THE UNIVERSITY OF CALGARY

# Soil – Pipeline Interaction In Slopes

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## ABSTRACT

Good engineering requires that economic designs be provided at acceptable levels of safety. This usually means predicting the system performance for which there exists little or no previous experience. The problem is often compounded by the variability of the raw data, on which the risk analysis is based.

In Canada, buried pipelines are used for economical transport of oil and natural gas. Due to circumstances such as difficult terrain, the pipelines sometimes may be constructed in unstable slopes. In such situation, the owner of the pipeline has an intrinsic interest in guaranteeing that his or her pipeline would not rupture or break due to unstable soil movements.

In this research, analytical and numerical solutions have been derived to determine the deflection profile of a buried pipeline in a slope subjected to a longitudinal, transverse and deep seated failure.

Application of statistical analysis on the relationships between the soil movement and the pipe deformation strain allows one to assess the risk of pipeline rupture with a given soil movement.

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To my parents who have provided support and encouragement throughout the program, I express grateful gratitude.

This thesis is dedicated to my parents whom I love very much.

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$A_{x\text{-section}}$ cross-sectional area of the pipe ($m^2$)

B transition point between the plastic and elastic soil reaction region (m)

c cohesive strength ($N/m^2$)

d pipe diameter (m)

$D_s$ soil displacement to reach ultimate reaction force (m)

E modulus of elasticity for steel (N/m)

$\varepsilon$ axial strain

$\varepsilon_p$ combined pipe strain for the plastic soil reaction region

$\varepsilon_e$ combined pipe strain for the elastic soil reaction region

F factor of safety

$f_m$ maximum axial force per unit of length at the soil pipe interface (N/m)

$F_{max}$ maximum axial force developed on the pipe (N)

$\gamma$ unit weight of soil ($N/m^3$)

H burial depth to pipe centerline (m)

$K_s$ elastic subgrade modulus or soil stiffness ($N/m^3$)

$\phi$ friction angle of soil (degrees)

L length of landslide (m)

$\mu$ mean value

n, r Ramberg-Osgood parameters

$N_c$ bearing capacity factor

| | |
|---|---|
| $R$ | radius of deep seated circular landslide (m) |
| $R_m$ | perpendicular distance of the center of circular landslide and the pipeline (m) |
| $R_f$ | ultimate reaction force (N) |
| $S_u$ | undrained shear strength ($N/m^2$) |
| $\sigma$ | axial stress ($N/m^2$) or standard deviation |
| $\sigma_y$ | effective yield stress ($N/m^2$) |
| $\sigma_{max}$ | maximum axial stress ($N/m^2$) |
| $t$ | pipe thickness (m) |
| $W$ | width of landslide (m) |

# Chapter 1

# Introduction

## 1.1 The nature and scope of the problem

Good engineering requires that economic designs be provided at acceptable levels of safety. This usually means predicting the system performance for which there exists little or no previous experience. The problem is often compounded by the variability of the raw data, on which the risk analysis is based.

In Canada, buried pipelines are used for economical transport of oil and natural gas. Due to circumstances such as difficult terrain, the pipelines sometimes may be constructed in unstable slopes. In such situation, the owner of the pipeline has an intrinsic interest in guaranteeing that his or her pipeline would not rupture or break due to unstable soil movements.

## 1.2 Contributions of this thesis

This thesis will present new analytical and numerical solutions for the design of pipelines subjected to transverse, longitudinal and deep-seated landslides.

The model for pipeline in a deep-seated landslide is completely new and has never been done before.

For the pipeline in transverse landslide, the stretching effect has been added to the bending strain to produce a new total strain, which has not been considered in other works. Previous works investigated transverse landslide of infinite width only. A transverse landslide of finite width has been modeled.

Several statistical analysis methods are also applied to a simplified pipeline strain model to assess the risk of probability of pipeline yielding on an unstable slope. These methods include First Order Second Moment method (FOSM), Rosenblueth's Point Estimate Method (PEM), and the Monte Carlo simulation method.

## 1.3 Organization of Thesis

This chapter has presented the nature of the problem. Chapter 2 will review the literature pertaining to soil-pipeline interaction and statistical analysis. Chapter 3 will summarize the new mathematical models for the longitudinal, transverse and deep-seated landslides and present the final resulting strain equations. The more detailed explanation of how the new models are derived is shown in appendices A, B and C. Chapter 4 describes the statistical methods to be used in assessing pipeline safety. Chapter 5 is a case study of a pipeline in an unstable slope, followed by conclusion and recommendations in Chapter 6.

# Chapter 2

# Literature review

## 2.1 Introduction

Buried steel pipelines have been and will continue to be damaged from permanent ground deformations (PGD). Permanent ground deformation refers to nonrecoverable soil movement such as a landslide. An understanding of how a pipeline behaves under such permanent ground deformations is essential to implementing a successful field monitoring and remediation program needed to prevent pipeline failures. The following literature review attempts to summarize the past work done in the pursuit of understanding soil-pipeline interaction subjected to permanent ground deformation.

## 2.2 Different types of landslides

A pipeline could be exposed to a planar landslide or a deep-seated landslide (see Figures 2.1, 2.2 and 2.3).

In a planar landslide, the sliding surface is parallel to the surface of the slope and often occurs when a soil has a specific lane of weakness. A pipeline would be exposed to some combination of longitudinal and transverse PGD depending on the pipeline orientation with respect to the direction of ground movement (see Figures 3.1 and 3.6). For longitudinal landslide, the soil movement is parallel to the pipeline axis, while for transverse landslide the soil movement is perpendicular to the pipeline axis.

In a deep-seated landslide, the sliding surface closely resembles arcs of circles. For a pipeline laid parallel to the direction of the slope, the deep-seated soil movement creates both a longitudinal and transverse force on the pipeline.

## 2.3 Previous studies on pipeline subjected to soil movements

Simplified design methods for pipelines subjected to transverse and longitudinal landslides have been proposed and developed by several researchers (e.g., O'Rourke and Nordberg, 1992; Flores-Berrones and O'Rourke (1992); Rajani *et al.*, 1995; O'Rourke *et al.*, 1995).

O'Rourke and Nordberg, (1992), and Flores-Berrones and O'Rourke (1992) studied the behavior of buried pipelines subjected to longitudinal permanent ground deformations (PGD). Five idealized longitudinal PGD patterns (see Figure 2.4) based on observed patterns from previous earthquakes were used and analytical relations for the axial strain in the pipe were developed. The five patterns considered were Block, Ramp, Ridge, Ramp-Block, and Asymmetrical Ridge. It was shown that the Block pattern produces the highest axial strain on the pipeline, but that the variations between different patterns are negligible compared to the effect of the length of the PGD. Their assumptions for their model are more fully described below as O'Rourke *et al.* (1995) continued and expanded on this work.

O'Rourke *et al.* (1995) adapted the Ramberg-Osgood (1943) model of uniaxial stress-strain behavior of steel (see Equation 2.1) for his study of steel pipe wrinkling due to longitudinal permanent ground deformation:

[2.1] $$\boxed{\varepsilon = \frac{\sigma}{E}\cdot\left[1+\frac{n}{1+r}\cdot\left(\frac{\sigma}{\sigma_y}\right)^r\right]}$$

where

$\varepsilon$ = axial strain,

$\sigma$ = axial stress,

E= modulus of elasticity for steel,

$\sigma_y$ = effective yield stress, and

n and r = Ramberg-Osgood parameters.

O'Rourke used an elastoplastic model (see Figure 2.5) for his force-deformation behavior at the soil-pipeline interface. This model is defined by two parameters: the maximum axial force per unit length at the soil pipe interface, $f_m$, and the relative displacement at which slippage between pipe and soil occurs. However, the slippage displacement is assumed small and neglected in his model.

The maximum axial force per unit length $f_m$ depends on the type of soil surrounding the pipe and the method of pipe installation. For the most general soil condition, when the soil surrounding the pipe has both friction and cohesive characteristics, $f_m$ is given by

[2.2] $$\boxed{f_m = (c+\gamma\cdot H\cdot(0.9\cdot\tan(\phi)))\cdot\pi\cdot d}$$

The maximum axial stress in the pipe (tensile in upper part and compressive in the lower part of the landslide), $\sigma_{max}$, developed on the pipeline in longitudinal landslides is given as follows:

$$\boxed{\sigma_{max} = \frac{F_{max}}{A_{x-section}} = \frac{[c+\gamma \cdot H \cdot (0.9 \cdot \tan(\phi))] \cdot (\pi \cdot d) \cdot (L/2)}{(\pi \cdot d) \cdot t}} \quad [2.3]$$

where

$F_{max}$ = maximum axial force developed on the pipe,

$A_{x\text{-}section}$ = cross-sectional area of the pipe,

c = cohesive strength of soil,

$\gamma$ = unit weight of soil,

H = burial depth to pipe centerline,

$\phi$ = friction angle of soil,

$0.9\tan(\phi)$ = coefficient of friction at the soil-pipe interface,

d = pipe diameter,

t = pipe thickness, and

L = length of longitudinal soil displacement zone.

O'Rourke *et al.* stated that steel pipe is able to deform in tension well beyond the effective yield strain without rupture. Therefore, wrinkling (local buckling) of the pipe wall in compression is taken as the failure mode of interest, particularly if the pipe is relatively free of corrosion and other defects. Based on laboratory tests on thin-wall cylinders, Hall and Newmark (1977) suggested that compressional wrinkling in a pipe begins at a strain calculated by equation [2.4]. O'Rourke used this equation as the pipe failure criteria in their analysis, which means a thicker or smaller radius pipe is more able to withstand compressional wrinkling.

[2.4] $$\boxed{\varepsilon_W = 0.175 \cdot \frac{\text{Pipe Wall Thickness}}{\text{Pipe Radius}}}$$

O'Rourke *et al.* found that behavior and onset of wrinkling are governed by the length of the Permanent Ground Deformation zone in longitudinal soil movements. Their model also showed that the maximum pipe strain developed from the soil movement increases if:

- the pipe was buried deeper,
- the pipe thickness was smaller,
- the soil cohesive strength and angle of friction was higher, and
- the soil density was higher.

Based on information on observed lateral spreads, O'Rourke and Nordberg (1990) concluded that the typical buried pipe subjected to transverse landslide between the margins of the Permanent Ground Displacement zone are compliant. This means the pipeline will displace at the same amount as the landslide.

Rajani *et al.*, (1995) presented some simplified design methods for pipelines subjected to transverse and longitudinal soil movements. Their model assumes a given pipe displacement, and does not take into account the soil-pipeline interaction in terms of their relative displacements and resulting force. They also assumed an infinite width of landslide. From their model, they showed that on initial landslide movement, the buried pipeline and the soil behaves elastically. As the landslide movement increases, ultimate passive resistance is developed in the surrounding soil medium, but the pipe will remain elastic. On further landslide movement, a plastic hinge or wrinkle begins to develop in the pipeline. They also showed that using cohesive soil in the model for the soil reaction produces the most conservative numbers because the undrained rapid response produces the highest soil resistance. They performed a parametric study for a pipeline subjected to transverse landslide, and they found that the soil strength has the most dominant effect on pipeline response. They found that the soil stiffness in terms of the elastic subgrade modulus, $K_s$, has large effect at small pipeline displacements (when the soil is elastic) but small effect for larger pipeline displacements (when the soil is plastic).

## 2.4 Previous probabilistic studies

The objective of statistical analysis, as applied to pipelines in unstable slopes, is to assess the risk and probability of pipeline failure. Three main techniques are used for statistical analysis, and they are the Monte Carlo simulations, the First Order Second Moment (FOSM), and the Rosenblueth's Point Estimate Method (PEM). The Monte Carlo technique involves generating random numbers using the mean and standard deviations for each variable of the function. FOSM is obtained from the Taylor series expansion of the function about the expectations of the random variables. This Taylor series approximation may impose excessive restrictions on the function (existence and continuity of the first or first few derivatives) and requires the computation of derivatives. These difficulties can be overcome with the use of point estimates (PEM) of the function, which leads to expressions akin to finite differences. Of these three methods, only the Monte Carlo simulation method uses the whole statistical distribution of the variables. The FOSM and PEM both only use one standard deviation from the mean for their calculations.

Appropriate actions could be taken for a given level of probability of pipeline failure. At low failure probability, perhaps implementing a monitoring program for the slope and the pipeline is all that is needed. At medium failure probability, some remedial action such as construction of a berm or improving drainage along the slope to stop the slope movement could be implemented. The induced pipe strain could be reduced by increasing the pipe wall thickness, decrease the burial depth of the pipe or use lower density soil with lower angle of frictional angle. It may also be a good idea to install block valves both upstream and downstream of the potential landslide zone to automatically shutoff the pipe in case of pipe failure. At high failure probability, major action could be needed such as rerouting the pipeline to a more stable area.

Nguyen and Chowdhury (1984) used Monte Carlo simulations and Rosenblueth's point estimate method for assessing slope stability of spoil piles in strip coal mines. Calculations for failure probability were first made using Monte Carlo simulations and assuming a potential two-wedge failure mechanism for the spoil piles. The Monte Carlo technique involves generating pseudo-random numbers based on the mean and standard deviations of the shear strength parameters. From 500 simulations, the frequency distribution of the factor of safety was obtained. The probability of failure was calculated as the ratio of area under the frequency distribution curve from the left-hand tail up the factor of safety of unity, and that bound by the whole curve.

An alternative procedure based on Rosenblueth's method of estimating moments was then used to compare the results with the Monte Carlo simulation method. Excellent agreement was found and it was recommended that, for practical purposes, the relatively quick Rosenblueth method should be used in estimating the probability of slope failures. Nguyen and Chowdhury also concluded that estimates of strength parameters of spoil piles based on test results and associated geomechanics considerations must be made for each particular mine. Due to the variation in soil parameters at each different site, it would not be feasible to construct design charts that are universally applicable to all spoil piles.

Christian et al. (1994) described how probabilistic descriptions of soil parameters can be derived from field and laboratory data and applied in stability analysis. The first-order, second-moment approach is explored and applied to the design of embankment dams. They promoted the use of reliability index ($\beta$) as defined in equation [2.5] as a way of normalizing the factor of safety with respect to its standard deviation:

$$[2.5] \quad \boxed{\beta = \frac{E[F]-1}{\sigma[F]}}$$

where

F = factor of safety,

E[F] = mean or expected value of the factor of safety, and

$\sigma$[F] = standard deviation of the factor of safety.

The reliability index describes safety by the number of standard deviations (i.e., the amount of uncertainty in the calculated value of F) separating the best estimate of F from its defined failure value of 1.0.

The uncertainty in the values of the soil properties due to scatter and systematic error was discussed. The former consisted of spatial variability and random measurement error. The latter includes both a statistical uncertainty in the expected values and the effects of bias – which is much more difficult to evaluate. The effect of spatial variability on the computed reliability index is reduced because the variability is averaged over a region – and only its average contribution to the uncertainty is of interest. In addition, the structure of the spatial variation can be used to estimate and then to eliminate the level of random noise in the soil property data. Since bias is often ignored in theoretical treatments of analytical procedures because it is difficult to quantify, the engineer must often rely on judgment to establish its contribution. Finally, they concluded that uncertainties in soil properties yield a lower bound estimate of probability of failure. An absolute probability of failure would require a more elaborate probabilistic risk analysis involving fault trees or other methods of evaluating risk due to all contingencies.

## 2.5 Critique

Obviously missing from previous research is the study of pipeline-soil interaction in a deep-seated landslide. All previous studies only have been for pipelines subjected to planar failure surfaces.

Flores-Berrones and O'Rourke (1992) and O'Rourke *et al.* (1995) ignored the relative displacement at which slippage between pipe and soil occurs. Their model ignores the elastic soil reaction by using only the maximum frictional force per unit length, $f_m$, to calculate the forces acting on the pipeline.

In addition, O'Rourke *et al.* assumed symmetrical force development between the stable and unstable soil regions. This could only be true if there is no elastic soil reaction, so that all force developed on the pipeline comes only from the maximum frictional force per unit length (plastic soil), $f_m$. If both plastic and elastic soil reaction were to be considered, then there should be a longer zone of plastic soil developed inside the landslide in order to balance the infinite length of stable soil.

Finally, O'Rourke's model is only set up for the simple case of finding the peak strain for a given length of landslide (L) with an unlimited amount of PGD ($\delta$) and vice versa. Solving both of these variables together at the same time would have allowed one to predict the peak strain for a given length of landslide by monitoring the PGD.

Rajani *et al.*, (1995) assumed a given pipe displacement for their model of pipeline subjected to transverse and longitudinal soil movements instead of basing their model for given amount of PGD. They ignored the soil-pipeline interaction in terms of their relative displacements and resulting force. They also did not consider transverse landslides of finite width. Finally, Rajani *et al.* ignored the effect of pipeline tensile strain because they assumed that flexure is the dominant behavior of the pipeline for small transverse displacements

This thesis will derive models for pipeline subjected to longitudinal, transverse, combined longitudinal and transverse, and deep-seated landslides. Transverse landslides of finite width will be studied, and tensile strain will be included as well. The models will allow one to predict the maximum pipeline strain for given dimensions of landslide by monitoring the amount of PGD. Finally, statistical analysis using the FOSM, PEM and Monte Carlo simulation methods will be applied to a pipeline subjected to longitudinal landslide in order to assess the probability of pipeline yielding.

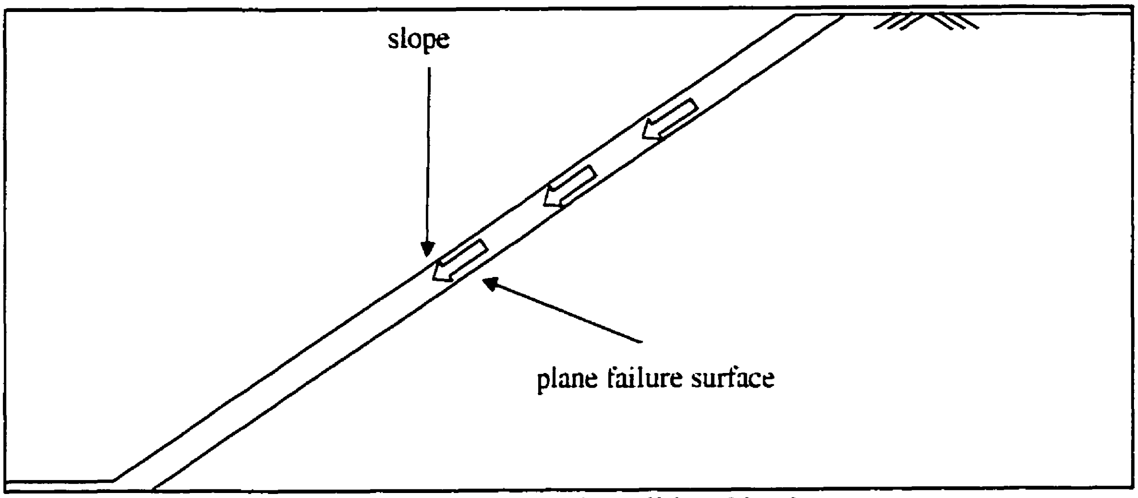



Fig. 2.1: Planar landslide (side view)

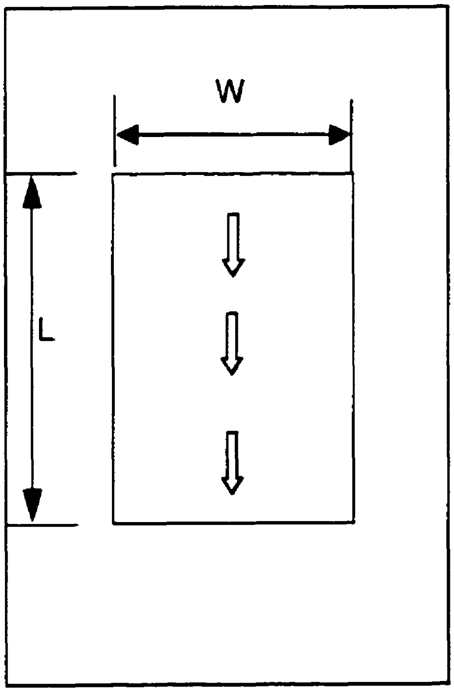



Fig. 2.2: Planar landslide (top view)

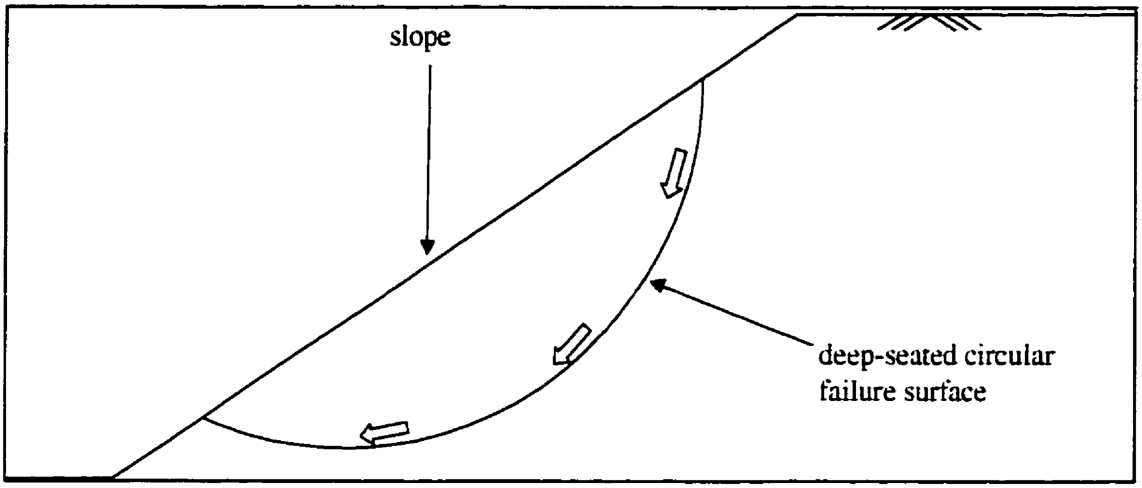



Fig. 2.3: Deep-seated landslide (side view)

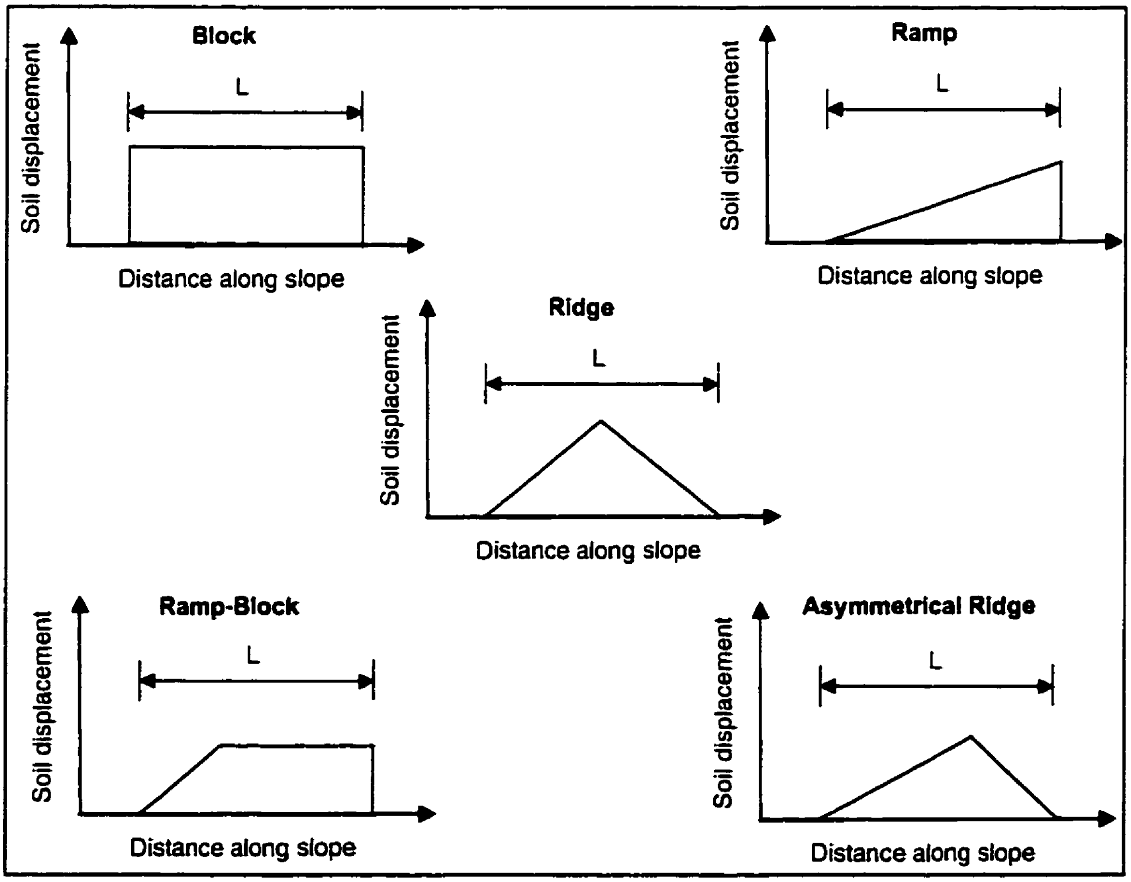



Fig. 2.4: Five idealized longitudinal PGD patterns

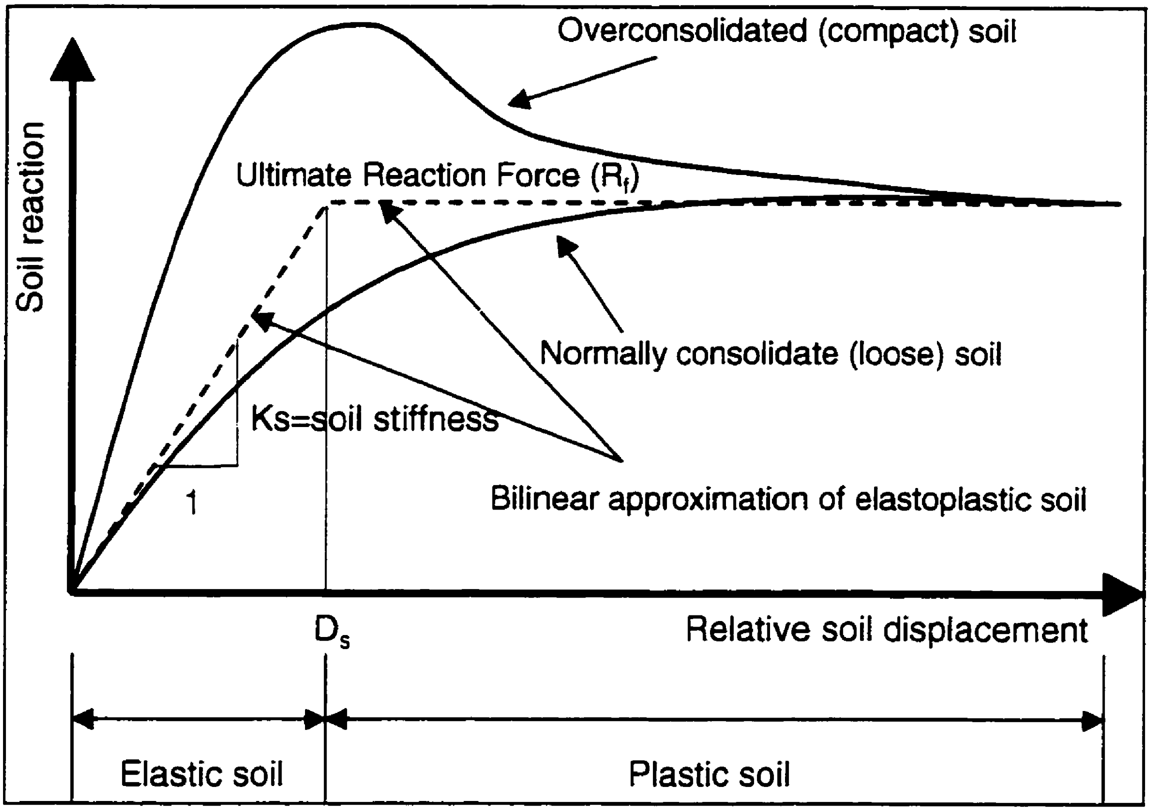



Fig. 2.5: Bilinear approximation of elastoplastic soil

# Chapter 3

# Soil - Pipeline Interaction on Slopes

## 3.1 Introduction

This chapter presents new analytical solutions for the design of pipelines subject to longitudinal, transverse and deep-seated landslides. The detailed derivation of equations presented in this chapter is shown in appendices A, B and C.

## 3.2 Longitudinal Landslide

For longitudinal landslide (where the landslide moves parallel to the pipeline – see Figure 3.1), new landslide displacement function and boundary conditions were used to eliminate discontinuity of the strain equation of elastic-only soil reaction. For the elastic/plastic soil reaction part, our derivation (see equations [A34] to [A61]) is very similar to the one proposed by O'Rourke and Nordberg's (1992). The only difference is that O'Rourke and Nordberg used the boundary condition that the pipe strain is zero at the center (L/2) of the landslide. This boundary condition is not true if the length of landslide (L) is large and amount of PGD ($\delta$) was small because pipe strain could reach zero well before the center of the landslide.

We assume elastoplastic soil/pipeline interaction using a bilinear approximation of soil's stress-strain curves (see Figures 2.5 and 3.2). This is like the stress-strain curves for steel with elastic and plastic ranges. The ultimate force developed as a function of soil displacement is expressed in equation [3.1a], where the subgrade modulus ($K_s$) is like the Young's Modulus of Elasticity (E) for steel. The ultimate force can also be expressed as a function of the horizontal bearing capacity and undrained strength for clay in equation [3.1b]. It is also possible to represent it as a function of the cohesive strength and frictional angle for a general soil type with cohesive and frictional characteristics in equations [3.1c].

[3.1a] $$\boxed{R_f = K_s \cdot d \cdot D_s}$$

[3.1b] $$\boxed{R_f = N_c \cdot d \cdot S_u \cdot (1\,\mathrm{m})}$$

[3.1c] $$\boxed{R_f = [c + \gamma \cdot H \cdot (0.9 \cdot \tan\varphi)] \cdot (\pi \cdot d) \cdot (1\,\mathrm{m})}$$

where

$N_c$ is the bearing capacity factor depending on the material properties.

$S_u$ is the undrained strength.

$K_s$ is the elastic subgrade modulus.

$D_s$ is the soil displacement to reach the ultimate reaction force (typically 5-10 mm according to Committee on Gas and Liquid Fuel Life Lines, 1984).

c is the soil cohesive strength,

$\gamma$ is unit weight of soil,

H is burial depth to pipe centerline,

$\phi$ is friction angle of soil, and

d is pipe diameter

The force equilibrium for a finite element piece of pipe (see Figure 3.3) is given in equation [3.2]. Combining this with the relationship between force and strain in equation [3.3], we obtain a second-order differential equilibrium equation in the stable and unstable regions in equation [3.4]:

[3.2] $$\boxed{N = N \quad \frac{d}{dx}N\cdot dx \; - \tau\cdot(\pi\cdot d)\cdot dx = K_s\cdot(u \quad \delta)\cdot(\pi\cdot d)\cdot dx}$$

[3.3] $$\boxed{N = E\cdot A\cdot\varepsilon = E\cdot A\cdot\frac{d}{dx}\; u \quad \delta}$$

[3.4] $$\boxed{u = C_1\cdot e^{\lambda\cdot x} \cdot C_2\cdot e^{\lambda\cdot x} \cdot \delta}$$

where,

N is the axial force on the pipe,

u is the pipe movement, and

$\delta$ is the soil movement.

We assume the longitudinal landslide movement function as indicated in Figure 3.4 (note that it is a plot of the *magnitude* of soil and pipeline displacement along the length of the pipe axis). This is identical to the model of pipeline subjected to rigid block PGD by O'Rourke and Nordberg (1992), but they had erroneously used the boundary condition of zero strain at the center of PGD (L/2).

The pipeline deflection is derived from solving the second-order differential equilibrium equation [3.4] with appropriate boundary conditions. We have a stable region from $x = -\infty$ to $x = 0$, and an unstable region from $x = 0$ to $x = L/2$. The pipeline from $x = -\infty$ to $L/2$ is in tension, and from $x = L/2$ and beyond is in compression.

### 3.2.1 Elastic soil reaction

For $\delta < 2D_s$, the soil is in elastic domain. The entire pipeline deflection profile can be determined by applying the second-order differential equilibrium equation in the stable and unstable regions (Figure 3.5). The continuity of strain at $x = 0$ cannot be satisfied using the step function indicated in Figure 3.4 for the soil displacement. Thus, we change the soil displacement profile to increase at an angle ($\alpha$) at $x = 0$, instead of instantaneously as for the case of the step function when solving for $\delta > 2D_s$. Also, the boundary condition of $\varepsilon = 0$ at $x = L/2$ is not used because this is not true if the length of the landslide (L) is very large. Instead, we replace the above boundary condition with the continuity condition of strain at the transition point between the plastic and elastic soil zones.

The analysis involves determination of 8 integration constants and position A. The 8 integration constants are denoted by ($C_1$ and $C_2$ in stable region; $k_1$, $k_2$, $k_3$, $k_4$, $k_5$, and $k_6$ in unstable region). We use the following boundary conditions and equation to determine the 9 constants:

At $x = -\infty$, $u = 0$ (1 constant) where u is the displacement;

At $x = 0$, u and u' are continuous (2 constants);

At x = A, u and u' are continuous where u' is the first derivative (2 constants);

At x = B, u and u' are continuous where u' is the first derivative (2 constants);

At x = A, u = $u_A$ where $u_A$ is the soil displacement at position A and equal to the pipeline deflection (1 constant);

Force equilibrium equation in longitudinal direction (1 constant).

The following are the equations to be used to solve the maximum strain in the pipe at $\delta < 2D_s$ (see the derivations in Appendix A.2 from equations [A12] to [A33]):

[3.2] $$\varepsilon_{max.} = \lambda \cdot \; u_o \; \frac{\tan(\alpha)}{2\cdot\lambda}\cdot e^{\lambda\cdot x_l} \; \frac{\tan(\alpha)}{2\cdot\lambda}\cdot e^{\lambda\cdot x_l} \cdot \tan(\alpha)$$

[3.3] $$x_l = A = \frac{\ln\left(1 \; \frac{2\cdot\lambda\cdot u_o}{\tan(\alpha)}\right)}{2\cdot\lambda}$$

[3.4] $$u_o = e^{\frac{1}{2}\cdot\lambda\cdot L} - e^{\frac{1}{2}\cdot\lambda\cdot L} \; e^{\frac{1}{2}\cdot\lambda\cdot L \; \frac{2\cdot\delta}{\tan(\alpha)}} \; e^{\frac{1}{2}\cdot\lambda\cdot L \; \frac{2\cdot\delta}{\tan(\alpha)}} \; \frac{\tan(\alpha)\cdot e^{\frac{1}{2}\cdot\lambda\cdot L}}{2\cdot\lambda}$$

[3.5] $$\lambda = \sqrt{\frac{K_s\cdot\pi\cdot d}{E\cdot A}}$$ and

[3.6] $$x_r = B = \frac{\delta}{\tan(\alpha)}$$

### 3.2.2 Elasto-plastic soil reaction

For $\delta > 2D_s$, the soil ultimate strength is mobilized. Four regions of soil-pipeline interaction are identified (see Figure 3.4):

Region 1 in stable soil with elastic soil resistance ($x = -\infty < x < x_a$);

Region 2 in stable soil with plastic soil resistance ($x = x_a < x < 0$);

Region 3 in unstable soil with plastic soil resistance ($x = 0 < x < x_b$);

Region 4 in unstable soil with elastic soil resistance ($x = x_b < x < L/2$ where L is the width of the landslide).

The entire pipeline deflection profile can be determined by applying the second-order differential equilibrium equation in each region. The analysis involves determination of 8 integration constants and 2 elasto-plastic interface locations, $x_a$ and $x_b$. The 8 integration constants are denoted by ($C_1$ and $C_2$ in Region 1; $C_3$ and $C_4$ in Region 2; $k_1$and $k_2$, in Region 3; $k_3$and $k_4$, in Region 4). We use the following boundary conditions and equation to determine the 10 constants:

At $x = -\infty$, $u = 0$ (1 constant) where u is the displacement;

At $x = x_a$, $u = D_s$ (1 constant);

At $x = x_a$, u and u' are continuous where u' is the first derivative (2 constants);

At $x = 0$, u and u' are continuous (2 constants);

At $x = x_b$, u and u' are continuous where u' is the first derivative (2 constants);

At $x = x_b$, $u = \delta - D_s$ where $\delta$ is the landslide movement (1 constant);

Force equilibrium equation in longitudinal direction (1 constant).

For the case of $\delta > 2D_s$, the maximum pipe strain occurs at interface between stable and unstable soil and is expressed in equation [3.7]. The expression is derived in Appendix A.3 from equations [A34] to [A62]. The strain is taken as the first derivative of the pipe deflection with respect to the pipe axis.

[3.7] $$\boxed{\varepsilon_{max.} = x_a \cdot \lambda^2 \cdot \lambda \cdot Ds}$$

where $x_a$ is solved by the secant or bisection method from the following equation:

[3.8] $$0 = e^{\lambda \cdot 2 \cdot x_a \cdot \frac{2}{\lambda} - 2 \cdot 2 \cdot x_a^2 \; \frac{4}{\lambda} \cdot x_a - \frac{5}{\lambda^2} \; \frac{2}{Ds \cdot \lambda^2} \cdot \delta - 1} - 1 \cdot 1 \quad 2 \cdot \lambda^2 \cdot x_a^2 \; 4 \cdot x_a \cdot \lambda - 5 \; \frac{2}{Ds} \cdot \delta \quad 2$$

## 3.3 Transverse Landslide

For transverse landslide (where the landslide is moving at 90° or perpendicular to the pipeline – see Figure 3.6), a new analytical solution has been developed to include both the bending and tensile strain on the pipe. The tensile strain is caused by the axial stretching of the pipe due to axial forces (see Figure 3.7). Bending strain is caused by the bending of the pipe such that the outer fiber of the pipe is stretched in comparison to its neutral axis (where the fiber length remains constant). See Equations [B31] and [B32] for the general tensile and bending strain equations. Figure 3.8 shows that the tensile stress is much higher than the bending stress near the interface between stable and unstable soil. It is therefore very important to account for the tensile stress as a part of the total stress.

Previous methods (e.g., Hetenyi, 1946; Rajani *et al.*, 1995) assumed a prior knowledge of the pipe displacement or the force acting on the pipeline at the edge of the landslide. They did not take into account the soil-pipeline interaction in terms of their relative displacements and resulting force. They also assumed an infinite landslide width.

Rajani *et al.* for example also employed the fourth-order differential equation of a beam on elastic foundation. However, they ignored relative soil-pipeline displacement, assumed infinite landslide width, and consequently they must also assume a double curvature at the interface between stable and unstable soil. They calculated the stress (strain) on a pipeline by finding the maximum moment developed due to an applied end load. The applied end load is derived as a function of end displacement of the pipe – which is assumed known.

Like previous methods, we also use the bilinear approximation of the elastoplastic soil. We have investigated transverse landslides with infinite and finite landslide width, and modeled the force acting on the pipeline as a function of relative soil-pipeline displacement. Equilibrium of forces between stable and unstable soil is also imposed. We also account for the tensile strain as a part of the total strain developed in the pipe – by adding the tensile strain to the tensile part of the bending strain.

For most practical or conservative case, the relative soil movement exceeds $D_s$. In such case, the ultimate soil resistance is mobilized. There are four different soil-pipeline interaction regions (see Figure 3.9):

Region 1 in stable soil with elastic soil resistance ($x = -\infty < x < A$);

Region 2 in stable soil with plastic soil resistance ($x = A < x < 0$);

Region 3 in unstable soil with plastic soil resistance ($x = 0 < x < B$);

Region 4 in unstable soil with elastic soil resistance ($x = B < x < (R^2 - Rm^2)^{0.5}$).

The entire pipeline deflection profile can be determined by applying the fourth-order differential equilibrium equation in each region (see equation [B4]). This equation is perfectly valid as long as the pipeline remains elastic. Once the soil's elastic displacement limit ($D_s$) is exceeded, then the ultimate reaction force ($R_f$) will be acting on that portion of the pipeline, while the rest of the pipeline-foundation is still elastic. Equation [B4] is solved to get pipeline displacement equation [B24] by assuming that the soil displacement is a constant along the width of the landslide and combining the homogeneous and nonhomogeneous solutions.

Using equation [B24], the analysis comes down to determining the 16 integration constants and 2 elasto-plastic interface locations, A and B. The 16 integration constants are denoted by ($C_1$, $C_2$, $C_3$, and $C_4$, in Region 1; $C_5$, $C_6$, $C_7$, and $C_8$, in Region 2; $k_1$, $k_2$, $k_3$, and $k_4$, in Region 3; $k_5$, $k_6$, $k_7$, and $k_8$, in Region 4). We use the following boundary conditions and equation to determine the 18 constants:

At $x = -\infty$, $u = 0$ (2 constants) where u is the displacement;

At $x = A$, $u = D_s$ (1 constant);

At $x = A$, $u$, $u'$, $u''$, and $u'''$ are continuous where $u'$, $u''$ and $u'''$ are first, second and third derivatives (4 constants);

At $x = 0$, $u$, $u'$, $u''$, and $u'''$ are continuous (4 constants);

At $x = B$, $u = u_p - D_s$ are continuous (1 constant);

At $x = B$, $u$, $u'$, $u''$, and $u'''$ are continuous (4 constants);

At $x = (R^2 - Rm^2)^{0.5}$, $u = 0$ (1 constant);

Force equilibrium equation in transverse direction (1 constant).

Parametric analysis shows that maximum critical strains are likely developed in Region 3. For most practical range of pipeline strains, the maximum strain due to both bending and stretching of the pipeline is given by the following equation expressed in terms of constant $k_1$, $k_2$, and $k_3$ and $R_f$.

$$[3.9] \quad \boxed{\varepsilon_p(x) = \frac{1}{2}\cdot\left[\frac{1}{E\cdot I}\cdot\left(\frac{1}{6}\cdot R_f\cdot x^3 + \frac{1}{2}\cdot k_1\cdot x^2 + k_2\cdot x + k_3\right)\right]^2 + \left|\frac{d}{2}\cdot\left[\frac{1}{E\cdot I}\cdot\left(\frac{1}{2}\cdot R_f\cdot x^2 + k_1\cdot x + k_2\right)\right]\right|}$$

where the constants $k_1$, $k_2$, and $k_3$ are solve as a set of 12 non-linear equations

### 3.3.1 Infinite width transverse landslide

Assuming the deflection profile follows Euler's Elastic Curve, and the width of landslide is infinite, we can state by symmetry that there is a double curvature at the edge of the landslide. We also assume the landslide takes place sufficiently slow enough so as not to shear the pipeline at the edge of the landslide.

As long as the relative landslide movement has not exceeded the limiting elastic displacement ($D_s$), the combined bending and stretching strain equation for a pipeline on elastic foundation is given in equation [3.9] and derived in equations [B100] to [B108]. Please note that the strain equation [3.9] is derived for the stable soil region ($x = -\infty$ to $x = 0$), and the values calculated would be a mirror imaged for the region ($x = 0$ to $x = \infty$).

[3.9] $$\boxed{\varepsilon = \frac{d}{2} \cdot \delta \cdot \sin(\lambda \cdot x) \cdot \lambda^2 \cdot e^{\lambda \cdot x} \quad \lambda^2 \cdot \frac{\delta^2}{4} \cdot e^{2 \cdot \lambda \cdot x} \cdot (1 \quad \sin(2 \cdot \lambda \cdot x))}$$

As the relative landslide movement exceeds $D_s$, then there develops a plastic region and an elastic region. The combined bending and stretching strains for the plastic region ($\varepsilon_p$), and the elastic region ($\varepsilon_e$), and the location of their transitions point (B) are given in equations [3.10], [3.11] and [3.12]. See the derivations in Appendix B from equations [B109] to [B131].

[3.10] $$\boxed{\begin{aligned} \varepsilon_p = \frac{d}{2} \cdot \frac{1}{2} \cdot x \cdot \frac{\lambda \cdot B^2 \cdot R_f - (2 - \lambda \cdot x) \cdot R_f \cdot B - 4 \cdot E \cdot I \cdot \lambda^3 \cdot D_s \quad R_f \cdot x}{((\lambda \cdot B - 1) \cdot (E \cdot I))} \ldots \\ + \frac{1}{1152} \cdot \left( \frac{\begin{array}{l} B^5 \cdot R_f \cdot \lambda - 3 \cdot R_f \cdot B^4 - \ 6 \cdot x^2 \cdot \lambda \cdot R_f - 8 \cdot E \cdot I \cdot \lambda^3 \cdot D_s \ \cdot B^3 \ldots \\ + \ 12 \cdot x^2 \cdot R_f - 4 \cdot R_f \cdot x^3 \cdot \lambda \ \cdot B^2 \ldots \\ + \ 24 \cdot x^2 \cdot E \cdot I \cdot \lambda^3 \cdot D_s \ \ 4 \cdot R_f \cdot x^3 - 24 \cdot E \cdot I \cdot \lambda \cdot D_s \ \ 24 \cdot E \cdot I \cdot \lambda \cdot \frac{\delta}{2} \ \cdot B \ldots \\ + 24 \cdot E \cdot I \cdot D_s \ \ 24 \cdot E \cdot I \cdot \frac{\delta}{2} \end{array}}{((\lambda \cdot B - 1) \cdot (B \cdot (E \cdot I)))} \right)^2 \end{aligned}}$$

[3.11]

$$
\begin{aligned}
\varepsilon_e = \frac{d}{2}\cdot\Bigg[ & \left[\begin{aligned} & (\sin(\lambda\cdot B)\cdot\sin(\lambda\cdot x) - \cos(\lambda\cdot B)\cdot\cos(\lambda\cdot x))\cdot R_f\cdot\frac{B^2}{4} \ldots \\ & + \left[\begin{aligned}&(\sin(\lambda\cdot B) - \cos(\lambda\cdot B))\cdot\sin(\lambda\cdot x) \ldots \\ &+ (\cos(\lambda\cdot B) - \sin(\lambda\cdot B))\cdot\cos(\lambda\cdot x)\end{aligned}\right]\cdot E\cdot I\cdot\lambda^3\cdot D_s\cdot B \ldots \\ & + (\sin(\lambda\cdot x)\cdot\cos(\lambda\cdot B) - \cos(\lambda\cdot x)\cdot\sin(\lambda\cdot B))\cdot E\cdot I\cdot\lambda^2\cdot D_s \end{aligned}\right]\cdot\frac{e^{\lambda\cdot B}\cdot 2}{((\lambda\cdot B - 1)\cdot(E\cdot I))\cdot e^{\lambda\cdot x}} \ldots \\
& + \frac{1}{2}\cdot\frac{\left[\begin{aligned} & \left[\begin{aligned}&(\cos(\lambda\cdot B) - \sin(\lambda\cdot B))\cdot\sin(\lambda\cdot x) \ldots \\ &+ 1\cdot(\sin(\lambda\cdot B) - \cos(\lambda\cdot B))\cdot\cos(\lambda\cdot x)\end{aligned}\right]\cdot\frac{R_f}{4}\cdot B^2 \ldots \\ & + (\cos(\lambda\cdot x)\cdot\cos(\lambda\cdot B) \;\; \sin(\lambda\cdot x)\cdot\sin(\lambda\cdot B))\cdot 2\cdot E\cdot I\cdot\lambda^3\cdot D_s\cdot B \ldots \\ & + \left[\begin{aligned}&(\sin(\lambda\cdot B) \;\; \cos(\lambda\cdot B))\cdot\cos(\lambda\cdot x) \ldots \\ &+ 1\cdot(\sin(\lambda\cdot B) - \cos(\lambda\cdot B))\cdot\sin(\lambda\cdot x)\end{aligned}\right]\cdot E\cdot I\cdot\lambda^2\cdot D_s \end{aligned}\right]^2}{(\lambda\cdot((\lambda\cdot B - 1)\cdot(E\cdot I)))\cdot e^{\lambda\cdot(B - x)}}\Bigg]
\end{aligned}
$$

[3.12]

$$
\begin{aligned}
0 = {} & R_f\cdot B^5\cdot\lambda^2 - 5\cdot\lambda\cdot R_f\cdot B^4 - 6\cdot R_f - 16\cdot E\cdot I\cdot\lambda^4\cdot D_s\cdot B^3 - 48\cdot E\cdot I\cdot\lambda^3\cdot D_s\cdot B^2 \ldots \\
& + 24\cdot\frac{\delta}{2} - 48\cdot D_s\cdot B\cdot E\cdot I\cdot\lambda^2 - D_s\cdot\frac{\delta}{2}\cdot 24\cdot E\cdot I\cdot\lambda
\end{aligned}
$$

[3.13]

$$
\lambda = \left(\frac{K_s\cdot d}{(4\cdot E\cdot I)}\right)^{\frac{1}{4}}
$$

In equation [3.12], B is the point of the transition between the plastic and elastic zones, and it may be solved using the secant or bisection method.

Some of the results of this investigation are listed below:

1) At small pipe deflections, the plastic zone is small. The displacement, tensile strain, and bending strain profiles of the elastoplastic equation look like that of the pure elastic equation.

2) At larger pipe deflections, the plastic zone increases. The elastoplastic soil deflects more than the pure elastic soil, but

the pipe in the elastoplastic soil curves less and subsequently has lower bending and tensile strains (see Figure 3.10).

3) Depending on the various soil and pipe characteristics, the tensile stress from the stretching of the pipe has a significant effect to the total strain.

4) The plastic zone and total strain increases with the increase of elastic modulus of the pipe (E), and the soil subgrade modulus ($K_s$).

5) The plastic zone increases but the total strain decreases with increasing pipe thickness (t), and pipe diameter (d).

6) If the soil density was less, the cohesive strength and frictional angle was lower, and the pipe was not buried as deep, then the total strain developed is less. In other words, a high soil resistance ($R_f$ is high) will produce larger stress in the pipe for any given ground displacement.

When the pipeline lies at an angle to the direction of landslide, the pipeline will be subjected to both longitudinal and transverse loading. Since the resulting displacement-strain relationships, as presented in this thesis, is modeled as elastic-plastic soil response, it is not possible to simply add the effects of combined transverse and longitudinal movements. However, it should be possible to take a combined longitudinal and transverse loading and divide the loading vectorially into each component, then determine the strain in the pipe due to each component of loading and sum up the effects on the pipe.

Figures 3.11 and 3.12 are plots of the maximum pipeline strain developed for a pipeline lying at 30 and 60 degrees to the landslide direction, respectively, for different lengths of landslide. At 60 degrees, the pipe is subjected to higher transverse loading and lower longitudinal loading than at 30 degrees. As the angle between the axis of the pipeline and the landslide direction increases, the bending strain component increases and become more dominant, while the tensile strain decreases.

### 3.3.2 Finite width transverse landslide

Figures 3.6 and 3.9 show an overhead view of a transverse landslide with finite width.

As long as the relative landslide movement has not exceeded the limiting elastic displacement ($D_s$), the combined bending and stretching strain equation for a pipeline on elastic foundation inside the landslide, where the maximum strain takes place, is

[3.14]

$$
\begin{aligned}
\varepsilon = \frac{1}{2}\cdot\Bigg[&(\sin(\lambda\cdot x)-\cos(\lambda\cdot x))\cdot\frac{1}{2}\cdot\frac{\sin\left(\frac{1}{2}\cdot\lambda\cdot W\right)^2 \quad \cos\left(\frac{1}{2}\cdot\lambda\cdot W\right)^2}{\sin\left(\frac{1}{2}\cdot\lambda\cdot W\right)^2-\cos\left(\frac{1}{2}\cdot\lambda\cdot W\right)^2}\cdot\delta\cdot e^{\lambda\cdot W}\ldots\\
&+(\sin(\lambda\cdot x)-\cos(\lambda\cdot x))\cdot\sin\left(\frac{1}{2}\cdot\lambda\cdot W\right)\cdot e^{\lambda\cdot W}\cdot\frac{\cos\left(\frac{1}{2}\cdot\lambda\cdot W\right)}{\sin\left(\frac{1}{2}\cdot\lambda\cdot W\right)^2-\cos\left(\frac{1}{2}\cdot\lambda\cdot W\right)^2}\cdot\delta\\
&+(\cos(\lambda\cdot x) \quad \sin(\lambda\cdot x))\cdot\frac{1}{2}\cdot\delta\cdot\cos(\lambda\cdot x)\Bigg]^2\cdot e^{\lambda\cdot x}\ldots\cdot\lambda\ldots\\
+\frac{d}{2}\cdot\Bigg[&1\cdot\sin(\lambda\cdot x)\cdot\frac{1}{2}\cdot\frac{\sin\left(\frac{1}{2}\cdot\lambda\cdot W\right)^2 \quad \cos\left(\frac{1}{2}\cdot\lambda\cdot W\right)^2}{\sin\left(\frac{1}{2}\cdot\lambda\cdot W\right)^2-\cos\left(\frac{1}{2}\cdot\lambda\cdot W\right)^2}\cdot\delta\cdot e^{\lambda\cdot W}\ldots\\
&+\cos(\lambda\cdot x)\cdot\sin\left(\frac{1}{2}\cdot\lambda\cdot W\right)\cdot e^{\lambda\cdot W}\cdot\frac{\cos\left(\frac{1}{2}\cdot\lambda\cdot W\right)}{\sin\left(\frac{1}{2}\cdot\lambda\cdot W\right)^2-\cos\left(\frac{1}{2}\cdot\lambda\cdot W\right)^2}\cdot\delta\\
&+\sin(\lambda\cdot x)\cdot\frac{1}{2}\cdot\delta\cdot\cos(\lambda\cdot x)\Bigg]\cdot e^{\lambda\cdot x}\ldots\cdot 2\cdot\lambda^2
\end{aligned}
$$

Equation [3.14] is derived by applying equations [B31] and [B32] to equations [B38] and [B39], and substituting in the constants from equations [B47] to [B52].

As the relative landslide movement exceeds $D_s$, then there develops a plastic region and an elastic region. For most practical cases, the maximum combined bending and stretching strains occurs inside the plastic region of the landslide (Region 3):

$$\boxed{\varepsilon_3(x) = \frac{1}{2}\cdot\left[\frac{1}{E\cdot I}\cdot\left(\frac{1}{6}\cdot R_f\cdot x^3 - \frac{1}{2}\cdot k_1\cdot x^2 - k_2\cdot x - k_3\right)\right]^2 - \frac{d}{2}\cdot\frac{1}{E\cdot I}\cdot\left(\frac{1}{2}\cdot R_f\cdot x^2 - k_1\cdot x - k_2\right)} \quad [3.15]$$

where $k_1$, $k_2$ and $k_3$ are constants to be solved numerically using the Newton-Raphson method (Press *et al.* 1986) as implemented in the Basic program shown in Appendix D and briefly described at the end of Appendix B. Equation [3.15] is derived by applying equations [B31] and [B32] to equations [B61] and [B62], and substituting in the constants solved from equations [B81] to [B96].

For a finite width of 20 m, the results are essentially identical to a transverse landslide of infinite width. For $\delta < 5D_s$, the maximum strain occurs at the elastic part (Region 4) of the landslide. Beyond $5D_s$, the maximum strain occurs in the plastic part (Region 3) of the landslide. Tensile strain begins to dominate at $\delta > 35D_s$, and the total strain will continue to increase from the tensile strain component, even though the bending strain begins to level off.

The results of a 5-m and 20-m width transverse landslide are plotted against the infinite width landslide in Figure 3.13. We can see that at 20-m width, the results are almost identical to the infinite width landslide. For the 5-m width landslide, both the bending and tensile strain is larger, and this produces a larger total strain in comparison to the 20-m and infinite width landslides. This means smaller width transverse landslides are more dangerous to the pipeline because they cause the pipeline to bend more sharply and stretch more than they would have for larger width landslides. They do this because the soil is relatively stiff, and a small width landslide will act like a concentrated load that tends to shear the pipeline along the interface between stable and unstable soil. The slope of the pipeline in the middle of the landslide must be zero because of symmetry, and to accomplish this for a small width landslide, the pipeline develops greater stretching and bending strain near the interface of the landslide and stable soil. If the soil was less dense, less cohesive and has lower internal frictional angle (lower soil resistance), then this problem would be reduced.

## 3.4 Deep-seated Landslide

In a deep-seated landslide, the sliding surface closely resembles arcs of circles (see Figure 2.3). Figure 3.14 shows a pipeline laid parallel to a slope of angle θ with a soil cover depth of h. There is a circular failure surface of radius R intersecting the pipeline. The distance Rm is the perpendicular distance of the center from the failure circle to the pipeline. The soil mass slides and rotates along the circular failure surface with a tangential displacement δ. The soil movement along the soil-pipeline interface is defined as u(x), which varies along the position of the pipeline. Its longitudinal ($u_L$) and perpendicular ($u_p$) components are given in equations [3.16] and [3.17]. The derivations are shown in equations [C3] through to [C11].

[3.16] $$\boxed{u_L = \frac{\delta}{R} \cdot Rm}$$

[3.17] $$\boxed{u_p(x) = \frac{\delta}{R} \cdot \quad R^2 \quad Rm^2 \quad x}$$

where x is the distance along the pipeline as defined in Figure 3.15.

### 3.4.1 Behavior of pipeline subjected to transverse displacement component

The transverse displacement component $u_L$ is maximum at the intersection of the pipeline and the circular failure (x = 0) surface, and varies linearly with distance along the pipeline within the circular landslide (see Figure 3.15). We assume that the minimum soil cover (h) is sufficient to permit the development of ultimate reaction force ($R_f$) in the soil on the upheave side of the circular failure surface. The derivations for the perpendicular component displacement component of the soil (equation [3.17]) as well as the equations of the pipeline displacement equations are shown in appendix C.

For most practical or conservative case, the relative soil movement exceeds $D_s$. In such case, the ultimate soil resistance is mobilized. There are four different soil-pipeline interaction regions (see Figure 3.15):

Region 1 in stable soil with elastic soil resistance ($x = -\infty < x < A$);

Region 2 in stable soil with plastic soil resistance ($x = A < x < 0$);

Region 3 in unstable soil with plastic soil resistance ($x = 0 < x < B$);

Region 4 in unstable soil with elastic soil resistance ($x = B < x < (R^2 - Rm^2)^{0.5}$).

The entire pipeline deflection profile can be determined by applying the 4th-order differential equilibrium equation in each region. The analysis involves determination of 16 integration constants and 2 elastoplastic interface locations, A and B. The 16 integration constants are denoted by ($C_1$, $C_2$, $C_3$, and $C_4$, in Region 1; $C_5$, $C_6$, $C_7$, and $C_8$, in Region 2; $k_1$, $k_2$, $k_3$, and $k_4$, in Region 3; $k_5$, $k_6$, $k_7$, and $k_8$, in Region 4). We use the following boundary conditions and equation to determine the 18 constants:

At $x = -\infty$, $u = 0$ (2 constants) where u is the displacement;

At $x = A$, $u = D_s$ (1 constant);

At $x = A$, $u$, $u'$, $u''$, and $u'''$ are continuous where $u'$, $u''$ and $u'''$ are first, second and third derivatives (4 constants);

At $x = 0$, $u$, $u'$, $u''$, and $u'''$ are continuous (4 constants);

At $x = B$, $u = up - Ds$ are continuous (1 constant);

At $x = B$, $u$, $u'$, $u''$, and $u'''$ are continuous (4 constants);

At $x = (R^2 - Rm^2)^{0.5}$, $u = 0$ (1 constant);

Force equilibrium equation in transverse direction (1 constant).

Parametric analysis shows that the maximum combined bending and tensile strain occurs in the plastic soil region of the soil movement (Region 3) as represented in equation [3.18]. This equation is derived by applying equations [B31] and [B32] to equation [C17]. The constants in the equation are solved simultaneously as a set of 18 non-linear equations [C21] to [C38].

[3.18]
$$\boxed{\begin{aligned}\varepsilon_3(x) = {} & \frac{1}{2}\cdot\left[\frac{1}{E\cdot I}\cdot\left(\frac{1}{6}\cdot R_f\cdot x^3 - \frac{1}{2}\cdot k_1\cdot x^2 - k_2\cdot x - k_3\right)\right]^2 \ldots \\ & + \frac{d}{2}\cdot\frac{1}{E\cdot I}\cdot\left(\frac{1}{2}\cdot R_f\cdot x^2 - k_1\cdot x - k_2\right)\end{aligned}}$$

### 3.4.2 Behavior of pipeline subjected to longitudinal displacement component

The longitudinal component of the soil movement from a circular deep-seated failure is constant along the pipeline. The soil-pipeline interaction problem is solved iteratively exactly using the method proposed by Chan and Wong (1997). The pipeline deflection is derived from solving the second-order differential equilibrium equation with appropriate boundary conditions as described already in section 3.2. The only difference is that in all the equations (for example, equation [3. 4]), the displacement magnitude ($\delta$) and landslide length (L) for the planar landslide is replaced by $((\delta/R)Rm)$ and $(2(R^2 - Rm^2)^{0.5})$ for the longitudinal component of deep-seated landslide (see Figure 3.16 and 3.17).

We have a stable region from $x = -\infty$ to $x = 0$, and an unstable region from $x = 0$ to $x = (\delta/R)Rm$. The pipeline from $x = -\infty$ to $(\delta/R)Rm$ is in compression, and from $x = (\delta/R)Rm$ to $\infty$ is in tension.

For $\delta < 2D_s$, the soil is in elastic domain. The entire pipeline deflection profile can be determined by applying the $2^{th}$-order differential equilibrium equation in the stable and unstable regions (Figure 3.17). The continuity of strain at $x = 0$ cannot be satisfied using the step function indicated in Figure 3.16 for the soil displacement. Thus, we change the soil displacement profile to increase at an angle ($\alpha$) at $x = 0$, instead of instantaneously as for the case of the step function when solving for $\delta > 2D_s$. Also, the boundary condition of $\varepsilon = 0$ at $x = (R^2 - Rm^2)^{0.5}$ is not used because this is not true if the length of the longitudinal component $(2(R^2 - Rm^2)^{0.5})$ is very large. The analysis involves determination of 8 integration constants and position A. The 8 integration constants are denoted by ($C_1$ and $C_2$ in stable region; $k_1$, $k_2$, $k_3$, $k_4$, $k_5$, and $k_6$ in unstable region). We use the following boundary conditions and equation to determine the 9 constants:

At $x = -\infty$, $u = 0$ (1 constant) where u is the displacement;

At $x = 0$, u and u' are continuous (2 constants);

At $x = A$, u and u' are continuous where u' is the first derivative (2 constants);

At $x = B$, u and u' are continuous where u' is the first derivative (2 constants);

At $x = A$, $u = u_A$ where $u_A$ is the soil displacement at position A and equal to the pipeline deflection (1 constant);

Force equilibrium equation in longitudinal direction (1 constant).

The following are the equations are used to solve the maximum strain in the pipe at $\delta < 2D_s$:

[3.19] $$\boxed{\varepsilon_{max.} = \lambda \cdot \; u_o \; \frac{\tan(\alpha)}{2\cdot\lambda} \cdot e^{\lambda \cdot x_1} \; \frac{\tan(\alpha)}{2\cdot\lambda} \cdot e^{\lambda \cdot x_1} - \tan(\alpha)}$$

[3.20] $$\boxed{x_1 = A = \frac{\ln\left(1 \; \frac{2\cdot\lambda\cdot u_o}{\tan(\alpha)}\right)}{2\cdot\lambda}}$$

[3.21] $$u_o := \left[ e^{\frac{1}{2}\cdot\lambda\cdot\left(R^2 - Rm^2\right)^{0.5}} - e^{\frac{1}{2}\cdot\lambda\cdot\left(R^2 - Rm^2\right)^{0.5} - \frac{2\cdot\frac{\delta}{R}\cdot Rm}{\tan(\alpha)}} \ldots + e^{-\frac{1}{2}\cdot\lambda\cdot\left(R^2 - Rm^2\right)^{0.5}} - e^{-\frac{1}{2}\cdot\lambda\cdot\left(R^2 - Rm^2\right)^{0.5} - \frac{2\cdot\frac{\delta}{R}\cdot Rm}{\tan(\alpha)}} \right] \cdot \frac{\tan(\alpha)\cdot e^{-\frac{1}{2}\cdot\lambda\cdot\left(R^2 - Rm^2\right)^{0.5}}}{2\cdot\lambda}$$

and

[3.22] $$\lambda := \sqrt{\frac{K_s\cdot\pi\cdot d}{E\cdot A}}$$

For $\delta > 2D_s$, the soil ultimate strength is mobilized. Similarly, four regions of soil-pipeline interaction are identified:

Region 1 in stable soil with elastic soil resistance ($x = -\infty < x < A$);

Region 2 in stable soil with plastic soil resistance ($x = A < x < 0$);

Region 3 in unstable soil with plastic soil resistance ($x = 0 < x < B$);

Region 4 in unstable soil with elastic soil resistance ($x = B < x < (R^2 - Rm^2)^{0.5}$).

The entire pipeline deflection profile can be determined by applying the $2^{th}$-order differential equilibrium equation in each region. The analysis involves determination of 8 integration constants and 2 elasto-plastic interface locations, A and B. The 8 integration constants are denoted by ($C_1$ and $C_2$ in Region 1; $C_3$ and $C_4$ in Region 2; $k_1$and $k_2$, in Region 3; $k_3$ and $k_4$, in Region 4). We use the following boundary conditions and equation to determine the 10 constants:

At $x = -\infty$, $u = 0$ (1 constant) where u is the displacement;

At $x = A$, $u = D_s$ (1 constant);

At $x = A$, u and u' are continuous where u' is the first derivative (2 constants);

At $x = 0$, u and u' are continuous (2 constants);

At $x = B$, u and u' are continuous where u' is the first derivative (2 constants);

At $x = B$, $u = \lambda - D_s$ where ld is the landslide movement (1 constant);

Force equilibrium equation in longitudinal direction (1 constant).

The maximum pipe strain is given by ($\delta > 2D_s$):

[3.23] $$\boxed{\varepsilon_{max} = A\cdot\lambda^2 - \lambda\cdot Ds}$$

where A is solved iteratively from the following equation:

[3.24]

$$0 = e^{-\left[2\cdot A\left(\frac{1}{\lambda} + 2\cdot A^2 + \frac{4}{\lambda}\cdot A + \frac{5}{\lambda^2}\right)\cdot\frac{2\cdot\frac{\delta}{R}\cdot Rm}{D_s\cdot\lambda^2} - R^2\, Rm^2\right]^{0.5}\cdot\lambda} - \left(1 + \frac{1}{\lambda}\right)\cdot\left(2\cdot A^2 + \frac{4}{\lambda}\cdot A + \frac{5}{\lambda^2}\right)\cdot\frac{2\cdot\frac{\delta}{R}\cdot Rm}{D_s\cdot\lambda^2}\cdot\lambda^2$$

Figure 3.18 is a plot of the maximum pipe strain developed from the perpendicular and longitudinal components. The strain calculated takes into account both the bending and tensile stress. For small soil movement, the strains induced by the longitudinal component are larger than those by perpendicular component. For large soil movement, the perpendicular component dominates the deformation mode, which indicates the most probable mode of failure is shearing near the intersection of the pipe and the circular failure surface.

## 3.5 Summary

Permanent Ground Displacement (PGD) refers to any type of a nonrecoverable soil movement such as a landslide. This thesis studied the soil-pipeline interaction for a pipeline in longitudinal, transverse and deep-seated landslides.

The tensile strain was added on top of the bending strain in the transverse landslide analysis, and it was shown to contribute significantly to the total strain developed in a pipeline.

It was shown that transverse landslides with a smaller width would produce greater strain on the pipeline than a landslide of larger width. Also shown was that pipelines in transverse landslides of greater than about 10-m width acts as if it was in a transverse landslide of infinite width. Our analysis indicates that if the width of the landslide (W) is greater than 10-m, the pipeline will act as a compliant pipe, that is, the pipeline will displace at the same amount as the landslide. This agrees with the conclusion made by O'Rourke and Nordberg (1990) based on information on observed lateral spreads.

Of the three different types of landslides, the deep-seated landslide is the most dangerous for pipeline failure. It does not take very much deep-seated soil movement to shear the pipeline near the interface between stable and unstable soil. It is wise to construct berms at the foot of slopes to prevent any such deep-seated landslides. The next most dangerous landslide is the transverse landslide because it can also shear the pipeline at the interface between stable and unstable soils.

The maximum strain developed in a pipeline in a longitudinal landslide reaches a plateau after and beyond a certain soil displacement. Since most longitudinal landslides are less than 100-m long, it would mean most pipelines could withstand most typical longitudinal landslides for any amount of soil displacement.

The most critical parameters in the safe design of pipelines in unstable slopes are the soil resistance ($R_t$), pipe modulus of elasticity (E), and pipe thickness (t). Using light weight aggregate (LWA) as a backfill not only provides drainage, but it will reduce the strain induced in the pipeline in a landslide. Using a more elastic steel pipe (lower modulus of elasticity) and thicker pipe would reduce the strain induced in the pipeline as well.

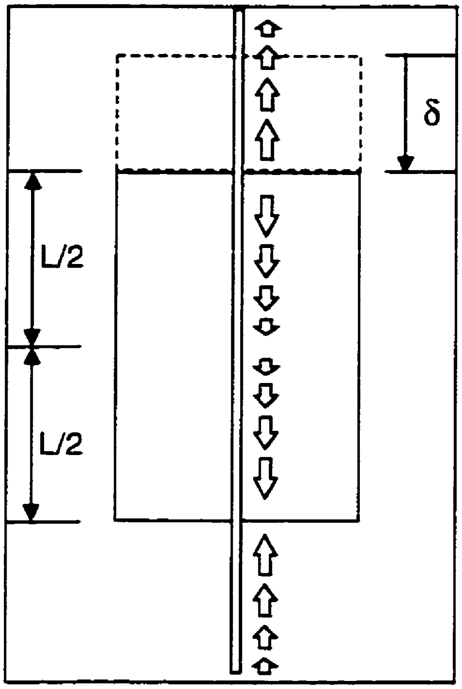



Fig. 3.1: Longitudinal landslide

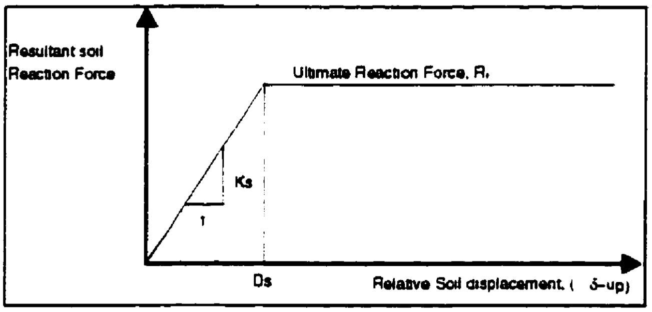



Fig. 3.2: Force-displacement relationship of soil

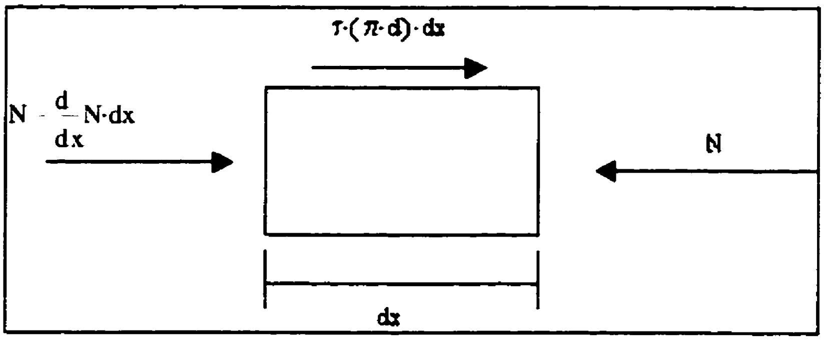



Fig. 3.3: Force equilibrium on a finite element of pipe

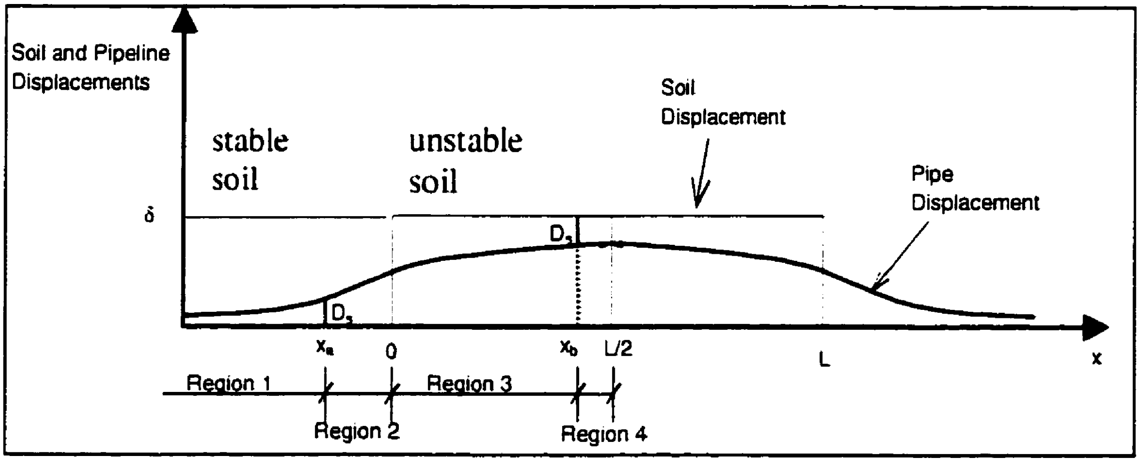



Fig. 3.4: Soil and pipeline displacement profile for longitudinal landslide

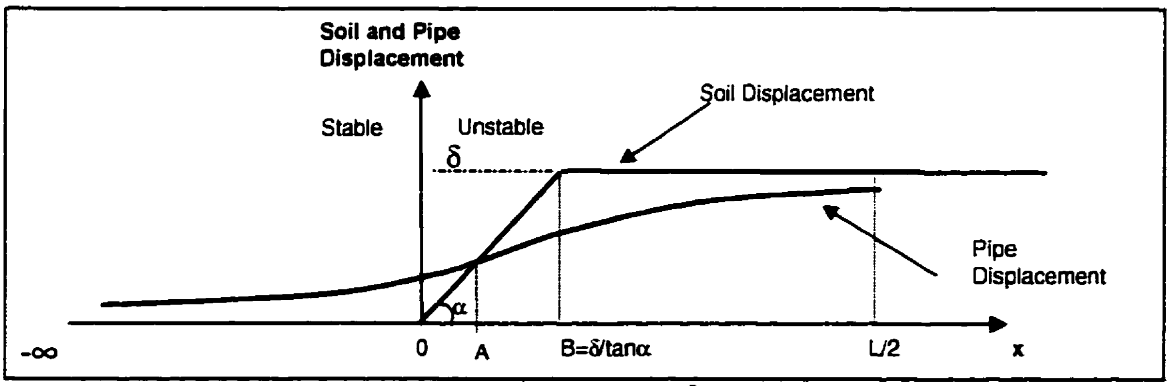



Fig. 3.5: Assumed soil displacement profile for $\delta < 2D_s$ in longitudinal landslide

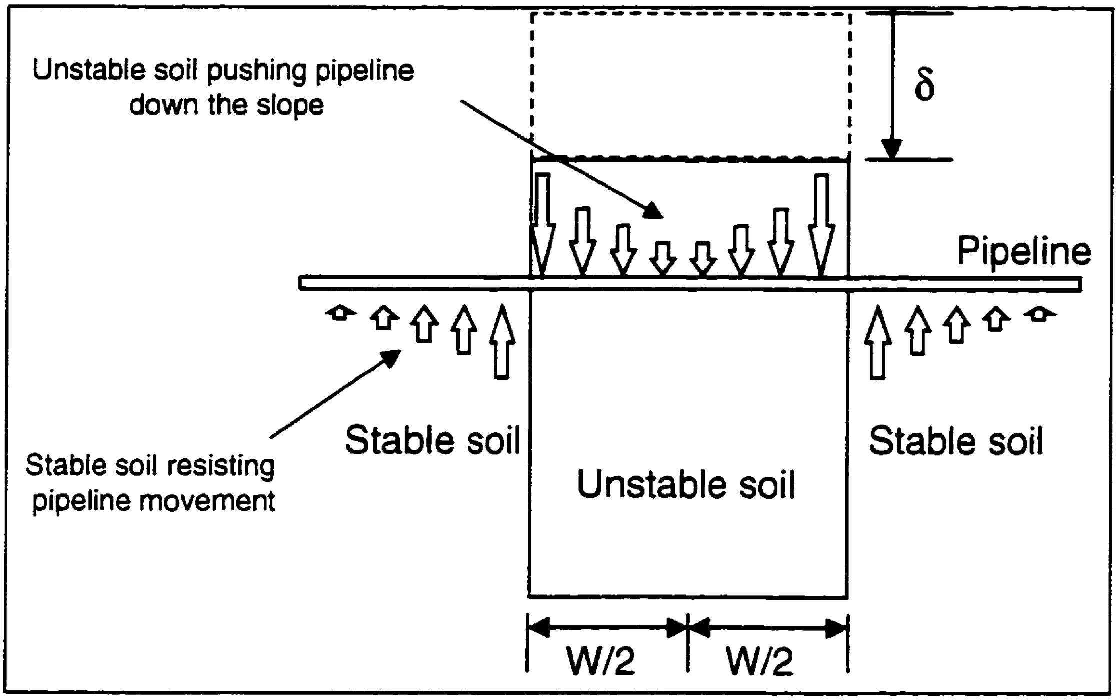



Fig. 3.6: Transverse landslide of finite width

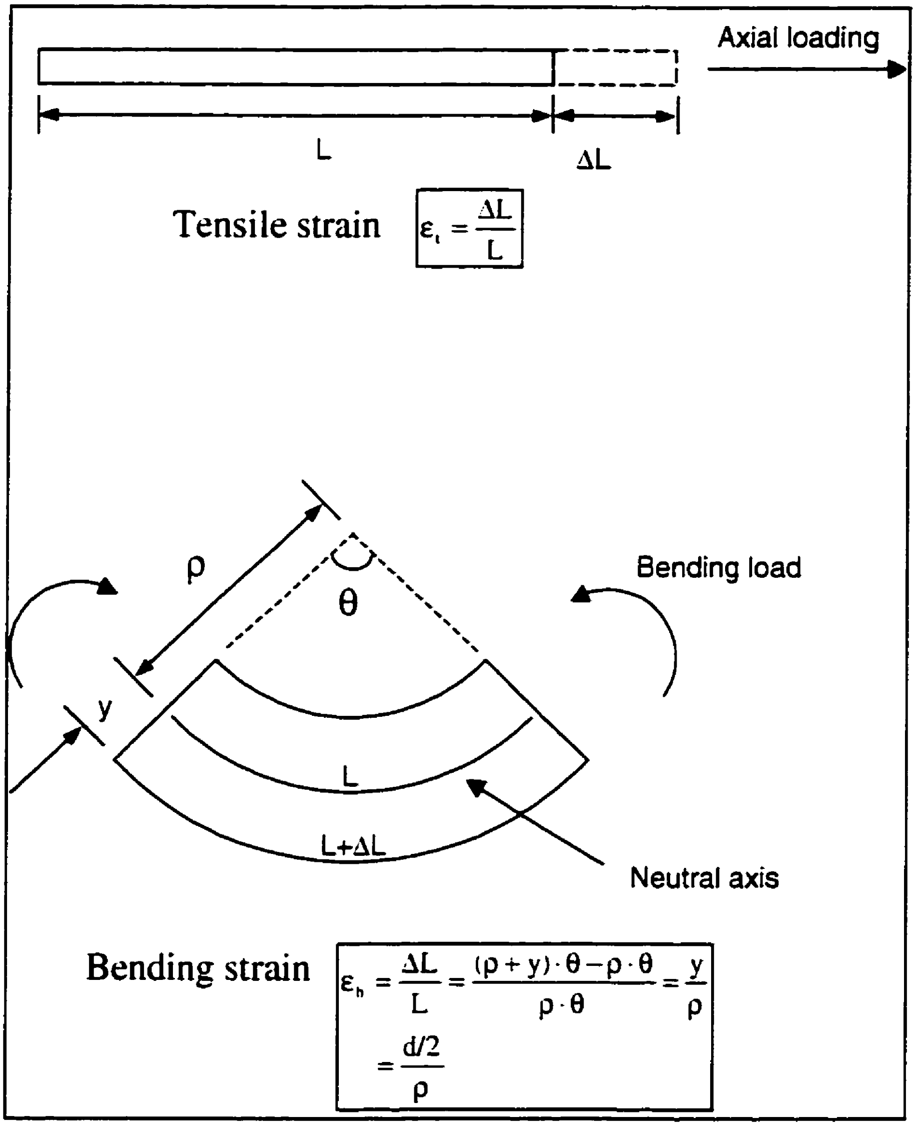



Fig. 3.7: Tensile and bending strain

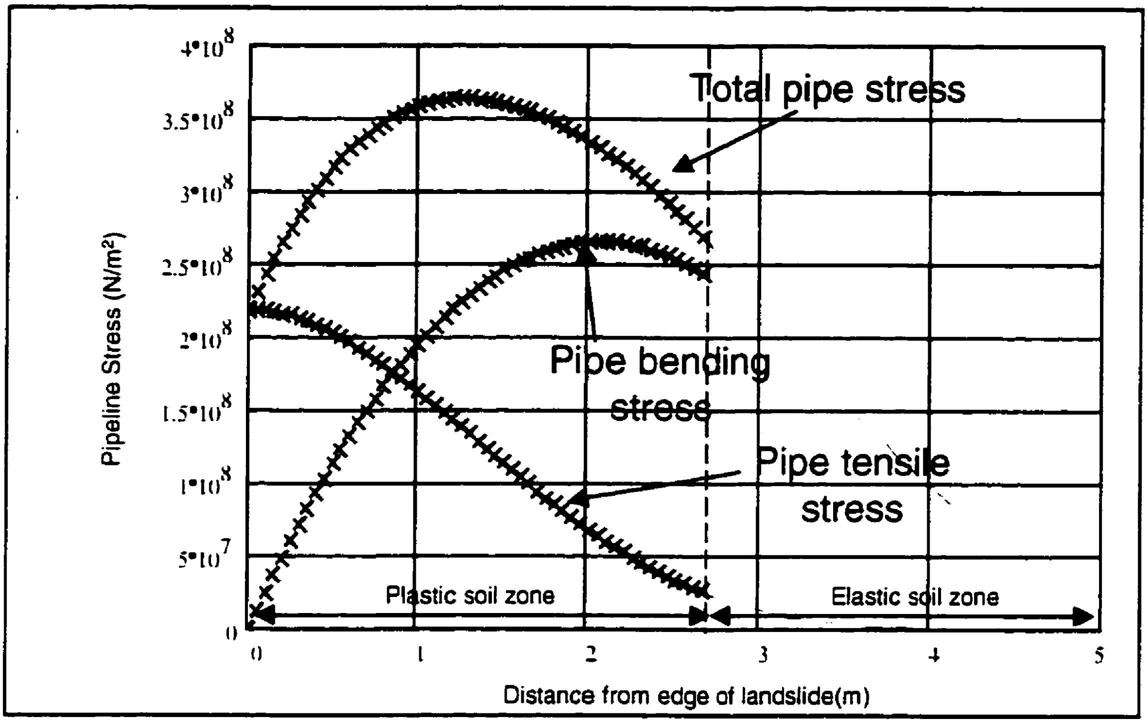



Fig. 3.8: Example of tensile. bending and total pipe stress in a transverse landslide

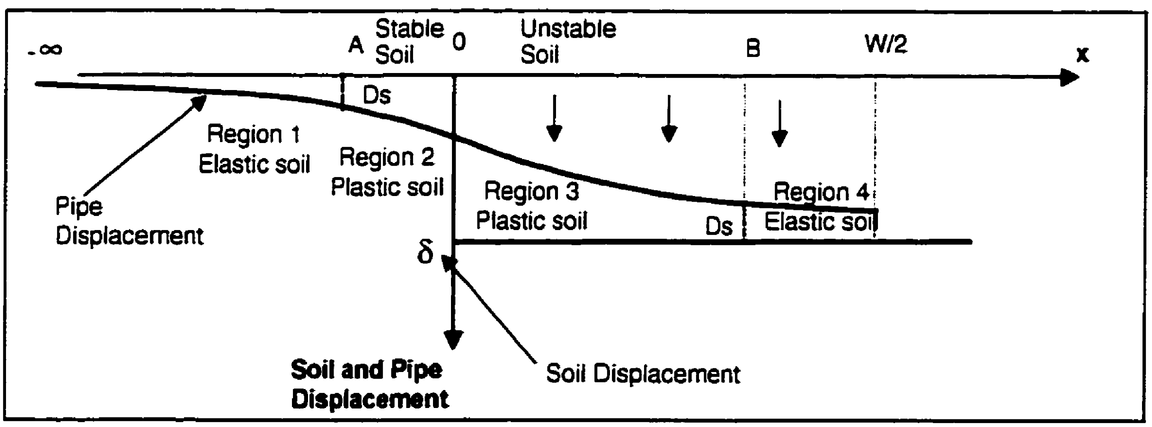



Fig. 3.9: The 4 regions of soil-pipeline interaction under transverse landslide of finite width

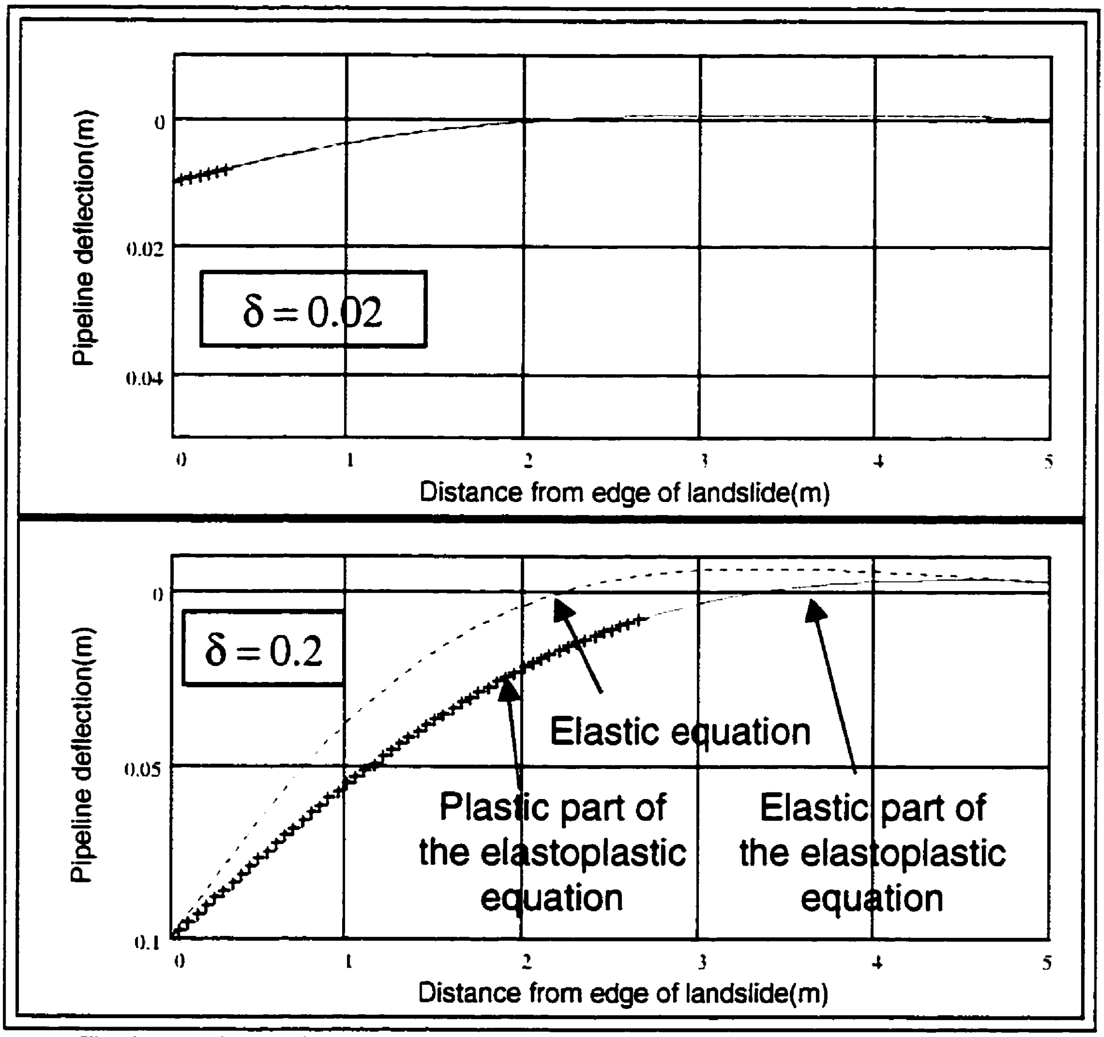



Fig 3.10: Pipe deflections using elastic soil and elastoplastic soil models

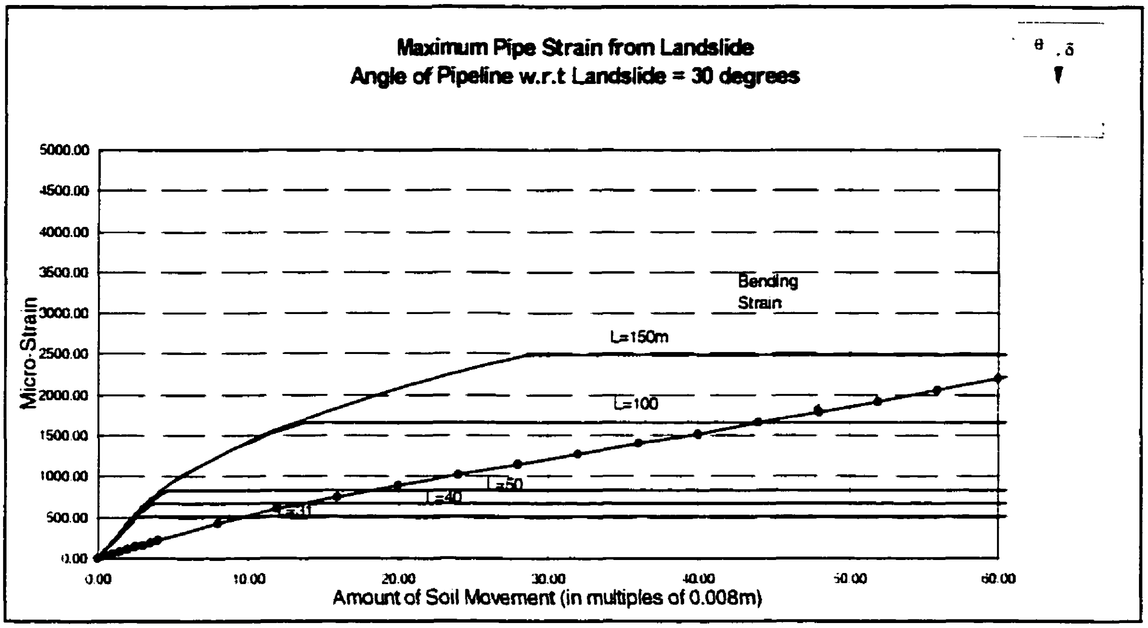



Fig. 3.11: Maximum pipe strains at 30 degrees to landslide direction

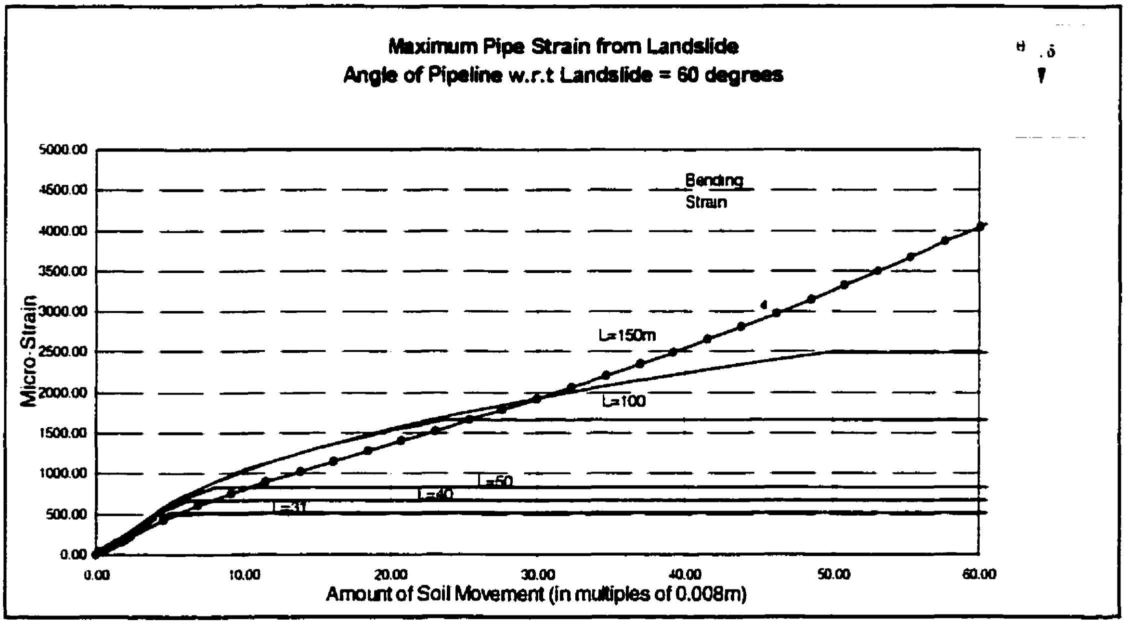



Fig. 3.12: Maximum pipe strains at 60 degrees to landslide direction

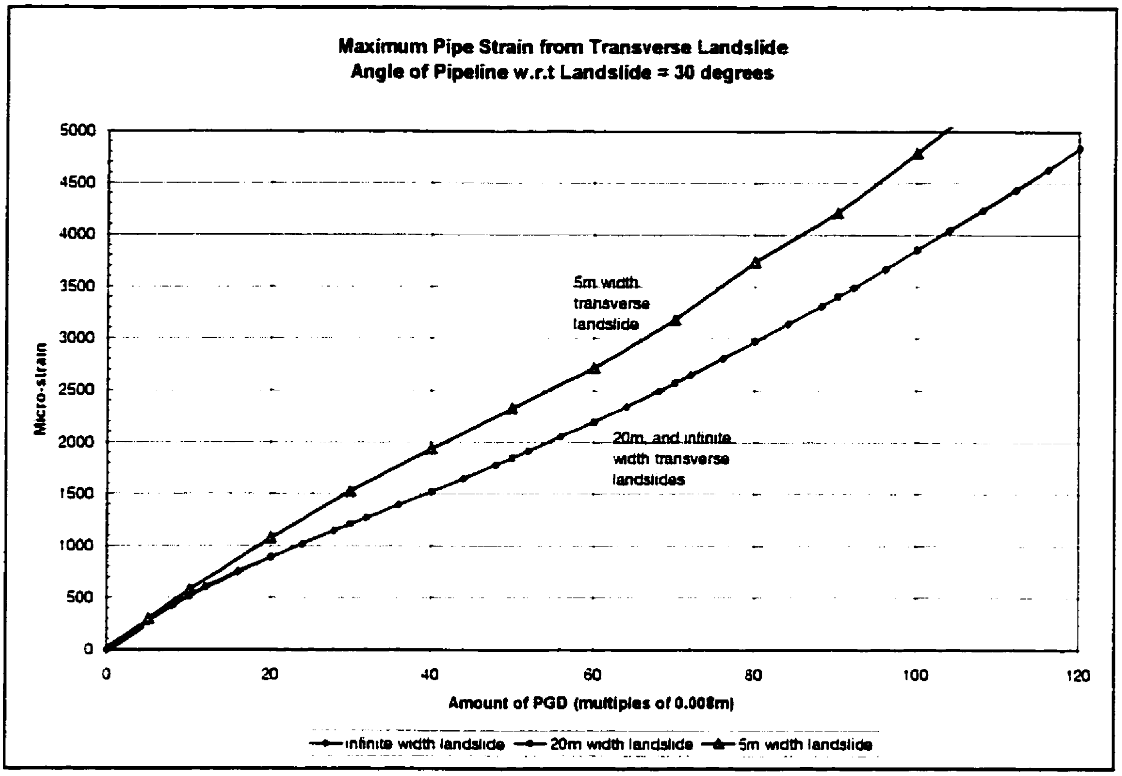



Fig. 3.13: Comparing maximum pipe strain at 30 degrees to landslide direction for infinite width and finite width transverse landslides

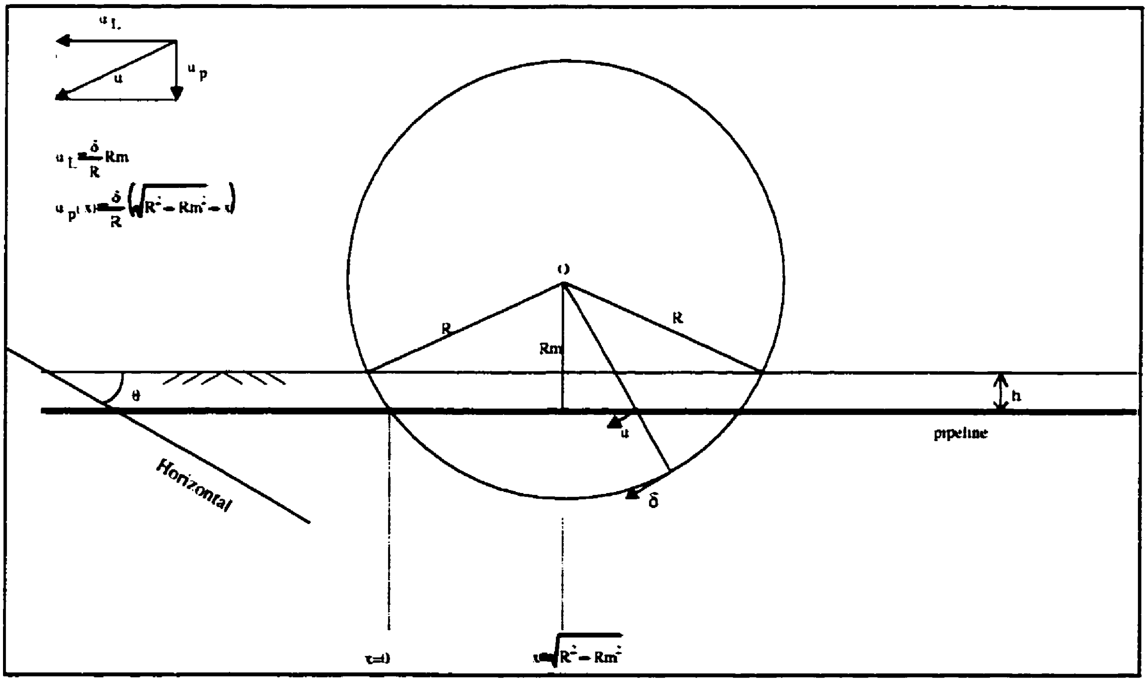



Fig. 3.14: Circular deep-seated landslide

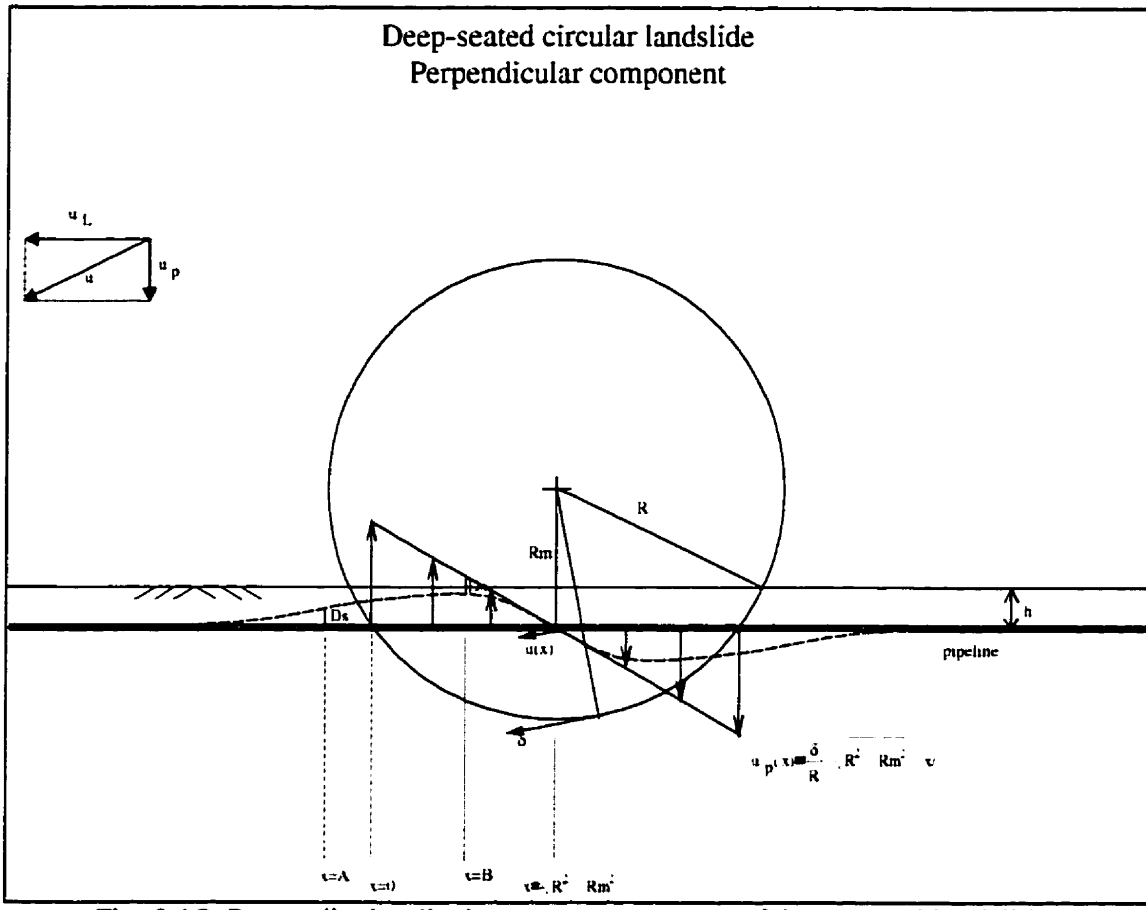



Fig. 3.15: Perpendicular displacement component of deep-seated landslide

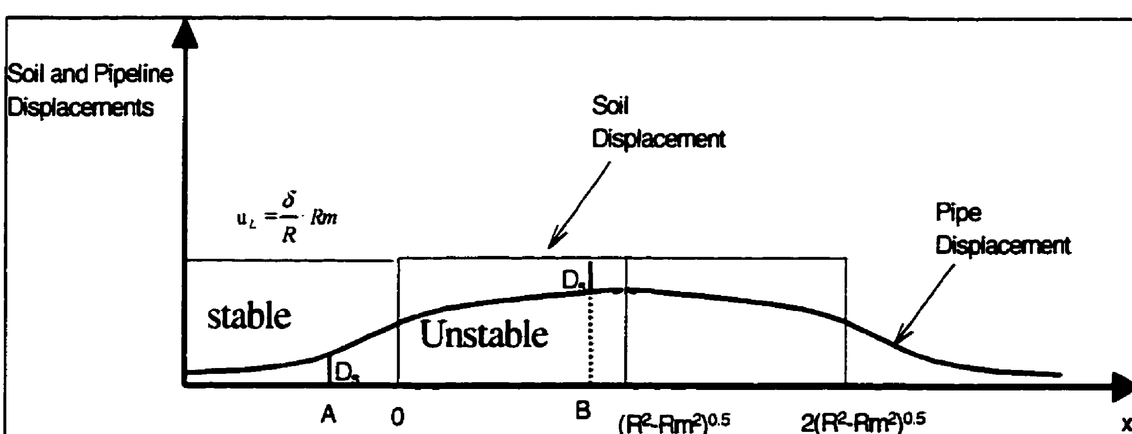



Fig. 3.16: Longitudinal displacement component of deep-seated landslide

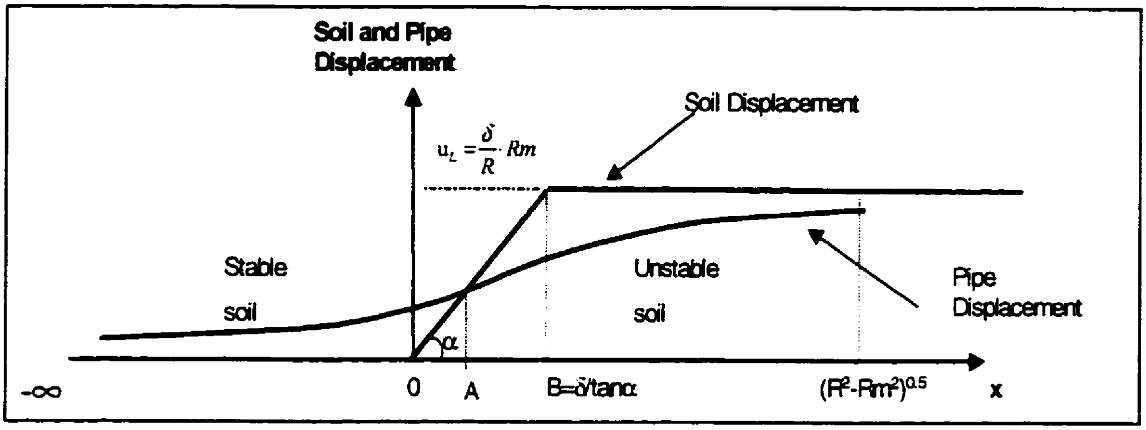



Fig. 3.17: Longitudinal displacement component of deep-seated landslide for $\delta < 2D_s$

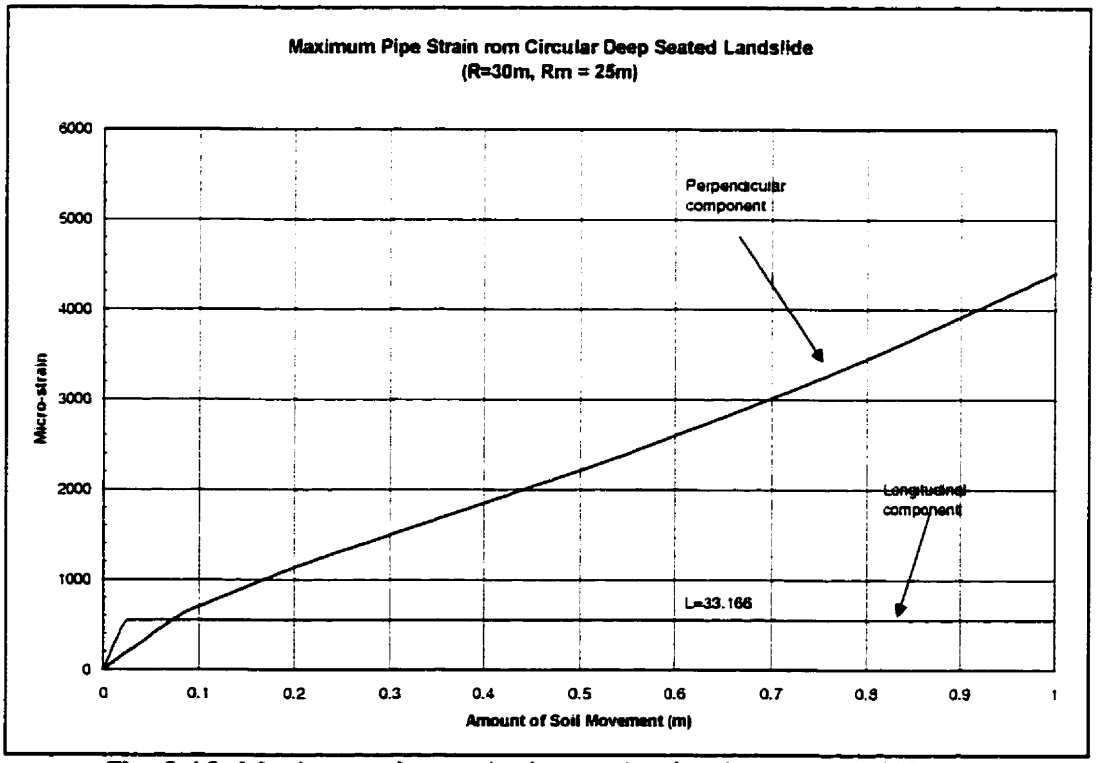



Fig. 3.18: Maximum pipe strain due to circular deep-seated failure.

# Chapter 4

# Statistical Analysis

## 4.1 Introduction

There is a trend in civil engineering today toward providing economical designs at specified levels of safety. Current design procedures, which are generally learned only after many trail-and-error iterations, often fall short of expectations in new situations. Even the raw data, on which problem solutions are based, themselves exhibit significant variability. The source of uncertainty in soil properties come from spatial variation of the soil properties, random testing errors, statistical estimation error due to a finite number of measurements, or bias in measurement procedures. Another source of uncertainty comes from the inability of man to completely model a physical system with 100% accuracy. The equations being used are often derived after making simplifying assumptions that approximate a real physical behavior. This chapter will only demonstrate how concepts of statistical analysis may be used to take into account the uncertainty of soil properties from their spatial variation. This would supplement the geotechnical engineer's judgment in assessing the risk of pipeline failure in unstable slope.

Three main techniques are used and they are the Monte Carlo simulations, the First Order Second Moment (FOSM), and the Rosenblueth's Point Estimate Method (PEM).

## 4.2 Fundamentals

In engineering applications, there are two primary definitions of the probability concept – they are relative frequency and subjective interpretation.

Relative frequency is a measure of the total favorable outcomes divided by total possible outcomes, and this is what historically has been offered as the measurement of probability. However, in geotechnical engineering, you cannot have repeated trails. A slope will fail or it won't. In the diverse topographic features of the earth's surface, it is not possible to have a large number of identical slopes with identical soil properties, geometry, and loading. Subjective interpretation of probability is therefore more useful for engineering applications.

There are 3 basic axioms governing the theory of probability. First, the probability of an event "A" ranges from zero to one, inclusive.

[4.1] $$\boxed{0 \le P(A) \le 1}$$

Second, if the probability of an outcome "B" is certain, then its probability is 1.

[4.2] $$\boxed{P(B) = 1}$$

Third and final axiom states that if the events $A_1$, $A_2$, ...,$A_n$ are mutually exclusive, or in other word, they can not occur simultaneously, then the probability of the occurrence of the sum of events $A_1$, $A_2$, ...,$A_n$ is the sum of their individual probabilities.

[4.3] $$\boxed{P[A_1 + A_2 + \cdots + A_n] = P[A_1] + P[A_2] + \cdots + P[A_n]}$$

The first moment E[x] is called the expected value, or mean of the variable x. It measures the central tendency and is like the center of gravity in statics. The second moment V[x] is called the variance. It measures the dispersion of the distribution and is like the moment of inertia in statics. Since variance gives dispersion in units of square of the random variable, a more meaningful measure of dispersion is the positive square root of its variance called the standard deviation, $\sigma[x]$. The coefficient of variation is used to measure the scatter of a random variable, and expresses the reliability of the central tendency.

| | **Population parameter** | **Sample statistic** |
|---|---|---|
| <u>Mid point</u><br>[4.4]Arithmetic Mean: | $\mu = E(x) = \int_{-\infty}^{\infty} x \cdot f(x)\,dx$ | $\bar{x} = \frac{1}{n} \cdot \sum_{i=1}^{n} x_i$ |
| <u>Variability</u><br>[4.5]Variance: | $\sigma^2 = E(x - \mu)^2$ | $s^2 = \frac{1}{n-1} \cdot \sum_{i=1}^{n} (x_i - \bar{x})^2$ |
| [4.6]Standard deviation: | $\sigma = \left[E(x - \mu)^2\right]^{\frac{1}{2}}$ | $s = \left[\frac{1}{n-1} \cdot \sum_{i=1}^{n} (x_i - \bar{x})^2\right]^{\frac{1}{2}}$ |
| [4.7]Coefficient of variation: | $CV = \frac{\sigma}{\mu}$ | $CV = \frac{s}{\bar{x}}$ |
| <u>Symmetry</u><br>[4.8]Coefficient of skewness: | $\gamma = \frac{E(x - \mu)^3}{\sigma^3}$ | $Cs = \frac{n \cdot \sum_{i=1}^{n} (x_i - \bar{x})^2}{(n-1)\cdot(n-2)\cdot s^3}$ |

Table 4.1: Basic Statistical Functions

According to Chow *et al.* (1988), the sample estimate of the variance is divided by "n-1" rather than "n" to ensure that the sample statistic is "unbiased", that is, not having a tendency, on average, to be higher or lower than the true value. The coefficient of skewness measures the symmetry of the data. For positive skewness ($Cs>0$), the data are skewed to the right, with a longer tail of data on the right of the mean.

## 4.3 First Order Second Moment (FOSM)

The "First Order" refers to the first order truncation of the Taylor series expansion, and the "Second Moment" refers to the use of variance. This method consists of expanding the system performance equation $h(x_1, x_2, ..., x_n)$ about $[E(x_1), E(x_2), ..., E(x_n)]$, the point at which each of the component variables takes on its expected values, by a multi-variable Taylor series expansion. We will expand it up to the 2nd order for illustration.

The Taylor series expansion of multi-variable about the component means is:

[4.9]
$$\boxed{\begin{aligned} h(x_1, x_2, \ldots, x_n) &= h\left[\mu_{x_1}, \mu_{x_2}, \ldots, \mu_{x_n}\right] + \sum_{i=1}^{n}(x_i - \mu_{x_i})\cdot\frac{\partial h}{\partial x_i} + \ldots \\ &\ldots \frac{1}{2}\cdot\left\{\sum_{i=1}^{n}(x_i-\mu_{x_i})^2\cdot\frac{\partial^2 h}{\partial x_i^2} + 2\sum_i\sum_j (x_i-\mu_{x_i})(x_j-\mu_{x_j})\frac{\partial^2 h}{\partial x_i \partial x_j}\right\}\end{aligned}}$$

Taking the expected values of both sides of the above equation:

$$E\left\{h\left[\mu_{x_1}, \mu_{x_2}, \ldots, \mu_{x_n}\right]\right\} = h\left[\mu_{x_1}, \mu_{x_2}, \ldots, \mu_{x_n}\right]$$

$$E\left\{\sum_{i=1}^{n}(x_i-\mu_{x_i})\cdot\frac{\partial h}{\partial x_i}\right\} = \sum_{i=1}^{n}\frac{\partial h}{\partial x_i}\cdot E(x_i-\mu_{x_i}) = 0$$

$$E\left\{\frac{1}{2}\cdot\sum_{i=1}^{n}(x_i-\mu_{x_i})^2\cdot\frac{\partial^2 h}{\partial x_i^2}\right\} = \frac{1}{2}\cdot\sum_{i=1}^{n}\frac{\partial^2 h}{\partial x_i^2}\cdot E(x_i-\mu_{x_i})^2 = \frac{1}{2}\cdot\sum_{i=1}^{n}\frac{\partial^2 h}{\partial x_i^2}\cdot \mathrm{Var}(x_i)$$

$$E\left\{\sum_i\sum_j(x_i-\mu_{x_i})(x_j-\mu_{x_j})\frac{\partial^2 h}{\partial x_i\partial x_j}\right\} = \sum_i\sum_j\frac{\partial^2 h}{\partial x_i\partial x_j}\cdot E(x_i-\mu_{x_i})\cdot E(x_j-\mu_{x_j}) = 0$$

Therefore Taylor expansion of multi-variable up to the second order is:

$$E(z) = E\{h(x_1, x_2, \ldots, x_n)\} = h\left[\mu_{x_1}, \mu_{x_2}, \ldots, \mu_{x_n}\right] + \frac{1}{2} \cdot \sum_{i=1}^{n} \frac{\partial^2 h}{\partial x_i^2} \cdot Var(x_i) \quad [4.10]$$

Only the first term is used in FOSM, so that the expected value of the system is simply the value of the system equation with mean component values. We get the variance by similarly expanding the Taylor series to the first order to get:

$$Var(z) = \sum_{i=1}^{n} \left(\frac{\partial h}{\partial x_i}\right)^2 \cdot Var(x_i) \quad [4.11]$$

To apply FOSM to computers, we need to find the numerical approximations of the first and second order derivatives. Expanding the Taylor series about x:

$$f(x+h) = f(x) + h \cdot f'(x) + \frac{1}{2} \cdot h^2 \cdot f''(x) + \frac{1}{6} \cdot h^3 \cdot f'''(x) + \ldots \quad [4.12]$$

$$f(x-h) = f(x) - h \cdot f'(x) + \frac{1}{2} \cdot h^2 \cdot f''(x) - \frac{1}{6} \cdot h^3 \cdot f'''(x) + \ldots \quad [4.13]$$

Numerical approximation of first derivative is derived by subtracting the above 2 equations, and it has a second order magnitude of error:

$$f'(x) = \frac{1}{2 \cdot h} \cdot \left[f(x+h) - f(x-h)\right] + O(h^2) \quad [4.14]$$

The second derivative is approximated by the following with a second order magnitude of error:

[4.15] $$\boxed{f''(x) = \frac{1}{h^2}\cdot\left[f(x+h) - 2\cdot f(x) + f(x-h)\right] + O(h^2)}$$

The first derivative can be improved by using Richardson's Extrapolation to get the order of magnitude of error down to 4.

[4.16] $$\boxed{f'(x) = \frac{4}{3}\cdot\phi\left(\frac{h}{2}\right) - \frac{1}{3}\cdot\phi(h) + O(h^4)}$$

where,

[4.17] $$\boxed{\phi(h) = \frac{1}{2\cdot h}\cdot\left[f(x+h) - f(x-h)\right]}$$

## 4.4 Rosenblueth's Point Estimate Method (PEM)

Point Estimate Method was first presented by Rosenblueth (1975), and later extended by him in 1981. It is a simple and versatile procedure to find the distribution of functions of random variables. For example, for a function of 3 random variables -- say, $y = y[x_1, x_2, x_3]$ and $r_{ij}$ is the correlation coefficient between variable $x_i$, and $x_j$.

[4.18] $$\boxed{E\left[y^n\right] = r_{+++} \cdot {y_{+++}}^n + r_{++-} \cdot {y_{++-}}^n + \ldots + r_{---} \cdot {y_{---}}^n}$$

where $y_{+++} = y(x_1 + \sigma_1, x_2 + \sigma_2, x_3 + \sigma_3)$

$$r_{+++} = r_{---} = \frac{1}{2^3} \cdot \left[1 + r_{12} + r_{23} + r_{31}\right]$$

$$r_{++-} = r_{--+} = \frac{1}{2^3} \cdot \left[1 + r_{12} - r_{23} - r_{31}\right]$$

The sign $r_{ij}$ is determined by the multiplication rule of i and j. For convenience, we will assume all correlation coefficients are zero in our example. The first 3 moments are as follows:

[4.19]
$$\boxed{\begin{array}{l} \text{mean} = m_1 = E\left[y^1\right] \\ \text{var} = m_2 = E\left[(y - \mu_y)^2\right] = E\left[y^2\right] - {m_1}^2 \\ m_3 = E\left[(y - \mu_y)^3\right] = E\left[y^3\right] - 3 \cdot m_1 \cdot m_2 - {m_1}^3 \\ \text{skewness} = m_3 / {m_2}^{1.5} \end{array}}$$

## 4.5 Monte Carlo Simulation

Monte Carlo Simulation is simply a repeated process of generating deterministic solutions to a given problem. The main element of a Monte Carlo simulation procedure is the generation of random numbers for a specific distribution. Previously, with slow computers, Monte Carlo simulations are costly in its application to complex problems, because it requires a large number of repetitions. With faster computers, this method can be readily used as a check for approximate methods of probability calculations.

If $u_1$ and $u_2$ are a pair of independent uniformly distributed random numbers, then a pair of independent random numbers from a normal distribution with mean $\mu$ and standard deviation $\sigma$, may be generated by:

$$\boxed{\begin{aligned} x_1 &= \mu + \sigma \cdot \sqrt{-2 \cdot \ln(u_1)} \cdot \cos(2 \cdot \pi \cdot u_2) \\ x_2 &= \mu + \sigma \cdot \sqrt{-2 \cdot \ln(u_1)} \cdot \sin(2 \cdot \pi \cdot u_2) \end{aligned}} \tag{4.20}$$

## 4.6 Other Methods

Another statistical method often used for slope stability analysis is the reliability index ($\beta$) as defined in equation [4.21]. It is a way of normalizing the factor of safety with respect to its standard deviation:

$$[4.21] \quad \boxed{\beta = \frac{E[F]-1}{\sigma[F]}}$$

where

F = factor of safety,

E[F] = mean or expected value of the factor of safety, and

$\sigma$[F] = standard deviation of the factor of safety.

The reliability index describes safety by the number of standard deviations (i.e., the amount of uncertainty in the calculated value of F) separating the best estimate of F from its defined failure value of 1.0. If the statistical distribution for the factor of safety is known, then the reliability index can be related to the probability of failure.

## 4.7 Summary

The objective of statistical analysis, as applied to pipelines in unstable slopes, is to assess the risk and probability of pipeline failure. Three main techniques are used for statistical analysis were shown.

The Monte Carlo simulation involves generating random numbers using the mean and standard deviations for each variable of the function. FOSM is obtained from the Taylor series expansion of the function about the expectations of the random variables. It may not always be possible to use the Taylor series approximation, because the function itself must satisfy the existence and continuity condition of the first or first few derivatives, and the computation of derivatives may be difficult. These difficulties can be overcome by using Rosenblueth's point estimates (PEM) of the function, which leads to expressions akin to finite differences.

Of these three methods, only the Monte Carlo simulation method uses the whole statistical distribution of the variables. The FOSM and PEM both only use one standard deviation from the mean for their calculations.

The PEM usually gives results very close to the Monte Carlo simulation. It is very easy to use, and the derivatives of the function need not be derived. It is recommended that PEM be used for analyzing soil-pipeline interaction, and the use the Monte Carlo simulation as a check on the results.

# Chapter 5

# Case History of a Pipeline in Unstable Slope

## 5.1 Background

Amoco Canada Resources Limited had a 6-inch (168.3mm) oil emulsion pipeline located in Willesden Green East near Rocky Mountain House, Calgary, Alberta. The Right of Way contains 6 pipelines. Active slope movements were affecting only the 6-inch oil emulsion line along the southern edge of the Right of Way.

The pipeline was laid about 2-m deep in a slope that is 43-m high and 185-m long. Landslide movements were occurring at depths of 2 to 2.5-m. Elastic stress calculations by Burt Engineering Limited (AGRA Earth & Environmental Limited, 1995) indicated that the oil emulsion line had not been stressed beyond the yield point. They determined the stress induced on the pipeline from the soil movement by multiplying the soil traction per meter by the total length of the unstable soil slab and divided by the cross-sectional area of the pipe. They also included the Posson effect and hoop pressure as a part of the longitudinal stress

The line was exposed in the landslide toe area. Some bending along-the-slope was observed in the pipeline. Approximate measurements and simple strain calculations indicated that the bending was within the tolerable limits. Excavations did not show signs of upwards trending shearing within the soil, and the line did not appear to be in immediate risk of being ruptured by landslide movements according to the 1995 report by AGRA Earth and Environmental Ltd.

A 3-D plot of the slope with a sketch of the pipeline location as well as the locations of the inclinometers and boreholes are shown in Figure 5.1. Figures 5.2 to 5.6 are photographs of the pipeline in the slope. The toe of the slope was separated by a 200-m wide flood plain from the North Saskatchewan River. The active failure appears to be about 100-m wide (across the slope) and 70 to 80-m long (down the slope).

Figures 5.7 to 5.9 are borehole logs 1 to 3, and Figures 5.12 and 5.13 are the grain size distributions for boreholes 2 and 3. The soil consists of 25% clay and 74% silt at 2.59 to 2.74-m depth, and 37% clay and 60% silt at 1.68-m depth. Boreholes indicate that the soil is made up of sand and clay at depth zero to 5.2-m.

Figures 5.14 to 5.17 are the inclinometer data from May 11 '95 to July 26 '95. Deflections direction A is towards West (downslope and parallel to the pipeline) and direction B is towards South. The slope has shown 1-mm/day movement parallel to the pipeline, and from May 11$^{th}$ to July 25$^{th}$, 1995, inclinometer "SI-2" registered a cumulative deflection of about 68-mm downslope. The surface of the slope indicator "SI-3" appears to be impacted between May 11 and May 18, 1995 resulting in the inclinometer showing up slope movement above 1.9-m. Slope indicators indicate that the possible slip plane is relatively shallow (less than 3m), and that there are no deep-seated movement.

The AGRA report does not contain any soil strength data. The soil strength parameters used in this chapter are based on empirical correlation.

## 5.2 Statistical Analysis Based on Simple Model (O'Rourke *et al.* 1995)

Table 5.1 consists of data used in the statistical analysis of a steel pipeline subjected to longitudinal landslide. The values were provided by AGRA outside of the AGRA report. Lumb (1966, and 1970) studied the statistical distributions of cohesion c and angle of friction $\phi$ for Hong Kong soils and found that $\tan\phi$ has an approximately normal distribution while c conforms more to a Beta distribution. The central region of this Beta distribution can be practically approximated by a normal distribution. Lumb's conclusions were later supported by Schultze (1972), and Harr (1977). For this case study, a normal distribution is assumed for the soil cohesion and frictional angle, since in the practical range of $\phi$ ($0° \le \phi \le 45°$), $\phi$ is approximately proportional to $\tan\phi$. Lumb (1980) and other workers have found that variability of the unit weight of soil $\gamma$ on the determinations statistical results to be insignificant, therefore the unit weight of soil is assumed to be a deterministic quantity for this case study.

The statistical analysis is based on the simplified models of uniaxial stress-strain behavior of steel by Ramberg-Osgood (1943) and the longitudinal strain equation from O'Rourke et al. (1995) (i.e., equations [2.1] and [2.3]).

| | mean | standard deviation |
|---|---|---|
| Length of PGD, L(m) | n/a | 0 |
| soil cohesion, c (N/m$^2$) | 2000 | 20 |
| depth to center of pipeline, H(m) | 2 | 0 |
| unit weight of soil, $\gamma$ (N/m$^3$) | 18000 | 0 |
| frictional angle, $\phi$ (degrees) | 30 | 3 |
| pipe thickness, t (m) | 3.18E-03 | 3.18E-05 |
| pipe modulus of elasticity, E (N/m$^2$) | 2.00E+11 | 0 |
| pipe yield strength, $\sigma_y$ (N/m$^2$) | 3.86E+08 | 0 |
| Ramberg-Osgood's n | 9.5 | 0 |
| Ramberg-Osgood's r | 11 | 0 |

Table 5.1: Input data for statistical analysis of a steel pipeline subjected to longitudinal landslide

The mean, standard deviation, and skewness values of maximum strain in pipeline subjected to longitudinal landslide -- estimated by three statistical methods (FOSM, PEM, and Monte Carlo simulation) -- are summarized in Table 5.2:

| Length of landslide (m) | mean | | | standard deviation | | | skewness | |
|---|---|---|---|---|---|---|---|---|
| | FOSM | PEM | Monte Carlo | FOSM | PEM | Monte Carlo | PEM | Monte Carlo |
| 10 | 162.78 | 163.34 | 163.08 | 17.86 | 17.89 | 12.67 | 0.0054 | 0.1400 |
| 20 | 325.57 | 326.68 | 325.98 | 35.72 | 35.79 | 25.47 | 0.0054 | 0.0960 |
| 30 | 488.35 | 490.02 | 489.24 | 53.57 | 53.68 | 37.82 | 0.0054 | 0.1357 |
| 40 | 651.14 | 653.36 | 652.53 | 71.43 | 71.58 | 50.65 | 0.0054 | 0.1184 |
| 50 | 813.97 | 816.79 | 815.31 | 89.35 | 89.55 | 63.54 | 0.0055 | 0.1263 |
| 60 | 977.14 | 980.88 | 979.27 | 107.71 | 108.09 | 77.00 | 0.0059 | 0.1399 |
| 70 | 1142.23 | 1148.71 | 1145.42 | 128.61 | 129.92 | 92.67 | 0.0077 | 0.2336 |
| 80 | 1315.88 | 1333.22 | 1325.14 | 160.78 | 166.32 | 120.90 | 0.0147 | 0.5766 |
| 90 | 1520.99 | 1578.97 | 1551.61 | 234.35 | 256.44 | 202.58 | 0.0325 | 1.9204 |
| 100 | 1825.88 | 2019.04 | 1926.81 | 439.28 | 517.39 | 431.85 | 0.0581 | 2.2433 |
| 110 | 2412.17 | 3007.05 | 2719.35 | 1014.64 | 1260.43 | 1145.64 | 0.0775 | 4.3466 |
| 120 | 3719.18 | 5398.51 | 4610.46 | 2538.77 | 3238.23 | 3126.22 | 0.0874 | 4.4026 |
| 130 | 6730.21 | 11108.16 | 9058.61 | 6306.09 | 8135.21 | 7974.48 | 0.0916 | 4.4775 |
| 140 | 13506.78 | 24150.37 | 19229.38 | 15030.43 | 19482.9 | 18419.67 | 0.0934 | 3.1189 |
| 150 | 28137.06 | 52485.88 | 41050.05 | 34093.46 | 44284.58 | 43274.24 | 0.0942 | 4.8989 |

Table 5.2: Results of Statistical Analysis Note: All Monte Carlo Simulations used 10,000 iterations

Figure 5.18 is a plot of the mean values of maximum strain as a function of the landslide length estimated by the three methods. The PEM provides the upper bound estimation on the mean value, whereas the FOSM gives the lower bound. The difference between these two bounds increases with increasing landslide length. Considering the coefficient of skewness, the Monte Carlo method yields a higher positive coefficient of skewness when L exceeds 90 m, as illustrated in Figures. 5.19 and 5.20. The coefficient of skewness yielded by PEM is small -- which may be due to the assumption of zero correlation coefficients. The probability density distributions as shown in Figures 5.19 and 5.20 can be used to find the probability of pipeline failure. For example, if we consider pipeline failure to occur at the start of pipe yielding at $\varepsilon_{yield} = 3458$ microstrains, then the probability of pipeline failure is about 100% at L = 130-m, and 1.7% at L = 100-m. The strains obtained from this simple model are similar to those found by Burt Engineering Limited.

The Monte Carlo simulation method is the most accurate of the three statistical methods. If the complex soil-pipeline interactions models, which are solved iteratively, are written into a computer program, then the Monte Carlo simulation method should be used for the statistical analysis.

Both FOSM and PEM do not require the large number of iterations used in Monte Carlo simulation. PEM results are very similar those from the Monte Carlo simulation for landslide lengths of up to 100-m, and they are more conservative than those from Monte Carlo simulation. For example, PEM produced a 4.8% higher strain value for a 100-m length landslide. Further, PEM does not require deriving derivatives of the function as required by FOSM. Therefore, if the complex soil-pipeline interaction models derived in this thesis were not implemented as a computer program, then it is recommended that PEM be used for the statistical analysis. The more time comsuming and computationally intensive Monte Carlo simulation can be used as a check on the PEM results.

## 5.3 Strain analyzed from new pipeline models

### 5.3.1 Strain from longitudinal landslide

The strains developed for different amount of soil movement and length of landslide is calculated from equations [3.2] and [3.7] and is shown in Figure 5.21.

For a pipeline diameter of 168.3 mm, the wrinkling strain, from equation [2.4], is $\varepsilon_w$ = 6613 microstrains. The yielding of the pipeline starts at $\varepsilon_{yield} = \sigma_{yield}/E_{pipe}$ = 1930 microstrains, or if we used the Ramberg-Osgood equation $\varepsilon_{yield}$ = 3458 microstrains. The result of longitudinal landslide analysis indicates that the pipeline would not yield for a landslide length of up to 100-m.

### 5.3.2 Strain from deep-seated landslide

Although there are no evidence of deep-seated failures developing in the actual slope itself, it is wise to calculate hypothetical deep-seated failures to determine the amount of such soil deep-seated movements needed to yield the pipeline.

If the slope were to have a circular deep-seated landslide with radius R = 45 m and Rm = 25 m. The length for the longitudinal displacement component is $2*(R^2-Rm^2)^{0.5}$ = 74.8 m. The maximum pipe strain developed from the transverse and longitudinal components of the deep-seated landslide, as analyzed using the method described in section 3.3, is presented in Figure 5.22. We see that there is no imminent danger of the pipeline yielding from the longitudinal component of the rotational soil movement. However, the pipe will begin to yield from the transverse component when the soil movement reaches $32D_s$ or only about 0.256 m. This indicates that any deep-seated soil movement is very dangerous for the pipeline because it would shear the pipeline near the intersection of the pipe and the circular failure surface, or at the very least , create wrinkles on the pipeline.

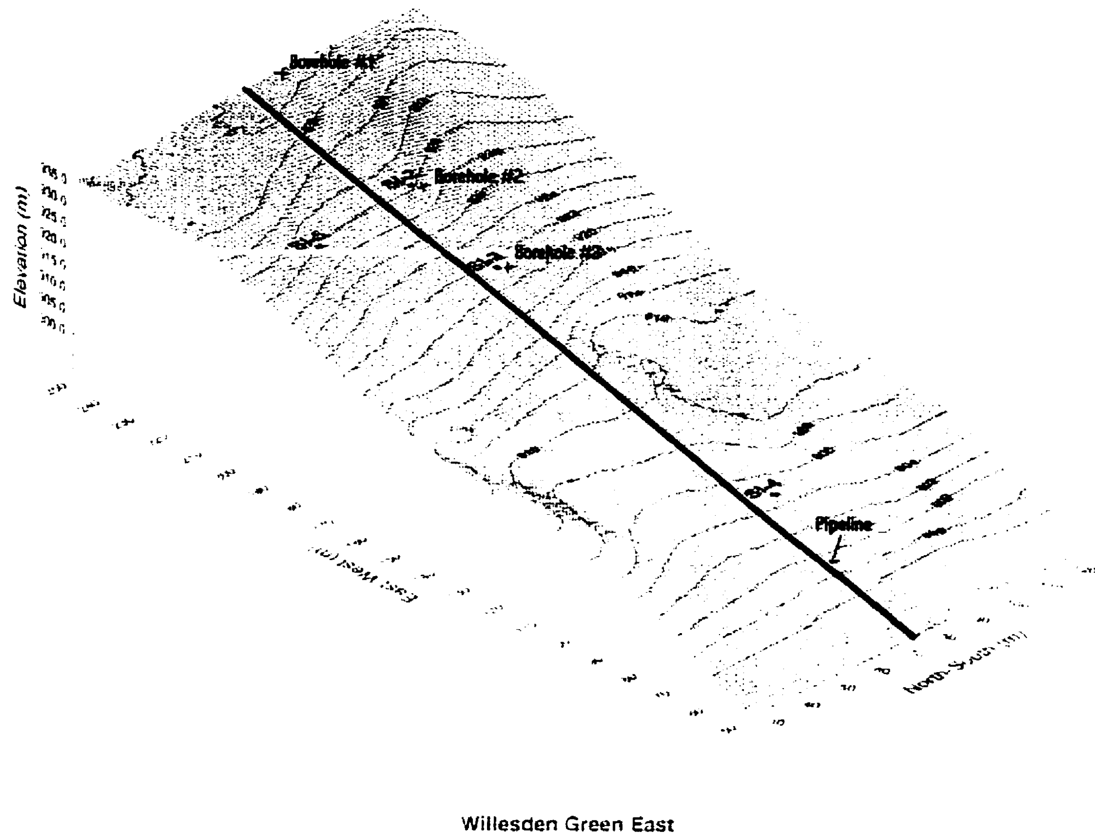



Fig. 5.1: 3-D Landslide surface contour of Willesden Green East with pipeline location

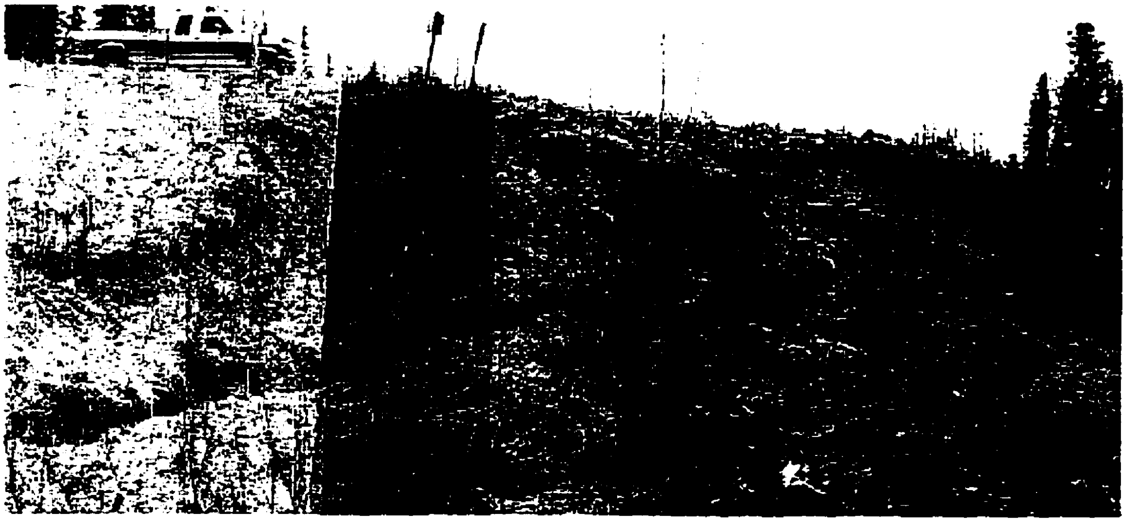


Fig. 5.2: Lateral edge of landslide at top of slope

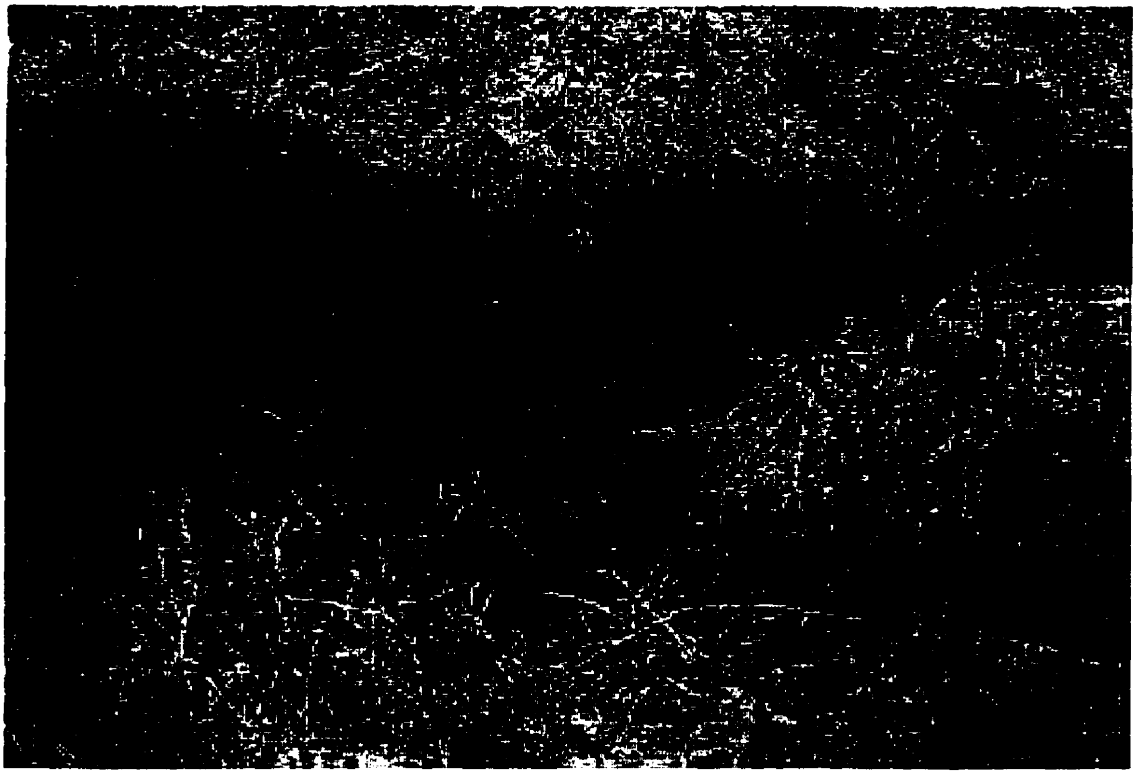


Fig. 5.3: Exposed 6 inch oil emulsion line near slope crest

Fig. 5.4: Groundwater in pipeline trench behind the slope crest

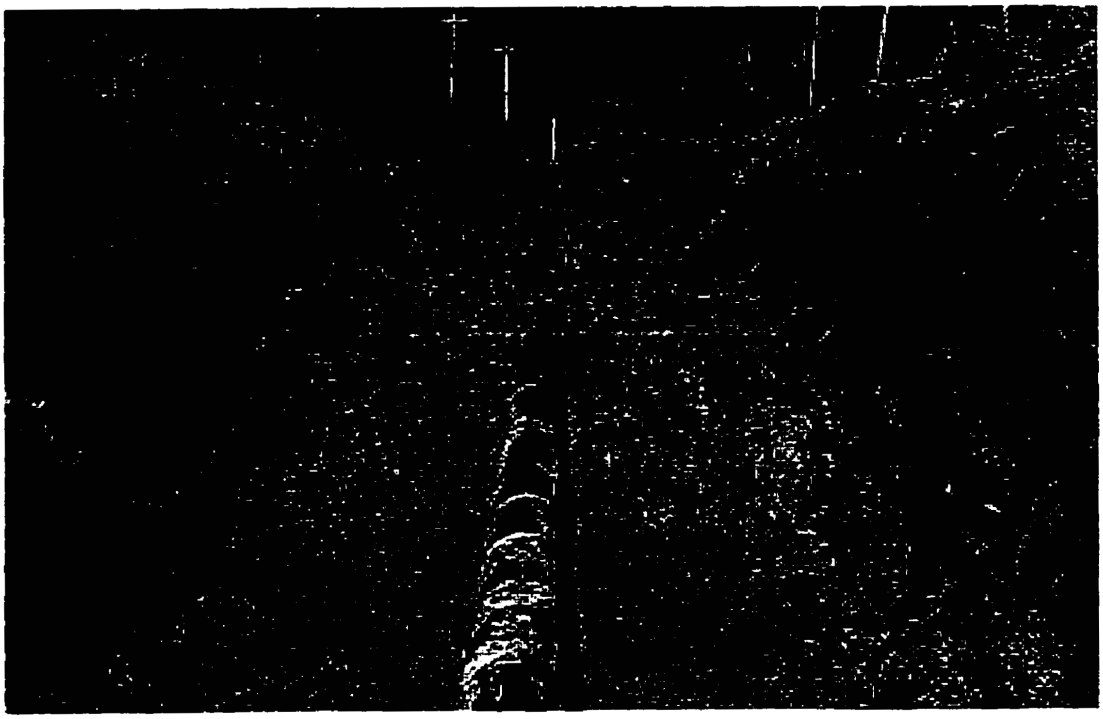


Fig. 5.5: 6 inch oil emulsion line

Fig. 5.6: View of oil emulsion line from top of slope

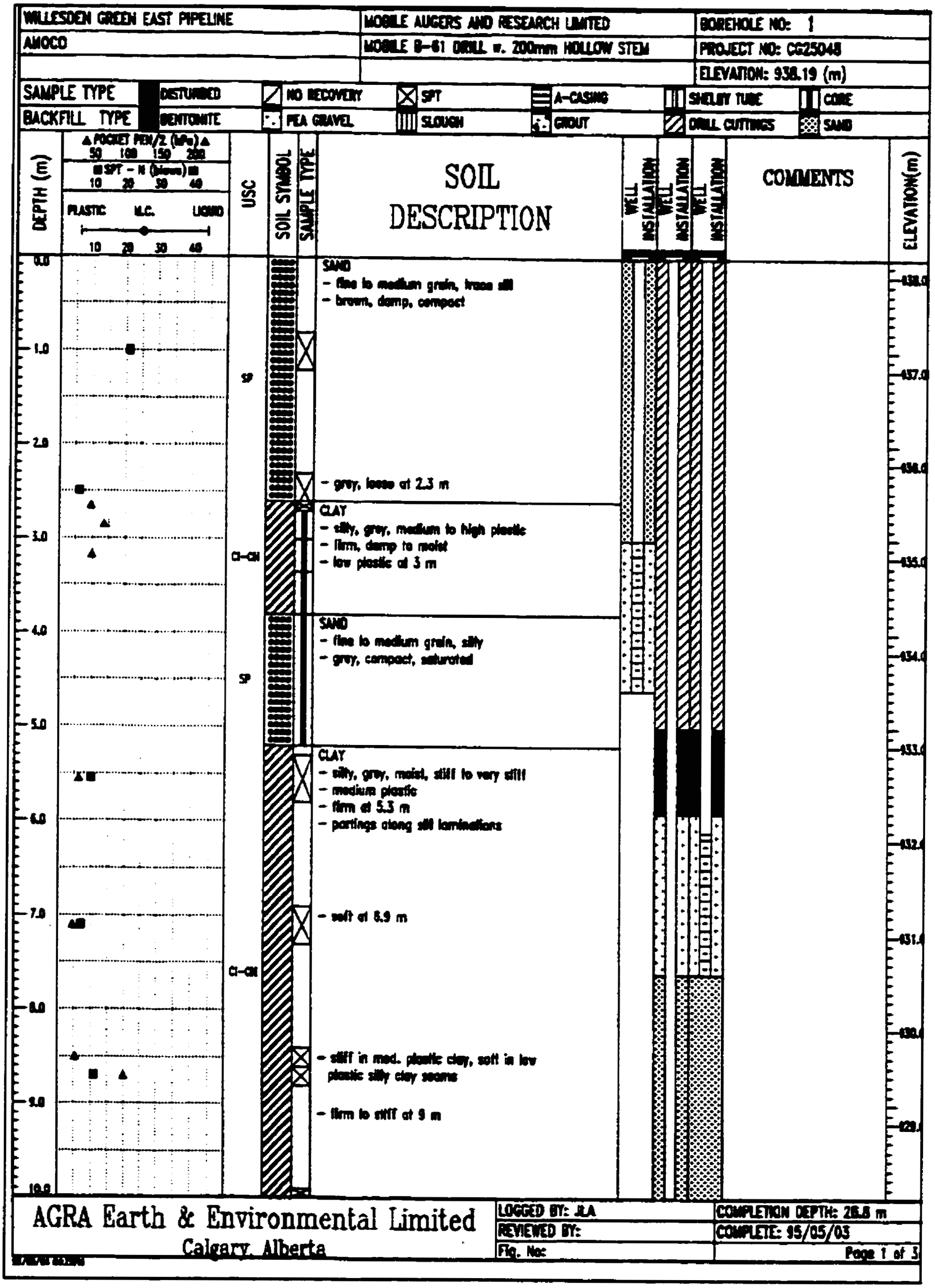



Fig. 5.7: Borehole No. 1

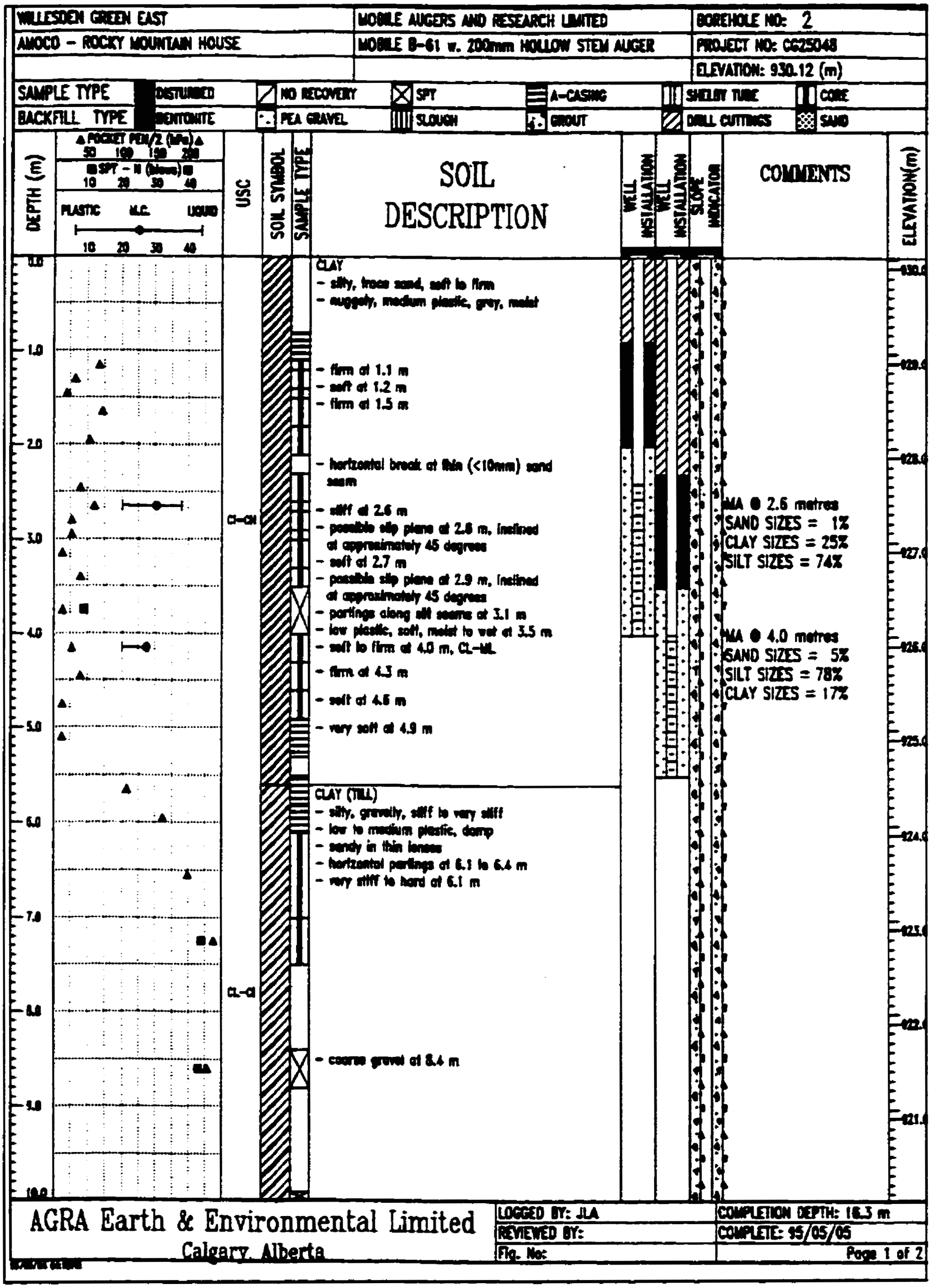



Fig. 5.8: Borehole No. 2

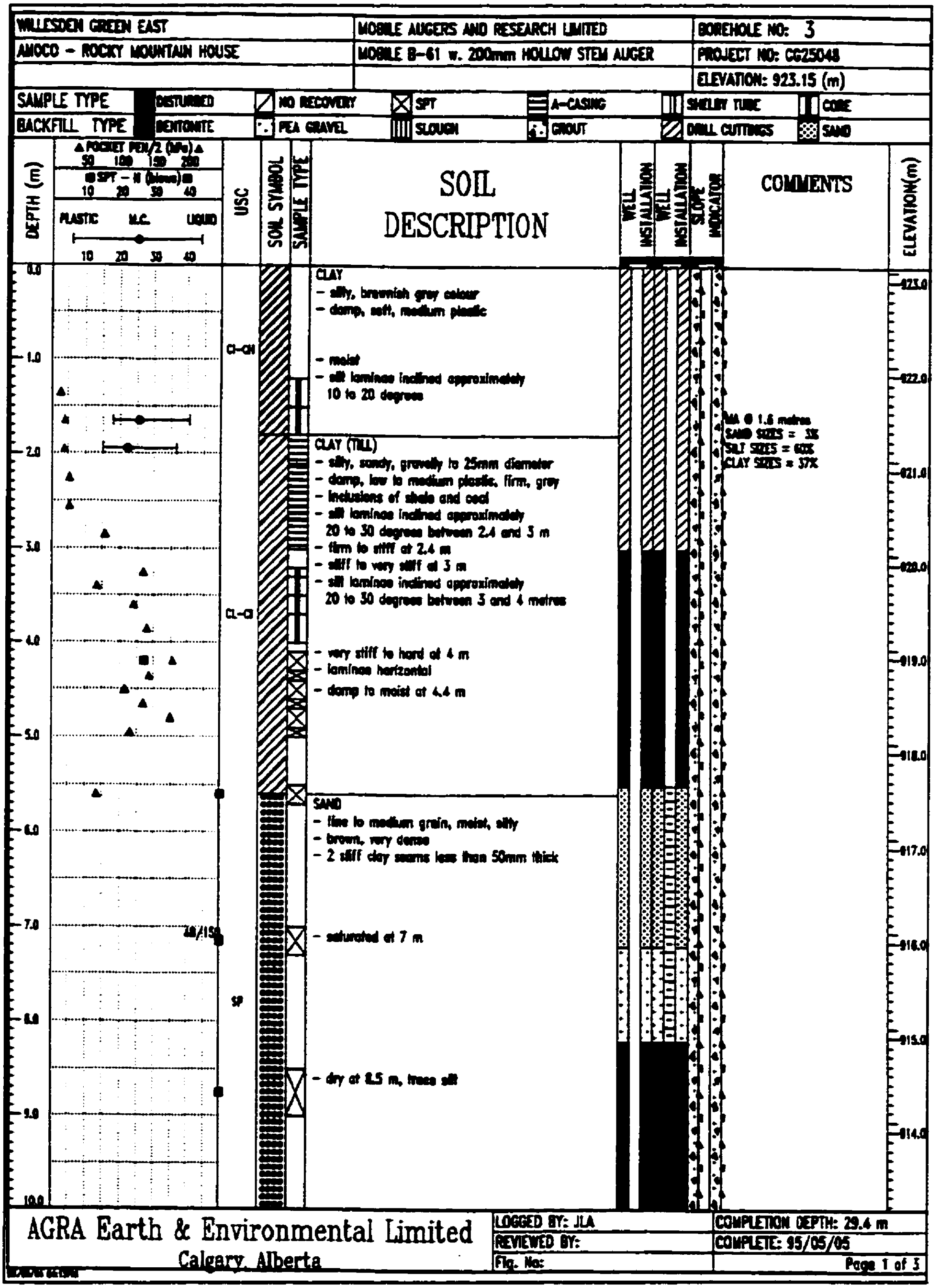



Fig. 5.9: Borehole No. 3

# EXPLANATION OF TERMS AND SYMBOLS

The terms and symbols used on the borehole logs to summarize the results of field investigation and subsequent laboratory testing are described in these pages.

It should be noted that materials, boundaries and conditions have been established only at the borehole locations at the time of investigation and are not necessarily representative of subsurface conditions elsewhere across the site.

**TEST DATA**

Data obtained during the field investigation and from laboratory testing are shown at the appropriate depth interval.

Abbreviations, graphic symbols, and relevant test method designations are as follows:

| | | | |
|---|---|---|---|
| *C | Consolidation test | *ST | Swelling test |
| $D_R$ | Relative density (formerly specific gravity) | TV | Torvane shear strength |
| Fines | Percentage by weight smaller than #200 sieve | VS | Vane shear strength (undisturbed-remolded) |
| k | Permeability coefficient | w | Natural water content (ASTM D 2216) |
| *MA | Mechanical grain size analysis and hydrometer test | $w_l$ | Liquid limit (ASTM D 423) |
| | | $w_p$ | Plastic limit (ASTM D 424) |
| N | Standard penetration test (CSA A119.1-60) | $\varepsilon_f$ | Unit strain at failure |
| $N_d$ | Dynamic cone penetration test | $\gamma$ | Unit weight of soil or rock |
| NP | Non plastic soil | $\gamma_d$ | Dry unit weight of soil or rock |
| pp | Pocket penetrometer strength | $\rho$ | Density of soil or rock |
| *q | Triaxial compression test | $\rho_d$ | Dry density of soil or rock |
| $q_u$ | Unconfined compressive strength | | |
| *SB | Shearbox test | → | seepage |
| $SO_4$ | Concentration of water-soluble sulphate | ▼ | observed water level |

**The results of these tests usually are reported separately*

**SOIL CLASSIFICATION AND DESCRIPTION**

Soils are classified and described according to their engineering properties and behaviour.

The soil of each stratum is described using the Unified Soil Classification System[1] modified slightly so that an inorganic clay of "medium plasticity" is recognized.

The use of modifying adjectives may be employed to define the actual or estimated percentage range by weight of minor components. This is similar to a system developed by D.M. Burmister.[2]

The soil classification system is shown in greater detail on page 2.

**SAMPLE TYPE** — The type of sample is shown at the appropriate depth interval using the following abbreviations:

A auger sample
B block sample
C rock core, or frozen soil core
D drive sample/SPT sample
P pitcher tube sample
U tube sample (usually thin-walled)
W wash or air return sample
O other (see report text)
⊠ indicates no sample recovery

Fig. 5.10: Explanation of terms and symbols used on borehole logs (part 1)

## MODIFIED UNIFIED CLASSIFICATION SYSTEM FOR SOILS

| MAJOR DIVISION | | | GROUP SYMBOL | GRAPH SYMBOL | TYPICAL DESCRIPTION | LABORATORY CLASSIFICATION CRITERIA | |
|---|---|---|---|---|---|---|---|
| COARSE-GRAINED SOILS (MORE THAN HALF BY WEIGHT LARGER THAN 200 SIEVE) | GRAVELS MORE THAN HALF COARSE GRAINS LARGER THAN NO 4 SIEVE | CLEAN GRAVELS (LITTLE OR NO FINES) | GW | | WELL GRADED GRAVELS, LITTLE OR NO FINES | $C_u = \frac{D_{60}}{D_{10}} > 4$ $C_c = \frac{(D_{30})^2}{D_{10} \times D_{60}} = 1$ to 3 | |
| | | | GP | | POORLY GRADED GRAVELS AND GRAVEL-SAND MIXTURES, LITTLE OR NO FINES | NOT MEETING ABOVE REQUIREMENTS | |
| | | DIRTY GRAVELS (WITH SOME FINES) | GM | | SILTY GRAVELS, GRAVEL-SAND-SILT MIXTURES | CONTENT OF FINES EXCEEDS 12% | ATTERBERG LIMITS BELOW "A" LINE OR P.I. LESS THAN 4 |
| | | | GC | | CLAYEY GRAVELS, GRAVEL-SAND-CLAY MIXTURES | | ATTERBERG LIMITS ABOVE "A" LINE P.I. MORE THAN 7 |
| | SANDS MORE THAN HALF FINE GRAINS SMALLER THAN NO 4 SIEVE | CLEAN SANDS (LITTLE OR NO FINES) | SW | | WELL GRADED SANDS, GRAVELLY SANDS, LITTLE OR NO FINES | $C_u = \frac{D_{60}}{D_{10}} > 6$ $C_c = \frac{(D_{30})^2}{D_{10} \times D_{60}} = 1$ to 3 | |
| | | | SP | | POORLY GRADED SANDS, LITTLE OR NO FINES | NOT MEETING ABOVE REQUIREMENTS | |
| | | DIRTY SANDS (WITH SOME FINES) | SM | | SILTY SANDS, SAND-SILT MIXTURES | CONTENT OF FINES EXCEEDS 12% | ATTERBERG LIMITS BELOW "A" LINE P.I. LESS THAN 4 |
| | | | SC | | CLAYEY SANDS, SAND-CLAY MIXTURES | | ATTERBERG LIMITS ABOVE "A" LINE P.I. MORE THAN 7 |
| FINE GRAINED SOILS (MORE THAN HALF BY WEIGHT PASSES 200 SIEVE) | SILTS BELOW A LINE NEGLIGIBLE ORGANIC CONTENT | $W_L < 50\%$ | ML | | INORGANIC SILTS AND VERY FINE SANDS, ROCK FLOUR, SILTY SANDS OF SLIGHT PLASTICITY | CLASSIFICATION IS BASED UPON PLASTICITY CHART (see below) | |
| | | $W_L > 50\%$ | MH | | INORGANIC SILTS, MICACEOUS OR DIATOMACEOUS, FINE SANDY OR SILTY SOILS | | |
| | CLAYS ABOVE A LINE ON PLASTICITY CHART NEGLIGIBLE ORGANIC CONTENT | $W_L < 30\%$ | CL | | INORGANIC CLAYS OF LOW PLASTICITY, GRAVELLY, SANDY, OR SILTY CLAYS, LEAN CLAYS | | |
| | | $30\% < W_L < 50\%$ | CI | | INORGANIC CLAYS OF MEDIUM PLASTICITY, SILTY CLAYS | | |
| | | $W_L > 50\%$ | CH | | INORGANIC CLAYS OF HIGH PLASTICITY, FAT CLAYS | | |
| | ORGANIC SILTS & CLAYS BELOW A LINE ON CHART | $W_L < 50\%$ | OL | | ORGANIC SILTS AND ORGANIC SILTY CLAYS OF LOW PLASTICITY | WHENEVER THE NATURE OF THE FINE CONTENT HAS NOT BEEN DETERMINED, IT IS DESIGNATED BY THE LETTER "F", E.G. SF IS A MIXTURE OF SAND WITH SILT OR CLAY | |
| | | $W_L > 50\%$ | OH | | ORGANIC CLAYS OF HIGH PLASTICITY | | |
| HIGHLY ORGANIC SOILS | | | Pt | | PEAT AND OTHER HIGHLY ORGANIC SOILS | STRONG COLOR OR ODOR, AND OFTEN FIBROUS TEXTURE | |

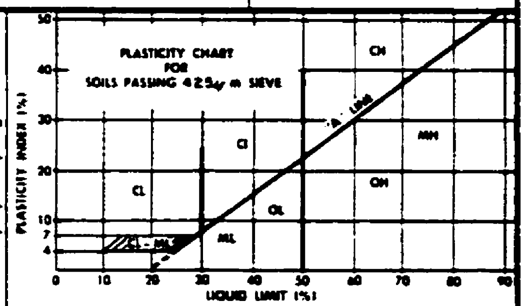
### SPECIAL SYMBOLS

BEDROCK (Undifferentiated)

VOLCANIC ASH

### SOIL COMPONENTS

| FRACTION | U S STANDARD SIEVE SIZE | | DEFINING RANGES OF PERCENTAGE BY WEIGHT OF MINOR COMPONENTS | |
|---|---|---|---|---|
| | PASSING | RETAINED | PERCENT | DESCRIPTOR |
| GRAVEL coarse | 75 mm | 19 mm | 50 - 35 | and |
| fine | 19 mm | 4.75 mm | 35 - 20 | some |
| SAND coarse | 4.75 mm | 2.00 mm | 20 - 10 | little |
| medium | 2.00 mm | 425 μm | 10 - 1 | trace |
| fine | 425 μm | 75 μm | | |
| SILT (non plastic) or CLAY (plastic) | 75 μm | | | |




1. ALL SIEVE SIZES MENTIONED ON THIS CHART ARE U.S. STANDARD, A.S.T.M. E.11.
2. BOUNDARY CLASSIFICATIONS POSSESSING CHARACTERISTICS OF TWO GROUPS ARE GIVEN COMBINED GROUP SYMBOLS, E.G. GW-GC IS A WELL GRADED GRAVEL SAND MIXTURE WITH CLAY BINDER BETWEEN 5% AND 12%.

Fig. 5.11: Explanation of terms and symbols used on borehole logs (part 2)

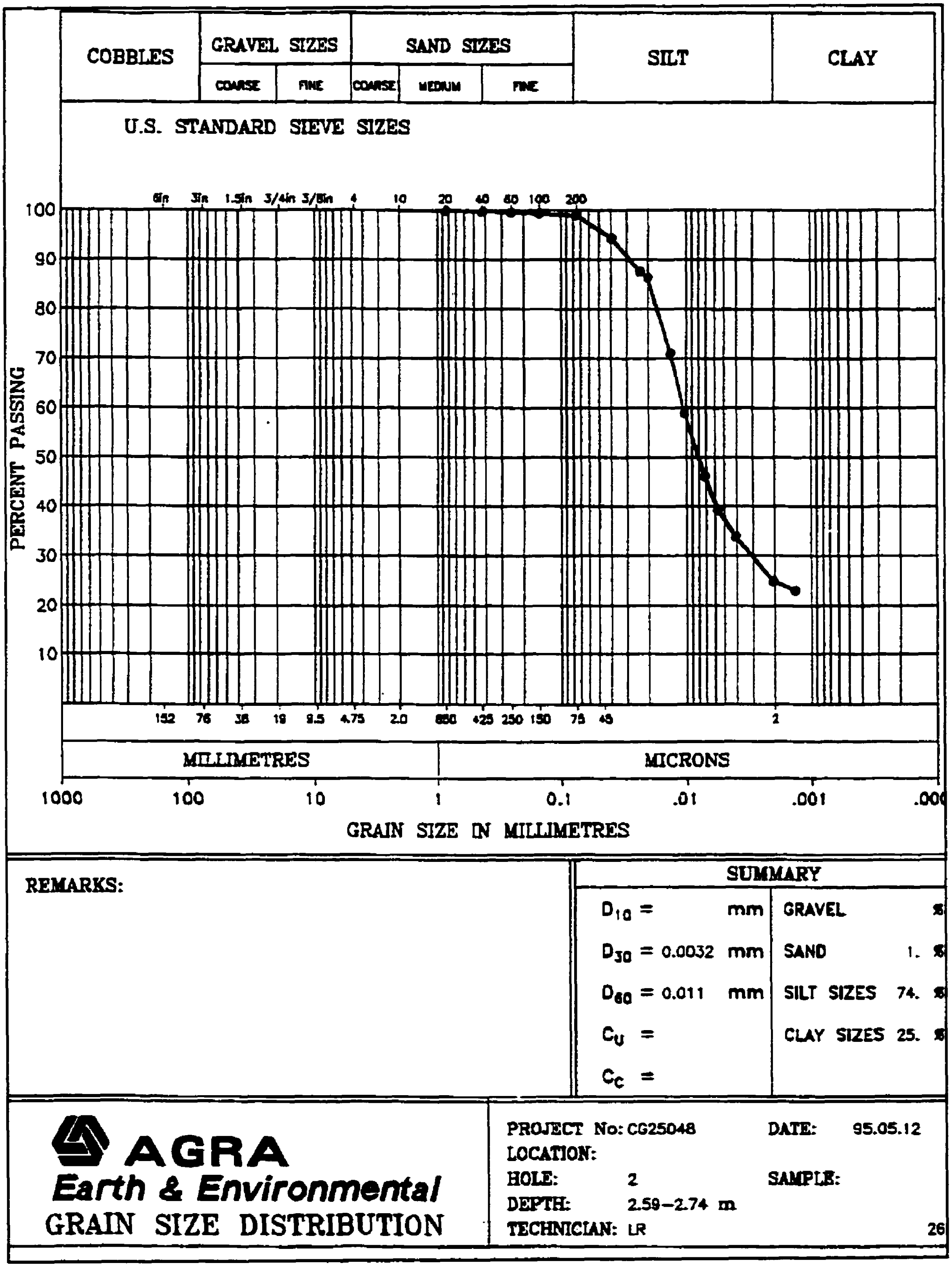
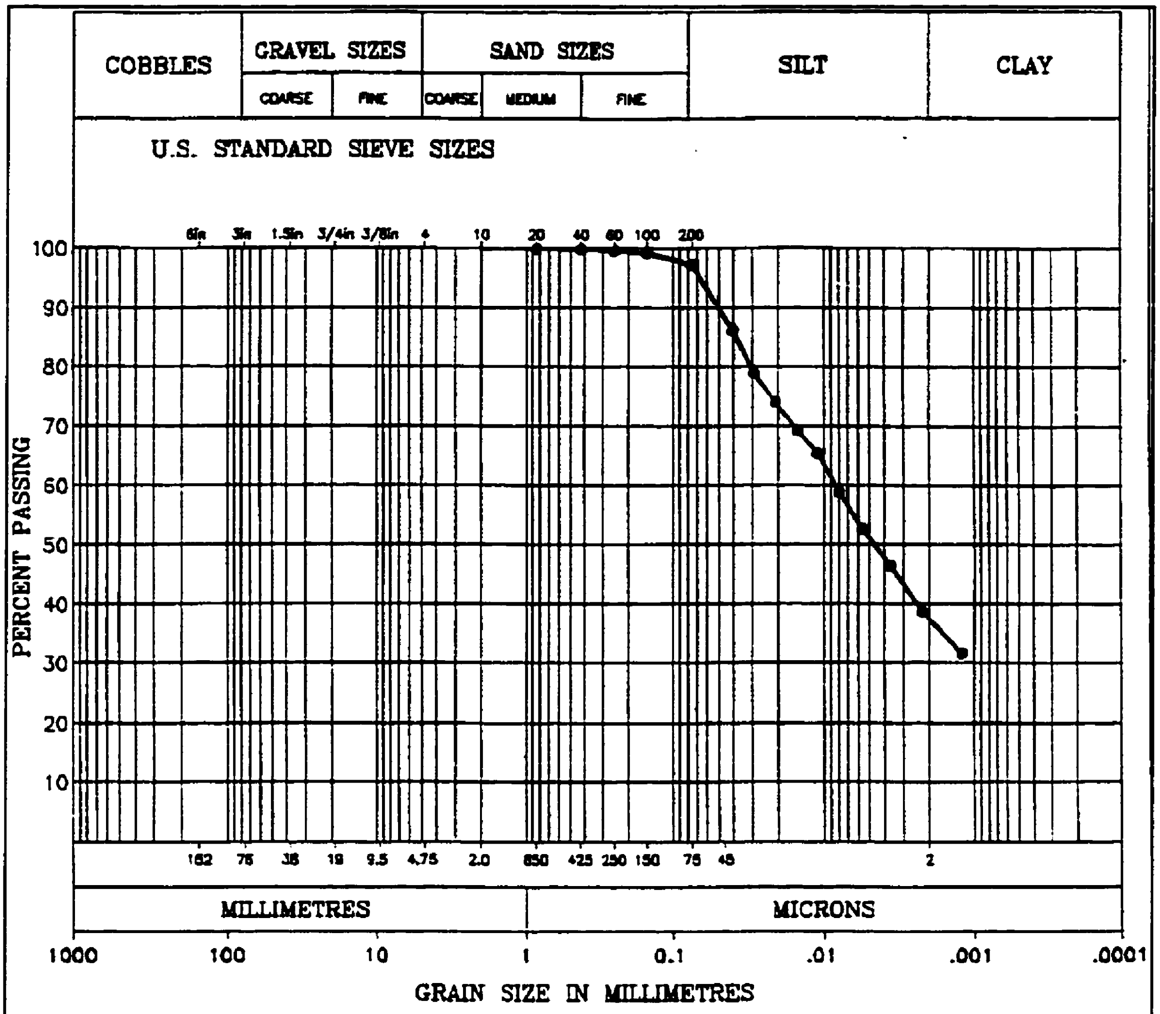



Fig. 5.12: Grain size distribution for borehole 2.




REMARKS: WL=41 WP=18 IP=23

| SUMMARY | | |
|---|---|---|
| $D_{10}$ = | mm | GRAVEL % |
| $D_{30}$ = | mm | SAND 3. % |
| $D_{60}$ = 0.0085 | mm | SILT SIZES 60. % |
| $C_U$ = | | CLAY SIZES 37. % |
| $C_C$ = | | |

AGRA Earth & Environmental

GRAIN SIZE DISTRIBUTION

PROJECT No: CG25048 DATE: 95.05.19
LOCATION:
HOLE: 3A SAMPLE:
DEPTH: 1.68 m
TECHNICIAN: LR 265

Fig. 5.13: Grain size distribution for borehole 3.

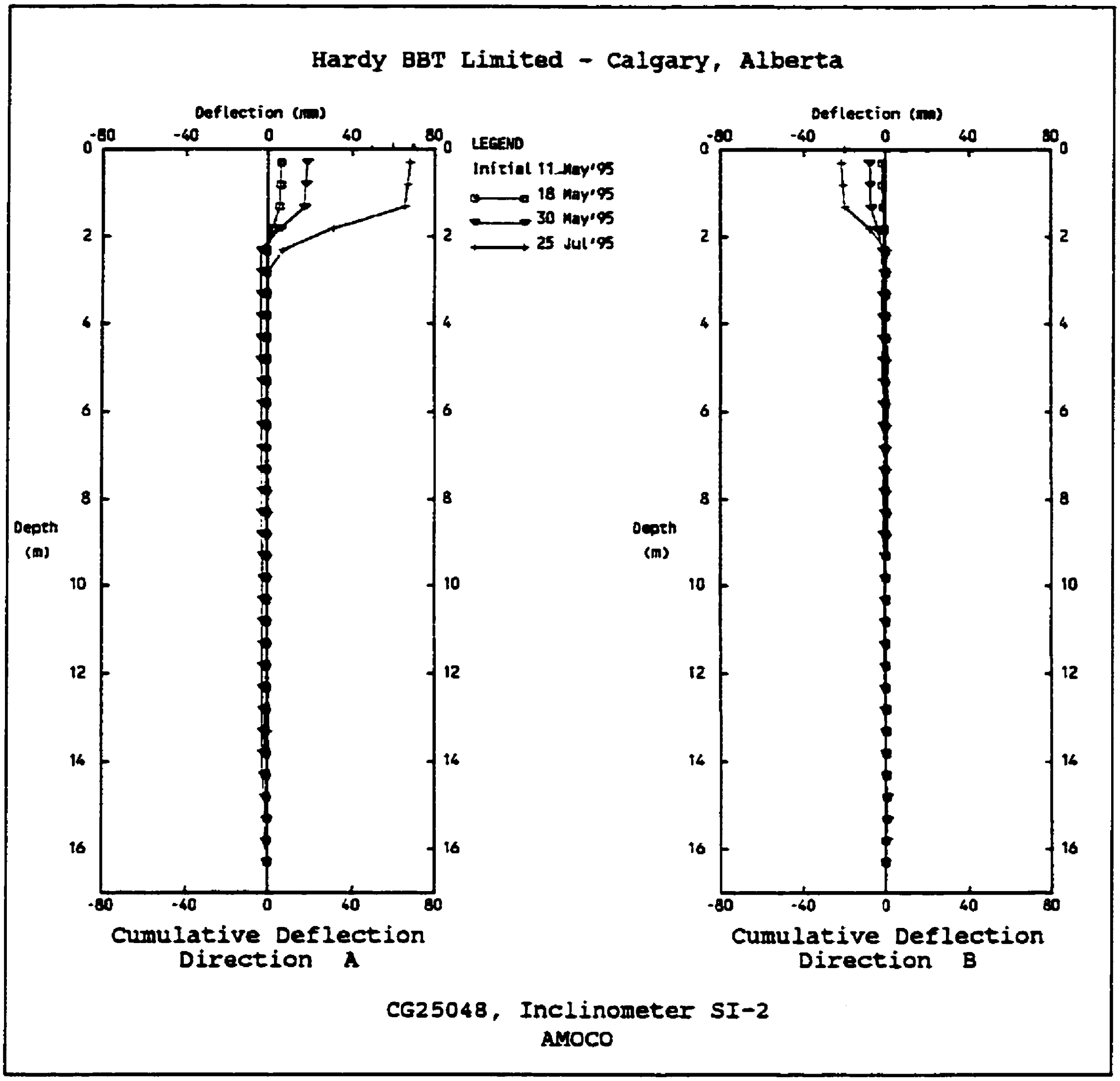



Fig. 5.14: Inclinometer SI-2

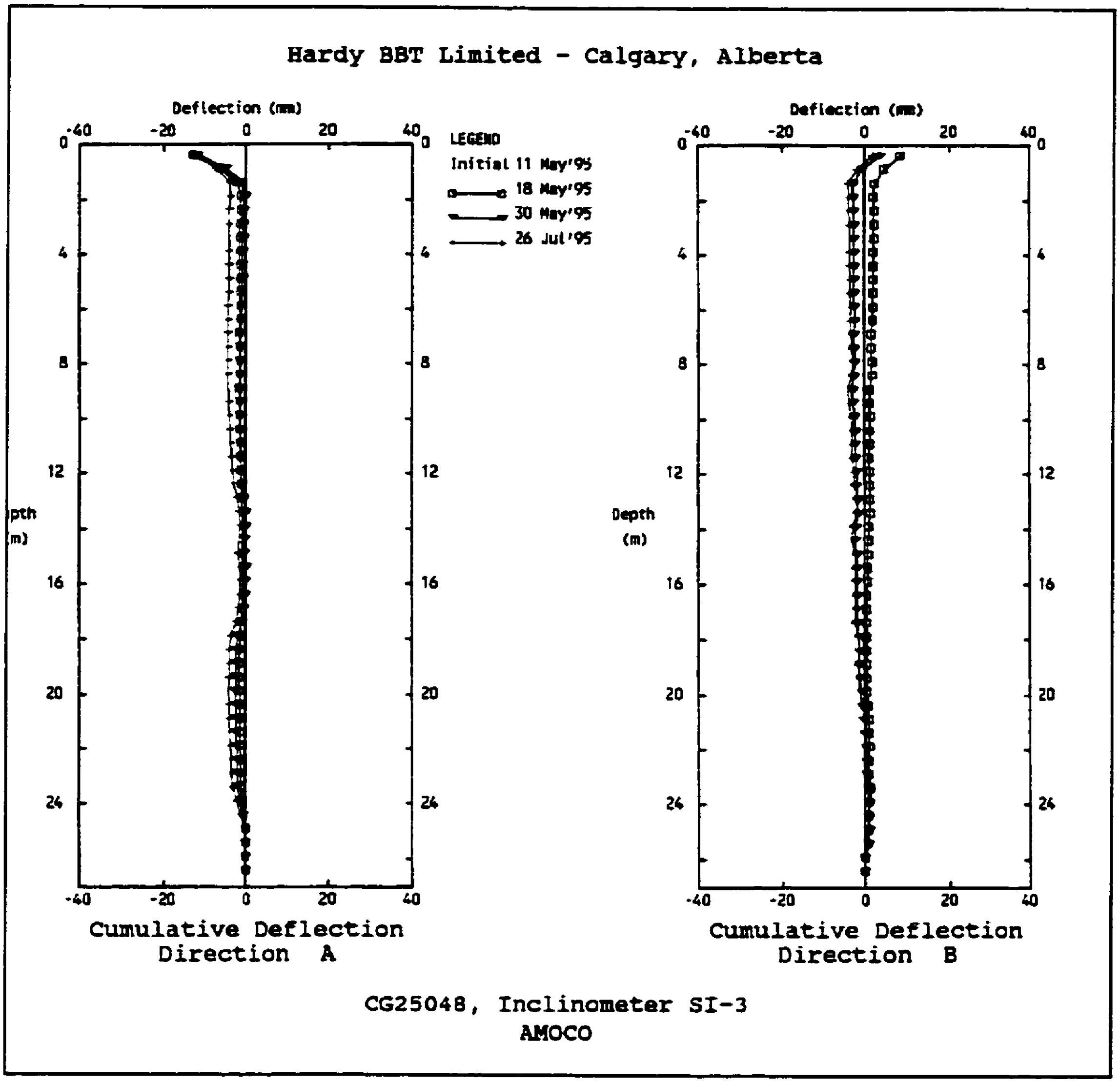



Fig. 5.15: Inclinometer SI-3

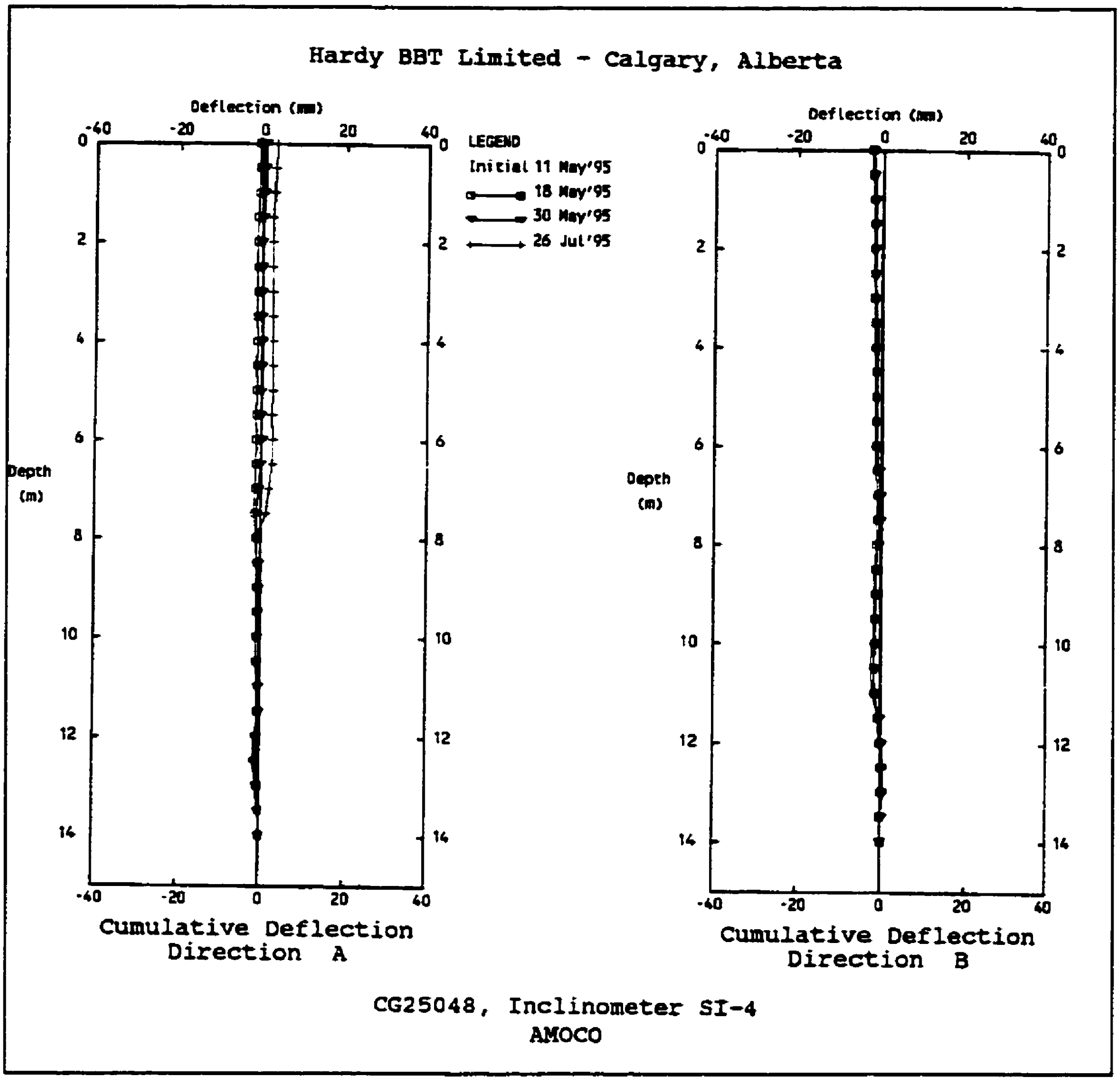



Fig. 5.16: Inclinometer SI-4

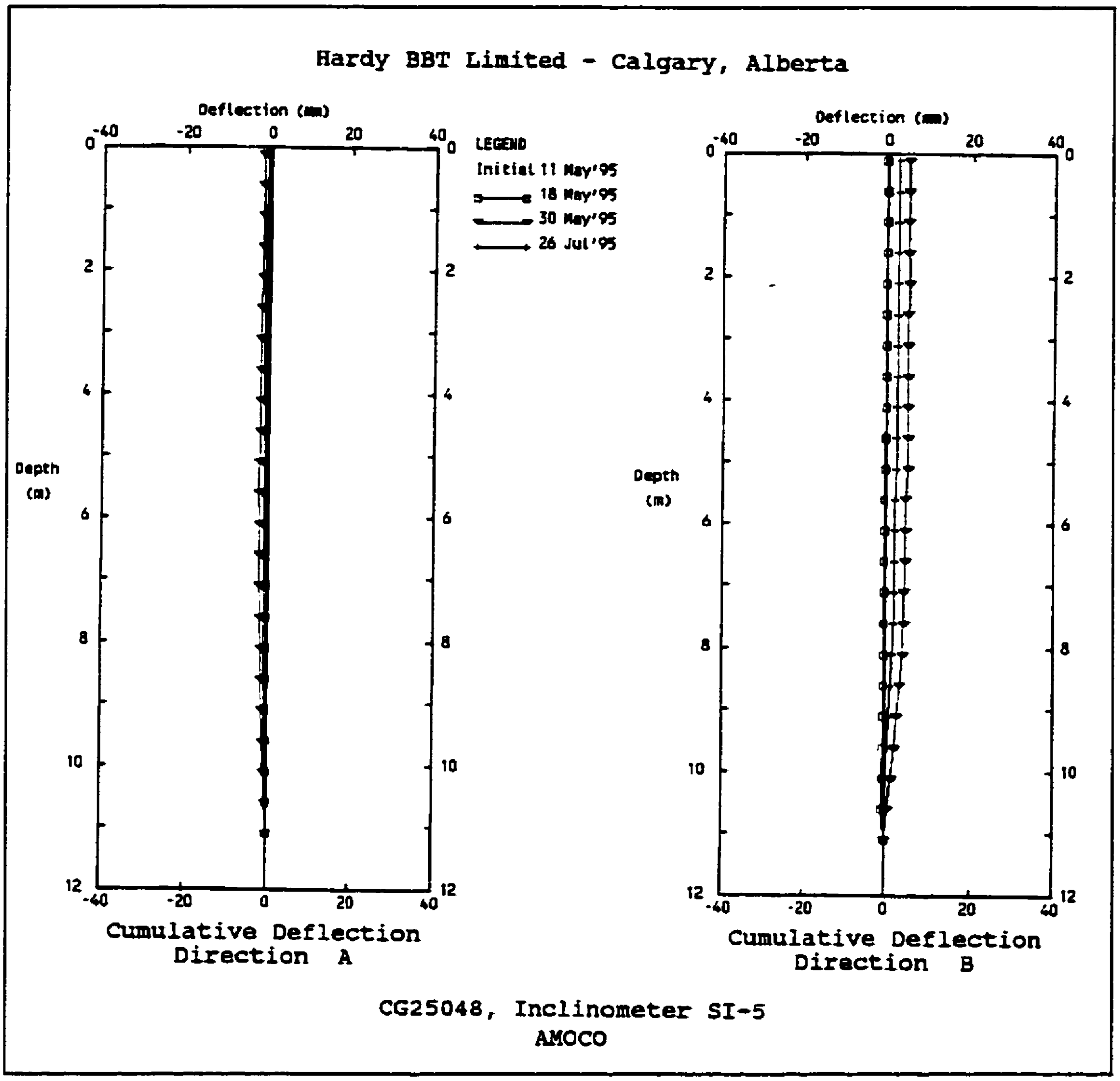



Fig. 5.17: Inclinometer SI-5

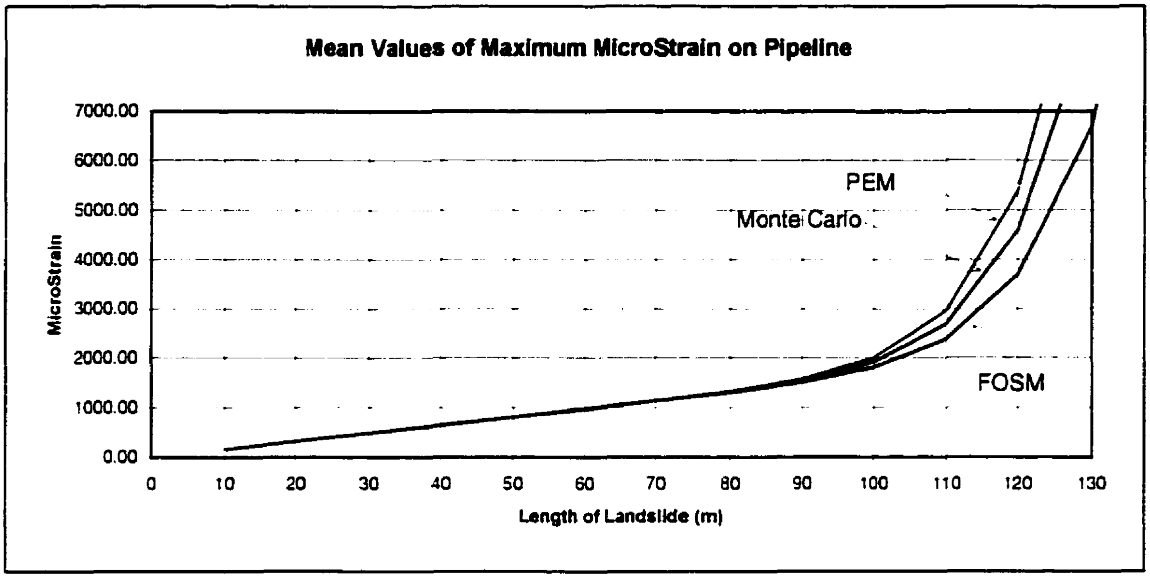



Fig. 5.18: Mean values of maximum microstrain on pipeline subjecteded to longitudinal landslide

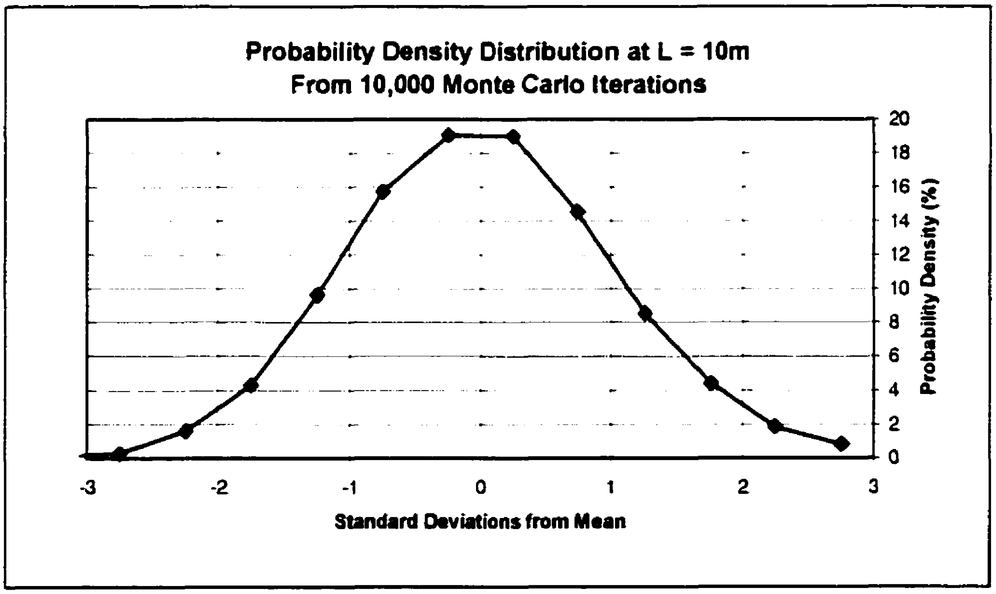



Fig.5.19: Probability density distribution at L=10m

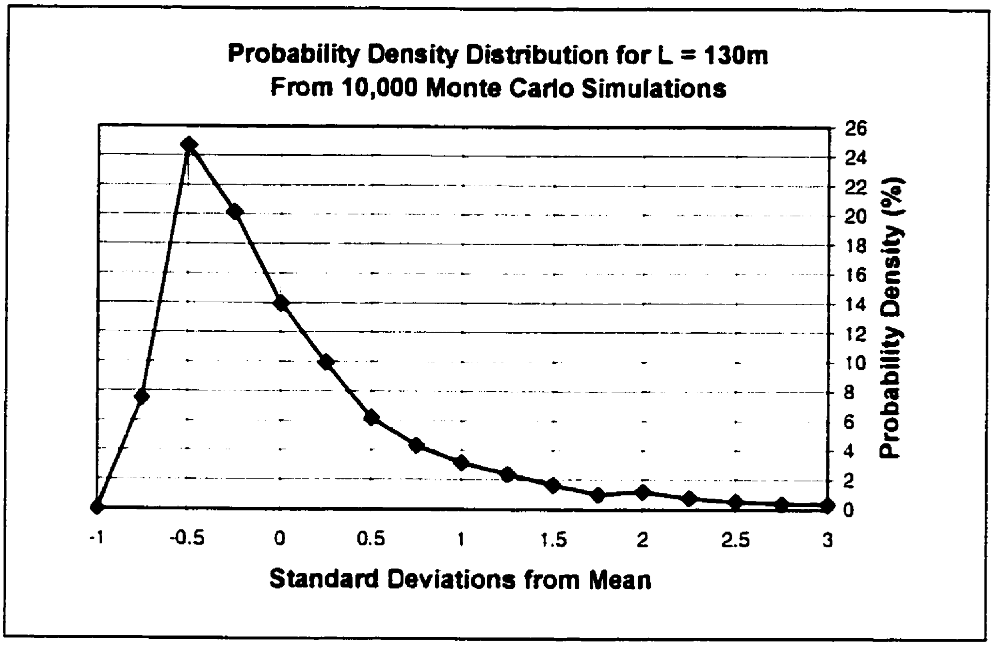



Fig.5.20: Probability density distribution at L=130m

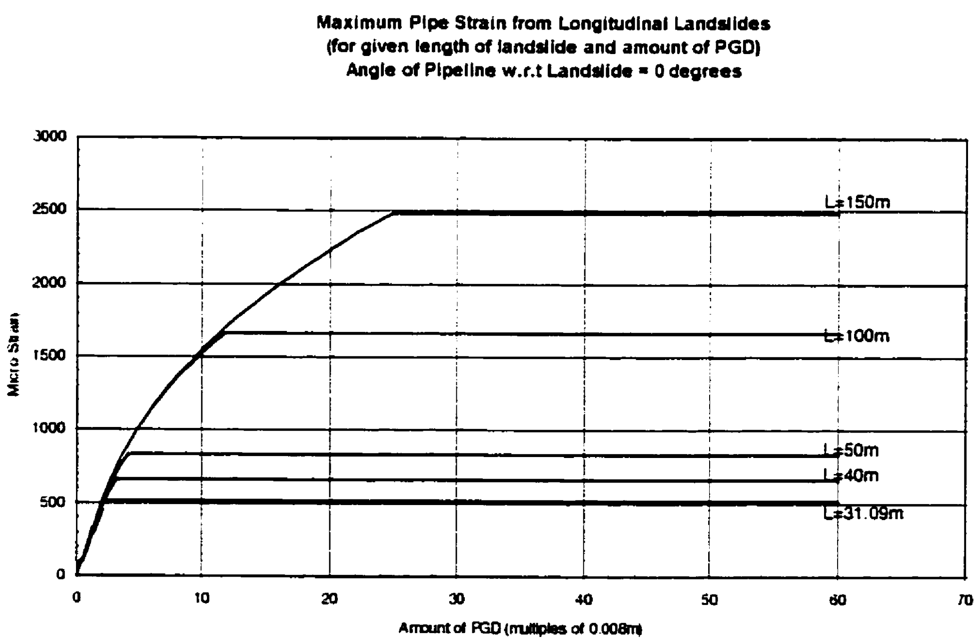



Fig. 5.21: Maximum strain for given length and amount of longitudinal landslide

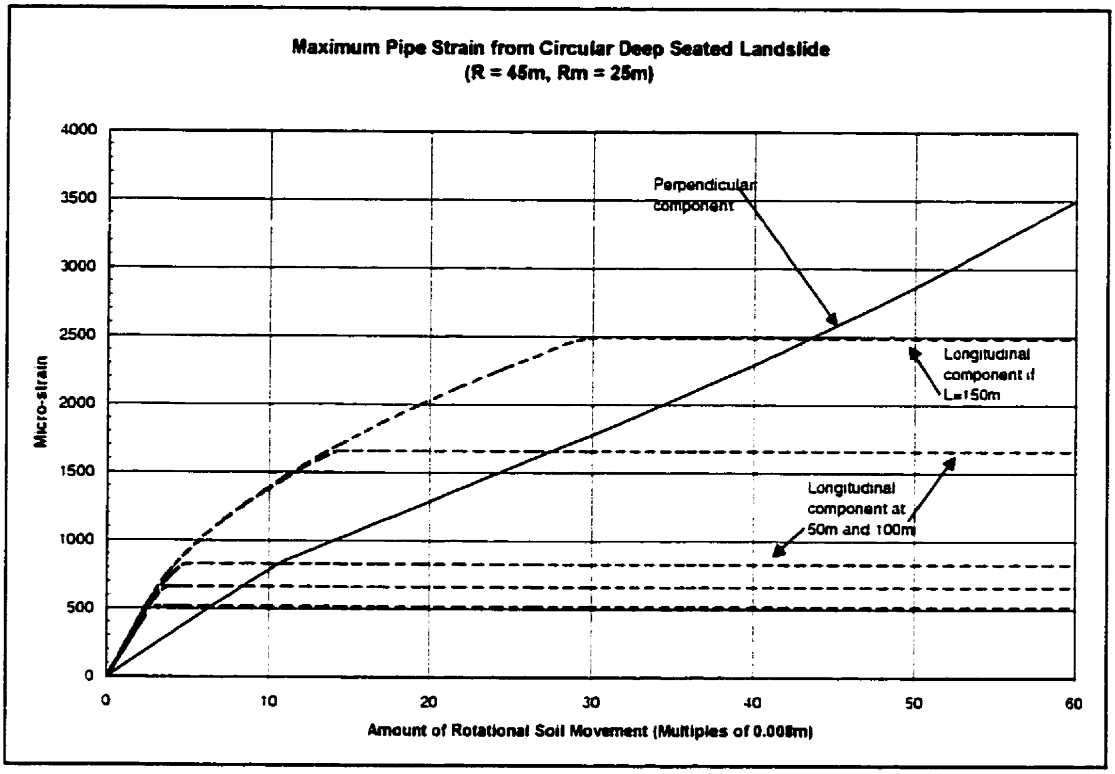



Fig. 5.22: Maximum strain for given deep-seated landslide with (R=45m. Rm = 25m) and amount of soil movement

# Chapter 6

# Conclusions and Recommendations

## 6.1 Conclusions

Analytical and numerical solutions have been derived to determine the deflection profile of a buried pipeline in a slope subjected to a longitudinal, transverse and deep-seated failure. Unlike previous methods, the methods presented in this thesis recognize that the force acting on the pipe is a function of the relative displacement between the pipeline and the landslide, and vice versa.

It was found that shear-type rupture and flexure bending are the two possible and dominant deformation modes of a buried steel pipe in a deep-seated slide. It was also found that for transverse landslide widths greater than about 10 m, the pipeline shows displacement and strain as if the width of the landslide is infinite. Relationships between the soil movement and the pipe deformation strain for different modes have been established so that one can assess the risk of pipeline rupture with a given soil movement. This allows consulting engineers to monitor the soil movements in a slope in order to predict pipeline strains.

The most critical parameters in the safe design of pipelines in unstable slopes are the soil resistance ($R_f$), pipe modulus of elasticity (E), and pipe thickness (t). Using light weight aggregate (LWA) as a backfill not only provides drainage, but it will reduce the strain induced in the pipeline in a landslide. Using a more elastic steel pipe (lower modulus of elasticity) and thicker pipe would reduce the strain induced in the pipeline as well. A small width transverse landslide produces higher strain than a larger width landslide – the difference is small at the beginning of soil movement but it becomes larger for larger amounts of PGD.

A large number of case studies are needed to compare the results of the statistical analysis so that realistic conclusions can be drawn in the future about the correlation between observed behavior and statistical predictions. Estimates of strength parameters of slopes based on test results and associated geomechanics considerations must be made for each particular slope – and these properties would vary from one site to another. Therefore, a quick method of calculation is needed to determine the probability of pipeline failure – such as the Monte Carlo Simulation and PEM method.

One must devise a remediation strategy based on the risk of pipeline failure from a statistical analysis. For low risk, one may improve the surface and near-surface drainage and establish a soil movement-monitoring program. At intermediate risk, one may try to bury the pipeline below the shallow landslide, but there would be a risk of inducing a higher strain on the pipeline if a deeper planar landslide or a deep-seated landslide were to develop. Another option is to install valves both upstream and downstream of the unstable slope for automatic shut-off of the pipeline. One may also try resloping and backfilling with LWA. If deep-seated landslides are possible, then constructing a toe berm is recommended. At high risk, one may reroute the pipeline to a stable slope.

## 6.2 Recommendations

It was assumed in this thesis that the soil properties such as cohesion c and frictional angle $\phi$ were all independent random variables. At present there are no available data concerning the magnitude of such correlations. A new area of research is needed to establish that such correlations exist and then to quantify them.

The statistical analysis, such as PEM, can be applied to the more complex pipeline strain equations presented in this thesis. However, it must be advised that it would be very cumbersome to implement FOSM, or require large computer power to apply Monte Carlo simulation.

The pipeline strain equations can be expanded to include the thermal strain, and pipeline operating pressure. The effect of pore pressure on the soil-pipeline interaction should also be investigated. Instead of the simple elastoplastic soil behavior, the model could be revised to use the stress-strain behavior of overconsolidated and normally consolidated soil.

The present model should be verified by using scaled physical model of a pipeline in a sliding mass of soil. Full scale testing requires suitable test sites and is costly. Centrifuge model testing is cheaper, but is limited by the size of the centrifuge available.

For example, it would be difficult to centrifuge model a pipeline with a distinct backfill material unless a large size centrifuge is available. Finite element modelling (FEM) is great for modelling complex slope and pipeline geometries. The parameters describing soil-pipeline interaction could be adopted from soil/structure interactions for foundations such as anchors and piles. However, FEM is limited by the assumptions used in modelling the soil/pipeline interaction as well.

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# Appendix A

# Derivations for Longitudinal Landslide

## A.1 General 2nd order equilibrium equation for pipeline:

From Figure A.1:

[A1] $$N = N + \frac{d}{dx}N\cdot dx - \tau\cdot(\pi\cdot d)\cdot dx = K_s\cdot(u - \delta)\cdot(\pi\cdot d)\cdot dx$$

[A2] $$N = E\cdot A\cdot\varepsilon = E\cdot A\cdot\frac{d}{dx}(u - \delta)$$

[A3] $$\frac{d}{dx}N = K_s\cdot(u - \delta)\cdot(\pi\cdot d)$$

From the stress-strain relationship:

[A4] $$\sigma = \frac{N}{A} = E\cdot\varepsilon$$

[A5] $$N = E\cdot A\cdot\varepsilon = E\cdot A\cdot\frac{d}{dx}(u - \delta)$$

Combine equations [A3] and [A5]

[A6] $$\frac{dN}{dx} = E\cdot A\cdot\frac{d}{dx}\frac{d}{dx}(u - \delta) = K_s\cdot\pi\cdot d\cdot(u - \delta)$$

[A7] $$\frac{d}{dx}\frac{d}{dx}u - \frac{K_s\cdot\pi\cdot d}{E\cdot A}\cdot u = \frac{K_s\cdot\pi\cdot d}{E\cdot A}\cdot\delta$$

Homogeneous solution of equation [A7]:

[A8] $$\boxed{u = C_1 \cdot e^{\lambda \cdot x} + C_2 \cdot e^{-\lambda \cdot x}}$$

where,

[A9] $$\boxed{\lambda = \left(\frac{K_s \cdot \pi \cdot d}{E \cdot A}\right)^{\frac{1}{2}}}$$

Nonhomogeneous solution of equation [A7]:

[A10] $$\boxed{u = \delta}$$

Therefore, the general $2^{nd}$ order equilibrium equation is as follows:

[A11] $$\boxed{u = C_1 \cdot e^{\lambda \cdot x} + C_2 \cdot e^{-\lambda \cdot x} + \delta}$$

## A.2 Pipeline equations for $\delta < 2D_s$ (see Figure A.2):

Region 1 $(-\infty < x < 0)$:

[A12] $$\frac{d}{dx}\frac{d}{dx}u_1 = \lambda^2 \cdot u_1 - 0$$

[A13] $$u_1 = u_o \cdot e^{\lambda \cdot x}$$

[A14] $$\varepsilon_1 = u_o \cdot \lambda \cdot e^{\lambda \cdot x}$$

Region 2 $(0 < x < A)$:

[A15] $$\frac{d}{dx}\frac{d}{dx}u_2 = \lambda^2 \cdot u_2 \quad \tan(\alpha) \cdot x$$

[A16] $$u_2 = k_1 \cdot e^{\lambda \cdot x} - k_2 \cdot e^{\lambda \cdot x} - \tan(\alpha) \cdot x$$

[A17] $$\varepsilon_2 = k_1 \cdot \lambda \cdot e^{\lambda \cdot x} \quad k_2 \cdot \lambda \cdot e^{\lambda \cdot x} \cdot \tan(\alpha)$$

Apply the continuity condition at x=0.

[A18] $$u_2 = u_o \quad \frac{\tan(\alpha)}{(2 \cdot \lambda)} \cdot e^{\lambda \cdot x} \quad \frac{\tan(\alpha)}{(2 \cdot \lambda)} \cdot e^{\lambda \cdot x} \cdot \tan(\alpha) \cdot x$$

[A19] $$\varepsilon_2 = u_o \quad \frac{\tan(\alpha)}{(2 \cdot \lambda)} \cdot \lambda \cdot e^{\lambda \cdot x} - \frac{\tan(\alpha)}{2} \cdot e^{\lambda \cdot x} - \tan(\alpha)$$

At x = A, $u_2 = u_{soil} = \tan(\alpha)(A)$

[A20] $$u_2(A) = u_o - \frac{\tan(\alpha)}{(2 \cdot \lambda)} \cdot e^{\lambda \cdot A} - \frac{\tan(\alpha)}{(2 \cdot \lambda)} \cdot e^{\lambda \cdot A} - \tan(\alpha) \cdot A$$

[A21] $$A = \frac{\cdot \ln\left(1 - \frac{2 \cdot \lambda \cdot u_o}{\tan(\alpha)}\right)}{2 \cdot \lambda}$$

Region 3 (A < x < B):

[A22] $$\frac{d}{dx}\frac{d}{dx}u_{soil} - u_3 = \lambda^2 \cdot \left(\tan(\alpha)\cdot x - u_3\right)$$

[A23] $$u_3 = k_3 \cdot e^{\lambda \cdot x} + k_4 \cdot e^{-\lambda \cdot x} + \tan(\alpha)\cdot x$$

[A24] $$\varepsilon_3 = k_3 \cdot \lambda \cdot e^{\lambda \cdot x} - k_4 \cdot \lambda \cdot e^{-\lambda \cdot x} + \tan(\alpha)$$

Apply the continuity condition at x = A

[A25] $$u_3 = u_o - \frac{\tan(\alpha)}{(2\cdot\lambda)}\cdot e^{\lambda\cdot x} - \frac{\tan(\alpha)}{(2\cdot\lambda)}\cdot e^{-\lambda\cdot x} + \tan(\alpha)\cdot x$$

[A26] $$\varepsilon_3 = u_o - \frac{\tan(\alpha)}{(2\cdot\lambda)}\cdot\lambda\cdot e^{\lambda\cdot x} + \frac{\tan(\alpha)}{2}\cdot e^{-\lambda\cdot x} + \tan(\alpha)$$

Region 4 (B < x < L/2):

[A27] $$\frac{d}{dx}\frac{d}{dx}u_{soil} - u_4 = \lambda^2 \cdot \left(\delta - u_4\right)$$

[A28] $$u_4 = k_5 \cdot e^{\lambda \cdot x} + k_6 \cdot e^{-\lambda \cdot x} + \delta$$

[A29] $$\varepsilon_4 = \left(k_5 \cdot e^{\lambda \cdot x} - k_6 \cdot e^{-\lambda \cdot x}\right)\cdot\lambda$$

Apply the continuity condition at x = B.

[A30] $$u_4 = u_o - \frac{1}{e^{\lambda\cdot x_r}}\cdot\left(1 - \frac{\tan(\alpha)}{2\cdot\lambda}\right)\cdot e^{\lambda\cdot x} + \left(1 - e^{\lambda\cdot x_r}\cdot\frac{\tan(\alpha)}{2\cdot\lambda}\right)\cdot e^{-\lambda\cdot x} + \delta$$

[A31] $$\varepsilon_4 = u_o - \frac{1}{e^{\lambda\cdot x_r}}\cdot\left(1 - \frac{\tan(\alpha)}{2\cdot\lambda}\right)\cdot\lambda\cdot e^{\lambda\cdot x} - \left(1 - e^{\lambda\cdot x_r}\cdot\frac{\tan(\alpha)}{2}\right)\cdot e^{-\lambda\cdot x}$$

Applying the force equilibrium condition on the 4 regions.

[A32] $$u_o := \left( e^{\frac{1}{2}\cdot\lambda\cdot L} - e^{\frac{-1}{2}\cdot\lambda\cdot L} - e^{\frac{1}{2}\cdot\lambda\cdot\left(L - \frac{2\cdot\delta}{\tan(\alpha)}\right)} - e^{\frac{1}{2}\cdot\lambda\cdot\left(L - \frac{2\cdot\delta}{\tan(\alpha)}\right)} \right) \cdot \frac{\tan(\alpha)\cdot e^{\frac{1}{2}\cdot\lambda\cdot L}}{2\cdot\lambda}$$

The maximum strain for $\delta < 2D_s$ occurs in regions 2 or 3 and is expressed as follows:

[A33] $$\varepsilon_{max} := u_o \cdot \frac{\tan(\alpha)}{(2\cdot\lambda)}\cdot\lambda\cdot e^{\lambda\cdot\frac{\ln\left(1 - \frac{2\cdot\lambda\cdot u_o}{\tan(\alpha)}\right)}{2\cdot\lambda}} \cdot \frac{\tan(\alpha)}{2}\cdot e^{\lambda\cdot\frac{\ln\left(1 - \frac{2\cdot\lambda\cdot u_o}{\tan(\alpha)}\right)}{2\cdot\lambda}} - \tan(\alpha)$$

## A.3 Pipeline equations for $\delta > 2D_s$ (see Figure A.3):

Region 1 ($-\infty < x < x_a$):

[A34] $$\frac{d}{dx}\frac{d}{dx}u_1 = \lambda^2 \cdot u_1 - 0$$

[A35] $$u_1 = C_1 \cdot e^{\lambda \cdot x} - C_2 \cdot e^{\lambda \cdot x}$$

[A36] $$\varepsilon_1 = C_1 \cdot \lambda \cdot e^{\lambda \cdot x} \quad C_2 \cdot \lambda \cdot e^{\lambda \cdot x}$$

Apply boundary conditions of zero pipe displacement at $x = -\infty$ and $\mathbf{D_s}$ pipe displacement at $x = x_a$.

[A37] $$u_1 = D_s \cdot e^{\lambda \cdot \; x \quad x_a}$$

[A38] $$\varepsilon_1 = D_s \cdot \lambda \cdot e^{\lambda \cdot \; x \quad x_a}$$

Region 2 ($x_a < x < 0$)

[A39] $$\frac{d}{dx}\frac{d}{dx}u_2 = \lambda^2 \cdot D_s$$

[A40] $$u_2 = \lambda^2 \cdot \frac{D_s}{2} \cdot x^2 - C_3 \cdot x - C_4$$

[A41] $$\varepsilon_2 = \lambda^2 \cdot D_s \cdot x - C_3$$

Region 3 ($0 < x < x_b$)

[A42] $$\frac{d}{dx}\frac{d}{dx}\; u_3 \quad \delta = \lambda^2 \cdot \; u_3 \quad D_s = \lambda^2 \cdot \; D_s$$

[A43] $$u_3 = \frac{1}{2} \cdot \lambda^2 \cdot D_s \cdot x^2 - k_1 \cdot x - k_2$$

[A44] $\boxed{\varepsilon_3 = \lambda^2 \cdot D_s \cdot x - k_1}$

Region 4 ($x_b < x < L/2$)

[A45] $\boxed{\frac{d}{dx}\frac{d}{dx} u_4 - \delta = \lambda^2 \cdot u_4 - \delta}$

[A46] $\boxed{u_4 = k_3 \cdot e^{\lambda \cdot x} - k_4 \cdot e^{\lambda \cdot x} - \delta}$

[A47] $\boxed{\varepsilon_4 = k_3 \cdot \lambda \cdot e^{\lambda \cdot x} \quad k_4 \cdot \lambda \cdot e^{\lambda \cdot x}}$

Applying the continuity condition at $x = x_a$, we get:

[A48] $\boxed{C_3 = \lambda^2 \cdot x_a - \lambda \cdot D_s}$

[A49] $\boxed{C_4 = \frac{1}{2} \cdot \lambda^2 \cdot x_a^2 \quad x_a \cdot \lambda \cdot 1 \cdot D_s}$

Applying the continuity condition at x equals zero, we get:

[A50] $\boxed{C_3 = k_1}$

[A51] $\boxed{C_4 = k_2}$

Applying the boundary condition that $u_3(x_b) = (\delta - D_s)$:

[A52] $\boxed{\delta - D_s = \frac{1}{2} \cdot \lambda^2 \cdot D_s \cdot x_b^2 - \lambda^2 \cdot x_a - \lambda \cdot D_s \cdot x_b - \frac{1}{2} \cdot \lambda^2 \cdot x_a^2 \quad x_a \cdot \lambda - 1 \cdot D_s}$

[A53] $\boxed{x_b = x_a - \frac{1}{\lambda} - \sqrt{2 \cdot x_a^2 - \frac{4}{\lambda} \cdot x_a - \frac{5}{\lambda^2} - \frac{2 \cdot \delta}{D_s \cdot \lambda^2}}}$

Applying the continuity condition at $x = x_b$, we get:

[A54] $$k_3 := \frac{1}{2} \cdot e^{-\lambda \cdot x_b} \cdot \left[ \left( x_a \cdot x_b - \frac{1}{2} \cdot x_b^2 - \frac{1}{2} \cdot x_a^2 \right) \cdot \lambda^2 - 2 \cdot x_a \cdot \lambda - 2 \right] \cdot D_s - \delta$$

[A55] $$k_4 := \frac{e^{\lambda \cdot x_b}}{2} \cdot \left[ \left( x_a \cdot x_b - \frac{1}{2} \cdot x_b^2 - \frac{1}{2} \cdot x_a^2 \right) \cdot \lambda^2 - 2 \cdot \lambda \cdot x_b \right] \cdot D_s - \delta$$

Combining equation [A52] to equations [A54] and [A55]:

[A56] $$k_3 := \frac{e^{\lambda \cdot x_b}}{2} \cdot \left( x_a - x_b \right) \cdot D_s \cdot \lambda$$

[A57] $$k_4 := \frac{e^{\lambda \cdot x_b}}{2} \cdot \left[ \left( x_b - x_a \right) \cdot \lambda - 2 \right] \cdot D_s$$

Applying the force equilibrium condition on the 4 different regions:

[A58 ] $$F := \int k_s \cdot \pi \cdot d \cdot \left( u_{soil} - u_{pipe} \right) dx$$

[A59] $$0 := F1 - F2 + F3 - F4$$

[A60] $$0 := e^{\left( 2 \cdot x_b - L \right) \cdot \lambda} - 1 + \left( x_a - x_b \right) \cdot \lambda - 2$$

Combine equations [A60] and [A53], we obtain equation [A61] which need to be solved for $x_a$:

[A61] $$0 := e^{\left[ 2 \cdot \left( x_a - \frac{1}{\lambda} + \sqrt{2 \cdot x_a^2 - \frac{4}{\lambda} \cdot x_a - \frac{5}{\lambda^2} - \frac{2 \cdot \delta}{D_s \cdot \lambda^2}} \right) - L \right] \cdot \lambda} - 1 + \left( \frac{1}{\lambda} - \sqrt{2 \cdot x_a^2 - \frac{4}{\lambda} \cdot x_a - \frac{5}{\lambda^2} - \frac{2 \cdot \delta}{D_s \cdot \lambda^2}} \right) \cdot \lambda - 2$$

The maximum strain for $\delta > 2D_s$ occurs in regions 2 or 3 at x = 0, and is expressed as follows:

[A62] $\boxed{\varepsilon_{max.} = x_a \cdot \lambda^2 - \lambda \cdot Ds}$

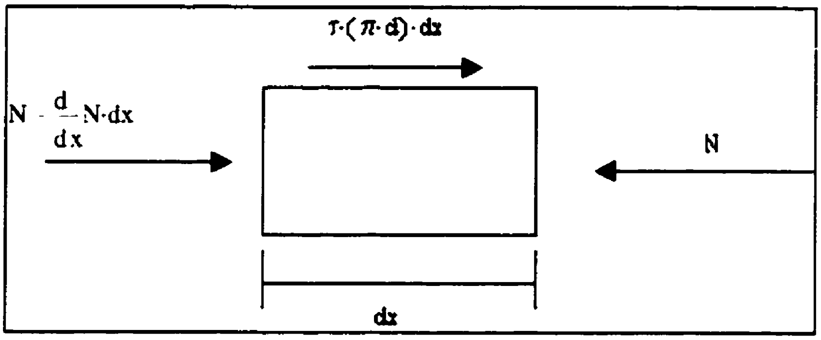


Fig. A.1: Force equilibrium on a finite element of pipe

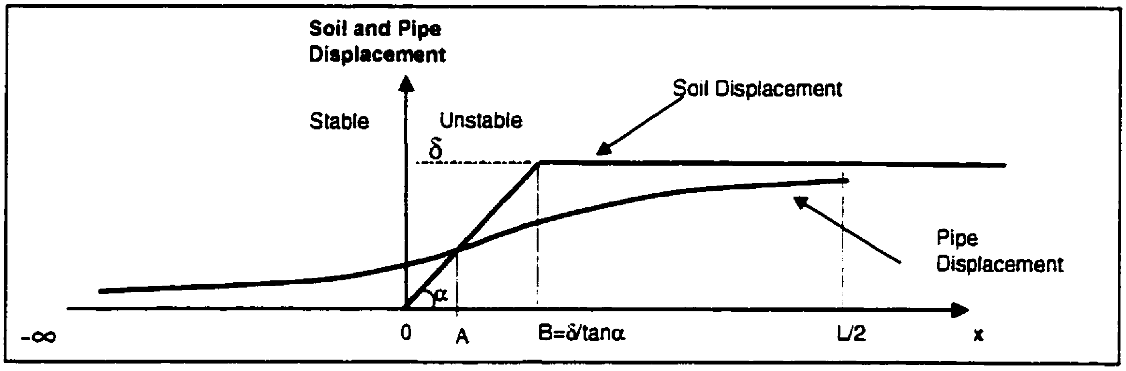


Fig. A.2: Ramp landslide movement

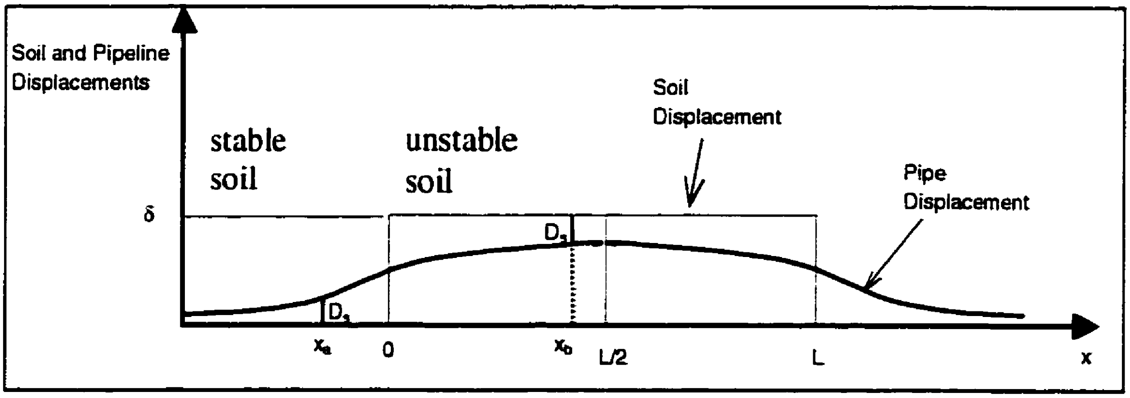


Fig. A.3: Step landslide movement

# Appendix B

# Derivations for Transverse Landslide

## B.1 Elastic curve equation:

The curvature of a plane curve is as

[B1] $$\boxed{\frac{1}{\rho}=\frac{d^2u/dx^2}{[1+(du/dx)^2]^{3/2}}}$$

For shallow curves, equation [B1] is approximated as

[B2] $$\boxed{\frac{1}{\rho}=\frac{d^2u}{dx^2}}$$

Within the elastic range, the curvature of the neutral surface may be expressed as

[B3] $$\boxed{\frac{1}{\rho}=\frac{M}{E\cdot I}=\frac{d^2u}{dx^2}}$$

The transverse load per unit length ($\omega$) is expressed as

[B4] $$\boxed{-\omega=\frac{\frac{d^2}{dx^2}(M)}{E\cdot I}=\frac{d^4u}{dx^4}}$$

where

$1/\rho$ is the curvature,

u is the transverse displacement,

M is the bending moment,

E is Young's modulus of elasticity, and

I is the moment of inertia.

# B.2 General pipeline displacement equation under transverse loading

From equation [B4]:

[B5] $$\boxed{\frac{d^4}{dx^4}(u-\delta)=\frac{-K_s\cdot d\cdot(u-\delta)}{E\cdot I}}$$

where.

d is the pipe diameter.

$\delta$ is the soil displacement. and

$K_s$ is the soil elastic subgrade modulus

[B6] $$\boxed{\frac{d^4}{dx^4}(u)-0=\frac{-K_s\cdot d\cdot(u-\delta)}{E\cdot I}}$$

Let $u=e^{m\cdot x}$

[B7] $$\boxed{m^4\cdot e^{m\cdot x}=\frac{K_s\cdot d}{E\cdot I}\cdot\delta-\frac{K_s\cdot d}{E\cdot I}\cdot e^{m\cdot x}}$$

### B.2.1 The homogeneous solution

[B8] $$\boxed{m^4\cdot e^{m\cdot x}+\frac{K_s\cdot d}{E\cdot I}\cdot e^{m\cdot x}=0}$$

[B9] $$\boxed{m^4+\frac{K_s\cdot d}{E\cdot I}=0}$$

Solving equation [B9]. we get:

[B10] $$\boxed{m_1=-m_3=(1+i)\cdot\left[\frac{K_s\cdot d}{E\cdot I}\right]^{1/4}=(1+i)\cdot\lambda}$$

[B11] $$\boxed{m_2 = -m_4 = (-1+i)\cdot\left[\frac{K_s \cdot d}{E \cdot I}\right]^{1/4} = (-1+i)\cdot\lambda}$$

where,

[B12] $$\boxed{\lambda = \left[\frac{K_s \cdot d}{E \cdot I}\right]^{1/4}}$$ and

[B13] $$\boxed{i = \sqrt{-1}}$$

[B14] $$\boxed{u = A_1 e^{m_1 x} + A_2 e^{m_2 x} + A_3 e^{m_3 x} + A_4 e^{m_4 x}}$$

[B15] $$\boxed{u = A_1 e^{(1+i)\cdot\lambda x} + A_2 e^{-(1+i)\cdot\lambda x} + A_3 e^{(-1+i)\cdot\lambda x} + A_4 e^{-(-1+i)\cdot\lambda x}}$$

Since

[B16] $$\boxed{e^{i\cdot\lambda x} = \cos(\lambda\cdot x) + i\cdot\sin(\lambda\cdot x)}$$ and

[B17] $$\boxed{e^{-i\lambda x} = \cos(\lambda\cdot x) - i\cdot\sin(\lambda\cdot x)}$$

Apply equations [B16] and [B17] to equation [B15].

[B18] $$\begin{aligned} u &= e^{\lambda x}\cdot\left[A_1\cdot(\cos(\lambda\cdot x)) + i\cdot\sin(\lambda\cdot x)) + A_4\cdot(\cos(\lambda\cdot x)) - i\cdot\sin(\lambda\cdot x))\right] + \cdots \\ &\cdots e^{-\lambda x}\cdot\left[A_3\cdot(\cos(\lambda\cdot x)) + i\cdot\sin(\lambda\cdot x)) + A_2\cdot(\cos(\lambda\cdot x)) - i\cdot\sin(\lambda\cdot x))\right] \end{aligned}$$

[B19] $$\begin{aligned} u &= e^{\lambda x}\cdot\left[(A_1 + A_4)\cdot\cos(\lambda\cdot x) + i\cdot(A_1 - A_4)\cdot\sin(\lambda\cdot x))\right] + \cdots \\ &\cdots e^{-\lambda x}\cdot\left[(A_3 + A_2)\cdot\cos(\lambda\cdot x) + i\cdot(A_3 - A_2)\cdot\sin(\lambda\cdot x))\right] \end{aligned}$$

Finally, the homogeneous solution is:

[B20] $$\boxed{u = e^{\lambda x}\cdot\left[(C_1\cdot\cos(\lambda\cdot x) + C_2\cdot\sin(\lambda\cdot x))\right] + e^{-\lambda x}\cdot\left[C_3\cdot\cos(\lambda\cdot x) + C_4\cdot\sin(\lambda\cdot x))\right]}$$

### B.2.1 The nonhomogeneous solution

[B21] $$\boxed{\frac{d^4u}{dx^4} = \frac{K_s \cdot d}{E \cdot I} \cdot (\delta - u)}$$

Let u be of a form of $u = A_0$.

[B22] $$\boxed{0 = \frac{K_s \cdot d}{E \cdot I} \cdot (\delta - A_0)}$$

Therefore the nonhomogeneous solution is

[B23] $$\boxed{A_0 = \delta}$$

Combining the homogeneous and nonhomogeneous solution, we have the displacement equation for the pipeline under transverse loading.

[B24] $$\boxed{\begin{array}{l} u = e^{\lambda x} \cdot [(C_1 \cdot \cos(\lambda \cdot x) + C_2 \cdot \sin(\lambda \cdot x))] + \cdots \\ \cdots + e^{-\lambda x} \cdot [C_3 \cdot \cos(\lambda \cdot x) + C_4 \cdot \sin(\lambda \cdot x))] + \delta \end{array}}$$

## B.3 Tensile strain of pipeline

From the Pythagorean theorem, we find the differential length of pipeline as follow:

[B25] $$\boxed{ds^2 = dx^2 + dy^2}$$

[B26] $$\boxed{ds = \sqrt{1 + \left(\frac{dy}{dx}\right)^2} \cdot dx}$$

[B27] $$\boxed{s = \int_0^{s_o} \sqrt{1 + \left(\frac{dy}{dx}\right)^2} \cdot dx}$$

Binomial expansion equation is

[B28] $$\boxed{(a + x)^n = a^n + n \cdot a^{n-1} + \frac{n \cdot (n-1)}{2!} \cdot a^{n-2} \cdot x^2 + \cdots}$$

Assuming shallow curve and applying binomial expansion, equation [B27] becomes:

[B29] $$\boxed{\begin{aligned} s &\approx \int_0^{s_o} \left[1 + \frac{1}{2} \cdot \left(\frac{dy}{dx}\right)^2\right] \cdot dx \\ &\approx s_o + \frac{1}{2} \int_0^{L} \left(\frac{dy}{dx}\right)^2 \cdot dx \end{aligned}}$$

Therefore, the differential length of pipeline is

[B30] $$\boxed{\Delta L \approx \frac{1}{2} \int_0^{s_o} \left(\frac{dy}{dx}\right)^2 \cdot dx}$$

In addition, the general equation for tensile strain is

[B31] $$\boxed{\varepsilon_t = \frac{\Delta L}{L} \approx \frac{1}{2} \cdot \left(\frac{dy}{dx}\right)^2}$$

# B.4 Bending strain of pipeline

From equation [B2] $\boxed{\frac{1}{\rho} = \frac{M}{E \cdot I} = \frac{d^2u}{dx^2}}$

As long as the stresses remain in the elastic range, the neutral axis pass through the centroid of the pipe section, and so the bending strain on the outer surface of the pipeline (d/2) is

[B32] $\boxed{\varepsilon_b\left(\frac{d}{2}\right) = \frac{d/2}{\rho} = \frac{d}{2} \cdot \frac{d^2u}{dx^2}}$

where d is the pipe outer diameter.

# B.5 Pipeline equations for elastic loading ($\delta < 2D_s$) with finite landslide width

Region 1 ( $-\infty < x < 0$):

We can reduce equation [B24] for the general pipe deflection profile, in the region from negative infinity to zero, by applying the boundary conditions at $x = -\infty$ ($C_3$ and $C_4$ equal 0).

[B33] $$\boxed{u_1 = e^{\lambda \cdot x} \cdot \left( C_1 \cdot \cos(\lambda \cdot x) - C_2 \cdot \sin(\lambda \cdot x) \right)}$$

[B34] $$\boxed{\frac{d}{dx} u_1 = \left[ (\cos(\lambda \cdot x) \quad \sin(\lambda \cdot x)) \cdot C_1 - (\sin(\lambda \cdot x) - \cos(\lambda \cdot x)) \cdot C_2 \right] \cdot e^{\lambda \cdot x} \cdot \lambda}$$

[B35] $$\boxed{\frac{d}{dx}\frac{d}{dx} u_1 = \left( 1 \cdot \sin(\lambda \cdot x) \cdot C_1 \cdot \cos(\lambda \cdot x) \cdot C_2 \right) \cdot 2 \cdot e^{\lambda \cdot x} \cdot \lambda^2}$$

[B36] $$\boxed{\frac{d}{dx}\frac{d}{dx}\frac{d}{dx} u_1 = \left[ ( 1 \cdot \cos(\lambda \cdot x) \quad 1 \cdot \sin(\lambda \cdot x)) \cdot C_1 - ( 1 \cdot \sin(\lambda \cdot x) - \cos(\lambda \cdot x)) \cdot C_2 \right] \cdot 2 \cdot e^{\lambda \cdot x} \cdot \lambda^3}$$

Region 2 ($0 < x < W/2$):

[B37] $$\boxed{u_2 = e^{\lambda \cdot x} \cdot \left( k_1 \cdot \cos(\lambda \cdot x) - k_2 \cdot \sin(\lambda \cdot x) \right) - e^{\lambda \cdot x} \cdot \left( k_3 \cdot \cos(\lambda \cdot x) - k_4 \cdot \sin(\lambda \cdot x) \right) - \delta}$$

[B38] $$\boxed{\begin{aligned} \frac{d}{dx} u_2 = \Big[ & \left[ (\sin(\lambda \cdot x) - \cos(\lambda \cdot x)) \cdot k_1 - (\sin(\lambda \cdot x) - \cos(\lambda \cdot x)) \cdot k_2 \right] \cdot e^{\lambda \cdot x} \ldots \\ & + \left[ (\cos(\lambda \cdot x) - \sin(\lambda \cdot x)) \cdot k_3 - (\sin(\lambda \cdot x) - \cos(\lambda \cdot x)) \cdot k_4 \right] \cdot e^{\lambda \cdot x} \Big] \cdot \lambda \end{aligned}}$$

[B39] $$\boxed{\frac{d}{dx}\frac{d}{dx} u_2 = \left[ \left( 1 \cdot \sin(\lambda \cdot x) \cdot k_1 - \cos(\lambda \cdot x) \cdot k_2 \right) \cdot e^{\lambda \cdot x} - \left( \sin(\lambda \cdot x) \cdot k_3 \quad \cos(\lambda \cdot x) \cdot k_4 \right) \cdot e^{\lambda \cdot x} \right] \cdot 2 \cdot \lambda^2}$$

[B40] $$\frac{d}{dx}\frac{d}{dx}\frac{d}{dx}u_2 = \left[(1\cdot\cos(\lambda\cdot x) - \sin(\lambda\cdot x))\cdot k_1 - (\cos(\lambda\cdot x) - \sin(\lambda\cdot x))\cdot k_2\right]\cdot e^{\lambda\cdot x} \ldots \cdot 2\cdot\lambda^3 \\ + \left[(\cos(\lambda\cdot x) \; \sin(\lambda\cdot x))\cdot k_3 - (\sin(\lambda\cdot x) - \cos(\lambda\cdot x))\cdot k_4\right]\cdot e^{-\lambda\cdot x}$$

Applying the following boundary conditions:

At x = 0, u, u', u'', u''' are continuous;

At x = W/2, u'=0;

Force equilibrium in the transverse direction:

We obtain the following 6 equations:

[B41] $$C_1 = k_1 - k_3 - \delta$$

[B42] $$k_1 \cdot k_2 \; k_3 \cdot k_4 = C_1 \cdot C_2$$

[B43] $$k_2 \; k_4 = C_2$$

[B44] $$-k_1 - k_2 - k_3 - k_4 = C_1 - C_2$$

[B45] $$0 = \left[\left(\sin\left(\frac{1}{2}\cdot\lambda\cdot W\right) \cdot \cos\left(\frac{1}{2}\cdot\lambda\cdot W\right)\right)\cdot k_1 \cdot \left(\sin\left(\frac{1}{2}\cdot\lambda\cdot W\right) \cdot \cos\left(\frac{1}{2}\cdot\lambda\cdot W\right)\right)\cdot k_2\right]\cdot e^{\lambda\cdot W} \ldots \\ + \left(\cos\left(\frac{1}{2}\cdot\lambda\cdot W\right) \; \sin\left(\frac{1}{2}\cdot\lambda\cdot W\right)\right)\cdot k_3 - \left(\sin\left(\frac{1}{2}\cdot\lambda\cdot W\right) - \cos\left(\frac{1}{2}\cdot\lambda\cdot W\right)\right)\cdot k_4$$

[B46] $$0 = \left(\sin\left(\frac{1}{2}\cdot\lambda\cdot W\right) - \cos\left(\frac{1}{2}\cdot\lambda\cdot W\right)\cdot e^{\frac{1}{2}\cdot\lambda\cdot W} - 1\right)\cdot k_1 \ldots \\ + \left(\cos\left(\frac{1}{2}\cdot\lambda\cdot W\right) \; \sin\left(\frac{1}{2}\cdot\lambda\cdot W\right)\cdot e^{\frac{1}{2}\cdot\lambda\cdot W} - 1\right)\cdot k_2 \ldots \\ + \left(\cos\left(\frac{1}{2}\cdot\lambda\cdot W\right) - \sin\left(\frac{1}{2}\cdot\lambda\cdot W\right)\cdot e^{\frac{1}{2}\cdot\lambda\cdot W} - 1\right)\cdot k_3 \ldots \\ + \left(\sin\left(\frac{1}{2}\cdot\lambda\cdot W\right) - \cos\left(\frac{1}{2}\cdot\lambda\cdot W\right)\cdot e^{\frac{1}{2}\cdot\lambda\cdot W} - 1\right)\cdot k_4 - C_1 - C_2$$

Solving for the 6 unknown constants, we get

[B47] $\boxed{k_1 = C_1 - \frac{1}{2}\cdot\delta}$

[B48] $\boxed{k_2 = C_2}$

[B49] $\boxed{k_3 = \frac{1}{2}\cdot\delta}$

[B50] $\boxed{k_4 = 0}$

[B51] $$\boxed{C_1 = \frac{1}{2}\cdot\frac{\sin\left(\frac{1}{2}\cdot\lambda\cdot W\right)^2 + \cos\left(\frac{1}{2}\cdot\lambda\cdot W\right)^2}{\sin\left(\frac{1}{2}\cdot\lambda\cdot W\right)^2 - \cos\left(\frac{1}{2}\cdot\lambda\cdot W\right)^2}\cdot e^{\lambda\cdot W} - \frac{1}{2}\cdot\delta}$$

[B52] $$\boxed{C_2 = \sin\left(\frac{1}{2}\cdot\lambda\cdot W\right)\cdot e^{\lambda\cdot W}\cdot\frac{\cos\left(\frac{1}{2}\cdot\lambda\cdot W\right)}{\sin\left(\frac{1}{2}\cdot\lambda\cdot W\right)^2 - \cos\left(\frac{1}{2}\cdot\lambda\cdot W\right)^2}\cdot\delta}$$

# B.6 Pipeline equations for $\delta > 2D_s$ with finite landslide width (see Figure B1)

Region 1 ($-\infty < x < A$)

[B53] $$\frac{d}{dx}\frac{d}{dx}\frac{d}{dx}\frac{d}{dx}u_1 \cdot 0 = \frac{k\cdot d\cdot u_1 \cdot 0}{E\cdot I}$$

[B54] $$u_1 = e^{\lambda\cdot x}\cdot C_1\cdot\cos(\lambda\cdot x)\cdot C_2\cdot\sin(\lambda\cdot x) - e^{\lambda\cdot x}\cdot C_3\cdot\cos(\lambda\cdot x)\cdot C_4\cdot\sin(\lambda\cdot x)$$

At $x = -\infty$, therefore $C_3$ and $C_4$ are both zero:

[B55] $$u_1 = e^{\lambda\cdot x}\cdot C_1\cdot\cos(\lambda\cdot x) - C_2\cdot\sin(\lambda\cdot x)$$

[B56] $$\frac{d}{dx}u_1 = (\cos(\lambda\cdot x)\quad\sin(\lambda\cdot x))\cdot C_1 - (\sin(\lambda\cdot x) - \cos(\lambda\cdot x))\cdot C_2\cdot e^{\lambda\cdot x}\cdot\lambda$$

[B57] $$\frac{d}{dx}\frac{d}{dx}u_1 = 1\cdot\sin(\lambda\cdot x)\cdot C_1 - \cos(\lambda\cdot x)\cdot C_2\cdot 2\cdot e^{\lambda\cdot x}\cdot\lambda^2$$

[B58] $$\frac{d}{dx}\frac{d}{dx}\frac{d}{dx}u_1 = (1\cdot\cos(\lambda\cdot x)\quad 1\cdot\sin(\lambda\cdot x))\cdot C_1\cdot(1\cdot\sin(\lambda\cdot x) - \cos(\lambda\cdot x))\cdot C_2\cdot 2\cdot e^{\lambda\cdot x}\cdot\lambda^3$$

Region 2 ($A < x < 0$):

[B59] $$\frac{d}{dx}\frac{d}{dx}\frac{d}{dx}\frac{d}{dx}u_2 - 0 = \frac{k\cdot d\cdot u_2 - 0}{E\cdot I} = \frac{R}{E\cdot I}$$

[B60] $$u_2 = \frac{1}{E\cdot I}\cdot\frac{R}{24}\cdot x^4 - \frac{C_5}{6}\cdot x^3 - \frac{C_6}{2}\cdot x^2 - C_7 x - C_8$$

[B61] $\frac{d}{dx}u_2 = \frac{1}{E\cdot I}\cdot -\frac{1}{6}\cdot R\cdot x^3 - \frac{1}{2}\cdot C_5\cdot x^2 - C_6\cdot x - C_7$

[B62] $\frac{d}{dx}\frac{d}{dx}u_2 = \frac{1}{E\cdot I}\cdot -\frac{1}{2}\cdot R\cdot x^2 - C_5\cdot x - C_6$

[B63] $\frac{d}{dx}\frac{d}{dx}\frac{d}{dx}u_2 = \frac{1}{E\cdot I}\cdot R\cdot x - C_5$

Region 3 ($0 < x < B$):

[B64] $\frac{d}{dx}\frac{d}{dx}\frac{d}{dx}\frac{d}{dx}u_3 - \delta + \frac{k\cdot d\cdot u_3 - \delta}{E\cdot I} = \frac{R}{E\cdot I}$

[B65] $u_3 = \frac{1}{E\cdot I}\cdot \frac{R}{24}\cdot x^4 - \frac{k_1}{6}\cdot x^3 - \frac{k_2}{2}\cdot x^2 - k_3\cdot x - k_4$

[B66] $\frac{d}{dx}u_3 = \frac{1}{E\cdot I}\cdot \frac{1}{6}\cdot R\cdot x^3 - \frac{1}{2}\cdot k_1\cdot x^2 - k_2\cdot x - k_3$

[B67] $\frac{d}{dx}\frac{d}{dx}u_3 = \frac{1}{E\cdot I}\cdot \frac{1}{2}\cdot R\cdot x^2 - k_1\cdot x - k_2$

[B68] $\frac{d}{dx}\frac{d}{dx}\frac{d}{dx}u_3 = \frac{1}{E\cdot I}\cdot R\cdot x - k_1$

Region 4 ($B < x < W/2$):

[B69] $\frac{d}{dx}\frac{d}{dx}\frac{d}{dx}\frac{d}{dx}u_4 - \delta = \frac{k\cdot d\cdot u_4 - \delta}{E\cdot I}$

[B70] $u_4 = e^{\lambda\cdot x}\cdot k_5\cdot\cos(\lambda\cdot x) - k_6\cdot\sin(\lambda\cdot x) - e^{\lambda\cdot x}\cdot k_7\cdot\cos(\lambda\cdot x) - k_8\cdot\sin(\lambda\cdot x) - \delta$

[B71]
$$\boxed{\frac{d}{dx}u_4 = \left[\left[(\sin(\lambda\cdot x) - \cos(\lambda\cdot x))\cdot k_5 - (\sin(\lambda\cdot x) - \cos(\lambda\cdot x))\cdot k_6\right]\cdot e^{\lambda\cdot x} \ldots \atop + \left[(\cos(\lambda\cdot x) - \sin(\lambda\cdot x))\cdot k_7 - (\sin(\lambda\cdot x) - \cos(\lambda\cdot x))\cdot k_8\right]\cdot e^{\lambda\cdot x}\right]\cdot\lambda}$$

[B72]
$$\boxed{\frac{d}{dx}\frac{d}{dx}u_4 = \left[\left(\sin(\lambda\cdot x)\cdot k_5 - \cos(\lambda\cdot x)\cdot k_6\right)\cdot e^{\lambda\cdot x} - \left(\sin(\lambda\cdot x)\cdot k_7 - \cos(\lambda\cdot x)\cdot k_8\right)\cdot e^{\lambda\cdot x}\right]\cdot 2\cdot\lambda^2}$$

[B73]
$$\boxed{\frac{d}{dx}\frac{d}{dx}\frac{d}{dx}u_4 = \left[\left[(\cos(\lambda\cdot x) \quad \sin(\lambda\cdot x))\cdot k_5 - (\cos(\lambda\cdot x) \quad \sin(\lambda\cdot x))\cdot k_6\right]\cdot e^{\lambda\cdot x} \ldots \atop + \left[(\cos(\lambda\cdot x) \quad \sin(\lambda\cdot x))\cdot k_7 - (\sin(\lambda\cdot x) - \cos(\lambda\cdot x))\cdot k_8\right]\cdot e^{\lambda\cdot x}\right]\cdot 2\cdot\lambda^3}$$

Apply the displacement boundary condition at x = A:

[B74]
$$\boxed{D_s = e^{\lambda\cdot B}\cdot\left(C_1\cdot\cos(\lambda\cdot A) - C_2\cdot\sin(\lambda\cdot A)\right)}$$

Apply the displacement boundary condition at x = B:

[B75]
$$\boxed{\delta \quad D_s = e^{\lambda\cdot B}\cdot\left(k_5\cdot\cos(\lambda\cdot B) - k_6\cdot\sin(\lambda\cdot B)\right) - e^{\lambda\cdot B}\cdot\left(k_7\cdot\cos(\lambda\cdot B) - k_8\cdot\sin(\lambda\cdot B)\right) - \delta}$$

Apply the strain boundary condition at x = W/2 (middle of landslide):

[B77]
$$\boxed{\frac{d}{dx}u_4\left(\frac{W}{2}\right) = 0 = \left[\left[\left(\sin\left(\lambda\cdot\frac{W}{2}\right) - \cos\left(\lambda\cdot\frac{W}{2}\right)\right)\cdot k_5 - \left(\sin\left(\lambda\cdot\frac{W}{2}\right) - \cos\left(\lambda\cdot\frac{W}{2}\right)\right)\cdot k_6\right]\cdot e^{\frac{1}{2}\cdot\lambda\cdot W} \ldots \atop + \left[\left(\cos\left(\lambda\cdot\frac{W}{2}\right) \quad \sin\left(\lambda\cdot\frac{W}{2}\right)\right)\cdot k_7 - \left(\sin\left(\lambda\cdot\frac{W}{2}\right) - \cos\left(\lambda\cdot\frac{W}{2}\right)\right)\cdot k_8\right]\cdot e^{\frac{1}{2}\cdot\lambda\cdot W}\right]\cdot\lambda}$$

Apply the force equilibrium condition for the 4 regions:

[B78]
$$\boxed{F = \int k_s\cdot\pi\cdot d\cdot\left(u_{soil} - u_{pipe}\right)\,dx}$$

[B79]
$$\boxed{0 = F1 - F2 - F3 - F4}$$

[B80]
$$\begin{aligned} 0 = & \,(\cos(\lambda\cdot A) - \sin(\lambda\cdot A))\cdot C_1 - (\cos(\lambda\cdot A) - \sin(\lambda\cdot A))\cdot C_2 \cdot e^{\lambda\cdot A} \quad D_s\cdot(A - B)\cdot 2\cdot\lambda \ldots \\ & + 1\cdot \quad (\cos(\lambda\cdot B) - \sin(\lambda\cdot B))\cdot k_5 - (\cos(\lambda\cdot B) - \sin(\lambda\cdot B))\cdot k_6 \cdot e^{\lambda\cdot B} \ldots \\ & + \cos\left(\frac{1}{2}\cdot\lambda\cdot W\right) - \sin\left(\frac{1}{2}\cdot\lambda\cdot W\right)\cdot k_5 - \sin\left(\frac{1}{2}\cdot\lambda\cdot W\right) - \cos\left(\frac{1}{2}\cdot\lambda\cdot W\right)\cdot k_6 \cdot e^{\frac{1}{2}\cdot\lambda\cdot W} \ldots \\ & + (\cos(\lambda\cdot B) \cdot \sin(\lambda\cdot B))\cdot k_7 - (\sin(\lambda\cdot B) \quad \cos(\lambda\cdot B))\cdot k_8 \cdot e^{\lambda\cdot B} \ldots \\ & + \sin\left(\frac{1}{2}\cdot\lambda\cdot W\right) - \cos\left(\frac{1}{2}\cdot\lambda\cdot W\right)\cdot k_7 - \cos\left(\frac{1}{2}\cdot\lambda\cdot W\right) - \sin\left(\frac{1}{2}\cdot\lambda\cdot W\right)\cdot k_8 \cdot e^{\frac{1}{2}\cdot\lambda\cdot W} \end{aligned}$$

Apply the continuity condition at $x = A$ – where $u_1 = u_2$, $u_1' = u_2'$, $u_1'' = u_2''$ and $u_1''' = u_2'''$. Four unknown constants are removed:

[B81] $C_8 = k_4$

[B82] $C_7 = k_3$

[B83] $C_6 = k_2$

[B84] $C_5 = k_1$

Applying the other 2 continuity boundary conditions at x = zero and x = B, we obtain 12 equations for 12 unknowns

[B85]
$$0 = e^{\lambda\cdot A}\cdot\left(C_1\cdot\cos(\lambda\cdot A) - C_2\cdot\sin(\lambda\cdot A)\right) - \frac{1}{E\cdot I}\cdot\frac{1}{24}\cdot R\cdot A^4 - \frac{1}{6}\cdot k_1\cdot A^3 - \frac{1}{2}\cdot k_2\cdot A^2 - k_3\cdot A - k_4$$

[B86]
$$\begin{aligned} 0 = & \,(\cos(\lambda\cdot A) \quad \sin(\lambda\cdot A))\cdot C_1 - (\sin(\lambda\cdot A) - \cos(\lambda\cdot A))\cdot C_2 \cdot e^{\lambda\cdot A}\cdot\lambda \ldots \\ & + \frac{1}{E\cdot I}\cdot\frac{1}{6}\cdot R\cdot A^3 - \frac{1}{2}\cdot k_1\cdot A^2 - k_2\cdot A - k_3 \end{aligned}$$

[B87]
$$0 = \left(1\cdot\sin(\lambda\cdot A)\cdot C_1 - \cos(\lambda\cdot A)\cdot C_2\right)\cdot 2\cdot e^{\lambda\cdot A}\cdot\lambda^2 \quad \frac{1}{E\cdot I}\cdot\frac{1}{2}\cdot R\cdot A^2 - k_1\cdot A - k_2$$

[B88]
$$0 = (\cos(\lambda\cdot A) \quad \sin(\lambda\cdot A))\cdot C_1 - (\cos(\lambda\cdot A) - \sin(\lambda\cdot A))\cdot C_2 \cdot 2\cdot e^{\lambda\cdot A}\cdot\lambda^3 \quad \frac{1}{E\cdot I}\cdot R\cdot A - k_1$$

[B89] $$\boxed{\frac{1}{E\cdot I}\cdot\frac{1}{24}\cdot R\cdot A^4-\frac{1}{6}\cdot k_1\cdot A^3-\frac{1}{2}\cdot k_2\cdot A^2-k_3\cdot A-k_4=D_s}$$

[B90] $$0=\frac{1}{E\cdot I}\cdot\frac{1}{24}\cdot R\cdot B^4-\frac{1}{6}\cdot k_1\cdot B^3\ldots+\frac{1}{2}\cdot k_2\cdot B^2-k_3\cdot B-k_4\quad-e^{\lambda\cdot B}\cdot k_5\cdot\cos(\lambda\cdot B)-k_6\cdot\sin(\lambda\cdot B)\ldots+e^{\lambda\cdot B}\cdot k_7\cdot\cos(\lambda\cdot B)-k_8\cdot\sin(\lambda\cdot B)-\delta$$

[B91] $$0=\frac{1}{E\cdot I}\cdot\frac{1}{6}\cdot R\cdot B^3-\frac{1}{2}\cdot k_1\cdot B^2-k_2\cdot B-k_3\quad(\sin(\lambda\cdot B)\cdot\cos(\lambda\cdot B))\cdot k_5\ldots+(\sin(\lambda\cdot B)-\cos(\lambda\cdot B))\cdot k_6\cdot e^{\lambda\cdot B}\ldots+(\cos(\lambda\cdot B)\quad\sin(\lambda\cdot B))\cdot k_7\ldots+(\sin(\lambda\cdot B)-\cos(\lambda\cdot B))\cdot k_8\cdot e^{\lambda\cdot B}\cdot\lambda$$

[B92] $$0=\frac{1}{E\cdot I}\cdot\frac{1}{2}\cdot R\cdot B^2-k_1\cdot B-k_2\quad\sin(\lambda\cdot B)\cdot k_5-\cos(\lambda\cdot B)\cdot k_6\cdot e^{\lambda\cdot B}\ldots+\sin(\lambda\cdot B)\cdot k_7\quad\cos(\lambda\cdot B)\cdot k_8\cdot e^{\lambda\cdot B}\cdot 2\cdot\lambda^2$$

[B93] $$0=\frac{1}{E\cdot I}\cdot R\cdot B\cdot k_1\quad(\cos(\lambda\cdot B)\quad\sin(\lambda\cdot B))\cdot k_5\cdot(\sin(\lambda\cdot B)\cdot\cos(\lambda\cdot B))\cdot k_6\cdot e^{\lambda\cdot B}\ldots+(\sin(\lambda\cdot B)\cdot\cos(\lambda\cdot B))\cdot k_7\cdot(\sin(\lambda\cdot B)\cdot\cos(\lambda\cdot B))\cdot k_8\cdot e^{\lambda\cdot B}\cdot 2\cdot\lambda^3$$

[B94] $$\boxed{\frac{1}{E\cdot I}\cdot\frac{1}{24}\cdot R\cdot B^4-\frac{1}{6}\cdot k_1\cdot B^3-\frac{1}{2}\cdot k_2\cdot B^2-k_3\cdot B-k_4=\delta\quad D_s}$$

[B95] $$0=\sin\left(\frac{1}{2}\cdot\lambda\cdot W\right)-\cos\left(\frac{1}{2}\cdot\lambda\cdot W\right)\cdot k_5\cdot\sin\left(\frac{1}{2}\cdot\lambda\cdot W\right)-\cos\left(\frac{1}{2}\cdot\lambda\cdot W\right)\cdot k_6\cdot e^{\frac{1}{2}\cdot\lambda\cdot W}\ldots+\cos\left(\frac{1}{2}\cdot\lambda\cdot W\right)\quad\sin\left(\frac{1}{2}\cdot\lambda\cdot W\right)\cdot k_7\cdot\sin\left(\frac{1}{2}\cdot\lambda\cdot W\right)-\cos\left(\frac{1}{2}\cdot\lambda\cdot W\right)\cdot k_8\cdot e^{\frac{1}{2}\cdot\lambda\cdot W}$$

[B96] $$\begin{aligned}0=\ &(\cos(\lambda\cdot A)-\sin(\lambda\cdot A))\cdot C_1-(\cos(\lambda\cdot A)-\sin(\lambda\cdot A))\cdot C_2\cdot e^{\lambda\cdot A}\quad D_s\cdot(A-B)\cdot 2\cdot\lambda\ldots\\&+1\cdot(\cos(\lambda\cdot B)-\sin(\lambda\cdot B))\cdot k_5-(\cos(\lambda\cdot B)-\sin(\lambda\cdot B))\cdot k_6\cdot e^{\lambda\cdot B}\ldots\\&+\cos\left(\frac{1}{2}\cdot\lambda\cdot W\right)-\sin\left(\frac{1}{2}\cdot\lambda\cdot W\right)\cdot k_5-\sin\left(\frac{1}{2}\cdot\lambda\cdot W\right)-\cos\left(\frac{1}{2}\cdot\lambda\cdot W\right)\cdot k_6\cdot e^{\frac{1}{2}\cdot\lambda\cdot W}\ldots\\&+(\cos(\lambda\cdot B)-\sin(\lambda\cdot B))\cdot k_7-(\sin(\lambda\cdot B)\quad\cos(\lambda\cdot B))\cdot k_8\cdot e^{\lambda\cdot B}\ldots\\&+\sin\left(\frac{1}{2}\cdot\lambda\cdot W\right)-\cos\left(\frac{1}{2}\cdot\lambda\cdot W\right)\cdot k_7-\cos\left(\frac{1}{2}\cdot\lambda\cdot W\right)-\sin\left(\frac{1}{2}\cdot\lambda\cdot W\right)\cdot k_8\cdot e^{\frac{1}{2}\cdot\lambda\cdot W}\end{aligned}$$

These 12 equations can be solved numerically by using Newton-Raphson Method for Nonlinear Systems of Equations with LU decomposition and back substitution.

[B97] $$\boxed{f_i(x+\delta x) = f_i(x) + \sum_{j=1}^{N} \frac{\partial f_i}{\partial x_j} \cdot \delta x_j + O'(dx^4)}$$

Richardson's extrapolation are used to obtain the first derivatives of each equation:

[B98] $$\boxed{f'(x) = \frac{4}{3} \cdot \phi\left(\frac{dx}{2}\right) - \frac{1}{3} \cdot \phi(dx) - O'(dx^4)}$$

where

[B99] $$\boxed{\phi(dx) = \frac{1}{2 \cdot dx} \cdot \left[f(x+dx) - f(x-dx)\right]}$$

See EPPE3.BAS program in appendix D for the implementation of Newton-Raphson method of solving system of nonlinear equations.

## B.7 Pipeline equations for $\delta < 2D_s$ with infinite landslide

We can reduce equation [B24] for the general pipe deflection profile, in the region from negative infinity to zero, by applying the boundary conditions at $x = -\infty$ ($C_3$ and $C_4$ equal 0).

[B100] $\boxed{u = e^{\lambda \cdot x} \cdot \left(C_1 \cdot \cos(\lambda \cdot x) - C_2 \cdot \sin(\lambda \cdot x)\right)}$

[B101] $\boxed{\frac{d}{dx} u = \left[\left(\cos(\lambda \cdot x) \cdot \lambda - \sin(\lambda \cdot x) \cdot \lambda\right) \cdot C_1 - \left(\sin(\lambda \cdot x) \cdot \lambda - \cos(\lambda \cdot x) \cdot \lambda\right) \cdot C_2\right] \cdot e^{\lambda \cdot x}}$

[B102] $\boxed{\frac{d}{dx}\frac{d}{dx} u = -2 \cdot \left(C_1 \cdot \sin(\lambda \cdot x) \cdot \lambda^2 + C_2 \cdot \cos(\lambda \cdot x) \cdot \lambda^2\right) \cdot e^{\lambda \cdot x}}$

Since there would be a double curvature for infinite width landslides, the bending moment at the interface between the stable and unstable soil ($x = 0$) would be zero. Thus from equation [B102] $C_2 = 0$.

[B103] $\boxed{u = e^{\lambda \cdot x} \cdot C_1 \cdot \cos(\lambda \cdot x)}$

The pipe displacement at $x = 0$ would be half of the landslide movement ($\delta$)

[B104] $\boxed{u = \frac{\delta}{2} \cdot e^{\lambda \cdot x} \cdot \cos(\lambda \cdot x)}$

[B105] $\boxed{\frac{d}{dx} u = \lambda \cdot \frac{\delta}{2} \cdot e^{\lambda \cdot x} \cdot \left(\cos(\lambda \cdot x) - \sin(\lambda \cdot x)\right)}$

[B106a] $\boxed{\left(\frac{d}{dx} u\right)^2 = \lambda^2 \cdot \frac{\delta^2}{4} \cdot e^{2 \cdot \lambda \cdot x} \cdot \left(1 - 2 \cdot \cos(\lambda \cdot x) \cdot \sin(\lambda \cdot x)\right)}$

[B106b] $$\boxed{\frac{d}{dx}u^{2} = \lambda^{2}\cdot\frac{\delta^{2}}{4}\cdot e^{2\cdot\lambda\cdot x}\cdot(1 \quad \sin(2\cdot\lambda\cdot x))}$$

[B107] $$\boxed{\frac{d}{dx}\frac{d}{dx}u = -\delta\cdot\sin(\lambda\cdot x)\cdot\lambda^{2}\cdot e^{\lambda\cdot x}}$$

From the general tensile strain and bending strain equations [B31] and [B32], we obtain the following strain equation for a pipeline in an infinite width transverse landslide for $\delta < 2D_s$:

[B108] $$\boxed{\varepsilon = \frac{d}{2}\cdot \quad \delta\cdot\sin(\lambda\cdot x)\cdot\lambda^{2}\cdot e^{\lambda\cdot x} \quad - \lambda^{2}\cdot\frac{\delta^{2}}{4}\cdot e^{2\cdot\lambda\cdot x}\cdot(1 \quad \sin(2\cdot\lambda\cdot x))}$$

Note that equation [B108] is derived from $x = -\infty$ to $x = 0$, but since infinite width landslide is assume, the strain values obtained are equivalent for the region from $x = 0$ to $x = \infty$.

## B.8 Pipeline equations for $\delta > 2D_s$ with infinite landslide (see Figure B.2)

Region 1 ($0 < x < B$)

[B109] $$\boxed{\frac{d}{dx}\frac{d}{dx}\frac{d}{dx}\frac{d}{dx}u_1 = \frac{R_f}{(E \cdot I)}}$$

[B110] $$\boxed{\frac{d}{dx}\frac{d}{dx}\frac{d}{dx}u_1 = \frac{R_f \cdot x - C_5}{(E \cdot I)}}$$

[B111] $$\boxed{\frac{d}{dx}\frac{d}{dx}u_1 = \frac{\frac{1}{2} \cdot R_f \cdot x^2 - C_5 \cdot x - C_6}{(E \cdot I)}}$$

[B112] $$\boxed{\frac{d}{dx}u_1 = \frac{\frac{1}{6} \cdot R_f \cdot x^3 - \frac{1}{2} \cdot C_5 \cdot x^2 - C_6 \cdot x - C_7}{(E \cdot I)}}$$

[B113] $$\boxed{u_1 = \frac{\frac{R_f}{24} \cdot x^4 - \frac{C_5}{6} \cdot x^3 - \frac{C_6}{2} \cdot x^2 - C_7 \cdot x - C_8}{(E \cdot I)}}$$

At $x = 0$, the pipe displacement is half that of the landslide displacement ($\delta$)

[B114] $\boxed{C_8 = 0}$

At $x = 0$, double curvature exists and the bending moment is zero

[B115] $\boxed{C_6 = 0}$

Applying equations [B114] and [B115], we have the following equations for the pipe in region 1:

[B116] $$\boxed{u_1 = \frac{\frac{R_f}{24} \cdot x^4 - \frac{C_5}{6} \cdot x^3 - C_7 \cdot x - \frac{\delta}{2} \cdot E \cdot I}{E \cdot I}}$$

[B117] $$\frac{d}{dx}u_1 = \frac{\frac{1}{6}\cdot R_f\cdot x^3 - \frac{1}{2}\cdot C_5\cdot x^2 - C_7}{(E\cdot I)}$$

[B118] $$\frac{d}{dx}\frac{d}{dx}u_1 = \frac{\frac{1}{2}\cdot R_f\cdot x^2 - C_5\cdot x}{(E\cdot I)}$$

[B119] $$\frac{d}{dx}\frac{d}{dx}\frac{d}{dx}u_1 = \frac{R_f\cdot x \cdot C_5}{(E\cdot I)}$$

Region 2 (B <x < ∞)

[B120] $$\begin{aligned} u_2 &= e^{\lambda x}\cdot\left[(C_1\cdot\cos(\lambda\cdot x) + C_2\cdot\sin(\lambda\cdot x))\right] + \cdots \\ &\cdots + e^{-\lambda x}\cdot\left[C_3\cdot\cos(\lambda\cdot x) + C_4\cdot\sin(\lambda\cdot x))\right] \end{aligned}$$

At x = ∞, pipe displacement is zero, and so $C_1$ and $C_2$ are zero

[B121] $$u_2 = \frac{C_3\cdot\cos(\lambda\cdot x) - C_4\cdot\sin(\lambda\cdot x)}{\exp(\lambda\cdot x)}$$

[B122] $$\frac{d}{dx}u_2 = ((\ \sin(\lambda\cdot x) - \cos(\lambda\cdot x))\cdot C_4 - (\ \cos(\lambda\cdot x)\ \ \sin(\lambda\cdot x))\cdot C_3\cdot\frac{\lambda}{\exp(\lambda\cdot x)}$$

[B123] $$\frac{d}{dx}\frac{d}{dx}u_2 = 2\cdot\lambda^2\cdot\frac{C_3\cdot\sin(\lambda\cdot x)\ \ C_4\cdot\cos(\lambda\cdot x)}{\exp(\lambda\cdot x)}$$

[B124] $$\frac{d}{dx}\frac{d}{dx}\frac{d}{dx}u_2 = ((\sin(\lambda\cdot x) - \cos(\lambda\cdot x))\cdot C_4 - (\cos(\lambda\cdot x) - \sin(\lambda\cdot x))\cdot C_3\cdot\frac{\lambda^3}{\exp(\lambda\cdot x)}\cdot 2$$

At x = B, the continuity boundary conditions for u, u', u'', and u''' are applied. In addition, the pipe displacement is equal to $D_s$ at x = B. The remaining constants of the pipe equation are solved:

[B125]

$$C_5 = \frac{1}{2}\cdot\frac{\lambda\cdot B^2\cdot R_f - 2\cdot R_f\cdot B - 4\cdot E\cdot I\cdot\lambda^3\cdot D_s}{(\lambda\cdot B - 1)}$$

[B126]

$$C_3 = \left[\begin{aligned}&\sin(\lambda\cdot B)\cdot B^2\cdot R_f \ldots\\ &+(\sin(\lambda\cdot B) - \cos(\lambda\cdot B))\cdot 4\cdot E\cdot I\cdot\lambda^3\cdot D_s\cdot B \ldots\\ &+4\cdot E\cdot I\cdot\lambda^2\cdot D_s\cdot\cos(\lambda\cdot B)\end{aligned}\right]\cdot\frac{\exp(\lambda\cdot B)}{(\lambda\cdot B - 1)\cdot(E\cdot I)\cdot 4\cdot\lambda^2}$$

[B127]

$$C_4 = \left[\begin{aligned}&\cos(\lambda\cdot B)\cdot R_f\cdot B^2 \ldots\\ &+(\cos(\lambda\cdot B) - \sin(\lambda\cdot B))\cdot 4\cdot E\cdot I\cdot\lambda^3\cdot D_s\cdot B \ldots\\ &+4\cdot E\cdot I\cdot\lambda^2\cdot D_s\cdot\sin(\lambda\cdot B)\end{aligned}\right]\cdot\frac{\exp(\lambda\cdot B)}{(\lambda\cdot B - 1)\cdot(E\cdot I)\cdot 4\cdot\lambda^2}$$

[B128]

$$C_7 = \frac{R_f\cdot B^5\cdot\lambda - 3\cdot R_f\cdot B^4 - 8\cdot B^3\cdot E\cdot I\cdot\lambda^3\cdot D_s - \left(D_s - \frac{\delta}{2}\right)\cdot 24\cdot B\cdot E\cdot I\cdot\lambda - \left(D_s - \frac{\delta}{2}\right)\cdot 24\cdot E\cdot I}{((\lambda\cdot B - 1)\cdot B\cdot 24)}$$

[B129]

$$\begin{aligned}0 = {} & R_f\cdot B^5\cdot\lambda^2 - 5\cdot\lambda\cdot R_f\cdot B^4 - (6\cdot R_f - 16\cdot E\cdot I\cdot\lambda^4\cdot D_s)\cdot B^3 - 48\cdot E\cdot I\cdot\lambda^3\cdot D_s\cdot B^2 \ldots\\ & + \left(24\cdot\frac{\delta}{2} - 48\cdot D_s\right)\cdot B\cdot E\cdot I\cdot\lambda^2 - \left(D_s - \frac{\delta}{2}\right)\cdot 24\cdot E\cdot I\cdot\lambda\end{aligned}$$

From the general tensile and bending strain equation [B31] and [B32], the total strain on the pipeline in the plastic and elastic soil regions are obtained:

[B130]

$$\varepsilon_p = \frac{d}{2}\cdot\left[\begin{aligned}&\frac{1}{2}\cdot x\cdot\frac{\lambda\cdot B^2\cdot R_f - (2 - \lambda\cdot x)\cdot R_f\cdot B - 4\cdot E\cdot I\cdot\lambda^3\cdot D_s - R_f\cdot x}{((\lambda\cdot B - 1)\cdot(E\cdot I))} \ldots\\ &+\frac{1}{1152}\cdot\frac{\begin{aligned}&(B^5\cdot R_f\cdot\lambda - 3\cdot R_f\cdot B^4 - 6\cdot x^2\cdot\lambda\cdot R_f - 8\cdot E\cdot I\cdot\lambda^3\cdot D_s)\cdot B^3 \ldots\\ &+(12\cdot x^2\cdot R_f - 4\cdot R_f\cdot x^3\cdot\lambda)\cdot B^2 \ldots\\ &+\left(24\cdot x^2\cdot E\cdot I\cdot\lambda^3\cdot D_s - 4\cdot R_f\cdot x^3 - 24\cdot E\cdot I\cdot\lambda\cdot D_s - 24\cdot E\cdot I\cdot\lambda\cdot\frac{\delta}{2}\right)\cdot B \ldots\\ &+24\cdot E\cdot I\cdot D_s - 24\cdot E\cdot I\cdot\frac{\delta}{2}\end{aligned}}{((\lambda\cdot B - 1)\cdot(B\cdot(E\cdot I)))}\end{aligned}\right]^2$$

[B131]

$$
\begin{aligned}
\varepsilon_e = \frac{d}{2}\cdot \Bigg[ & \bigg[ (\sin(\lambda\cdot B)\cdot\sin(\lambda\cdot x) - \cos(\lambda\cdot B)\cdot\cos(\lambda\cdot x))\cdot R_f\cdot\frac{B^2}{4} \ldots \\
& + \begin{pmatrix} (\sin(\lambda\cdot B) - \cos(\lambda\cdot B))\cdot\sin(\lambda\cdot x) \ldots \\ +(\cos(\lambda\cdot B) - \sin(\lambda\cdot B))\cdot\cos(\lambda\cdot x) \end{pmatrix} \cdot E\cdot I\cdot\lambda^3\cdot D_s\cdot B \ldots \\
& + (\sin(\lambda\cdot x)\cdot\cos(\lambda\cdot B) \quad \cos(\lambda\cdot x)\cdot\sin(\lambda\cdot B))\cdot E\cdot I\cdot\lambda^2\cdot D_s \bigg] \cdot \frac{e^{\lambda\cdot B}\cdot 2}{((\lambda\cdot B - 1)\cdot(E\cdot I))\cdot e^{\lambda\cdot x}} \ldots \Bigg] \\
& + \frac{1}{2}\cdot \frac{\left[\begin{aligned} & \begin{pmatrix} (\cos(\lambda\cdot B) \quad \sin(\lambda\cdot B))\cdot\sin(\lambda\cdot x) \ldots \\ + 1\cdot(\sin(\lambda\cdot B) - \cos(\lambda\cdot B))\cdot\cos(\lambda\cdot x) \end{pmatrix} \cdot \frac{R_f}{4}\cdot B^2 \ldots \\ & +(\cos(\lambda\cdot x)\cdot\cos(\lambda\cdot B) \quad \sin(\lambda\cdot x)\cdot\sin(\lambda\cdot B))\cdot 2\cdot E\cdot I\cdot\lambda^3\cdot D_s\cdot B \ldots \\ & + \begin{pmatrix} (\sin(\lambda\cdot B) \quad \cos(\lambda\cdot B))\cdot\cos(\lambda\cdot x) \ldots \\ + 1\cdot(\sin(\lambda\cdot B) - \cos(\lambda\cdot B))\cdot\sin(\lambda\cdot x) \end{pmatrix} \cdot E\cdot I\cdot\lambda^2\cdot D_s \end{aligned}\right]^2}{(\lambda\cdot((\lambda\cdot B - 1)\cdot(E\cdot I)))\cdot e^{\lambda\cdot(B - x)}}
\end{aligned}
$$

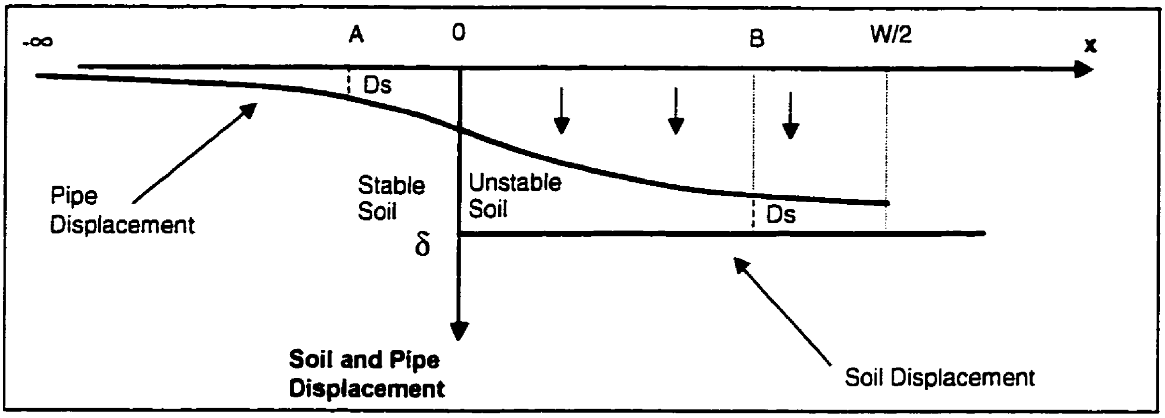


Fig. B.1: Transverse landslide with finite width

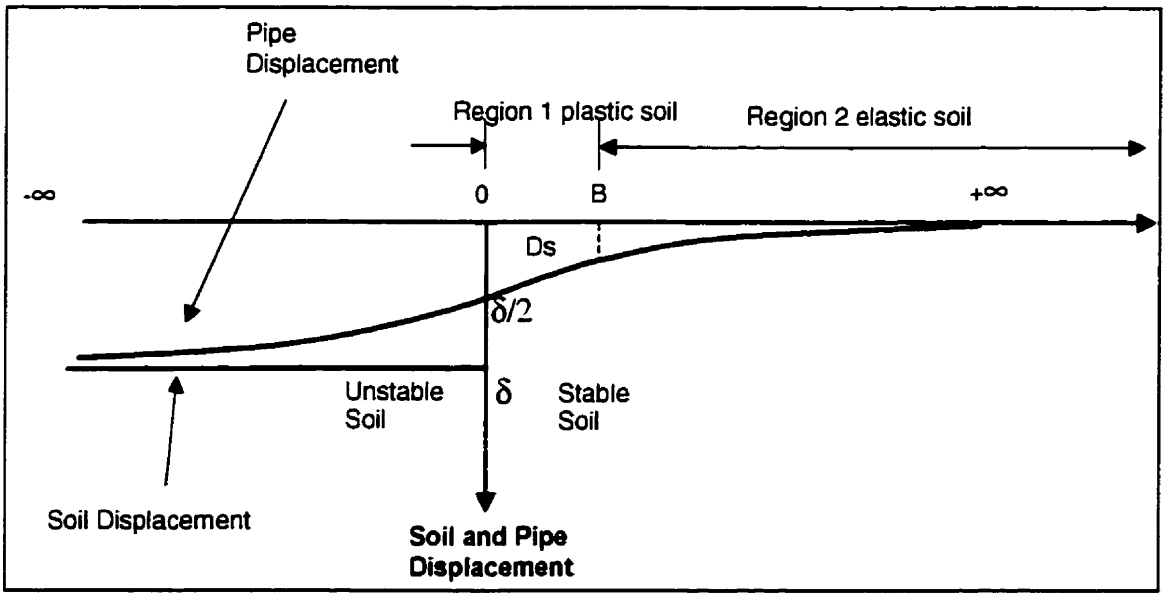


Fig. B.2: Transverse landslide with infinite width

# Appendix C

# Derivations for the Longitudinal and Perpendicular Components of Soil Movement in Deep-seated Circular Landslide

This appendix shows the derivations for the longitudinal ($u_L$) and perpendicular ($u_p$) components of the soil movement along the soil-pipeline interface as defined as $u(x)$:

[C1] $$\boxed{u_L = \frac{\delta}{R} \cdot Rm}$$

[C2] $$\boxed{u_p(x) = \frac{\delta}{R} \cdot \sqrt{R^2 - Rm^2 - x^2}}$$

The soil mass slides and rotates along the circular failure surface with a tangential displacement $\delta$. Assume that the amount of soil movement varies uniformly from the center of the failure circle (see Figure C1):

[C3] $$\boxed{u(r) = \delta \cdot \frac{r}{R}}$$

Set the coordinate system so that the x-axis coincides with the pipeline(see Figure C2).

Let $\beta = \beta(x)$ = the direction that u(r) is acting along the failure surface with respect to the pipeline. (see Figure C3).

[C4] $$\beta = \beta(x) = \mathrm{atan}\left(\frac{dy}{dx}\right) = \mathrm{atan}\left[\frac{\pm x}{\left(R^2 - x^2\right)^{.5}}\right] = \mathrm{atan}\left[\frac{x}{\left(R^2 - x^2\right)^{.5}}\right]$$

The longitudinal and perpendicular components of the soil movements are given below and shown in Figure C4.

[C5] $$u_L(r) = u(r)\cdot\cos(\beta)$$

[C6] $$u_p(r) = u(r)\cdot\sin(\beta)$$

Since $\cos(\beta)$ and $\sin(\beta)$ can be written as follows:

[C7] $$\cos(\beta) = \frac{Rm}{r} = \frac{\left(r^2 - x^2\right)^{.5}}{r}$$

[C8] $$\sin(\beta) = \frac{x}{r}$$

We can re-write the longitudinal and perpendicular soil movement components as follows:

[C9] $$u_L(r) = u(r)\cdot\cos(\beta) = \delta\cdot\frac{r}{R}\cdot\frac{\left(r^2 - x^2\right)^{.5}}{r} = \delta\cdot\frac{Rm}{R} = \text{constant}$$

[C10] $$u_p(r) = u(r)\cdot\sin(\beta) = \delta\cdot\frac{r}{R}\cdot\frac{x}{r} = \delta\cdot\frac{x}{R}$$

Moving the origin of the coordinate system so that x starts at the left interface between the pipeline and the failure surface (see Figure C5), we change the perpendicular component $u_p(r)$ as follows:

[C11] $$\boxed{u_p(r) = \delta \cdot \frac{\left(R^2 - Rm^2\right)^{.5} - x}{R}}$$

# C.1 Pipeline equations for perpendicular component of deep-seated landslide

Region 1 ($-\infty < x < A$):

[C12] $$\frac{d}{dx}\frac{d}{dx}\frac{d}{dx}\frac{d}{dx}u_1 - 0 = \frac{k\cdot d\cdot u_1 - 0}{E\cdot I}$$

[C13] $$u_1 = e^{\lambda\cdot x}\cdot C_1\cdot\cos(\lambda\cdot x) - C_2\cdot\sin(\lambda\cdot x) - e^{\lambda\cdot x}\cdot C_3\cdot\cos(\lambda\cdot x) - C_4\cdot\sin(\lambda\cdot x)$$

Region 2 ($A < x < 0$):

[C14] $$\frac{d}{dx}\frac{d}{dx}\frac{d}{dx}\frac{d}{dx}u_2 \quad 0 = \frac{k\cdot d\cdot u_2 \quad 0}{E\cdot I} = \frac{R_f}{E\cdot I}$$

[C15] $$u_2 = \frac{1}{E\cdot I}\cdot\frac{R_f}{24}\cdot x^4 - \frac{C_5}{6}\cdot x^3 - \frac{C_6}{2}\cdot x^2 - C_7\cdot x - C_8$$

Region 3 ($0 < x < B$)

[C16] $$\frac{d}{dx}\frac{d}{dx}\frac{d}{dx}\frac{d}{dx}u_3 \quad u_p = \frac{k\cdot d\cdot u_3 \quad u_p}{E\cdot I} = \frac{R_f}{E\cdot I}$$

[C17] $$u_3 = \frac{1}{E\cdot I}\cdot\frac{R_f}{24}\cdot x^4 - \frac{k_1}{6}\cdot x^3 - \frac{k_2}{2}\cdot x^2 - k_3\cdot x - k_4$$

Region 4 ($B < x < (R^2 - Rm^2)^{0.5}$)

[C18] $$\frac{d}{dx}\frac{d}{dx}\frac{d}{dx}\frac{d}{dx}u_4 \cdot u_p = \frac{k\cdot d\cdot u_4 \quad u_p}{E\cdot I}$$

[C19] $$u_4 = e^{\lambda\cdot x}\cdot k_5\cdot\cos(\lambda\cdot x) - k_6\cdot\sin(\lambda\cdot x) - e^{\lambda\cdot x}\cdot k_7\cdot\cos(\lambda\cdot x) - k_8\cdot\sin(\lambda\cdot x) - u_p(x)$$

[C20] $$u_p(x) = \frac{\delta}{R}\cdot\sqrt{R^2 - Rm^2} - x$$

The following boundary conditions are applied:

At $x = -\infty$, $u = 0$;

At $x = A$, $u = D_s$;

At $x = A$, $u$, $u'$, $u''$, and $u'''$ are continuous;

At $x = 0$, $u$, $u'$, $u''$, and $u'''$ are continuous;

At $x = B$, $u = u_p - D_s$

At $x = B$, $u$, $u'$, $u''$, and $u'''$ are continuous;

At $x = (R^2 - Rm^2)^{1/2}$, $u = 0$;

Force equilibrium.

After applying the boundary conditions, we get the following equations:

[C21] $\boxed{C_1 = 0}$

[C22] $\boxed{C_2 = 0}$

[C23] $\boxed{C_8 = k_4}$

[C24] $\boxed{C_7 = k_3}$

[C25] $\boxed{C_6 = k_2}$

[C26] $\boxed{C_5 = k_1}$

[C27] $\boxed{0 = e^{\lambda \cdot A} \cdot \left(C_1 \cdot \cos(\lambda \cdot A) - C_2 \cdot \sin(\lambda \cdot A)\right) - \frac{1}{E \cdot I} \cdot \left(\frac{1}{24} \cdot R_f \cdot A^4 - \frac{1}{6} \cdot C_5 \cdot A^3 - \frac{1}{2} \cdot C_6 \cdot A^2 - C_7 \cdot A - C_8\right)}$

[C28] $\boxed{\begin{bmatrix} (\cos(\lambda \cdot A) - \sin(\lambda \cdot A)) \cdot C_1 \ldots \\ + (\sin(\lambda \cdot A) - \cos(\lambda \cdot A)) \cdot C_2 \end{bmatrix} \cdot e^{\lambda \cdot A} \cdot \lambda = \frac{1}{E \cdot I} \cdot \left(\frac{1}{6} \cdot R_f \cdot A^3 - \frac{1}{2} \cdot C_5 \cdot A^2 - C_6 \cdot A - C_7\right)}$

[C29] $\boxed{0 = (-1 \cdot \sin(\lambda \cdot A) \cdot C_1 - \cos(\lambda \cdot A) \cdot C_2) \cdot 2 \cdot e^{\lambda \cdot A} \cdot \lambda^2 - \frac{1}{E \cdot I} \cdot \left(\frac{1}{2} \cdot R_f \cdot A^2 - C_5 \cdot A - C_6\right)}$

[C30]
$$\left[\begin{array}{l}(-\cos(\lambda\cdot A)-\sin(\lambda\cdot A))\cdot C_1\ ... \\ +(\cos(\lambda\cdot A)-\sin(\lambda\cdot A))\cdot C_2\end{array}\right]\cdot 2\cdot e^{\lambda\cdot A}\cdot\lambda^3 = \frac{1}{E\cdot I}\cdot R_f\cdot A - C_5$$

[C31]
$$e^{\lambda\cdot A}\cdot\left(C_1\cdot\cos(\lambda\cdot A) - C_2\cdot\sin(\lambda\cdot A)\right) = Ds$$

[C32]
$$0 = \frac{1}{E\cdot I}\cdot\left(\frac{1}{24}\cdot R_f\cdot B^4 - \frac{1}{6}\cdot k_1\cdot B^3\ ... \\ + \frac{1}{2}\cdot k_2\cdot B^2 - k_3\cdot B - k_4\right) - \left(e^{\lambda\cdot B}\cdot\left(k_5\cdot\cos(\lambda\cdot B) - k_6\cdot\sin(\lambda\cdot B)\right)\ ... \\ + e^{\lambda\cdot B}\cdot\left(k_7\cdot\cos(\lambda\cdot B) - k_8\cdot\sin(\lambda\cdot B)\right) - u_p(B)\right)$$

[C33]
$$0 = \frac{1}{E\cdot I}\cdot\left(\frac{1}{6}\cdot R_f\cdot B^3 - \frac{1}{2}\cdot k_1\cdot B^2 - k_2\cdot B - k_3\right) - \left[\begin{array}{l}\left[(\sin(\lambda\cdot B)-\cos(\lambda\cdot B))\cdot k_5\ ... \\ +(\sin(\lambda\cdot B)-\cos(\lambda\cdot B))\cdot k_6\right]\cdot e^{\lambda\cdot B}\ ... \\ +\left[(\cos(\lambda\cdot B)-\sin(\lambda\cdot B))\cdot k_7\ ... \\ +(\sin(\lambda\cdot B)-\cos(\lambda\cdot B))\cdot k_8\right]\cdot e^{\lambda\cdot B}\end{array}\right]\cdot\lambda - \frac{\delta}{R}$$

[C34]
$$0 = \frac{1}{E\cdot I}\cdot\left(\frac{1}{2}\cdot R_f\cdot B^2 - k_1\cdot B - k_2\right) - \left[(\sin(\lambda\cdot B)\cdot k_5 - \cos(\lambda\cdot B)\cdot k_6)\cdot e^{\lambda\cdot B}\ ... \\ +(\sin(\lambda\cdot B)\cdot k_7 - \cos(\lambda\cdot B)\cdot k_8)\cdot e^{\lambda\cdot B}\right]\cdot 2\cdot\lambda^2$$

[C35]
$$0 = \frac{1}{E\cdot I}\cdot\left(R_f\cdot B - k_1\right) - \left[\left((\cos(\lambda\cdot B)-\sin(\lambda\cdot B))\cdot k_5 - (\sin(\lambda\cdot B)-\cos(\lambda\cdot B))\cdot k_6\right)\cdot e^{\lambda\cdot B}\ ... \\ +\left((\sin(\lambda\cdot B)-\cos(\lambda\cdot B))\cdot k_7 - (\sin(\lambda\cdot B)-\cos(\lambda\cdot B))\cdot k_8\right)\cdot e^{\lambda\cdot B}\right]\cdot 2\cdot\lambda^3$$

[C36]
$$\left(e^{\lambda\cdot B}\cdot\left(k_5\cdot\cos(\lambda\cdot B) - k_6\cdot\sin(\lambda\cdot B)\right)\ ... \\ + e^{\lambda\cdot B}\cdot\left(k_7\cdot\cos(\lambda\cdot B) - k_8\cdot\sin(\lambda\cdot B)\right) - u_p(B)\right) = u_p(B) - Ds$$

[C37]
$$0 = e^{\lambda\cdot D}\cdot\left(k_5\cdot\cos(\lambda\cdot D) - k_6\cdot\sin(\lambda\cdot D)\right) - e^{\lambda\cdot D}\cdot\left(k_7\cdot\cos(\lambda\cdot D) - k_8\cdot\sin(\lambda\cdot D)\right) - u_p(D)$$

[C38]
$$0 = \left[(\cos(\lambda\cdot A)-\sin(\lambda\cdot A))\cdot C_1 - (\cos(\lambda\cdot A)-\sin(\lambda\cdot A))\cdot C_2\right]\cdot e^{\lambda\cdot A} - Ds\cdot(A-B)\cdot 2\cdot\lambda\ ... \\ + 1\cdot\left[\begin{array}{l}\left((\cos(\lambda\cdot B)-\sin(\lambda\cdot B))\cdot k_5 - (\cos(\lambda\cdot B)-\sin(\lambda\cdot B))\cdot k_6\right)\cdot e^{\lambda\cdot B}\ ... \\ +\left((\cos(\lambda\cdot D)-\sin(\lambda\cdot D))\cdot k_5 - (\sin(\lambda\cdot D)-\cos(\lambda\cdot D))\cdot k_6\right)\cdot e^{\lambda\cdot D}\ ... \\ +\left((\cos(\lambda\cdot B)-\sin(\lambda\cdot B))\cdot k_7 - (\sin(\lambda\cdot B)-\cos(\lambda\cdot B))\cdot k_8\right)\cdot e^{\lambda\cdot B}\ ... \\ +\left((\sin(\lambda\cdot D)-\cos(\lambda\cdot D))\cdot k_7 - (\cos(\lambda\cdot D)-\sin(\lambda\cdot D))\cdot k_8\right)\cdot e^{\lambda\cdot D}\end{array}\right]$$

These 18 equations ([C21] to [C38]) can be solved numerically by using Newton-Raphson Method for Nonlinear Systems of Equations with LU decomposition and back substitution as described in appendix B.

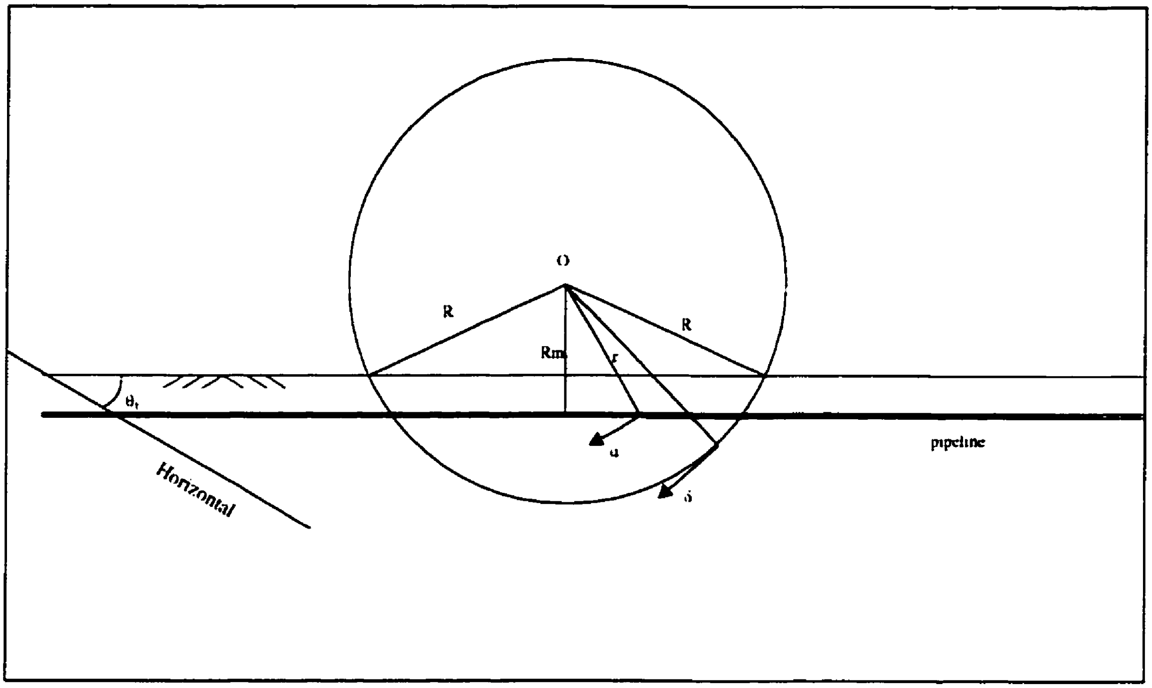


Fig. C1: Soil movement

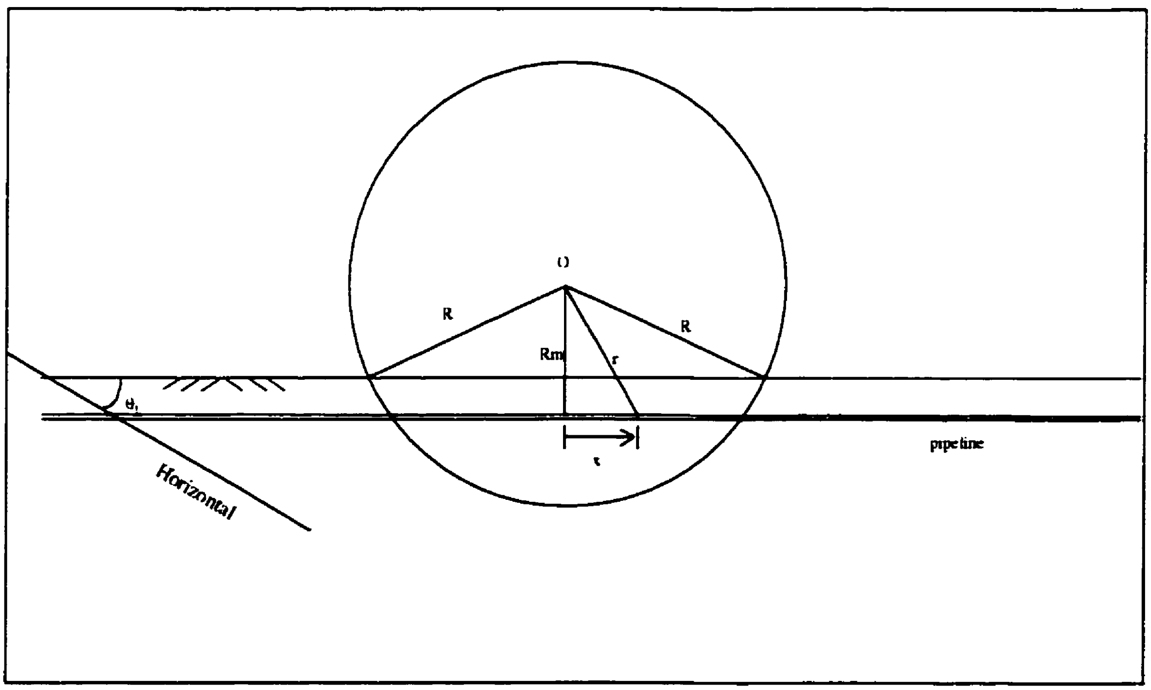


Fig. C2: Coordinate system

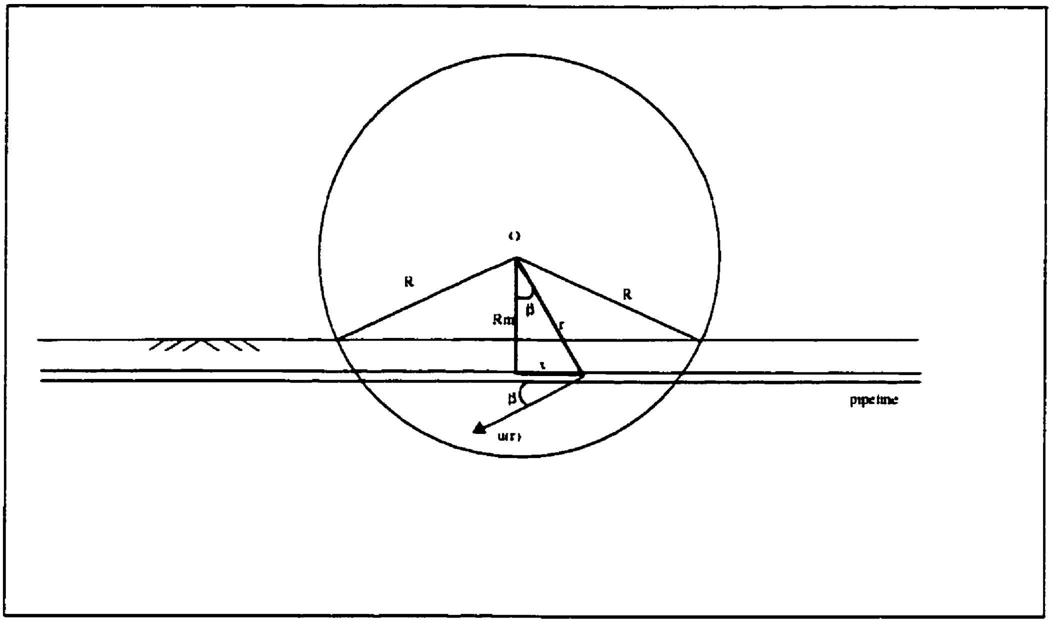


Fig. C3: Direction of soil movement

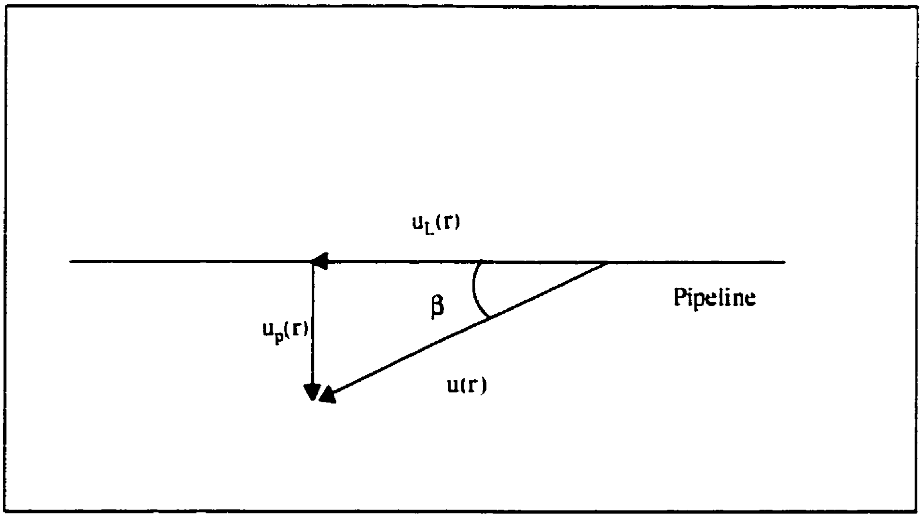


Fig. C4: components of soil movement

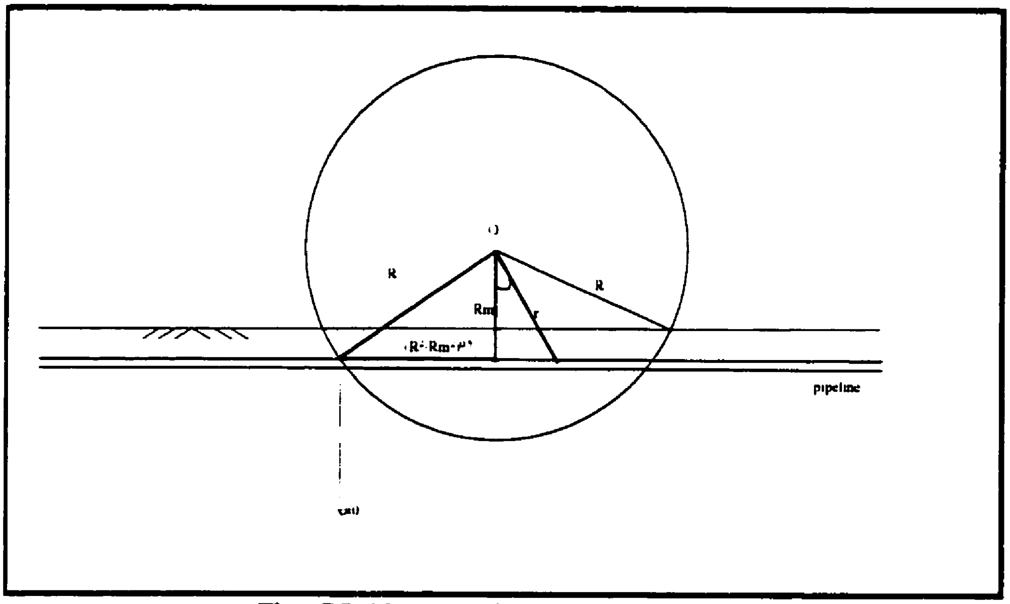


Fig. C5: New starting position of x-axis

# Appendix D

# Numerical Solution of Multiple Nonlinear Equations Using Newton-Raphson Method

```
'--EPPE3.BAS
'--- We use partial pivoting in LUDCMP and LUBKSB.
'---Multiple non-linear roots solved by Newton-Raphson method and LU decomposition
'-- and back substitution – for Pipe bending with E/P and E/P regions.
'--(c) PETER D.S. CHAN APR. 1997
'-----------------------------------------------------------------
DEFINT I-K
'-----------------------------------------------------------------
DECLARE SUB MNewton (x#(), n, df#(), f#())
DECLARE SUB LUDCMP (x#(), n, df#(), f#(), index%())
DECLARE SUB LUBKSB (x#(), n, df#(), f#(), index%())
DECLARE SUB printdf (n, df#())
DECLARE SUB printx (x#(), f#(), n)
DECLARE SUB printf (n, f#())
DECLARE SUB pause ()
DECLARE SUB UsrEqns (x#(), n, df#(), f#())
DECLARE FUNCTION x1givenW (x#())
DECLARE FUNCTION x2givenW (x#())
DECLARE FUNCTION x3givenW (x#())
DECLARE FUNCTION StrainEqns# (i, x#(), x)
DECLARE FUNCTION dStrainEqns# (i, j, x#(), x)
DECLARE FUNCTION FirstDerv# (i, j, x#(), x)
CLS
CONST PI# = 3.141592653589793#
CONST expon# = 2.718281828459045#
CONST No = 0, Yes = 1

CONST Ks# = 6500000#, d# = .1683#, t# = .00318#
CONST Ep# = 200000000000#, sigy = 3.86E+08, Ds# = .008#
CONST TINY = 1E-20
CONST TOL = 1E-12

CONST W = 20, delta = 55.901
```

```
CONST ntrial = 1000, n = 12
DIM x#(n), df#(n, n), f#(n)
x#(1) = -768.709372#
x#(2) = -1256.94#
x#(3) = -70131.02#
x#(4) = -37199.12#
x#(5) = 2420488#
x#(6) = 19480160#
x#(7) = .0000938643#
x#(8) = -.00008014427#
x#(9) = -89.336369#
x#(10) = 126.822848#
x#(11) = -14.05369#
x#(12) = 9.440185#
CALL MNewton(x#(), n, df#(), f#())

END

FUNCTION dStrainEqns# (i, j, x#(), x)
'---------------------------------------------------------------------
C1# = x#(1)
C2# = x#(2)
k1# = x#(3)
k2# = x#(4)
k3# = x#(5)
k4# = x#(6)
B# = x#(7)

Ip# = PI# / 4 * ((d# / 2) ^ 4 - (d# / 2 - t#) ^ 4)
Id# = (Ks# * d# / (4 * Ep# * Ip#)) ^ .25
R# = Ks# * d# * Ds#
C# = COS(Id# * x)
S# = SIN(Id# * x)
E# = expon# ^ (Id# * x)

SELECT CASE i
  CASE IS = 1
    temp# = (-C# * S# + 1 - C# ^ 2) * C1# ^ 2
    temp# = temp# + (-2 * C# * S# + 2 * C# ^ 2 - 1) * C2# * C1#
    temp# = temp# + (C# ^ 2 + C# * S#) * C2# ^ 2
    temp# = temp# * 2 * E#
    dStrainEqns# = temp# + (-(C# + S#) * C1# - (S# - C#) * C2#) * d
  CASE IS = 2
    temp# = 1 / (Ep# * Ip#) * (-1 / 6 * R# * x ^ 3 + .5 * k1# * x ^ 2 + k2# * x + k3#)
    temp# = temp# * (-.5 * R# * x ^ 2 + k1# * x + k2#)
    IF j = 1 THEN
      dStrainEqns# = temp# + .5 * d# * (-R# * x + k1#)
    ELSEIF j = 2 THEN
      dStrainEqns# = temp# - .5 * d# * (-R# * x + k1#)
    END IF
  CASE IS = 3
    temp# = 1 / (Ep# * Ip#) * (1 / 6 * R# * x ^ 3 + .5 * k1# * x ^ 2 + k2# * x + k3#)
```

```
    temp# = temp# * (.5 * R# * x ^ 2 + k1# * x + k2#)
    IF j = 1 THEN
      dStrainEqns# = temp# + .5 * d# * (R# * x + k1#)
    ELSEIF j = 2 THEN
      dStrainEqns# = temp# - .5 * d# * (R# * x + k1#)
    END IF

END SELECT

END FUNCTION

FUNCTION FirstDerv# (i, j, x#(), x)
'---f'(x)=1/(2*dx)*[f(x+dx)-f(x-dx)]-O'(dx^2)
'
'---Using Richardson's Extrapolation:
'---f'(x)=4/3*phi(dx/2)-1/3*phi(dx) - O'(dx^4)
'--- where phi(dx) = 1/(2*dx)*[f(x+dx)-f(x-dx)]
'
'--df=[df1/dx1 df1/dx2]  = [dfi/dxj ...]
'--  [df2/dx1 df2/dx2]   [...........]
'-->"i" refers to the ith f eqn, & "j" refers to the jth partial derv.
'--ie: df1/dx2 = df(1)/dy = 2y, and df2/dx1 == df(2)/dx = 1.
'------------------------------------------------------------------

  dx = x / 10 ^ 2 / 2
  xt = x + dx
  xt2 = x - dx
  phi1 = (dStrainEqns#(i, j, x#(), xt) - dStrainEqns#(i, j, x#(), xt2)) / (2 * dx)

  dx = x / 10 ^ 2
  xt = x + dx
  xt2 = x - dx

  phi2 = (dStrainEqns#(i, j, x#(), xt) - dStrainEqns#(i, j, x#(), xt2)) / (2 * dx)


  FirstDerv# = 4 / 3 * phi1 - 1 / 3 * phi2

END FUNCTION

SUB LUBKSB (x#(), n, df#(), f#(), index%())
i2 = 0

FOR i = 1 TO n
  k = index%(i)  '<---unscrambling the permutation as we go.
  sum# = f#(k)    '---W/O pivoting this would simply be y(1)=f(1) and
  f#(k) = f#(i)   '---y(2)=f(2)-df(2,1)*y(1) or y(i)=f(i)-sum{fromj=1 to i-1
                  '--- of df(i,j)*y(j)}.
                  '---Note we don't need the vector y(), we simply place y(i)
                  '--- into f(i)

  IF (i2 <> 0) THEN
    FOR j = i2 TO i - 1
```

```
      sum# = sum# - df#(i, j) * f#(j)
    NEXT j
  ELSEIF sum# <> 0 THEN
    i2 = i      '---A nonzero element was encountered, so from now on we
  END IF        '---will have to do the sums in the loop above.  This is
                '---just some optimization because if the first sum was 0,
                '---then we don't have to do the loop for the second sum.
                '---ie: y(2) = f(2)-df(2,1)*y(1) and y(1) = 0.
  f#(i) = sum#
NEXT i

'---use f(i) to store the deltaX(i):
'---for the Newton-Raphson iteration of xnew(i) = xold(i) + deltaX(i),
'---we now have                    x(i) = x(i) + f(i)
FOR i = n TO 1 STEP -1    '---Back Substitution
  sum# = f#(i)
  IF (i < n) THEN
    FOR j = i + 1 TO n
      sum# = sum# - df#(i, j) * f#(j)
    NEXT j
  END IF
  f#(i) = sum# / df#(i, i)
NEXT i

END SUB

SUB LUDCMP (x#(), n, df#(), f#(), index%())
'--instead of decomposing df() into L() and U(), we will decompose df() back
'--into itself (since we only use df() once for each iteration):
'--ie:[1   0] and [u11 u12]  into [u11 u12]
'--   [L12 1]     [ 0  u22]       [L12 u22]
'-----------------------------------------------------------------------
DIM VV#(n)
flag = 1    '---no row interchange yet
'---loop over rows to get the implicit scaling information
FOR i = 1 TO n
  AAmax# = 0
  FOR j = 1 TO n
    IF ABS(df#(i, j)) > AAmax# THEN
      AAmax# = ABS(df#(i, j))
    END IF
  NEXT j
  IF (AAmax# = 0) THEN
    PRINT "Singular matrix"  '---no nonzero largest element
    AAmax# = TINY
  END IF
  VV#(i) = 1# / AAmax#      '<---VV(i) = 1/AAmax of the ith row
NEXT i

FOR j = 1 TO n   '---loop over the columns of Crout's method
  FOR i = 1 TO j - 1   '---U(ij) = df(ij) - sum(k=1 to i-1 of L(ik)*U(kj))
    sum# = df#(i, j)
    FOR k = 1 TO i - 1
```

```
      sum# = sum# - df#(i, k) * df#(k, j)
    NEXT k
    df#(i, j) = sum#   '<---U(i,j)
  NEXT i

  AAmax# = 0      '---initialize AAmax for the search for largest pivot element

'---Following are i=j     of U(ij) =          df(ij) - sum(k=1 to i-1 of L(ik)*U(kj))
'------------and i=j+1..n of L(ij)= 1/U(jj) * [df(ij) - sum(k=1 to j-1 of L(ik)*U(kj)]
  FOR i = j TO n
    sum# = df#(i, j)
    FOR k = 1 TO j - 1
      sum# = sum# - df#(i, k) * df#(k, j)
    NEXT
    df#(i, j) = sum#   '---haven't divide by U(jj) yet for the "i>j" cases

    dum# = VV#(i) * ABS(sum#)   '<---(VV(i) = 1/AAmax).
    IF (dum# >= AAmax#) THEN   '---Check the elements below i=j for a larger
      imax = i             '--- AAmax, and mark this new AAmax row with
      AAmax# = dum#           '--- "imax".
    END IF
  NEXT i

  IF j <> imax THEN   '---Do we need to interchange rows?  We need i=j for
    FOR k = 1 TO n   '---the pivot element.
      dum# = df#(imax, k)        '<---Switching the rows between (imax,...)
      df#(imax, k) = df#(j, k)    '---and (j,...): remember imax is found by
      df#(j, k) = dum#          '---looking for larger AAmax below i=j element.
    NEXT k                   '---So we use "j", since "i" has been looped
    flag = -flag
    VV#(imax) = VV#(j)  '---Also interchange the scale factor vector VV.
  END IF

  index%(j) = imax     '<---recording for each jth row, the swap between j
                 '----and imax rows.

  IF df#(j, j) = 0 THEN df#(j, j) = TINY

  IF (j <> n) THEN    '---Finall, we divide by the pivot element for i=j+1...n
    dum# = 1# / df#(j, j)
    FOR i = j + 1 TO n
      df#(i, j) = df#(i, j) * dum#
    NEXT i
  END IF

NEXT j

END SUB

SUB MNewton (x#(), n, df#(), f#())
'---Newton-Raphson Method for 1 variable:
'---f(x+h)= 0 = f(x)+h*f'(x) + O('h^2)
'---h = -f(x)/f'(x)
```

```
'---xln+1 = xln + h
'--------------------------------------------------------------------
DIM index%(n), flag, VV#(n), SumF#
FOR k = 1 TO ntrial

  CALL UsrEqns(x#(), n, df#(), f#())
  LOCATE 1, 1
  PRINT USING "after UsrEqns,  W=###.##, delta = ##.######, trial = ####"; W; delta; k
  CALL printx(x#(), f#(), n)
  SumF# = 0
  FOR i = 1 TO n
    SumF# = SumF# + ABS(f#(i))
  NEXT i
  PRINT USING "total abs(f(i))= ##.#########^^^^"; SumF#
  IF SumF# < TOL THEN  '---SumF# is calculated in the printdf sub.
    PRINT "Convergence completed. Tol= "; TOL
    CALL pause
    EXIT SUB
  END IF
  CALL LUDCMP(x#(), n, df#(), f#(), index%())
  CALL LUBKSB(x#(), n, df#(), f#(), index%()) '---the deltaX(i) is stored in f(i)
  FOR i = 1 TO n
    x#(i) = x#(i) + f#(i)     '---f(i) is the deltaX(i) returned from LUBKSB
    ErrF# = ErrF# + ABS(f#(i))
  NEXT i
NEXT k

END SUB

SUB pause
  col = CSRLIN
  row = POS(0)
  LOCATE 24, 1
  PRINT "press any key...";
  DO WHILE INKEY$ = ""
  LOOP
  LOCATE row, col
END SUB

SUB printdf (n, df#())

  FOR i = 1 TO n
    PRINT USING "##.##^^^^  ##.##^^^^  ##.##^^^^  ##.##^^^^  ##.##^^^^  ##.##^^^^
##.##^^^^"; df#(i, 1); df#(i, 2); df#(i, 3); df#(i, 4); df#(i, 5); df#(i, 6); df#(i, 7)
  NEXT i
END SUB

SUB printf (n, f#())
  FOR i = 1 TO n
    PRINT USING "f(##)= ##.######^^^^ "; i; f#(i)
  NEXT i
END SUB
```

```
SUB printx (x#(), f#(), n)
  LOCATE 2, 1
  FOR i = 1 TO n
    PRINT USING "x(##)=   #########,###.#########      f(##)=  ##.######^^^^        "; i; x#(i);
i; f#(i)
  NEXT i
END SUB

FUNCTION StrainEqns# (i, x#(), x)
'---tensile Strain = 1/2*(dy/dx)^2
'---Bending Strain = d/2*(d^2y/dx^2)
'------------------------------------------------------------------
C1# = x#(1)
C2# = x#(2)
k1# = x#(3)
k2# = x#(4)
k3# = x#(5)
k4# = x#(6)
B# = x#(7)

Ip# = PI# / 4 * ((d# / 2) ^ 4 - (d# / 2 - t#) ^ 4)
Id# = (Ks# * d# / (4 * Ep# * Ip#)) ^ .25
R# = Ks# * d# * Ds#
C# = COS(Id# * x)
S# = SIN(Id# * x)
E# = expon# ^ (Id# * x)

SELECT CASE i
  CASE IS = 1
    temp# = .5 * (((C# - S#) * C1# + (S# + C#) * C2#) * E# * Id#) ^ 2
    temp# = temp# + d# / 2 * ((-S# * C1# + C# * C2#) * 2 * E# * Id# ^ 2)
  CASE IS = 2
    temp# = .5 * ((-R# * x ^ 3 / 6 + k1# * x ^ 2 / 2 + k2# * x + k3#) / (Ep# * Ip#)) ^ 2
    temp2# = d# / 2 * ((-R# * x ^ 2 / 2 + k1# * x + k2#) / (Ep# * Ip#))
    temp# = temp# + ABS(temp2#)
  CASE IS = 3
    temp# = .5 * ((R# * x ^ 3 / 6 + k1# * x ^ 2 / 2 + k2# * x + k3#) / (Ep# * Ip#)) ^ 2
    temp2# = d# / 2 * ((R# * x ^ 2 / 2 + k1# * x + k2#) / (Ep# * Ip#))
    temp# = temp# + ABS(temp2#)
END SELECT
StrainEqns# = temp# * 10 ^ 6
END FUNCTION

SUB UsrEqns (x#(), n, df#(), f#())
'---Example:
'---f(1)= x^2+y^2-1=0
'---f(2)= x-y=0
'-----------------
'--df=[df1/dx1 df1/dx2]  = [dfi/dxj ...]
'--  [df2/dx1 df2/dx2]    [...........]
'-->"i" refers to the ith f eqn, & "j" refers to the jth partial derv.
'--ie: df1/dx2 = df(1)/dy = 2y, and df2/dx1 == df(2)/dx = 1.
'-----------------------------------------------------------------
```

```
'f(1) = -(x(1) ^ 2 + x(2) ^ 2 - 1)
'f(2) = -(x(1) - x(2))
'df(1, 1) = 2 * x(1): df(1, 2) = 2 * x(2)
'df(2, 1) = 1: df(2, 2) = -1
'----------------------------------------------------------------
C1# = x#(1)
C2# = x#(2)
k1# = x#(3)
k2# = x#(4)
k3# = x#(5)
k4# = x#(6)
k5# = x#(7)
k6# = x#(8)
k7# = x#(9)
k8# = x#(10)
A# = x#(11)
B# = x#(12)

IF A# > 0 OR A# < -W * 2 THEN
  A# = -W
  x#(11) = A#
END IF
IF B# > W / 2 OR B# < 0 THEN
  B# = W / 2
  x#(12) = B#
END IF


Ip# = PI# / 4 * ((d# / 2) ^ 4 - (d# / 2 - t#) ^ 4)
ld# = (Ks# * d# / (4 * Ep# * Ip#)) ^ .25
R# = Ks# * d# * Ds#
epsy = sigy / Ep#
EB# = expon# ^ (ld# * B#)
CB# = COS(ld# * B#)
SB# = SIN(ld# * B#)
EA# = expon# ^ (ld# * A#)
CA# = COS(ld# * A#)
SA# = SIN(ld# * A#)
EW# = expon# ^ (ld# * W / 2)
CW# = COS(ld# * W / 2)
SW# = SIN(ld# * W / 2)

temp# = EA# * (C1# * CA# + C2# * SA#)
temp2# = (-R# * A# ^ 4 / 24 + k1# * A# ^ 3 / 6 + k2# * A# ^ 2 / 2 + k3# * A# + k4#)
temp2# = temp2# / (Ep# * Ip#)
f#(1) = temp# - temp2#

temp# = ((CA# - SA#) * C1# + (SA# + CA#) * C2#) * EA# * ld#
temp2# = (-R# * A# ^ 3 / 6 + k1# * A# ^ 2 / 2 + k2# * A# + k3#) / (Ep# * Ip#)
f#(2) = temp# - temp2#

temp# = (-SA# * C1# + CA# * C2#) * 2 * EA# * ld# ^ 2
temp2# = (-R# * A# ^ 2 / 2 + k1# * A# + k2#) / (Ep# * Ip#)
```

```
f#(3) = temp# - temp2#

temp# = ((-CA# - SA#) * C1# + (CA# - SA#) * C2#) * 2 * EA# * Id# ^ 3
temp2# = (-R# * A# + k1#) / (Ep# * Ip#)
f#(4) = temp# - temp2#

temp# = (-R# * A# ^ 4 / 24 + k1# * A# ^ 3 / 6 + k2# * A# ^ 2 / 2 + k3# * A# + k4#)
f#(5) = temp# / (Ep# * Ip#) - Ds#

temp# = (R# * B# ^ 4 / 24 + k1# * B# ^ 3 / 6 + k2# * B# ^ 2 / 2 + k3# * B# + k4#)
temp# = temp# / (Ep# * Ip#)
temp2# = ((CB# * k5# + SB# * k6#) * EB# + (CB# * k7# + SB# * k8#) / EB# + delta)
f#(6) = temp# - temp2#

temp# = (R# * B# ^ 3 / 6 + k1# * B# ^ 2 / 2 + k2# * B# + k3#) / (Ep# * Ip#)
temp2# = ((-SB# + CB#) * k5# + (SB# + CB#) * k6#) * EB#
temp2# = (temp2# + ((-CB# - SB#) * k7# + (-SB# + CB#) * k8#) / EB#) * Id#
f#(7) = temp# - temp2#

temp# = (R# * B# ^ 2 / 2 + k1# * B# + k2#) / (Ep# * Ip#)
temp2# = ((-SB# * k5# + CB# * k6#) * EB# + (SB# * k7# - CB# * k8#) / EB#) * 2 * Id# ^ 2
f#(8) = temp# - temp2#

temp# = (R# * B# + k1#) / (Ep# * Ip#)
temp2# = ((-CB# - SB#) * k5# + (-SB# + CB#) * k6#) * EB#
temp2# = (temp2# + ((-SB# + CB#) * k7# + (SB# + CB#) * k8#) / EB#) * 2 * Id# ^ 3
f#(9) = temp# - temp2#

temp# = (R# * B# ^ 4 / 24 + k1# * B# ^ 3 / 6 + k2# * B# ^ 2 / 2 + k3# * B# + k4#)
temp# = temp# / (Ep# * Ip#)
f#(10) = temp# - (delta - Ds#)

temp# = ((-SW# + CW#) * k5# + (SW# + CW#) * k6#) * EW#
f#(11) = (temp# + ((-CW# - SW#) * k7# + (-SW# + CW#) * k8#) / EW#)

temp# = ((CA# + SA#) * C1# + (-CA# + SA#) * C2#) * EA# - (Ds# * (A# + B#) * 2 * Id#)
temp2# = ((CB# + SB#) * k5# + (-CB# + SB#) * k6#) * EB#
temp2# = temp2# + ((-CB# + SB#) * k7# + (-SB# - CB#) * k8#) / EB#
temp2# = temp2# + ((-CW# - SW#) * k5# + (-SW# + CW#) * k6#) * EW#
temp2# = temp2# + ((-SW# + CW#) * k7# + (CW# + SW#) * k8#) / EW#
f#(12) = temp# - temp2#

'-----------------------------------------------------------------
f#(1) = -f#(1)
f#(2) = -f#(2)
f#(3) = -f#(3)
f#(4) = -f#(4)
f#(5) = -f#(5)
f#(6) = -f#(6)
f#(7) = -f#(7)
f#(8) = -f#(8)
f#(9) = -f#(9)
f#(10) = -f#(10)
```

```
f#(11) = -f#(11)
f#(12) = -f#(12)

'-----------------------------------------------

df#(1, 1) = EA# * CA#
df#(1, 2) = EA# * SA#
df#(1, 3) = -A# ^ 3 / 6 / (Ep# * Ip#)
df#(1, 4) = -A# ^ 2 / 2 / (Ep# * Ip#)
df#(1, 5) = -A# / (Ep# * Ip#)
df#(1, 6) = -1 / (Ep# * Ip#)
df#(1, 7) = 0
df#(1, 8) = 0
df#(1, 9) = 0
df#(1, 10) = 0
df#(1, 11) = f#(2)
df#(1, 12) = 0

df#(2, 1) = (CA# - SA#) * EA# * Id#
df#(2, 2) = (SA# + CA#) * EA# * Id#
df#(2, 3) = -A# ^ 2 / 2 / (Ep# * Ip#)
df#(2, 4) = -A# / (Ep# * Ip#)
df#(2, 5) = -1 / (Ep# * Ip#)
df#(2, 6) = 0
df#(2, 7) = 0
df#(2, 8) = 0
df#(2, 9) = 0
df#(2, 10) = 0
df#(2, 11) = f#(3)
df#(2, 12) = 0

df#(3, 1) = -2 * SA# * EA# * Id# ^ 2
df#(3, 2) = 2 * CA# * EA# * Id# ^ 2
df#(3, 3) = -A# / (Ep# * Ip#)
df#(3, 4) = -1 / (Ep# * Ip#)
df#(3, 5) = 0
df#(3, 6) = 0
df#(3, 7) = 0
df#(3, 8) = 0
df#(3, 9) = 0
df#(3, 10) = 0
df#(3, 11) = f#(4)
df#(3, 12) = 0

df#(4, 1) = -(CA# + SA#) * 2 * EA# * Id# ^ 3
df#(4, 2) = (-SA# + CA#) * 2 * EA# * Id# ^ 3
df#(4, 3) = -1 / (Ep# * Ip#)
df#(4, 4) = 0
df#(4, 5) = 0
df#(4, 6) = 0
df#(4, 7) = 0
df#(4, 8) = 0
df#(4, 9) = 0
df#(4, 10) = 0
```

```
df#(4, 11) = (-CA# * C1# - SA# * C2#) * 4 * EA# * Id# ^ 4
df#(4, 11) = df#(4, 11) + R# / (Ep# * Ip#)
df#(4, 12) = 0

df#(5, 1) = 0
df#(5, 2) = 0
df#(5, 3) = A# ^ 3 / 6 / (Ep# * Ip#)
df#(5, 4) = A# ^ 2 / 2 / (Ep# * Ip#)
df#(5, 5) = A# / (Ep# * Ip#)
df#(5, 6) = 1 / (Ep# * Ip#)
df#(5, 7) = 0
df#(5, 8) = 0
df#(5, 9) = 0
df#(5, 10) = 0
df#(5, 11) = (-R# * A# ^ 3 / 6 + k1# * A# ^ 2 / 2 + k2# * A# + k3#) / (Ep# * Ip#)
df#(5, 12) = 0

df#(6, 1) = 0
df#(6, 2) = 0
df#(6, 3) = B# ^ 3 / (6 * Ep# * Ip#)
df#(6, 4) = B# ^ 2 / (2 * Ep# * Ip#)
df#(6, 5) = B# / (Ep# * Ip#)
df#(6, 6) = 1 / (Ep# * Ip#)
df#(6, 7) = -EB# * CB#
df#(6, 8) = -EB# * SB#
df#(6, 9) = -CB# / EB#
df#(6, 10) = -SB# / EB#
df#(6, 11) = 0
df#(6, 12) = f#(7)

df#(7, 1) = 0
df#(7, 2) = 0
df#(7, 3) = B# ^ 2 / (2 * Ep# * Ip#)
df#(7, 4) = B# / (Ep# * Ip#)
df#(7, 5) = 1 / (Ep# * Ip#)
df#(7, 6) = 0
df#(7, 7) = (SB# - CB#) * EB# * Id#
df#(7, 8) = -(CB# + SB#) * EB# * Id#
df#(7, 9) = (SB# + CB#) / EB# * Id#
df#(7, 10) = (SB# - CB#) / EB# * Id#
df#(7, 11) = 0
df#(7, 12) = f#(8)

df#(8, 1) = 0
df#(8, 2) = 0
df#(8, 3) = B# / (Ep# * Ip#)
df#(8, 4) = 1 / (Ep# * Ip#)
df#(8, 5) = 0
df#(8, 6) = 0
df#(8, 7) = (SB#) * 2 * EB# * Id# ^ 2
df#(8, 8) = -(CB#) * 2 * EB# * Id# ^ 2
df#(8, 9) = -(SB#) * 2 / EB# * Id# ^ 2
df#(8, 10) = (CB#) * 2 / EB# * Id# ^ 2
```

```
df#(8, 11) = 0
df#(8, 12) = f#(9)

df#(9, 1) = 0
df#(9, 2) = 0
df#(9, 3) = 1 / (Ep# * Ip#)
df#(9, 4) = 0
df#(9, 5) = 0
df#(9, 6) = 0
df#(9, 7) = (CB# + SB#) * 2 * EB# * Id# ^ 3
df#(9, 8) = (-CB# + SB#) * 2 * EB# * Id# ^ 3
df#(9, 9) = (-CB# + SB#) * 2 / EB# * Id# ^ 3
df#(9, 10) = (-CB# - SB#) * 2 / EB# * Id# ^ 3
df#(9, 11) = 0
temp# = R# / (Ep# * Ip#)
temp2# = (CB# * k5# + SB# * k6#) * 4 * EB# * Id# ^ 4
temp2# = temp2# + (CB# * k7# + SB# * k8#) * 4 / EB# * Id# ^ 4
df#(9, 12) = temp# + temp2#

df#(10, 1) = 0
df#(10, 2) = 0
df#(10, 3) = B# ^ 3 / (6 * Ep# * Ip#)
df#(10, 4) = B# ^ 2 / (2 * Ep# * Ip#)
df#(10, 5) = B# / (Ep# * Ip#)
df#(10, 6) = 1 / (Ep# * Ip#)
df#(10, 7) = 0
df#(10, 8) = 0
df#(10, 9) = 0
df#(10, 10) = 0
df#(10, 11) = 0
df#(10, 12) = (R# * B# ^ 3 / 6 + k1# * B# ^ 2 / 2 + k2# * B# + k3#) / (Ep# * Ip#)

df#(11, 1) = 0
df#(11, 2) = 0
df#(11, 3) = 0
df#(11, 4) = 0
df#(11, 5) = 0
df#(11, 6) = 0
df#(11, 7) = (-SW# + CW#) * EW#
df#(11, 8) = (SW# + CW#) * EW#
df#(11, 9) = (-CW# - SW#) / EW#
df#(11, 10) = (-SW# + CW#) / EW#
df#(11, 11) = 0
df#(11, 12) = 0

df#(12, 1) = (CA# + SA#) * EA#
df#(12, 2) = (-CA# + SA#) * EA#
df#(12, 3) = 0
df#(12, 4) = 0
df#(12, 5) = 0
df#(12, 6) = 0
df#(12, 7) = -(CB# + SB#) * EB# + (CW# + SW#) * EW#
df#(12, 8) = (CB# - SB#) * EB# + (SW# - CW#) * EW#
```

```
df#(12, 9) = (CB# - SB#) / EB# + (SW# - CW#) / EW#
df#(12, 10) = (CB# + SB#) / EB# - (CW# + SW#) / EW#
df#(12, 11) = (CA# * C1# + SA# * C2#) * 2 * Id# * EA# - (2 * Ds# * Id#)
temp# = (-CB# * k5# - SB# * k6#) * 2 * Id# * EB#
temp# = temp# - (CB# * k7# + SB# * k8#) * 2 * Id# / EB#
df#(12, 12) = temp# - 2 * Ds# * Id#

END SUB

FUNCTION x1givenW (x#())
'---Find the "x" for maximum values of strains.
'--- This means using Newton-Raphson to find "x" at which the "derivative
'--- of the strain equations (dStrainEqns#())" is equal to 0.
'---Tensile Strain = 1/2*(dy/dx)^2  \
'---Bending Strain = d/2*(d^2y/dx^2)  \--> function StrainEqns#()
'-----------------------------------------------------------------
'---Newton-Raphson Method for 1 variable:
'---f(x+h)= 0 = f(x)+h*f'(x) + O('h^2)
'---h = -f(x)/f'(x)
'---xln+1 = xln + h
'-----------------------------------------------------------------
B# = x#(7)
x = B#
CLS
FOR i = 1 TO ntrial
  f# = dStrainEqns#(1, 0, x#(), x)
  df# = FirstDerv#(1, 0, x#(), x)
  x = x - f# / df#
  LOCATE 1, 1
  PRINT "i="; i
  PRINT "x1="; x; " Strain*10^6 = "; StrainEqns#(1, x#(), x)
  IF ABS(f# / df#) < 10 ^ -5 THEN EXIT FOR
  IF ABS(x) > 100 THEN
    x = B#
    EXIT FOR
  END IF
NEXT i


x1givenW = x
END FUNCTION

FUNCTION x2givenW (x#())
'---Find the "x" for maximum values of strains.
'--- This means using Newton-Raphson to find "x" at which the "derivative
'--- of the strain equations (dStrainEqns#())" is equal to 0.
'-----------------------------------------------------------------
'---Newton-Raphson Method for 1 variable:
'---f(x+h)= 0 = f(x)+h*f'(x) + O('h^2)
'---h = -f(x)/f'(x)
'---xln+1 = xln + h
'-----------------------------------------------------------------
B# = x#(7)
```

```
x = .1 * B#
x2 = .1 * B#
CLS
FOR i = 1 TO ntrial
  f# = dStrainEqns#(2, 1, x#(), x)
  df# = FirstDerv#(2, 1, x#(), x)
  x = x - f# / df#
  LOCATE 1, 1
  PRINT "i="; i
  PRINT "x2a="; x; " Strain*10^6 = "; StrainEqns#(2, x#(), x)
  IF ABS(f# / df#) < 10 ^ -5 THEN EXIT FOR
NEXT i
CALL pause
FOR i = 1 TO ntrial
  f# = dStrainEqns#(2, 2, x#(), x2)
  df# = FirstDerv#(2, 2, x#(), x2)
  x2 = x2 - f# / df#
  LOCATE 1, 1
  PRINT "i="; i
  PRINT "x2b="; x2; " Strain*10^6 = "; StrainEqns#(2, x#(), x2)
  IF ABS(f# / df#) < 10 ^ -5 THEN EXIT FOR
NEXT i
CALL pause
IF StrainEqns#(2, x#(), x) > StrainEqns#(2, x#(), x2) THEN
  x2givenW = x
ELSE
  x2givenW = x2
END IF
END FUNCTION

FUNCTION x3givenW (x#())
'-----------------------------------------------------------------
B# = x#(7)
x = W / 4
x2 = W / 4
CLS
FOR i = 1 TO ntrial
  f# = dStrainEqns#(3, 1, x#(), x)
  df# = FirstDerv#(3, 1, x#(), x)
  x = x - f# / df#
  LOCATE 1, 1
  PRINT "i="; i
  PRINT "x3a="; x; " Strain*10^6 = "; StrainEqns#(3, x#(), x)
  IF ABS(f# / df#) < 10 ^ -5 THEN EXIT FOR
NEXT i
CALL pause
CLS
FOR i = 1 TO ntrial
  f# = dStrainEqns#(3, 2, x#(), x2)
  df# = FirstDerv#(3, 2, x#(), x2)
  x2 = x2 - f# / df#
  LOCATE 1, 1
  PRINT "i="; i
```

```
  PRINT "x3b="; x2; " Strain*10^6 = "; StrainEqns#(3, x#(), x2)
  IF ABS(f# / df#) < 10 ^ -5 THEN EXIT FOR
NEXT i
CALL pause
IF StrainEqns#(3, x#(), x) > StrainEqns#(3, x#(), x2) THEN
  x3givenW = x
ELSE
  x3givenW = x2
END IF

END FUNCTION
```

# Appendix E

# Computer Implementation of the Longitudinal Landslide

```
DEFINT A-Z
'PROGRAM: PLANAR LANDSLIDE:
'CREATOR: PETER D.S. CHAN
'DATE: OCT. 18 '97
'PURPOSE: to find max strain of pipe buried in planar landslide.
'----------------------------------------------------------------
'd# = .1683#
't# = .00318#
'E# = 200 * 10 ^ 9
'H = 2
'coh# = 2 * 10 ^ 3
'gamma# = 18 * 10 ^ 3
'phi# = 30 * pi# / 180
'Ds# = .008#
'L# = 1000#
'delta.unit# = 2.5 * Ds#
'----------------------------------------------------------------
DECLARE SUB LongSlide ()
DECLARE SUB graph (delta.ds#, emax#, delta.min#)
DECLARE FUNCTION next.higher.value# (given.value#)
DECLARE FUNCTION interval.step! (given.value, max.num.divisions)
DECLARE SUB ReadInputFile ()
'---for 2Ds < delta.min < delta < delta.max
DECLARE SUB BisectXa (a#, b#, fa#, fb#)
DECLARE FUNCTION GetDelta.min# (L#, ld#, Ds#)
DECLARE FUNCTION f.xa# (y#)     '---f.xa = eq'm+comp. eqn in terms of xa
DECLARE FUNCTION xb.xa# (y#)    '---xb.xa = xb compatibility eqn in terms of xa
DECLARE FUNCTION emax.xa# (y#)   '---emax.xa = max strain eqn in terms of xa
DECLARE FUNCTION f.deltamin# (y#)  '---eqn for find delta.min
DECLARE SUB initvalues ()
'---for delta < 2Ds
DECLARE SUB BisectUo (a#, b#, fa#, fb#)
DECLARE FUNCTION f.uo# (y#)
```

```
DECLARE FUNCTION emax.uo# (y#)   '—emax.uo = max strain eqn in terms of uo
'—Menu Handling—
DECLARE SUB Initialize ()
DECLARE SUB Menu1 ()
DECLARE SUB CheckScreen ()
DECLARE SUB DrawFrame (TopSide, BottomSide, LeftSide, RightSide)
DECLARE SUB BoxInit ()
DECLARE SUB printmenustring (menustring$, char.pos%, num.chars.hilited%)
'--Values for keys on the numeric keypad and the spacebar:
CONST UP = 72, DOWN = 80, LFT = 75, RGHT = 77
CONST UPLFT = 71, UPRGHT = 73, DOWNLFT = 79, DOWNRGHT = 81
CONST SPACEBAR = " "
CONST F1 = 59, F2 = 60, F3 = 61, F4 = 62, F5 = 63
CONST F6 = 64, F7 = 65, F8 = 66, F9 = 67, F10 = 68

'-Null$ is the first character of the two-character INKEY$
'-value returned for direction keys such as UP and DOWN:
'NULL$ = CHR$(0): ESCAPE$ = CHR$(27): ENTER$ = CHR$(13)
CONST ESCAPE = 27, ENTER = 13, NULL = 0
CONST FALSE = 0, TRUE = NOT FALSE

DIM SHARED InitRow, MaxRow, MaxColors, Foreground, Background, Hilite, selection
DIM SHARED d#, t#, E#, H, coh#, gamma#, phi.deg#, phi.rad#, Ds#, L#, L.min#
DIM SHARED delta.ds#, delta.unit#, delta.min#, delta.max#
DIM SHARED angle.rad#, angle.deg#, delta.unit.parallel#, delta.ds.parallel#
DIM SHARED ks#, ld#, Area#, runlong3f, runlong6b
DIM SHARED outfilename$, DeltaSwitch
CLS

ReadInputFile
initvalues

CONST mnuStringLength = 51
'==========================================================
TYPE mnuType
   mnuString AS STRING * mnuStringLength
   mnuHotKey AS INTEGER       '—position of the hotkey
   mnuHiliteWidth AS INTEGER  '—number of chars displayed for the hotkey
END TYPE
'----main menu items: string and position of hilite----
CONST MainMenuItems = 17
DIM SHARED MainMenu(MainMenuItems) AS mnuType
DATA "1)                       pipe diameter(m): ",1,1
DATA "2)                      pipe thickness(m): ",1,1
DATA "3)        pipe modulus of elasticity(Pa/m): ",1,1
DATA "4)               depth to center of pipe(m): ",1,1
DATA "5)                  soil cohesion(N/m^2): ",1,1
DATA "6)                   soil density(N/m^3): ",1,1
DATA "7)     soil internal frictional angle(degrees): ",1,1
DATA "8)  limit elastic displacement of soil(Ds (m)): ",1,1
DATA "9)                  length of landslide (m): ",1,1
DATA "0) landslide magnitude(in multiples of Ds)      : ",1,1
DATA "F1) landslide magnitude entry toggle            : ",1,2
```

```
DATA "a)ngle between landslide and pipe orientation(deg): ",1,1
DATA "s)ave input values to 'long.dat'          : ",1,1
DATA "f)ilename in which the output is stored         : ",1,1
DATA "g)raph                              : ",1,1
DATA "F10) Calculate                          : ",1,3
DATA "q)uit                              : ",1,1
FOR i = 1 TO MainMenuItems
   READ MainMenu(i).mnuString: READ MainMenu(i).mnuHotKey
   READ MainMenu(i).mnuHiliteWidth
NEXT i
'---Box for main menu---
CONST Box.Top = 2, Box.Bottom = MainMenuItems + Box.Top + 1, Box.Left = 1, Box.Right = 78

CONST PI# = 3.141592653589793#
CONST expon# = 2.718281828459045#
CONST No = 0, Yes = 1

Initialize
Menu1
WIDTH 80, InitRow      '-restore original number of rows
COLOR 7, 0           '-restore default color
CLS

END

'------------------------------------------------------------
' GetRow, MonoTrap, and RowTrap are error-handling routines invoked by
' the CheckScreen SUB procedure.  GetRow determines whether the program
' started with 25, 43, or 50 lines.  MonoTrap determines the current
' video adapter is monochrome.  RowTrap sets the maximum possible
' number of rows (43 or 25).

GetRow:
  IF InitRow = 50 THEN
    InitRow = 43
    RESUME
  ELSE
    InitRow = 25
    RESUME NEXT
  END IF

MonoTrap:
  MaxColors = 2
  RESUME NEXT

RowTrap:
  MaxRow = 25
  RESUME

SUB BisectUo (a#, b#, fa#, fb#)
'---for delta < 2Ds
'-------------------------------------------------------
SHARED TOL#, xa.max#, L#, Ds#, delta.ds#, angle.deg#
```

```
runflag1 = 1

fa# = f.uo#(a#)
fb# = f.uo#(b#)
wide# = b# - a#

'---Do bisection once the root is bracketed.---
'CLS
LOCATE 1, 1
PRINT USING "bisecting... L = #######.##, delta/Ds = #######.###, angle = ##.##"; L#;
delta.ds#; angle.deg#
IF (fa# = fb#) THEN
   runflag1 = 0
END IF
WHILE runflag1 <> 0
   LOCATE 2, 1
   PRINT "iteration ="; runflag1
   wide# = wide# / 2
   c# = (b# + a#) / 2
   LOCATE 3, 1
   PRINT USING "c =###.###############"; c#
   fc# = f.uo#(c#)
   IF fa# * fc# <= 0 THEN
      LOCATE 4, 1
      PRINT "root exists inside interval:    a and c"
      PRINT USING "a =###.###############   fa =#.###############    "; a#; fa#
      PRINT USING "c =###.###############   fc =#.###############    "; c#; fc#
      PRINT USING "b =###.###############   fb =#.###############    "; b#; fb#
      IF (fa# = 0 OR fb# = 0) THEN
        IF fa# = 0 THEN
           b# = a#
        ELSE
        END IF
      ELSE
        b# = c#
        fb# = fc#
        LOCATE 8, 1
        PRINT "New..."
        PRINT USING "b =###.###############   fb =#.###############    "; b#; fb#
      END IF
   ELSEIF fc# * fb# <= 0 THEN
      LOCATE 4, 1
      PRINT "root exists inside interval:    c and b"
      PRINT USING "a =###.###############   fa =#.###############    "; a#; fa#
      PRINT USING "c =###.###############   fc =#.###############    "; c#; fc#
      PRINT USING "b =###.###############   fb =#.###############    "; b#; fb#
      a# = c#
      fa# = fc#
      LOCATE 8, 1
      PRINT "New..."
      PRINT USING "a =###.###############   fa =#.###############    "; a#; fa#
   ELSE
```

```
            LOCATE 8, 1
            PRINT "root is not bracketed..."
            END
        END IF
        IF ABS(fb# - 0) < TOL# THEN
            runflag1 = 0
        ELSE
            runflag1 = runflag1 + 1
        END IF
        IF wide# < 10 ^ -15 THEN
            LOCATE 9, 1
            PRINT USING "a=b= ###.############### =  ###.###############"; a#; b#
            runflag1 = 0
        END IF

    WEND

END SUB

SUB BisectXa (a#, b#, fa#, fb#)
SHARED TOL#, xa.max#, L#, Ds#, delta.ds#, angle.deg#
'---For delta >2Ds
'---This subroutine finds the xa for which the force+comp. equation(in function
'---f.xa) is =0.
'---To avoid imaginary numbers inside the sqrt terms of the governing equation
'---(see function "f.xa"), xa < 1/ld*(1-sqrt(1-(5/2-delta/Ds))) --->this is
'--- the "xa.max"
'-----------------------------------------------------------------------
'----NOTE: a < c < b

    runflag2 = 1
    runflag1 = 1

    fa# = f.xa#(a#)
    fb# = f.xa#(b#)
    wide# = b# - a#

    '---reset TOL#: double precision gives ones followed by 15 decimal places
    '---, tens with 14 decimal places, and hundreds with 13 decimal places...etc

    '---check bracketing of zero by "a" and "b"---
    'CLS
    LOCATE 1, 1
    PRINT USING "bracketing...   xa.max= ###.###############              "; xa.max#
    WHILE runflag2 <> 0
        IF (fa# * fb# <= 0 OR fa# = fb#) THEN '---zero is bracketed by "a" and "b"
            runflag2 = 0
        ELSE                    '---zero is not bracketed
            wide# = b# - a#
            IF fa# > 0 THEN     '---both fa# and fb# are positive
                IF fa# > fb# THEN      '---graph is sloping down to right
                    'b# = b# + wide#       '---try to drive fb# into negative
                    b# = b# * .999999999999999#
```

```
            WHILE b# > xa.max#
               b# = b# * 1.01
            WEND
         ELSE                  '---graph is climbing up to right
            a# = a# - wide#
         END IF
      ELSE                   '---both fa# and fb# are negative
         IF fa# > fb# THEN     '---graph is sloping down to right
            a# = a# - wide#
         ELSE
            'b# = b# + wide#
            b# = b# * .999999999999999#
            WHILE b# > xa.max#
               b# = b# * 1.000000000000001#
            WEND
         END IF
      END IF
      fa# = f.xa#(a#)
      fb# = f.xa#(b#)
      runflag2# = runflag2# + 1
      LOCATE 2, 3
      PRINT "iteration: "; runflag2#
      PRINT "a="; a#; "   fa ="; fa#
      PRINT "b="; b#; "   fb ="; fb#
   END IF
   IF wide# < 10 ^ -15 THEN
      LOCATE 9, 1
      PRINT USING "a=b= ###.############### =  ###.###############"; a#; b#
      'BEEP
      runflag2 = 0
      'END
   END IF
WEND

'---Do bisection once the root is bracketed.---
'CLS
LOCATE 1, 1
PRINT USING "bisecting...  L = #######.##, delta/Ds = #######.###, angle = ##.##"; L#;
delta.ds#; angle.deg#
IF (fa# = fb#) THEN
   runflag1 = 0
END IF
WHILE runflag1 <> 0
   LOCATE 2, 1
   PRINT "iteration ="; runflag1
   wide# = wide# / 2
   c# = (b# + a#) / 2
   LOCATE 3, 1
   PRINT USING "c =###.###############"; c#
   fc# = f.xa#(c#)
   IF fa# * fc# <= 0 THEN
      LOCATE 4, 1
      PRINT "root exists inside interval:   a and c"
```

```
        PRINT USING "a =###.###############   fa =#.###############   "; a#; fa#
        PRINT USING "c =###.###############   fc =#.###############   "; c#; fc#
        PRINT USING "b =###.###############   fb =#.###############   "; b#; fb#
        IF (fa# = 0 OR fb# = 0) THEN
          IF fa# = 0 THEN
             b# = a#
          ELSE
          END IF
        ELSE
          b# = c#
          fb# = fc#
          LOCATE 8, 1
          PRINT "New ..."
          PRINT USING "b =###.###############   fb =#.###############   "; b#; fb#
        END IF
     ELSEIF fc# * fb# <= 0 THEN
        LOCATE 4, 1
        PRINT "root exists inside interval:    c and b"
        PRINT USING "a =###.###############   fa =#.###############   "; a#; fa#
        PRINT USING "c =###.###############   fc =#.###############   "; c#; fc#
        PRINT USING "b =###.###############   fb =#.###############   "; b#; fb#
        a# = c#
        fa# = fc#
        LOCATE 8, 1
        PRINT "New..."
        PRINT USING "a =###.###############   fa =#.###############   "; a#; fa#
     ELSE
        LOCATE 8, 1
        PRINT "root is not bracketed..."
        END
     END IF
     IF ABS(fb# - 0) < TOL# THEN
       runflag1 = 0
     ELSE
       runflag1 = runflag1 + 1
     END IF
     IF wide# < 10 ^ -15 THEN
       LOCATE 9, 1
       PRINT USING "a=b= ###.############### =  ###.###############"; a#; b#
       'BEEP
       'END
       runflag1 = 0
     END IF

  WEND

END SUB

SUB BoxInit STATIC
'============================ BoxInit
================================
'  Calls the DrawFrame procedure to draw the frame around the sort menu,
'  then prints the different options stored in the OptionTitle array.
```

```
'
=====================================================================
===
'
  DrawFrame Box.Top, Box.Bottom, Box.Left, Box.Right

  LOCATE Box.Top - 1, Box.Left + 5: PRINT "Pipe Strain Program (copyrighted 1996 Peter D.S.
Chan)";
  LOCATE Box.Top + 1
  FOR i = 1 TO MainMenuItems
    LOCATE , Box.Left + 2
    IF i = selection THEN
      Foreground = 0: Background = 7: Hilite = 10 '---reverse video
    ELSE
      Foreground = 7: Background = 0: Hilite = 15
    END IF
    printmenustring MainMenu(i).mnuString, MainMenu(i).mnuHotKey,
MainMenu(i).mnuHiliteWidth
  NEXT i
  Foreground = 7: Background = 0  '---Turn off reverse video
  COLOR Foreground, Background
'-------------------------------
  LOCATE Box.Top + 1, mnuStringLength + 3
   PRINT USING "#######.######"; d#
  LOCATE Box.Top + 2, mnuStringLength + 3
   PRINT USING "#######.######"; t#
  LOCATE Box.Top + 3, mnuStringLength + 3
   PRINT USING "    ###,.##^^^^"; E#
  LOCATE Box.Top + 4, mnuStringLength + 3
   PRINT USING "#######.##"; H
  LOCATE Box.Top + 5, mnuStringLength + 3
   PRINT USING "######,.##"; coh#
  LOCATE Box.Top + 6, mnuStringLength + 3
   PRINT USING "######,.##"; gamma#
  LOCATE Box.Top + 7, mnuStringLength + 3
   PRINT USING "#######.##"; phi.deg#
  LOCATE Box.Top + 8, mnuStringLength + 3
   PRINT USING "#######.######"; Ds#
  LOCATE Box.Top + 9, mnuStringLength + 3
   PRINT USING "#######.##"; L#
  LOCATE Box.Top + 10, mnuStringLength + 3
   IF DeltaSwitch = FALSE THEN
      PRINT USING "#######.###"; delta.ds#
      LOCATE Box.Top + 11, mnuStringLength + 3
      PRINT "{manual |      }"
   ELSE
      PRINT USING "0.00 to ###.######"; delta.max# / Ds#
      LOCATE Box.Top + 11, mnuStringLength + 3
      PRINT "{       | auto }"
   END IF
  LOCATE Box.Top + 12, mnuStringLength + 3
   PRINT USING "    ##.##"; angle.deg#
  LOCATE Box.Top + 14, mnuStringLength + 3
```

```
    PRINT USING " &"; outfilename$

    '---for printing input statements-----
    LOCATE 23, 1
    PRINT "                                        ";
    LOCATE 23, 1

END SUB


SUB CheckScreen STATIC
'============================== CheckScreen
==================================
'   Checks for type of monitor (VGA, EGA, CGA, or monochrome) and
'   starting number of screen lines (50, 43, or 25).
'
================================================================
===
'
   'SCREEN 0
   'WIDTH 80, 50
   '---Find out what how many rows the user was using initially.
   '-Try locating to the 50th row; if that fails, try the 43rd. Finally,
   '-if that fails, the user was using 25-line mode:
   InitRow = 50
   ON ERROR GOTO GetRow
   LOCATE InitRow, 1

   ' Try a SCREEN 11 statement to see if the current adapter has color
   ' graphics; if that causes an error, reset MaxColors to 2:
   MaxColors = 15
   ON ERROR GOTO MonoTrap
   SCREEN 11
   SCREEN 0

   '-See if 43-line mode is accepted; if not, run this program in 25-line
   '-mode:
   'MaxRow = 43
   MaxRow = 25
   ON ERROR GOTO RowTrap
   WIDTH 80, MaxRow
   ON ERROR GOTO 0          ' Turn off error trapping.

END SUB

SUB DrawFrame (TopSide, BottomSide, LeftSide, RightSide)
'============================== DrawFrame
====================================
'  Draws a rectangular frame using the high-order ASCII characters É (201) ,
'  » (187) , È (200) , ¼ (188) , º (186) , and Í (205). The parameters
'  TopSide, BottomSide, LeftSide, and RightSide are the row and column
'  arguments for the upper-left and lower-right corners of the frame.
'  Additional:Ì (204), ¹(185)
```

```
'
'==================================================================
===
'
  CONST ULEFT = 201, URIGHT = 187, LLEFT = 200, LRIGHT = 188
  CONST VERTICAL = 186, HORIZONTAL = 205
  CONST BEAMLEFT = 204, BEAMRIGHT = 185

  FrameWidth = RightSide - LeftSide - 1
  LOCATE TopSide, LeftSide
  PRINT CHR$(ULEFT); STRING$(FrameWidth, HORIZONTAL); CHR$(URIGHT);
  FOR Row = TopSide + 1 TO BottomSide - 1
    LOCATE Row, LeftSide
    PRINT CHR$(VERTICAL); SPC(FrameWidth); CHR$(VERTICAL);
  NEXT Row
  LOCATE BottomSide, LeftSide
  PRINT CHR$(LLEFT); STRING$(FrameWidth, HORIZONTAL); CHR$(LRIGHT);

END SUB

FUNCTION emax.uo# (uo#)
'---for delta < 2Ds
'--------------------------------------------------
SHARED ld#, Ds#

alfa# = 89.98999999999999# * (PI# / 180#)
xi# = -LOG(1 - 2# * uo# * ld# / TAN(alfa#)) / (2# * ld#)

p1# = (uo# - TAN(alfa#) / (2# * ld#)) * (expon# ^ (ld# * xi#))
p2# = TAN(alfa#) / (2# * ld#) * expon# ^ (-ld# * xi#)
emax.uo# = ld# * (p1# - p2#) + TAN(alfa#)
END FUNCTION

FUNCTION emax.xa# (xa#)
'---the max strain for delta > 2Ds
'--------------------------------
SHARED ld#, Ds#
   emax.xa# = Ds# * (-xa# * ld# ^ 2 + ld#)
END FUNCTION

FUNCTION f.deltamin# (delta.min#)
'---For delta >2Ds
'---This is the force+comp. equation with xa=0 substituted in.
'------------------------------------------------------
SHARED L#, ld#, Ds#
   b# = (5 - 2 * delta.min# / Ds#) ^ .5
   f.deltamin# = (expon# ^ (2 * b# - 2 + L# * ld#) + 1) * (1 - b#) - 2
END FUNCTION

FUNCTION f.uo# (uo#)
'---for delta < 2Ds
'---note: e = 2.718281828459045
'--------------------------------------------------
```

```
SHARED ld#, Ds#, delta.unit.parallel#, L#

alfa# = 89.98999999999999# * (PI# / 180#)
'xr# = delta.unit# / TAN(alfa#)
xr# = delta.unit.parallel# / TAN(alfa#)
xi# = -LOG(1# - 2# * uo# * ld# / TAN(alfa#)) / (2# * ld#)
'---note: LOG is basic is the natural logarithm "LN"
p1# = uo# * expon# ^ (ld# * (L# + xr#))
'p1# = uo#

p2a# = ld# ^ 2 * xi# ^ 2 * expon# ^ (ld# * (L# / 2# + xr#))
p2b# = -expon# ^ (ld# * (xr# + L#))
p2c# = expon# ^ (ld# * L#) + expon# ^ (2# * ld# * xr#) - expon# ^ (ld# * xr#)

p2# = TAN(alfa#) / (2 * ld#) * (p2a# + p2b# + p2c#)
'p2# = TAN(alfa#) / (2 * ld#) * (p2a# + p2b# + p2c#) / expon#^(ld# * (L# + xr#))

f.uo# = p1# + p2#

END FUNCTION

FUNCTION f.xa# (xa#)
'---For delta >2Ds
'---this is the force+comp. equation in terms of 'xa'
'---Idea is to balance the forces with the comp. conditions in finding the xa
'---for which this function 'f.xa#' = 0
'-----------------------------------------------------------------------
SHARED ld#, Ds#, delta.unit.parallel#, L#

'p1# = (2 * xa# ^ 2) - (4 / ld# * xa#) + (5 / ld# ^ 2) - (2 / (Ds# * ld# ^ 2) * delta.unit#)
p1# = (2 * xa# ^ 2) - (4 / ld# * xa#) + (5 / ld# ^ 2) - (2 / (Ds# * ld# ^ 2) * delta.unit.parallel#)
'IF (p1#) < 10 ^ -(13 - LOG(L#) / LOG(10)) THEN
IF p1# < 0 THEN
      p1# = 0
END IF
'---limitation EXP(n) has the limit of n <= 88.02969
'---expon#(n) =2.718281828459045#(n) has the limit of n <= 709.78
p1b# = ld# * (2 * xa# - 2 / ld# + 2 * (p1#) ^ .5 + L#)
IF p1b# > 709.78 THEN
   LOCATE 23, 1
   PRINT "                                                    ";
   LOCATE 23, 1
   PRINT "Overflow: L is too large!"
   LOCATE 24, 1
   PRINT "Press any key to continue...";
   DO
   LOOP WHILE INKEY$ = ""
   END
END IF

eq# = expon# ^ (p1b#) + 1

p2# = (2 * ld# ^ 2 * xa# ^ 2) - (4 * xa# * ld#) + (5) - (2 / Ds# * delta.unit.parallel#)
```

```
IF p2# < 0 THEN
      p2# = 0
END IF
f.xa# = eq# * (1 - (p2#) ^ .5) - 2


END FUNCTION

FUNCTION GetDelta.min# (L#, Id#, Ds#)
'---For delta >2Ds
'---this function is to find the delta.min (at which xa=0)
'---It finds the root of the force+comp. equation for which xa=0 (equation is
'---located in f.deltamin function.
'---Note: the min. L for which forces will balance is 2/Id#: this is found
'---from the force+comp. equation with xa=0 and xb=L/2 --> subst and solve for L.
'-----------------------------------------------------------------------
   a# = 2.5 * Ds# '---this is the absolute max delta for xa=0: above this you'ld get complex
numbers
   b# = 2 * Ds#   '---this is the abs min delta for xa=0
   fa# = f.deltamin#(a#)
   fb# = f.deltamin#(b#)
   wide# = a# - b#
   T "getting delta.min..."
   runflag1 = 1
   IF (fa# = fb#) THEN
      runflag1 = 0
   END IF
   WHILE runflag1 <> 0
      wide# = wide# / 2
      c# = (b# + a#) / 2
      fc# = f.deltamin#(c#)
      IF fa# * fc# <= 0 THEN
         IF (fa# = 0 OR fb# = 0) THEN
           IF fa# = 0 THEN
              b# = a#
           ELSE
           END IF
         ELSE
           b# = c#
           fb# = fc#
         END IF
      ELSEIF fc# * fb# <= 0 THEN
         a# = c#
         fa# = fc#
      END IF
      IF ABS(fb# - 0) < TOL# THEN
         runflag1 = 0
      ELSE
         runflag1 = runflag1 + 1
      END IF
      IF wide# < 10 ^ -15 THEN
         runflag1 = 0
      END IF
```

```
   WEND

   IF ABS(fa#) < ABS(fb#) THEN
      GetDelta.min# = a#
   ELSE
      GetDelta.min# = b#
   END IF

END FUNCTION

SUB graph (delta.ds#, emax#, delta.min#)
SHARED Ds#, outfilename$
Style% = &HFF00               ' Use to make dashed line.

'---get maximum of all delta.ds# and emax# (i.e. the last data):begin
OPEN outfilename$ FOR INPUT AS #1
blank$ = INPUT$(44, #1)
INPUT #1, landslide#  '---...reading the length of landslide.
FOR n = 2 TO 9
  LINE INPUT #1, blank$
NEXT n
blank$ = INPUT$(51, #1)
INPUT #1, angle.deg#  '---...reading the angle between landslide and pipeline.
FOR n = 11 TO 12
  LINE INPUT #1, blank$
NEXT n

n = 0
DO
  INPUT #1, delta.ds#, emax#
LOOP UNTIL EOF(1)
CLOSE #1
delta.ds.max# = delta.ds#
emax.max# = emax#
'---get maximum of all delta.ds# and emax# (i.e. the last data):end


SCREEN 12
WIDTH 80, 60

' View port sized to proper scale for graph:
'---VIEW makes: (0,0) at top left of screen
'VIEW (60, 30)-(620, 400), , 16
view.x.start = 60
view.y.start = 50
view.x.end = 600
view.y.end = 390
VIEW (view.x.start, view.y.start)-(view.x.end, view.y.end), , 16

'---Make window large enough for strain from 0 to emax, and delta from 0 to delta.ds.max
'---WINDOW makes: (0,0) at bottom left of screen
window.x.end = delta.ds.max#
```

```
window.y.end = next.higher.value(INT(emax.max#) + 1)

WINDOW (0, 0)-(window.x.end, window.y.end)


OPEN outfilename$ FOR INPUT AS #1
FOR n = 1 TO 11
  LINE INPUT #1, blank$
NEXT n
n = 0

CLS
LINE (0, 0)-(0, 0)
DO
  INPUT #1, delta.ds#, emax#
  x# = delta.ds#
  y# = emax#   ' Calculate the y coordinate.
  IF ABS(delta.ds# - delta.min# / Ds#) <= 10 ^ -4 THEN
     LINE (x#, y#)-(x#, y#), 11
  ELSE
     LINE -(x#, y#), 11     ' Draw a line from the last
                            ' point to the new point.
  END IF
LOOP UNTIL EOF(1)
CLOSE #1

'---draw x-axis ticks----------------------------------------------
'---height of tick is 1/100 of the height of window y-axis
window.x.tick.height = window.y.end
window.x.tick.step! = interval.step!(window.x.end, 15)
window.x.tick.begin! = window.x.tick.step!
FOR n = window.x.tick.begin! TO window.x.end STEP window.x.tick.step!
   LINE (n, 0)-(n, window.x.tick.height), 7, , Style%
NEXT n

'---draw y-axis ticks----------------------------------------------
'---height of tick is 1/100 of the width of window y-axis
window.y.tick.height = window.x.end    '---draw dashed line across graph.
window.y.tick.step! = interval.step!(window.y.end, 15)
window.y.tick.begin! = window.y.tick.step!
FOR n = window.y.tick.begin! TO window.y.end STEP window.y.tick.step!
   LINE (0, n)-(window.y.tick.height, n), 7, , Style%
NEXT n

'---Label x-axis data values----------------------------------------
'---note: in screen mode 11 or 12
'---x-axis: 640pixels/80text = 8pixels/txt
'---y-axis: 480pixels/30text = 16pixels/txt or  480pixels/60text = 8pixels/txt
LOCATE 51, 8: PRINT "0"
prev.col = 8
FOR n! = window.x.tick.begin! TO window.x.end STEP window.x.tick.step!
   col = (view.x.start + n! / window.x.end * (view.x.end - view.x.start)) / 8
   LOCATE 51, col
```

```
   IF prev.col <> col THEN
      'PRINT USING "##"; n!
      PRINT n!
      prev.col = col
   END IF
NEXT n!

'---Label y-axis data values-------------------------------------------
prev.row = 25
FOR n! = window.y.tick.begin! TO window.y.end STEP window.y.tick.step!
   Row = (view.y.end - (n! / window.y.end) * (view.y.end - view.y.start)) / 8 + 1
   LOCATE Row, 1
   IF prev.row <> Row THEN
      PRINT n!
      prev.row = Row
   END IF
NEXT n!

'---label x and y axis, and title.
LOCATE 2, 10: PRINT "Maximum Pipe uStrain(longitudinal) vs Amount of Landslide"
LOCATE 5, 25: PRINT USING "L=#### , angle = ##.## "; landslide#; angle.deg#
LOCATE 55, 15: PRINT USING "Amount of Landslide in Multiples of #.#### (m)"; Ds#
LOCATE 5, 1: PRINT "uStrain"

DO
LOOP WHILE INKEY$ = ""
SCREEN 0
END SUB

SUB Initialize
   selection = 1
   CheckScreen
   BoxInit

END SUB

SUB initvalues
'SHARED d#, t#, E#, H, coh#, gamma#, phi.deg#, phi.rad#, Ds#, L#, L.min#
'SHARED delta.ds#, delta.unit#, delta.min#, delta.max#
'SHARED angle.rad#, angle.deg#, delta.unit.parallel#, delta.ds.parallel#
'SHARED ks#, ld#, Area#, runlong3f, runlong6b
      angle.rad# = angle.deg# * PI# / 180
      phi.rad# = phi.deg# * PI# / 180
      'delta.unit# = in meters, delta.ds# = in multiples of Ds
      delta.unit# = (delta.ds# * Ds#)
      delta.unit.parallel# = delta.unit# * COS(angle.rad#)
      delta.ds.parallel# = delta.unit.parallel# / Ds#
      'I# = pi# / 4 * ((d# / 2) ^ 4 - (d# / 2 - t#) ^ 4)
      Area# = PI# * ((d# / 2) ^ 2 - (d# / 2 - t#) ^ 2)
      ks# = (coh# + gamma# * H * (.9# * TAN(phi.rad#))) / Ds#
      ld# = ((ks# * PI# * d#) / (E# * Area#)) ^ .5
      L.min# = 2 / ld#
      IF L# < L.min# THEN L# = L.min#
```

```
    '---for a pipe lying at an angle, the delta required to achieve the
    '---same delta(for pipe lying parallel to delta) is delta/cos(angle)
    '--- -- which is larger.
    delta.max# = 1 / 4 * (L# ^ 2 * ld# ^ 2 + 6) * Ds# / COS(angle.rad#)
    delta.min# = GetDelta.min#(L#, ld#, Ds#) / COS(angle.rad#)

    runlong3f = FALSE
    runlong6b = TRUE '---TRUE = on and FALSE = off
    IF delta.ds# * COS(angle.rad#) - 2 <= 10 ^ -14 THEN
       runlong3f = TRUE
       runlong6b = FALSE
    ELSEIF delta.unit# < delta.min# THEN
       delta.unit# = delta.min#
    ELSEIF delta.unit# > delta.max# THEN
       delta.unit# = delta.max#
    END IF
    delta.unit.parallel# = delta.unit# * COS(angle.rad#)
    delta.ds.parallel# = delta.unit.parallel# / Ds#
    delta.ds# = delta.unit# / Ds#
END SUB

FUNCTION interval.step! (given.value, max.num.divisions)
'---Num of divisions should be <= 15
FOR n = 0 TO 10
   num.divisions! = given.value / 10 ^ n       '---  10^n =1,10,100...etc
   IF num.divisions! <= max.num.divisions THEN
      interval.step! = 10 ^ n
      EXIT FOR
   END IF
   num.divisions! = given.value / (2 * 10 ^ n)  '---2*10^n =2,20,200...etc
   IF num.divisions! <= max.num.divisions THEN
      interval.step! = 2 * 10 ^ n
      EXIT FOR
   END IF
   num.divisions! = given.value / (5 * 10 ^ n)  '---5*10^n =5,50,500...etc
   IF num.divisions! <= max.num.divisions THEN
      interval.step! = 5 * 10 ^ n
      EXIT FOR
   END IF
NEXT n

END FUNCTION

SUB LongSlide

'---max. precision for double precision is 15 decimal places---
'---so, the max TOL# =10^-14, because the program will not be able to compare
'---the 16th decimal places if you set TOL#=10^-15.
TOL# = 10 ^ -13#

IF DeltaSwitch = FALSE THEN '---manual delta input
  CLS
  IF runlong6b THEN
```

```
    '------test f.xa#------
    '---to avoid imaginary numbers inside the sqrt terms of the governing equation
    '---(see function "f.xa"), xa < 1/ld*(1-sqrt(1-(5/2-delta/Ds)))
    xa.max# = (1 / ld#) * (1 - (delta.unit.parallel# / Ds# - 1.5) ^ .5)
    xa.min# = 1 / ld# - L# / 2
    b# = xa.max#
    a# = xa.min#
    CALL BisectXa(a#, b#, fa#, fb#)

    IF ABS(fa#) < ABS(fb#) THEN
      xa# = a#
    ELSE
      xa# = b#
    END IF
    PRINT "after BisectXa..."
    PRINT "   a="; a#; "      fa="; fa#
    PRINT "   b="; b#; "      fb="; fb#
    PRINT "  xa="; xa#; "   f(xa)="; f.xa#(xa#)
    PRINT
    PRINT "L= "; L#
    PRINT "Delta.min/Ds ="; delta.min# / Ds#; ",    Delta.max/Ds ="; delta.max# / Ds#;
    PRINT "Delta/Ds ="; delta.ds#
    PRINT "xa.min ="; xa.min#; ",  xa.max ="; xa.max#
    PRINT "xa= "; xa#; ",   f(xa)= "; f.xa#(xa#)
    PRINT "xb ="; xb.xa#(xa#)
    PRINT "emax * 10^6 ="; emax.xa#(xa#) * 10 ^ 6
  ELSEIF runlong3f THEN
    b# = delta.unit.parallel#
    a# = 0#
    CALL BisectUo(a#, b#, fa#, fb#)

    IF ABS(fa#) < ABS(fb#) THEN
      uo# = a#
    ELSE
      uo# = b#
    END IF
    PRINT "after BisectUo..."
    PRINT "   a="; a#; "      fa="; fa#
    PRINT "   b="; b#; "      fb="; fb#
    PRINT "  uo="; uo#; "   f(uo)="; f.uo#(uo#)
    PRINT
    PRINT "L= "; L#
    PRINT "Delta/Ds ="; delta.ds#
    PRINT "uo= "; uo#; ",   f(uo)= "; f.uo#(uo#)
    PRINT "emax * 10^6 ="; emax.uo#(uo#) * 10 ^ 6
  END IF
END IF
OPEN outfilename$ FOR OUTPUT AS #3
      PRINT #3, USING "L                                   = #######.##"; L#
      PRINT #3, USING "delta.max/Ds                            = #######.#####"; delta.max# / Ds#
      PRINT #3, USING "delta/Ds                              = #######.#####"; delta.ds#
      PRINT #3, USING "pipe diameter(m)                        = #######.######"; d#
      PRINT #3, USING "pipe thickness(m)                       = #######.######"; t#
```

```
      PRINT #3, USING "pipe modulus of elasticity(Pa/m)          =    ###,.##^^^^"; E#
      PRINT #3, USING "depth to center of pipe(m)               = #######.##"; H
      PRINT #3, USING "soil cohesion(N/m^2)                     = ######,.##"; coh#
      PRINT #3, USING "soil density(N/m^3)                      = ######,.##"; gamma#
      PRINT #3, USING "soil internal frictional angle(degrees)   = #######.##"; phi.deg#
      PRINT #3, USING "limit elastic displacement of soil(Ds (m)) = #######.######"; Ds#
      PRINT #3, USING "angle between landslide and pipe orientation(deg) = ##.##"; angle.deg#
      PRINT #3, "xa.min ="; xa.min#; "  xa.max ="; xa.max#
      PRINT #3, "xa= "; xa#; ",   f(xa)= "; f.xa#(b#)
      PRINT #3, "xb ="; xb.xa#(xa#)
      PRINT #3, "emax * 10^6 ="; emax.xa#(xa#) * 10 ^ 6
CLOSE #3
IF DeltaSwitch = TRUE THEN   '---auto delta input
   CLS
   OPEN outfilename$ FOR OUTPUT AS #3  '---to clear content of file for new
   CLOSE #3                           '---storage.
   OPEN outfilename$ FOR APPEND AS #3
      PRINT #3, USING "L                                        = #######.##"; L#
      PRINT #3, USING "pipe diameter(m)                         = #######.######"; d#
      PRINT #3, USING "pipe thickness(m)                        = #######.######"; t#
      PRINT #3, USING "pipe modulus of elasticity(Pa/m)          =    ###,.##^^^^"; E#
      PRINT #3, USING "depth to center of pipe(m)               = #######.##"; H
      PRINT #3, USING "soil cohesion(N/m^2)                     = ######,.##"; coh#
      PRINT #3, USING "soil density(N/m^3)                      = ######,.##"; gamma#
      PRINT #3, USING "soil internal frictional angle(degrees)   = #######.##"; phi.deg#

      PRINT #3, USING "limit elastic displacement of soil(Ds (m)) = #######.######"; Ds#
      PRINT #3, USING "angle between landslide and pipe orientation(deg) = ##.##"; angle.deg#
      PRINT #3,
      PRINT #3, "  delta/Ds      Max pipe strain*10^6"

   '---0 <delta < 2Ds=2*0.008
   FOR n# = 0 TO (2 * Ds#) / COS(angle.rad#) STEP ((Ds# / 4) / COS(angle.rad#))
      delta.ds# = n# / Ds#
      CALL initvalues
      b# = delta.unit.parallel#
      a# = 0#
      CALL BisectUo(a#, b#, fa#, fb#)
      IF ABS(fa#) < ABS(fb#) THEN
         uo# = a#
      ELSE
         uo# = b#
      END IF
      LOCATE 20, 1
      PRINT "  delta/Ds     Max pipe strain*10^6"
      PRINT USING "#####.#####     #####.#####"; delta.ds#; emax.uo#(uo#) * 10 ^ 6
      PRINT #3, USING "#####.#####     #####.#####"; delta.ds#; emax.uo#(uo#) * 10 ^ 6
   NEXT n#
   delta.ds.min.int.rnd.dn% = INT(delta.min# / Ds#)
   delta.ds.max.int.rnd.dn% = INT(delta.max# / Ds#)
   n.end% = delta.ds.max.int.rnd.dn% - (delta.ds.min.int.rnd.dn%) + 2
   SpeedUp.Flag = No
   '---delta.min < delta < delta.max
```

```
  FOR n = 1 TO n.end% STEP (1 / COS(angle.rad#))
    '---speed up calculations after 20 iterations:begin
    IF n > 20 AND SpeedUp.Flag = No THEN
       SpeedUp.Flag = Yes
       SpeedUp.Step = interval.step!(n.end% - n, 20)
       IF SpeedUp.Step > 20 THEN SpeedUp.Step = 20
    END IF
    IF (SpeedUp.Flag = Yes) AND (n.end% - n > SpeedUp.Step) THEN
       n = n + SpeedUp.Step
    END IF
    '---speed up calculations after 20 iterations:end

    SELECT CASE n
    CASE IS = 1
      delta.ds# = delta.min# / Ds#
    CASE IS = n.end%
      delta.ds# = delta.max# / Ds#
    CASE ELSE
      delta.ds# = delta.ds.min.int.rnd.dn% + (n - 1)
    END SELECT
    CALL initvalues
    xa.max# = (1 / ld#) * (1 - (delta.unit.parallel# / Ds# - 1.5) ^ .5)
    xa.min# = 1 / ld# - L# / 2
    b# = xa.max#
    a# = xa.min#
    CALL BisectXa(a#, b#, fa#, fb#)

    IF ABS(fa#) < ABS(fb#) THEN
      xa# = a#
    ELSE
      xa# = b#
    END IF
    LOCATE 20, 1
    PRINT "   delta/Ds     Max pipe strain*10^6"
    emax# = emax.xa#(xa#) * 10 ^ 6
    PRINT USING "#####.#####     #####.#####"; delta.ds#; emax#
    PRINT #3, USING "#####.#####     #####.#####"; delta.ds#; emax#

  NEXT n
  '---get one more delta.ds# to show that max emax# has been reached:start
   delta.ds# = next.higher.value(INT(delta.ds#) + 1)
   emax# = emax#
   PRINT #3, USING "#####.#####     #####.#####"; delta.ds#; emax#
  '---get one more delta.ds# to show that max emax# has been reached:end

  CLOSE #3
END IF

LOCATE 23, 1
PRINT "Press any key to continue...";
DO
LOOP WHILE INKEY$ = ""
```

```
END SUB

SUB Menu1
      'cursor.col = POS(0)
      'cursor.row = CSRLIN

DO
   initvalues.flag = FALSE
   BoxInit
   menuflag = Yes  '---used to refresh menu display until enter key is pressed.
   DO
    KeyVal$ = INKEY$
   LOOP UNTIL KeyVal$ <> ""
   SELECT CASE LEN(KeyVal$)
   CASE 0
     keycode = 0
   CASE 1
     keycode = ASC(KeyVal$)  '---single character
   CASE ELSE
     keycode = -ASC(RIGHT$(KeyVal$, 1))      '---extended character
   END SELECT

  IF KeyVal$ = CHR$(ENTER) THEN
     KeyVal$ = MID$(MainMenu(selection).mnuString, MainMenu(selection).mnuHotKey,
MainMenu(selection).mnuHiliteWidth)
     SELECT CASE KeyVal$
       CASE "F1"
          KeyVal$ = CHR$(NULL) + CHR$(F1)
       CASE "F2"
          KeyVal$ = CHR$(NULL) + CHR$(F2)
       CASE "F3"
          KeyVal$ = CHR$(NULL) + CHR$(F3)
       CASE "F4"
          KeyVal$ = CHR$(NULL) + CHR$(F4)
       CASE "F5"
          KeyVal$ = CHR$(NULL) + CHR$(F5)
       CASE "F6"
          KeyVal$ = CHR$(NULL) + CHR$(F6)
       CASE "F7"
          KeyVal$ = CHR$(NULL) + CHR$(F7)
       CASE "F8"
          KeyVal$ = CHR$(NULL) + CHR$(F8)
       CASE "F9"
          KeyVal$ = CHR$(NULL) + CHR$(F9)
       CASE "F10"
          KeyVal$ = CHR$(NULL) + CHR$(F10)
       CASE ELSE
     END SELECT
  END IF
  SELECT CASE UCASE$(KeyVal$)
    CASE CHR$(NULL) + CHR$(UP)
     selection = selection - 1
     IF selection = 0 THEN selection = MainMenuItems '---jump from top to bottom
```

```
CASE CHR$(NULL) + CHR$(DOWN)
 selection = selection + 1
 IF selection > MainMenuItems THEN selection = 1 '---jump from bottom to top
CASE "1"
 initvalues.flag = TRUE
 selection = 1
 BoxInit
 back# = d#
 INPUT "pipe diameter(m): "; in$
 IF in$ = "" THEN d# = back# ELSE d# = VAL(in$)
CASE "2"
 initvalues.flag = TRUE
 selection = 2
 BoxInit
 back# = t#
 INPUT "pipe thickness(m): "; in$
 IF in$ = "" THEN t# = back# ELSE t# = VAL(in$)
CASE "3"
 initvalues.flag = TRUE
 selection = 3
 BoxInit
 back# = E#
 INPUT "pipe modulus of elasticity(Pa/m): "; in$
 IF in$ = "" THEN E# = back# ELSE E# = VAL(in$)
CASE "4"
 initvalues.flag = TRUE
 selection = 4
 BoxInit
 back# = H
 INPUT "depth to center of pipe(m): "; in$
 IF in$ = "" THEN H = back# ELSE H = VAL(in$)
CASE "5"
 initvalues.flag = TRUE
 selection = 5
 BoxInit
 back# = coh#
 INPUT "soil cohesion(N/m^2): "; in$
 IF in$ = "" THEN coh# = back# ELSE coh# = VAL(in$)
CASE "6"
 initvalues.flag = TRUE
 selection = 6
 BoxInit
 back# = gamma#
 INPUT "soil density(N/m^3): "; in$
 IF in$ = "" THEN gamma# = back# ELSE gamma# = VAL(in$)
CASE "7"
 initvalues.flag = TRUE
 selection = 7
 BoxInit
 back# = phi.deg#
 INPUT "soil internal frictional angle(degrees): "; in$
 IF in$ = "" THEN phi.deg# = back# ELSE phi.deg# = VAL(in$)
CASE "8"
```

```
  initvalues.flag = TRUE
  selection = 8
  BoxInit
  back# = Ds#
  INPUT "limit elastic displacement of soil(Ds (m)): "; in$
  IF in$ = "" THEN Ds# = back# ELSE Ds# = VAL(in$)
CASE "9"
  initvalues.flag = TRUE
  selection = 9
  BoxInit
  back# = L#
  INPUT "length of landslide (m): "; in$
  IF in$ = "" THEN L# = back# ELSE L# = VAL(in$)
CASE "0"
  initvalues.flag = TRUE
  selection = 10
  BoxInit
  back# = delta.ds#
  INPUT "amount of landslide(in multiples of Ds (m): "; in$
  IF in$ = "" THEN delta.ds# = back# ELSE delta.ds# = VAL(in$)
  IF DeltaSwitch = TRUE THEN  '---if was at auto delta input, set it to
    DeltaSwitch = FALSE  '---manual input(as if pressing F1)
  END IF
CASE (CHR$(NULL) + CHR$(F1))
  selection = 11
  BoxInit
  DeltaSwitch = NOT (DeltaSwitch)  '---TOGGLE THE DeltaSwitch
CASE "A"
  initvalues.flag = TRUE
  selection = 12
  BoxInit
  back# = angle.deg#
  INPUT "angle between direction of landslide and the pipeline(degrees): "; in$
  IF in$ = "" THEN angle.deg# = back# ELSE angle.deg# = VAL(in$)
CASE "S"
  selection = 13
  BoxInit
  OPEN "long.dat" FOR OUTPUT AS #2
    PRINT #2, outfilename$
    PRINT #2, d#
    PRINT #2, t#
    PRINT #2, E#
    PRINT #2, H
    PRINT #2, coh#
    PRINT #2, gamma#
    PRINT #2, phi.deg#
    PRINT #2, Ds#
    PRINT #2, L#
    PRINT #2, delta.ds#
    PRINT #2, angle.deg#
    PRINT #2, DeltaSwitch
  CLOSE #2
CASE "F"
```

```
      selection = 14
      BoxInit
      back$ = outfilename$
      INPUT "output filename:"; outfilename$
      IF outfilename$ = "" THEN outfilename$ = back$
    CASE "G"
      selection = 15
      BoxInit
      CALL graph(delta.ds#, emax#, delta.min#)
    CASE CHR$(NULL) + CHR$(F10)
      selection = 16
      BoxInit
      LongSlide
    CASE "Q"
      selection = 17
      BoxInit
      END
    CASE CHR$(ESCAPE)
      selection = 17
      BoxInit
      END
    CASE SPACEBAR
    CASE ELSE
  END SELECT

  IF initvalues.flag = TRUE THEN
      CALL initvalues
  END IF

'------------------
LOOP WHILE menuflag = Yes

END SUB

FUNCTION next.higher.value# (given.value#)
'---function to get the next higher value from the given value in an easy
'---interval for plotting: .5,1,1.5,...10,15,20,...,100,150,200,...etc
FOR power = 0 TO 10
    IF 10 ^ power > given.value# THEN EXIT FOR
NEXT power
power = power - 1
FOR coef! = .5 TO 10 STEP .5
    IF coef! * 10 ^ power >= given.value# THEN EXIT FOR
NEXT coef!
next.higher.value# = coef! * 10 ^ power
END FUNCTION

SUB printmenustring (menustring$, char.pos%, num.chars.hilited%)
  COLOR Foreground, Background: PRINT LEFT$(menustring$, char.pos% - 1);
  COLOR Hilite, Background: PRINT MID$(menustring$, char.pos%, num.chars.hilited%);
  COLOR Foreground, Background: PRINT RIGHT$(menustring$, LEN(menustring$) -
char.pos% - (num.chars.hilited% - 1))
```

```
END SUB

SUB ReadInputFile
PRINT "opening file long.dat"
OPEN "long.dat" FOR INPUT AS #1
   INPUT #1, outfilename$
   INPUT #1, d#
   INPUT #1, t#
   INPUT #1, E#
   INPUT #1, H
   INPUT #1, coh#
   INPUT #1, gamma#
   INPUT #1, phi.deg#
   INPUT #1, Ds#
   INPUT #1, L#
   INPUT #1, delta.ds#
   INPUT #1, angle.deg#  '---angle between direction of landslide and pipeline(deg)
   INPUT #1, DeltaSwitch  '---used to switch between Manual calculation(-1) or Auto
calculation(1)

CLOSE #1

END SUB

FUNCTION xb.xa# (xa#)
'---For delta >2Ds
'-------------------
SHARED ld#, Ds#, delta.unit.parallel#, L#
p1# = 2 * xa# ^ 2 - 4 * xa# / ld# + 1 / ld# ^ 2 * (5 - 2 * delta.unit.parallel# / Ds#)
IF p1# < 0 THEN
      p1# = 0
END IF
xb# = 1 / ld# - xa# - (p1#) ^ .5
xb.xa# = xb#

END FUNCTION
```