker is used. The racker first of all fills the keg with CO_2 at the same pressure as in the racker tank. Next, the beer is allowed to displace the gas in the keg. This procedure ensures that there is no foaming and that the keg is completely filled. After filling, the filling tube is removed and keg is bunged. It is stored at 34° - $38^{\circ}F$ and should be consumed as quickly as possible as it is not pasteruized.

The annual racking capacity was 152,000 barrels.

2.3.6. Bottling

The bottleshop is the most mechanized department in the brewery. The procedure starts with the empty bottles being fed through the automatic uncasing machines from where they are carried by conveyor to the "soaker" or bottle washer. Here they are mechanically fed into the carriers which will take them through various caustic solutions ranging from 2% to 6% and at a temperature of $85^{\circ}F$

to 170°F. Some machines use brushes to wash the inside of the bottles whereas others use high pressure sprays. After the bottles have been washed and sterilized they are thoroughly rinsed to ensure that all the caustic has been removed.

From the soaker, the bottles pass through an electronic bottle inspector which automatically rejects any bottles that were not completely cleaned. Following this, a second visual inspection is given where the operator checks for chipped necks, glass flaws and any foreign matter on the side of the bottles.

After the inspections, the bottles go to the filler where they are picked off the line and set under the filling tubes. The tube seals off the bottle and it is filled with CO_2 at the same pressure as the beer in the filler barrel. The CO_2 is then displaced by the beer and the bottle is filled. As the bottles leave the filler they are given a sharp rap. This causes the beer to foam and push out any air. Finally, the crowener seals the bottles just as the foam overflows the top of the bottle.

After the crown has been applied, the bottles move to the pasteurizer where the temperature is gradually increased to 145°F where it is held for a controlled period of time. The temperature is then gradually reduced so that the bottles are discharged at approximately 70°F.

From the pasteurizer, the bottles move to a final inspection station where they are checked to ensure toat all bottles have been filled to the proper level and that no inferior bottles have been filled. Following this, the label is applied and the bottles are packaged in quantities of either six, twelve or twenty-four bottles. The packages are then sealed, stacked on pallets and moved into the warehouse in preparation for shipping.

The annual capacity of the bottleshop at that time was 316,800 barrels. This is the production area that that has the least capacity with the exception of racking. Even when the bottling and racking capacities are combined their total is still less than any of the other production areas. Therefore, if production is optimally scheduled for the bottleshop it is an easy matter to schedule production in the other areas such that beer will always be available for bottling.

Chapter 3

THE PRODUCTION SCHEDULING AND INVENTORY CONTROL PROBLEM

3.1. Introduction

Now that the production process has been examined, it should be easier to understand the problem i.e. to schedule production in such a way that the sum of production and inventory costs are minimized over a one-year period with the constraint that demand must always be satisfied.

For the particular brewery under consideration, the capacities of the bottleshop and warehouse were the bottlenecks in the production process. For this reason and also because of the complixity of the problem, it was assumed that beer would always be available for bottling purposes. Therefore, in this problem the only relevant part of the production process is the bottleshop[1] and correspondingly the only relevant production costs are those incurred in this area. Further, only the cost of labour is considered as it is the only relevant variable cost that can be accurately measured. Other costs such as materials are not relevant for this problem as these materials have to be used to meet demand.

The first inventory cost considered is that of inventory carrying cost. This cost is calculated by assuming that if money

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[1]Draught beer was not considered as it is brewed along with the beer to be bottled and the capacity of the racker was more than adequate to meet the demand.

was not tied up in inventory it could be earning interest elsewhere. Also, because of the limited warehousing capacity, the cost of storing inventory at a location other than the brewery has to be taken into account.

In the following sections, the demand, production and inventory equations are discussed, after which the equation for the objective function is dealt with.

3.2. Demand Equations

The brewery under consideration markets three brands of beer which we shall label as Brand A, Brand B and Brand C. All of these brands are available in three package sizes, namely one dozen bottles, one-half dozen bottles and one dozen cans. However, for the purposes of this study, it was assumed that the only package size available was one dozen bottles, and therefore all of the demand for the other packages was converted to these units.

One-half dozens could have easily been considered, but it would have meant doubling the size of the problem. But one dozen cans are not quite as easily included in the model as the packaging takes place on a separate can line, and this can line employs manpower from the bottling line during periods when bottling activity is slack.

The best measure of demand for these products is shipments from the brewery although in some cases the product goes to outside warehouses before being shipped to the retail outlets. Shipments are on a daily Monday to Friday basis and sometimes Saturday, but does not take place on Sundays or statutory holidays. However, for this study the daily demand was combined into a weekly demand in order to reduce the problem size and secondly because bottleshop production has to be scheduled for an entire week at a time.

Before stating the demand equations, the last thing to note is that because of limited warehousing capacity at the brewery, warehousing can also take place outside of the brewery. Therefore shipments can originate from both internal brewery storage and external brewery storage so that the demand for each product in the ith time period (i = 1 to 52) is the sum of these internal and external shipments. In the following equations, DEMAi = Demand of Product A in period i, OSINTAi = Outshipments from Internal Inventory of Product A in period i and OSEXTAi = Outshipments from External Inventory of Product A in period i. Similarily equation (2.2) relates to product B and (2.3) to product C.

(2.1.) DEMAi = OSINTAi + OSEXTAi

(2.2.) DEMBi = OSINTBi + OSEXTBi

(2.3.) DEMCi = OSINTCi + OSEXTCi

3.3. Production Equations

As previously mentioned, the only aspect of the production process that we are concerned with in this study is the bottleshop. In the previous chapter, the bottling operation was dealt with in a very brief manner as it was felt that a detailed discussion would be more appropriate in this section. A typical bottleshop operation might take place as follows:

1. At 6:00 a.m., one man would begin his shift, his function being to start up the bottling line.

2. At 7:00 a.m., an additional nine men would start; one fork lift operator to move pallets of empty bottles from storage to the automatic uncasers, one man to receive the empty bottles, four men to feed the bottles into the automatic uncasers, two men to operate the automatic uncasers and one man to ensure that the bottles are properly fed into the soakers (bottle washer).
3. At 7:30 a.m., the bottles start coming out of the soaker at which time one man begins his shift, his function being to ensure that the bottles are being discharged properly from the soaker. At 7:35 a.m., the empty bottles reach the filler at which time one filler operator begins his shift.
4. From the filler, the bottles go to the crowner and then to the pasteurizer. Thirty-five minutes later, i.e. at 8:10 a.m., the bottles start coming out of the pasteurizer at which time an additional eight men begin. Three of these men are inspectors who check the bottles to ensure that they have been filled to the proper level and that no dirty bottles have been filled. From here the bottles go to the labeler where one labeler operator is required and then to the packagers where two operators are necessary. Finally, the cartons are palletized, this function requiring one man. There is one additional man that begins at

8:10 a.m., who is the housekeeper.

As can be seen, the total manpower requirement for this particular bottleshop is twenty-one men. However, there is a problem with this type of operation by coffee breaks and lunch breaks in that portions of the line would have to shut down during these times. Therefore, to overcome this problem we have what is called a relief crew consisting of five men. This crew also begins at 8:10 a.m. and relieves men throughout the line all during the day, the end result being that the bottling line will operate for more than an eight hour day.

The shift described above, consisting of twenty-six men in total, can be called a single shift with a relief crew. In addition to this type of shift, there are a tremendous number of other shifts that can be worked up to a maximum of twenty hours per day, leaving four hours for cleanup and maintenance. However, to determine all of the possible combinations is a major study in itself, and so therefore for the purposes of this study, it is assumed that there are only five shift types available, being those that were actually employed. In the accompanying table, the following information is provided for each of these shifts:

1. The running time in minutes which is determined by the length of time that the filler is operating. For Shift 1 it can be seen that this running time is 500. If there were no relief crew, the filler would only run for 430 minutes so that the effect of the relief crew is to add

Table 1

Bottling Shift Production Capacities

	Shift	Running Time in Minutes (Filler)	B.P.M.	Total Possible Bottles Filled	Dozen Per Day[1] to Shipping @ 90%	Dozens Per Week to Shipping	Manpower Required Per Shift	Total Hours Worked Per Week	Total Labour Cost	Dozens Per[2] Man Hour Per Week	Cost Per Dozen
1.	Single w/relief crew	500	550	275,000	20,600	103,000	26	1,040	5,005	99	$.049
2.	Single w/relief crew extended	560	550	308,000	23,100	115,500	31	1,240	5,250	93	.045
3.	Double no relief crew	850	550	467,500	35,100	175,500	41	1,640	5,740	107	.033
4.	Double 1 relief crew	925	550	508,750	38,200	191,000	45	1,800	6,300	106	.033
5.	Double 2 relief crews	1,010	550	555,500	41,700	208,500	50	2,000	7,000	104	.034

[1] 90% efficiency assumed to take account of set-up costs, brand and package size changes, etc.

[2] Calculated on basis of 8 hours per man.

70 minutes running time as well as eliminating the additional shut-down and set-up costs.

2. The next heading is Bottles per Minute (B.P.M.) which is the number of bottles that the bottling line is capable of producing during every minute of production. This is constant at 550 B.P.M. for all shifts; however, it should be noted that it is possible to run the line at slower speeds than this.
3. The maximum number of bottles filled on each shift is simply the running time multiplied by the 550 B.P.M. rated capacity.
4. The dozens per day to shipping is the maximum number of bottles filled divided by 12 and then multiplied by a factor of 90%. This 90% factor has been determined through actual experience and is designed to take account of such things as set-up costs and brand and package size changes.
5. The dozens per week to shipping is simply the dozens per day to shipping multiplied by 5 which is the normal working week. Naturally, overtime can be worked to increase this level but this will be discussed later.
6. The next heading is the manpower required per shift and is self-explanatory.
7. The total hours worked per week is the manpower required multiplied by 40 hours, being the number of

hours each employee works in a week.

8. The total labour cost is the number of hours worked in a week multiplied by an hourly rate of $3.50. However, there is what is called a permanent work force which for this study is designated as 41 men or shift 3. Every time shifts 1 or 2 are worked, there are permanent employees laid off and these employees receive a guaranteed wage which amounts to 65% of their normal wage entitlement. Therefore, to the normal labour costs of shifts 1 and 2, we have to add the guaranteed wages of the employees on layoff. As an example, when Shift 1 is worked, there are 15 men laid off times 40 hours per man times $3.50 per hour times 65%. In total, this amounts to $1,365.00. One further point to note is that overtime cannot be worked when there are men on layoff. This means that because there are men laid off when shifts 1 and 2 are worked overtime cannot be worked on these shifts but can only be worked on shifts 3, 4 and 5.
9. The dozens per man-hour per week is calculated by dividing the total dozens per week to shipping by the total man-hours worked per week.
10. Finally, the cost per dozen is calculated by dividing the total cost per week by the total dozens per week to shipping.

As discussed, there are five different shifts available for production. Only one of these shifts may be used during any one week so that the shifts are 0,1 integer variables and are designated as N1i, N2i, etc., where N1i = shift 1 in period i, N2i = shift 2 in period i and so on. In addition, there is also the constraint that the sum of the shifts must be equal to 1[2] which in turn means that the upper bound for all shifts is equal to 1.

(3.1) N1i, N2i, N3i, N4i, N5i = 0,1

(3.2) N1i + N2i + N3i + N4i + N5i = 1

As mentioned earlier, overtime can be worked but only on shifts 3, 4 and 5 as overtime cannot be worked if there are any permanent employees on layoff. Further, overtime can only be worked up to a maximum of 4 hours per day as well as an 8 hour shift on Saturdays, giving a total potential of 28 overtime hours per week or .7 of a normal 40-hour week. The overtime rate of pay is $1\frac{1}{2}$ times the normal rate of pay or $5.25 per hour. The overtime equations are as follows where M3i, M4i and M5i are equal to the overtime production on shifts 3, 4 and 5 in period i.

(3.3) M3i $\leq$ .7 N3i

(3.4) M4i $\leq$ .7 N4i

(3.5) M5i $\leq$ .7 N5i

The total production during any given week is equal to the sum of the production of product A plus product B plus product C

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[2] As will be seen later, this constraint should have been equal to or less than 1.

where PRODAi, PRODBi and PRODCi is equal to the production of products A, B and C in period i. The values 103,000, 115,500, etc. can be interpreted as the capacities of shift 1, shift 2 and so on.

(3.6) PRODAi + PRODBi + PRODCi = 103,000 N_1i + 115,500 N_2i + 175,000 (N_3i + M_3i) + 191,000 (N_4i + M_4i) + 208,500 (N_5i + M_5i)

3.4. Inventory Equations

Inventory can be stored both internally, i.e. at the plant and externally. However, if the inventory is stored outside the plant, there is a cost of 3¢ per dozen associated with this storage. The internal inventory of each product in time period i (i = 1 - 52) is defined as the internal inventory in time period i - 1 plus the production in time period i minus the shipments to the market in time period i minus the shipments to external inventory in time period i. The internal inventory equations are as follows where INTINVAi, Bi and Ci are equal to internal inventory of products A, B and C in period i and SEXTAi, Bi and Ci are equal to shipments of products A, B and C to external storage in period i.

(4.1) INTINVAi = INTINVAi-1 + PRODAi - OSINTAi - SEXTAi

(4.2) INTINVBi = INTINVBi-1 + PRODBi - OSINTBi - SEXTBi

(4.3) INTINVCi = INTINVCi-1 + PRODCi - OSINTCi - SEXTCi

The total internal inventory (TINTINVi) that is held is the sum of the internal inventory of each product and cannot exceed 70,000 dozen.

(4.4) TINTINVi = INTINVAi + INTINVBi + INTINVCi

(4.5) TINTINVi $\leq$ 70,000

The external inventory of each product in time period i is defined as the external inventory in period i - 1 plus shipments to external inventory in period i minus shipments to the market place in period i. The equations are as follows:

(4.6) EXTINVAi = EXTINVAi-1 + SEXTAi - OSEXTAi

(4.7) EXTINVBi = EXTINVBi-1 + SEXTBi - OSEXTBi

(4.8) EXTINVCi = EXTINVCi-1 + SEXTBi - OSEXTCi

The total external inventory held is the sum of the external inventory of the three products. There is no upper bound on external inventory.

(4.9) TEXTINVi = EXTINVAi + EXTINVBi + EXTINVCi

The total inventory of each product is the sum of the internal and external inventories. Further to recognizing the requirement of a safety stock level of inventory, it was assumed that the inventory level for each product had to be equal to at least 1/5th of the demand for the next time period. The equations are as follows where TINVAi, Bi and Ci is eqial to the total inventory of products A, B and C.

(4.10) TINVAi = INTINVAi + EXTINVAi = 0.2 DEMAi+1

(4.11) TINVBi = INTINVBi + EXTINVBi = 0.2 DEMBi+1

(4.12) TINVCi = INTINVCi + EXTINVCi = 0.2 DEMCi+1

Finally, the total inventory of all products is equal to the sum of the total internal inventory (equation 4.4) and the total external inventory (equation 4.9). In the following equation TINVi denotes this total inventory.

(4.13) TINVi = TINTINVi + TEXTINVi

3.5. Objective Function

Our objective is to minimize the sum of all production and inventory costs stated above as follows:

(5.1) MINF = 5005 N1i + 5250 N2i + 5740 (N3i + 1.5M3i) +

6300 (N4i + 1.5M4i) + 7000 (N5i + 1.5M5i) + .0005 TINVi

+ .03 (SEXTAi + SEXTBi + SEXTCi)

In this equation, the figures 5005, 5250, etc. are the costs of the various shifts as calculated in Table 1. It will be noticed that for shifts 3, 4 and 5 this cost is multiplied by 1.5 if the overtime option is selected as if overtime is worked the cost is 1.5 times that of normal time. To determine the inventory carrying costs, the total inventory of all products is multiplied by a factor of .0005 which is the cost of money invested in inventory for one period. Similarily, the external inventory costs are determined by multiplying the sum of all shipments to external inventory by a factor of 3¢ per dozen.

Chapter 4

THE CONTINUOUS SOLUTION

4.1. Introduction

In Chapter 1, it was pointed out that the computer package employed in this study first of all arrives at a solution to the general linear programming problem i.e. a continuous solution before solving the problem with integer constraints. Therefore, the continuous solution will be discussed before examining the integer case. However, this solution will be examined for only one four week period because of the problem that optimal solutions were obtained for 13 groups of 4-week periods rather than the full 52 weeks together.

4.2. Demand

As previously mentioned, the daily demand for each product was summed so as to arrive at a weekly demand for each of the three products A, B and C. In the following table these demands have been listed by product and by time period as Demand A1, Demand A2, etc., which is to be interpreted as the demand for product A in time period 1 etc. In addition, the "dual values" are also shown, which tell us that if the demand is increased by 1 unit the marginal cost will be, in the case of product A in time period 1, increased by $ .03950.[1]

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[1]These dual values will be discussed further in the production section.

Table 2

Demand Values and Dual Values

Product and Period	Constraint Value	Dual Value
Demand A1	166,173	$.03950
Demand A2	183,786	.04000
Demand A3	177,254	.04050
Demand A4	210,100	.04100
Demand B1	10,782	.03950
Demand B2	10,983	.04000
Demand B3	13,918	.04050
Demand B4	14,709	.04100
Demand C1	4,741	.03950
Demand C2	4,923	.04000
Demand C3	5,536	.04050
Demand C4	6,543	.04100

4.3. Production

In review, it was previously pointed out that production can take place on any one (or more in the continuous case) of 5 shifts. Further, overtime can only be worked on shifts 3, 4 and 5 as shift 3 was selected as the permanent shift and overtime cannot be worked if there are permanent employees on lay-off. In the following section both overtime and normal time production will be discussed for the 4-week period under review.

4.3.1. Normal Time Production

It was pointed out in Chapter 3 that one of the constraints on production was that the sum of the shifts must be equal to one which means that either all of one shift or parts of shifts which in total sum to one must be chosen in each period.[2] This constraint is necessary in the integer case but is not in the continuous case, so that if the constraint in the continuous situation had been equal to or less than 1 rather than just equal to one, cost would have been reduced, as portions of the higher productivity shifts could have been utilized in the lower volume situations. In addition to satisfying this constraint, the model also selects the shifts to be used for production in such a way as to:

1. Satisfy the requirement that demand must be met
2. Satisfy the minimum inventory requirement
3. Satisfy the requirement of minimizing production and inventory costs while still satisfying constraints 1 and 2.

The following table lists all of the potential shift selections, with those that were selected showing a value equal to or less than one but not zero. In addition, the table also shows the reduced costs associated with the shifts which is to be interpreted as the marginal cost of employing a particular shift.

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[2] The dual values accociated with this constraint will be discussed later.

Table 3

Shift Selections and Reduced Costs

Shift and Period	Value	Reduced Cost
Shift 11	.00000	$2,172.25
Shift 21	.00000	1,923.50
Shift 31	.00000	43.50
Shift 41	1.00000	-8.75
Shift 51	.00000	.00
Shift 12	.00000	2,225.00
Shift 22	.00000	1,970.00
Shift 32	.00000	60.00
Shift 42	.21463	.00
Shift 52	.78537	.00
Shift 13	.00000	2,269.00
Shift 23	.00000	2,007.75
Shift 33	.00000	.00
Shift 43	.00000	.00
Shift 53	1.00000	-8.75
Shift 14	.00000	2,313.00
Shift 24	.00000	2,045.50
Shift 34	.00000	75.50
Shift 44	.00000	.00
Shift 54	1.00000	-17.50

In Table 3 above, it can be seen that in period 1 all of shift 4 was employed which means that shift 4 is at its upper bound of 1. As a result of shift 4 being at its upper bound, shift 5 is the basic variable in period 1. The reduced cost of shift 4 is -$8.75 which means that if the upper bound could have been exceeded by 1 unit costs would have been reduced by $8.75. If shift one had been selected, the extra cost would have been $2,172.00 as compared to $1,924.00 for shift 2 and $43.50 for shift 3.

In period 2, it can be seen that 21.463% of shift 4 was utilized along with 78.537% of shift 5. Since neither of these shifts are at their upper bound shifts 4 and 5 are both basic variables in period 2. Had shifts 1, 2 or 3 been utilized, the reduced costs would have been $2,225.00, $1,970.00 and $60.00 respectively.

Table 3 shows that in period 3 all of shift 5 was utilized which in turn means that shift 5 was at its upper bound of 1. In this case both shift 3 and shift 4 are basic variables. The reduced cost of shift 5 is -$8.75, which again means that if the upper bound could have been exceeded by 1 unit costs would have been reduced by $8.75. Had shifts one or two been utilized the additional costs would have been $2,269.00 and $2,007.75 respectively.

In period 4 shift 5 was again fully utilized with a reduced cost of -$17.50. The basic variable in this period was shift 4 and the reduced costs for shifts 1, 2 and 3 were $2,313.00, $2,045.50 and $75.50 respectively.

Having examined the shift selections and their related reduced costs we will now examine the dual values associated with the total production equation as listed in Table 4.

Table 4

Total Production Dual Values

Production and Period	Dual Value
Total Production 1	$.03950
Total Production 2	.04000
Total Production 3	.04050
Total Production 4	.04100

Here, it is noticed that the dual values are identical to those associated with a one-unit increase in demand, the reason being that if demand increases by one unit, production necessarily increases by one unit. Another interesting thing to note about these dual values is that they are increasing by an amount equal to \$.00500 from one period to the next, an amount which is equal to the cost of holding one unit of inventory for one time period. This situation does not occur in many of the other four-week periods examined. The explanation must be that in this particular four-week period if say an additional unit of production were required in period 4 the least costly way of providing it would be to produce it in period 1 and pay inventory costs through to period 4.

The discussion of the dual values associated with the constraint that the sum of the shifts must equal one has purposely been delayed until now as these dual values are related to both the shift selections and production costs. These dual values are shown in Table 5 and can be interpreted as the cost of one full shift or a combination of shifts making up one shift.

Table 5

Shift Values and Dual Values

Sum of Shifts in Period	Constraint Value	Dual Value
Period 1	1	\$1,236
Period 2	1	1,340
Period 3	1	1,436
Period 4	1	1,531

As can be seen, these costs are substantial, ranging from \$1,236.00 in period 1 to \$1,531.00 in period 4. If these costs are added to the cost of the basic shift and the result divided by the capacity of the basic shift, the production costs are obtained. For example, in period 1 it will be recalled that shift 4 was selected but since it was at its upper bound the basic shift was shift 5. By adding the dual value of \$1,236.00 from Table 6 to the \$7,000.00 cost of shift 5 (see Table 1) and dividing the result by the 208,500 dozen capacity of shift 5 (see Table 1) we arrive at the production cost of \$.0395 as shown in Table 5. This is substantially higher than the cost per dozen of \$.033 as calculated in Table 1.

In period 2, both shifts 4 and 5 were utilized and are therefore both basic. For shift 4 we have \$1,340.00 (Table 5) added to the total cost of shift 4 of \$6,300.00 all divided by the 191,000 capacity of shift 4. The result is \$.040 which is identical to the value shown in Table 4 for period 2. Similarily, for shift 5 we have (\$1,340.00 + \$7,000.00)/208,500 = \$.040. In fact, the dual variables may be computed from these shifts by solving the following equations

$$\text{Shift 42:} \quad 6{,}300 = U_1 + 191{,}000\ U_2$$

$$\text{Shift 52:} \quad 7{,}000 = U1 + 208{,}500\ U_2$$

where $U_1 \gtreqless 0$, and $U_2 \geqq 0$. The dual value of the shift constraint equation is U_1 and is equal to \$1,340. Similarily, the dual value of the toal production equation is U_2 and is equal to .04.

Finally, we are in a position to look at the actual levels of production in each period.

Table 6

Levels of Production

Period	Product A	Product B	Product C	Total
1	169,695	16,527	4,778	191,000
2	182,480	17,219	5,045	204,744
3	193,455	2,723	12,322	208,500
4	194,469	14,031	---	208,500

In period 1 the total production equals 191,000 which is equal to the total normal time production available on shift 4. Similarily, in periods three and four the production was 208,500 which represents 100% of shift 5. In period 3 however, 21.463% of shift 4 was utilized along with 78.537% of shift 5 which means that of the total production of 204,744, 191,000 x .21463 or 40,944 units were produced on shift 4 and 208,500 x .78537 or 163,750 units on shift 5.

The final area to examine before concluding this section on normal time production[3] is to ensure that the requirements with respect to demand and inventory were met. This can best be done by examining the following table.

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[3]For the four-week period under consideration, normal time production represents total production as the overtime options did not have to be utilized.

Table 7

Analysis of Production

Period and Product	Opening Inventory	Production	Demand	Closing Inventory	Minimum Inventory
Product A1	33,235	169,695	166,173	36,757	36,757
Product B1	2,156	16,527	10,782	7,901	2,197
Product C1	948	4,778	4,741	985	985
Product A2	36,757	182,480	183,786	35,451	35,451
Product B2	7,901	17,219	10,983	14,137	2,784
Product C2	985	5,045	4,923	1,107	1,107
Product A3	35,451	193,455	177,254	51,652	42,022
Product B3	14,137	2,723	13,918	2,942	2,942
Product C3	1,107	12,322	5,536	7,893	1,309
Product A4	51,652	194,469	210,110	36,011	36,011
Product B4	2,942	14,031	14,709	2,264	2,264
Product C4	7,893	--	6,543	1,350	1,350

The closing inventory was calculated by subtracting the demand from the sum of the opening inventory for the period and production during the period. As can be seen, in all cases the closing inventory was at least equal to the minimum inventory requirement which in turn means that the levels of production were always sufficient to meet the demand and inventory requirements.

4.3.2. Overtime Production

In the previous chapter, it was defined that overtime production could not exceed 70% or .7 of the normal productive capacities of shifts 3, 4 and 5. In the four week period under examination, overtime production was not utilized[4] and therefore the values of the overtime variables are all zero as shown in Table 8. If, however, the overtime option had been utilized, the costs would have been substantially increased as shown be the reduced costs listed in the same table.

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[4] Overtime production was not utilized in any of the continuous solutions but it was used four times in the integer solutions.

Table 8

Overtime Reduced Costs

Shift and Period	Value	Reduced Costs
Shift 31	.00000	$1,678
Shift 32	.00000	1,590
Shift 33	.00000	1,599
Shift 34	.00000	1,415
Shift 41	.00000	1,905
Shift 42	.00000	1,810
Shift 43	.00000	1,715
Shift 44	.00000	1,619
Shift 51	.00000	2,264
Shift 52	.00000	2,160
Shift 53	.00000	2,056
Shift 54	.00000	1,952

The constraint values and dual values associated with the overtime equations are shown in Table 9. In periods 1, 3 and 4 it can be seen that overtime could have been worked up to a maximum of .7 or 70% of shifts 4 and 5 respectively. The dual values attatched to these constraint values of .7 were in all cases equal to zero which means that the overtime constraints are nonbinding. In fact, the only instance in which there is a dual value which is not zero is corresponding with shift 3 in period 3. Here, the value is $96.78571 but since shift 3 was not utilized in period 3 this value has no meaning.

In period 2 where there was a utilization of 21.463% of shift 4 and 78.537% of shift 5, the table shows constraint values of .15024 and .54976 which are exactly equal to .7 of the percentage utilization of shifts 4 and 5.

Table 9

Overtime Values and Dual Values

Overtime Shift and Period	Constraint Value	Dual Value
Shift 31	.00000	.00000
Shift 32	.00000	.00000
Shift 33	.00000	96.78571
Shift 34	.00000	.00000
Shift 41	.70000	.00000
Shift 42	.15024	.00000
Shift 43	.00000	.00000
Shift 44	.00000	.00000
Shift 51	.00000	.00000
Shift 52	.54976	.00000
Shift 53	.70000	.00000
Shift 54	.70000	.00000

4.4. Inventory

4.4.1. Minimum Inventory Requirements

As previously mentioned, the minimum inventory requirement for each product at the end of time period i is equal to 1/5 of the demand in period i + 1. The following table lists these lower bound constraints together with the actual inventory levels and dual values beginning with the opening inventory in period 1 or in other words the closing inventory in period 1 - 1.

Table 10

Minimum Inventory Values and Dual Values

Product and Period	Constraint Value	Constraint Lower Bound	Dual Value
Inventory A0	33,235	33,235	$.01064
Inventory A1	36,757	36,757	.00000
Inventory A2	35,451	35,451	.00000
Inventory A3	51,652	42,022	.00000
Inventory A4	36,011	36,011	.04150
Inventory B0	2,156	2,156	.01064
Inventory B1	7,901	2,197	.00000
Inventory B2	14,137	2,784	.00000
Inventory B3	2,942	2,942	.00000
Inventory B4	2,264	2,264	.04150
Inventory C0	948	948	.01064
Inventory C1	985	985	.00000
Inventory C2	1,107	1,107	.00000
Inventory C3	7,893	1,309	.00000
Inventory C4	1,350	1,350	.04150

In the above table it can be seen that minimum requirements have been met in all cases and in some exceeded.

The dual values here are rather interesting as they are always zero except for period 0 (which is really the end of a previous 4-week period) and period 4, the end of the 4-week period under discussion. This serves to point out that the optimization was not achieved for the full 52 week cycle but only for the thirteen individual four-week periods, as if we had optimized for the entire 52 weeks it is highly likely that the inventory of at least one of the products would have been higher than the minimum requirement. Also, this is the key to the dual values being zero in all but the ending periods as it can be seen that the inventory level of one of the products exceeded the minimum requirement so that if the lower bound constraint were increased by one for any or all of the products the inventory level of the product that exceeded the minimum could be reduced accordingly. In period 4, however, this is not possible as all products were at their minimum acceptable level and therefore if the minimum requirement were increased by 1 unit for any one product costs would accordingly be increased by $.04150.

4.4.2. Internal Inventories

Internal inventories are defined as being up to a maximum level of 70,000 dozen due to capacity limitations at the plant being considered. Since the internal inventory of each product has been listed in both the previous section (4.4.1.) and the "Production" section (4.3.) it is not felt necessary to list them again. However, it is worth noting that the dual values are identical to those associated with demand and production, ranging from $.03950 in period 1 to $.04100 in period 4. The reason, of course, is that if we require an additional unit of inventory we have to produce it. Regarding the reduced costs associated with the internal inventory variables we find that they are equal to zero.

4.4.3. External Inventories

External inventories are those inventories in excess of 70,000 and are stored at a premium cost. As was the case with overtime production, we find that the external inventory option was not utilized during the four-week period under consideration. However, if it were utilized, the dual values would be identical to those associated with the internal inventories. But, when the reduced costs associated with the external variables are examined, it is seen that if this external option had been utilized, costs would have increased by \$.03000 per unit.

4.5. Objective Function

For the four-week period under consideration, the value of the objective function turned out to be \$27,249. This is made up of both production and inventory costs as outlined in the following brief section.

4.5.1. Production Costs

By considering the shifts utilized and the cost of each shift as shown in Table 1 of Chapter 2, the following table can be developed.

Table 11

Production Costs

Period	Shift	Cost	Percent Utilized	Total Cost
1	4	$6,300	100.000	$6,300
2	4	6,300	21.463	1,352
2	5	7,000	78.537	5,498
3	5	7,000	100.000	7,000
4	5	7,000	100.000	7,000

If the total cost column is summed it is found that of the total cost of $27,249, production costs represent $27,150 or 99.6%.

4.5.2. Inventory Costs

Inventory carrying costs were assumed to be $.00050 per unit per period. If this is multiplied by the total inventory held during the four-week period of 198,450 dozen it is found that inventory carrying costs would have totalled $99.00. When this is added to the production costs of $27,150, we arrive at a grand total of $27,249, the value of the objective function. Because internal inventory costs represent only .36% of the total, it can be concluded that they are insignificant.

Chapter 5

THE INTEGER SOLUTION

5.1. Introduction

The integer problem differs from the continuous problem in that in the integer case certain variables are required to take on the values of either 0 or 1, whereas in the continuous case the variables can assume any integer value within specified bounds. Specifically, it is the production variables N1, N2, N3, N4 and N5 that are defined as integer variables, meaning that they can only assume the values 0 or 1. By further defining that the sum of these variables must be equal to 1, the situation arises where a shift must be chosen for production during a period but only one of the shifts may be selected.[1] In the continuous case, it will be recalled that it is possible to use say ½ of shift 1 and ½ of shift 2, providing that the sum of the partial shifts selected equals one.

Again in an attempt to reduce the problem size and thereby reduce computer costs, the problem was run for thirteen separate groups of four weeks. Therefore, the solution for the total 52 weeks is not a true optimal one but it is felt to be sufficiently

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[1] In hindsight, it would have been preferable to have defined the sum of the shifts as equal to or less than 1 as this would have permitted the situation of having no production in a given period. A second way of doing this would be to define a shift with a cost of 65% of 41 men and zero production.

close to permit discussion of the entire 52 weeks in this section. However, the discussion will be limited to the solution obtained for the toal of all products rather than for each individual product, as we are primarily interested in overall trends.

5.2. Demand

Demand was taken to be the same as shipments from the brewery and was summed to establish demand on a weekly basis. The demand for beer is extremely seasonal, peaking in the summer months and December and falling off to its lowest levels in the winter months of January and February.

Figure 1[2] shows these weekly demands for 52 weeks beginning with the first week in April and clearly points out the wide seasonal fluctuations, reaching its highest level of 231,362 in period 12 and its lowest level of 91,900 in period 41. When these high and low figures are related to the average weekly demand of 146,911 dozen we find that the demand in period 12 is 157% of the average and in week 41 it is 63%. As can be expected these seasonal fluctuations, combined with the limited shelf life of beer, create severe problems in the scheduling of production and utilization of capacities.

5.3. Production

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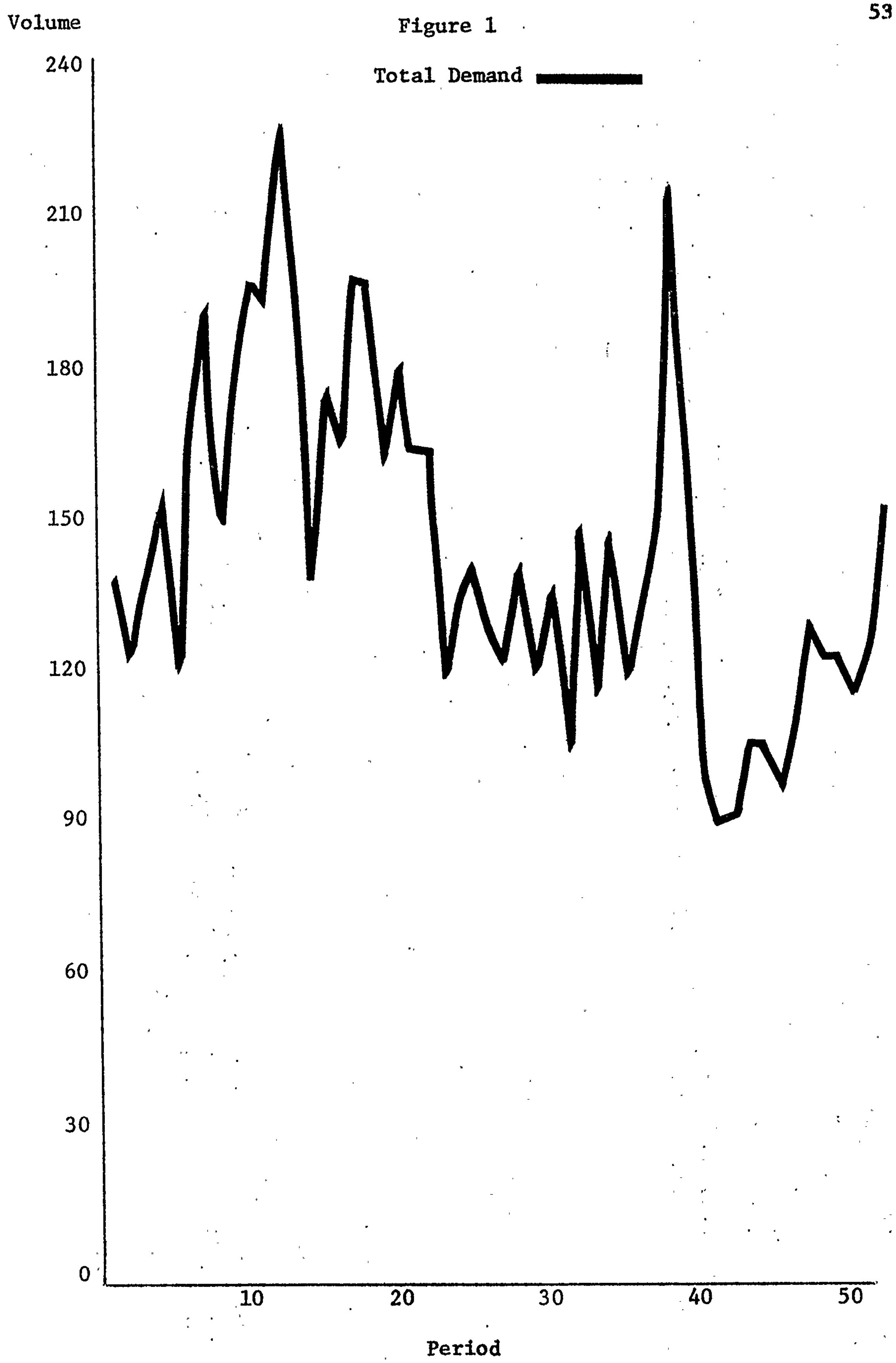
[2]For the actual demand figures by week and product see Table 1 Appendix 1.

Figure 1


Volume
Total Demand
240
210
180
150
120
90
60
30
0
10
20
30
40
50
Period

5.3.1. Actual Production Vs. Demand.

Actual production[3] is plotted against actual demand in figure 2. As can be expected, the graph shows a fairly definite relationship between demand and production levels. Generally speaking, as demand increases production also increases and vice versa. However, this relationship is not perfect as there is often times a lag between the time when demand changes and production reacts. For example, in periods 1 and 2, production levels are below demand, resulting in much higher production levels in periods 3, 4, 5 and 6. In period 7, production drops away off whereas demand continues to increase. Again, we see the reaction of production in periods 8, 9 and 10.

In periods 33, 43 and 45 it is noted that there is no production taking place, as during these periods the bottling line was being overhauled. These periods are planned well in advance and take place when forecasted demand is at its lowest. Examination of production levels reveals that for the 2 or 3 periods preceeding these shutdowns, production is substantially above demand in order to build up inventories.

5.3.2. Integer Production Vs. Demand.[4]

In the integer case, it can be seen from Figure 3 that there is a much more direct relationship between demand and produc-

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[3]Total weekly production is listed in Table 3, Appendix 1. For weekly production by product see Table 2, Appendix 1.

[4]The total weekly integer production is listed in Table 3, Appendix 1. For weekly production by product see Table 2, Appendix 1.

Figure 2

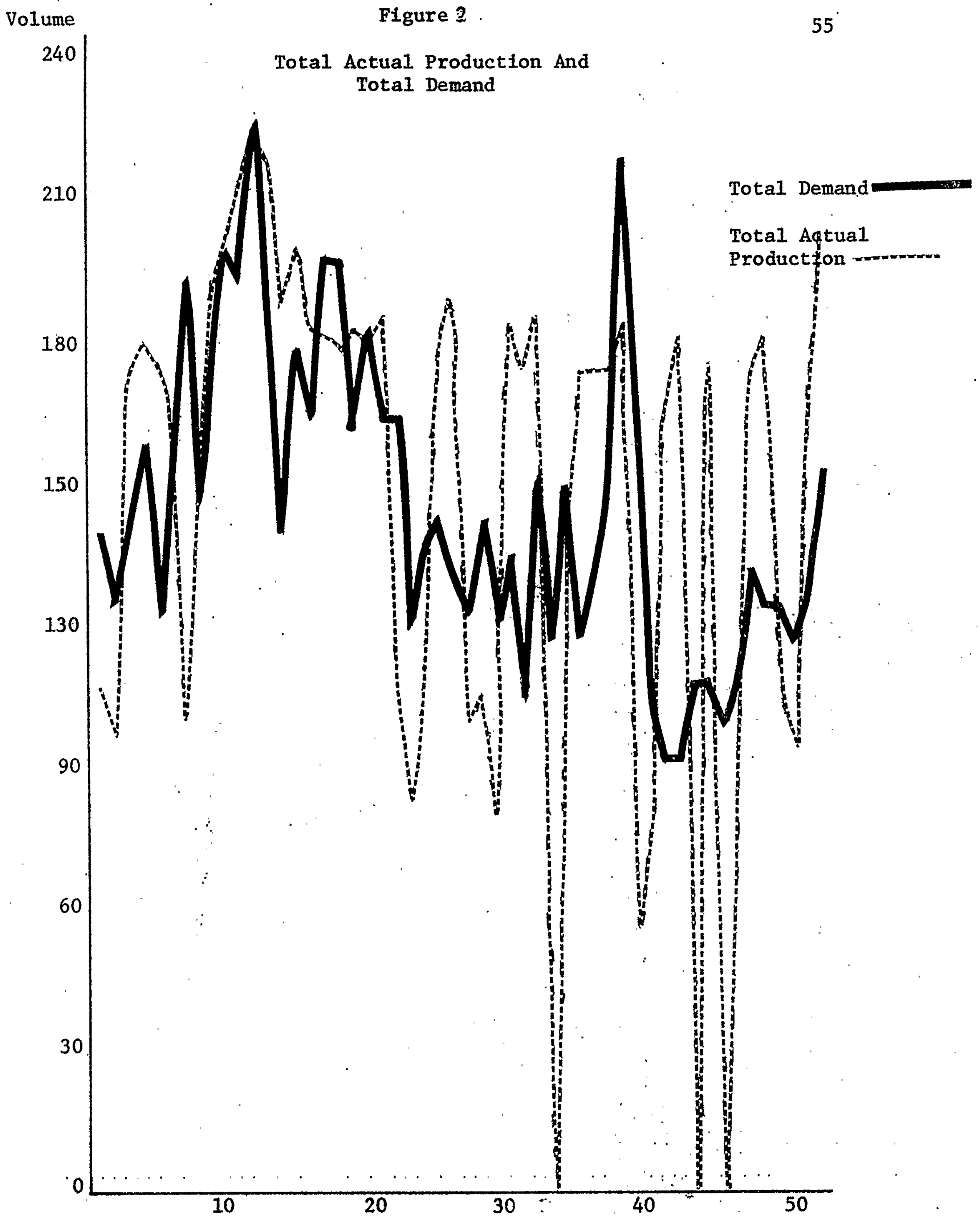
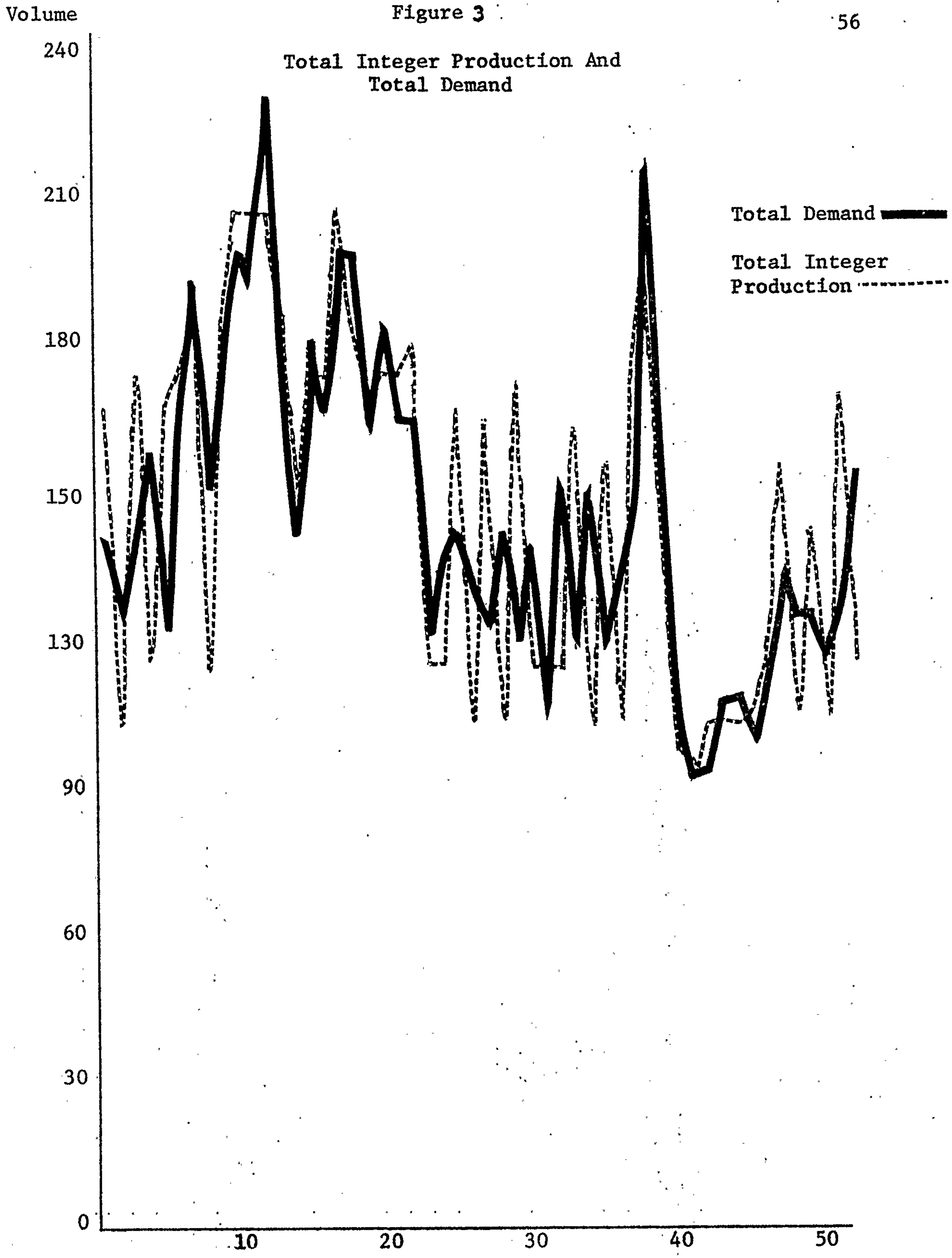

Volume
Total Actual Production And
Total Demand
240
210
180
150
130
90
60
30
0
10
20
30
40
50
Period
Total Demand
Total Actual Production

Figure 3

Total Integer Production And Total Demand



tion levels. Further in the integer case it can be seen that production reacts prior to demand changes rather than after as in the actual case. This is because in the integer case demand was known in advance of production whereas under actual conditions production is based on forecasts which can easily be in error. A further benefit of this perfect knowledge of demand is that in the integer case production deviates from demand to a much lesser extent than in the actual situation. As will be seen shortly, this in turn means that lower inventory levels are required in the integer case.

The dual values associated with the production variables in the integer case are almost always equal to .0005, .001 and .00150 which means that a one unit increase in demand and hence production can be accommodated for the cost of internal inventory only. As will be recalled, this is much lower than in the continuous case. But it should be pointed out that there are exceptions to this when demand and hence production and inventory are at peak levels.

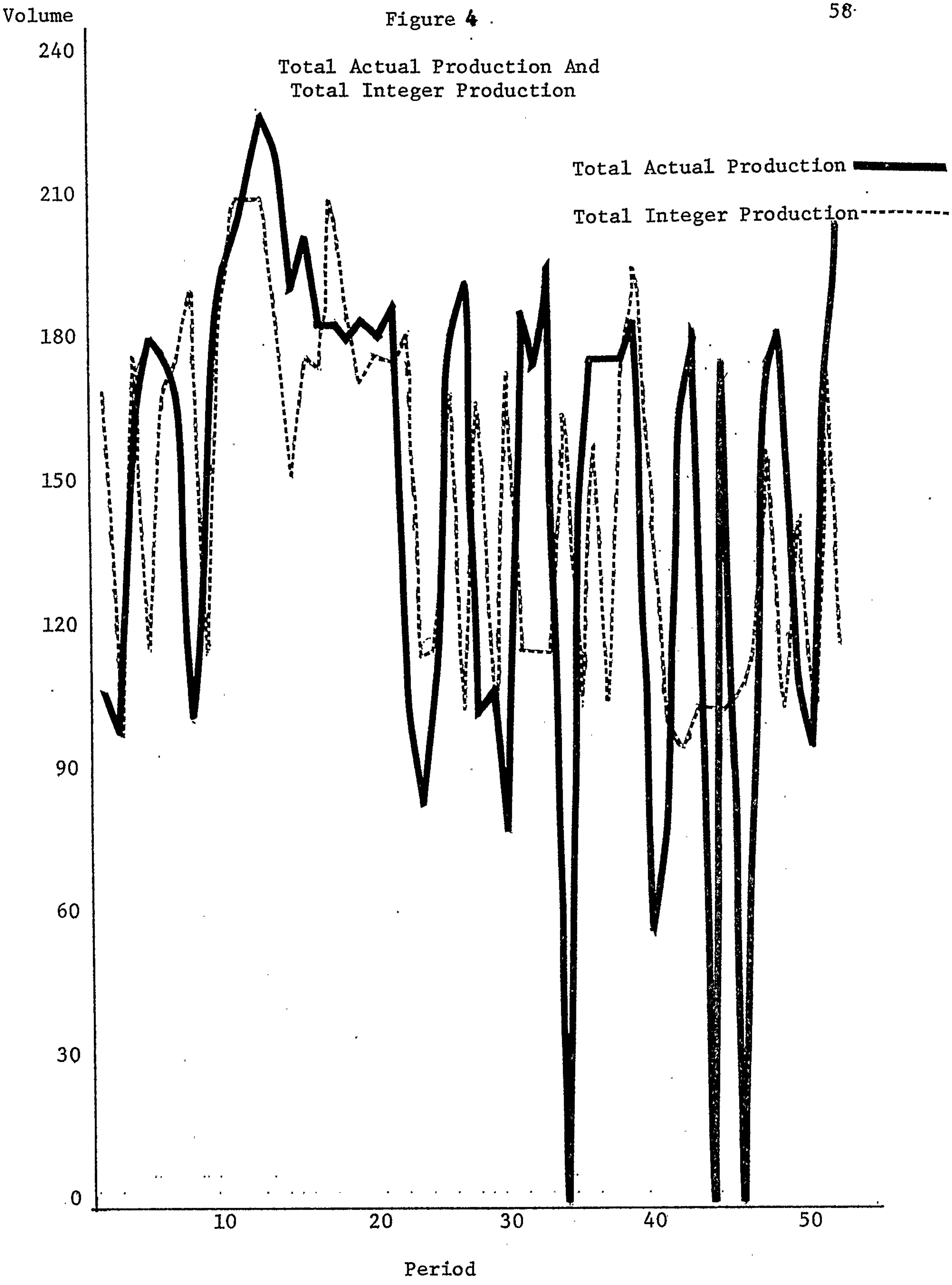
5.3.3. Actual Production Vs. Integer Production.

In Figure 4, actual production is plotted against integer production. The general trend is much the same but there is a marked difference in variations between weeks, the integer production clearly being much more stable than the actual.

Again, it is noticed that in the actual case there were three periods in which no production took place due to line overhauls. In contrast, there were no periods of zero production in the integer case which is simply because these shutdowns were not taken into account when formulating the constraints. In fact, they should

Figure 4

Total Actual Production And
Total Integer Production



have been and as a result production and inventory costs are slightly understated as production and inventory levels prior to these shutdowns would have had to be higher.

5.3.4. Actual Overtime Production Vs. Integer Overtime Production.

In the integer case, overtime production was at a minimum, being utilized in only four time periods. Table 12 below lists the period, the shifts in which overtime was worked and the amount produced.

Table 12

Integer Overtime Production

Period	Shift	Production
13	3	11,090
18	3	8,494
22	3	5,398
38	4	1,564

Actual overtime production is not available but actual overtime hours worked are. During the 52 weeks under consideration, the actual overtime hours worked totalled 2,990. By converting the integer production into hours it is found that in contrast only 248 overtime hours were worked in the integer case.

5.3.5. Actual Shift Selection Vs. Integer Shift Slection.[5]

Figure 5 shows the frequency with which shifts 1 - 5 were selected under actual conditions.[6] Similarily, Figure 6 shows the number of times each shift was selected in the integer case. In the actual case shift 4 was selected the most at 16 times out of a possible 49, with shift 3 being selected 12 times, shift 1 ten times, shift 5 seven times and shift 2 four times.

In the integer case, we see a much higher concentration of production in shifts 1, 2 and 3 with shifts 4 and 5 only being selected 3 and 5 times respectively. Shift 3 is definitely the dominant shift being selected a total of 22 out of a possible 52 times. One reason for this is that shift 3 was selected as the permanent shift and therefore if shifts 1 and 2 were worked a penalty in the form of a guaranteed annual wage had to be paid. Even so, these two shifts combined were selected a total of 22 times which indicates the possibility that shifts one or two should have the permanent shift rather than shift 3. However, this could only be determined by re-running the problem with first shift 1 and then shift 2 as the permanent shift and comparing the results. As mentioned before, this was not possible.

In concluding this section, figures 7 and 8 clearly demonstrate that under the integer case there was a much greater stability of production. But again, it must be pointed out that in

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[5] Both the actual and integer shift selections by week can be found in Table 3, Appendix 1.

[6] In the actual case, the toal shift selections are only 49 vs. 52 in the integer case because of the three periods when no production took place.

Figure 5

Actual Shift Selections

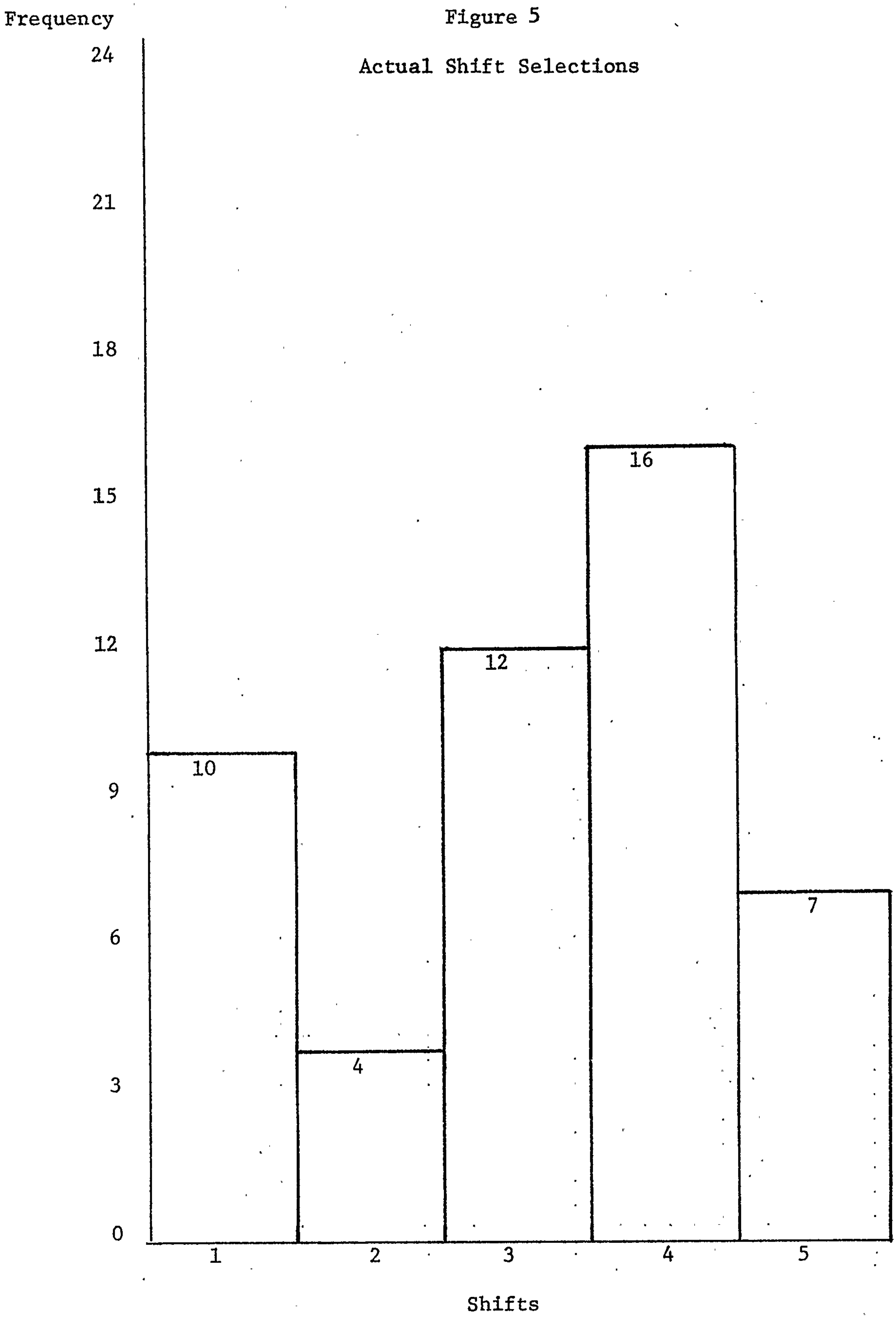
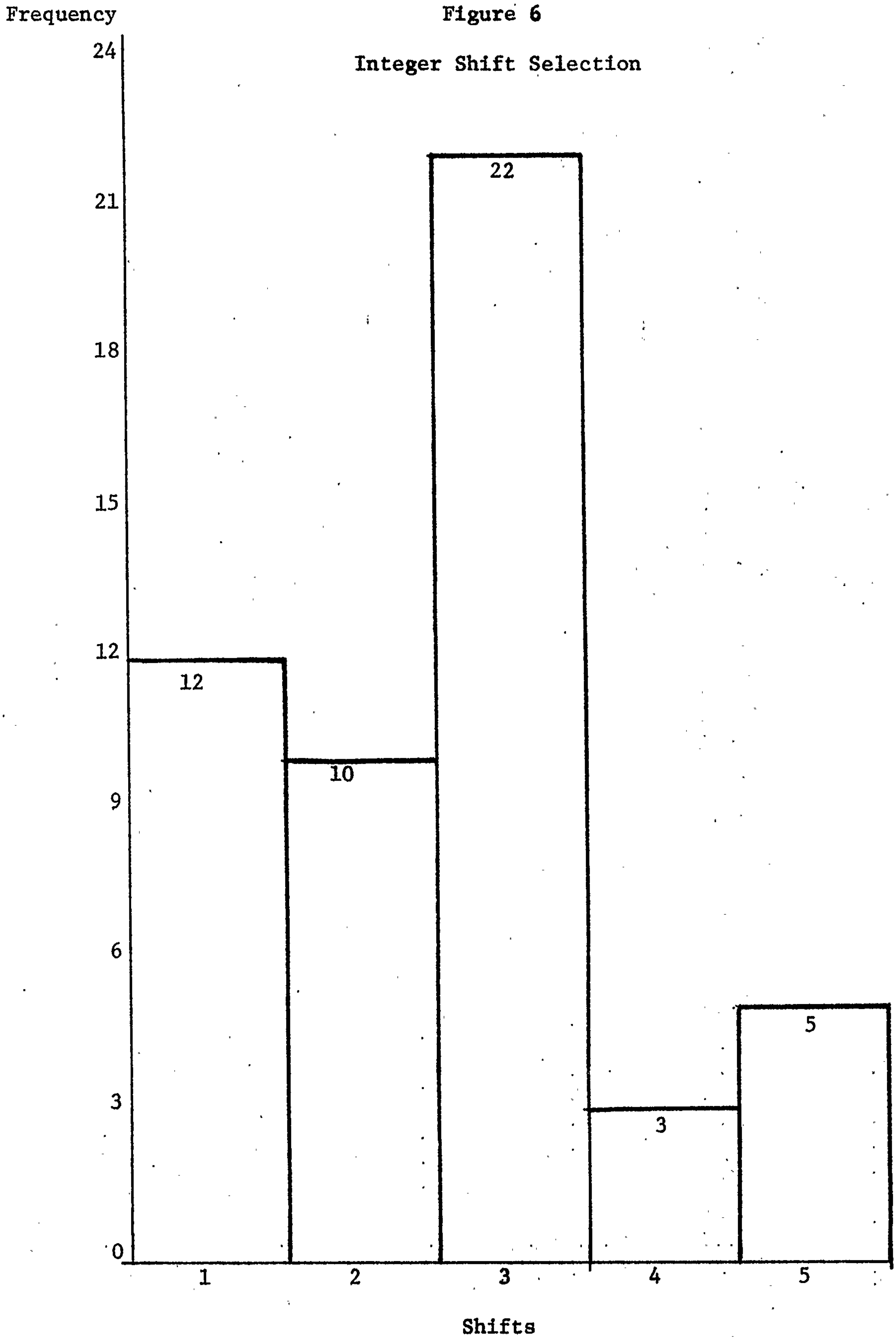


Figure 6

Integer Shift Selection



the integer case there was perfect knowledge of demand patterns whereas in the actual case there was not.

5.4. Inventory

5.4.1. Actual Inventory[7] Vs. Demand.

In Figure 7, total actual inventory is plotted against total demand. Here, it is very difficult to determine any definite relationship between demand and inventory although some general comments can be made. For example, when demand is increasing there is often a buildup of inventory such as periods 10 through 17 and 35 - 38. In contrast, there are also occasions such as period 25 in which there is a substantial drop in inventory when demand increases. The reason for this is likely that this increased demand was not anticipated and therefore production had not reacted. In support of this statement we see that inventory is again built up in periods 26 and 27 in response to this increased demand. One further point to note is that in periods 33, 43 and 45 the inventory levels are much higher than demand which is because of the planned shutdown of the bottleshop during these periods.

5.4.2. Integer Inventory[8] Vs. Demand.

The total integer inventory is compared with demand in Figure 8. In contrast to the actual case, a definite relationship

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[7] Actual weekly inventory levels by product can be found in Table 5, Appendix 1. Total actual inventories are listed in Table 6, Appendix 1.

[8] Integer inventories by week and product and in total can also be found in Tables 5 and 6, Appendix 1.

Figure 7

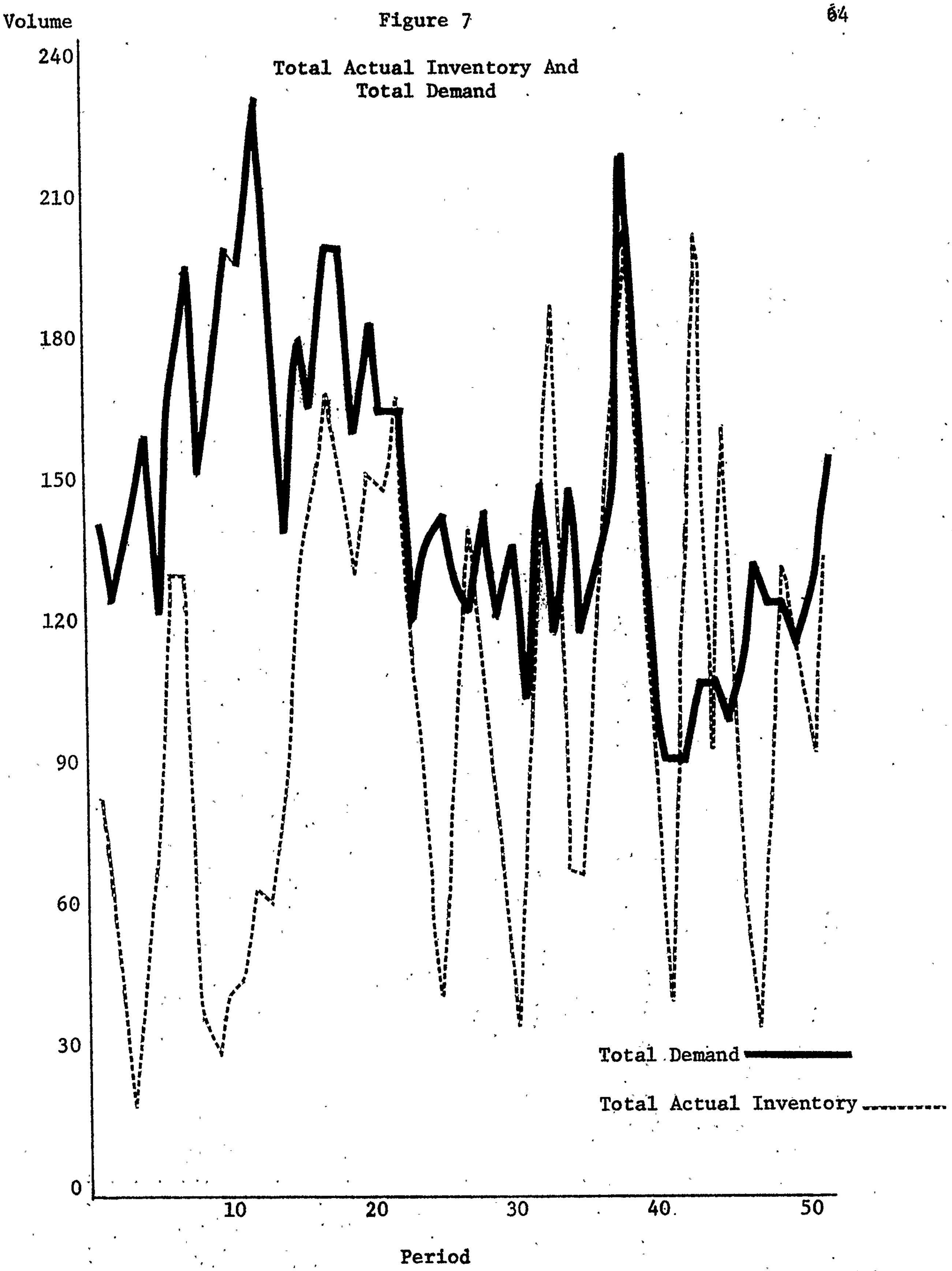
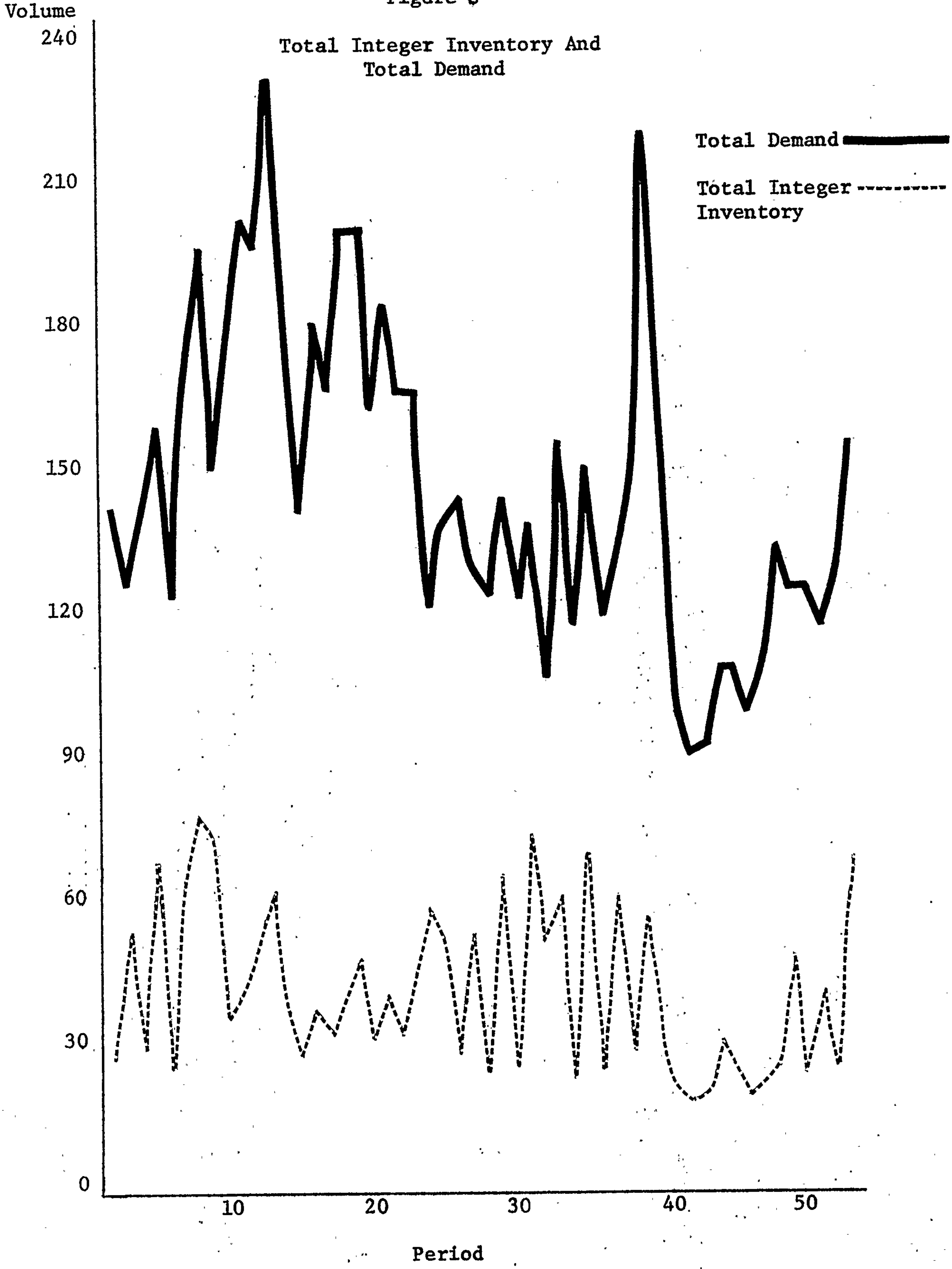

Volume
240
210
180
150
120
90
60
30
0
Total Actual Inventory And
Total Demand
Total Demand
Total Actual Inventory
10
20
30
40
50
Period

Figure 8

Total Integer Inventory And Total Demand



between demand and inventory can be found. Examination of the graph reveals that in almost every instance the inventory level moves in exactly the same manner as demand i.e. when demand goes down inventory goes down. Again, however this is because in the integer case there existed perfect knowledge of demand therefore enabling production and inventory levels to be planned accordingly.

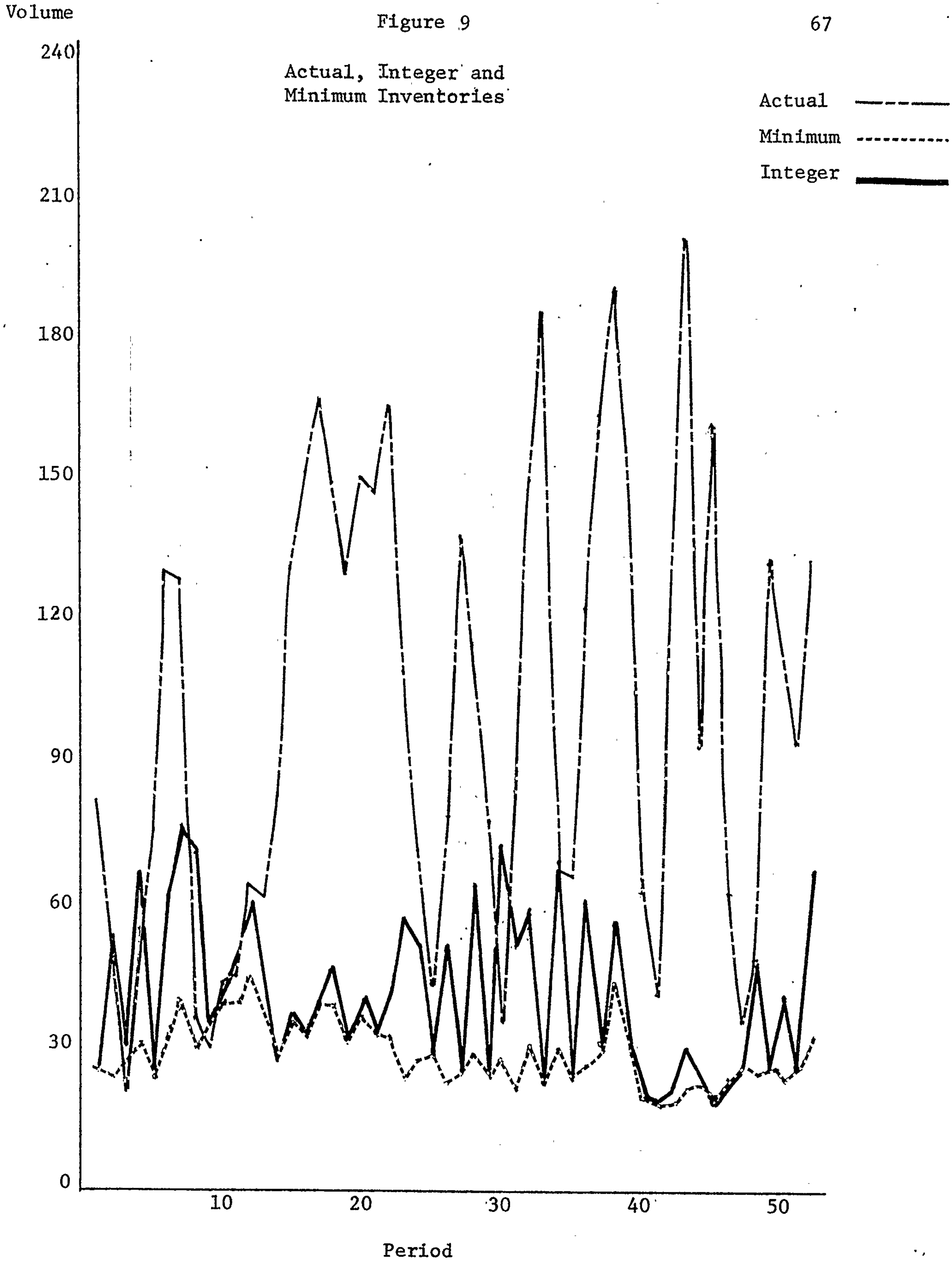
5.4.3. Actual, Integer and Minimum Inventories.

In figure 9, the total actual and integer inventories are plotted against the minimum inventory requirements. As will be recalled, the minimum inventory requirement in period i is equal to 1/5 of the demand in period i and 1. However, it should be noted that this level has no statistical validity and in a realistic situation would naturally have to be based on calculated safety stock requirements.

Looking first at the actual inventory, it can be seen that with the exception of periods 3 and 9, in which the actual inventory falls below the assumed minimum, the actual levels tend to be substantially higher than the assumed minimum. To further illustrate this point, it was calculated from Tables 4 and 6, Appendix 1 that on average, the minimum inventory requirement was 29,340 dozen as compared with an actual average of 102,631 dozen. However, when these levels are compared with the average weekly demand of 146,911 dozen the actual level appears much more favourable as it amounts to only 3.5 days average demand. When it is considered that it is extremely difficult to accurately forecast demands due to weather patterns it would seem that this level is not at all unreasonable.

Figure 9

Actual, Integer and Minimum Inventories



As can be expected, the integer level of inventory is almost always at a much lower level than is the actual and further is never less than the minimum requirement.[8] On average, the integer inventory was at a level of 41,701 dozen as compared to the average minimum of 29,340 dozen and the actual average of 102,631 dozen. Again, however, it must be pointed out that in the integer case there was perfect knowledge of demand. If demand were uncertain as it was in the actual case it is highly likely that the integer inventory levels would have been much higher.

5.4.4. Actual External Inventory Vs. Integer External Inventory.

The total external inventories for both the actual and integer cases can be found in Table 4, Appendix 1. In 35 of the 52 periods considered, external inventory i.e. inventory in excess of 70,000 dozen was actually carried. In sharp contrast, external inventory was necessary only 3 times in the integer case. During the periods in which external inventory was carried, the average level in the actual case was 58,709 dozen as compared to a mere 4,704 dozen in the integer. As will be seen in the next section, this amounts to a substantial dollar difference. But, if the perfect knowledge of demand had not existed in the integer case, it is likely that the difference would have been sharply reduced.

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[8] It should also be noted that every fourth period, the integer inventory drops to the same level as the minimum requirement, again demonstrating that an optimum solution was arrived at for thirteen four week periods as opposed to an optimum solution for the full 52 weeks.

5.5. Objective Function

5.5.1. Actual Production Costs Vs. Integer Production Costs.

For the purposes of this study, production costs are assumed to be labour costs only. For the 52 weeks under consideration the records show that there were 82,412 normal time and 2,990 overtime hours worked. Based on a rate of $3.50 per hour for normal time and $5.25 for overtime, this works out to a total actual cost of $304,140.

If production had been scheduled as it was in the integer case, it is found that the normal time hours worked would have been 76,360 along with 248 hours of overtime. Based on the same rates as above, this amounts to a total cost of $268,562. However, it will be recalled that if shifts 1 or 2 are utilized you still have to pay the men on lay-off 65% of their normal entitlement. Therefore, when this fact is considered, the total production cost is increased by $25,480 to $294,042, a saving of only $10,098 or 3.3% over the actual case.

5.5.2. Actual Inventory Costs Vs. Integer Inventory Costs.

In this study we are concerned with both inventory carrying costs and the costs of storing inventory in external storage. The inventory carrying costs are calculated by multiplying the total inventory (both internal and external) by a rate of $.0005 per dozen.[9] The total actual inventory carried over the period amounted to 5,336779 dozen. When this is multiplied by $.0005 it is found

[9] This rate is somewhat low but the difference would be minimal in terms of dollars.

that the total inventory carrying cost was $2,668.

While the carrying costs are minimal, the same cannot be said of the external inventory costs. Based on estimated shipments to external inventory of 798,819 dozen at a cost of $.03 per dozen, the total actual cost amounted to $23,965, giving a total actual inventory cost of $26,633.

In the integer case, the total inventory carried amounted to 2,168,437 dozen. At a rate of $.0005 the total carrying cost was $1,084. For the external inventory costs, only 10,899 dozen were shipped to external storage at a total cost of $327. The total inventory cost under the integer case amounted to $1,411, a saving of $25,222 over the actual case.

5.5.3. Total Actual Costs Vs. Total Integer Costs.

In the actual case, the total production and inventory costs amounted to $330,773. In the integer case they were $295,453, a saving of $35,320 or 10.7%. At first glance these savings appear interesting. But it should be obvious by now that a great deal more investigations would have to be undertaken before any solid conclusions can be reached.

Chapter 6

CONCLUSIONS

6.1. Results of Model

6.1.1. Production costs

Section 5.5.1. concluded that if production had been scheduled using the model that was developed in chapter 3, savings in production costs of $10,089 or 3.3% would have resulted. This is extremely interesting as at the outset of the study it was felt that the savings would be much greater. In actual fact, this may be the case. As will be recalled, the assumption was made that shift 3 was the permanent shift and therefore every time shifts 1 and 2 were worked,65% of the wages of those men on layoff had to be paid. Had shift 1 been the permanent shift and if the same shifts were selected, additional savings of $25,480 would have resulted. However, these savings can only be verified by running the problem using the new assumptions.

A second area that may or may not result in further savings is that the problem was optimized for each 4 week period as opposed to the entire 52 weeks. Again, the effect of this error can only be determined by rerunning the problem.

6.1.2. Inventory Costs

The total savings in the integer case amounted to $35,320 or 10.7%. Of this,savings in external inventory costs accounted for $23,638 or 67%. While these savings might appear interesting, they have to be considered with caution. For example, the main reason that external storage was not utilized to any great extent in the integer case was because of the perfect knowledge of future demand that existed. Under actual operating conditions this advantage would not exist and, because of the difficulties in accurately forecasting demand and the variances that can be caused by weather, it is almost certain that inventory levels would be substantially higher.

A second area that could cause problems is that of minimum inventory requirements. The safety stock level of one average day's demand would in all probability not be sufficient, as there are wide variations in daily demand patterns. If this safety stock level were higher, this would again have the effect of increasing inventory levels and hence the utilization of external storage.

In summary, it is likely that under actual operating conditions the savings resulting from the low utilization of external storage would be sharply reduced.

6.2. Structure of Model

If a computer model were to be utilized in the scheduling of production at the particular brewery being considered here, close attention would have to be paid to its structure. Computer costs for a linear programming model with integer constraints can be very high and therefore the number of the integer variables should be kept to a minimum. In developing this model the number of integer variables was kept to a minimum; however, it is likely that they could be further reduced by undertaking a study to determine the optimum utilization of manpower and thereby reduce the potential shift selections available.

To further reduce the problem size, the assumption could be made that there is only one brand as opposed to the three. In actual fact, one brand accounted for 91% of the total demand and therefore it is felt that it would be relatively easy to schedule the production of the other two brands by hand.

It may be however, that it is not necessary to have a model with integer constraints at all. The results of the continuous case are very similar to those of the integer case and, because of this, it is felt that the use of a straight linear programming model with no integer constraints would yield information that was just as valuable to management but at a lower cost. The only real difficulty that might be encountered is that in the continuous model the situation often arises where parts of two shifts are selected for production. However under actual operating conditions it should not be too difficult to intelligently choose which of the shifts to employ in its entirety.

Finally, it would be very interesting to analyse the impact of having a larger warehouse capable of carrying inventories sufficiently high to enable production to take place on one shift only for the entire year. But in this case, constraints would have to to built into the model to protect against the beer becoming too old.

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APPENDIX I

Table 1

WEEKLY DEMAND BY PRODUCT

(STATED IN EQUIVALENT DOZENS)

Period	PRODUCT A Actual Value	PRODUCT B Actual Value	PRODUCT C Actual Value	Total
1	129,389	8,410	4,390	142,189
2	113,431	8,883	3,814	126,128
3	128,237	7,138	3,380	138,755
4	143,480	11,641	4,029	159,150
5	111,290	5,975	5,519	122,784
6	145,741	13,545	9,404	168,690
7	176,775	13,108	5,512	195,395
8	132,172	14,951	5,330	152,453
9	166,173	10,782	4,741	181,696
10	183,786	10,983	4,923	199,692
11	177,254	13,918	5,536	196,708
12	210,110	14,709	6,543	231,362
13	180,054	11,321	6,748	198,123
14	130,478	7,580	4,399	142,457
15	160,728	13,389	5,938	180,055
16	148,578	14,425	4,426	167,429
17	182,463	12,125	5,651	200,239
18	179,229	14,460	6,031	199,720
19	146,679	11,487	4,750	162,916
20	164,791	10,298	7,913	183,002
21	145,568	14,193	6,786	166,547
22	150,504	9,737	4,318	164,559
23	107,272	10,765	3,483	121,520
24	126,659	9,019	3,724	139,402
25	130,914	7,518	4,964	143,396
26	120,223	6,770	3,923	130,916

(Cont'd next page)

Table 1 (Cont'd)

WEEKLY DEMAND BY PRODUCT

(STATED IN EQUIVALENT DOZENS)

Period	PRODUCT A Actual Value	PRODUCT B Actual Value	PRODUCT C Actual Value	Total
27	113,717	6,432	4,041	124,190
28	132,022	8,166	3,370	143,558
29	113,570	5,665	3,277	122,512
30	125,807	7,230	4,120	137,157
31	97,732	5,922	3,147	106,801
32	138,592	9,660	3,892	152,144
33	104,216	7,780	6,053	118,049
34	137,163	7,514	4,375	149,052
35	110,536	4,744	3,419	118,699
36	122,770	8,127	3,735	134,632
37	133,780	10,679	4,298	148,757
38	199,942	13,640	5,451	219,033
39	136,137	10,272	3,724	150,133
40	93,346	5,258	2,189	100,793
41	84,424	4,931	2,545	91,900
42	83,390	4,332	5,707	93,429
43	100,511	4,547	3,099	108,157
44	99,449	6,083	2,708	108,240
45	90,924	5,154	2,487	99,565
46	101,402	7,876	3,309	112,587
47	119,050	9,469	4,084	132,603
48	115,598	5,424	4,496	125,518
49	116,300	6,085	3,381	125,766
50	108,134	6,763	3,194	118,091
51	118,051	6,375	3,676	128,102
52	141,200	7,889	5,516	154,605

Table 2

PRODUCTION OF EACH PRODUCT
(STATED IN EQUIVALENT DOZENS)

Period	PRODUCT A		PRODUCT B		PRODUCT C	
	Actual	Integer	Actual	Integer	Actual	Integer
1	84,161	155,561	18,403	8,505	5,029	4,275
2	91,346	90,739	--	8,534	6,651	3,727
3	149,355	163,952	15,767	8,038	8,135	3,510
4	165,178	100,665	15,451	10,508	--	4,327
5	166,765	119,760	--	34,423	10,408	14,921
6	147,844	175,500	16,168	--	4,306	--
7	79,303	185,524	16,318	--	6,167	5,476
8	127,433	96,171	12,169	14,117	5,867	5,212
9	178,461	169,695	16,178	12,771	--	4,778
10	191,295	182,480	--	20,975	11,142	5,045
11	189,368	193,455	15,934	2,723	10,543	12,322
12	201,439	194,469	12,912	14,031	11,158	--
13	202,988	170,139	16,740	10,173	--	6,278
14	174,270	136,528	16,463	10,771	--	4,707
15	184,553	158,298	--	11,567	16,026	5,635
16	175,854	155,355	7,300	13,965	--	4,671
17	166,157	190,181	16,617	12,592	--	5,727
18	154,501	164,354	15,232	13,865	10,197	5,775
19	168,682	150,301	14,527	11,250	--	9,594
20	168,839	160,947	--	11,077	11,364	3,476
21	164,042	155,906	16,200	13,301	5,459	6,293
22	90,744	166,804	16,677	9,943	--	4,151
23	59,125	101,553	15,148	10,416	8,417	3,531
24	110,246	102,809	--	8,719	--	3,972
25	157,289	155,345	16,270	7,368	6,565	4,756
26	190,684	92,352	--	6,702	--	3,946

(Cont'd next page)

Table 2 (Cont'd)

PRODUCTION OF EACH PRODUCT
(STATED IN EQUIVALENT DOZENS)

Period	PRODUCT A Actual	PRODUCT A Integer	PRODUCT B Actual	PRODUCT B Integer	PRODUCT C Actual	PRODUCT C Integer
27	84,107	153,727	11,625	6,779	5,453	3,907
28	103,662	91,983	--	7,666	2,456	3,351
29	64,347	161,798	9,676	5,798	5,435	3,446
30	178,378	104,607	--	6,968	7,410	3,925
31	163,905	105,534	10,370	6,670	--	3,296
32	160,104	101,891	16,072	9,284	10,486	4,325
33	--	150,787	--	7,727	--	5,717
34	125,538	91,856	16,898	6,960	4,765	4,184
35	164,696	147,741	10,946	5,420	--	3,482
36	167,452	90,514	--	8,638	8,099	3,848
37	156,597	147,012	11,773	23,960	7,892	4,528
38	167,387	187,181	16,321	277	--	5,106
39	56,716	127,579	--	9,270	--	3,417
40	67,150	91,562	--	5,192	10,979	2,260
41	156,187	84,217	9,344	6,865	--	3,177
42	158,819	95,493	12,137	2,321	10,814	5,186
43	--	91,620	--	8,359	--	3,021
44	159,158	97,744	10,842	2,593	5,859	2,663
45	--	93,019	--	7,589	--	2,652
46	72,243	104,932	12,492	7,104	--	3,464
47	170,143	140,927	--	8,660	4,614	4,166
48	170,475	93,171	10,310	5,556	--	4,273
49	83,840	131,749	16,621	6,221	5,198	3,354
50	93,615	93,035	--	6,685	6,273	3,280
51	167,586	159,303	--	6,678	--	4,044
52	184,686	102,216	10,981	7,993	8,496	5,291

Table 3

TOTAL PRODUCTION AND

SHIFT SELECTIONS

	ACTUAL		INTEGER	
PERIOD	TOTAL PRODUCTION	SHIFT SELECTION	TOTAL PRODUCTION	SHIFT SELECTION
1	107,593	2	168,341	3
2	97,997	1	103,000	1
3	173,257	3	175,500	3
4	180,629	4	115,500	2
5	177,173	4	169,104	3
6	168,318	3	175,500	3
7	101,788	1	191,000	4
8	145,469	3	115,500	2
9	194,639	5	187,244	4
10	202,437	5	208,500	5
11	215,845	5	208,500	5
12	225,509	5	208,500	5
13	219,728	5	186,590	3
14	190,733	4	152,006	3
15	200,579	5	175,500	3
16	183,154	4	173,991	3
17	182,774	4	208,500	5
18	179,930	4	183,994	3
19	183,209	4	171,145	3
20	180,203	4	175,500	3
21	185,701	4	175,500	3
22	107,421	2	180,898	3
23	82,690	1	115,500	2
24	110,246	2	115,500	2
25	180,124	4	167,469	3
26	190,684	4	103,000	1

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Table 3 (Cont'd)

TOTAL PRODUCTION AND

SHIFT SELECTIONS

	ACTUAL		INTEGER	
PERIOD	TOTAL PRODUCTION	SHIFT SELECTION	TOTAL PRODUCTION	SHIFT SELECTION
27	101,185	1	164,413	3
28	106,118	2	103,000	1
29	79,458	1	171,042	3
30	185,788	4	115,500	2
31	174,275	3	115,500	2
32	186,662	4	115,500	2
33	--	-	164,231	3
34	147,201	3	103,000	1
35	175,642	3	156,343	3
36	175,551	3	103,000	1
37	176,262	3	175,500	3
38	183,708	4	192,564	4
39	56,716	1	140,266	3
40	78,129	1	99,014	1
41	165,531	3	94,259	1
42	181,770	4	103,000	1
43	--	-	103,000	1
44	175,859	3	103,000	1
45	--	-	106,260	2
46	84,735	1	115,500	2
47	174,757	3	153,753	5
48	180,785	4	103,000	1
49	105,659	1	141,324	3
50	93,615	1	103,000	1
51	167,586	3	170,025	3
52	204,163	5	115,500	2

Table 4

MINIMUM INVENTORY REQUIREMENTS

BY PRODUCT FOR START OF EACH PERIOD

(STATED IN EQUIVALENT DOZENS)

Period	Product A	Product B	Product C	Total
1	25,878	1,682	878	26,184
2	22,686	1,777	763	25,226
3	25,647	1,428	676	27,751
4	28,696	2,328	806	31,830
5	22,258	1,195	1,104	24,557
6	29,148	2,709	1,881	33,738
7	35,355	2,622	1,102	39,079
8	26,434	2,990	1,066	30,490
9	33,235	2,156	948	36,339
10	36,757	2,197	985	39,939
11	35,451	2,784	1,107	39,342
12	42,022	2,942	1,309	46,273
13	36,011	2,264	1,350	39,625
14	26,096	1,516	880	28,492
15	32,146	2,678	1,188	36,012
16	29,716	2,885	885	33,486
17	36,493	2,425	1,130	40,048
18	35,846	2,892	1,206	39,944
19	29,336	2,297	950	32,583
20	32,958	2,060	1,583	36,601
21	29,114	2,839	1,357	33,310
22	30,101	1,947	864	32,912
23	21,454	2,153	697	24,304
24	25,332	1,804	745	27,881
25	26,183	1,504	993	28,680
26	24,045	1,354	785	26,184

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Table 4 (Cont'd)

MINIMUM INVENTORY REQUIREMENTS

BY PRODUCT FOR START OF EACH PERIOD

(STATED IN EQUIVALENT DOZENS)

Period	Product A	Product B	Product C	Total
27	22,743	1,286	808	24,837
28	26,404	1,633	674	28,711
29	22,714	1,133	655	24,502
30	25,161	1,446	824	27,431
31	19,576	1,184	629	21,389
32	27,718	1,932	778	30,428
33	20,843	1,556	1,211	23,610
34	27,433	1,503	875	29,811
35	22,107	949	684	23,740
36	24,554	1,625	747	26,926
37	26,756	2,136	860	29,752
38	39,988	2,728	1,090	43,806
39	27,227	2,054	745	30,026
40	18,669	1,052	438	20,159
41	16,885	986	509	18,380
42	16,678	866	1,141	18,685
43	20,102	909	620	21,631
44	19,890	1,217	542	21,649
45	18,185	1,231	497	19,913
46	20,280	1,575	662	22,517
47	23,810	1,894	817	26,521
48	23,120	1,085	899	25,104
49	23,260	1,217	676	25,153
50	21,627	1,353	649	23,629
51	23,610	1,275	735	25,620
52	28,240	1,578	1,103	30,921

Table 5

OPENING INTERNAL INVENTORY
(STATED IN EQUIVALENT DOZENS)

Period	PRODUCT A Actual	PRODUCT A Integer	PRODUCT B Actual	PRODUCT B Integer	PRODUCT C Actual	PRODUCT C Integer
1	64,432	25,878	3,919	1,682	1,649	878
2	33,463	52,050	13,912	1,777	2,288	763
3	11,378	29,358	5,029	1,428	5,125	676
4	32,496	65,073	13,658	2,328	9,880	806
5	46,681	22,258	17,468	1,195	5,851	1,104
6	47,767	23,041	11,493	29,643	10,740	10,506
7	50,242	52,800	14,116	16,098	5,642	1,102
8	14,300	65,944	17,326	2,990	6,297	1,066
9	9,561	33,235	14,544	2,156	6,834	948
10	21,849	36,757	19,940	4,145	2,093	985
11	29,358	35,451	8,957	14,137	7,699	1,107
12	41,472	51,652	10,973	2,942	12,706	7,893
13	32,801	36,011	12,564	2,664	17,321	1,350
14	41,444	26,096	17,983	1,516	10,573	880
15	36,960	32,146	26,866	4,707	6,174	1,188
16	40,261	29,716	13,477	2,885	16,262	885
17	51,787	36,493	6,352	2,425	11,861	1,130
18	52,776	44,211	10,844	2,892	6,380	1,206
19	47,838	29,336	11,616	2,297	10,546	950
20	49,548	32,958	14,656	2,060	5,796	5,794
21	56,395	29,114	4,358	2,839	9,247	1,357
22	55,715	39,452	6,365	1,947	7,920	864
23	53,093	55,752	13,305	2,153	3,602	697
24	43,776	50,033	17,688	1,804	8,536	745
25	29,799	26,183	8,669	1,504	4,812	993
26	46,166	50,614	17,421	1,354	6,413	785

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Table 5 (Cont'd)

OPENING INTERNAL INVENTORY
(STATED IN EQUIVALENT DOZENS)

Period	PRODUCT A Actual	PRODUCT A Integer	PRODUCT B Actual	PRODUCT B Integer	PRODUCT C Actual	PRODUCT C Integer
27	56,859	22,743	10,651	1,286	2,490	808
28	50,254	62,753	15,844	1,633	3,902	674
29	59,334	22,714	7,678	1,133	2,988	655
30	19,442	67,730	11,689	1,446	5,146	824
31	57,105	49,742	4,459	1,184	8,436	629
32	55,804	57,544	8,907	1,932	5,289	778
33	42,798	20,843	15,319	1,556	11,883	1,211
34	55,482	67,414	7,539	1,503	5,830	875
35	43,857	22,107	16,923	949	6,220	684
36	44,074	59,012	23,125	1,625	2,801	747
37	47,812	26,756	15,023	2,136	7,165	860
38	43,124	39,988	16,117	15,417	10,759	1,090
39	45,894	27,227	18,798	2,054	5,308	745
40	53,540	18,669	8,546	1,052	1,584	438
41	27,344	16,885	3,288	986	10,374	509
42	54,387	16,678	7,701	2,920	7,912	1,141
43	41,475	28,781	15,506	909	13,019	620
44	49,121	19,890	10,959	4,721	9,920	542
45	41,211	18,185	15,718	1,231	13,071	497
46	42,810	20,280	9,568	2,666	10,584	662
47	13,651	23,810	14,184	1,894	7,275	817
48	57,480	45,687	4,715	1,085	7,805	899
49	56,994	23,260	9,601	1,217	3,405	676
50	44,641	38,709	20,137	1,353	5,222	649
51	48,325	23,610	13,374	1,275	8,301	735
52	58,360	64,862	7,011	1,578	4,629	1,103

Table 6

TOTAL OPENING INVENTORY

(STATED IN EQUIVALENT DOZENS)

Period	TOTAL OPENING INTERNAL Actual	TOTAL OPENING INTERNAL Integer	TOTAL OPENING EXTERNAL Actual	TOTAL OPENING EXTERNAL Integer	TOTAL Actual	TOTAL Integer
1	70,000	28,438	14,259	--	84,259	28,438
2	49,663	54,590	--	--	49,663	54,590
3	21,532	31,462	--	--	21,532	31,462
4	56,034	68,207	--	--	56,034	68,207
5	70,000	24,557	7,513	--	77,513	24,557
6	70,000	63,190	61,902	--	131,902	63,190
7	70,000	70,000	61,530	7,687	131,530	77,687
8	37,923	70,000	--	3,212	37,923	73,212
9	30,939	36,339	--	--	30,939	36,339
10	43,882	41,887	--	--	43,882	41,887
11	46,014	50,695	--	--	46,014	50,695
12	65,151	62,487	--	--	65,151	62,487
13	62,686	40,025	--	--	62,686	40,025
14	70,000	28,492	14,291	--	84,291	28,492
15	70,000	38,041	62,567	--	132,567	38,041
16	70,000	33,486	83,091	--	153,091	33,486
17	70,000	40,048	98,841	--	168,841	40,048
18	70,000	48,309	81,546	--	151,546	48,309
19	70,000	32,583	61,756	--	131,756	32,583
20	70,000	40,812	82,049	--	152,049	40,812
21	70,000	33,310	79,250	--	149,250	33,310
22	70,000	42,263	98,404	--	168,404	42,263
23	70,000	58,602	41,266	--	111,266	58,602
24	70,000	52,582	2,436	--	72,436	52,582
25	43,280	28,680	--	--	43,280	28,680
26	70,000	52,753	10,008	--	80,008	52,753

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APPENDIX II

SUMMARY OF COMPUTER RUNS

In the preceding chapters, only the results of the final computer runs are presented. However, to arrive at these results took approximately 2 years and many, many failures.

At the outset, a system of equations was set up and, using simplified data an attempt was made to arrive at the continuous solution only. As can be expected this took some time due to being unfamiliar with the package and also because of certain assumptions made in the equations. As an example of the latter point, a constraint was set up such that there was an option as to whether or not a particular brand was to be produced. However, if a brand was produced, then there was a minimum quantity that had to be produced no matter what the demand and inventory situation was. This constraint was very realistic but for some reason the package would not accept it, therefore, it had to be relaxed. However, what may not be obvious here is that it took several runs and a great deal of time to discover that it was this particular constraint causing the problem.

The first plateau reached was that of arrived at a continuous solution for a simplified set of data. Following this, the integer constraints were introduced for this simplified set of data and naturally, the problem wouldn't work. After a great deal of time and experimentation, it was discovered that the problem was caused because the sum of the shifts (equation 3.2, page 27) was defined as being equal to or less than one. This constraint caused no problem in the continuous case but would not run in the integer case. Finally, however, an integer solution was reached.

Up to this point, the matrix generator had not been employed but, because of the large number of constraints it was decided to utilize it. After a trial and error period this tool was successfully utilized and it was felt the time had come to attempt to solve the problem using actual data. However, the matric generator wouldn't work, the reason being that the computer company has dropped the matrix generator described in the manual and supported a new one. Unfortunately, no one knew how to make this new matrix generator run and two months had elapsed before the problem was finally solved.

After nearly two years, an external solution using actual data was finally obtained for one four week period. In terms of computing time it took slightly less than 1 minute or approximately $16.00 for each four week solution. As was mentioned earlier, it was decided to run the problem 13 times in groups of four weeks as it was feared that the problem time and therefore costs would increase exponentially if an attempt were made to solve the problem for the full 52 weeks.

In retrospect, it would have been desirable to make changes and run the problem again under new assumptions. But this would have meant several additional runs and since the problem could not be run on university computers this was not possible.