GRODECS , THE GRODEC ACTION POTENTIAL , AND GRODEC-BASED NEURAL NETWORKS

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Abtract Grodecs are abstract growth decay entities that are related to oscillators. The switching behaviour of grodecs has been analysed in detail and it is shown that grodecs can be used as the basis of both an unconventional type of neural net called a grodec stack machine, as well as conventional neural nets.

A single grodec has the most of the switching properties of a short section of biological membrane, and can give rise to action potentials when properly set. A pair of grodecs can generate a much sharper action potential than can a single grodec, and this mechanism is very like that of a neuron with dual Na and K ion switching, so that a neuron membrane can be regarded conceptually as a Na grodec coupled to a K grodec. Grodecs can be coupled in positive (excitory) or negative (inhibitory) manner to form logic circuits. Examples of such circuits are given, and show that grodecs can be the basis of conventional neural nets and sequential machines in general. Grodecs can also be coupled in an unconventional manner to form grodec stack machines. In such machines the action potentials sum in periods of growth and decay, so that a central machine pressure parameter exhibits time function behaviour similar to that of a fractal growth-decay time function, such as an En:m function. Although the manner of the summation is much simpler than in grodec stack machines, action potential summation does occur in nature, and causes the large voltages generated by electric fish.

1.0 Introduction

Although grodecs, or growth decay entities, were developed as abstract entities for the construction of grodec stack machines [2] with time fractal behaviour [1, 9, 10, 14], it turns out that the grodecs defined also have the properties of neuron membranes or neuramines [6, 12]. Grodec stack machines are thus neural nets, although of a special kind. The difference between conventional neural nets and grodec stack machines is that in the grodec stack machines the pressure potentials or action potentials are summed instead of being allowed to propagate only as discrete action potentials in a network that permits logical operations. In this paper we analyse grodecs and compare grodec stack machines with conventional neural networks. As we shall see, grodecs are related to oscillators, but have switching properties.

1.1 Oscillator versus grodec concepts

A simple oscillator generates a sinusoidal magnitude with a single natural frequency of oscillation, and a complex oscillator generates a complex repeating pattern, which is the summation of a collection of simple oscillations. A simple oscillator requires two energy stores that contain a fluid and an inertial connection. Some well known examples are :

(a) Two electrical capacitors with an inductive link:

An initially positively charged capacitor is one store, and an initially negatively charged capacitor is the other, connected in series through an inductor. The two capacitors will initially try to neutralize each other, giving rise to a current of electric charge, which overshoots because of the inertia of the inductance, causing the charges of the two capacitors to reverse; then the whole process resumes in the opposite direction giving rise to the oscillation.

(b) A closed cylinder with a piston dividing the cylinder into two partitions or stores of gas.

If one gas compartment (one store) is initially compressed by the piston, on release the piston will recoil; because of inertia it will then compress the gas of the other compartment (the other store), and so on in an oscillation. In the limit the piston can be weightless, so that it is the inertia of the fluid (the gas) that gives rise to the oscillation.

(c) A U-tube containing a liquid.

If initially one of the two tubes (the stores) has a higher level of liquid that the other, fluid will flow to the other tube, with inertia causing an overshoot, so that an oscillation results.

Oscillators have been intensively studied in engineering. A closely related entity is the *growth decay entity*, or *grodec.* For every kind of oscillator there seems to be the possibility of constructing a grodec. Like the oscillator a grodec has two stores that can each hold a fluid, and energy that tends to drive the fluid between the stores. But unlike the oscillator the link is not two way inertial, but one-way switched, resulting in behaviour that is quite different from that of an oscillator.

1.3 Growth-decay entities or grodecs

A growth decay entity or grodec, as envisaged in this paper, is an abstraction of the simplest examples of the various kinds of physical entity - electrical, hydraulic, gaseous - that can exhibit repeated growth and decay phenomena. We can define two distinct types of grodecs: active grodecs and passive grodecs, as will be explained presently.

A simple grodec is formally a 6-tuple $<c,L,R,g,k,i>$, capable of reacting to a flow F of some abstract fluid. L and R are variables; the other attributes are constants for a specific grodec. The attributes of the tuple of the grodec are always positive. The flow F can be either positive or negative.

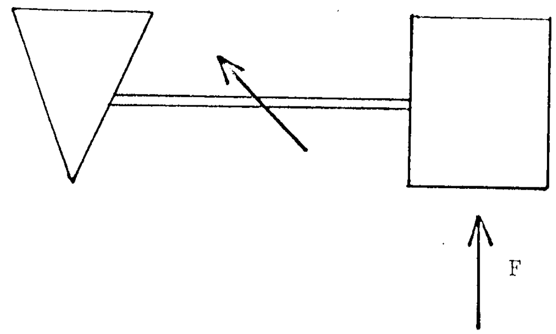


Figure 1a

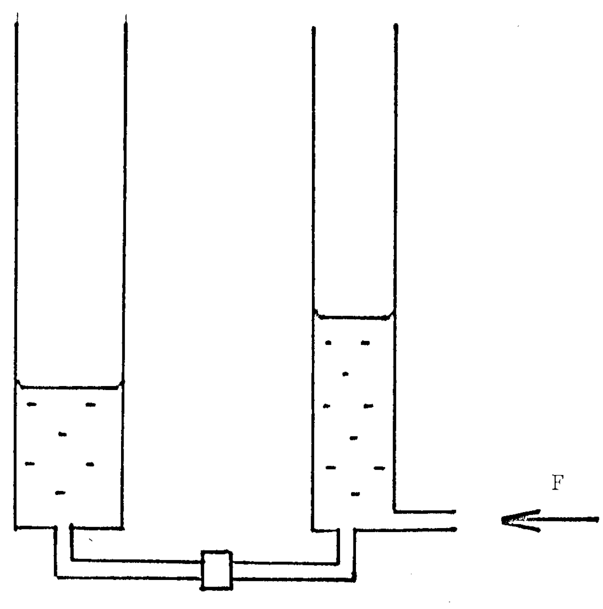


Figure 1b

A grodec can be visualized as two balancing stores - a right store and a left store, connected by a flow line in which there is a switch. In diagram (Figure 1a), the left store is depicted as a triangle and the right store as a square, to make distinction easy. R is the measure of fluid quantity for the right store and L for the left store. The capacitance of the right store can be taken as unity in arbitrary units; the capacitance of the left store is c in the same units, so that essentially c is the relative capacitance of the left store with respect to the right store. Capacitance is to taken as the measure of how much fluid is required to generate a unit of pressure on a store, so that quantity of fluid Q relates to pressure P by $Q = cP$. Thus the pressure of the right store must be R and that of the left must be L/c. When the pressures on the two stores are equal, which occurs when $L/c = R$, there is normally no inclination for fluid to flow between the stores. If the pressure on the right store exceeds that of the left there is an inclination for fluid to move from right to left, and vice versa.

Fluid can move from right to left stores only if the pressure difference between right and left stores has climbed to a trigger value g. Thus g is the trigger pressure that opens the switch, and which prevents the pressure in the right store rising by more than g with respect to the left store. Once the switch has been triggered a steady flow i from the right store ensues, until both (a) the right store pressure has fallen to the level where $R - L/c = 0$, and (b) the original pressure of the right store has decayed by a fraction 1-k of the original pressure difference, that is, by a pressure drop of g(1-k), at which point the switch closes. Thus following triggering of the switch because of a pressure difference g, the two stores will come into pressure equilibrium, that is, $R = L/c$, and the right store pressure will have fallen by g(1-k) and the left store pressure will have risen by gk.

We define k as the decay fraction for a grodec, that is, k is that fraction of the trigger pressure g remaining in the right store following a pressure decay.

1.4 Grodec implementations

It should be understood that a passive grodec is an abstraction, and that no physical entity is likely to function exactly like a grodec. A grodec merely has the essential properties of similar physical entities.

A physical implementation, whether electrical, hydraulic or gaseous, of a grodec may differ in detail, but not in principle, from the abstract grodec. It is the essential features of these physical systems that are abstracted in a grodec, namely that each store can hold a fluid, that the pressure in a store increases with the quantity of fluid, and that a specific threshold pressure difference between the two stores will cause a switch to open and allow fluid to flow from right to left, the flow stopping when (a) the pressure difference falls to a specific level to a specific, which can be zero, and (b) a specific pressure drop of the right store has occurred. In an implementation, the fluid could be an electron gas or hole gas as in a metal or semiconductor, a mobile ion concentration as in an electrolyte or membrane, a liquid, or a gas. In the grodec the pressure in a store is assumed for simplicity to increase linearly with quantity of fluid, but linear increase is not an essential feature; similarly, in a grodec the flow between stores is assumed for simplicity to be constant, but this constant flow also is not an essential feature.

As mentioned above grodecs relate to oscillators. The grodecs dealt with in this paper are abstractions, but to help visualize one, the hydraulic version in Figure 1b is useful. The left and right store are vertical columns containing a liquid under gravitational pressure. The columns are connected by a tube containing a pressure switch. When the pressure difference between the right and left colums reaches g, the switch opens, and fluid flows from right to left, until the height (proportional to pressure) in each is equilibrated. Store capacitance is proportional to column cross-section area.

A grodec can be a component in a system of grodecs, and can give rise to phenomena that can be both complex and appealing. For example, pressure in a stack of passive grodecs can trace out a growth decay fractal time function that is an En:1 fractal time function. A grodec also has most of the properties of a short section of neuron membrane so that grodecs can form conventional neural nets

1.5 The neuron action potential and the grodec model of a neuron.

The most obvious example of grodecs in nature is in the grodecs of neurons. But first a brief summary of neuron structure. A typical neuron has a main body or the *soma*, with protruding *dendrites* that recieve signals, at locations called *synapses*, from other neurons. Signals received at dendrites are integrated in the soma to trigger an action potential that travels along a long *axon* emanating from the soma. At the end of the axon away from the soma and dendrites the axon typically branches. The axon branches, called *terminations*, end on the dendrites of other neurons (or at muscles), the joint between an axon termination and a dendrite of another neuron being the synapse or coupling location between two neurons. The long axon propagates action potentials, and to speed up propagation, the axon is often enclosed in an insulating *myelin sheath* [6].

Consider a myelinated neuron axon, that is, an axon sheathed in myelin except for gaps, like an insulated copper wire where the insulation has been removed (at the gaps) at regular intervals. [These gaps are known as the *nodes of Ranvier*.] In essense, a myelinated neuron can be considered as a sequence of grodec pairs, one grodec of the pair having sodium ion concentration as the fluid and the other having potassium ion concentration as the fluid. There is a grodec pair at each sheathing gap or node where the membrane is exposed to both inter and intra cellular fluids, which, because of different ionic conductances through the membrane give rise to electrical potentials accross the membrane.

Outside the membrane the concentration of Na is high and of K is low and inside Na is low and K is high, so that Na tends to diffuse inward along with other matching negative Cl ions, while K tends to migrate out [6]. However the membrane at a gap in the myelin sheath is relatively impermeable to Na and K but permeable to negative CL ions, so that negative ions can migrate inwards only in sufficient quantity to make the inside of the membrane electrically negative with respect to the outside. Seen as an Na grodec the inside is the left store, the outside is the right store, and the membrane is the connection link containing an electrical pressure (millivoltage) activated switch; the converse is true for the K grodec. The Na and K pressures are set close to the trigger pressures, and when a small increase in pressure accross the membrane is occasioned by passing a small (microamp) current, the Na and K switches trigger, and the membrane becomes permeable first to Na and then to K, causing Na ions to flow strongly in and K to flow out, which abruptly causes the inside electrical potential to go positive. The local equilibration of ion concentrations close to the membrane then causes the Na and K switches to turn off, Na and K ion flows from the background concentrations further from the membrane ensue to restore the K and Na concentrations close to the membrane to what they where before the switches triggered and the grodecs are reset, ready to be triggered again. During the action potential, at a myelin gap, the inside postive potential causes a current to flow to the next myelin gap, triggering the membrane at that gap, which in turn causes the next gap to trigger, and so on in an ionic domino effect as the action potential travels from sheathing gap to sheathing gap along the neuron.

A equivalent neuron model can be constructed using just one grodec type. In fact the reader not versed in the nuances of elecrochemical potentials accross membranes and membrane ionic conductances will likely get a much clearer insight into the working of a neuron from the grodec model, which is almost trivially simple.

Suppose a grodec is set with pressure difference R-L/c just under the trigger pressure g, so that a small amount of fluid entering the right store will trigger the grodec. So, suppose we add some fluid to R. The switch triggers, fluid flows from right to left as the pressure difference collapses, and the two pressures equilibrate.

Such a model is not quire complete, however, for it will permit just one firing, since once triggered it can not fire again unless the pressure differences are reset. This will happen automatically if the right store is connected through a poorer fluid-conducting link to a large pressure source or reservoir at the same pressure R as the right store. Similarly, the left store can be connected via a poorer fluid-conducting link to a large pressure sink at the same pressure L/c as the left store. Repeating our experiment once more, if R-L/c is again just less than g, adding a small quantity of fluid to R will trigger the grodec, and the pressure will collapse; and then the switch will close when the pressure difference R-L/c reaches zero. But now the source and sink will respectively add further fluid to the right store and remove fluid with the left store while the switch remains closed, thus resetting the initial grodec pressure difference close to g. As a result, the grodec can be triggered repeatedly, just like a neuron, and each time it will automatically reset itself, and when the grodec is triggered it will execute a pressure trace very similar to the millivoltage trace of the action potential in many biological membranes, that is, a quick initial collapse in pressure and a slower recovery. A model with a faster recovery can be constructed using two grodecs, as we shall see later.

It goes almost without saying that since neurons and logical equivalents are the building blocks of biological brains and conventional neural nets, then grodecs must be fundamental. We begin by showing how grodecs and simple grodec stacks can exibit time fractal behaviour, then we analyse grodec action potentials, and finally consider how grodecs can be combined to form logic circuits, that is, to form conventional neural nets.

2.0 En:1 and En:m functions and the n-level grodec stack

To begin, we examine in detail the behavior of a single passive grodec under an inflow F. Then we examine a one-sided 2-level grodec stack, and then extrapolate to a one sided n-level grodec stack, where n is a large integer. As we shall see, under inflow F, depending on the setting for the trigger pressure g of the grodecs of the stack, the pressure R of the right store of the grodec stack can trace out the pattern of an En:m function for which m = 1. An En:1 function is one made up of growth and decay segments, in which each growth segment subdivides into n segments [1,2] in an endless fractal manner. A decay segment does not subdivide, as we illustrate presently. The particular value of n in the function depends on the values chosen for g in each grodec of the stack. Finally we look at how grodec machines can exhibit En:m function behaveiour, where m is not 1, that is where each decay segment subdivides into m segments.

2.1 A single passive grodec under constant inflow F

A grodec, when fed from the right by a steady flow F, will experience successive periods of growing right store pressure R each terminating when the switch opens, causing a subsequent period of decaying right store pressure R, which in turn terminates when the switch closes, allowing growth to resume. Both the growth and the decay periods are periods of linear pressure change with time (Figure 2). During a decay period fluid flows at a steady rate i from right to left, so that during this period the fluid in the right store decays at a rate i - F, if F remains on while the switch is open. However, in practice we shall only consider grodecs where the inflow F turns off while any grodec switch is open.

The growth curve for R with regular periods of decay, as in Figure 2, can be computed precisely from the attributes of the grodec. Suppose we initialize L = R = 0. At time t = 0, the fluid inflow F is turned on positively. The value of R increases at a rate of F units per second for a period of g/F seconds, so that R peaks at a value g at time t = g/F. At this point the switch opens, causing the right store pressure R to decay at a rate i-F and the value of L to increase at a rate i. The decay will stop when the right and left store pressures are equal, that is, at time T, measured from the beginning of the decay period, given by:

$$g - T(i - F) = Ti/c$$

that is, at time $T = cg/(ic + i - Fc)$

At this point $R = L/c = Ti/c = gi/(ic + i - Fc)$

Measured from the beginning of the initial growth of the right store the time t at the end of the decay period is

$$t = g/F + cg/(ic + i - Fc)$$

Following the decay, growth in R resumes at rate F until $R = g + gi/(ic + i - Fc)$, at time $t = 2g/F + cg/(ic + i - Fc)$. The ensuing decay period stops at the level given by $R = 2gi/(ic + i - Fc)$, at time $t = 2g/F + 2cg/(ic + i - Fc)$.

In general, following n periods of growth, the peak coordinates for the right store are:

$$R = g + (n-1)gi/(ic + i - Fc) \quad \text{....(1)}$$

$$t = ng/F + (n-1)cg/(ic + i - Fc) \quad \text{... (2)}$$

Following n periods of decay, the trough coordinates for the right store are:

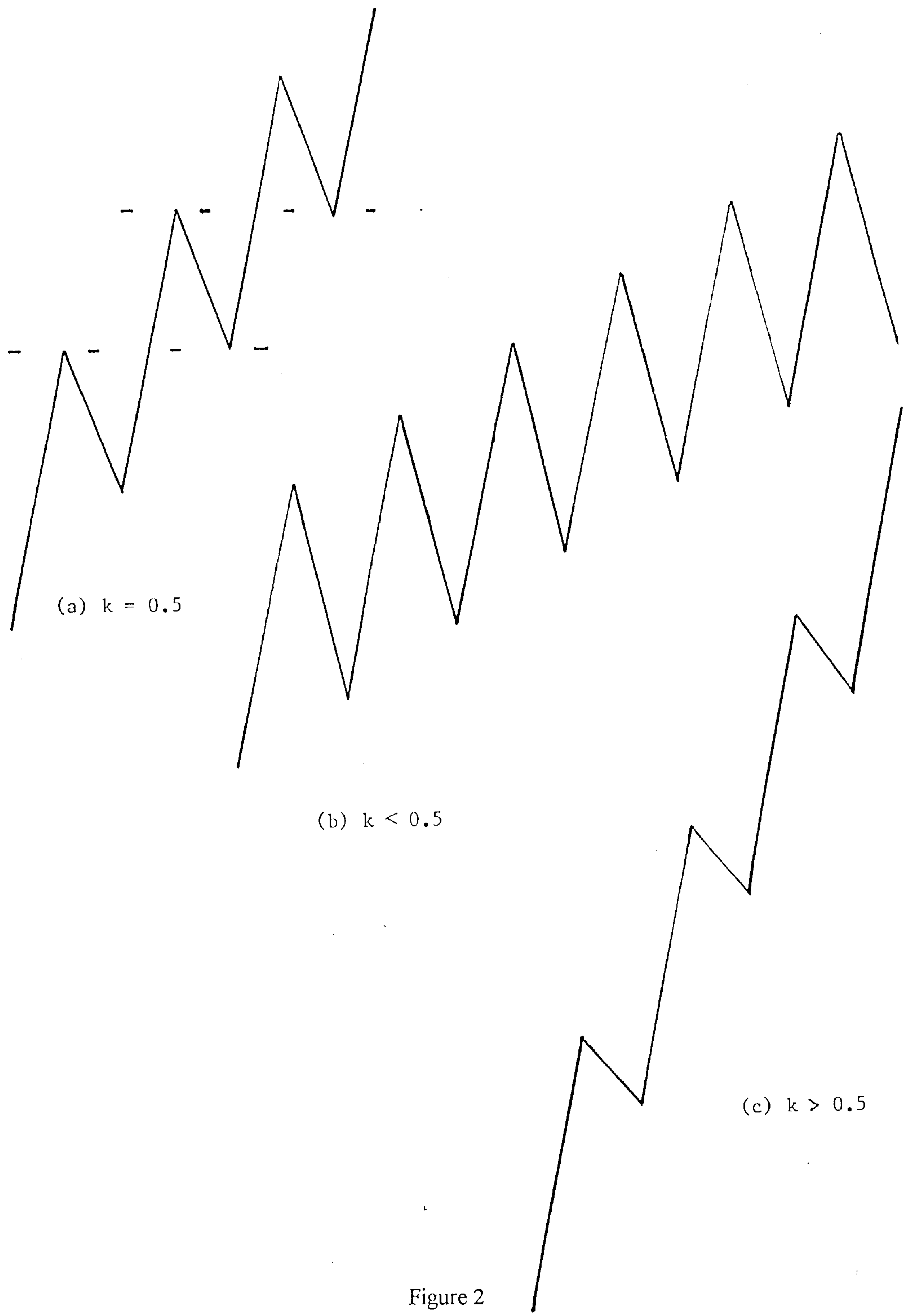


Figure 2

$$R = ngi/(ic + i - Fc) \quad \dots (3)$$
$$t = ng/F + ncg/(ic + i - Fc) \quad \dots (4)$$

A simpler and more appealing situation ensues if we assume that F is turned off during a decay period, and this will be assumed in all cases henceforth. If this is the case, following n periods of growth, the peak coordinates for the right store are:

$$R = g + (n-1)gi/(ic + i) = g + (n-1)g/(1+c) \quad \dots (5)$$
$$t = ng/F + (n-1)cg/(ic + i) = ng/F + (n-1)cg/i(c+1) \quad \dots (6)$$

Following n periods of decay, the trough coordinates for the right store are:

$$R = ngi/(ic + i) = ng/(1+c) \quad \dots (7)$$
$$t = ng/F + ncg/(ic + i) = ng/F + ncg/i(1+c) \dots (8)$$

Since a decay period reduces an initial pressure g to $g/(1+c)$, which must also be kg, we must have:

$$k = 1/(1+c)$$

so that we must make the setting on the grodec

$$c = (1-k)/k \quad \dots (9)$$

If k and c in the grodec are set to obey this condition, then the grodec right store pressure will always be reduced by exactly $g(1-k)$ in moving to equilibrium with the left store when the switch is triggered. This condition is not important right now in the more elementary parts of our development, but becomes critical later on. Notice that if $k = 0.5$, then $c = 1$, so that R retraces exactly half of g following a switch trigger (Figure 2a), which is what one would expect. Also if $c > 1$, then the decay fraction k is less than 50% (Figure 2b), and greater than 50% if $c < 1$ (Figure 2c).

Notice that if the flow F is turned off during decay periods the pressure peak and trough pressures R do not depend on F, but only on the attributes of the grodec. Notice also that if $c = 1$, equations (5) and (7) for peak and trough pressures reduce to:

$$R = g(1 + (n-1)/2) \quad \dots (10)$$
$$R = ng/2 \quad \dots (11)$$

This gives us a series of peak and trough pressures:

$$g, g/2, 3g/2, g, 2g, 3g/2, \dots$$

so that each decay period retraces exactly half of the previous growth period (Figure 2a).

2.2 A one-sided 2 level-grodec stack

In this section we look briefly at a simple system of grodecs, namely a one-sided grodec stack, involving just 2 grodecs. In such a stack the right stores are common, such that the pressures of the right stores and their fluid quantities are equalized at all times. In such a stack one grodec can be said to be inner with respect to the other or outer grodec. The inner grodec is grodec 1 and the outer grodec 2 (Figure 3a). The left stores have the same pressures only when the system is in equilibrium. Remembering that we are dealing with

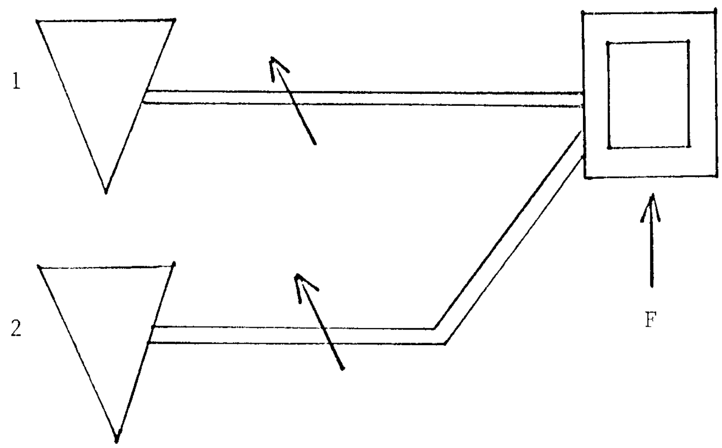


Figure 3a

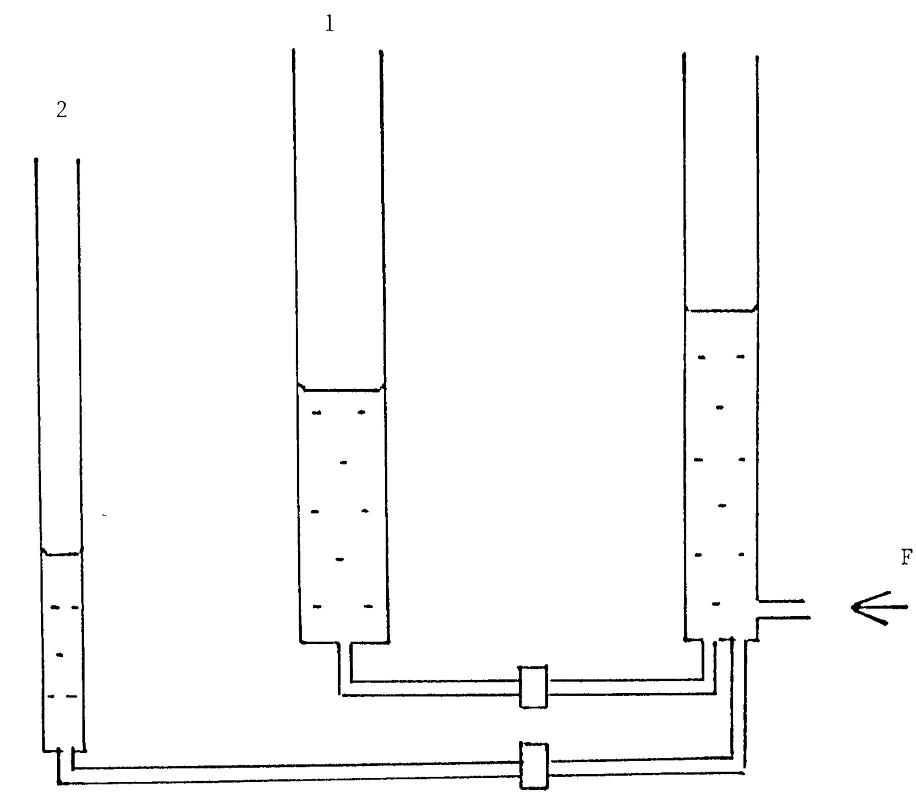


Figure 3b

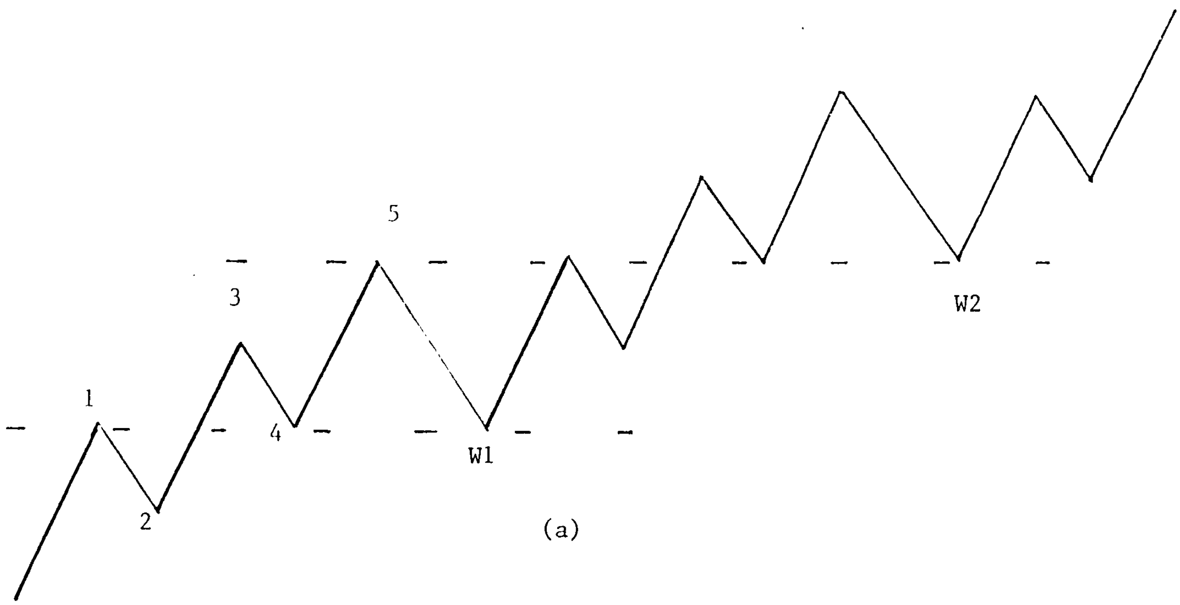
an abstraction with specific rules for the behaviour of that abstraction, the reader may nevertheless be helped by visualizing a grodec in terms of the columns of liquid in Figure 2b. We can put two of these liquid column grodecs together to form a 2-grodec stack by merging the right colums and having two separate left colums or stores each connected to the common right column or store by a tube containing a pressure switch (Figure 3b).

If fluid is fed into the right common store, the outer right store and the inner right store of the two grodecs automatically have the same increased pressure, but the pressures in the left stores do not change until the appropriate switches open and fluid flows from right to left. Furthermore, the grodecs are so selected that threshold pressure differences for opening switches are quite different in the inner and outer grodecs. In the inner grodec, the threshold switching pressure difference g_1 is considerably smaller than g_2 for the outer grodec. This means that as the pressure in the right stores increases under a positive inflow F, the threshold pressure difference g_1 for opening the switch to allow the inner left store to equilibrate will be reached long before the threshold pressure difference g_2 is reached for opening the switch for allowing fluid to flow into the outer left store and equilibrate the pressure there.

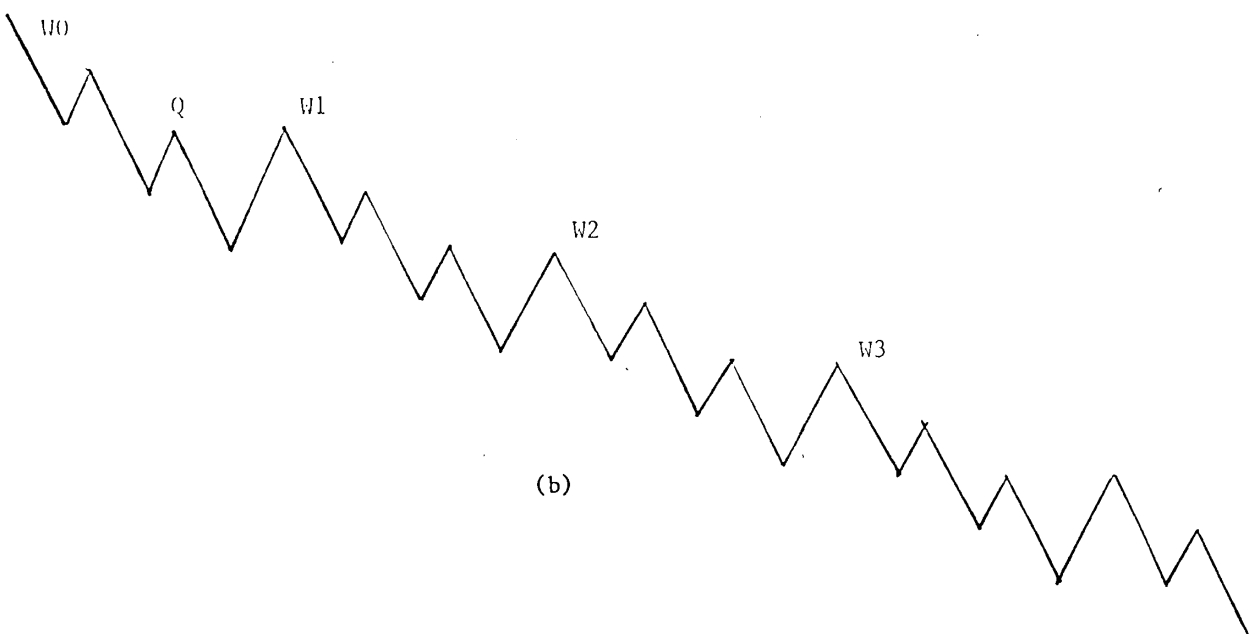
With this two-grodec stack, the inner grodec 1 can be denoted by $<c_1, L_1, R, g_1, k_1, i_1>$, and the outer grodec 2 is denoted by $<c_2, L_2, R, g_2, k_2, i_2>$. We assume that initially right and left pressures in the grodecs are equal , so that
$L_1/c_1 = L_2/c_2 = R = H$ where H is the initial pressure (which may also be taken as zero). Suppose now, we begin feeding fluid into a right store at a rate F. A possible trajectory of R is shown in Figure 4a. For this trajectory to actually occur, the constants c_1, g_1, i_1 and c_2, g_2, i_2 must be appropriately set. However, for the moment merely assume that g_2 is much larger than g_1.

A full analysis of this machine is beyond the scope of this paper [see], but the essentials are as follows. Initially the inner and outer grodec switches are closed and the only fluid flowing into the right stores is due to F. This fluid flowing into the right raises the pressure R until it reaches g_1, causing the inner switch to open, thus causing the pressure R to fall and L_1 to rise until equilibrium is reached. At this point the inflowing fluid F causes R to resume its increase until the pressure difference between right and left for grodec 1 again reaches g_1 and again we have a decay period, following which R resumes its climb until the pressure difference between right and left store of the inner grodec is just under g_1. At this point we have reached the third peak (P) in Figure 4a.

The pressure in the left store of grodec 2 has been so far unaffected and is still at the initial value H. But now, suppose that the pressure R at the third peak is such that the pressure difference between the right and left stores of grodec 2, that is, $R - L_2/c_2$, is equal to g_2, the threshold for opening the switch in outer grodec 2, to equilibrate the pressure in both stores of grodec 2. If g_2 is thus set, the switch in the outer grodec may be set to open either (a) just before the switch in the inner grodec can open (the "just before opening" or JBO case) or (b) at the same time as the switch in the inner grodec opens (the "same time opening" or STO case" case). We consider only JBO, which is the technically simpler case, in this paper. The opening of the outer switch causes a much longer decay period and a deeper decay of the pressure R, than was the case for the decays associated with the



(a)



(b)

Figure 4

opening of the inner switch as illustrated in Figure 4a, for k equal to 50%. We can also have k less than or equal to 50%, depending on grodec settings.

At the end of this more prolonged decay period (point W1 in Figure 4a), the outer left and right stores of both grodecs have equilibrated and the switch in the outer line closes, as does the switch in the inner. At this point (point W1 in the diagram) the pressures in all four store are in equilibrium.

At point W1, conditions are the same as at point W0, except that overall pressure levels are higher. If the right stores continue to be fed by F, the pressure trajectory from W0 to W1 will exactly replicate, from W1 to W2, and continue to replicate, from W2 to W3, and so on. All that has happened is the the inner grodec has modulated the growth and decay periods of the outer grodec, the extent of the modulation depending on the attributes of the grodec

In Figure 4a the grodec attributes are assumed to be such that we get two decay periods in each outer grodec growth period. With different grodec parameters we would get different modulation, such as 3 decay periods within a growth period.

For the growth period modulation in Figure 4a to occur, the following, from equation (5), must hold

$$g_2 = g_1 + (3-1)g_1/(c_1 + 1) \qquad \text{... (12)}$$

This also ensures that the decay factor in the trace (fraction of a previous segment that does not decay) is the same everywhere in the trace in Figure 4a, and that the modulation will replicate exactly when F resumes after equilibrium is reached. The trace in Figure 4a is two subdivision levels of the trace of an E5:1 function. In an E5:1 function growth segments subdivide into 5 smaller segments, 3 growth and 2 decay, and this subdivision continues for ever in fractal manner. We can also have E7:1 functions where each growth segment subdivides into 4 growth segments and 3 decay segments. The general fractal time-function is En:1 with each growth segment dividing into (n+1)/2 growth segments and (n-1)/2 decay segments, with no subdivision of decay segments. Here we are dealing with equisegment functions, where a segment sudivides into (n+1)/2 equal growth segments and (n-1)/2 equal decay segments. Non equisegment functions also exist, but the equisegment functions are the simplest and are the only ones considered in this paper. Other En:m functions are covered in [2].

Using equation (11), the condition for any equisegment En:1 function to be traced out, at two levels of decomposition, must be:

$$g_2 = g_1 + ((n+1)/2 - 1)g_1/(c_1 + 1) \qquad \text{...(15)}$$

Notice that we have substituted (n+1)/2 for n in the expression on the right in equation (1), because an En:1 function has a sequence (n+1)/2 growth periods before the onset of a larger retracement. In terms of a grodec stack tracing an En:1 function, there are (n+1)/2 growth periods involving pressure on g_1 before a larger decay period where g_2 opens. The above equation assumes that F is turned off during decay periods.

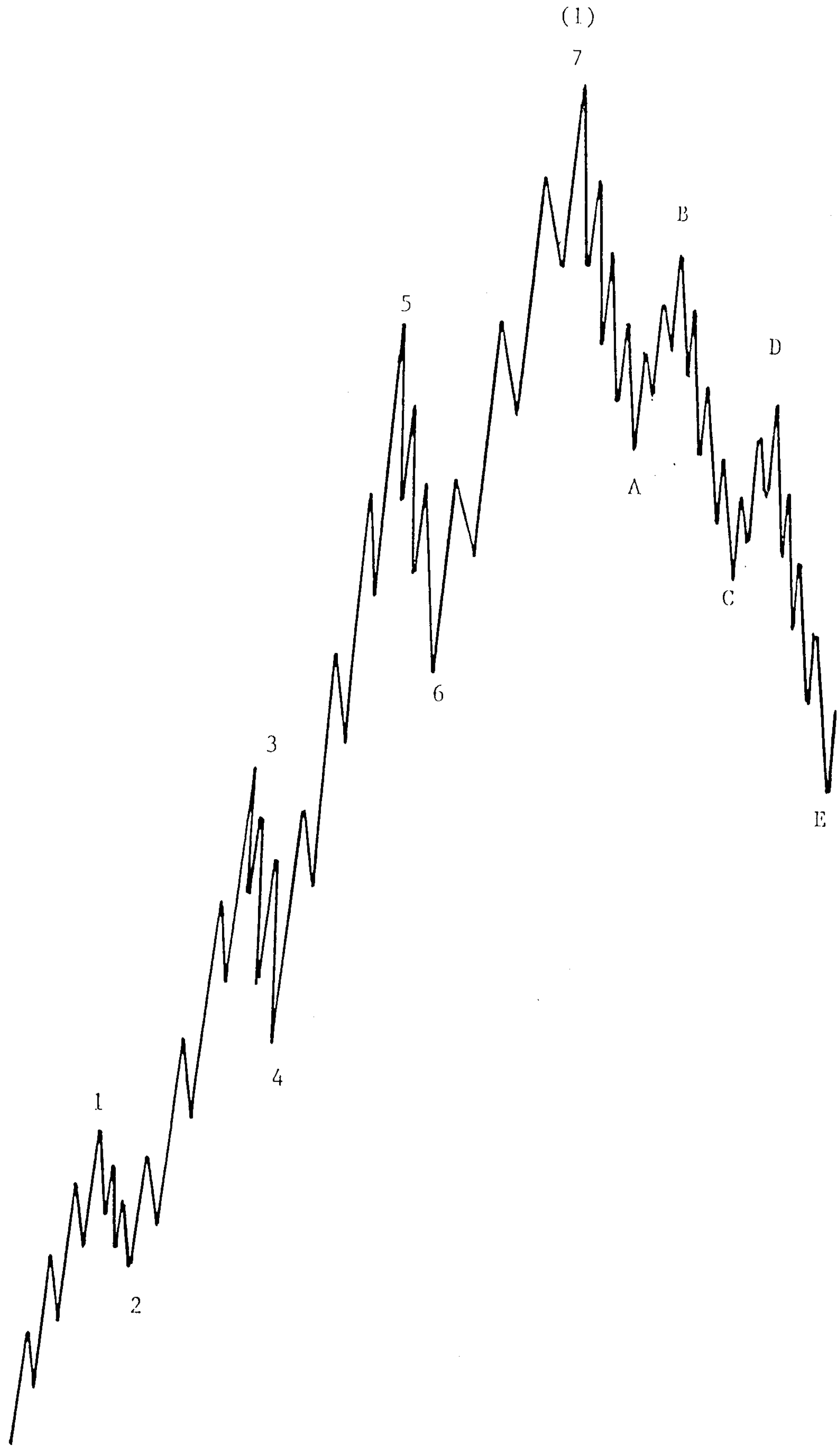


Figure 5 E7:5

2.3 One-sided n-level grodec stack machine

For each additional grodec with the proper settings, we can get a further level of decomposition in the equisegment En:1 function, so that an n-level grodec stack would be needed to exhibit behaviour consistent with an En:1 function to n levels of decomposition. The analysis of such machines is beyond the scope of this paper.

Notice that En:1 functions do not have any decomposition in the decay segments. It is possible to construct a grodec machine where the pressures trace is like that in Figure 4a (see Figure 4b) except that it grows downwards. In other words the decay segments decompose but the growth segments do not. The grodec stack machines described above for upward or positive growing En:1 functions are called left grodec stack machines. Those for downwards growing or negative En:1 functions are called right grodec stack machines and are very similar in operation. We can combine these two types of machines to form a balanced growdec stack machine, in which the pressure trace follows an En:m function, that is, a growth-decay fractal time function in which each growth segment subdivides into n segments and each decay segment subdivides into m segments. The details of balanced grodec stack machines are beyond the scope of this paper. They are very complex and exhibit sensitive dependence on initial conditions [4,5, 15].

Generators and a generation to a few levels of decomposition for E5:3 function and the E7:5 function are given below. The E7:5 function is illustrated graphically in Figure 5.

E5:3 generation

```
Axiom   Ud
U ->    UdUdU
d ->    DuD
D ->    DuDuD
u ->    UdU
```

```
                U                               d                          1  1
  U     d       U      d      U         D      u      D                    5  3
UdUdU DuD UdUdU DuD UdDdU   DuDuD UdU DuDuD                                13  8
...
```

E7:5 generation

```
Axiom   Ud
U ->    UdUdUdUd
d ->    DuDuD
D ->    DuDuDuD
u ->    UdUdU
```

```
                     U                      d
          U  d  U   d  U   d  U   D  u  D   u  D
UdUdUdUDuDuDUdUdUdUDuDuDUdUdUdUDuDuDUdUdU+
DuDuDuDUdUdUDuDuDuDUdUdUDuDuDuD...
```

In the above En:m functions m had the values 3 (in E5:3) and 5 (in E7:5), which means that the decay segments futher subdivide. If m = 1, as in E5:1 and E7:1 functions, the sudivision is simpler, since the decay segments no longer subdivide:

E5:1 static generation Axiom Ud

U -> UdUdU

d -> d

```
                U                              d
        U   d   U   d   U                      d
   UdUdU  d  UdUdU  d UdDdU                    d
   ...
```

E7:1 generation Axium Ud

U -> UdUdUdUd

d -> d

```
                       U                               d
      U     d      U     d      U     d     U          d
 UdUdUdU d UdUdUdU d UdUdUdU d UdUdUdU                 d
 ...
```

The above generations are essentially static or fractal generations [6, 7], since we take an existing pair of growth (U) and decay (d) segments and endlessly subdivide them. But an En:m function can grow endlessly in time. It thus belongs to the general class of self-affine functions [5]. The *dynamic or forward time* generator is somewhat different from the static generator shown above [2].

To end this section it is worth pointing out that biological systems do make use of summation of the switching potentials, although not in as involved a manner as with grodec stack machines. These biological systems are the high voltage generators of electric fish, where the voltages accross membranes are summed when action potentials occur in selected membranes, giving voltages high enough to generate sufficient external current external to stun other fish [6].

3.0 The grodec action potentials

As mentioned earlier, both a single grodec and a pair of grodecs can exhibit action potential phenomena. The action potential of the single grodec is like that common to many biological membranes, with a sharp initial pressure change and a slower recovery. The action potential with a pair of grodecs is more like that of a neuron with both a sharp initial pressure change and a sharp recovery. Most biological membranes have only one switch, usually a calcium switch, corresponding to one grodec, while neurons have two switches, namely the sodium and potassium switches [6,12], corresponding to two grodecs.

3.1 Analysis of the action potential in single grodecs

Referring to Figure 2a, suppose that the pressure difference R-/L/c is just slightly less than g. Now suppose that a small amount of fluid is pumped into the right side, so that R-L/c = g and the grodec triggers. Fluid will flow at rate i from right to left, the pressure difference initially between the two stores being g. If t is the time for the two stores to reach pressure quilibrium then:

$$g - ti = ti/c$$

so that

$$t = cg/i(c+1)$$

The drop in pressure for the right store, or the action potential magnitude A, must be

$$A = ti = cg/(c+1)$$

This shows that if c, the capacitance of the left store relative to the right is very large the action potential is almost equal to g; if c is very small the action potential tends to zero. This also makes sense in terms of an hydraulic model, in which c is the cross section of the left colum relative to that of the right. It also makes sense in terms an electrochemical model, but not so obviously, for the capacitance (not conventional electrical charge capacitance but rather ion concentration capacitance, in terms of the amount of concentration required to change the pressure - electrochemical potential - by a unit amount) is not a linear function of ion concentration, since ions in a solvent behave like a gas, causing the electrochemical potential to vary with the logarithm of the ion concentration, that is, [RTln([Na])]/F, R and T being the gas constant and temperature respectively, and F being the Faraday constant. This means that electrochemical potential, or pressure difference accross a membrane, increases 60 millvolts for each 10-fold change in ion concentration difference at room temperature; ion concentration differences accross biological membranes are of the order of 10-50 times, giving electrical potentials accross membranes from 0 to about 90 millivolts.

Assume that the relative capacitance c of the left reservoir is very large, so that the action potential is approximately g. Once the right and left stores of the grodec have equilibrated, the switch closes, and the right side is replenished from the right reservoir, and the grodec is thus reset, ready to trigger again.. If we assume this replenishment is at a constant rate j, then the time to reset the grodec will be g/j, if j is considerably less than i. This model action potential trace is shown in Figure 6a. In practical implementations of such a model the flows i and j will not be constant but will fall as the pressure difference falls, so that i and j must be taken as average values, where the flows begin large and end small as equilibrium is approaches. Thus the action potential will in practice look more as shown in Figure 6b as opposed to that in Figure 6a. The i and j flows will follow a profile in time as shown in Figure 6c, from which it is apparent that although j begins abruptly large following the switch triggering, j begins to increase as soon as the pressure in R begins to fall. This will have the affect of making the action potential less of a spike and more of a round pulse. Such action potentials occur in many biological cells and are usually due to a single Ca++ ion switch.

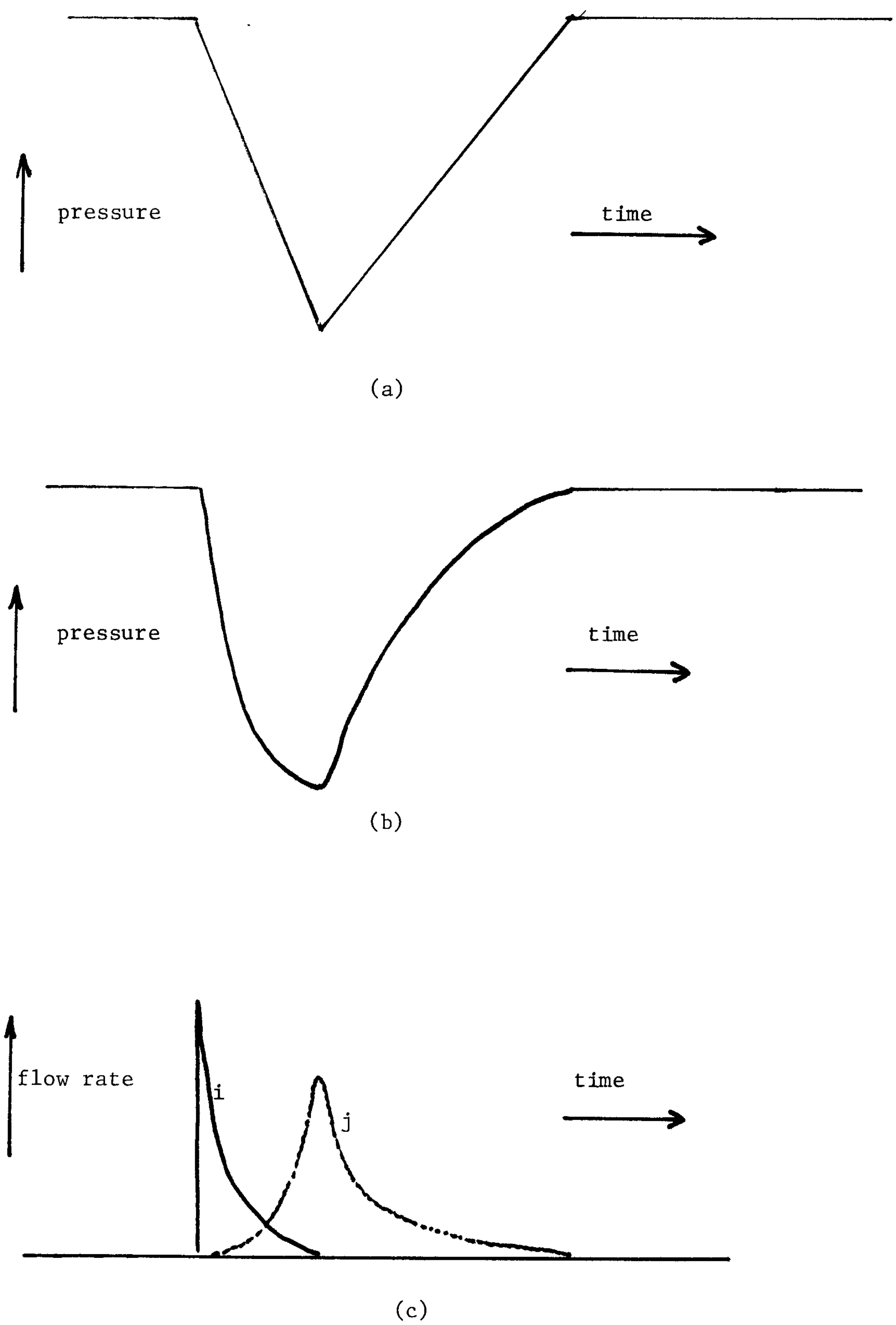


Figure 6

3.2 Analysis of the action potential with two grodecs

If we want an action potential with a sharp spike we must use two grodecs (corresponding to the Na and K switches of neurons). This is illustated by the grodec arrangement in Figure 7a. There are two grodecs, a left grodec (grodec N) and a right grodec (grodec K). These will function in much the same way as the Na and K switches of a neuron, so that it is economical of thought to refer to the grodecs as grodec N and grodec K. The left store of the grodec N and the right store of the grodec K are very large compared with the other stores and can be regarded as reservors. The left store of grodec N and the right store of grodec K are common and have resting pressure R, equal to that of the right store of grodec K. The left store of grodec N is at a pressure slightly higher than R-g.

The action potential in this grodec pair functions as follows. A small initial flow of fluid into the common store will raise the pressure difference between the stores of grodec N to g, causing the pressure in the common store to drop fairly abruptly to

$$R - cg/(c + 1)$$

under the influence of the flow i from right to left store. Since c is large for grodec N, this pressure drop is close to: $R - g$

Following this pressure drop the two stores of grodec N are in equilibrium. The time time taken to reach equilibrium is

$$cg/i(c + 1)$$

which, since c is very large is close to : g/i

The pressure trace is illustrated in Figure 7b.

At pressure equilibrium in grodec N the switch now turns off. But the common store is now at pressure R-g and the right store of grodec K is at pressure R, so that there is a pressure difference g between them. If g is also the trigger pressure of grodec K, the switch of grodec K will now abruptly open and the process will be reversed with fluid flowing from the right store of grodec K to the left store of grodec K. If r is the capacitance of the right store of grodec k relative to the left store of grodec K, the two stores come to equilibrium in a time t given by:

$$tj = g - tj/r$$

or $$t = gr/j(r+1)$$

and at a pressure $$R - g + tj$$

or $$R-g + gr/(r + 1)$$

If the relative capacitance r is very large, that is, if the right store of grodec K is a reservoir, the time tends to g/j and the pressure of the common store to R, which brings things back to where they started, thus resetting the system. The abrupt opening of the right switch will bring the pressure of the central store as quickly back to R as it was initially brought to R-g by the left switch, if j and r are comparable - which they can be. Thus we get a spike action potential as illustrated in Figure 7b. In practice the flows would be large initially and small as equilibrium approached, so that i and j are really averages; in such a practical implementation the spike action potentail would have slightly more curved shape as shown in Figure 7c.

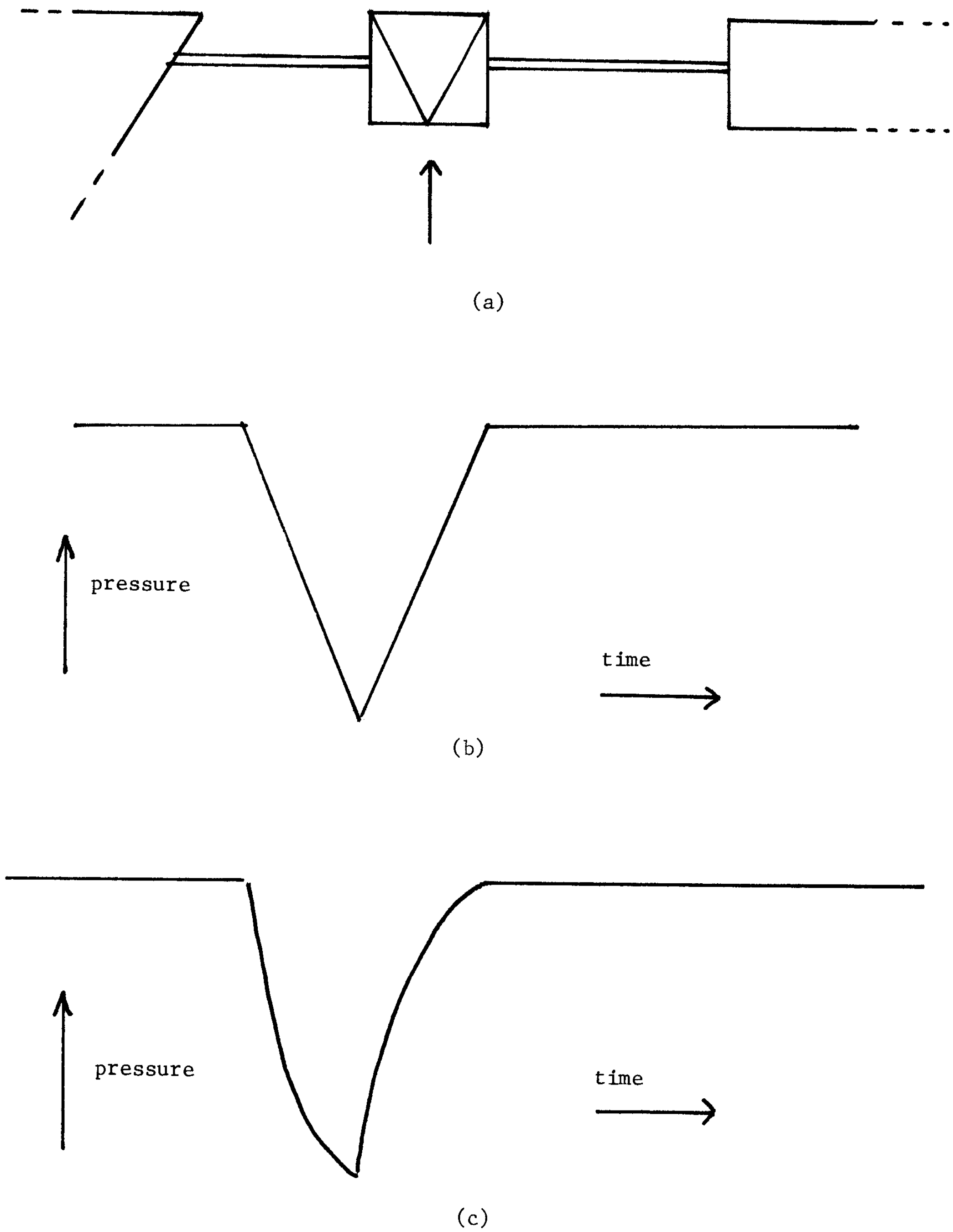


Figure 7

This analysis shows why nature probably evolved the use of two grodecs in neurons, namely the Na+ grodec and the K+ grodec. Triggering of the Na+ switch alone would be sufficient to produce an action potential. However the recovery period would have to be slow relative to the time taken by the initial membrane potential collapse. The action potentials of the shape in Figure 6a or 6b would therefore not permit a high frequency of pulses to be generated. To solve this problem nature use the second grodec, the K grodec, so that when the K switch triggers it has the effect of bringing the membrane potential back to the original resting potential much more quickly than would otherwise be the case, thus allowing the highest possible pulse frequency in neurons.

4.0 Grodec logic circuits as a neural net model

We have shown above that a single grodec has the switching properties of a neuron, except that recovery is slow, and that a pair of grodecs can also have the switching properties of a neuron, but with comparably fast recovery. Either can thus be used as a model of a neuron. It remains to show how these grodecs can be used to construct logic circuits. We shall use the single grodec model of a neuron for the sake of simplicity. The logic of the discussion to follow would not be materially different if a pair of grodecs were iused as a neuron model.

4.1 Stimulation and inhibition of one grodec by another

When used as a neuron model the single grodec, in its "resting" or non active state, is set with its pressure difference just under the switching pressure g. Now when the switch triggers energy is released, which will normally be dissipated as waste heat. However, in a neuron this energy is usually utilized to trigger neighbouring grodecs in a trivially simple manner, as follows. If the membrane potential has switched at one myelin gap G1, there is an electromotive force (about 100 millivolts) around a circuit through G1 and a neigbouring gap G2. The resulting current that flows in this circuit must flow accross the neuron membrane in neigbouring gap G2 and is sufficient to trigger the membrane in that gap. That action potential in G2 will trigger one in the next gap, and so on. [The principle is the same if there is no myelin sheath surrounding the nerve, although the stimulating current from one excited neuron membrane section must now be distributed in a non linear manner to its neighbouring neuron membrane section, according to transmission line principles.]

This is one way in which an action potential is triggered in a neigbouring membrane section in nature, namely use the energy released during the action potential to change the pressure in a neigbouring grodec suficiently to trigger it. Another way is to use the energy released to alter the trigger pressure of a neighbouing grodec. This is what is done at a neuron synapse, where a neurotransmitter chemical is relased by one neuron at is terminal (presynaptic), which substance then migrates to the neighbouring neuron dendrite membrane (postsynaptic), alters its trigger pressure downward, and so triggers an action potential localized to where the neurotranmitter is accepted [6].

We can use both methods to form artificial neural nets from grodecs, that is, either use of released energy of an action potential to alter the pressure of another grodec and so trigger it, or use of released energy to alter the trigger pressure g of another grodec and so trigger it.

In the discussion of neural net model in the next section, we shall assume that the first method is being used to couple two or more grodecs. The details of the coupling do not matter. We might imagine for example that the fluid flow during the pressure collapse generates a small amount of electrical power which is used to pump a small quantity of fluid into the right store of a neighbouring grodec, thus increasing its pressure enough to trigger it. This is positive or stimulative coupling of grodecs, where an active grodec causes a quantity of fluid q to be pumpled into the right store of a resting grodec.

We can also have negative or inhibitive coupling of two grodecs where the energy released by one grodec is used to pump a small quantity of fluid out of athe right store resting grodec, reducing its pressure and making it harder to trigger. The type of coupling is indicated in the diagrams in Figure 8 by + or - inside a circle. Note that because of the nature of the resting equilibrium in the grodec, a delivery of a quantity q that does not stimulate the grodec will result in the removal of q to the reservoir shortly afterwards, since delivery of q cause the right store to be out of equilibrium with the reservoir, and cause fluid to move from the right store to the reservoir. Similarly, removal of a quantity q from the right store will result in it being replaced from the reservor in a short time. This temporary nature of the coupling affect of one grodec on another due to an action potential is necessary if the logic circuits formed from grodecs are to work repeatedly.

4.2 Grodec logic gates

Basic And circuit

If grodecs A, B and C are each positively coupled to grodec D such each can deliver a fluid quantity q D, and it take a quantity 3q to stimulate the grodec D, then the action potentials A, B and C must all go off at the same time in order to trigger grodec D. (Figure 8a). In this way an And circuit can be constructed any number of inputs.

Majority circuit

If 5 grodecs are positively coupled to a sixth, where the sixth needs 3q to trigger, and each of the 5 can deliver q, then the sixth will trigger if a majority, at least 3, of the 5 are triggered (Figure 8b). In this way a Majority circuit can be constructed with any number of inputs.

A and not B circuit

If A and B grodecs are coupled to grodec C, which needs q to trigger, and if A is positively coupled to deliver q and B negatively coupled to remove q, then if both A and B are triggered, C will not be triggered. C will be triggered only if A alone is triggered, because of the inhibition coming from grodec B (Figure 8c).

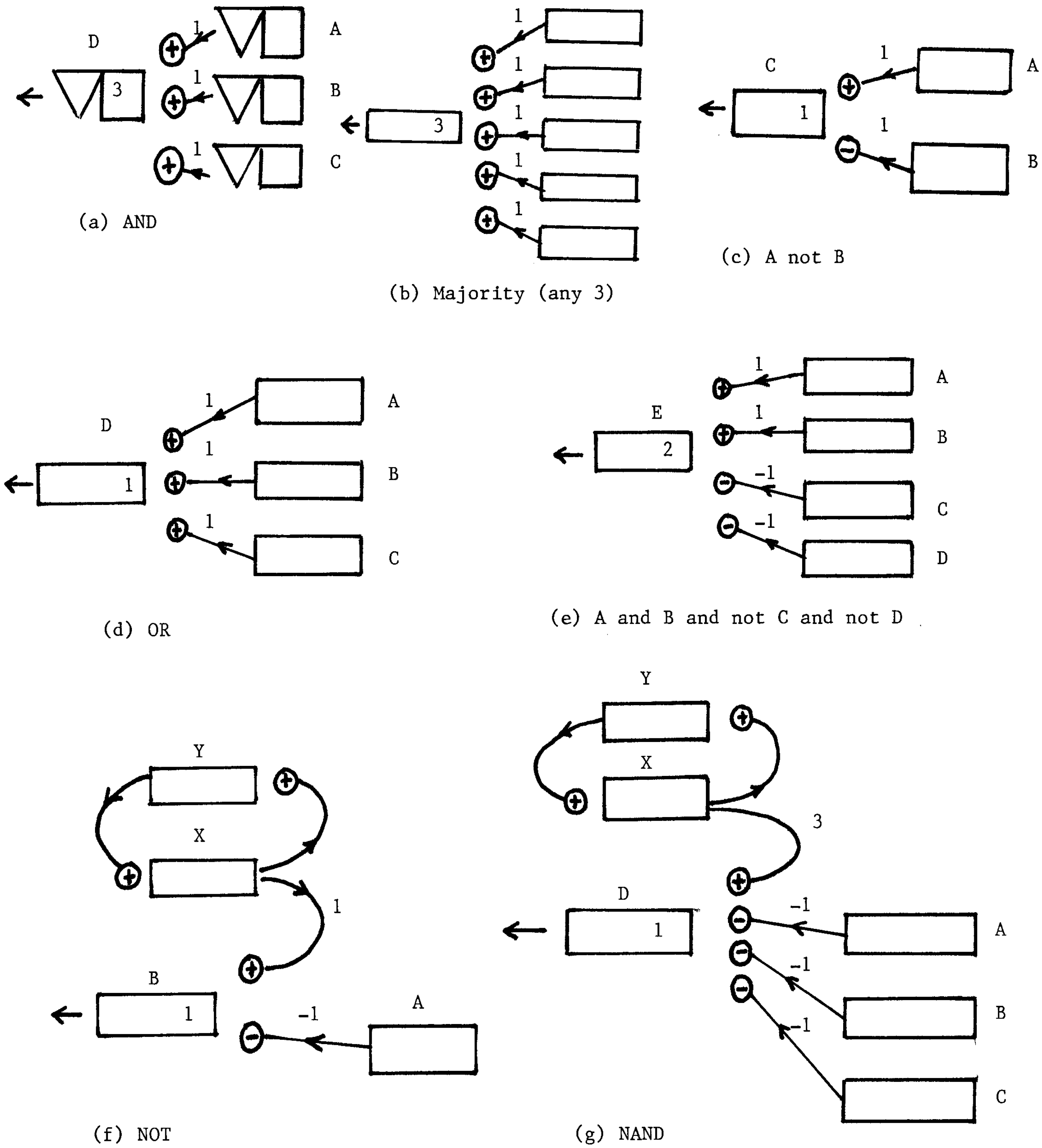


Figure 8 Grodec-based logic elements

OR circuit

If A, B, and C grodecs are coupled positively to grodec D, each delivering q to D, and if D needs q to trigger, then D will trigger if any one of A, B or C triggers (Figure 8d).

A and B but neither C or D

We have this circuit if A and B are each positively coupled to E and each deliver q to E while C and D are each inhibitively coupled and each remove q from E, and E needs 2q to trigger. If A and B are triggered, then E will trigger. But if A and B trigger and either C or D also trigger, or both C and D trigger, then E will not trigger (Figure 8e).

Not circuit

This one is slightly less obvious, for we need a set of oscillating grodecs X and Y, in addition to two grodecs A and B for input and output. Grodec A is negatively coupled to B, and can remove q units from B. Grodecs X and Y constitute a separate system. X triggers Y and after a delay Y triggers X which, after a delay triggers Y, and so on. But not only does X deliver enough fluid to triggrer Y, it is also positively coupled to B and delivers q units to B each time it triggers; B is set so that q units is enough to trigger it.

Thus since the oscilartor XY triggers continually, when A is not triggered B will continually trigger, that is, if A false, B is true. However if A triggers, this will inhibit B and so B cannot trigger. But as soon as A stops being triggered B will resume being riggered, that is, if A is true B is false. This gives us a Not circuit (Figure 8f)

Not And or Nand

Grodecs A, B and C are negatively coupled to D. An oscillator grodec X is positively coupled to D, delivering 3q to D, where q is just enough to trigger D. Each of A, B, and C remove q units from B. Thus D will trigger continually, so that D is true, except if all of A, B and C are triggered, that is, except when A, B and C are all true. This is Nand logic, where the output is (1-ABC), when A, B and C can have values 1 or 0 (Figure 8g).

4.3 Finite-state grodec machines

It has been shown by Minsky [7, 8, 13] and by others in the field of finite-state sequential machines, that relatively few logic elements of the type shown above, which in function are similar to those originally developed by McCulloch and Pitts [11], can be used to build simple networks to carry out conventional computing functions, including short-term memory, with encoding and decoding, as well as arithmetic functions.

A finite state machine [3] can be abstracted as a 6-tuple M = (Q, S, R, f(Q,S), g(Q,S), q0) where the set Q is the finite set of internal machine states, the set S is the set of possible inputs to the machine, and R is the set of possible outputs. The function q = f(Q,S) is the state-transition function in which q is a member of Q; it determines the next state of the machine as a function of the current state and the current input, that is, the next state depends on the current input and the current state The output function r = g(Q,S), where r is a member of R, determines the next output of the machine as a function of the current input and current state. Thus the next output is determined by the current input and the curent state. At any time the machine can be reset to an initial state q0.

In practical terms a grodec logical machine built from the above logic elements will have an array of input grodecs at the input interface, some of which will be triggered by an input that is a member s of S. Also at the output interface of the machine will be an array of grodecs, some of which will be triggered to give an output that is a member r of R. To be specific about the operation of a grode logical machine, we need to consider (a) the state transition time and (b) the nature of the set of allowed inputs S.

Take any pair of coupled grodecs of the machine, G1 and G2, such that when G1 triggers it triggers G2. If G1 being triggered at time zero always results in G2 being triggered at the same later time p for every grodec pair, such that at time p G1 has been reset and could be triggered again, and if this is true for every coupled grodec pair in the machine, then p is the propagation time of the machine. When the machine is operating, every p units of time action potentials will be triggered in different grodecs of the machine, even with looped grodecs or oscillators. For example if G1 is coupled to G2, which is in turn coupled to G1, then if this pair is oscillating it will generate action potentials in either grodec G1 or G2 at a frequency of 1/2p spikes per second, and in both grodecs at a frequency of 1/p spikes per second. Finally, if the machine has m grodecs in sequence between input and output grodec interfaces inclusive, an output action potential will be dected at k = mp units of time after an input stimulus; k is therefore also the time required, on receipt of an input, for the machine to switch to a new state, or the state transition time of the machine, ready to accept a new input.

Consider now the inputs to a grodec logical machine. Suppose the machine has input interface grodecs A-K and output interface grodecs L-Z. A specific input s5, coming with the machine in state q4, could involve stimulus of input interface grodecs A, C, D and F, for example, at time 4k. [The initial input s1 comes at time zero, with the machine in state q0.] The stimulus on each input interface grodec due to any input sn would be a single stimulus (being a transfer of fluid to the grodec sufficient to trigger it) at a specific time nk, where k is the machine's state transition time and n is an integer. Each of the input interface grodecs A, C, D and F would respond to s5 at time 4k with an action potential at time 4k + p. The next specific input s6 would be at time 5k and could be stimulus of input interface grodecs B, C, D, G, and K so that each of these grodecs would generate an action potential at time 5k + p, and so on. A specific input s5 will propagate through the logic elements of the machine, in time k = mp, turning on some oscillating loops and turning off others turned on by earlier inputs, thus placing the machine in a new state q5 at time 5k + mp = 6k and stimulating an action potential at specific output interface grodecs, say N, P and U, that is, the output r5. This output r5 obviously must depend on the nature of the input s5 and the state of the machine at the time of the input, that is, q4. Also the new state of the machine q5 must obviously depend on the imput s5 and the original state q4. Thus a grodec logic machine of the type described, with a well defined state transition time, satisfies all the conditions for a finite state machine.

Note that the above discussion implies a design feature of action potential based logic elements not found in logic elements based on conventional electronic gates. Suppose a grodec logic machine could be economically designed so that a specific input s8 could be processed with g gates in sequence (for a time gp) while all other inputs needed x + i gates in sequence for a time (x + i)p. This would mean that one type of input could result in an output much more quickly than would be the case with other inputs. But action potentials do not endure, unlike the voltage states of conventional electronic gates, so that if the machine were to be used as a subcomponent of a larger machine, this design would cause a malfunction, since outputs would be inputs to another machine that would expect them at intervals of k. It would thus be necessary to add i grodecs in sequence in an appropriate way to the original machine to ensure that the output from s8 appeared exactly k units of time after the input.

4.4 Grodec memory

We have not discussed memory units, although it is clear that short term memory units can be constructed as oscilators, On for True and Off for False, from the logic elements presented above. However, in biological neural networks the evidence is compelling that long term memory is due to changes in synaptic efficiency, that is, coupling efficiency between neurons [6, 12], rather than to conventional logic circuitry.

In terms of grodecs, synaptic efficiency is the measure of how many units of fluid are transferred to a coupled grodec by an excited grodec. The more intriguing aspect of synaptic efficiency memory, however, is that in biological systems it happens because of use. If a synapse is used more it becomes more efficient in terms of making a greater contribution to generating an action potential in the axon [6]. We can use grodecs to illustrate what this means.

Suppose we have an And grodec circuit with 4 input grodecs A, B C and D and one output grodec E, where A, B, C and D each transfer q units to E and E needs q units to trigger. Suppose now, and this is the key point, that the coupling system in the grodecs is designed to increase the quantity of fluid transferred to E if the coupling is used much more than usual. Assume an unusually large use of the A and B grodecs on the input of the And circuit so that eventually each of these can transfer 2q units of fluid to E instead of the original q units each. This will mean that now any two inputs alone will trigger E, except C and D. In this state we can say a 1 is being stored, and something is remembered. However, suppose now that A and B falling into subsequent abnormal disuse causes the coupling efficiency mechanism to go into reverse. Once more A and B deliver q units each to E, and once more the circuit is a 4 input And. Here there is a process of forgetting, and 0 is stored once more. It is probably because of this kind of adaptability memory, as well as massively parallel operation, that neural machines in nature are so profoundly different in operation from conventional digital computing machines [7,8]. This may ultimately have a great deal to do with biological machine intelligence and conciousness. However, it should be at least clear that grodecs and grodec machines can serve as the building blocks of conventional neural networks for researching these problems, even if only with thought experiments.

SUMMARY

Grodecs are growth decay entities in which there are two stores that contain a fluid under pressure. The two stores are connected by a flow line in which there is a switch that opens abruptly when the pressure difference reaches a specific trigger pressure. We have analysed the switching behaviour of grodecs in detail and we have demonstrated that grodecs can be used as the basis of both an unconventional type of neural net called a grodec stack machine, and conventional neural nets.

A single grodec has the most of the switching properties of a section of biological membrane, and can give rise to action potentials when properly set. However, a pair of grodecs can generate a much sharper action potential, like those of a neuron with dual Na and K ion switching, so that a neuron membrane can be regarded conceptually as a Na grodec coupled to a K grodec.

Grodecs can be coupled in positive (excitory) or negative (inhibitory) manner to form logic circuits and example of such common circuits have been given, so that grodecs can be the basis of conventional neural nets.

However, grodecs can be coupled quite differently, in a manner that is quite unconventional in terms of neural nets, to form grodec stack machines. In such machines the action potentials accumulate in periods of growth and decay, so that a central machine pressure parameter exhibits time function behaviour similar to that of a fractal growth-decay time function, such as an En:m function. The mechanism of how the pressure changes can be accummulated in a grodec stack machine to follow the swing pattern of a fractal En:m functions is also presented. Grodec stack machine are complex, and can exhibit sensitive dependence on initial conditions, and thus chaotic behaviour. A consequence of this complexity is that the extent to which a grodec stack machine could be designed to follow an En:m function to arbitrary level of fractal subdivision remains unknown.

REFERENCES

1. Bradley, J. A complete L-system specification for generating an exact self-affine growth-decay function with a random walk scaling property. (1994) Computer J., (37), 1, pp 57-66

2. Bradley, L. (1995) "En:m Fractal Time Functions, Grodecs and Grodec Stack Machines", University of Calgary Research Report No. 95/578/30.

3. Denning, P.J., Dennis, J.B., and J. E. Qualitz, (1978) "Machines, Languages and Computation", Prentice-Hall, N.J.

4. Devaney, R. L. (1986) "An introduction to Chaotic Dynamical Systems. Benjamin Cummings Publishing Co., Menlo Park, Ca.

5. Devaney, R. L. (1987) Chaotic bursts in non linear dynamic systems. Science 235, 342-345.

6. Eckert, R., Randal., and G. Augustine (1988) "Animal Physiology", W. H. Freeman, New York.

7. Feldman, J.A. (1982) Dynamic conections in neural networks, Biol. Cybernet. 46, 27-39.

8. Feldman J.A., and D. H. Ballard (1982) Connectionist models and their properties, Cognit. Sci., 6, 205-254

9. Mandelbrot, B. B. (1984) Fractals in Physics: Squig clusters, diffusions, fractal measures, and the unicity of fractal dimensionality. J. Statistical Physics, (34) pp895-930.

10. Mandelbrot, B.B (1985) Self-affine fractals and the fractal dimension. Phys. Scr., (32) pp 257-260.

11. McCulloch, W.S., and W. Pitts (1943). A logical calculus of the ideas immanent in nervous activity. Bull. Math. Biophysics.

12. McGregor, R.J (1987). "Neural and Brain Modeling", Academic Press, San Diego, Cal.

13. Minsky, M.L. (1967) "Computation: Finite and Infinite machines", Prentice-Hall, Englewood Cliffs, N.J.

14. Peitgen, H. et al. (1988) The science of fractal images, Springer-Verlag, Berlin.

15. Peitgen, H.., et al. (1992) Chaos and fractals. Springer-Verlag, Berlin.
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