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THE OPTIMUM DESIGN OF REINFORCED CONCRETE CONTINUOUS BEAMS

by

EUGENE JOHN O'BRIEN

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The undersigned certify that they have read and recommend to the Faculty of Graduate Studies for acceptance, a thesis entitled THE OPTIMUM DESIGN OF REINFORCED CONCRETE CONTINUOUS BEAMS submitted by Eugene J. O'Brien in partial fulfillment of the requirements for the degree of Doctor of Philosophy in Engineering.

Robert Loov

Dr. R.E. Loov (Supervisor)
Department of Civil Engineering

Dr. P.A. Binding
Department of Mathematics and Statistics

Dr. R.I. Gilbert (External Examiner)
University of New South Wales

Dr. N.E. Kalogerakis
Department of Chemical Engineering

Dr. A.E. McMullen
Department of Civil Engineering

N.G. Shrive

Dr. ~~N.G.~~ Shrive
Department of Civil Engineering

Oct 31st 1985

ABSTRACT

The optimum design of reinforced concrete continuous beams is considered. A method is developed for design in accordance with the currently accepted deterministic approach and a new probabilistic approach is proposed. In both cases, the variables are the cross-section dimensions and the areas of longitudinal and transverse reinforcement at all sections in each beam span. The constraints are the requirements of serviceability and the ultimate limit state.

Mathematical programming techniques have not traditionally been favoured for reinforced concrete design due to the associated high computational costs. This problem is largely overcome by the development of a method of optimization by decomposition. The beam is decomposed into a number of two-span substructures, each subjected to all loading conditions. The large number of variables associated with each sub-substructure degenerates conveniently into a very small number. This facilitates the use of robust direct search programming methods. The sub-problem designs are combined in an iterative sequence to give the optimum design of the beam. Examples illustrate the use of the program and the sensitivity of beam cost to certain parameters. An efficient method of determining a near-optimal solution is described.

A new probabilistic approach to design is developed for design office use. The method is based on the assumption that loading is uniformly distributed in each span and is of random intensity. The method is shown to be more rational than the existing approach and to yield less conservative results.

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a	=	depth of assumed rectangular stress block
a_i	=	constant defined by Eqn. 5.2
A	=	constant defined by Eqn. 4.15
A_{bn}	=	area of bottom steel
A_I	=	influence area
A_s	=	area of tension steel
A'_s	=	area of compression steel
A_{Sl}	=	area of one slab
A_t	=	tributary area
A'_{tn}	=	equivalent area of compression reinforcement available through extension of bottom steel from side n
b_1	=	breadth of rectangle of concrete surrounding the tension steel
b_2	=	breadth of rectangle of concrete surrounding the compression steel
b_i	=	constant defined by Eqn. 5.2
b_v	=	minimum effective web width for shear
b_w	=	band width
B	=	tributary area
c	=	depth to neutral axis
c_i	=	constant defined by Eqn. 5.2
$c_{i,i-1}$	=	constant defined by Eqn. 5.2
C_i	=	unit costs
cov(.)	=	covariance
d_b	=	effective depth to bottom steel
d_t	=	effective depth to top steel (from bottom of member)

d_v = effective depth for shear

d_1 = effective depth to (top or bottom) tension steel

d_2 = effective depth to (top or bottom) compression steel

dH_{Rn} = portion of total required moment capacity to be provided by steel n

$\underline{D}_i^j$ = difference between starting and finishing point in j^{th} optimization of stage i

e = constant defined by Eqn. 4.2

f = constant defined by Eqn. 4.3

f_c = value of objective function at $\underline{x}_c$

f'_c = characteristic strength of concrete

f_i = stage objective function

f_m = value of objective function at $\underline{x}_m$

f_n = value of objective function at $\underline{x}_n$

f_s = stress in tension steel

f'_s = stress in compression steel

f_y = yield strength of steel

F = correction factor for probabilistic shear design

$F(\underline{x})$ = objective function evaluated at $\underline{x}$

F_q = objective function at q^{th} iteration

g_1, g_2 = functions

g_i = inequality constraint

G = function defined by the "maximum" operation

G_s = "smooth" maximum function (Appendix C)

G'_s = smooth maximum function with continuous second derivatives

h = total section depth

h_i = equality constraint

[H] = Hessian matrix of second derivatives

H_n = moment at n^{th} support of span, hogging positive

H_{pn} = moment capacity at point P_n on side n of support

H_{xo} = maximum moment (hogging positive) in absence of compression reinforcement

I_{SXY} = influence surface coefficient for shear

I_X, I_Y = influence line coefficients for moment

I_{XY} = influence surface coefficient for moment

$\hat{j}$ = number of loading which is critical for y_1

$\hat{k}$ = number of loading which is critical for $\bar{y}_1$

k_i = constant defined by Eqn. 5.10

ℓ = span length

ℓ_d = full development length of reinforcement

ℓ'_d = development length required for moment, M_A

ℓ_{dn} = full development length of steel at end n

ℓ_i = constant defined by Eqn. 5.11

m_I = integral of (normalized) influence surface for moment

m_{SI} = integral of (normalized) influence surface for shear

$\bar{M}(X_o)$ = mean value of moment at X_o

$(M_A)_{jk}$ = moment at intersection of bending moment diagrams associated with loadings j and k

M_B = moment due to applied loads at Point B

M_C = moment due to applied loads at Point C

M_{det} = moment found using deterministic procedure

M_D = moment due to applied loads at Point D

M_{DB} = design moment at Point B

M_{DC} = design moment at Point C

M_{DD} = design moment at Point D

M_{EB} = moment as implied by elastic analysis at Point B

M_{EC} = moment as implied by elastic analysis at Point C

M_{ED} = moment as implied by elastic analysis at Point D

M_f = factored moment of resistance

M_f^t = factored moment of resistance at commencement of T-section behaviour

M_i = vector modulus for i^{th} stage optimization

M_i^* = optimum value of vector modulus, M_i

M_{prob} = moment found using probabilistic procedure

M_o = moment (sag positive) illustrated in Fig. 4.2

M_{RB} = moment of resistance at Point B

M_{RC} = moment of resistance at Point C

M_{RD} = moment of resistance at Point D

n = subscript to denote end of span. n = 1 denotes the left end of the span (right of support) and n = 2 denotes the right end of the span

n_g = number of inequality constraints

n_h = number of equality constraints

n_i = number of variables at Stage i in a multi-stage problem

n_ℓ = number of loading conditions

n_s = number of spans

p, p_i = power to which term is raised

q = uniform loading intensity

q_{Dni} = span dependent portion of uniform dead loading intensity

$\bar{q}_{Dni}$ = mean value of q_{Dni}

q_{Do} = span independent portion of uniform dead loading intensity

$\bar{q}_{Do}$ = mean value of q_{Do}

q_{Di} = uniform dead loading intensity in i^{th} span

q_{Li} = uniform live loading intensity in i^{th} span

q_{Ln} = span dependent portion of uniform live loading intensity

q_{Lni} = span dependent portion of uniform live loading intensity in i^{th} span

q_{LR} = reduced uniform live loading intensity

q_{Lo} = span independent portion of uniform live loading intensity

r = distance between points, (x_i,y_i) and (x_j,y_j)

r_{qM} = correlation coefficient between q and M

s_i = extent of boundary region

s_{Ln1} = factored deviation of q_{Ln1}

S = moment of resistance for sag

$S(X_o)$ = shear force at X_o

$\bar{S}(X_o)$ = mean shear force at X_o

S_i = moment of resistance for sag in i^{th} span

s_M = factored deviation of moment

t = constant defined by Eqn. 4.8

t_1 = total depth of section of concrete surrounding tension steel

t_2 = total depth of section of concrete surrounding compression steel

V_f = factored shear force

w = constant defined by Eqn. 4.10

$w(x,y)$ = load intensity at a point (x,y)

x = distance along span measured from left-hand support

$\underline{x}$ = vector of variables for optimization

$\underline{x}_b$ = centroid of simplex

$\underline{x}_c$ = point determined by contraction operation

$\underline{x}_h$ = point in simplex with highest value of objective function

$\underline{x}_i$ = vector of variables in i^{th} stage of a multi-stage problem

$\underline{x}_i^o$ = initial value of $\underline{x}_i$

$\underline{x}_i^t$ = transpose of $\underline{x}_i$

$\underline{x}_i^*$ = optimum value of $\underline{x}_i$

x_{in} = coordinate of first point of intersection of applied bending moment and moment capacity diagrams

$\underline{x}_j$ = vertex of simplex for sequential simplex method

x_{jk} = k^{th} component of $\underline{x}_j$

$\underline{x}_\ell$ = point in simplex with least value of objective function

$\underline{x}_m$ = point determined by expansion operation

x_n = coordinate measured on side n from support inwards towards the centre of the span

$\underline{x}_n$ = point determined by reflection operation

x_{qn} = coordinate of point at which bottom steel is cut-off

x_{sn} = coordinate of point at which bottom steel is no longer required for resistance of sag moment

x_{tn} = coordinate of point of tangency of applied bending moment and moment capacity diagrams

X_o, X = ratio of distance from left-hand span support to span length

X_i = random variable

$\bar{X}_i$ = mean value of X_i

$X_{i-1,j}$ = variable moment at i^{th} support for j^{th} loading

y_a, $\bar{y}_a$ = dimensions illustrated in Fig. 4.3b

$y_{i-1,i}$ = interconnection variables

y_n = length in n^{th} end of span in which top reinforcement is required to resist flexure

y_{pn} = extents illustrated in Fig. 4.6b

y_{xn} = the extent in span n in which moment (hogging positive) exceeds H_{xo}

Y = random variable

Y_1 = extent of reinforcement to be provided at Support B

$\bar{Y}$ = mean value of Y

Y_n = total length in n^{th} span in which reinforcement is required

z_o = amount by which the extent of bottom steel must exceed that theoretically required

z_n = minimum amount by which top steel on side n must exceed that required for flexure

$\bar{z}_n$ = minimum amount by which top steel on side opposite to side n must exceed that required for flexure

α = scalar used for the minimization of a function along a specified direction

β = number of standard deviations (a measure of safety)

γ_D = dead load factor

γ_L = live load factor

δA = small increment of area

δA_t = the additional area of top steel required to resist moment in excess of H_{xo}

δ_i = immediate deflection due to live load

δ_i^j = change in M_i resulting from j^{th} optimization of stage i

δ_t = sum of the long-time deflection due to sustained load and immediate deflection due to additional live load

Δ_i^j = change in i^{th} stage objective resulting from its j^{th} optimization

ε_x = longitudinal strain at member mid-depth

ε_o = area independent component of loading intensity

$\bar{\varepsilon}_o$ = mean value of ε_o

$\varepsilon_1(x,y)$ = area dependent component of loading intensity

θ = angle of shear cracks in concrete

κ = constant defined by Eqns. 6.23 and 6.24

κ' = constant defined by Eqn. 6.34

$\underline{\lambda}$ = vector of coordinating variables

ρ = ratio of deflection at which equivalent cost is first applied to maximum allowable deflection

σ_{Dni} = standard deviation of dead loading intensity, q_{Dni}

σ_{Do} = standard deviation of dead loading intensity, q_{Do}

σ_{Lni} = standard deviation of uniform loading intensity, q_{Lni}

σ_{Lo} = standard deviation of uniform loading intensity, q_{Lo}

σ_M = standard deviation of moment, M

σ_{qL} = standard deviation of live loading intensity, q_L

σ_S = standard deviation of shear force, S

σ_{Xi} = standard deviation of the random variable, X_i

σ_Y = standard deviation of the random variable, Y

$\sigma_{\varepsilon o}$ = standard deviation of load intensity, ε_o

$\sigma_{\varepsilon 1}$ = standard deviation of load intensity, ε_1

ϕ_c = material reduction factor for concrete

ϕ_s = material reduction factor for steel

∇F = the gradient vector for the function F

CHAPTER 1

INTRODUCTION

1.1 GENERAL

Methods of structural optimization have received a great deal of attention in recent years. Closed form solutions have been developed in some areas (Brown, 1975; O'Brien and O'Keeffe, 1985) but are, in general, only applicable to simple structures or substructures. For larger problems in structural optimization, it is necessary to apply methods of mathematical programming (MP). Some highly efficient MP algorithms are widely available. However, when applied to large structural problems, considerable practical difficulties have been encountered (Kirsch, 1975). Structural design problems, being based on the specifications of codes of practice, often involve discontinuities in the objective function and its derivatives (Surtees and Tordoff, 1977; Douty, 1976). This precludes the use of the more efficient gradient based algorithms (Kirsch, 1981). Direct search methods are less sensitive to discontinuities but are relatively inefficient when large numbers of variables are involved.

The complex behaviour of reinforced concrete (RC) under stress has resulted in extensive design and detailing specifications in the code of practice (Canadian Standards Association, 1984). Thus, the optimum RC design problem is a complex one, even for relatively small structures. Much research effort has been devoted to the optimization of reinforced concrete elements (Brown, 1975; Chou, 1977; Friel, 1974, Naamen, 1982). However, there is, in general, considerable interaction between the elements which make up a large structure. The result is that the optimum designs of elements, considered in isolation, are often quite different from the optimum designs of the same elements when part of a larger structure. This is especially true for optimum continuous beam design. The behaviour of a two-span continuous beam is fundamentally different than that of two simply supported beams acting independently on the same spans. In this dissertation, a method for the optimum design of continuous beams is developed. Despite the popularity of this form of construction, no other method appears to have been published for accurate optimum design. The problem involves the selection of the section dimensions and the areas of longitudinal and transverse reinforcement for all sections in each beam span. Topology is assumed to have been prespecified. The design is required to satisfy the constraints of serviceability and the ultimate limit state. Elastic analysis with (optimum amounts of) "plastic moment redistribution" is used to determine the bending moment and shear force envelopes.

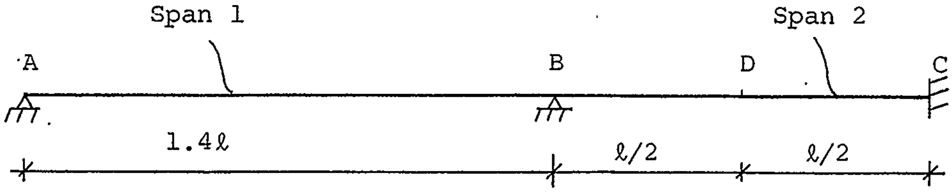
Two approaches to optimum continuous beam design have been developed. The first is a method of design in accordance with the requirements of the current code of practice (Canadian Standards Association, 1984). The second approach is to suggest an alternative method of design to that specified in the code which provides a more rational basis for design and which facilitates optimization.

1.2 OPTIMUM DESIGN FOR MULTIPLE LOADS

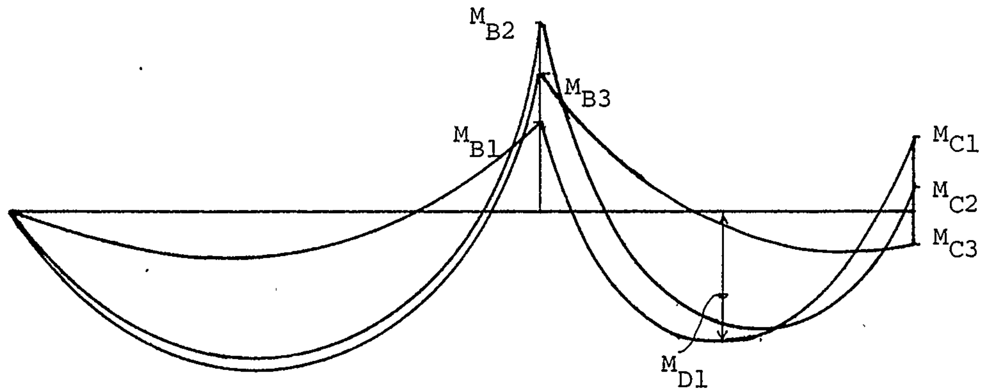
It is currently required in the Canadian code of practice (Canadian Standards Association, 1984) that continuous beams be designed to support all possible combinations of specified maximum and minimum loads. A major portion of this study has involved the development of a method of optimum design in accordance with this and the other requirements of the code.

The study involved the development of an algorithm by which optimum designs could be efficiently determined. Thus, the primary result of this research on optimum design is an efficient computer design program. Much of the content of this dissertation involves the development of the strategies which have been incorporated into the program. It was felt that the program should be versatile and easy to use. For example, the user must be permitted to keep certain section dimensions fixed while allowing others to take their optimum values. Of those that are allowed to vary, it must be possible to constrain some groups of dimensions to vary as one. Further, the input data which transmits these user requirements must not be overly complex.

In addition to the section dimensions, the variables for optimization include the amounts of plastic moment redistribution to be made after the elastic analysis. The detailed treatment of these parameters distinguishes this development from other studies on the optimum design of reinforced concrete structures that the author has studied. For the (elastic) bending moment envelope illustrated in Fig. 1.1b, an optimum design may involve reductions or increases in any or all of M_{B1}, M_{B2}, M_{B3}, M_{C1}, M_{C2} and M_{C3}. Increasing M_{C1} may decrease the



(a) Continuous Beam



(b) Bending Moment Diagrams

Fig. 1.1. - Example Illustrating Parameters for Optimization

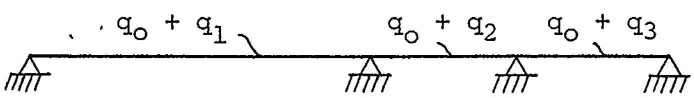


Fig. 1.2. - Example with Probabilistic Loading

amount of bottom steel required in Span 2 but may increase the amount of top steel required at C. Increasing M_{B1} may also decrease the amount of bottom steel required in Span 2 but this is countered by the fact that it may increase the length of top steel required at B.

Studies have indicated that many of the difficulties associated with the optimization of large structural systems are overcome by methods of decomposition (Kirsch, 1975). Decomposition involves the resolution of the primary problem into a number of subproblems, each involving a lesser number of variables. Continuous beam design for multiple loadings is a large optimization problem involving large numbers of variables. Due to the "ill conditioned" nature of the problem, it can not be readily solved using the efficient gradient based optimization algorithms. As direct search algorithms are only efficient for problems involving small numbers of variables, a method of decomposition has been applied to resolve the overall problem into a number of small subproblems. Each of the subproblems is sufficiently small to be solved quite efficiently using direct search methods.

Due to the high degree of interaction between the elements of a continuous beam, none of the existing methods of decomposition is suitable for this problem. Hence, it was necessary to develop a method which will be called sequential decomposition (SD). This method was developed for the specific problem in hand. However, it can be applied to any problem with a similar serial structure. The SD procedure involves the consideration of the overall problem as being composed of a number of subproblems. The subproblems are solved in a sequence which converges to the global optimum. Due to the integral nature of a continuous beam, the subproblems are not simply supported single-span beams but are smaller continuous beams of two spans or more.

A series of test runs demonstrates the relative efficiency of the optimization procedure. A further series of examples indicates the sensitivity of cost to some of the various input parameters and leads to some interesting conclusions for optimum beam design.

1.3 PROBABILISTIC APPROACH TO DESIGN

A simple probabilistic model is developed for the applied loading on buildings. It is proposed that this model be used for design in lieu of the traditional considerations of multiple deterministic loading conditions. While the model was developed primarily for live loading, it is shown to be equally applicable to dead loads. A primary aim in the development of the new procedure was that it be simple, both conceptually and in its use. For design, it is assumed that the loading in each span is uniformly distributed and that the intensity consists of two parts (see example, Fig. 1.2, p. 4). The first part, q_o, is constant for all spans in the beam while the second part is peculiar to that particular span. Each of the components of the live loading intensities (q_o, q_1, q_2, q_3 in Fig. 1.2) are considered to be statistically independent random variables, that is, the value of each is uninfluenced by the value of the others. The probabilistic design procedure consists of analyzing the beam to determine the stresses as linear combinations of the random variables. A simple formula is then applied for the determination of the means and standard deviations of these stresses from which extreme values can readily be determined.

The probabilistic procedure is a more rational basis for design than the traditional method of considering a number of deterministic load combinations. A comparison is made between the stresses which result from actual loading and those implied by the new procedure. It is shown to provide excellent agreement. Further, the new procedure is shown to provide considerably better consistency in levels of safety than is provided by the traditional deterministic approach. Design examples illustrate the use of the new procedure and the inconsistencies implicit in the traditional method.

CHAPTER 2

REVIEW OF PREVIOUS WORK

2.1 OPTIMUM STRUCTURAL DESIGN

2.1.1 General

Methods of optimization have received a great deal of attention over the past three decades and an impressive array of techniques has been developed. Several excellent books on the subject are available. Koo (1977), Luenberger (1973) and in particular, Gill, Murray and Wright (1981) provide the reader with an insight into optimization in general terms. More specialized books include those of Rao (1978) and Zahradnik (1971) on engineering optimization and Spunt (1971), Gallagher and Zienkiewicz (1973), Majid (1974) and Kirsch (1981) on structural optimization.

Despite an abundance of research and the fact that optimization has been shown to result in substantial savings (Johnson, 1984), there has been some hesitancy in the industry in the adoption of optimization techniques for structural design (Goble and Moses, 1975; Frind and Wright, 1975; Spillers and Kountouris, 1980). With the increasing use of computers in design offices, one would expect a great deal of software to be available for optimum design. However, as indicated by Firmin, Gilmor and Collins (1977) this may not be the case. At their time of writing, there were in fact very few design programs (much less optimum design programs) readily available nationwide in Canada. This was in contrast to a large number of analysis programs. They attribute the shortage of design programs, among other things, to the facts that design is code-specific and that much of design is a subjective process.

Two approaches to structural optimization have been developed, namely, optimality criteria (OC) methods and mathematical programming (MP) methods. The former are based on a defined criterion for optimality, the premise being that the optimum is found once that criterion is satisfied (Kirsch, 1981). Popular optimality criteria are simultaneous failure (all modes of failure should occur simultaneously (Schmit, 1981)) and fully stressed design (all members should be fully stressed for some loading condition). Gellatly and Berke (1971) and Gorzynski and Thornton (1975) are among those who have applied these methods to trusses but they can also be used for continuous (Stelzer, 1981) and cable (Cinquini and Contro, 1981) structures. It is well known that OC may not result in designs of minimum weight or cost (Kirsch, 1981; Gellatly and Berke, 1973). Nevertheless, the methods allow the engineer to take advantage of the special nature of structural optimization problems (Khot and Berke, 1981) which results in substantial savings in computational effort.

The other, more direct, approach to structural optimization is mathematical programming. These are the general optimization methods which seek to find the true optimum solution. Despite considerable improvements in the past thirty years, however, the direct application of general MP subroutines to large structural problems can require prohibitively large amounts of computer time (see Section 5.3).

There are a number of different kinds of MP, each suitable for different kinds of problems. Linear programming is among the most popular due to its relatively low computer time requirements. However, almost all forms of MP have been applied to structural optimization, dynamic programming to trusses and planar structures (Raj and Durrant, 1976; Twisdale and Khachaturian, 1975), geometric programming to the design of beams and slabs (Khalil, 1977), exterior penalty function methods to the design of bridges (Sargious and Badawy, 1976) and geometric programming to the design of factory structures (Bradley, Brown and Feeney, 1974).

While most approaches to structural optimization are based either on methods of OC or MP, there are some exceptions. Arora, Haug and Rim (1975) have applied an optimal control algorithm to the design of plane frames. Also, Wright and Feng (1971) and Arora and Haug (1978) have developed "hybrid methods", seeking to use the best features of both OC and MP simultaneously. The former have sequentially applied the stress ratio iteration method, the basic iteration method and the gradient projection method to the minimum weight design of plane frames. The latter have examined the features of both OC and MP and have recommended a combination of these for a hybrid method which they apply to the design of a truss. In both cases, it is assumed that the gradients of the objective function are available.

A persistent difficulty with methods of optimization has been the selection of an appropriate objective function, namely, the function to be minimized or maximized. McAdam (1983) emphasises the need to minimize the formwork and construction time costs. Brondum-Nielson (1985) is among those who consider reinforcement only as the objective function. However, most work has concentrated on the minimization of either structural weight or structural cost as reflected in unit prices of materials "in place". For structures such as trusses with specified topology, the material cost may be the only significant cost that is variable and, accordingly, minimum weight designs can give realistic results. There are also some structural applications where the minimization of weight and cost are almost synonymous. Examples include aircraft structures and structures located in remote areas (Fairweather, 1985). However, abundant evidence has been provided to indicate that, in general, minimum weight designs are not necessarily of minimum cost (Naamen, 1976; Kirsch, 1981). Nevertheless, a considerable amount of effort has been expended on the development of methods of minimum weight design even for prestressed structures (Rozvany, 1964; Birkeland, 1974) and composite structures (McNeely, Sneep and Smith, 1985). The problem with cost as an objective is that the cost coefficients are not readily quantifiable (Schmit, 1969; Russell and Choudhary, 1980). Construction costs alone depend on market situations and on the preferences of individual contractors. Fortunately, optimum solutions are, in general, not highly sensitive to these coefficients (Templeman, 1983). Thus, if exact values are not known, the use of typical values is reasonable. In accordance with this, the more recent work on optimum reinforced concrete design has tended to use cost rather than weight as the objective function.

Substantial progress has been made in optimum structural design over the past fifteen years. In 1969, Schmit stated that current trends were characterized by:

(a) "efforts to generate large scale capabilities involving drastic idealization and consideration of a limited class of failure modes";

(b) "efforts to generate optimization capabilities for relatively small special problems considering complex failure mode analyses involving less idealization".

While these comments still hold some relevance today, considerable improvements have occurred since then. Optimum truss design techniques can treat problems with multiple loadings (Lev, 1978; Cassis and Sepulveda, 1985) and variable geometry (Imai and Schmit, 1981; Lipson and Gwin, 1977). Methods of optimal frame design are available although few algorithms for configurational optimization with comprehensive design constraints yet exist (Topping, 1983). Recent developments in structural design include the use of artificial intelligence (Rooney and Smith, 1982 and 1983). The optimum design of structures with control has also received some attention (Kirsch and Moses, 1977) and would appear to have future potential.

Templeman (1983) summarizes the current status of optimum structural design:

(a) "The ability and methodology to write good practical computer software for optimum design has existed for more than ten years."

(b) "At present, very little software exists to implement practical optimum design, and the rapid growth in design office computers must suffer from a lack of appropriate software."

He concludes that researchers must step back from the research frontiers of structural optimization and become involved in providing practical design software.

2.1.2 Application of Optimization to Reinforced Concrete Structures

Element Optimization: As stated by Schmit, a great deal of the research effort in structural optimization has been focussed on small specialized problems (1969). This comment is given validity today by the profusion of publications on reinforced concrete element design. The relative simplicity of element design is especially attractive. It has meant that, for some elements, when simplifying assumptions or approximations are made, closed-form solutions can be obtained (Kaliszky, 1965; Timleck, 1972; Friel, 1974; Brown, 1975; Loov and Bhatia, 1978; Chou, 1977; Loov and Khalil, 1980; Somayaji, 1982; Loov, 1984). These solutions are in the form of sequences of equations and checks. Khalil (1977) gives a detailed explanation of how a constrained problem can be solved in this manner. Another approach, adopted by Khalil (1977) and Salinas (1980) is the solution of large numbers of examples by a mathematical programming method and the use of these to derive rules and/or equations to direct the design to the optimum.

Recent developments involve more accurate and/or more sophisticated analysis of element behaviour than can be accommodated in a closed-form solution. Accordingly, the problems are simply expressed in a form suitable for solution using methods of MP. Of the methods of MP, linear programming (LP) appears to be most popular. This is due to the widespread availability of reliable LP computer packages rather than to the linearity of element optimization problems. Morris (1978) has applied LP to prestressed flexural members and Naamen (1982) to prestressed tension members. Other methods of MP have also been used for element optimization. Salinas (1974) has applied geometric programming to the optimum design of flexural members while Cohn and MacRae (1984) have applied a general method of non-linear programming to the design of partially prestressed flexural members.

As can be seen, an abundance of methods is available for the optimum design of reinforced concrete elements. Unfortunately, for cast-in-place structures, there remains the important problem of integrating the element designs into the total structure (Russell and Choudhary, 1980). There are, in fact, not many simply supported flexural members in these structures. Accordingly, this author and others (Templeman, 1983) feel that an excess of sophistication in an element optimization method tends to limit its use.

Optimization of Large Concrete Structures: Structural Optimization procedures for large structures have tended in the past to fit into the first of the categories given by Schmit (1969), namely, they involved "drastic idealization". The consequence of this, in Schmit's words, is that "the engineer runs the risk of treating an inadequate representation of the right problem". Nevertheless, approximate procedures for large concrete structures are useful tools for preliminary design. As early as 1966, Hill proposed a method for the optimization of high-rise reinforced concrete buildings. More recently, Beaufait and Gerlein (1979) and Gerlein and Beaufait (1980) have proposed sophisticated methods for the preliminary design of large reinforced concrete frames. Andam and Knapton (1980) have also reported results obtained from a suite of programs for the optimum design of a family of precast concrete frames.

Two researchers have developed comprehensive methods for the design of reinforced concrete continuous beams. The first of these is Cauvin (1979) who has written a program for the nonlinear optimum design of (continuous beams and) frames. The advantage of the method is that it performs a non-linear analysis thereby removing the necessity for considerations of optimum plastic moment redistribution. The disadvantage is that sections are treated as independent. Hence, solutions involve non-prismatic beams, each section (in general) having a different geometry. Further, deflection considerations can not be included in the optimization process.

The second procedure for optimum continuous beam design was proposed by Kirsch (1983). This author has applied a "multilevel" strategy to the optimization problem. Beams are assumed to be prismatic and the dimensions to vary independently from one span to the next. An elastic analysis is carried out for multiple loading conditions. Subsequently, an optimization strategy is applied to determine the amount of plastic moment redistribution. However, only the maximum ("envelope") moments are treated in the redistribution calculations. The implication for the example illustrated in Fig. 1.1. (p. 4) is that the design moments, M_{DB}, M_{DC} and M_{DD}, are satisfactory provided that,

(a) they are sufficiently close to M_{B2}, M_{C1} and M_{D1}, respectively, to satisfy the requirements of ductility and,

(b) they satisfy the equilibrium requirement:

$$M_{DD} - (M_{DB} + M_{DC})/2 \geqq q_{max}\ell^2/8 \tag{2.1}$$

where q_{max} is the maximum factored loading intensity in Span 2. In this example, in fact,

$$M_{D1} - (M_{B1} + M_{C1})/2 = q_{max}\ell^2/8 \qquad (2.2)$$

As $M_{B1} < M_{B2}$, it would be possible using this design procedure, to reduce M_{D1} without considering the additional length of top reinforcement at B that this implies. Accordingly, this procedure, while having the advantage of simplicity, is based on a non-conservative assumption. The problem is compounded by the fact that M_{D1} would almost certainly be reduced in such an example as no increase in cost results from this while a decrease in the area of bottom steel in Span 2 does.

2.1.3 Multi-Level Optimum Design Procedures

Decomposition is the process of breaking down large optimization problems into smaller subproblems and combining the subproblem solutions is some way to determine the solution to the original problem. The method has received considerable attention because the computational effort required to solve an optimization problem tends to increase exponentially with the number of variables considered. In 1970, Lasdon suggested that decomposition was mandatory for truly large problems because of time and/or storage limitations (at that time). Optimal design of complete structures is, in general, a problem involving large numbers of variables. Further, it is well known that stresses in elastic structures are insensitive to changes in the dimensions of components at remote locations (Cauvin, 1979; Kirsch, 1983).

As early as 1969, Schmit stated that the study of formalized schemes for the decomposition of structural problems warranted attention. At this time, Lasdon (1968) had already published what

Kirsch (1975) refers to as the "goal coordination method", a generalized method of decomposition for mathematical programming problems. The method is applicable to problems in which the objective and constraints are additively separable. It involves the determination of the saddle point of the Lagrangian function (Hakkala and Hirvonen, 1978) and depends critically on its existence. Unfortunately, the existence of this point cannot be guaranteed for non-convex problems (Lasdon, 1968) and elastic structural design problems are commonly non-convex (Reinschmidt and Norabhoompipat, 1975). Kaweko and Ha (1983) appear to have overcome this problem of saddle point existence. They have solved the dual problem using repeated solution of a dual augmented with highly convex terms. They apply this procedure to the optimum design of perfectly plastic structures under multiple loadings.

In 1979, Kirsch and Moses applied the "model coordination" method of decomposition to elastic structures. The method is limited by the fact that the number of variables common to more than one subproblem (coordinating variables) must be small for efficient solution. They use the example of two 12-bar trusses connected by one bar. Nevertheless, there are many situations in which this method can be quite effective. As mentioned in the previous subsection, Kirsch (1983) has applied a multi-level approach to optimum continuous beam design. Despite the non-conservative assumption referred to above, the procedure is a demonstration of the effective use of a decomposition approach for optimum structural design.

2.2 LIVE LOADING ON BUILDINGS

2.2.1 Historical Review

As early as 1947 it was "common practice" (Dunham) to reduce the live load intensity by an amount which depended in some way on the area supported. At this time, Dunham used the results of two live load surveys to investigate empirically the nature of the relationship between load intensity and tributary area. He suggested a linear approximation for the relationship which was adopted for use in the United States (Peir and Cornell, 1973). Horne (1951) seems to have been the first researcher to model floor loading theoretically. He considered the load intensity at a given time and location to be independent of the intensity at the same time and at a different location. On this basis, he established a relationship between loading intensity and area for a given time. The current Canadian code (National Research Council of Canada, 1980) is based on his work (Peir and Cornell, 1973). It is suggested in this code that the live load is a function of tributary area to the power of 0.5. Peir and Cornell (1973) have stated that this functional form is better than the linear relationship specified in the U.S. code.

In the 1960's, the probabilistic approach to structural design enjoyed increasing popularity. In 1960, Brown reported the considerable use of a statistical approach to design in the aircraft industry and suggested its use for the design of buildings. In 1969, Cornell applied the second moment probabilistic approach to a structural design problem. Since then this method has been used for the assessment of levels of uncertainty pertaining to live loading.

A shortage of statistical information on live loading levels has long been a problem (Woolson, 1923). This has been rectified in part by the publication in the 1960's and 1970's of a number of major load surveys (Bryson and Gross, 1968; Karman, 1969; Mitchell and Woodgate, 1971; Culver, 1976). However, further information is still required (Culver, 1976; Harris, Corotis and Bova, 1981).

A number of researchers have developed methods by which the overall safety levels implied by code specifications can be assessed. Allen reported a comprehensive study in 1975 assuming all variables to be log-normally distributed. He recognized that the instantaneous live load intensity has a lower mean and higher variance than the maximum-in-lifetime intensity but did not consider temporal variations in detail. Also in that year, MacGregor published a state-of-the-art review of the limit state design of reinforced concrete structures. In 1978, Turkstra and Daly assessed a number of two-moment criteria for structural safety analysis. More recently, Galambos, Ellingwood, MacGregor and Cornell (1982) and Ellingwood, MacGregor, Galambos and Cornell (1982) have published the results of an extensive study on probability based load criteria. The purpose of this study was to establish load factors and combinations for use with all building materials and to provide a methodology for the selection of resistance criteria.

2.2.2 Probabilistic Live Loading Models

Live load models are theoretical hypotheses used to predict the statistical properties of live loads and their effects (stress, deflection, etc.). Peir and Cornell (1973) expound the virtues of such models:

(1) "they can guide the efficient collection, analysis and interpretation of observed data";

(2) "they extend the utility of experiments (load surveys) by making it possible to make predictions about loads and load effects on building types not observed in the data set";

(3) "they can help bring order, insight and economy of thought into understanding the nature and critical influencing factors of structural engineering".

Peir and Cornell developed a model for the behaviour of sustained load, namely, that portion of live loading which remains invariant with time for long periods (e.g. office furniture and personnel). They calibrate this theory with results obtained from the survey of Mitchell and Woodgate (1971). Loading intensity is assumed to consist of a point-invariant and a point varying portion. This latter portion is assumed to be spatially correlated, i.e. if load is higher than average at a particular location, then it is considered likely that the load at nearby locations is also high. The authors derive expressions for the behaviour of the maximum-in-lifetime sustained load effects and compare their results to the specifications of the North American codes of practice.

McGuire and Cornell (1974) use a live load model to compare the effects of gravity live loads with the building design loads specified in Canadian and American codes. They look at a number of common design cases (moments, shears, axial forces). They assume that the point-varying portion of sustained load is uncorrelated and incorporate considerations of extraordinary loads, namely, that portion of live loading that occurs effectively instantaneously in time (e.g. large grouping of people). They derive formulae for the mean and variance of the maximum-in-lifetime total load by fitting an extreme value distribution to results obtained from their model. A significant conclusion of this work is that for load reduction with area, the tributary area is a less consistent parameter than the "influence area". The influence area is that area over which the influence surface is significantly non-zero.

Ellingwood and Culver (1977) outline another total (sustained plus extraordinary) live load model. They point out that no data are available for certain parameters associated with extraordinary loading. Accordingly, they assume values for these and assume a relationship with area. The authors determine that the total design load at smaller areas is quite sensitive to the values assumed and suggest that additional research is necessary for their determination.

Chalk and Corotis (1980) developed a theory for the combination of the effects of sustained and extraordinary loads. They state that while their theory is not as general as others, it has the advantage of simplicity for the given application and they have shown that it agrees well with simulated results.

Harris, Corotis and Bova (1981) extend the previously developed models with a more comprehensive representation of extraordinary loads. They consider these loads to have three possible causes: remodeling, unusual gatherings of people and less usual larger gatherings of people. Parameter values were derived from engineering judgement.

Corotis and Tsay (1983) developed a procedure for the computation of load duration statistics as a function of load level. These quantities may be required for the calculation of long-term deflections and settlement of structures. Again, the need for additional data is emphasized for the authors found load duration statistics to be more sensitive to the underlying load process parameters than lifetime maximum load.

2.2.3 General

It can be seen that attention to the area of building live loads has been increasing steadily in recent years. It is true that, before these developments, a great deal was known about the behaviour of structures under specified loads but very little was known about what loads should be specified. This unsatisfactory situation has been remedied in part by recent developments. However, it appears that all of the research to date has focussed on attempts to assess the levels of safety implicit in current design procedures. It is the opinion of this author that more attention must be paid to the design procedures themselves. Current design procedures, no matter how comprehensive, suffer from the fact that the behaviour of structures under live load is not a deterministic process. The simple probabilistic design procedure developed in this dissertation is fundamentally different, in the author's opinion, from all of the research reported here. The difference is that the new method is not intended to be used to assess the level of safety implicit in the current policy of considering multiple loading combinations. It is instead put forward as an alternative to that procedure.

CHAPTER 3

SOME OPTIMIZATION THEORY

3.1 INTRODUCTION

In engineering design, there are often several satisfactory solutions to a given problem. For example, there are usually many different reinforced concrete beam designs that are adequate for any given situation. Optimization is the process of selecting the "best" adequate design. The function used to measure how "good" a design is, is an objective function and the criteria used to test its "adequacy" are constraints. The latter may be in the form of either equations or inequalities. Accordingly, an optimization problem may be formulated mathematically as follows:

Minimize the objective function

$$F(\underline{x})$$

subject to the constraints,

$$g_i(\underline{x}) \geqq 0 \quad , \quad i = 1,2,\ldots, n_g$$

and,

$$h_i(\underline{x}) = 0 \quad , \quad i = 1,2,\ldots, n_h$$

where,

$$\underline{x} = (x_1, x_2, \ldots, x_n)$$

Both minimization and maximization problems can be expressed in this form because the maximization of an objective, $F(\underline{x})$, is equivalent to the minimization of the function, $-F(\underline{x})$.

Traditionally most attention has been given to problems in which the constraints are precisely defined. Several approaches have been developed, each suitable for certain kinds of problem. A number of these approaches are referred to in subsequent chapters as their suitability for the problems of continuous beam design is reported. In this chapter, the more important methods are reviewed. It was also found necessary to consider those optimization problems in which some constraints are not "crisply" defined. These methods of "fuzzy programming" are reviewed in Section 3.5.

3.2 ZEROTH ORDER METHODS OF UNCONSTRAINED OPTIMIZATION

Zeroth order methods are those which do not require the calculation of objective function derivatives. They are suitable for problems in which discontinuities of gradient exist. Two of the most useful zeroth order methods are the Sequential Simplex Method and the Method of Conjugate Directions.

3.2.1 Sequential Simplex Method

The Sequential Simplex Method was first proposed by Spendley, Hext and Himsworth (1962) and was subsequently developed by Nelder and Mead (1965). It is important in that it does not require the objective function to be differentiable. This requirement is implicit in the exact line searches of the conjugate directions method (Haftka and Kamat, 1984). For an n-variable minimization problem, the Sequential Simplex method is based on a comparison of the objective function values at the (n+1) vertices of a simplex. At each stage in the optimization procedure, the vertex with the highest value is replaced with a new point. New points are sought by a process of reflection in the centroid of the vertices. If the reflection is favourable, an "expansion" is considered. If unfavourable, a method of contraction is employed.

The method is particularly suitable for problems in which the variables can have only discrete values. For such problems, the discrete nature of the variables can be ignored except in the calculation of the objective function. This is calculated using the discrete values nearest to the components of the design point. The resulting objective function is discontinuous but, as is now shown, this does not prevent solution using the Sequential Simplex method.

A program has been written in Fortran IV for the solution of discrete optimization problems using the procedure described. The flowchart is given in Appendix B. The program was applied to the solution of the following problem:

Minimize $F(x_1, x_2) = 100[(x_1 - 3)^2 + x_2^2]$

where x_1 and x_2 are multiples of 0.01.

A portion of the search is illustrated in Fig. 3.1. In this figure, the three vertices of each simplex are labelled with the simplex number and are joined by lines to form a triangle. An expansion is used between Simplexes 3 and 4 and a contraction is required between Simplexes 6 and 7. For the seven simplexes illustrated, the solution can be seen to be converging in the general direction of the minimum. Restarts are necessary to prevent the simplexes from becoming too small prematurely. In general, the method appears to be effective for discrete problems but requires a considerable number of function evaluations.

3.2.2 Method of Conjugate Directions

The Method of Conjugate Directions has been developed by Powell (1964). It is applicable to optimization problems with a

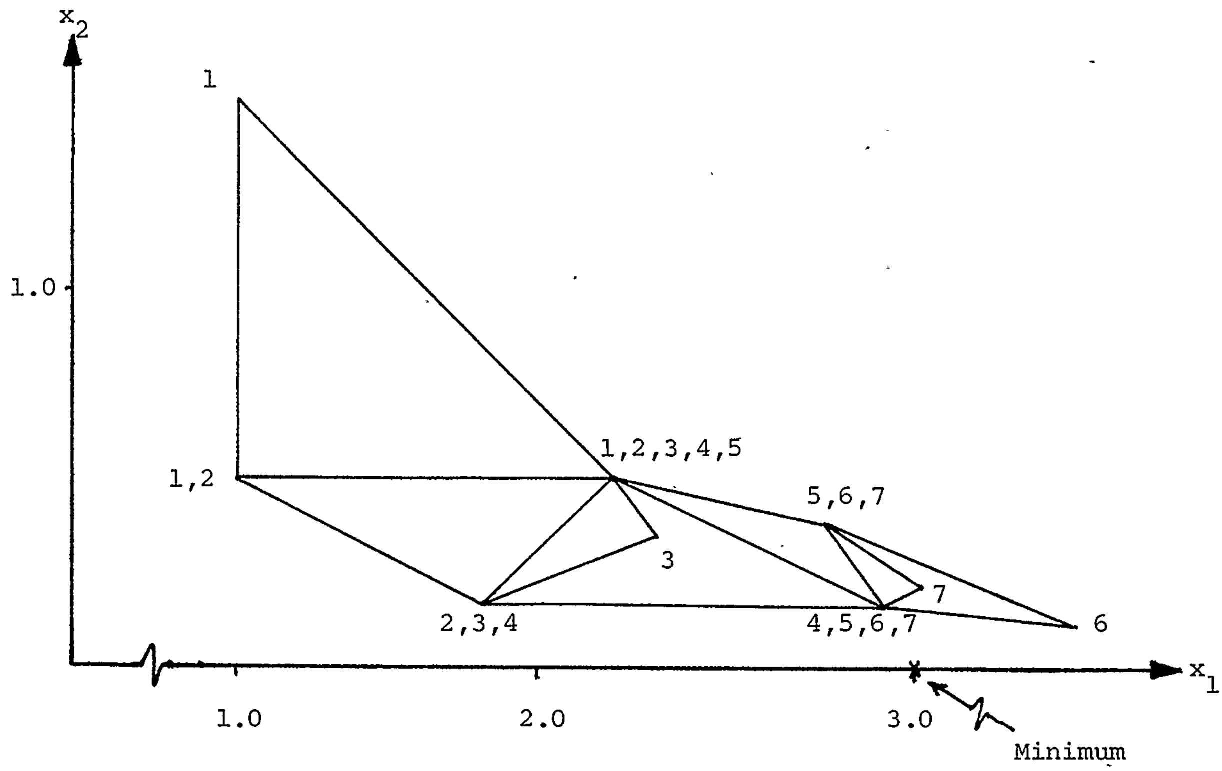


Fig. 3.1. - Sequential Simplex Method

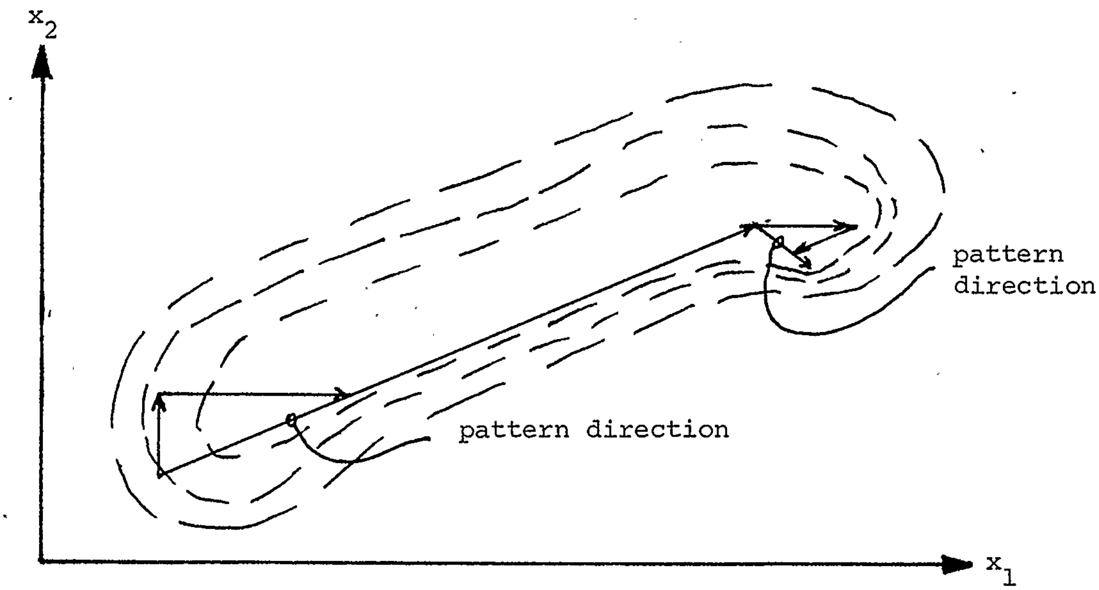


Fig. 3.2. - Method of Conjugate Directions

continuous objective and with or without continuous gradients. The method is quadratically convergent, that is, the minimum of a quadratic function of n variables is found in n function evaluations. Searches are initially made along each of n conjugate directions. Then, a search is made along the "pattern direction", that is, along the line passing through the first and last points (see Fig. 3.2). It can be shown (Kirsch, 1981) that the pattern direction is conjugate. Thus, a new set of conjugate directions is formed when the first is replaced with the pattern direction. This whole series of (n+1) line searches is repeated until convergence is achieved. As for the Sequential Simplex method, restarts are necessary; in this case, to prevent the conjugate directions from approaching dependence.

A computer program has been written in Fortran IV for optimization by the method of conjugate directions. A quadratic approximation based on three sample points is used for the line searches. Search is continued along the line in steps of increasing size until three equally spaced points are found such that the objective function at the centre point has the least value. When the conjugate directions approach dependence, the complete process is restarted with a reduced initial step length for the line searches. The flowcharts for the program are given in Appendix A (Figs. A.7 and A.8). The program has proven to be quite efficient in the solution of a number of test problems with continuous objective functions and small numbers of variables.

3.3 SECOND ORDER METHODS OF UNCONSTRAINED OPTIMIZATION

Second order methods of optimization are those which require the second derivatives of the objective function or approximations to them. For "well-behaved" functions, these methods are extremely powerful. They are based on a Taylor series expansion of the objective function:

$$F(\underline{x}) = F_q + \nabla F_q^t(\underline{x} - \underline{x}_q) + \tfrac{1}{2}(\underline{x} - \underline{x}_q)^t[H_q](\underline{x} - \underline{x}_q)$$

where F_q is the objective function value, ∇F_q is the gradient vector and $[H_q]$ is the Hessian matrix at the q^{th} iteration. Differentiating and setting the first derivatives to zero gives the Newton-Raphson equation:

$$\underline{x}_{q+1} = \underline{x}_q - [H_q]^{-1}\nabla F_q$$

It has been found that a line search along the direction, $([H_q]^{-1}\nabla F_q)$, gives better results. Thus,

$$\underline{x}_{q+1} = \underline{x}_q - \alpha^*[H_q]^{-1}\nabla F_q$$

where α^* is the scalar which minimizes the objective function along this direction. There are two difficulties associated with this Newton method of optimization:

(1) For problems with large numbers of variables, the calculation of the components of the Hessian matrix can require excessive computational labour.

(2) The inversion of the Hessian matrix is impractical for large problems.

The quasi-Newton methods have been successful in overcoming both of the problems mentioned above. They are based on the use of an estimate of the inverse Hessian matrix derived from information of objective function values and gradients. They are extremely powerful methods, particularly for large problems with well-behaved objective functions. However, as is reported in Chapter 5, some quasi-Newton methods are sensitive to gradient discontinuities, regions with zero gradients and to sudden (although continuous) changes in gradient.

3.4 DECOMPOSITION IN NON-LINEAR PROGRAMMING

Many large optimization problems are impractical to solve without decomposition into a number of smaller subproblems. The computational labour required to solve an optimization problem tends to increase rapidly with the number of variables. Thus, while the solution to one large problem may be impractical, the repeated solution of a number of smaller independent problems may not be. Of course, an integrated optimization problem is not, in general, composed of independent subproblems. However, certain subproblems can sometimes be found that can be combined in a sequence which leads to the solution of the overall problem. There are currently two basic approaches to decomposition which may be referred to as the "model" and the "goal" coordination methods (Mesarovic, Macko and Takahara, 1969). Both methods are described briefly in this section.

3.4.1 Model Coordination Method

For many optimization problems, there are certain variables which, if fixed, result in degeneration of the problem into a number of independent subproblems. Such variables are known as coordinating variables. The model coordination method consists of a two-level (or, in general, multi-level) approach. At the first level, all coordinating variables are fixed and the independent subproblems are solved. At the second level of a two-level problem, the optimum values of all of the coordinating variables are sought. For each given combination of these variables, the "objective function evaluation" consists of solving all first level subproblems and calculating the corresponding value of the total objective. Thus, the second-level problem solution requires several solutions of each of the first level subproblems.

The following example (after Kirsch, 1981) illustrates the use of the method. For the two bar truss of Fig. 3.3, the values of the dimension, y, and the cross-sectional areas of the bars, x_1 and x_2, are sought for minimum volume of steel. The constraints are that stress should not exceed 100 MPa and that y should be less than 3000 mm. Thus, the problem can be formulated as:

$$\text{Minimize } F = x_1[16 \times 10^6 + y^2]^{\frac{1}{2}} + x_2[10^6 + y^2]^{\frac{1}{2}}$$

$$\text{subject to} \quad 200[16 \times 10^6 + y^2]^{\frac{1}{2}} - x_1 y \leq 0 \quad ,$$

$$800[10^6 + y^2]^{\frac{1}{2}} - x_2 y \leq 0 \quad ,$$

$$\text{and} \quad y - 3000 \leq 0.$$

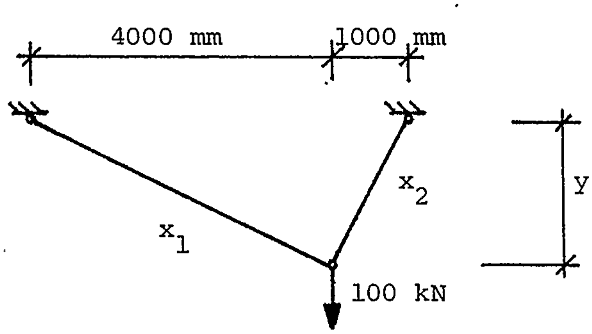


Fig. 3.3. - Decomposition Example

For this problem, the single coordinating variable is y and the first level subproblems are:

$$\text{Minimize } f_1 = x_1[16 \times 10^6 + y_o^2]^{\frac{1}{2}}$$

$$\text{subject to } 200[16 \times 10^6 + y_o^2]^{\frac{1}{2}} - x_1 y_o \leqq 0$$

and,

$$\text{Minimize } f_2 = x_2[10^6 + y_o^2]^{\frac{1}{2}}$$

$$\text{subject to } 800[10^6 + y_o^2]^{\frac{1}{2}} - x_2 y_o \leqq 0$$

where y_o is a fixed value for y. The second level problem consists of finding a value for y less than 3000 mm that minimizes F. The solution is,

$$(x_1, x_2, y) = (447, 894, 2000)$$

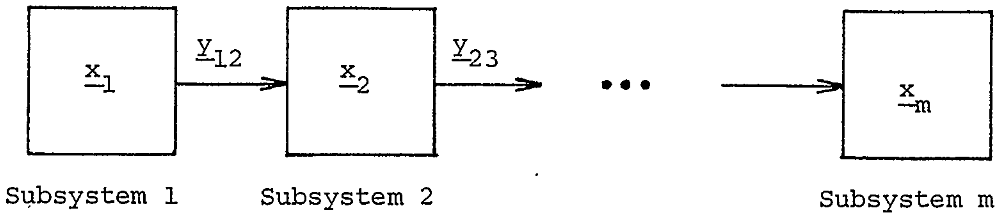
in units of mm^2 and mm.

It can be seen that the model coordination method is simple to apply. However, as the second level function evaluation requires such a considerable amount of computational labour, it is not suitable for problems involving large number of coordinating variables.

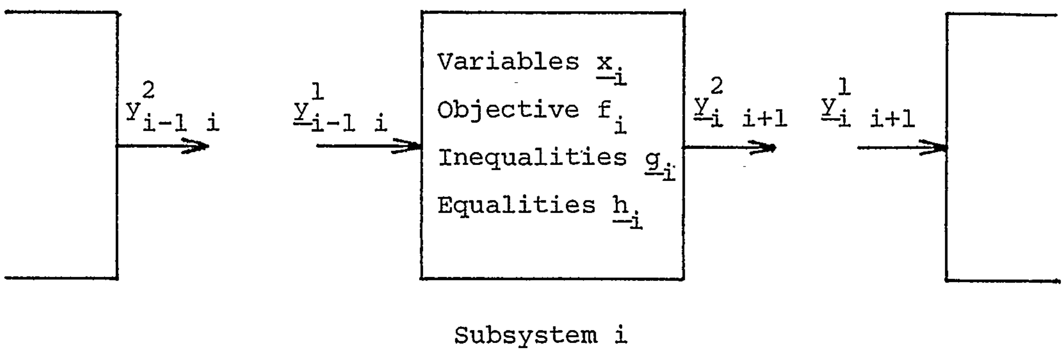
3.4.2 Goal Coordination Method

The goal coordination method of decomposition is based on the principle of duality. It is particularly well suited to serial problems, as illustrated in Fig. 3.4a. Such a problem can be formulated as:

$$\text{Minimize } \sum_{i=1}^{m} f_i(\underline{x}_i, \underline{y}_{i-1\ i}, \underline{y}_{i\ i+1})$$



(a) Serial Problem



(b) Subsystem i with Arbitrary Interconnection Variables

Fig. 3.4. - Decomposition of Serial Problems by Goal Coordination

subject to $\underline{g}_i\ (\underline{x}_i,\ \underline{y}_{i-1\ i},\ \underline{y}_{i\ i+1}) \geq \underline{0}$, $i = 1,2,\ldots, m$

and $\underline{h}_i\ (\underline{x}_i,\ \underline{y}_{i-1\ i},\ \underline{y}_{i\ i+1}) = \underline{0}$, $i = 1,2,\ldots, m$

where f_i is the objective, $\underline{x}_i$ are the variables and $\underline{g}_i$ and $\underline{h}_i$ are the inequality and equality constraints respectively for the i^{th} subsystem. The vector $\underline{y}_{i\ i+1}$ represents the "interconnection variables" common to Stages i and (i+1). Naturally, these variables have the same value regardless of which stage is being considered. However, an alternative formulation considers two values for each variable and uses a constraint to ensure their equality (see Fig. 3.4b). This formulation is:

$$\text{Minimize } \{ \sum_{i=1}^{m} f_i(\underline{x}_i,\ \underline{y}^1_{i-1\ i},\ \underline{y}^2_{i\ i+1}) + \sum_{i=1}^{m-1} \underline{\lambda}^t_i(\underline{y}^1_{i\ i+1} - \underline{y}^2_{i\ i+1}) \}$$

subject to $\underline{g}_i(\underline{x}_i,\ \underline{y}^1_{i-1\ i},\ \underline{y}^2_{i\ i+1}) \geq \underline{0}$, $i = 1,2,\ldots, m$

and $\underline{h}_i(\underline{x}_i,\ \underline{y}^1_{i-1\ i},\ \underline{y}^2_{i\ i+1}) = \underline{0}$, $i = 1,2,\ldots, m$

An equivalent dual form of this problem (Luenberger, 1973) is:

$$\text{Maximize } \{ \sum_{i=1}^{m} H_i(\underline{\lambda}_i) \}$$

$$\text{where } H_i(\underline{\lambda}_i) = \text{Minimum } \{f_i(\underline{x}_i,\ y^1_{i-1\ i},\ y^2_{i\ i+1}) + \underline{\lambda}^t_{i-1}\ \underline{y}^1_{i-1\ i} - \underline{\lambda}^t_i\ \underline{y}^2_{i\ i+1}\}$$

subject to $\underline{g}_i(\underline{x}_i,\ \underline{y}^1_{i-1\ i},\ \underline{y}^2_{i\ i+1}) \geq 0$

and $\underline{h}_i(\underline{x}_i,\ y^1_{i-1\ i},\ y^2_{i\ i+1}) = \underline{0}$

for $i = 1,2,\ldots, m$

As each of these minimizations is independent, they can be solved with relatively little computational effort.

Applying the goal coordination procedure to the problem of Fig. 3.3, the alternative formulation is:

$$\text{Minimize } x_1[16 \times 10^6 + (y^2_{12})^2]^{\frac{1}{2}} + x_2[10^6 + (y^1_{12})^2]^{\frac{1}{2}} + \lambda(y^1_{12} - y^2_{12})$$

$$\text{subject to } 200[16 \times 10^6 + (y^2_{12})^2]^{\frac{1}{2}} - x_1(y^2_{12}) \leqq 0$$

$$800[10^6 + (y^1_{12})^2]^{\frac{1}{2}} - x_2(y^1_{12}) \leqq 0$$

$$(y^1_{12}) - 3000 \leqq 0$$

$$(y^2_{12}) - 3000 \leqq 0$$

$$\text{and} \quad y^1_{12} - y^2_{12} = 0$$

The dual form of this is:

$$\text{Maximize } \{H_1(\lambda) + H_2(\lambda)\}$$

$$\text{where} \quad H_1(\lambda) = \text{Minimum } \{x_1[16 \times 10^6 + (y^2_{12})^2]^{\frac{1}{2}} - \lambda y^2_{12}\}$$

$$\text{subject to } 200[16 \times 10^6 + (y^2_{12})^2]^{\frac{1}{2}} - x_1(y^2_{12}) \leqq 0$$

$$\text{and} \quad (y^2_{12}) - 3000 \leqq 0$$

$$\text{and where } H_2(\lambda) = \text{Minimum } \{x_2[10^6 + (y^1_{12})^2]^{\frac{1}{2}} + \lambda\, y^1_{12}$$

$$\text{subject to } 800[10^6 + (y^1_{12})^2]^{\frac{1}{2}} - x_2(y^2_{12}) \leqq 0$$

$$\text{and} \quad y^1_{12} - 3000 \leqq 0$$

The saddle point is at, $\lambda = -600$, the optimal values for the other variables being as before.

The goal coordination formulation is mathematically elegant but there are some disadvantages to the method (Kirsch, 1981):

(1) The saddle point does not always exist. Thus, the method is not effective for all problems.

(2) The total number of variables involved in the solution process is greater than the original number.

(3) The most serious disadvantage to the method, for structural applications, is that intermediate solutions do not represent feasible designs. A method in which "approximately optimal" designs can be found at little cost would be more useful in many situations.

3.5 FUZZY PROGRAMMING

3.5.1 Introduction and Review of Previous Work

The conventional optimization problem involves the minimization of a single given objective function subject to certain constraints. In most real design situations however, constraints can not be precisely defined (Dlesk and Liebman, 1983) and are said to be "fuzzy". Zimmermann (1976) gives an example of how imprecision due to fuzziness is different from imprecision due to randomness: "While 'Tomorrow it will rain with a probability of 0.7 and the sun will shine with a probability of 0.3' is imprecise because of randomness, the statement 'I like all good looking girls' is imprecise because of the fuzzy meaning of 'good looking girls'". Research in this field has been in one of two areas. The first is the quantification of verbal expressions such as the one above. Elms (1982) is among those who have published work in this area. The other is the quantification of imprecision resulting from imprecise objectives. Only this latter area is considered further here.

Zimmermann (1978) has stated that there are indications that human decision makers generally neither combine their individual objective functions linearly nor apply the minimum operator to combine them. A mathematical fuzzy set theory has been developed as an aid to the understanding of fuzzy objectives and their combination (Zadeh, 1965). (An example of two such fuzzy sets are $x \cong 3$ and $x \gtrsim 4$.) Kam and Brown (1983) have used fuzzy set theory for the problem of combining statistical information with a measure of imprecision based on engineering experience. Soyster (1973) and Zimmermann (1976) have incorporated fuzzy concepts into linear programming. Chuang and Munro have applied fuzzy linear programming to a water quality management problem (1983). Carmichael (1980) has applied a non-linear fuzzy programming procedure to the optimum design of a truss.

For the continuous beam design problem, the design must be such that deflections at working loads are within acceptable limits. While codes of practice do specify exact limits for given situations, it is clearly not desirable to have all deflections at the limiting values. This is particularly so if a design with considerably lower deflections is only slightly more expensive. Accordingly, an "acceptable" limit for a deflection should be represented by a band of values of varying levels of acceptability rather than by one single value. Different formulations are applicable depending on whether the problem is linear or non-linear. Both cases are reviewed in the subsections following.

3.5.2 Fuzzy Linear Programming

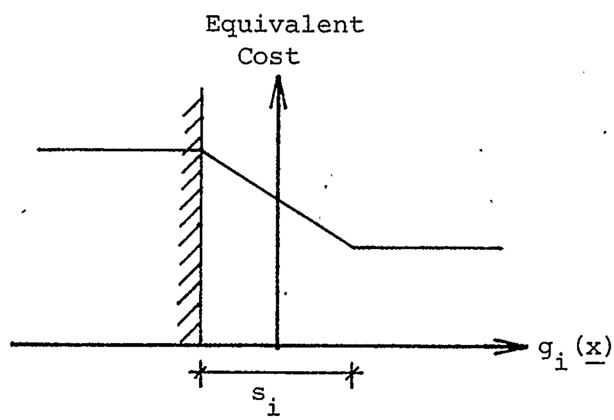
A linear programming problem can be represented by:

Minimize $z = F(\underline{x})$

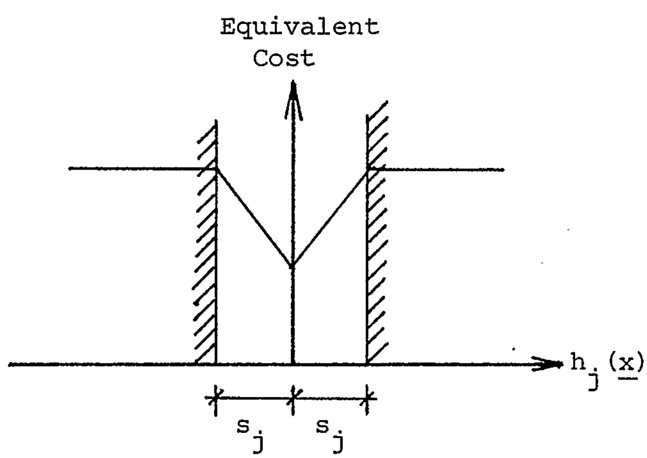
subject to $\underline{g}(\underline{x}) \geqq 0$ (m inequalities)

and $\underline{h}(\underline{x}) = 0$ (n equations)

where $\underline{g}$ and $\underline{h}$ are vectors whose components are linear functions of $\underline{x}$. If a constraint is soft, the point at which it is just satisfied is replaced by a boundary region. At each point within this region, an equivalent cost is assigned as a measure of the degree of satisfaction of the constraint (see Figs. 3.5a and 3.5b). Using the Bellman-Zadeh (Bellman and Zadeh, 1970) criterion, the cost used as a measure of the degree of satisfaction of the constraints is the maximum of the



(a)



(b)

Fig. 3.5. - Equivalent Cost Functions for Soft Constraints
(a) Inequality Constraint, (b) Equality Constraint

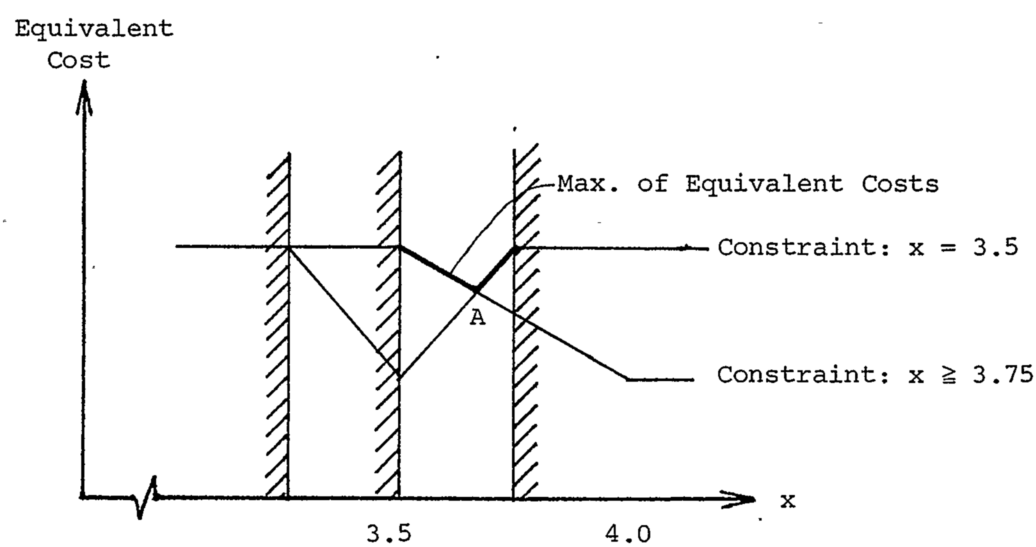


Fig. 3.6. - Maximum of Equivalent Costs Function

equivalent costs for all constraints. This cost function is illustrated for a given example in Fig. 3.6. The objective is included among the equivalent costs when using this criterion. A point often selected for design (Munro, 1984) is the minimum over $\underline{x}$ of the maximum of the equivalent costs (point A in Fig. 3.6).

The concept of considering only the maximum of the equivalent costs is unreasonable for many cases. However, to add all the equivalent costs results in a linear function and an optimum at an extreme point of the common boundary region. More reasonable results can be obtained when non-linear equivalent costs are used.

3.5.3 Fuzzy Non-Linear Programming

The same principles are applied to non-linear problems with soft constraints as to linear problems. The difference for non-linear problems is that non-linear equivalent cost functions can be readily incorporated. The problem is one of multiple objectives, one additional objective corresponding to each soft constraint. A considerable amount of research has been done on multicriterion optimization and a number of methods exist for the conversion of such problems into ones involving a single objective only. Of these, the weighting objectives method has received most attention (Osyczka, 1984). This method simply consists of the weighted addition of all equivalent costs and the minimization of the resulting function. An equivalent cost function that is convex and quite versatile is

$$F_i = C_i \left| \frac{h_i(\underline{x}) - s_i}{s_i} \right|^{p_i}$$

where $h_i(\underline{x}) = 0$ is an equality constraint, $2s_i$ is the extent of the boundary region and C_i and p_i are constants. A similar function can be used for inequality constraints, the only difference being that the function is given a value of zero at feasible points outside the boundary region.

3.6 CONCLUSIONS

A number of topics relating to optimization are reviewed. The zeroth order methods for unconstrained problems are useful in that they are insensitive to gradient discontinuities. Two of the better known methods are described. For large problems of a less sensitive nature, the second order methods are extremely powerful. The basic principle is described together with the difficulties and the methods used to overcome them.

Decomposition in one form or another is a very useful method for solving large optimization problems. The conventional methods are described and the advantages and disadvantages of each are outlined.

Finally, the subject of fuzzy programming is reviewed. This is the subject of representing mathematically, problems which involve "soft" constraints. It is pointed out that many design problems can not be represented accurately by fuzzy linear programs. No such problem exists however with fuzzy non-linear programming. An equivalent cost function is proposed that is versatile and convex.

CHAPTER 4

DESIGN OF REINFORCEMENT

4.1 INTRODUCTION

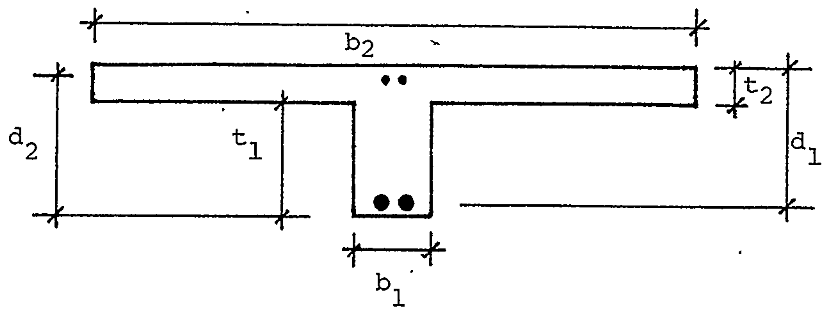
A major part of the reinforced concrete beam design process occurs after the bending moment and shear force envelopes have been specified. The areas of longitudinal tension and compression reinforcement must be determined together with the bar lengths. The areas and spacings of transverse reinforcement must also be found.

Subroutines have been written for the determination of all these quantities. The calculations are in accordance with the Canadian code of practice, CAN3-A23.3-M84 (Canadian Standards Association, 1984). The analysis is more rigorous than that which would normally be done by hand. Consequently, designs found by these calculations can be safer and/or less expensive.

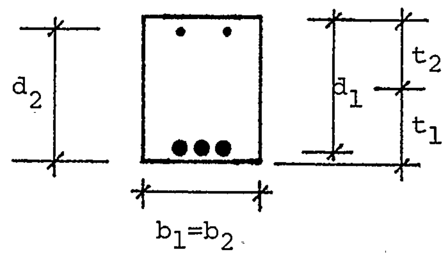
4.2 TENSION REINFORCEMENT

A subroutine has been written for the determination of the area of tension reinforcement required at a section to resist a specified moment. The concrete geometry and the area of compression reinforcement are assumed to be known. The subroutine is applicable to beams of T-, rectangular and inverted T-section.

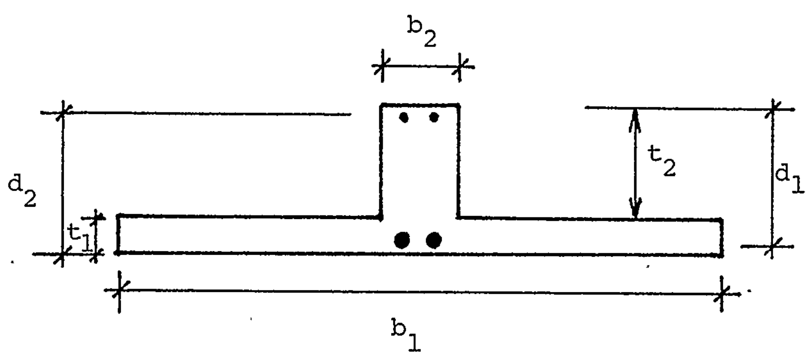
The section geometries are illustrated in Figs. 4.1a, b and c. The assumed stress distribution, for all cases, is given in Fig. 4.1d. The shape depends only on the relative values of the breadths, b_1 and b_2. In all three cases, the tension reinforcement, of area, A_s, is at a depth, d_1, from the extreme fibre in compression. Similarly, the



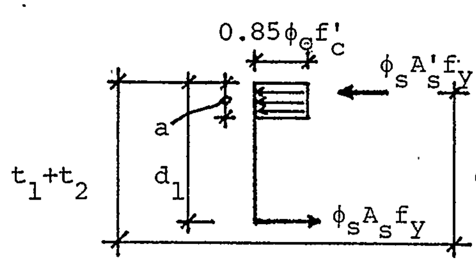
(a) T-Section



(b) Rectangular Section



(c) Inverted T-Section



(d) Stress Distribution

Fig. 4.1. - Section Geometries and Stress Distribution

compression reinforcement, of area, A_s', is at a depth, d_2, from the extreme fibre in tension. The portion of concrete of breadth, b_1 and thickness, t_1 surrounds the tension reinforcement while that of breadth, b_2 and thickness, t_2, surrounds the compression reinforcement. The area of tension steel, A_s, is sought that just provides an ultimate moment of resistance of M_f.

When the depth of the assumed rectangular stress block, a, is less than t_2, rectangular behaviour results and the equilibrium equation is,

$$0.85\ \phi_c f_c'\ a\ b_2 = \phi_s f_y (A_s - A_s') \tag{4.1}$$

In this equation, f_c' is the characteristic strength of concrete, f_y is the yield strength of steel and ϕ_s and ϕ_c are the capacity reduction factors for steel and concrete respectively. The constants, e and f, are defined by:

$$e = \phi_s f_y, \tag{4.2}$$

and

$$f = \frac{\phi_s^2\ f_y^2}{1.7\ \phi_c\ f_c'} . \tag{4.3}$$

Hence, for rectangular behaviour,

$$\frac{a}{d_1} = \frac{2f}{e} \frac{(A_s - A_s')}{b_2 d_1} \tag{4.4}$$

The ultimate moment of resistance can then be shown to be:

$$M_f = ed_1(A_s - A_s') - (f/b_2)(A_s - A_s')^2 + eA_s'(d_1 + d_2 - t_1 - t_2) \tag{4.5}$$

The minimum area of steel required to provide this moment of resistance is:

$$A_s = A_s' + \frac{b_2 d_1}{2f} \{e - [e^2 - 4f(M_f - eA_s'(d_1 + d_2 - t_1 - t_2))/(b_2 d_1^2)]^{\frac{1}{2}}\} \tag{4.6}$$

Rectangular behaviour is assumed to cease when $a = t_2$. The moment at this point is:

$$M_f^t = \frac{e^2 t\, b_2 d_1^{\;2} (2 \;\; - t)}{4f} + A_s' \, e \; (d_1 + d_2 - t_1 - t_2) \tag{4.7}$$

where

$$t = t_2/d_1. \tag{4.8}$$

For $M_f > M_f^t$, force equilibrium gives:

$$\frac{a}{d_1} = \frac{2f}{e} \frac{(A_s - A_s')}{b_1 d_1} + \frac{t(w - 1)}{w} \tag{4.9}$$

where

$$w = b_1/b_2 \tag{4.10}$$

Moment equilibrium gives:

$$M_f = eA_s'(d_1 + d_2 - t_1 - t_2) + \frac{e^2 t^2 b_2 d_1^2 (w - 1)}{4wf}$$

$$+ \frac{(A_s - A_s')ed_1}{w} [t + w - wt] - \frac{(A_s - A_s')^2 f}{b_1} \tag{4.11}$$

from which the required area of steel is:

$$A_s = A_s' + \frac{b_2 d_1}{2f} \{e(t + w - wt) - [e^2 w((1 - t)^2 (w - 1) + 1)$$

$$- \frac{4fw(M_f - eA_s'(d_1 + d_2 - t_1 - t_2))}{b_2 d_1^2}]^{\frac{1}{2}}\} \tag{4.12}$$

The requirements of minimum area of reinforcement are included in this subroutine.

For the usual case in which the beam is cast integrally with the slab, the tension reinforcement in areas of positive (sagging) moment can be found by considering the section to have a T-shape ($b_1 < b_2$). The tension reinforcement in areas of negative (hogging) moment can be found by considering the section to have an inverted T-shape ($b_1 > b_2$). Thus, this single subroutine can be used for all sections of such a beam.

4.3 THEORETICAL CUT-OFF POINTS

A function subprogram has been written for the determination of the cut-off points in a span subjected to uniformly distributed loading. The extents of the regions in which moment (sag positive) is less than zero are calculated. If, at one support, the moment is positive, then this extent is zero. If the moment is negative at all points in the span, then, the cut-off points are both set equal to the point of maximum moment. Thus, negative moment reinforcement is provided throughout the span.

For a span of length, ℓ, subjected to a uniformly distributed load, q per unit length, the moment at a distance x from the left hand support is:

$$M(x) = (q\ell^2/2)[x/\ell - (x/\ell)^2] - H_1 - (H_2 - H_1)\ x/\ell \tag{4.13}$$

In this equation, H_1 and H_2 are the hogging moments at the left and right hand supports respectively. The points at which this moment equals zero, if such points exist, are defined by:

$$x = \{A \pm [A^2 - 2q\ell^2 H_1]^{\frac{1}{2}}\}/(q\ell) \tag{4.14}$$

where

$$A = q\ell^2/2 + H_1 - H_2 \tag{4.15}$$

If H_1 and H_2 are both positive and

$$A^2 \geqq 2q\ell^2 H_1, \tag{4.16}$$

then two regions in the span exist where moment is negative. These regions are of length y_1 and y_2 where

$$y_1 = \ell\{(A + [A^2 - 2q\ell^2 H_1]^{\frac{1}{2}})/(q\ell^2)\}, \tag{4.17}$$

and

$$y_2 = \ell\{1 - (A + [A^2 - 2q\ell^2 H_1]^{\frac{1}{2}})/(q\ell^2)\} \tag{4.18}$$

In general, the extents of the regions where moment is less than zero, are given by:

$$y_n = y_n\{n,\ q\ell^2,\ \ell,\ H_1,\ H_2\} \quad , \quad n = 1,2, \tag{4.19}$$

where,

$$y_n\{n,\ q\ell^2,\ \ell,\ H_1,\ H_2\} = \ell\{n-1 + [\frac{(3-2n)\ A - (A^2 - 2q\ell^2 H_1)^{\frac{1}{2}}}{q\ell^2}]\} \tag{4.20}$$

for $H_1 \geqq 0$, $H_2 \geqq 0$ and $A^2 \geqq 2q\ell^2 H_1$. Similarly, the extents of the regions in the span where the moment is less than a specified moment, M_o, are:

$$y_n\{n,\ q\ell^2,\ \ell,\ H_1 + M_o,\ H_2 + M_o\} \quad , \quad n = 1,2$$

for $H_1 + M_o \geqq 0$, $H_2 + M_o \geqq 0$ and $A^2 \geqq 2q\ell^2(H_1 + M_o)$.

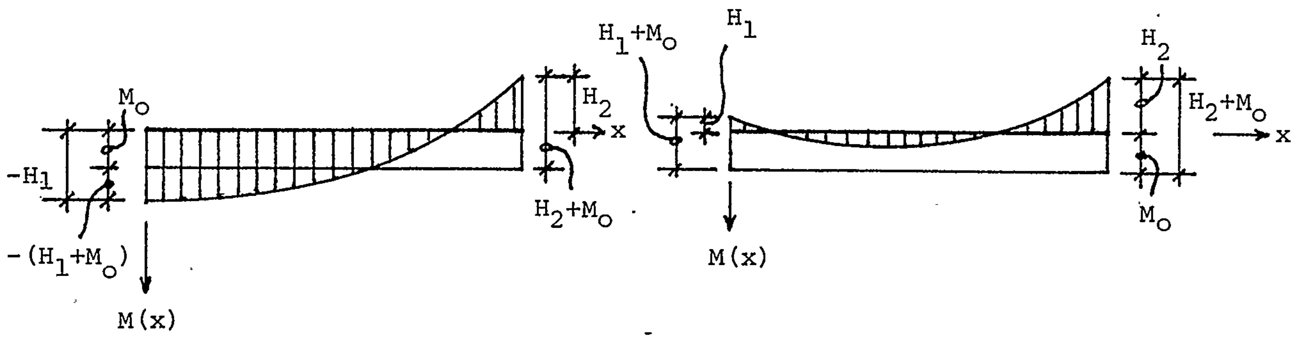
If $(H_n + M_o)$ is negative for some n, the region, y_n, in which M(x) is less than M_o, is of zero extent (see Fig. 4.2a). Accordingly,

$$y_n\{n,\ q\ell^2,\ \ell,\ H_1 + M_o,\ H_2 + M_o\} = 0,$$

for $(H_n + M_o) < 0$. Finally, if $A^2 < 2q\ell^2(H_1 + M_o)$, no point exists where $M(x) = M_o$ (Fig. 4.2b). Clearly, when this is the case, the moment, M(x), is less than M_o throughout the span. A point may still be required in the span at which to splice the negative moment reinforcement. An appropriate such point is the point of minimum (absolute) moment. When this point is used, the theoretical extents of reinforcement are given by:

$$y_n\{n,\ q\ell^2,\ \ell,\ H_1 + M_o,\ H_2 + M_o\} = \ell\{n-1 + \frac{(3-2n)A}{q\ell^2}\} \quad , \ n=1,2 \qquad (4.21)$$

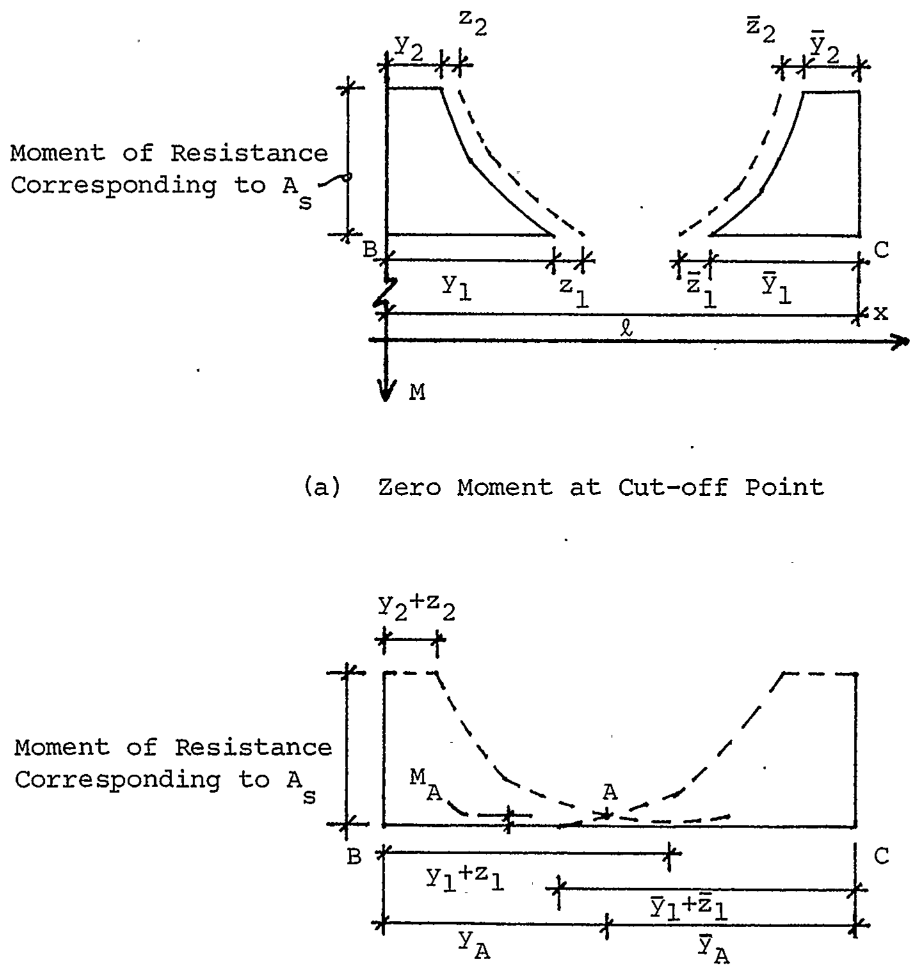
This function subprogram is used to determine the extents of the top reinforcement. When only one cut-off point is specified it is used to determine the points where no top reinforcement is needed. When two cut-off points are specified, it is also used to determine the points where a specified area of reinforcement is no longer required (theoretically). In all cases, the theoretical extents of top reinforcement are calculated for all loading conditions and the most critical values are taken.



(a) (H_1+M_o) negative

(b) $M(x) < M_o$ for all x

Fig. 4.2. - Theoretical Cut-off Points



(a) Zero Moment at Cut-off Point

(b) Non-Zero Moment at Cut-off Point

Fig. 4.3. - Full Extent of Top Reinforcement

4.4 FULL EXTENT OF TOP REINFORCEMENT

The full extent of top reinforcement is the theoretical extent, described above, plus the anchorage and development lengths required for flexural resistance. Additional top reinforcement for shear resistance is considered separately in Section 4.7.

A general segment of a bending moment envelope is presented in Fig. 4.3 (previous page). The area of reinforcement, A_s, resists moment at Support B. The theoretical cut-off points are y_1 and y_2. Also associated with Support B are the theoretical cut-off points at the opposite support, $\bar{y}_1$ and $\bar{y}_2$. When there is only one cut-off point at B or when the first of two cut-off points is being considered, y_2 is zero. The amounts by which the extents of reinforcement must exceed those required for flexure, z_1, z_2, $\bar{z}_1$ and $\bar{z}_2$, are illustrated in the figure (see, for example, Clause 12.12.3, Canadian Standards Association, 1984).

When the cut-off points are such that $(y_1 + z_1 + \bar{y}_1 + \bar{z}_1)$ is less than or equal to the span length, ℓ, the calculation of full extent is relatively simple. The extent of reinforcement to be provided at support B, Y_1, is the greater of $(y_2 + z_2 + \ell_d)$ or $(y_1 + z_1)$, where ℓ_d is the development length for A_s. This can be written:

$$Y_1 = \max(y_1 + z_1,\ y_2 + z_2 + \ell_d) \tag{4.22}$$

When $(y_1 + z_1 + \bar{y}_1 + \bar{z}_1)$ exceeds the span length, (Fig. 4.3b), Y_1 is the greater of $(y_2 + z_2 + \ell_d)$ or $y_A + \ell_d'$, where ℓ_d' is an appropriate development length for the moment, M_A, illustrated in Fig. 4.3b. If there are n_ℓ loading conditions, M_A is given by:

$$M_A = \max_{j=1}^{n_\ell} \{\max_{k=1}^{n_\ell} (M_A)_{jk}\} \quad , \tag{4.23}$$

where $(M_A)_{jk}$ is the moment at the intersection of the bending moment diagrams for the j^{th} and the k^{th} loading conditions. It is anticipated that a splice will be specified only when M_A is relatively small. Thus, it is assumed that the differences between the full and theoretical extents at the point of intersection are z_1 and $\bar{z}_1$. Accordingly, $(M_A)_{jk}$, is defined by the equation,

$$2a_5 + 2a_2a_3 - a_1a_4 - 4(M_A)_{jk}(a_3 - 1)/q_k =$$

$$\{[a_1^2 - 4a_2 + 8(M_A)_{jk}/q_k][a_4^2 - 4a_3a_5 + 8a_3(M_A)_{jk}/q_k]\}^{\frac{1}{2}} \qquad (4.24)$$

where,

$$a_1 = \ell[\frac{2(H_{kC} - H_{kB})}{q_k\ell^2} - 1] + 2\bar{z}_1 \qquad (4.25)$$

$$a_2 = \bar{z}_1{}^2 + \bar{z}_1\ \ell[\frac{2(H_{kC} - H_{kB})}{q_k\ell^2} - 1] + \frac{2\ell^2\ H_{kB}}{q_k\ell^2} \qquad (4.26)$$

$$a_3 = \frac{q_j}{q_k} \qquad (4.27)$$

$$a_4 = a_3\{\ell[\frac{2(H_{jC} - H_{jB})}{q_j\ell^2} - 1] - 2z_1\} \qquad (4.28)$$

and

$$a_5 = a_3\{z_1^2 - z_1[\frac{2(H_{jC} - H_{jB})}{q_j\ell^2} - 1] + \frac{2\ell^2 H_{jB}}{q_j\ell^2}\} \qquad (4.29)$$

In these equations, H_{iB} and H_{iC} are the hogging moments at supports B and C respectively and q_i is the uniformly distributed loading, for the i^{th} loading condition, $i = 1, n_\ell$. The computational labour involved in making n_ℓ^2 evaluations of the term $(M_A)_{jk}$, is considered by the author to be inappropriate in view of the approximate nature of reinforced concrete design. Accordingly, it is assumed that

$$M_A = (M_A)_{\hat{j}\hat{k}}$$

where $\hat{j}$ and $\hat{k}$ are the loadings that define y_1 and $\bar{y}_1$ respectively. If $\hat{j} = \hat{k}$ and $z_1 = \bar{z}_1$, then,

$$a_1 = a_4, \tag{4.30}$$

$$a_2 = a_5, \tag{4.31}$$

and

$$a_3 = 1. \tag{4.32}$$

Hence, Eqn. 4.24 implies,

$$M_A = \frac{q_k}{2}\{a_2 - \frac{a_1^2}{4}\} \tag{4.33}$$

In the general case, Eqn. 4.24 implies a quadratic equation in $(M_A)_{\hat{j}\hat{k}}$:

$$\begin{aligned}(2M_A/q_k)^2(a_3 - 1)^2 &+ (2M_A/q_k)\{2(a_3 - 1)(a_5 - a_2a_3)\\ &+ (a_4 - a_1)(a_1a_3 - a_4)\} + \{(a_5 - a_2a_3)^2\\ &+ (a_2a_4 - a_1a_5)(a_4 - a_1a_3)\} = 0\end{aligned} \tag{4.34}$$

For $q_{\hat{j}} = q_{\hat{k}}$, a_3 has a value of unity and Eqn. 4.34 gives,

$$M_A = \frac{q_k}{2}\,\frac{\{(a_5 - a_2)^2 + (a_2a_4 - a_1a_5)(a_4 - a_1)\}}{(a_4 - a_1)^2} \tag{4.35}$$

When $q_{\hat{j}}$ and $q_{\hat{k}}$ are not equal,

$$M_A = \frac{-q_k[2(a_3 - 1)(a_5 - a_2a_3) + (a_4 - a_1)(a_1a_3 - a_4) + \sqrt{a_6}]}{4(a_3 - 1)^2} \quad (4.36)$$

where,

$$a_6 = (a_1a_3 - a_4)[4(a_3 - 1)^2(a_2a_4 - a_1a_5) + (a_1 - a_4)^2(a_1a_3 - a_4)$$

$$+ 4(a_3 - 1)(a_1 - a_4)(a_2a_3 - a_5)] \quad (4.37)$$

It can be shown by substitution into Eqn. 4.24 that the lesser root of the quadratic is the correct one. Having found M_A, we can readily calculate y_A. It is the extent of the span over which the bending moment exceeds M_A. As such, it can be found using the subprogram described in Section 4.2. In this calculation also, it is assumed that the $\hat{j}^{th}$ loading is appropriate. The development length for M_A is,

$$\ell_d' = \frac{\ell_d \, M_A}{M_R(A_s)} \quad (4.38)$$

where $M_R(A_s)$ is the moment of resistance associated with A_s.

If A_s is the area of steel at an interior support, then the total extent of A_s is the sum of the extents in the spans adjacent to it. If, on the other hand, A_s is the area of steel at an external support which is attached to an external concrete member, an additional extent of reinforcement is required to anchor the steel in that member. This extent is taken as $(z_1 + \ell_d)$. Thus, at all supports, the full extent of top reinforcement for flexural resistance can be calculated.

4.5 FULL EXTENT OF BOTTOM REINFORCEMENT

The full extent of bottom reinforcement required for flexural resistance is determined. In each span of the continuous beam, the bottom reinforcement is required to extend at least 150 mm beyond the ends of that span. The only exception is at a simple support when the reinforcement is required to extend only as far as the end. These requirements are specified in Clause 12.11.1 for one-fourth of the reinforcement. It is assumed here that all of the bottom reinforcement is cut-off at the same point. This simplifies construction and provides useful longitudinal reinforcement for shear resistance.

The bending moment distributions for given loading conditions are illustrated in Fig. 4.4. Local co-ordinate axes are located at the two ends. In both cases, the direction is inward from the support towards the centre of the span. The subscript n refers to the end of the span being considered; n = 1 refers to the left hand end and n = 2 refers to the right. The required moment of resistance of the bottom reinforcement, S, is assumed to be developed linearly over a length, ℓ_d, starting at the point $x = x_{sn}$. For the case illustrated in Fig. 4.4a, the extent of reinforcement is minimum when the required moment capacity function is tangent to the applied moment function M(x). The point of tangency is given by:

$$x_{tn} = \ell\{0.5 + (3 - 2n)(H_1 - H_2)/(q\ell^2) - S\ell/(q\ell^2\ell_d)\} \tag{4.39}$$

where q is the loading intensity, ℓ is the span length and H_n is the hogging moment at the n^{th} end. For this case, the point of tangency of the two moment functions, x_{tn}, is their point of intersection, x_{in}. For the case illustrated in Fig. 4.4b, there is no point of tangency between

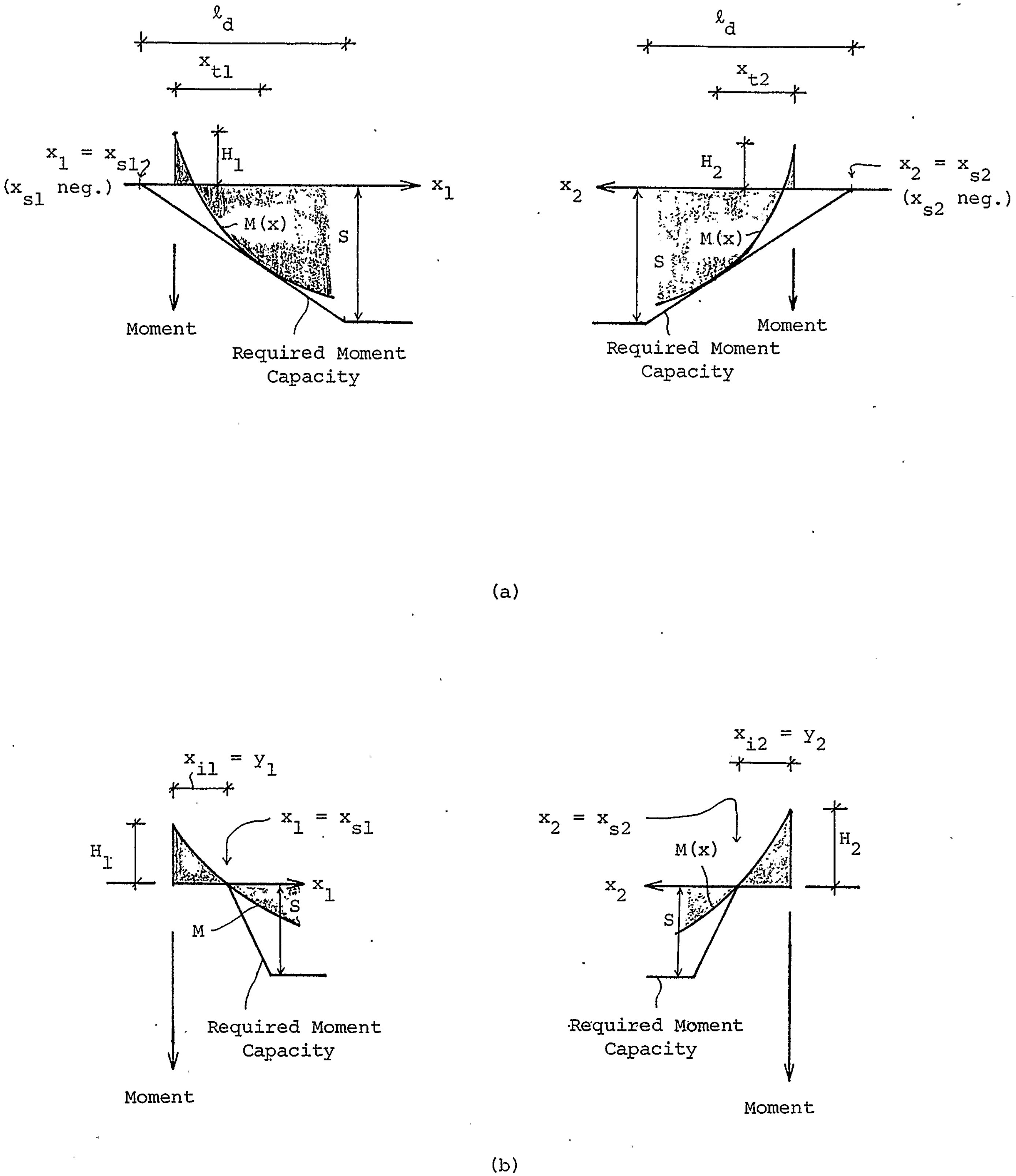


Fig. 4.4 - Full Extent of Bottom Reinforcement

the two functions in the region of positive M(x). The extent of reinforcement is minimum, in this case, at the first point of intersection, x_{in}, of the required capacity function with the moment function. It can be seen that, in the latter case, x_{in} equals the theoretical extent of top reinforcement, y_n. In general,

$$x_{in} = \max[x_{tn}, y_n] \tag{4.40}$$

The point at which the required moment capacity is zero is, in general, given by,

$$x_{sn} = x_{in} - \frac{\ell_d \, M(x_{in})}{S} \tag{4.41}$$

The dimension x_{sn} is calculated for all loadings. If $\hat{x}_{sn}$ is the minimum value calculated, then the bottom steel extends to the point,

$$x_n = \min\{-150, \hat{x}_{sn} - z_o\} \tag{4.42}$$

where z_o is the amount by which the extent of bottom reinforcement must extend beyond that theoretically required and where $\hat{x}_{sn}$ and z_o are measured in millimetres.

In this section, the full extent of bottom reinforcement required for flexural resistance is calculated. In most cases, the reinforcement extends 150 mm into the support. However, when the situation occurs that additional reinforcement is necessary, the calculation of the tangent point location ensures that no more than the minimum required for safety is provided.

4.6 COMPRESSION REINFORCEMENT

4.6.1 Introduction

Compression reinforcement is sometimes required to ensure a ductile section while resisting a large moment. While it is recognised that it is also sometimes used to reduce deflections, it is not specified here unless required for ductility. In the usual case of a T-section, the area of concrete in the upper part of the section is large. Accordingly, there is almost always sufficient ductility for positive moment and no compression reinforcement is required. On the other hand, it is quite reasonable to expect that ductility for negative moment will occasionally be inadequate unless compression reinforcement is provided. When this occurs, compression reinforcement is provided in the form of an extension to the existing bottom steel.

To ensure ductility, the Canadian code, CAN3-A23.3-M84, specifies an upper limit on the ratio (c/d) where c and d are the depths from the extreme compression fibre to the neutral axis and the centroid of tension steel, respectively. The program has been written however, so the user may specify a more stringent limit if desired. The moment corresponding to maximum (c/d) with only tension reinforcement present, is H_{xo}. When the moment, H is less than H_{xo}, no compression reinforcement is provided. When H exceeds H_{xo}, the area of reinforcement is first calculated that will provide resistance to H_{xo} in the absence of compression reinforcement. Then, the additional area required to provide resistance to $(H - H_{xo})$ in the presence of the compression reinforcement is found and added to the first.

Initially, the possibility of extending the bottom steel from only one of the spans meeting at the support is considered. If this provides insufficient ductility, the steel from both spans may be extended.

4.6.2 Extension of Reinforcement From One Span

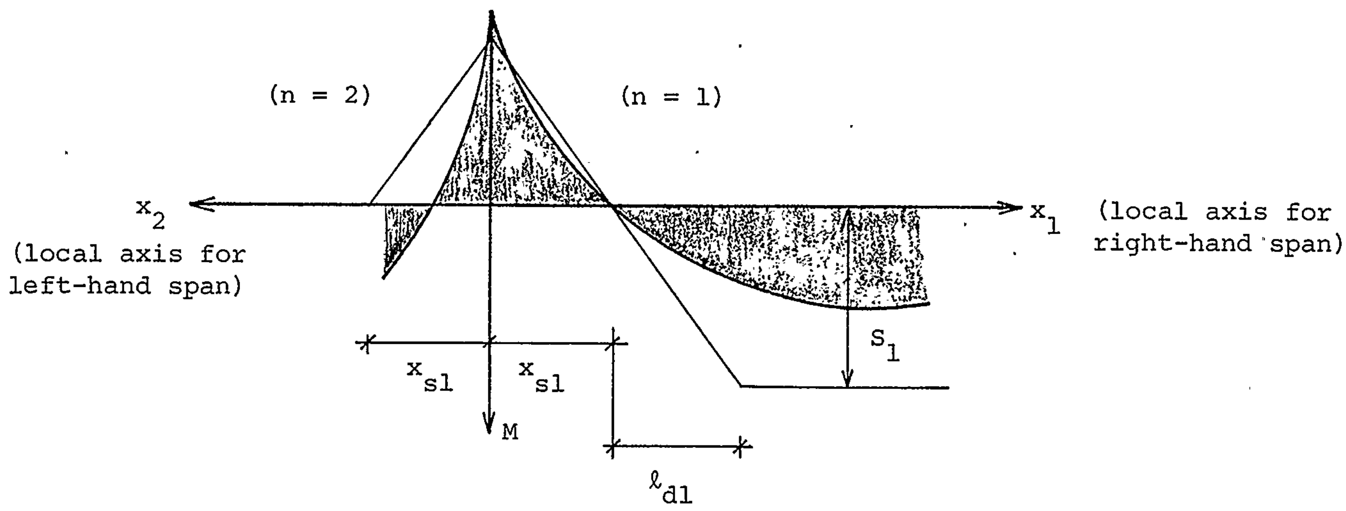
In many cases, the extension of the reinforcement from one span is sufficient to provide the ductility necessary for the provision of adequate reinforcement at a support. Each span is considered in turn and the extension requiring the least overall volume of steel is selected.

The portion of the bottom steel from any one span that can be used in compression is limited by the amount of compressive stress that can be developed in the bars near the support. From a consideration of the force equilibrium at a section, the ratio (c/d) can be shown to be a linear function of $(A_s f_s - A_s' f_s')$, where A_s and A_s' are the areas of steel and f_s and f_s' are the stresses, in tension and compression respectively. Thus, for specified (c/d), the term $(A_s f_s - A_s' f_s')$ is constant; an increase in A_s is limited by the magnitude of f_s' that can be developed.

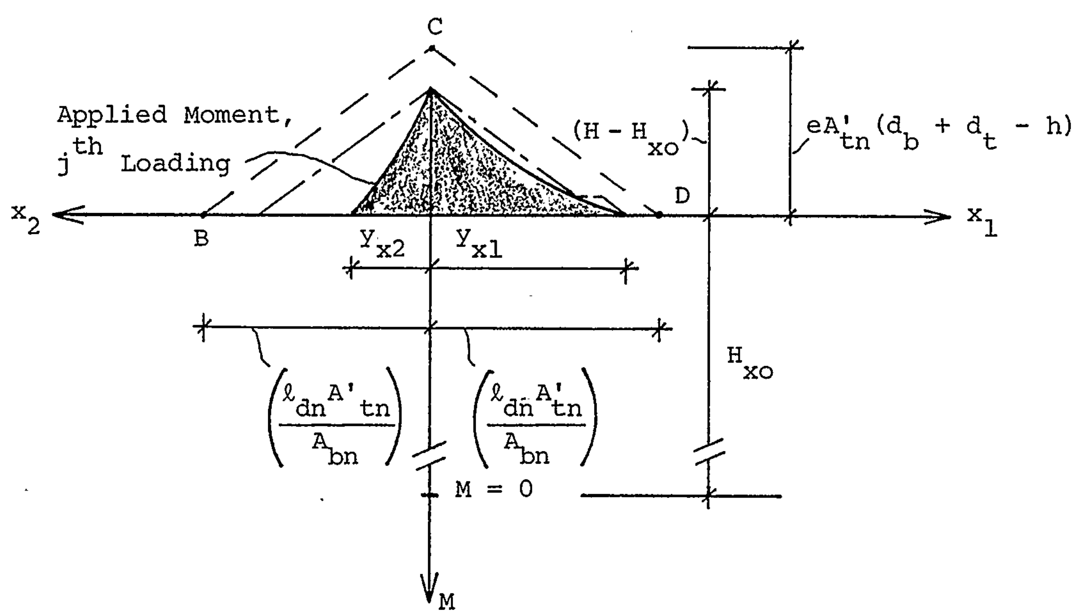
The bending moment diagram for the j^{th} loading condition is illustrated in Fig. 4.5a. As in Section 4.5, local co-ordinate directions are measured inwards from the support towards the centres of the spans. The sag moment in the right-hand span, S_1, has a development length ℓ_{d1}. In this span, the bottom steel is no longer required for resistance of sagging moment at the point,

$$x_1 = x_{s1}$$

An expression for x_{sn}, n = 1,2, is derived in Section 4.5 (Eqn. 4.41). At the point, $x_1 = 0$, the maximum compressive stress that can be



(a) Development of Compression Reinforcement



(b) Moment Capacity with Compression Reinforcement

Fig. 4.5. - Compression Reinforcement at Support

$$\min\{f_y, f_y\, x_{s1}/\ell_{d1}\} \quad , \quad \text{for } x_{s1} \geqq 0 \tag{4.43}$$

where f_y is the yield strength of the steel. When x_{s1} is negative, no compressive stress can be developed at this point. This compressive stress in the bottom steel is equivalent to compression reinforcement, whose full yield strength can be developed, of area:

$$A'_{t1} = \begin{cases} \min\{A_{b1}, A_{b1}x_{s1}/\ell_{d1}\} \quad , \quad \text{for } x_{s1} \geqq 0 \\ \\ 0 \quad , \quad \text{for } x_{s1} \leqq 0 \end{cases} \tag{4.44}$$

In general, the equivalent area of compression reinforcement available through extension of the bottom steel from side n, is

$$A'_{tn} = \begin{cases} \min\{A_{bn}, A_{bn}\, x_{sn}/\ell_{dn}\} \quad , \quad \text{for } x_{sn} \geqq 0 \\ \\ 0 \quad , \quad \text{for } x_{sn} \leqq 0 \end{cases} \tag{4.45}$$

The amount by which the area of tension reinforcement A_t, can be increased to resist a moment greater than H_{xo}, is equal to the equivalent area of compression reinforcement, A'_{tn}. From Eqns. 4.5 and 4.11, the new maximum moment capacity at the support is,

$$H_{xo} + e\, A'_{tn}(d_b + d_t - h) \tag{4.46}$$

where d_b and d_t are the effective depths to the bottom and top reinforcements respectively and h is the total beam depth. The new capacity in the region of the support is now bounded by the line segments BC and CD in Fig. 4.5b. The extents in each span, y_{x1} and y_{x2}, for which the hogging moment near the support exceeds H_{xo}, are calculated using the subprogram described in Section 4.3. Due to the concavity of the bending moment diagram, there are only three critical

Due to the concavity of the bending moment diagram, there are only three critical points near the support,

$$x_1 = 0$$

$$x_1 = y_{x1} \tag{4.47}$$

$$x_1 = -y_{x2}$$

When the support moment, H, satisfies,

$$H \leq H_{xo} + e\, A'_{tn}(d_b + d_t - h), \tag{4.48}$$

and

$$y_{xn} \leq x_{sn}, \tag{4.49}$$

then the moment near the support can be resisted by an extension of A_{bn} and an increase of A_t. The bottom reinforcement, A_{bn}, must then extend to the point,

$$x_n = \min\{-y_{x\ 3-n}, -(H - H_{xo})\ell_{dn}/[e\ A_{bn}(d_b + d_t - h)]\} - z_{on} \tag{4.50}$$

where z_{on} is the amount by which the bottom reinforcement on Side n must exceed that theoretically required. The amount by which A_t must be increased is,

$$\delta A_t = \frac{(H - H_{xo})}{e(d_b + d_t - h)} \tag{4.51}$$

These calculations are repeated for all loading conditions for which $H \geq H_{xo}$, and the maximum values of δA_{tn} and x_n are selected. If the extensions associated with both sides are feasible, the extra volume of steel is calculated in each case and the extension corresponding to least volume is taken.

4.6.3 Extension of Reinforcement From Both Spans

In cases of very large negative moment at a support, an extension of the bottom steel from one span alone may not provide sufficient ductility. When this is the case, extensions to both of the bottom steels meeting at the support are considered.

The maximum area of bottom steel available for compression reinforcement at a support is,

$$A'_{t1} + A'_{t2},$$

where A'_{tn} is given by Eqn. 4.45, for n = 1,2. The corresponding maximum moment capacity at the support is

$$H_{xo} + e(d_b + d_t - h)(A'_{t1} + A'_{t2})$$

When extension of the reinforcement from both spans is considered, several different designs can be shown to provide sufficient resistance. The following approach to this complex problem is adopted here:

(1) The equivalent areas of compression reinforcement, A'_{tn}; n = 1,2, are calculated. These areas can be assumed to be available at the support.

(2) The capacity at the support is determined. If the capacity is found to be insufficient, the section is deemed infeasible and the compression reinforcement calculations are terminated.

(3) The following portions of the total capacity at the support are assigned to the bottom reinforcement from the spans:

$$dH_{Rn} = \frac{(H - H_{xo})A'_{tn}}{(A'_{t1} + A'_{t2})} \quad ; \quad n = 1,2 \tag{4.52}$$

This moment can be developed at or (more probably) before the support. The reinforcement from Side n, n = 1,2 is extended to, $x_n = -x_{qn}$, where x_{qn} is such that a moment of dH_{Rn} can be developed at the support. Thus,

$$x_{qn} = 150 + \frac{dH_{Rn}\,\ell_{dn}}{eA_{bn}(d_b + d_t - h)} \quad , \tag{4.53}$$

where all linear measurements are expressed in millimetres. The dotted and chained lines in Fig. 4.6a represent conservative bounds on the capacity in the region of the support. The combined capacity is represented in Fig. 4.6b.

(4) If, $\max(x_{sn}, x_{q\ 3-n}) < y_{xn}$, then $A_{b\ 3-n}$ is extended to the point, $x_n = y_{xn} + z_{o\ 3-n}$.

(5) A further check is required at the points P_1 and P_2 in Fig. 4.6b, where the moment capacity diagram is concave. The point P_n (n = 1,2) is defined by, $\min(x_{sn}, x_{q\ 3-n})$. If, $x_{sn} > x_{q\ 3-n}$, then the moment capacity is defined by the steel from Side n. If, on the other hand, $x_{sn} < x_{q\ 3-n}$, the steel from Side 3-n defines the moment capacity at P_n. In general, the moment capacity at P_n is:

$$H_{Pn} = H_{xo} + eA_{bm}(d_b + d_t - h)\,\min\{1, \frac{|x_{sn} - x_{q\ 3-n}|}{\ell_{dm}}\} \tag{4.54}$$

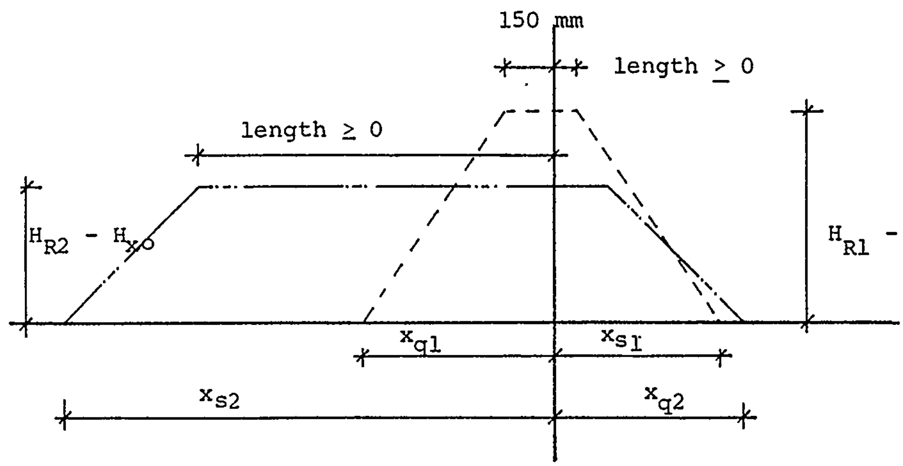
where,

$$m = n \qquad \text{for} \qquad x_{sn} > x_{q\ 3-n} \tag{4.55}$$

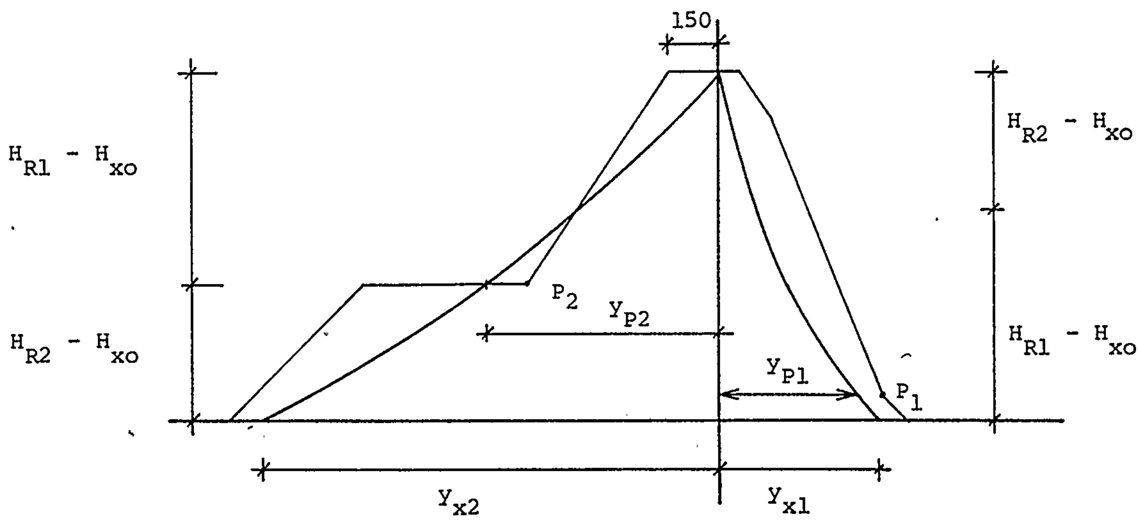
and,

$$m = 3\text{-}n \qquad \text{for} \qquad x_{sn} < x_{q\ 3-n} \tag{4.56}$$

Using the subprogram described in Section 4.3, the extents, y_{Pn};



(a) Lower Bounds on Capacities of Reinforcement from Individual Spans



(b) Lower Bound on Total Combined Capacity

Fig. 4.6. - Development of Compressive Stresses for Large Hogging Moment at Support

$n = 1,2$, over which hogging moment exceeds H_{Pn}, are calculated. If,

$$y_{Pn} > \min(x_{sn}, x_{q\ 3-n}),$$

(as illustrated in Fig. 4.6b), then the extent of bottom reinforcement from Side (3-n) is increased by,

$$y_{Pn} - \min(x_{sn}, x_{q\ 3-n}).$$

The amount by which the area of top steel must be increased is given by Eqn. 4.51. The maximum values for this increase and for the increases in the extents of bottom steel, are taken, for all loading conditions.

4.7 DESIGN OF SHEAR REINFORCEMENT

4.7.1 Introduction

New provisions for the resistance of shear stresses have been included in the new Canadian code of practice, CAN3-A23.3-M84. These provisions are based on the truss analogy in which diagonal cracks are assumed to form in the concrete. The designer, subject to certain constraints, may select the slopes of these cracks. An approximate procedure is described here for the determination of appropriate crack angles, spacing of transverse reinforcement and areas and extents of longitudinal reinforcement.

4.7.2 Ductility and Concrete Crushing

Crack angles at all sections must be large enough to prevent diagonal crushing of the concrete and yet small enough to ensure that failure, should it occur, is ductile. If no angle can be found to satisfy these two constraints, then section geometry is deemed inadequate. The longitudinal strain at the member mid-depth, is assumed in accordance with Clause 11.4.2.5, to be,

$$\varepsilon_x = 0.002.$$

Then, it follows from Clause 11.4.3 that, for ductile failure, the crack angles, θ, must satisfy:

$$\begin{aligned} \tan\theta &\leq 1 \quad , \quad f_y = 400 \text{ MPa} \\ \tan\theta &\leq 1.07 \ , \quad f_y = 300 \text{ MPa} \end{aligned} \tag{4.57}$$

Also, it follows from Clause 11.4.2 that, to prevent concrete crushing, the crack angles must exceed the values defined by:

$$1.14 \tan\theta + 1.82 \cot\theta + 0.68 \cot^3\theta = \frac{\phi_c\ f'_c\ b_v d_v}{V_f(x)} \tag{4.58}$$

where,

b_v is the minimum effective web width for shear,

d_v is the effective shear depth,

f'_c is the compressive strength of the concrete,

$V_f(x)$ is the shear force at a distance x from the left support,

ϕ_c is the resistance factor for concrete.

This relationship is plotted in Fig. 4.7 for $V_f(x)/(\phi_c f'_c\ b_v\ d_v)$ versus $\tan\theta$. The best fit linear approximation of this function for, $\tan 15° \leq \tan\theta \leq 1.07$, (also plotted), is:

$$\frac{V_f(x)}{\phi_c\ f'_c\ b_v\ d_v} = 0.359 \tan\theta - 0.0738 \tag{4.59}$$

Thus, an accurate estimate for the concrete crushing constraint on θ is given by:

$$\tan\theta \geq \frac{2.79\ V_f(x)}{\phi_c\ f'_c\ b_v\ d_v} + 0.206 \tag{4.60}$$

The simple constraints given by Ineqs. 4.57 and 4.60 are used to ensure that the crack angles assumed are adequate in terms of concrete stresses and the requirements of ductility.

4.7.3 Required Transverse Reinforcement

Comprehensive Design Procedure: The crack angle selected at a section implies the required amounts of both longitudinal and transverse reinforcement. Increasing the crack angle reduces the amount of longitudinal reinforcement required while increasing the quantity of transverse. Thus, if the area and extent of longitudinal reinforcement is fixed, the limitation implies a minimum feasible crack angle. At

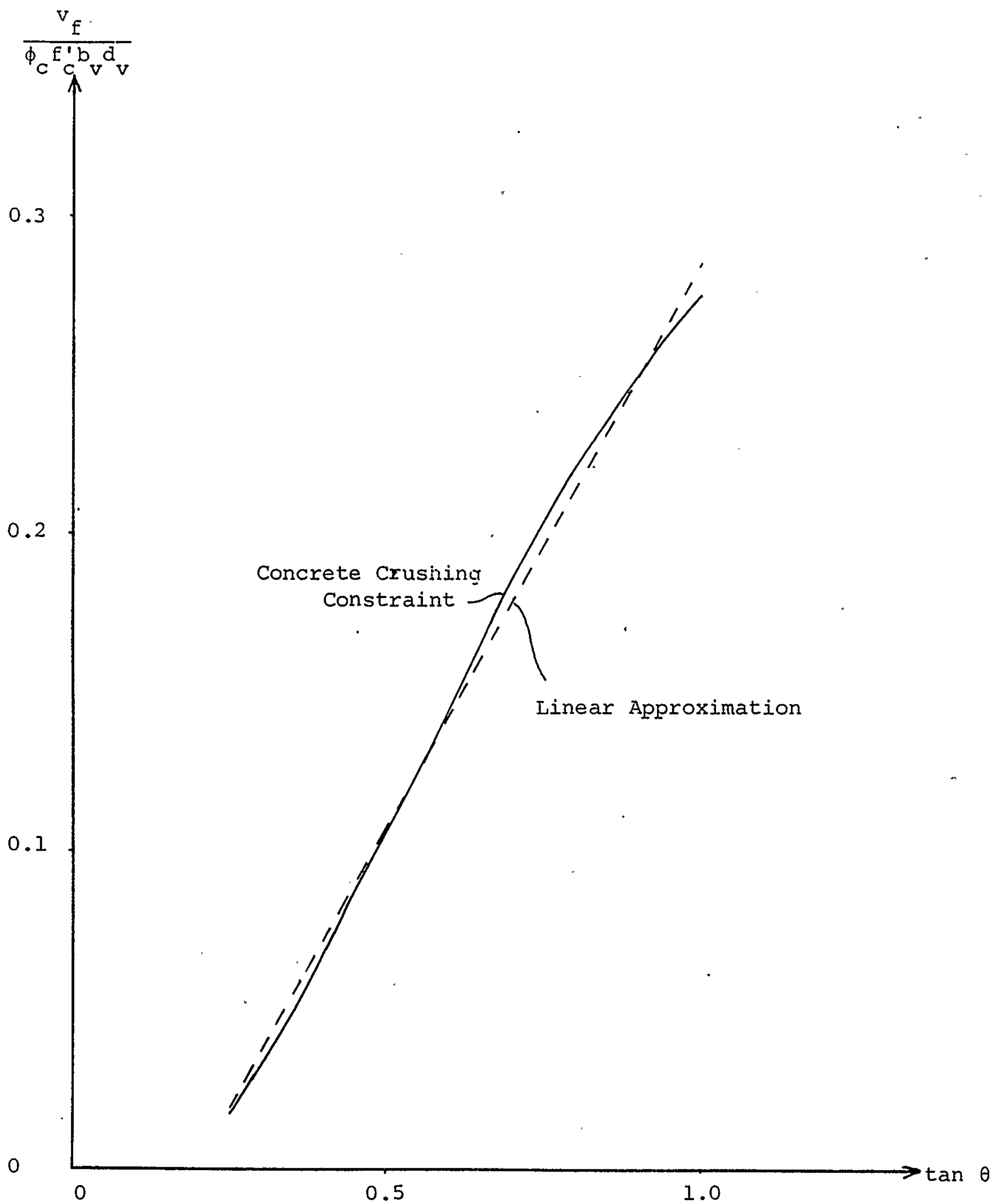


Fig. 4.7. - Minimum Crack Angle to Prevent Concrete Crushing

each section, the angle must exceed this minimum and the minimum required to prevent crushing of the concrete. The larger of these minima at each section is assigned to θ and the corresponding stirrup spacing calculated. This process is repeated at appropriately spaced sections to determine the volume of transverse reinforcement required in the region of each support. The volume of longitudinal reinforcement is already known. Thus, assuming costs per unit volume for longitudinal and transverse reinforcement, we can calculate the total cost. If this process is repeated for three different values of a cut-off point, and a quadratic equation is fitted to the results, an estimate can be made of the cut-off point corresponding to least cost. This process is repeated for each cut-off point for the steel at a support. It is felt that the interaction between these variables will not significantly affect the results. Accordingly, for top reinforcement with two cut-off points per span, an estimate of the shear reinforcement of least cost can be found by twelve calculations of total (shear reinforcement) cost in the region of the support.

Simplified Design Procedure: It is usually economical to minimize the amount of transverse reinforcement (Canadian Portland Cement Association, 1985; Dilger, 1981; see also Section 5.4). The shear reinforcement calculations, on the basis of this assumption, are very simple. The minimum $\theta(x)$ allowed by Ineq. 4.60 is selected. This implies the required amounts of longitudinal and transverse reinforcement at all sections.

Only the latter (simplified) design procedure for the calculation of shear reinforcement has been programmed.

4.8 CONCLUSIONS

In this chapter consideration is given to the design procedure after the specification of the concrete geometry and the stress envelopes. Algorithms are described for the calculation of the areas, lengths and locations of the longitudinal and transverse steels. Rigorous calculations are employed in a reasonable interpretation of the latest Canadian code of practice.

For T-, rectangular and inverted T-, sections, equations are developed for the calculation of the minimum required area of tension steel. For top reinforcement, an algorithm is described for the calculation of the theoretical cut-off points. For the locations of the actual cut-off points, anchorage and development requirements must also be considered. When the moment at a cut-off point is non-zero, the minimum required development length is calculated. It has been assumed throughout this work that bottom longitudinal steel should extend at least 150 mm into all internal supports. There may be some cases when it is required to extend this reinforcement further to provide sufficient development length for a particular loading condition. The minimum required amount is calculated that provides adequate resistance for all loading conditions and extends at least 150 mm into the support.

When the maximum allowable area of tension reinforcement is insufficient to provide adequate resistance at a support, the possibility of providing compression reinforcement is considered. The provision of compression reinforcement allows an increase in the area of tension reinforcement without compromising the requirements of ductility. The compression reinforcement is provided in the form of an extension to one or both of the bottom steels meeting at the support. A comprehensive series of checks ensures that adequate resistance is provided at all points in the region.

Changes to the provisions for shear have been made in the new code of practice. A linear approximation for the fourth order concrete crushing constraint has been determined. For the calculation of the crack angles which minimize the total cost, comprehensive and simplified procedures are described. The simplified procedures have been incorporated into a computer program.

Subroutines have been written based on all the algorithms described in this chapter. Due to the detail and rigour of the calculations, the subroutines result in designs that are safer and/or less expensive than those normally calculated by hand.

CHAPTER 5

A DECOMPOSITION APPROACH TO OPTIMUM DESIGN FOR MULTIPLE LOADINGS

5.1 INTRODUCTION

The problem of optimum design of reinforced concrete continuous beams subject to multiple loading conditions is considered. The cross-section dimensions and the areas of longitudinal and transverse reinforcement are sought for all sections in each beam span. The design is required to satisfy the constraints of serviceability and the ultimate limit state. Elastic analysis with "plastic moment redistribution" is used to determine the bending moment and shear force envelopes.

This optimization problem would be computationally prohibitively expensive to solve by straight application of conventional mathematical programming techniques. Fortunately, the problem is serial in nature. While no efficient conventional method is available which can exploit this serial nature, it was possible to develop a method of "sequential decomposition" (SD), for the optimization of serial problems. The method is more general than similar approaches (eg., Kirsch, Reiss and Shamir, 1972) of which the author has knowledge and is applicable to a broad range of problems. The SD procedure involves the consideration of the overall problem as being composed of a number of subproblems. The subproblems are solved in a sequence which leads to convergence to the global optimum. A quadratic fitting technique is used to speed this convergence. Examples are given as illustrations of the procedure and as indicators of the relative efficiencies of various SD approaches.

SD is shown to be particularly effective for the continuous beam design problem. A straightforward application of the procedure to a typical example results in considerable savings in computer time. However, it is shown that substantial additional savings arise from the fact that the subproblems degenerate to ones involving only two to three variables. This small number is independent of the number of loading conditions considered. The simplicity of the subproblems makes possible the use of robust direct search optimization routines.

Computer programs have been developed for the application of the methods of SD to the continuous beam design problem. Programs with and without the degenerate form of the subproblem solution, programs allowing alternative SD approaches and programs using optimization routines with and without gradient calculations, are described.

5.2 OPTIMIZATION BY SEQUENTIAL DECOMPOSITION

5.2.1 Multi-Stage Problems

Multi-stage problems are those that can be divided into stages such that the variables at any stage are affected only by those in immediately adjacent stages. This includes serial multi-stage systems and systems with recycle. The multi-stage problem considered here is one with m stages. The variables associated with the i^{th} stage are

$$\underline{x}_i = (x_{i1}, x_{i2}, \ldots, x_{in_i}).$$

The objective function is a sum of terms involving components from (a) the vector $\underline{x}_i$, (b) vectors, $\underline{x}_{i-1}$ and $\underline{x}_i$ or (c) vectors $\underline{x}_i$ and $\underline{x}_{i+1}$ for all i. Similarly, constraints involve components of $\underline{x}_i$ and $\underline{x}_j$ only, where

$|i - j| \leq 1$. In a multi-stage system with recycle, the first and last stages are considered to be adjacent.

The i^{th} stage objective is the sum of all of the terms of the objective function which involve components of $\underline{x}_i$. Accordingly, for the objective function;

$$F = 50x_{11} - x_{12}x_{11}^2 + (x_{11} + x_{12})x_{21} + 50x_{21}x_{22}^2 + x_{22}x_{31}$$

$$+ 50(x_{31} + x_{32} + x_{33})^2 + [x_{31}^3 - x_{31}x_{11}]^{\frac{1}{2}} \quad ,$$

the stage vectors are,

$$\underline{x}_1 = (x_{11}, x_{12})$$

$$\underline{x}_2 = (x_{21}, x_{22})$$

$$\underline{x}_3 = (x_{31}, x_{32}, x_{33})$$

and the stage objectives are,

$$f_1 = 50x_{11} - x_{12}x_{11}^2 + (x_{11} + x_{12})x_{21} + [x_{31}^3 - x_{31}x_{11}]^{\frac{1}{2}}$$

$$f_2 = (x_{11} + x_{12})x_{21} + 50x_{21}x_{22}^2 + x_{22}x_{31}$$

$$f_3 = x_{22}x_{31} + 50(x_{31} + x_{32} + x_{33})^2 + [x_{31}^3 - x_{31}x_{11}]^{\frac{1}{2}}$$

It can be noted that the evaluation of all the stage objectives requires, in general, less computational labour than two full objective function evaluations.

Some multi-stage problems can be solved by dynamic programming. However, this method is unsuitable when the numbers of variables at a stage, n_i, is large and is not applicable to problems with recycle.

5.2.2 Second Order Methods

Methods of optimization based on the calculation of second derivatives are extremely powerful for a broad range of problems. However, they become inefficient for problems involving a large number of variables for the following two reasons (Kirsch, 1981):

(1) The number of components in the Hessian matrix of second derivatives increases with the square of the number of problem variables. For large problems, the computational effort required for their calculation can become excessive.

(2) Inversion of the Hessian becomes computationally more expensive as the size of the matrix increases.

Multi-stage problems are far less sensitive than others to both of these computational difficulties.

In the first instance, far fewer components of the Hessian matrix need to be calculated if a problem is recognized as being of a multi-stage type. This follows from the fact that there are no terms of the objective function which involve components of $\underline{x}_i$ and $\underline{x}_j$ for $|i - j| > 1$. Thus, all of the derivatives involving these components are known to be zero. Additional advantage is gained from the fact that stage objectives can be used for numerical evaluation of the components of the Hessian. As an example, consider a 10-stage problem with three components per stage. The Hessian matrix calculations involve 171 second derivative evaluations. If numerical approximations are being used, stage objectives can be evaluated instead of the full function objective at one fifth the computational labour or less. Thus, the calculations involve the equivalent of less than 35 full objective function second derivative calculations. If the multi-stage nature of the problem is ignored, 900 full objective function second derivative calculations are required. Clearly, the Hessian matrix is far more accessible to multi-stage problems.

The second major difficulty in the use of second order methods for large problems is in the inversion of the Hessian matrix. In fact, for large matrices, the Hessian is not inverted directly. Instead, the set of simultaneous equations for which the Hessian matrix components are coefficients, is solved, The computer time for this process can be shown to be a linear function of the square of the half band width of the matrix (Ghali and Neville, 1979). For the example outlined above, the half band width of the Hessian is only six and thus, the computer time is proportional to 36. In general, the half band width of the Hessian can equal the number of problem variables. Accordingly, for a general 30-variable problem, the computer time can be proportional to as much as 900. Clearly, Hessian matrix inversion is a relatively small task for multi-stage problems without recycle.

The quasi-Newton method (Section 3.3) also overcomes some of the problems associated with the conventional second order approach. However, if exact determination of the inverse Hessian matrix is possible at similar computational cost to an approximation, then the exact approach would surely produce substantial savings overall. In addition to expressing the problem in multi-stage form, provision should be made in the algorithm to calculate only the non-zero components of the Hessian matrix and to use only the stage objectives if numerical approximations are being made. Clearly, the banded nature of the matrix should also be fully exploited in its inversion. For many multi-stage problems with continuous gradients, such an algorithm may well be the most efficient.

5.2.3 Sequential Decomposition

The method of sequential decomposition (SD) has been developed for the solution of multi-stage optimization problems either with or without recycle. It is applicable to all deterministic optimization problems, no continuity of the objective function or its gradients being necessary. The method involves partial searches of design space in a sequence which is repeated until convergence is achieved. Convergence to the optimum can not be guaranteed when the objective function is "ill-conditioned". However, excellent results have been obtained with all of the problems considered. In its simplest form, the procedure for a (4k+1)-stage system ($k \geq 1$), is as follows:

Elementary Method:

(1) The vector $\underline{x}_{2k+1}$ is fixed. The portion of the objective function which involves components of vectors $\underline{x}_1$ to $\underline{x}_{2k}$ is minimized in a search of the sub-space defined by these vectors. Similarly, a minimum is sought in the sub-space defined by vectors $\underline{x}_{2k+2}$ to $\underline{x}_{4k+1}$. Because $\underline{x}_{2k+1}$ is fixed, these two searches are independent of one another.

(2) The vectors $\underline{x}_{k+1}$ and $\underline{x}_{3k+1}$ are fixed and minima are sought in the subspaces defined by vectors $\underline{x}_1$ to $\underline{x}_k$, $\underline{x}_{k+2}$ to $\underline{x}_{3k}$ and $\underline{x}_{3k+2}$ to $\underline{x}_{4k+1}$.

(3) Steps (1) and (2) are repeated until convergence is reached.

In a problem well suited for SD in which variables at a stage are not greatly influenced by variables in remote stages, convergence is rapid.

An example of the application of the procedure to the continuous beam design problem is reported in Section 5.3.2.

An alternative SD method involves the reduction of the subproblems to the minimization of stage objectives. The simplest way to achieve this for a (4k+1) stage system is as follows:

Even-Odd Alternation:

(1) Fix all of $\underline{x}_i$; i = 2,4, ..., 4k-2, 4k and minimize each of $f_i(\underline{x}_i)$; i = 1,3, ..., 4k-1, 4k+1.

(2) Fix all of $\underline{x}_i$; i = 1,3, ..., 4k-1, 4k+1 and minimize each of $f_i(\underline{x}_i)$; i = 2,4, ..., 4k-2, 4k.

(3) Repeat steps (1) and (2) to convergence.

This procedure has been applied successfully to the continuous beam design problem (Section 5.3.2). A slightly more complex version of it involves simultaneous solution of Steps (1) and (2) above. The procedure is as follows:

Simple Progression:

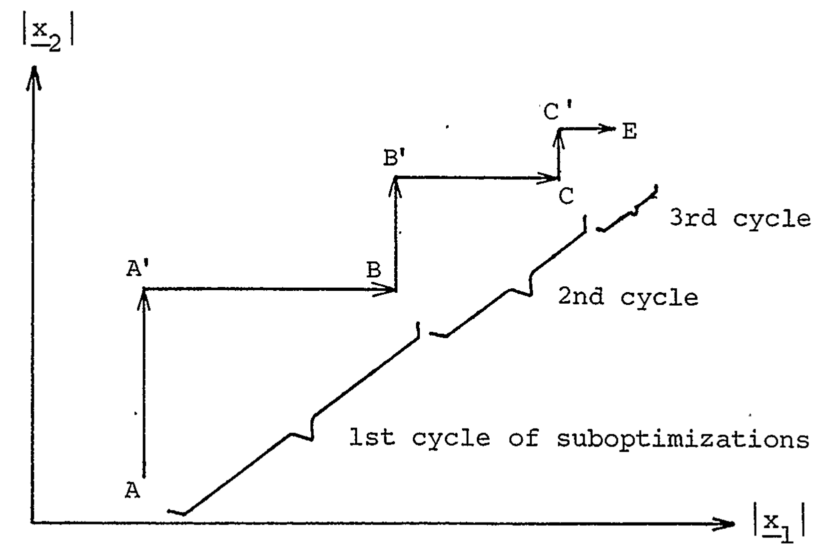
(1) For i = 1,4k-1, fix $\underline{x}_i$ and $\underline{x}_{i+2}$ and minimize $f_{i+1}(\underline{x}_{i+1})$. Finally, fix $\underline{x}_{4k}$ and minimize $f_{4k+1}(\underline{x}_{4k+1})$.

(2) Repeat step (1) until convergence is reached.

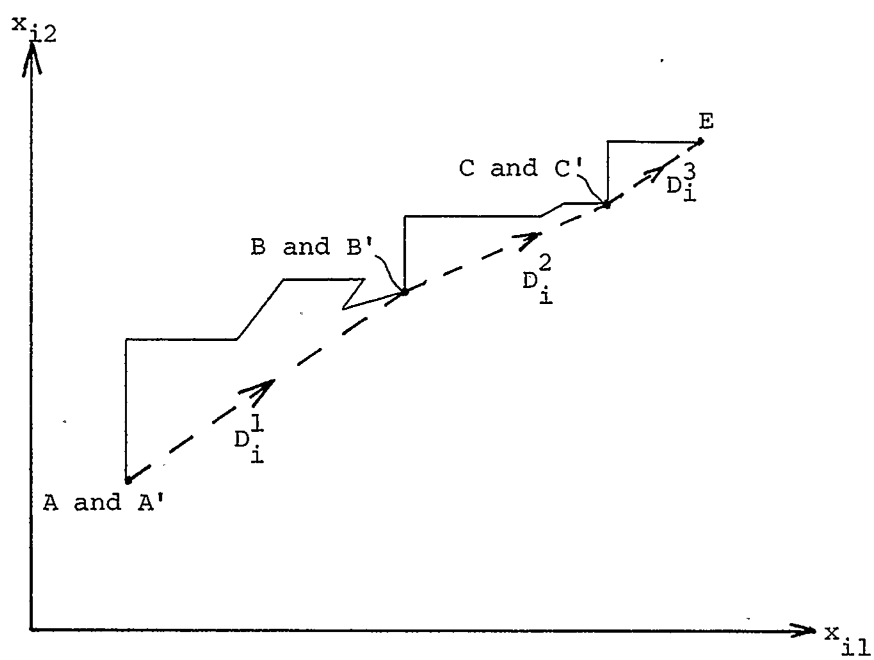
This procedure is illustrated by example in Section 5.2.5.

5.2.4 Quadratic Projection:

In the methods of SD outlined above, convergence can be slow for some problems. The search tends to follow a "zig-zag" pattern as illustrated in Fig. 5.1a. Convergence might be improved by a search in global vector space in a direction such as AB in that figure. However, for continuous beam design, degeneration of the stage optimization



(a) Global Search Without Quadratic Projection



(b) Successive Searches of Subspace $\underline{x}_i$

Fig. 5.1. - Search Patterns with Sequential Decomposition

subproblem makes a purely subspace-based method very efficient. Accordingly, this development is based on the results of subspace searches only. It is applicable only to systems without recycle.

An example with three successive stage optimizations is illustrated in Fig. 5.1b. In this figure, the vector $\underline{D}_i^j$ is the change in the stage vector, $\underline{x}_i$, resulting from the j^{th} optimization of this stage. If the difference vectors for three successive Stage i optimizations, $\underline{D}_i^j$, $\underline{D}_i^{j+1}$ and $\underline{D}_i^{j+2}$ are parallel for all i and the (full) objective function is quadratic, then the exact solution to the subproblems can be calculated. The equations derived on the basis of these assumptions are used regardless of whether or not the assumptions are true.

For an m-stage system, the vector modulus,

$$M_i = \left| \underline{x}_i - \underline{x}_i^o \right| \tag{5.1}$$

is used as a measure of the change in $\underline{x}_i$ from an initial point, $\underline{x}_i^o$. This initial design point is the design point at the start of a sequence of optimizations leading to quadratic projection; it is not necessarily the design point at the initiation of the SD procedure. Similarly, the "j^{th} optimization of Stage i" refers to the j^{th} optimization of this stage in the sequence. When the difference vectors are parallel, M_i is the distance in design space from $\underline{x}_i^o$. If the objective function is quadratic, then all stage objectives, f_i, must also be quadratic. Furthermore, f_i are quadratic functions of distance moved in any fixed direction in design subspace. Thus,

$$f_i = a_i + (b_i + c_{i-1,i} M_{i-1} + c_{i,i+1} M_{i+1}) M_i + c_i (M_i)^2 \tag{5.2}$$

where a_i, b_i, c_i and $c_{i,i+1}$ are constants for i=1,2,..., m and both c_{01} and $c_{m,m+1}$ are zero. Initially,

$$M_i = 0 \quad ; \quad i = 1,2, \ldots, m \tag{5.3}$$

Hence, the initial values of the stage objectives are given by,

$$f_i^o = a_i \quad ; \quad i = 1,2, \ldots, m \tag{5.4}$$

When the method of "simple progression" described in the preceding section is used, the i^{th} stage objective after its first optimization is,

$$f_i^1 = a_i + (b_i + c_{i-1,i}\ \delta_{i-1}^1)\ \delta_i^1 + c_i(\delta_i^1)^2, \tag{5.5}$$

where δ_i^j is the change in M_i resulting from the j^{th} optimization of Stage i. Hence, the change in the i^{th} stage objective resulting from its first optimization is

$$\Delta_i^1 = (b_i + c_{i-1,i}\ \delta_{i-1}^1)\ \delta_i^1 + c_i(\delta_i^1)^2 \quad ; \quad i = 1,2, \ldots, m \tag{5.6}$$

There is no term in δ_{i+1}^1, in the above equation for the method of simple progression involves no change in $\underline{x}_{i+1}$ until after the optimization of Stage i. In optimizations subsequent to the first,

$$\Delta_i^j = [b_i + c_{i-1,i} \sum_{k=1}^{j} (\delta_{i-1}^k) + c_{i,i+1} \sum_{k=1}^{j-1} (\delta_{i+1}^k)]\delta_i^j$$

$$+ c_i\delta_i^j[2 \sum_{k=1}^{j-1} (\delta_i^k) + \delta_i^j] \quad ; \quad i = 1,2,\ldots, m \quad ; \quad j > 1 \tag{5.7}$$

After three cycles of optimizations, sufficient information is available to evaluate the constants and to determine the location of the minimum of the quadratic function. The (3m) equations are:

$$\begin{bmatrix} 1 & \delta_i^1 & 0 \\ 1 & \delta_i^2 + 2\delta_i^1 & \delta_{i+1}^1 \\ 1 & \delta_i^3 + 2\delta_i^1 + 2\delta_i^2 & \delta_{i+1}^1 + \delta_{i+1}^2 \end{bmatrix} \begin{Bmatrix} b_i \\ c_i \\ c_{i,i+1} \end{Bmatrix} = \begin{Bmatrix} \Delta_i^1/\delta_i^1 - c_{i-1,i}\ \delta_{i-1}^1 \\ \Delta_i^2/\delta_i^2 - c_{i-1,i}(\delta_{i-1}^1 + \delta_{i-1}^2) \\ \Delta_i^3/\delta_i^3 - c_{i-1,i}(\delta_{i-1}^1 + \delta_{i-1}^2 + \delta_{i-1}^3) \end{Bmatrix} \tag{5.8}$$

If (δ_i^j) is zero for some j, then the term (Δ_i^j/δ_i^j) is indeterminate. However, this only occurs when optimization of Stage i results in no change in M_i in which case quadratic projection is inappropriate. Inversion of Eqns. 5.8 gives:

$$\begin{Bmatrix} b_i \\ \\ c_i \\ c_{i,i+1} \\ \\ \end{Bmatrix} = \begin{Bmatrix} [\delta_i^1\delta_{i+1}^1(\ell_i^3 - \ell_i^2 - c_{i-1,i}\delta_{i-1}^3) + \delta_i^1\delta_{i+1}^2\ (2\ell_i^1 - \ell_i^2 - c_{i-1,i}(\delta_{i-1}^1 - \delta_{i-1}^2)) \\ \quad + (\delta_i^2(\delta_{i+1}^2 - \delta_{i+1}^1) - \delta_i^3\delta_{i+1}^1)(\ell_i^1 - c_{i-1,i}\delta_{i-1}^1)]/k_i \\ [\delta_{i+1}^1(\ell_i^2 - \ell_i^3 + c_{i-1,i}\delta_{i-1}^3) + \delta_{i+1}^2(\ell_i^2 - \ell_i^1 - c_{i-1,i}\delta_{i-1}^2)]/k_i \\ [\delta_i^1(\ell_i^3 - \ell_i^2 - c_{i-1,i}\delta_{i-1}^3) + \delta_i^2(\ell_i^1 - 2\ell_i^2 + \ell_i^3 + c_{i-1,i}(\delta_{i-1}^2 - \delta_{i-1}^3)) \\ \quad + \delta_i^3(\ell_i^1 - \ell_i^2 + c_{i-1,i}\delta_{i-1}^2)]/k_i \end{Bmatrix} \tag{5.9}$$

where,

$$k_i = \delta_{i+1}^2(\delta_i^1 + \delta_i^2) - \delta_{i+1}^1(\delta_i^2 + \delta_i^3) \tag{5.10}$$

and,

$$\ell_i^j = \Delta_i^j/\delta_i^j \quad ; \quad j = 1,3 \tag{5.11}$$

Applying these equations at each stage starting with the first, all the constants, b_i, c_i, $c_{i,i+1}$; $i = 1,2,\ldots, m$, are found. The partial derivative of f_i with respect to M_i is, from Eqn. 5.2:

$$\frac{\partial f_i}{\partial M_i} = b_i + c_{i-1,i}M^*_{i-1} + c_{i,i+1}M^*_{i+1} + 2c_iM^*_i \tag{5.12}$$

Setting all partial derivatives to zero gives,

$$c_{i-1,i}M^*_{i-1} + 2c_iM^*_i + c_{i,i+1}M^*_{i+1} = -b_i \quad ; \quad i = 1,2,\ldots, m \tag{5.13}$$

Solving these m equations simultaneously gives the optimum moduli for the m stage vectors. The quadratic projection is along the direction defined by the most recent move in the stage vector, $\underline{x}_i$, namely, from $\underline{x}_i^2$ to $\underline{x}_i^3$. Thus,

$$\underline{x}_i^* = \underline{x}_i^2 + (\underline{x}_i^3 - \underline{x}_i^2)(M^*_i - \delta_i^1 - \delta_i^2)/\delta_i^3 \quad ; \quad i = 1,2,\ldots, m \tag{5.14}$$

It is anticipated that three successive stage optimizations will often involve progressively smaller moves in roughly the same direction and that projection will move the design point slightly further in that direction. If a component value is alternately increasing and decreasing in successive cycles, then Eqn. 5.14 is not applied. If, as anticipated, successive component changes are in the same direction, then further projection in that direction only is allowed. Quadratic projection is illustrated in the example following and its effectiveness is demonstrated for continuous beam design in Section 5.3.

5.2.5 Example

A method of SD is applied to a 4-stage unconstrained problem with two variables in the first stage and one in each of the others. The objective function is:

$$F = (x_{11} + x_{12})^2 + (x_{11} - x_{12})^2 + 2(x_{11} + x_{12} - x_{21})^2$$
$$+ (x_{21} - x_{31})^2 + x_{41}^2 \tag{5.15}$$

The stage objectives can be taken as:

$$\begin{bmatrix} f_1 \\ \\ f_2 \\ \\ f_3 \\ \\ f_4 \end{bmatrix} = \begin{bmatrix} (x_{11} + x_{12})^2 + (x_{11} - x_{12})^2 + 2(x_{11} + x_{12} - x_{21})^2 \\ \\ 2(x_{11} + x_{12} - x_{21})^2 + (x_{21} - x_{31})^2 \\ \\ (x_{21} - x_{31})^2 \\ \\ x_{41}^2 \end{bmatrix} \tag{5.16}$$

Solving the first derivative equations (independently) gives the stage optima:

$$\left.\begin{aligned} \underline{x}_1 &= \begin{bmatrix} x_{21}/3 \\ x_{21}/3 \end{bmatrix}, \quad \underline{x}_2 = \{[2(x_{11} + x_{12}) + x_{31}]/3\} \\ \underline{x}_3 &= \{x_{21}\}, \quad \underline{x}_4 = \{0\} \end{aligned}\right\} \tag{5.17}$$

Thus, for this simple example, the SD procedure involves a search for the simultaneous solution of Eqns. 5.17. Starting at an initial point of,

$$(\underline{x}_1^t, \underline{x}_2^t, \underline{x}_3^t, \underline{x}_4^t) = ((10, 10), 10, 10, 10) \tag{5.18}$$

three cycles of optimizations give:

$$\begin{bmatrix} \underline{x}_1 \\ \\ \underline{x}_2 \\ \underline{x}_3 \\ \underline{x}_4 \end{bmatrix} : \begin{bmatrix} \begin{bmatrix} 10 \\ 10 \end{bmatrix} \\ 10 \\ 10 \\ 10 \end{bmatrix} \rightarrow \begin{bmatrix} 3.333 \\ 3.333 \\ 7.778 \\ 7.778 \\ 0 \end{bmatrix} \rightarrow \begin{bmatrix} 2.593 \\ 2.593 \\ 6.049 \\ 6.049 \\ 0 \end{bmatrix} \rightarrow \begin{bmatrix} 2.016 \\ 2.016 \\ 4.705 \\ 4.705 \\ 0 \end{bmatrix}$$

The corresponding changes in the stage objectives are:

$$\begin{bmatrix} \Delta_1^1 & \Delta_1^2 & \Delta_1^3 \\ \Delta_2^1 & \Delta_2^2 & \Delta_2^3 \\ \Delta_3^1 & \Delta_3^2 & \Delta_3^3 \\ \Delta_4^1 & \Delta_4^2 & \Delta_4^3 \end{bmatrix} = \begin{bmatrix} -533 & -6.58 & -3.98 \\ -14.8 & -8.96 & -5.42 \\ -4.94 & -2.99 & -1.81 \\ -100 & 0 & 0 \end{bmatrix} \tag{5.19}$$

Hence,

$$[\delta_i^j] = \begin{bmatrix} -6.667 & -.741 & -.576 \\ -2.222 & -1.728 & -1.344 \\ -2.222 & -1.728 & -1.344 \\ -10 & 0 & 0 \end{bmatrix} \tag{5.20}$$

and

$$[\ell_i^j] = \begin{bmatrix} 80 & 8.89 & 6.91 \\ 6.67 & 5.19 & 4.03 \\ 2.22 & 1.73 & 1.34 \\ 10 & - & - \end{bmatrix} \tag{5.21}$$

Applying Eqns. 5.9 at Stages 1, 2 and 3 gives, respectively:

$$\begin{bmatrix} b_i \\ c_1 \\ c_{12} \end{bmatrix} = \begin{bmatrix} 160 \\ 12 \\ -8 \end{bmatrix}, \quad \begin{bmatrix} b_2 \\ c_2 \\ c_{23} \end{bmatrix} = \begin{bmatrix} -40 \\ 3 \\ -2 \end{bmatrix}, \quad \begin{bmatrix} b_3 \\ c_3 \\ c_{34} \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \tag{5.22}$$

As $\delta_4^2 = 0$, quadratic projection is not applied at this stage. Eqns. 5.13 are, for this example,

$$\begin{bmatrix} 24 & -8 & 0 \\ -8 & 6 & -2 \\ 0 & -2 & 2 \end{bmatrix} \begin{bmatrix} M_1 \\ M_2 \\ M_3 \end{bmatrix} = \begin{bmatrix} -160 \\ 40 \\ 0 \end{bmatrix} \tag{5.23}$$

Solving Eqns. 5.23 simultaneously gives:

$$(M_1^*, M_2^*, M_3^*,) = (-10, -10, -10) \tag{5.24}$$

These are the optimal changes of the moduli from the starting point, ((10, 10), 10, 10, 10). Finally, Eqn. 5.14 gives:

$$\begin{bmatrix} \underline{x}_1^* \\ \\ \underline{x}_2^* \\ \underline{x}_3^* \end{bmatrix} = \begin{bmatrix} \begin{bmatrix} 2.593 \\ 2.593 \end{bmatrix} \\ 6.049 \\ 6.049 \end{bmatrix} + \begin{bmatrix} \begin{bmatrix} -0.576 \\ -0.576 \end{bmatrix} \text{ x } (-2.593)/(-.576) \\ -1.344 \text{ x } (-6.049)/(-1.344) \\ -1.344 \text{ x } (-6.049)/(-1.344) \end{bmatrix} \tag{5.25}$$

$$\Rightarrow \qquad (\underline{x}_1^t, \underline{x}_2^t, \underline{x}_3^t, \underline{x}_4^t)^* = ((0, 0), 0, 0, 0) \tag{5.26}$$

In this example, the directions of change in successive Stage 1 optimizations are equal for, in each case,

$$x_{11} = x_{12} \tag{5.27}$$

The objective function is also quadratic. Accordingly, the solution found by this first quadratic projection is the exact solution. It will be found to simultaneously satisfy all of Eqns. 5.17.

5.3 CONTINUOUS BEAM DESIGN FOR MULTIPLE LOADINGS

5.3.1 Problem Formulation

Most reinforced concrete design codes require consideration of a number of different loading conditions. For a continuous beam, the most adverse effects of all loading conditions must be provided for, at each section. The resulting optimum design problem is large and complex. The designer generally has the power to decide on the following parameters: the concrete geometry in each beam span, the areas of top and bottom reinforcement and their cut-off points and the area and spacing of transverse reinforcement. Accordingly, these constitute the true variables of the optimization problem.

The constraints are the requirements of serviceability and the ultimate limit state. The ultimate strength requirements are that all sections be adequate to resist the moments and forces resulting from each of the loading conditions. The optimization problem is complicated at this point by the fact that, within certain limits, the designer is free to select the moments and forces that result from a given loading condition. Most reinforced concrete design codes provide for some "plastic moment redistribution" after an elastic analysis. This is in recognition of a degree of ductility that exists in the members and their ability to redistribute stresses before failure. Thus, the moments and forces to be resisted can have any values subject to the limitations:

(1) The conditions of equilibrium must be satisfied.

(2) The deviation of the moments selected from the moments found by elastic analysis must be less than the maximum deviation allowed.

This maximum allowable deviation is found from considerations of section ductility.

Suitable values for these moments are not readily apparent. A moment just equal to the moment of resistance at a support may imply, by equilibrium, a violation of a cut-off point constraint in an adjacent span (see Fig. 5.2). On the other hand, a lesser value may imply a sagging moment, in the other span, in excess of that moment of resistance.

The problem of selecting a feasible combination of the moments to be resisted is overcome, if these moments are considered to be the variables. In an alternative formulation, the support moments for all the loading conditions are considered as independent variables along with the section dimensions. Then, the steel areas and cut-off points are dependent variables; they are the minimum amounts required to resist the most adverse stresses at all sections. High support moments tend to imply large volumes of top reinforcement while small values can imply large volumes of bottom reinforcement. As the number of independently variable section dimensions is usually small, these variables can be treated as second level in a model co-ordination decomposition approach (Section 3.4.1). This still leaves a very large number of first level variables, more than $(n_s-1)n_\ell$ for a beam with n_s spans and n_ℓ loading conditions. However, the problem is in multi-stage problem form, each support corresponding to a stage. Accordingly, the first level problem is suitable for solution by methods of SD.

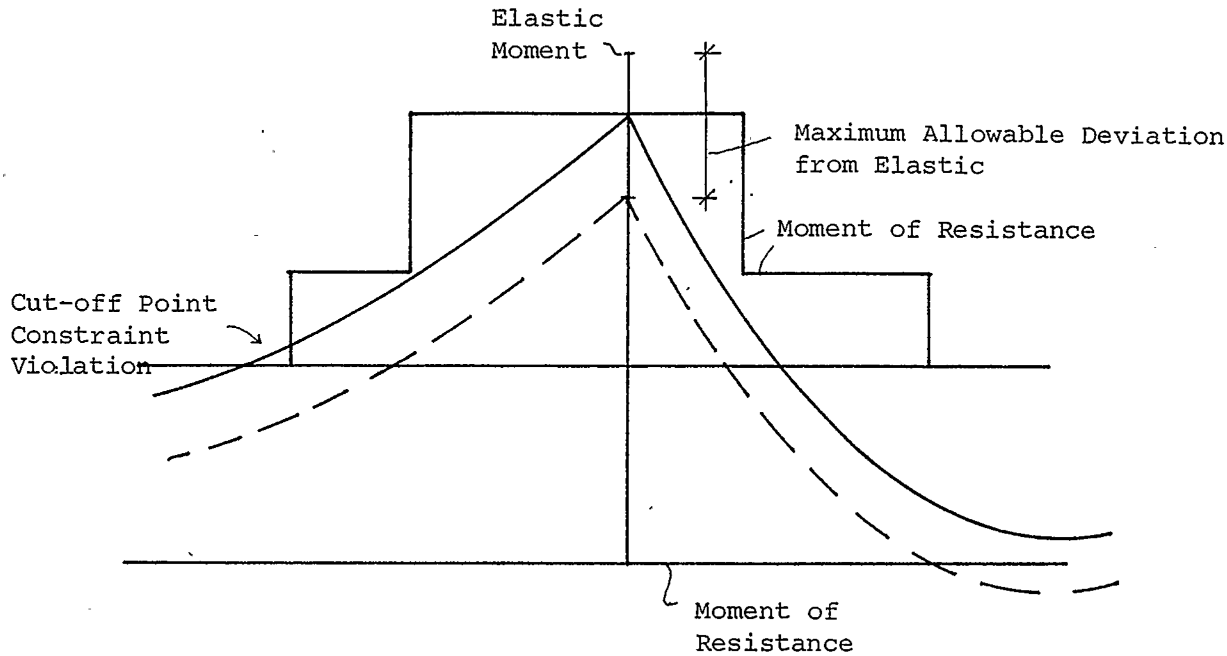


Fig. 5.2. - Bending Moment Diagram at a Support

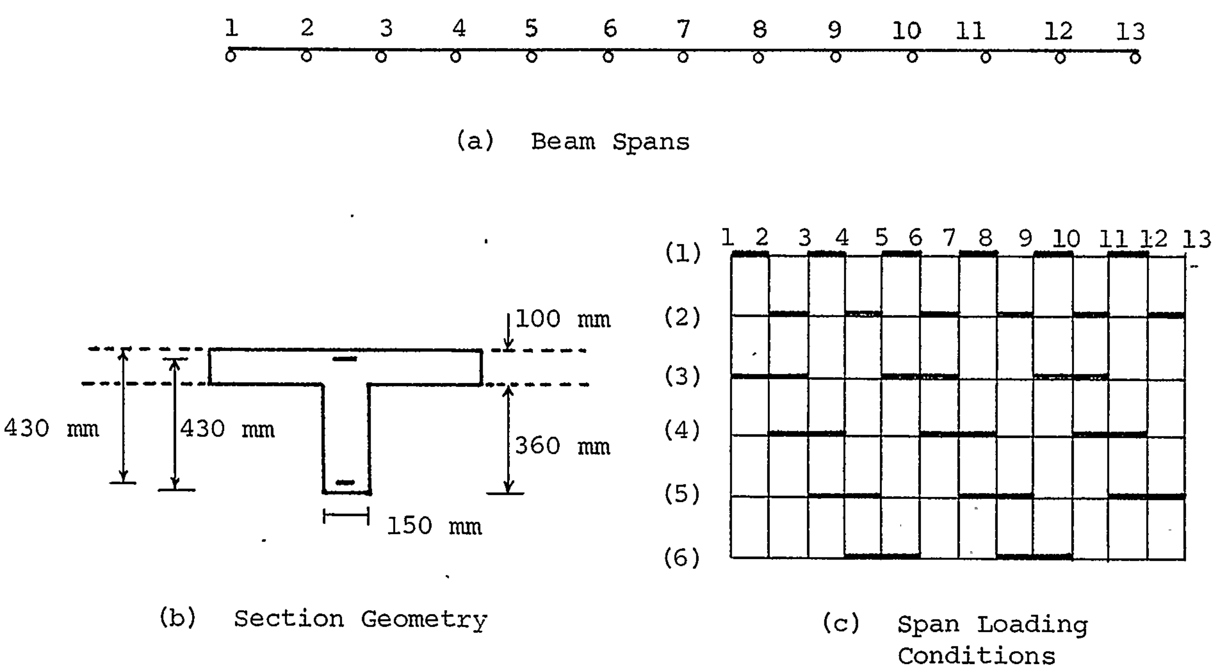


Fig. 5.3. - Twelve-Span Beam Example

5.3.2 Application of Sequential Decomposition

Methods of SD have been applied to the optimum design of the continuous beam illustrated in Fig. 5.3a. The beam consists of twelve 6 m spans. Such a long beam is not intended to reflect a realistic situation but rather to facilitate exact division into 2-, 3-, 4- and 6-span substructures. The section geometry is as illustrated in Fig. 5.3b for all sections. The beam is cast integrally with a slab with the clear distance from beams on either side being 3 m. Accordingly, the top breadth is not specified but is calculated in the program in accordance with the provisions of CAN3-A23.3-M84. Similarly, the amounts of deviation of the moments from those found by elastic analysis are calculated in accordance with the code equation. The shear provisions of Clause 11.3 are applied to this example rendering shear design independent of design for flexure. For the purpose of calculation of anchorage lengths, all bar diameters are specified as 20 mm. It is also specified that the areas of top reinforcement at each support and of bottom reinforcement in each span vary independently. Only one cut-off point is used for top reinforcement in each case.

The six loading conditions for the twelve-span beam are represented in Fig. 5.3c. In this figure, the heavy lines refer to the maximum uniformly distributed loading of $1.25\ q_D + 1.5\ q_L$. The light lines refer to the minimum factored loading of $0.85\ q_D$. When all moments at all supports due to the six loading conditions are considered, the first level problem involves 66 variables. When it is considered that each function evaluation involves complete specification of the minimum requirements for steel at all points (as described in

Chapter 4), it can be seen that this optimization problem could be computationally prohibitively expensive.

A number of alternative solution schemes were tried for this example to test the feasibility of methods of SD. A quasi-Newton optimization algorithm, ZXMIN, from the IMSL library (IMSL, 1982) was first used. Initially, this algorithm was applied directly to the problem. When an accuracy to only two significant digits was specified in ZXMIN, no solution was found in 10,000 seconds of central processing unit (CPU), time on a Control Data Corporation, Cyber 175 computer. When accuracy to one significant digit was specified, a solution was found in 9,013 seconds of CPU time after 11,493 full beam designs. An elementary method of SD was found to be more efficient. The procedure was as follows:

(1) Fix all moments at Support 7 (Fig. 5.3a). Solve the subproblems comprising Supports 2 to 6 and Supports 8 to 12 using ZXMIN with accuracy to two significant digits.

(2) Fix all moments at Supports 4 and 10 and solve the subproblem comprising Supports 5 to 9.

(3) Repeat Steps (1) and (2) until convergence of all moments to within 1% is achieved.

With this decomposition scheme, a solution was found in 8,747 seconds of CPU time. Details of both solutions are given in Table 5.1 together with a diagrammatic representation of the decomposition scheme for the latter. The results from four other solution schemes are also reported in the table. In all cases, the two families of subproblems illustrated were solved alternately. ZXMIN was employed for all these subproblem

Table 5.1. - Executions with SD and Various Substructure Sizes

No. of Spans in Typical Substructure	One Complete Cycle of Subproblem Optimization	No. of Variables for Typical Subproblem	No. of Subproblems per Cycle	No. of Accurate Significant Digits in Subproblem Solutions	Average No. of Function Evaluations per Subproblem	Total No. of Function Evaluations	Total CPU Time	Total No. Cycles
12	1 2 3 4 5 6 7 8 9 10 11 12 13	66	1	1	11,493	11,493	9,013	1
6	1 2 3 4 5 6 7 8 9 10 11 12 13	30	3	2	2,491	22,417	8,747	3
5	1 2 3 4 5 6 7 8 9 10 11 12 13	24	5	2	1,359	20,385	6,525	3
4	1 2 3 4 5 6 7 8 9 10 11 12 13	18	5	2	1,013	15,201	3,956	3
3	1 2 3 4 5 6 7 8 9 10 11 12 13	12	7	2	503	17,622	3,430	5
2	1 2 3 4 5 6 7 8 9 10 11 12 13	6	11	2	173	11,406	1,475	6

solutions and neither "Simple Progression" nor "Quadratic Projection" as described in Section 5.2 were used. There is a marked reduction of total CPU time with subproblem size. The scheme composed mostly of 5-span substructures required only 6,525 seconds. Both this and the scheme composed of 4-span substructures converged in three cycles. Naturally, the latter required by far the lesser time at 3,956. The average number of function evaluations per subproblem for the scheme with 3-span substructures is less than half that for the 4-span substructure scheme. This results in a reduction to 3,430 seconds despite the fact that two extra cycles of optimization were required. The scheme requiring least CPU time was that composed of 2-span substructures for which the total was only 1,475 seconds. This scheme constitutes "Even-Odd Alternation" as described in Section 5.2. The saving in CPU by use of two-span subproblems is so pronounced that schemes involving subproblems of this size only, have been given further consideration.

Applying the methods of Simple Progression and Quadratic projection as outlined in Section 5.2, the CPU time is still further reduced to 1,211 seconds. Details of this execution are given in Table 5.2. The computation time saved is due to a reduction in the number of cycles. The additional time required for the quadratic projection calculations is not significant. The deviations of the moments at Supports 2 and 3 from their optimum values are given in Table 5.3 before and after quadratic projection. It can be seen that the projection is quite effective in reducing the deviations for many cases.

Table 5.2. - Executions with Two-Span Subproblems and Simple Progression

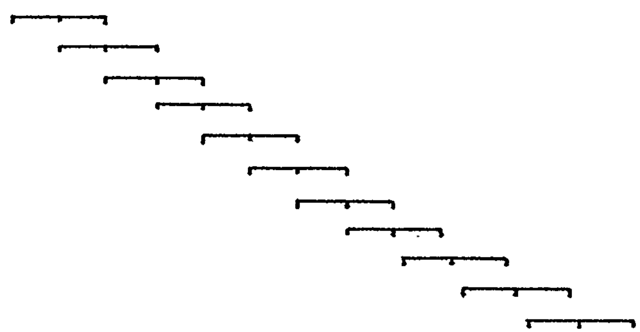
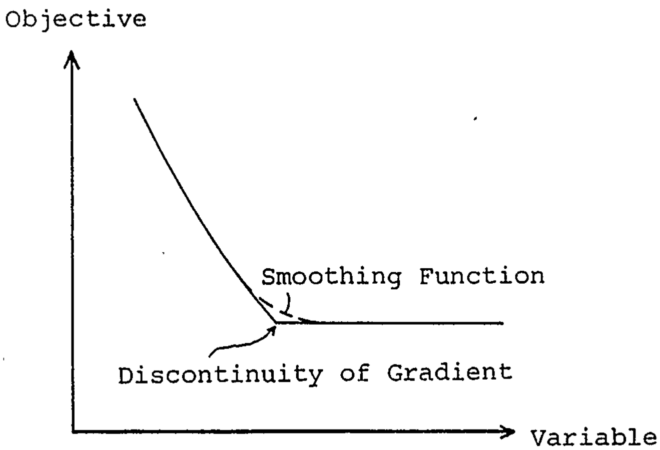
Problem Number	One Complete Cycle of Subproblem Optimization (the same for all three examples)	No. of Variables per Subproblem	Optimization Algorithm Used for Subproblems	No. of Subproblems per Cycle	Average No. of Function Evaluations per Subproblem	Total No. of Function Evaluations	Total CPU Time	Total No. Cycles
1	1 2 3 4 5 6 7 8 9 10 11 12 13 	6	ZXMIN	11	171	9,411	1211	5
2		6	CDMIN	11	152	16,763	2053	10
3		2	CDMIN	11	25	1,401	81	5

Table 5.3. - Results of Quadratic Projection

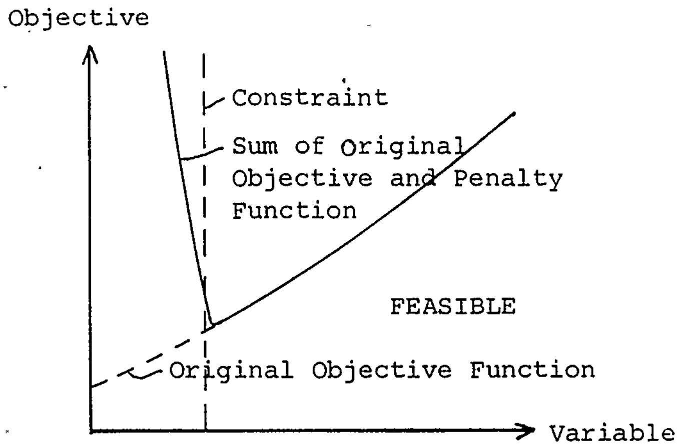
	Support 2						Support 3					
Loading	1	2	3	4	5	6	1	2	3	4	5	6
Simple Progression	0.29	-0.01	0.15	0.01	0.02	-0.21	-0.16	0.35	-0.17	0.07	-0.02	0.23
Quadratic Projection	-0.05	-0.01	0.15	0.01	0.01	-0.21	0.02	0.10	-0.17	0.07	-0.01	0.10

5.3.3 Two-Span Substructure Optimization

The quasi-Newton optimization algorithm, ZXMIN, was used for the two-span substructure optimizations in all of the executions described above. This algorithm is quite efficient for many problems but was found to be sensitive to discontinuities in the cost function gradients. The program, as first written, contained many such discontinuities. For example, cost is proportional to area of bottom steel which equals the maximum of that required for flexural resistance or the minimum allowable value as specified in the code. A gradient discontinuity exists at the point where the area required for flexural resistance equals the minimum area (see Fig. 5.4a). The use of a "smoothing function" for all such situations solved this problem with an insignificant loss of accuracy (see Appendix C). ZXMIN is also sensitive to regions of zero gradient in design space. This problem was overcome by incorporating a very small equivalent cost for support moments not equal to those found by elastic analysis. The most serious problem with ZXMIN, however, is that it is sensitive to regions of rapid (although continuous) change in the gradients of the objective function. Such regions sometimes occur in the application of a penalty function (see Fig. 5.4b). For example, ZXMIN was found to fail for problems with two independently variable areas of top steel at a support where the first area was required, by exterior penalty function, to exceed one half of the second (Clause 12.12.3, CAN3-A23.3-M84). When no such situation is encountered, as in the executions reported above, the algorithm works well. However, it is clearly not suitable for general use on the continuous beam design problem as formulated here.



(a) Discontinuity and Smoothing Function



(b) Objective with Region of Rapid Gradient Change

Fig. 5.4 - Potentially Problematic Objective Functions for Quasi-Newton Optimization Methods

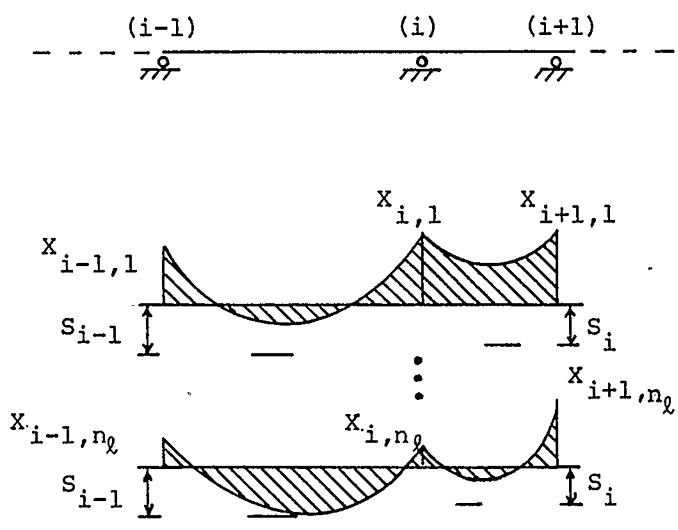


Fig. 5.5. - Substructure and Bending Moment Diagrams

For problems involving discontinuities of gradient, the method of conjugate directions has been recommended (Powell, 1964). Unfortunately, this method tends to be relatively inefficient for problems involving large numbers of variables. As stated in Chapter 3, an algorithm, CDMIN, has been written for optimization by the method of conjugate directions. While the difficulties encountered with library subroutines could perhaps have been overcome by the use of an alternative library subroutine or by reformulation of the problem, it was found more convenient to use CDMIN in lieu of ZXMIN. The use of the former greatly expedited the error detection process because of its relative simplicity and its insensitivity to "ill-conditioning" of the objective. When CDMIN was used in lieu of ZXMIN with the Simple Progression method of SD, the total CPU time was increased from 1211 to 2053 seconds (see Table 5.2). However, it is difficult to compare the two results. In both cases, cycles of subproblem solution were continued until the support moments, after two consecutive cycles, were not changed by more than 1%. However, with ZXMIN, suboptimization was continued until the variables, after consecutive iterations, agreed to two significant digits. On the other hand, with CDMIN, iteration was continued until convergence to within 0.1%. Despite the more stringent convergence criterion for CDMIN, the results of the suboptimizations were found to be less accurate than those found using ZXMIN. This is a consequence of the nature of the two iterative procedures. It was because of this inaccuracy that solution using CDMIN required twice as many cycles of optimization as that using ZXMIN (Table 5.2). The degrees of accuracy in the final results were similar using both algorithms but, in general, the moments found using ZXMIN were closer to the exact values.

Accordingly, solution using CDMIN appears to be slightly less accurate and considerably more computationally expensive than solution using ZXMIN. Fortunately however, it is possible to reduce considerably the number of variables in the subproblems. Consider the substructure illustrated in Fig. 5.5 (page 97) for a problem involving n_ℓ loading conditions. The n_ℓ variables for this subproblem are the i^{th} support moments, X_{ij} ; $j = 1,2, \ldots, n_\ell$, all other support moments being fixed. The design moment for sag in the i^{th} span is S_i (see Fig. 5.5). This is the maximum for all loadings of the maximum moment in the i^{th} span. Thus, for the j^{th} loading condition, X_{ij} must be sufficiently large that the moment in both spans of the substructure is everywhere less than or equal to the design moment for sag. An interesting situation exists if, in both spans, the moment is less than the design moment for sag. If this is the case, then decreasing X_{ij} may imply a decrease in the area of top steel required at Support i and/or a decrease in the lengths of top steel at any or all of Supports i-1, i and i+1. It has been assumed throughout this thesis that all bottom steel extends into the supports by at least 150 mm (see Clause 12.11.1). Accordingly, it is reasonable to assume that decreasing X_{ij} in this situation will not lead to an increase in the length of bottom steel required. The validity of the assumption can be tested in the final design. Thus, for as long as the sagging moment in both spans is less than the respective design moments, a reduction in X_{ij} can not imply an increase in the required volume of steel. Hence, no loss of accuracy results from assigning to X_{ij} the maximum of the following values:

(1) that moment which would imply a maximum sagging moment in Span i-1 of S_{i-1};

(2) that moment which would imply a maximum sagging moment in Span i of S_i;

(3) the minimum allowable moment at Support i as dictated by the moment found by elastic analysis and the allowed deviation from that moment.

Thus, regardless of the number of loading conditions, n_ℓ, the two-span substructure variables, X_{ij}; $j = 1,2, \ldots, n_\ell$ can be regarded as being dependent on the two variables S_{i-1} and S_i. Similarly, for a one-span substructure (required for a beam with ends encastre), the variable moments can be regarded as being dependent on the design moment for sag in the span.

This reduction in problem size greatly improves the efficiency of the program using CDMIN. The twelve-span example of Fig. 5.3, when designed using this strategy for subproblem solution, required only 81 seconds of CPU time (Table 5.2). Again, it is difficult to compare this time with the others in the table. Double precision calculations of the objective were recommended for accurate results using ZXMIN. This precision was retained for the six-variable example using CDMIN but was suspended for the two-variable problem. There is a corresponding halving in the CPU time per function evaluation. Also incorporated into the two-variable program is a three-tier accuracy for subproblem solution. For this 12-span example in which a convergence criterion of 1% has been specified, the convergence criterion for the subproblems is

10% in the first cycle, 1% in the second and 0.1% in subsequent cycles. The termination criterion is not tested until after the fourth cycle. These changes in the programming, combined with the reduction in the number of variables, have resulted in a program that is more than an order of magnitude more efficient than before. The new program required an average of only 25 function evaluations per subproblem solution.

In summary, it has been found that the algorithm ZXMIN is not suitable for general use on the continuous beam design problem as formulated here. However, degeneration of the two-span subproblem enables the more robust algorithm CDMIN to be used with even less computational labour than ZXMIN.

5.3.4 Program Description

The end result of the theory and tests outlined above is the computer program BOD (Beam Optimum Design). BOD is a comprehensive program containing fifty-four subprograms and three to four thousand fortran statements. It has been written in Fortran IV rather than a more advanced fortran language because many standard library subroutines are still widely available only in this language. This latest edition of the beam design program makes use of one standard subroutine, namely, LEQT1F, which is from the IMSL library (IMSL, 1982) and is used to solve a set of simultaneous equations. The program was written on the Control Data Corporation, Cyber 175 computer.

BOD involves analysis and design at several levels. At one of the most basic levels is the subroutine CONANA. This involves the specification of the required amounts of steel for one- or two-span substructures and the calculation of the associated costs. At a higher level, the subroutine SUBOPT directs a search for the optimum amounts of moment redistribution in the substructures. The subroutine ST involves the complete steel design process, combining the substructure designs by means of the sequential decomposition method. At the highest level, the main program in BOD is used for the determination of the optimum values of the geometric variables. For given values of these variables, full analyses are carried out in FAN. In FAN, ST is called for the steel design, DEFL is called for the calculation of deflections and the total cost of the continuous beam is calculated.

A detailed description of BOD is given in Appendix A with flowcharts for all the major subprograms.

5.4 SENSITIVITY ANALYSIS FOR CONTINUOUS BEAMS

Sensitivity of beam cost to a number of input parameters is assessed for certain examples. These results provide a valuable indication of what areas of research may be fruitful in terms of cost savings to the construction industry as well as indicating how near-optimal designs can be found with relatively little computational effort.

5.4.1 Objective Function

Unit costs have been determined on the basis of information received from a major Canadian construction firm (Note 1) and informal conversations with others involved in the industry.

Concrete: The material cost of concrete is a function of the characteristic strength, f'_c:

Material Cost of Concrete: $\$\ 81.75/m^3$ (f'_c = 20 MPa) (5.28)

$\$\ 82.00/m^3$ (f'_c = 30 MPa) (5.29)

$\$102.50/m^3$ (f'_c = 40 MPa) (5.30)

The very small difference in cost between 20 MPa and 30 MPa concrete may be due to a low demand for the former. In view of the small difference, it seems reasonable to assume that it would always be more cost-effective to use the latter. Linear interpolation gives a cost of $\$92.25/m^3$ for f'_c = 35 MPa. The additional costs for additives ($\$3.50/m^3$ for "Winter Heat" and $\$1.50/m^3$ for Air Entrainment) are not considered here. The labour associated with the pouring of concrete is 0.553 manhours per cubic metre and the current crew rate (for a unionized workforce) is $18.46 per manhour. Hence,

Labour Cost of Concrete = $\$10.21/m^3$ (5.31)

While these figures are exactly as used by the major contractor, it is observed that their accuracy, in general terms, is far less than might be inferred from the number of significant digits given.

Forms: The total material cost of forms is,

$$\text{Material Cost of Forms} = \$7.50/\text{m}^2 \tag{5.32}$$

The labour cost is calculated at 1.367 manhours per square metre at a present crew rate of \$19.88 per manhour. Thus,

$$\text{Labour Cost of Forms} = \$27.18/\text{m}^2 \tag{5.33}$$

In the case of a beam cast integrally with a slab, form-work must be provided for both the beam and the slab. Hence, increasing the web width has no effect on the total area required. Accordingly, the variable cost of forms is the cost required for the beam sides only.

Steel: Based on prices given by the major construction firm, the current supply cost of steel is \$0.44/kg, and the installation cost is \$0.42/kg. The corresponding volume costs (using a density of 7850 kg/m^3) are:

$$\text{Steel Supply Cost} = \$3454/\text{m}^3 \tag{5.34}$$

$$\text{Steel Installation Cost} = \$3297/\text{m}^3 \tag{5.35}$$

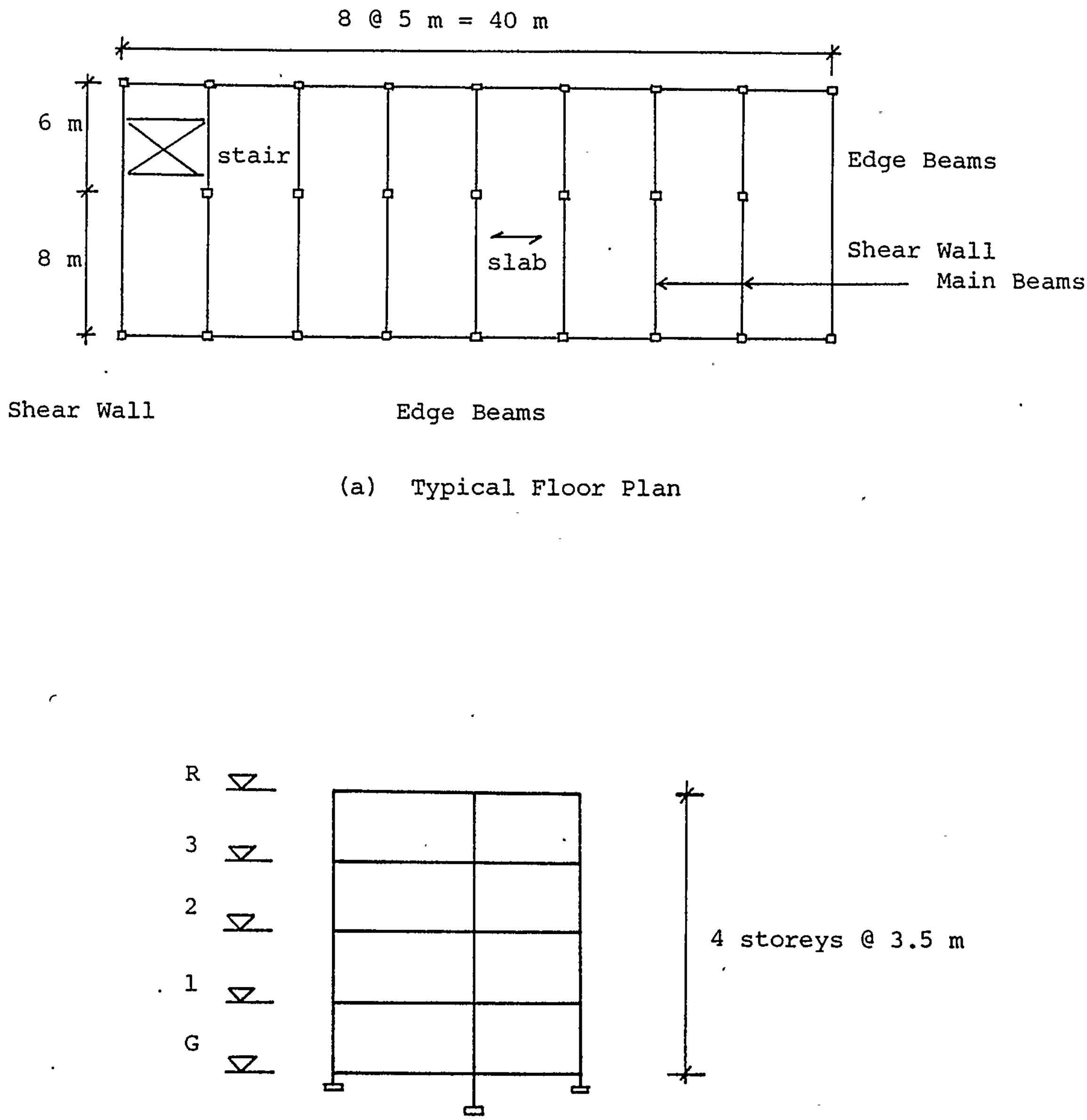
The major construction firm consulted did not differentiate between the unit prices of longitudinal and transverse steel. A steel supplier stated that their charges for cut and bent bars were \$0.540/kg and \$0.507/kg for transverse and longitudinal respectively. A steel placing firm estimated that a given volume of No. 10 bars would require four times as long to install as No. 30 bars and that No. 10 transverse reinforcement would require about three times as much installation time as No. 10 longitudinal reinforcement. Of course, not all longitudinal reinforcement can be in the form of large bars but the chances of reducing the number of bars is higher than for transverse reinforcement. Accordingly, it is proposed to specify a unit price for the installation of transverse equal to 7.5 times that for the installation of longitudinal. (Values for this ratio of 5 and 10 are later used to test its effect on cost.) If it is then assumed that the volume of longitudinal reinforcement in a typical beam is ten times the volume of transverse and the weighted average is required to equal \$3297, the unit costs for the installation of reinforcement become:

$$\text{Installation Cost of Longitudinal Steel} = \$2,100/\text{m}^3 \tag{5.36}$$

$$\text{Installation Cost of Transverse Steel} = \$15,500/\text{m}^3 \tag{5.37}$$

Clearly, these figures are based on a number of subjective decisions and are far from accurate. Accordingly, subsequent users of the program BOD should view the logic used in their calculation critically before adopting it for general use.

Depth Penalty: From a purely structural viewpoint, narrow deep beams tend to be most efficient. This is a result of the associated large lever arm. However, excessively deep beams require space and the resulting increase in the height of the building can significantly affect the overall cost. This increased height can result in greater wind loads with the result that other structural elements will need to be strengthened. Increased height also creates a need for additional cladding. Only the latter of these two factors is considered here. A cladding cost of $\$16/\text{m}^2$ is assumed and the simple layout illustrated in Fig. 5.6 (after Higgins and Hollington, 1973) is adopted. It is recognized that this layout may certainly not be typical. However, such is the



(a) Typical Floor Plan

(b) Typical Cross-Section

Fig. 5.6. - Typical Building (after Higgins and Hollington, 1973)

diversity of layouts among buildings that it is difficult to find one that is "average". If the layout is as illustrated in Fig. 5.6, there are only two beams to be designed, namely, the edge beam and the main beams. If the depth of both of these is increased by δd, then the cost of additional cladding required is \$1728 δd, where δd is in metres or \$1.7 δd where δd is in millimetres. If this cost is shared between the two types of beams,

$$\text{Cost Penalty} = \$0.86\ \delta d \text{ per beam} \tag{5.38}$$

One might argue that increasing the depth of the edge beams would not necessarily require the building height to be increased but for the purposes of this study, it is assumed that it would. Clearly, this cost term is only a first estimate of what the term might be in a given situation. For specific buildings, the engineer will know the building layout and may be in a better position to estimate the value. In view of the approximate nature of the estimate made here, the sensitivity of beam cost to this value is tested at a later stage.

Equivalent Cost of Deflection: In the program BOD, the limitation on deflection is treated as a fuzzy constraint (see Section 3.5). The equivalent cost of deflections close to the maximum allowable value is,

$$\text{Equiv. Cost} = k \left| \frac{(\delta/\delta_{max} - \rho)}{(1 - \rho)} \right|^p \tag{5.39}$$

this term being applied only when δ is in excess of $(\rho\ \delta_{max})$. In this equation, δ_{max} is the maximum allowable deflection as specified in the code (Canadian Standards Association, 1984). Equivalent costs are applied for immediate deflection due to live load, δ_i, and for the sum of the long-time deflection due to all sustained loads and the immediate deflection due to the additional live load, δ_t. For the examples considered here, the cost is taken to be zero for deflections of less than half the maximum allowable and to increase as a cubic function thereafter. Hence,

$$\rho = 0.5 \tag{5.40}$$

$$p = 3 \tag{5.41}$$

The constant, k, in Eqn. 5.39 represents the equivalent cost when the deflection equals the maximum allowable value. If deflections at working loads are excessive, some form of levelling operation may be required. The cost of such a process (for the major construction firm consulted) is $17.15 per square metre. The equivalent cost when deflection equals its maximum value, k, reflects the probability of such an operation being required. It also reflects the reduced value of a building when deflections in a number of beams are high but do not require repair. For the purposes of this work, a value of k equal to 50% of the cost of levelling the tributary area has been selected. This reflects the author's opinion that if the predicted deflection equals the maximum value, then there is about a 50% chance that the actual deflection will exceed the maximum and that the levelling operation will be required. It is assumed that if the beam deflection is acceptable, then the slab deflection will be such that no levelling will be necessary. Thus,

$$k = \$8.60\ A_t \tag{5.42}$$

where A_t is the tributary area in square metres. As before, the subjectivity of this logic is emphasized.

Objective Function: It is assumed that beams are cast integrally with slabs of specified depth. Accordingly, the variable beam dimensions are the breadth of the web and the depth below slab level (b_1 and t_1 in Fig. 4.1a). These will be referred to here as b_b and t_b respectively. The variable cost of concrete (material plus labour) is proportional to the product of these and the beam length, i.e. ($b_b t_b \ell$). The variable cost of forms is proportional to the surface area, namely, ($2t_b \ell$). The depth penalty is applied to t_b and the equivalent cost of deflections is additive to the other costs. Hence, the objective function to be minimized is, for f'_c = 30 MPa:

$$\begin{aligned} \text{Obj.} = {} & (82 + 10.21)10^{-9}(b_b t_b \ell - V_t - V_\ell) + (7.5 + 27.18)10^{-6}(2t_b \ell) \\ & + (3{,}454 + 15{,}500)10^{-9} V_t + (3{,}454 + 2{,}100)10^{-9} V_\ell + 0.86 t_b \\ & + 8.6\{(360\delta_i/\ell - 0.5)/0.5\}^3 + 8.6\{240\delta_t/\ell - 0.5)/0.5\}^3 \end{aligned} \quad (5.43)$$

where all linear dimensions are in millimetres, V_t and V_ℓ are the volumes of transverse and longitudinal steel respectively (mm^3) and maximum deflections of $\ell/360$ and $\ell/240$ are being used for δ_i and δ_t respectively. Similar equations can be derived for other strengths of concrete. It is of interest to note that the depth penalty associated with the increased cladding requirements is of the same order of magnitude as the depth cost due to the formwork requirements.

5.4.2 Design Examples and Results

All of the following examples have been designed in accordance with the requirements of the Canadian Code of Practice (Canadian Standards Association, 1984). For ease of construction it has been specified that the section geometries be the same in all spans. A beam slab system has been specified with a distance between spans of 5 m. The flange width of the resulting T-beam is selected in accordance with the Code requirements. The depth of slab has been specified as 100 mm except where otherwise stated. The breadth of the web and the depth-below-slab of the beam are allowed to vary but minimum values of 200 mm and 260 mm respectively have been specified. These minima are required to ensure that sufficient anchorage is available for stirrups and sufficient breadth for the spacing of the reinforcement. This combination of breadth and depth was found to be adequate (for the examples considered) to prevent spalling due to high shear stresses.

The dead loading was taken to be the weight of the slab plus self weight plus 1 kN/m^2 throughout the tributary area to account for other dead loading. The live loading intensity was that specified for office occupancy, i.e.,

$$2.4(0.3 + [9.8/A_t]^{\frac{1}{2}}) \ kN/m^2$$

where A_t is the tributary area. For loading applied over two spans, the tributary area was taken to be the sum of the tributary areas for each span and the intensity was reduced accordingly. This may not be in keeping with normal office practice. However, the reduction formula is intended to reflect the lesser probability of large areas having a high average load than small ones. Therefore, it is felt that it is appropriate to apply it for the complete area on which the loading is being applied. The maximum loading was obtained by applying factors of 1.25 and 1.5 to dead and live load respectively and the minimum by applying a factor of 0.85 to the dead load. It is recognized that, again, it may not be normal office practice to apply a factor of 0.85 to the dead load. However it is felt that this reasonably reflects the possibility of adjacent spans having lower-than-expected and higher-than-expected dead loading.

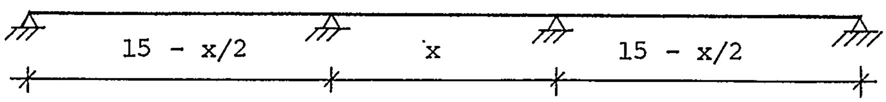
The specified yield strength of steel was 400 MPa for both longitudinal and transverse reinforcement and the characteristic strength of concrete was 30 MPa except where otherwise stated. For all of these examples, No. 30 bars were used for the calculation of the development lengths of all longitudinal reinforcement and a cover of 40 mm was specified. The three span beam of Fig. 5.7a was considered subject to the loading conditions illustrated in Fig. 5.7b. Except where otherwise stated, the span lengths are equal at 10 m.

Series 1 - Variable Span Lengths: For the beam illustrated in Fig. 5.7, the program BOD was used to determine the optimum designs for values of x, equal to 8 m, 10 m and 12 m. In this initial series it was found for the three examples,

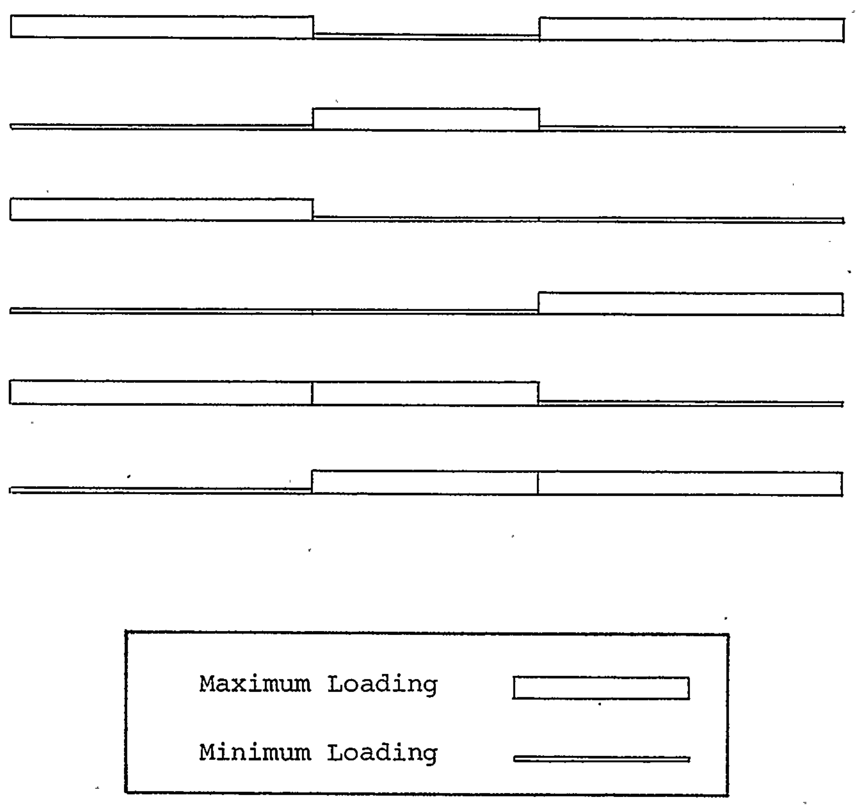
(a) that the objective was an average of 3.4% from the minimum after optimization of the section geometry only, and

(b) that fixing the geometry at the values found in (a) above and determining the optimum amounts of plastic moment redistribution brought the objective to within 1.1% of the optimum.

In contrast, only about 10% of the total computational effort is required for the implementation of these two steps. In view of this, the program BOD was adapted to allow the user to terminate the optimization process at this stage. This approximation has also been used in all of the remaining examples. All components of the cost



(a) Beam Geometry



(b) Beam Loadings

Fig. 5.7. - Optimization Example

function for these examples and an example with, x = 14, are given in the first four columns of Table 5.4.

In addition to the cost components, the optimum depth (below slab), breadth and maximum allowed plastic moment redistribution (measure of ductility) are reported in the table. While the exact optimal dimensions are reported, it is recognised that these would normally be "rounded off" by the designer. The various components of the objective are illustrated in Fig. 5.8. As x increases, the outer span lengths decrease. It can be seen that, as this occurs, deflections become less of a problem initially and depth decreases. This is reflected by a substantial drop in the depth penalty and the cost of forms. Corresponding to the reductions in depth are increases in breadth. However, the total volume of concrete decreases. The volume of longitudinal steel increases and ductility decreases. As x gets larger, the outer span lengths become small relative to that of the centre span. The equivalent cost of deflections in the outer spans becomes zero while that in the centre span increases. This results in a sharp rise in the required section depth with a corresponding fall in breadth. The volume of concrete increases once more while that of longitudinal steel decreases.

In general, the cost components which are most sensitive to the centre-span length are those associated with depth. While increases in depth are associated with decreases in other costs, the net result is an increase in the total objective.

Series 2 - Variable Concrete Strength: The beam of Fig. 5.7, with a centre-span length of 10 m is designed with a characteristic concrete

Table 5.4. - Optimum Beam Design Solutions

	Series 1 - Variable Span Lengths (m)				Series 2	Series 3 - Variable Slab Depth (mm)				Series 4 Variable Steel Cost Ratio		Series 5 Variable Depth Penalty	
	x = 8	x = 10	x = 12	x = 14	$f'_c = 35$	80	90	110	120	5	10	low	high
Total Objective ($)	3491	3197	3039	3681	3167	3227	3202	3194	3195	3198	3188	2284	6648
Cost of Forms ($)	969	858	764	1011	843	885	863	856	854	860	856	991	580
Cost of Concrete ($)	594	573	553	615	572	572	568	569	565	563	571	398	1437
Cost of Long. Steel ($)	493	509	603	554	514	477	498	518	527	535	481	462	731
Cost of Transv. Steel ($)	162	139	127	174	131	138	137	140	142	114	157	150	207
Equiv. Cost of Deflns. ($)	72	56	46	73	63	57	66	51	48	62	60	37	100
Depth Penalty ($)	1201	1064	948	1254	1045	1097	1070	1061	1059	1066	1062	1228	719
Optimum Depth (mm)	466	412	367	486	405	425	415	411	411	413	412	476	279
Optimum Breadth (mm)	468	510	555	465	468	493	503	508	505	500	510	309	1882
Maximum Plastic Distribution Allowed (%)	18.3	15.7	9.7	14.5	15.6	16.2	15.8	15.8	15.7	15.5	15.7	9.5	20

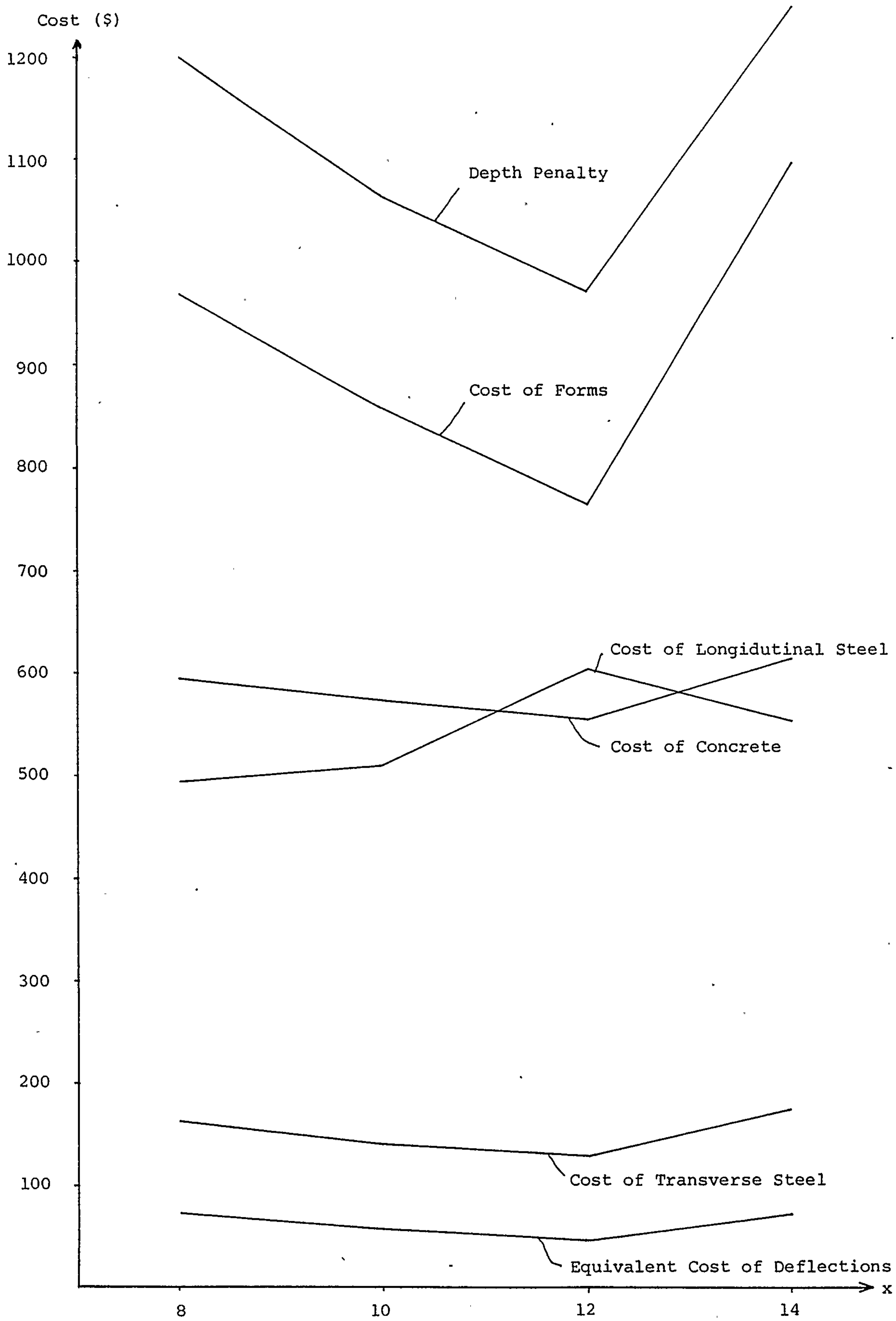


Fig. 5.8. - Variation of Cost Components with Centre-Span Length

strength of, f'_c = 35 MPa. Surprisingly enough, this results in a slight cost saving although the difference is less than 1%. It is important to mention at this point that some iterative calculations in the program were terminated when accuracy to within 0.5% was obtained. The volume of concrete, as expected, decreases when the unit cost rises with the end result that the total concrete cost is unchanged. Depth decreases by only 7 mm but there is a significant 78 mm decrease in breadth. As might be expected, the volume of longitudinal steel increases while the required volume of transverse steel decreases. There is a slight increase in the equivalent cost of deflections due to increased deflections resulting, presumably, from the reduced depth and breadth.

Series 3 - Variable Slab Depth: While it is recognized that the reasons for the selection of the slab depth are generally external to the beam design, it is of interest to see the influence of this parameter on cost. In this series the beam of Fig. 5.7 is used with the length of the centre span fixed at 10 m. The various cost components are given in Table 5.4 and are plotted in Fig. 5.9. Perhaps the most surprising result is that the objective function decreases initially with increasing slab depth. It then levels off and is approximately constant for slabs with depths of 100 mm - 120 mm. Thus, the additional loading due to the increased weight of slab is initially more than offset by its contribution to the strength and stiffness of the beam. The reader is reminded that the cost of concrete considered in the objective, only includes the concrete below the level of the bottom of the slab. Thus, increasing the slab depth provides the beam with additional "free" concrete. The presence of the extra depth of concrete has almost

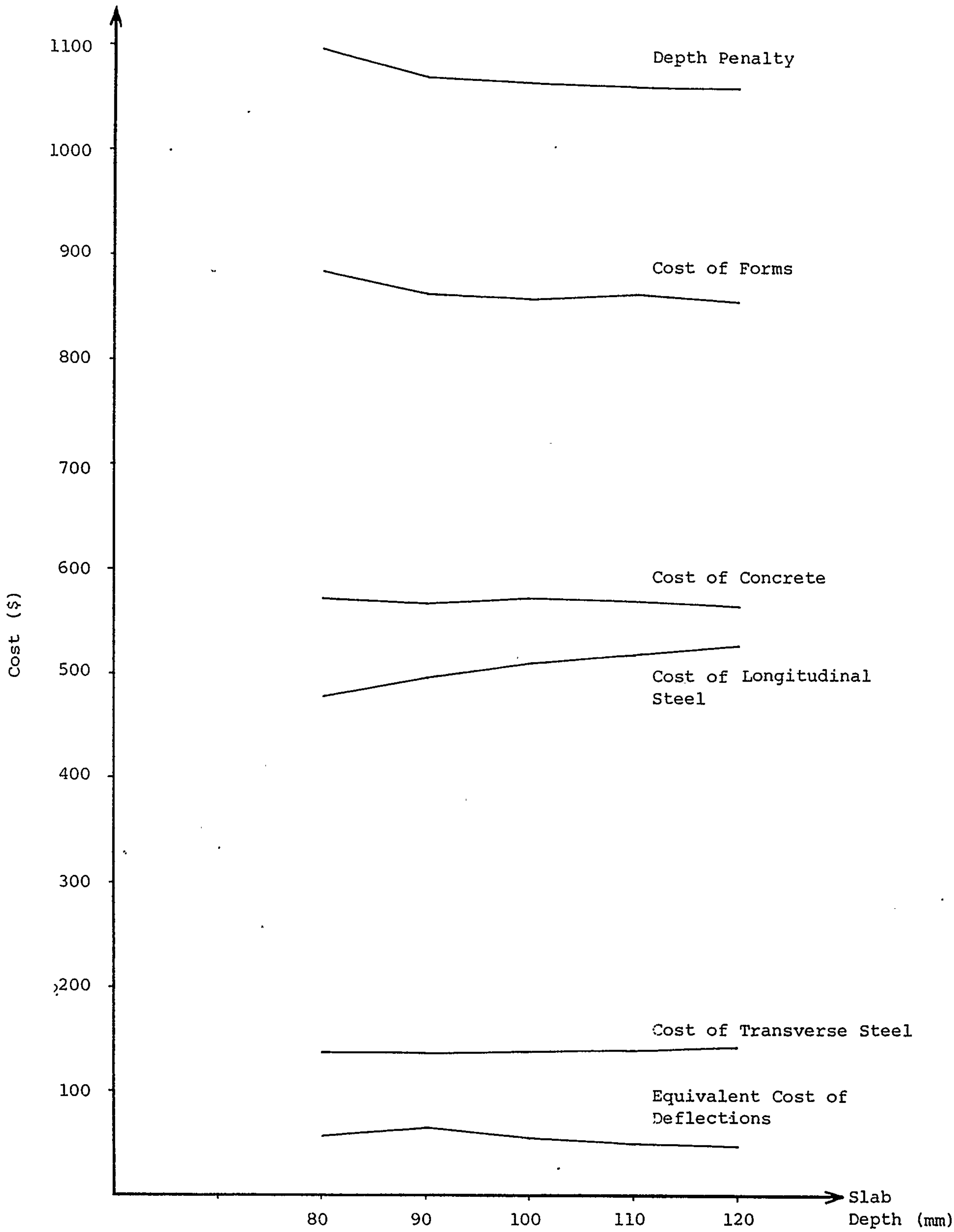


Fig. 5.9. - Variation of Cost Components with Slab Depth

exactly offset the changes in the concrete geometry which the increased loading implies. Thus, the beam breadth and depth-below-slab are almost constant. The volume of longitudinal steel does increase steadily as slab depth is increased but the section ductility remains approximately constant.

Series 4 - Variable Transverse to Longitudinal Steel Cost Ratio: It has been stated (Eqn. 5.35) that the installation cost of steel for the major construction firm consulted is \$3297/m^3. Assuming a transverse to longitudinal steel cost ratio of 7.5 and a volume ratio of 0.1, and requiring the weighted mean to equal \$3297/m^3 implies an installation cost of transverse steel of \$15,500/m^3 and of longitudinal \$2,100/m^3. In order to assess the effects of the assumed cost ratio on the optimum solution, the ratio is changed to 5 and 10 and the corresponding unit costs used in the program. For a ratio of 5,

Installation Cost of Longitudinal Steel = \$2,400/m^3

Installation Cost of Transverse Steel = \$12,100/m^3

Similarly, for a cost ratio of 10, the installation costs are \$1,800/m^3 and \$18,100/m^3 for longitudinal and transverse respectively. The results of using these ratios are given in Table 5.4. It can be seen that the total objective function varies by 0.3% which is less than the level of accuracy specified for these examples (0.5%). As expected, the total cost of transverse steel gets larger as the unit cost increases. There is a corresponding decrease in the total cost of longitudinal steel. However, there was no significant change in the volumes of transverse and longitudinal steel.

Series 5 - Variable Depth Penalty: As for the case of the transverse to longitudinal steel cost ratio, the choice of a depth penalty is somewhat subjective. Accordingly, it is varied here by a factor of 5. This has an enormous effect on the optimum design, the optimum depth varying between 279 mm and 476 mm. When the depth penalty is at its highest, the optimum depth-below-slab is small but still above the minimum allowable value. In order to provide sufficient stiffness and shear resistance, breadth is increased to the remarkable 1882 mm and higher deflections are tolerated. Additional steel, both longitudinal and transverse is provided.

The very high sensitivity of the optimum solution to the depth penalty selected, suggests a need for a detailed consideration of this term. The designer must consider the layout of the building to be designed, the cost of cladding and the other costs associated with the building height. It has been assumed here that depth-related costs are shared between the number of different beams in the building. Of course, the actual storey height is more likely to be a function of the maximum of the beam depths. This can be accurately considered by simultaneous design of all beams in a storey.

5.5 CONCLUSIONS

In this chapter, multi-stage problems are described and illustrated. It is shown that the second order methods of optimization can be used for these problems without the difficulties which sometimes render those methods inefficient. However, sequential decomposition (SD) is a more versatile efficient method for the solution of multi-stage problems. Three forms of this optimization philosophy are described and a process of quadratic projection is developed to improve the convergence properties of the procedure.

The model co-ordination method is applied to the optimum beam design problem. The first level subproblem then involves finding the support moments for fixed section dimensions. It is to this subproblem that methods of SD are applied. By far the most efficient of the SD methods tried, involved the repeated optimization of a series of two-span substructures. A further considerable advantage of this substructure size results from a degeneration in the substructure optimization problem. Both sequential decomposition and this degeneration contribute to an enormous reduction in the time required for the solution of the optimum beam design problem.

A program for optimum beam design, BOD, has been written based on the principles outlined in this chapter. This has been used to test the sensitivity of the optimum design to the various parameters which affect the solution. A significant finding of this series of tests is that an excellent near-optimal solution can be found by a two step approach. First, the optimum section dimensions are found while allowing no optimization of the amounts of plastic moment redistribution.

Then, with these dimensions fixed, the optimum amount of redistribution is determined. Cost was found to be significantly affected by the ratio of the span lengths in a three-span beam. The slab depth had little effect on the beam cost and low slab depths can be seen to actually result in a slightly more expensive beam despite the reduction in dead loading. The concrete strength, f'_c, and the transverse:longitudinal steel cost ratio did not significantly affect beam cost while the depth penalty selected had a substantial effect.

CHAPTER 6

A PROBABILISTIC APPROACH TO THE LOADING OF BEAMS

6.1 INTRODUCTION

A major complicating factor in the design of continuous beams is the large number of loading conditions that must be considered. A number of researchers have suggested simplifications (Peir and Cornell, 1973; Beeby, 1981) but these, by neglecting some possible loading combinations, could result in non-conservative designs. Several probabilistic loading models have been developed (Peir and Cornell, 1973; McGuire and Cornell, 1974; Ellingwood and Culver, 1977; Chalk and Corotis, 1980) but these are all, in the author's opinion, too complex for everyday design office use. In this chapter, a probabilistic loading model is presented which is more rational than the traditional deterministic approach but is sufficiently simple for design.

In design by the traditional method, the extreme loading situations are all treated as if equally likely to occur. This is clearly not the case, however, as extremely large loading intensity in alternate spans (and these spans only) is clearly less probable than the same loading intensity in one span only. Yet, it would be non-conservative to ignore the fact that, of two adjacent spans, one may have high loading intensity while the other may have a low value. All such loading situations are treated in a rational manner when the loading intensity in each span is considered to be a random variable. This is the basis of the probabilistic model presented in this chapter. The model is justified by a comparison with the effects of actual loading. Determination of design bending moment and shear force diagrams by the new method is described and illustrated.

6.2 TEMPORAL DISTRIBUTION OF LOADING

An impressive array of analytical techniques are currently available for the determination of the effect of a given loading. In this context, a load "effect" is a "mode of structural response" produced by a load such as a force, moment or deflection (Peir and Cornell, 1973). It is the load effects rather than the loads themselves that are of interest to the engineer. Live loading consists of a complex combination of loads of various intensities and distributed over various areas. Thus, to determine the load effects due to actual live loads is a complex process, too tedious for general design office use. A far simpler method of design involves the concept of an equivalent uniformly distributed load (EUDL). The EUDL is that uniformly distributed load, which would produce the same load effects as the actual loading, if applied over the same area. Hence, for a given load effect, the relationship between actual loading and the EUDL is determined from a consideration of the influence surface.

Live loading intensity is a statistical parameter which varies spatially and temporally. The variation of load with time is considered by many (Peir and Cornell, 1973; McGuire and Cornell, 1974; Chalk and Corotis, 1980; Corotis and Tsay, 1983) to be of the form illustrated in Fig. 6.1c. The load is made up of two components. The first is the "sustained load" (Fig. 6.1a) which stays approximately constant for long periods of time, changes perhaps corresponding to changes of occupancy in the area. This includes normal office loading such as furniture and

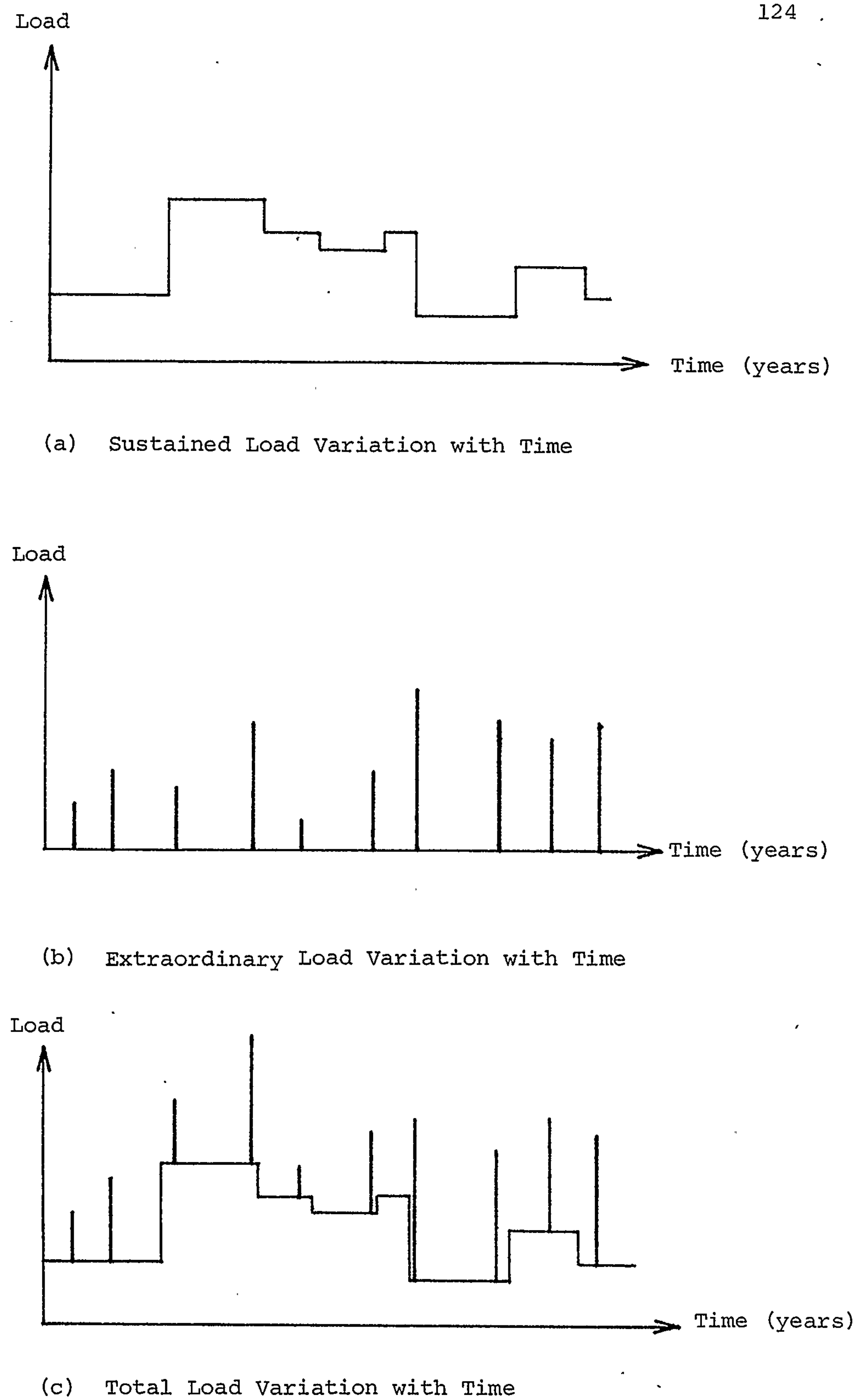


Fig. 6.1. - Variation of Load with Time

personnel. The second component is the loading "event" (Fig. 6.1b) which occurs occasionally during the lifetime of a structure. This is assumed to be additive to the sustained loading. It corresponds to incidences of extraordinary loading such as the presence of large numbers of people or large quantities of furniture (during decorating, for example).

What is of interest to the engineer is the maximum total load effect that occurs in the design lifetime of the structure. The EUDL for this maximum-in-lifetime load effect is a random variable, varying from one building to the next. A safe design is achieved by determining and designing for the value in this distribution that has a sufficiently low probability of occurrence.

6.3 ELASTIC DESIGN PROCEDURE FOR MOMENT

A new method is proposed for the design of structures subjected to live loading. It is presented here without justification or proof, the accuracy of the method being demonstrated in Section 6.4. Currently, a structure is analysed under the influence of a number of extreme loading situations. For this, minimum and maximum loading intensities are specified for live loading by the application of load factors. These correspond to the extreme values of the maximum-in-lifetime EUDL. In the new procedure, the EUDL is considered to be a random variable with a span-independent and a span-dependent portion. Thus, the EUDL for live loading in the i^{th} span is,

$$q_{Li} = q_{Lo} + q_{Lni} \quad ,$$

where q_{Lo} is span-independent, that is, it is constant for all spans and q_{Lni} is the span-dependent component. Thus, the component, q_{Lni}, is exclusive to the i^{th} span and is statistically independent of q_{Lnj}, $j \neq i$. The means and standard deviations of both components are specified.

Design Procedure: The analysis of the structure is carried out for arbitrary loading intensities in each span. For the matrix methods of analysis, this involves full inversion of the flexibility/stiffness matrix; the more efficient algorithms for the solution of sets of simultaneous equations may not be used. This numerical disadvantage is countered by the fact that with the new method, only one series of calculations is required for the complete spectrum of loadings being considered. Analysis for arbitrary loading intensities gives the load effects (moment and shear force) at all points as a linear function of the independent statistical parameters. It can readily be shown (Ang and Tang, 1975) that if,

$$Y = \sum_{i=1}^{n} a_i X_i, \tag{6.1}$$

where a_i are constants and X_i are statistically independent random variables, then the mean and standard deviation of Y are, respectively:

$$\bar{Y} = \sum_{i=1}^{n} a_i \bar{X}_i \tag{6.2}$$

$$\sigma_Y = \left(\sum_{i=1}^{n} a_i^2 \sigma_{Xi}^2 \right)^{\frac{1}{2}} \tag{6.3}$$

In these equations, $\bar{X}_i$ is the mean value of X_i and σ_{Xi} is its standard deviation. Eqns. 6.2 and 6.3 are used to determine the mean and standard deviation of the load effects at each point. For design, the sum of the mean and a certain multiple of the standard deviation is evaluated for each load effect.

6.4 ASSESSMENT OF DESIGN METHODS FOR MOMENT

6.4.1 General

Loading is not distributed uniformly in space. Hence, there is no single EUDL that gives exact results for all load effects. However, for convenience of design, a single intensity of EUDL for all load effects is clearly desirable. It is shown in this section that a considerable discrepancy arises when one fixed EUDL is used for the moment at all points in a beam span. In contrast, when the new probabilistic approach to design is used, excellent results are achieved.

6.4.2 Statistical Distribution of Moment

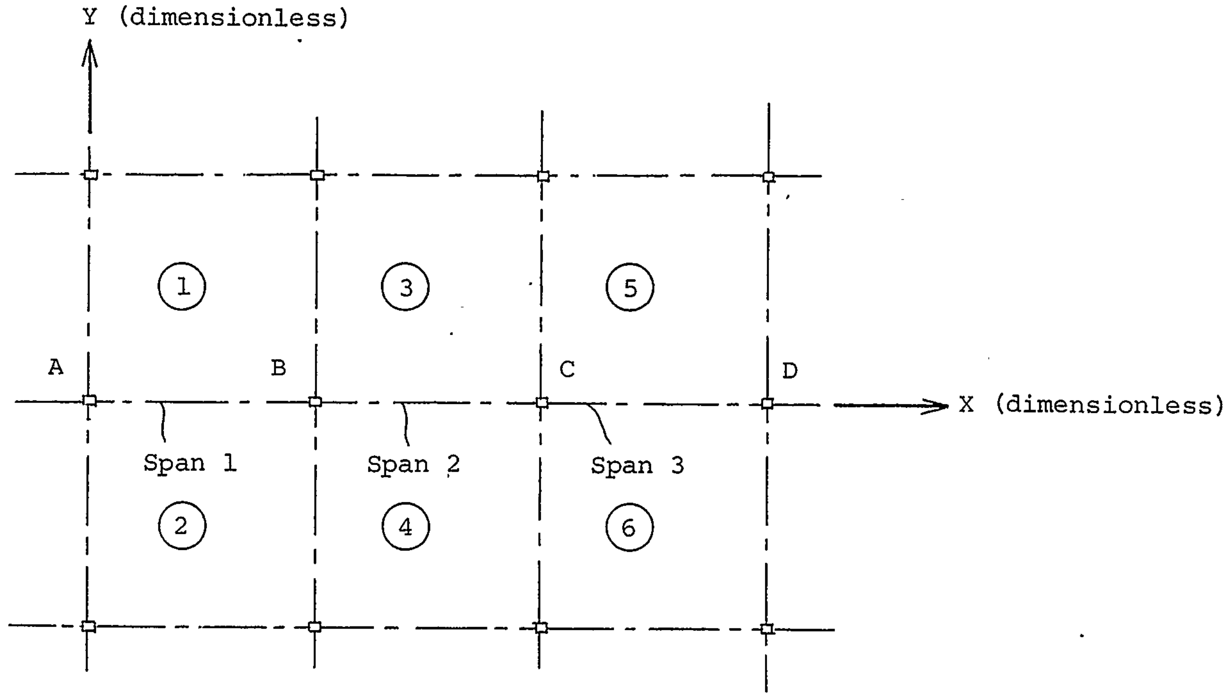
The statistical properties of the moment at a point in a beam are determined by a consideration of the area of floor (roof) illustrated in Fig. 6.2a. This section includes all areas in which the influence surfaces for moment in the beam, BC, are (significantly) non-zero. Such areas have been termed "influence areas" (McGuire and Cornell, 1974). It is assumed that the moments in the beam BC, found using this portion of a "square grid" structure, will provide a reasonably accurate estimate of the moments in a typical interior beam of a building. The equations for the influence line for moment at a distance, (ℓX_o) from support B (Fig. 6.2b) are as follows:

(1) for a unit load a distance $X\ell$ to the right of A,

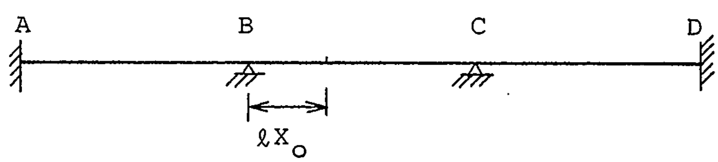
$$I_X = \ell X^2(1 - X)(9X_o - 7)/15 \qquad (6.4)$$

(2) for a unit load a distance $X\ell$ to the right of B,

$$I_X = \ell(-6X^3 + 14X^2 + 7X)/15 + \ell X_o/5(4X^3 - 6X^2 - 3X) \quad (X < X_o) \qquad (6.5)$$



(a) Section of Floor or Roof Including the Influence Areas for the Moments at all points in BC.



(b) Section Through Beam BC

Fig. 6.2. - Influence Areas for Moment in a Beam

$$I_X = \ell(-6X^3 + 14X^2 - 8X)/15 + \ell X_o/5(4X^3 - 6X^2 - 3X + 5) \quad (X > X_o) \quad (6.6)$$

(3) for a unit load a distance $X\ell$ to the right of C,

$$I_X = \ell X(1 - X)^2(2 - 9X_o)/15 \quad (6.7)$$

The equation for the deflection of a beam subjected to a unit displacement at one end is:

$$I_Y = \ell Y^2(3 - 2Y) \quad (6.8)$$

A convenient approximation (Ayer and Cornell, 1968; Peir and Cornell, 1973) for the influence surface function is the product of the influence line function I_X, and the function I_Y:

$$I_{XY} = I_X I_Y \quad (6.9)$$

The error involved in this approximation has been found to be small (Ayer and Cornell, 1968).

The load intensity at a point during the maximum-in-lifetime total loading consists of a sustained and an extraordinary component. It is widely assumed (Peir and Cornell, 1973; McGuire and Cornell, 1974; Ellingwood and Culver, 1977; Chalk and Corotis, 1980) that the sustained portion consists of a point-independent and a point-dependent portion, while the extraordinary portion consists of a random number of randomly sized load cells occurring randomly in space. For the purposes of comparing load with load effect, the extraordinary load, like the sustained, has been treated as if composed of a point-dependent and a point-independent portion. In the development, the maximum-in-lifetime total load is assumed to consist of two such components:

$$w(x, y) = \varepsilon_o + \varepsilon_1(x, y) \tag{6.10}$$

where ε_o is constant for all x and y and $\varepsilon_1(x, y)$ is a random variable with a mean of zero which varies from one point to the next. This latter random variable, $\varepsilon_1(x, y)$, may be spatially correlated or uncorrelated. The two cases are considered in turn:

Spatially Uncorrelated $\varepsilon_1(x, y)$: The intensity $\varepsilon_1(x, y)$, is considered to a totally uncorrelated "white noise" process, that is, the correlation between the intensities at two points is zero if the points are separated by any significant distance. The moment due to the loading at a point (x, y) is,

$$M(X_o) = \int_{A_I} w(x, y) I_{XY}(x, y)\, dA \tag{6.11}$$

where A_I is the area of the six slabs illustrated in Fig. 6.2a. Hence,

$$M(X_o) = \int_{A_I} [\varepsilon_o + \varepsilon_1(x, y)] I_{XY}(x, y) dA \tag{6.12}$$

$$M(X_o) = \varepsilon_o \int_{A_I} I_{XY}(x, y) dA + \int_{A_I} \varepsilon_1(x, y) I_{XY}(x, y) dA \tag{6.13}$$

It follows that the mean moment is:

$$\bar{M}(X_o) = \bar{\varepsilon}_o\, m_I A_{S1} \tag{6.14}$$

where m_I is the integral over the six slabs of the (normalized) slab surface and A_{S1} is the area of one slab. The variance of the moment is:

$$\sigma_M^2(X_o) = \sigma_{\varepsilon o}^2\, m_I^2\, A_{S1}^2 + \delta A\, \sigma_{\varepsilon 1}^2\, A_{S1} \int_6 I_{XY}^2(X, Y) dA. \tag{6.15}$$

where $\sigma_{\varepsilon o}^2$ and $\sigma_{\varepsilon 1}^2$ are variances, δA is the (small) area over which the loading intensity is constant and integration is over all six slabs (normalized).

Spatially Correlated $\varepsilon_1(x, y)$: Peir and Cornell (1973) have suggested the following covariance function "because it apparently produces good results and because it proves convenient to use":

$$\mathrm{cov}[\varepsilon_1(x_i, y_i), \varepsilon_1(x_j, y_j)] = \sigma_{\varepsilon 1}^2 e^{-r^2/d} \tag{6.16}$$

where d is constant and r is the distance between (x_i, y_i) and (x_j, y_j). The mean moment is unchanged (Eqn. 6.14). However, the variance becomes, from Eqn. 6.13:

$$\sigma_M^2(X_o) = \sigma_{\varepsilon o}^2 m_I^2 A_{S1}^2 + \int_{A_i}\int_{A_j}\{\mathrm{cov}[\varepsilon_1(x_i,y_i),\varepsilon_1(x_j,y_j)]$$

$$I_{XY}(x_i,y_i) I_{XY}(x_j,y_j)\}dA_j dA_i \tag{6.17}$$

$$\sigma_M^2(X_o) = \sigma_{\varepsilon o}^2 m_I^2 A_{S1}^2 + \sigma_{\varepsilon 1}^2 \int_{A_i}\int_{A_j}\{\sigma_{\varepsilon 1}^2 (e^{-r^2/d})_{i,j}$$

$$I_{XY}(x_i,y_i) I_{XY}(x_j,y_j)\}dA_j dA_i \tag{6.18}$$

6.4.3 Assessment of Traditional Method of Design

A reasonable design moment for the actual loading is given by the sum of the mean moment due to that loading and a specified multiple of the standard deviation. Hence,

$$\text{Required Design Moment} = \bar{M}(X_o) + \beta\sigma_M(X_o), \tag{6.19}$$

where β is a specified constant reflecting the desired level of safety. Under the current practice of considering a factored equivalent uniformly distributed live loading, $\gamma_L q_L$, the design moment is:

$$\text{Currently Provided Design Moment} = \gamma_L q_L A_{S1} \int_n I_{XY}(X, Y)\,dA \qquad (6.20)$$

where the number of slabs, n, over which the integration is carried out, depends on the loading condition being considered. The factored loading, $\gamma_L q_L$, represents an extreme value for the uniform loading intensity. The corresponding mean moment is not the mean required moment as integration is only over those spans which adversely influence the load effect being considered. Neither is the corresponding variance equal to the variance of the actual required moment. The difference between the design moment provided and that which is actually required, depends on the moment being considered. Thus, the current design procedure results in different levels of safety being provided at different locations in the beam. The variance of moment implicit in the current design procedure is:

$$\text{Currently Implied Variance} = \gamma_L^2\, \sigma_{qL}^2\, A_{S1}^2 \left(\int_n I_{XY}(X, Y)\,dA\right)^2 \qquad (6.21)$$

Thus,

$$\frac{\text{Required Moment Variance}}{\text{Provided Moment Variance}} = \frac{\sigma_{\varepsilon o}^2\, m_I^2}{\gamma_L^2 \sigma_{qL}^2 \left(\int_n I_{XY}(X, Y)\,dA\right)^2} + \frac{\sigma_{\varepsilon 1}^2\, \delta A}{\gamma_L^2 \sigma_{qL}^2 A_{S1}}\,\kappa \qquad (6.22)$$

where, when $\varepsilon_1(x, y)$ is spatially uncorrelated,

$$\kappa = \frac{\int_6 I_{XY}^2(X, Y)\,dA}{\left(\int_n I_{XY}(X, Y)\,dA\right)^2}\,, \qquad (6.23)$$

and when $\varepsilon_1(x, y)$ is spatially correlated,

$$\kappa = \frac{1}{A_{S1}(\delta A)} \frac{\int_{A_i} \int_{A_j} \sigma^2_{\varepsilon 1} (e^{-r^2/d})_{i,j} I_{XY}(x_i, y_i) I_{XY}(x_j, y_j) dA_i dA_j}{(\int_n I_{XY}(X, Y) dA)^2} \quad (6.24)$$

The first term in Eqn. 6.22 is, for all X_o, proportional to,

$$\frac{m_I^2}{(\int_n I_{XY}(X, Y) dA)^2}$$

which equals,

$$\left\{ \frac{\int_6 I_{XY}(X, Y) dA}{\int_n I_{XY}(X, Y) dA} \right\}^2$$

where n depends on the loading condition being considered. For considerations of support moment, n = 4, for it is usual practice to consider loadings in the spans adjacent to the support only. While the integral in the numerator is over six slabs, the ratio for support moment will be close to unity for the contribution from the two remote slabs will be small. For moments at other points, the deviation of this term from unity may be slightly larger.

The second term of the equation involves the function, κ, which has been widely treated in the literature as the sole contributor to the inconsistencies resulting from the use of equivalent uniformly distributed loads. The values of κ reported by McGuire and Cornell (1974) and Ellingwood and Culver (1977) appear to have been derived on the basis of integration over a single slab for both the numerator and the denominator. Here, all 6 slabs are used for the numerator and, for the denominator, the loading situations normally considered for design are used. This is to simulate the actual contribution to moment variance in the former case and to simulate the contribution to variance currently implied by the Codes in the latter.

For consistent levels of safety, the ratio of required to provided moment variance should be constant throughout a span. That this is not the case can be seen in Fig. 6.3. Plotted here are the numerators and denominators of κ. The three denominators correspond to the three loading cases appropriate to the left-hand half of the span, AB. These correspond to the cases when the first, the first and second or the first and third span(s) has (have) maximum factored loading while all others have minimum loading. Two denominators are plotted depending on whether or not it is assumed that $\varepsilon_1(x, y)$ is spatially correlated. It can be seen that this assumption has little effect on the moment variance. In contrast, the relationship between this variance and the location, X_o, being considered, differs substantially from the corresponding relationship implicit in the traditional design procedure.

6.4.4 Assessment of Probabilistic Method of Design

Whether the actual live load is a "white noise" process or is spatially correlated in accordance with the relationship assumed above, it is clear that the levels of safety implicit in current practice are inconsistent. It will be seen that the proposed model provides an excellent fit with the general shape of the actual design bending moment distribution. This is within the bounds of error which result from the assumption on correlation. The bending moment resulting from live loading intensities of q_{L1}, q_{L2} and q_{L3} (force/area) in Spans 1, 2 and 3

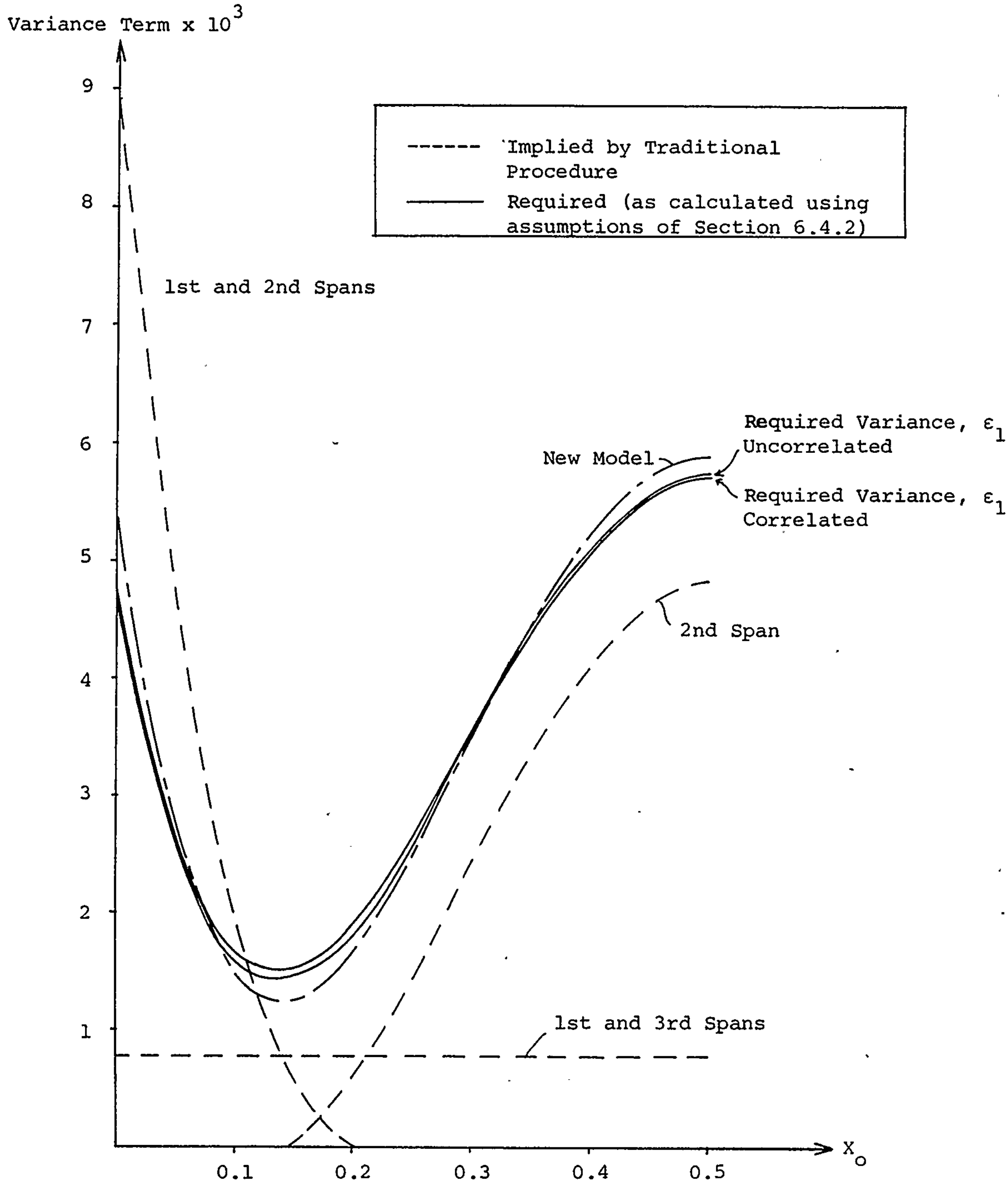


Fig. 6.3. - Variance of Moment Versus Location in Span

respectively is:

$$M(X_o) = \frac{A_S \ell}{180}\{q_{L1}(9X_o - 7) + q_{L2}(-90X_o^2 + 90X_o - 10) + q_{L3}(2 - 9X_o)\} \tag{6.25}$$

The uniform live loading intensity in each span is assumed to have two component parts:

$$q_{Li} = q_{Lo} + q_{Lni} \tag{6.26}$$

where q_{Lo} is constant for all spans and q_{Lni} are statistically independent, identically distributed random variables, one for each span. Hence,

$$M(X_o) = A_S\ell/180\{15q_{Lo}(-6x_o^2 + 6X_o - 1) + q_{LN1}(9X_o - 7)$$

$$+ 10q_{Ln2}(-9X_o^2 + 9X_o - 1) + q_{Ln3}(2 - 9X) \tag{6.27}$$

$$\sigma_M^2(X_o) = A_S^2\ell^2/180^2\{225\sigma_{Lo}^2(-6X_o^2 + 6X_o - 1)^2$$

$$+ \sigma_{Ln}^2[(9X_o - 7)^2 + 100(-9X_o^2 + 9X_o - 1)^2 + (2 - 9X_o)^2]\} \tag{6.28}$$

where σ_{Lo} and σ_{Ln} are the standard deviations of q_{Lo} and all q_{Lni} respectively. When analysis is by the new procedure,

$$\frac{\text{Required Moment Variance}}{\text{Provided Moment Variance}} =$$

$$\frac{\sigma_{\varepsilon o}^2\, m_I^2\, A_{S1}^2 + \delta A \sigma_{\varepsilon 1}^2\, A_{S1} \int_6 I_{XY}^2(X, Y)\,dA}{\frac{A_{S1}^2\ell^2}{180^2}\{225\sigma_{Lo}^2(-6X_o^2 + 6X_o - 1)^2 + \sigma_{Ln}^2[(9X_o - 7)^2 + 100(-9X_o^2 + 9X_o - 1)^2 + (2 - 9X_o)^2]\}} \tag{6.29}$$

where,

$$m_I^2 = \ell^2(-6X_o^2 + 6X_o - 1)^2/144 \tag{6.30}$$

It is proposed to select σ_{Lo} such that

$$\sigma_{\varepsilon o}^2 m_I^2 A_{S1}^2 = A_{S1}^2 \ell^2/180^2[225\sigma_{Lo}^2(-6X_o^2 + 6X_o - 1)^2] \quad (6.31)$$

which implies,

$$\sigma_{Lo} = \sigma_{\varepsilon o} \quad (6.32)$$

and to select σ_{Ln} such that

$$\frac{\sigma_{Ln}^2 A_{S1}^2 \ell^2}{180^2}[(9X_o - 7)^2 + 100(-9X_o^2 + 9X_o - 1)^2 + (2 - 9X_o)^2]$$

$$= \delta A \sigma_{\varepsilon 1}^2 A_{S1} \int_6 I_{XY}^2(X, Y)\,dA \quad (6.33)$$

This would cause the required moment variance to equal that provided. However, in order for Eqn. 6.33 to be satisfied everywhere, it is necessary that κ' be constant for all X_o, where,

$$\kappa' = \frac{\int_6 I_{XY}^2(X, Y)\,dXdY}{\ell^2[(9X_o - 7)^2 + 100(-9X_o^2 + 9X_o - 1)^2 + (2 - 9X_o)^2]} \quad (6.34)$$

The denominator of κ^1 multiplied by an appropriate constant is plotted in Fig. 6.3. The reason for the good fit is that all the assumptions of the model are similar to those used in the calculation of the effects of the actual loading. The only difference is in the size of the segments of area assumed to have independent loading intensities. Thus, the difference is that between $\delta A \int I_{XY}^2(X, Y)\,dA$, (segment of area infinitesimal) and $[\int I_{XY}(X, Y)\,dA]^2$ (segment = slab area). The error, such as it is, is of the form of an exaggeration at the critical points of the curve.

6.5 VARIATION OF LOADING PARAMETERS WITH AREA

6.5.1 General

A number of comprehensive loading surveys (Mitchell and Woodgate, 1971; Culver, 1976) have provided useful data for the determination of the distribution of sustained live loading in buildings and its variation with area. However, there appear to be no reliable data available at the present time for extraordinary loading. A number of studies have been carried out on the effects of extraordinary loading and the combination of this with sustained load effects (McGuire and Cornell, 1974; Ellingwood and Culver, 1977; Chalk and Corotis, 1980; Harris, Corotis and Bova, 1981). However, each of these has been done on the basis of assumed statistical properties derived from "engineering judgement". Unfortunately, extraordinary loads do appear to make a significant contribution to expected maximum-in-lifetime total load effects (Ellingwood and Culver, 1977). In view of this problem, the selection of the form of the functions by which mean and standard deviation vary with area, is a subjective process. The functions adopted here (after McGuire and Cornell, 1974) are but some of a number that may be appropriate. However, it is felt that the use of these functions together with the more rational probabilistic model will provide more reliable results than those found using the existing Code equations and procedures.

6.5.2 Code Specifications

The National Building Code of Canada (National Research Council of Canada, 1980) currently specifies that the live loading cases to be considered are a uniformly distributed load whose intensity is a function of area and a fixed concentrated load applied on an area of 750 mm square. For buildings used for purposes other than storage, manufacturing, retail, garage or assembly, the reduced uniform load intensity is specified as:

$$q_{LR} = q_L[0.3 + (9.8/B)^{\frac{1}{2}}] \quad ; \quad B > 20 \text{ m}^2 \tag{6.35}$$

$$q_{LR} = q_L \quad ; \quad B \leqq 20 \text{ m}^2 \tag{6.36}$$

where B is the tributary area in square metres and q_L is the characteristic live loading intensity.

If the intensity were assumed to vary with area regardless of the magnitude of that area, then the total load on a portion of the beam of area A, would be:

$$\text{Load} = q_L A[0.3 + (9.8/A)^{\frac{1}{2}}] \tag{6.37}$$

For office occupancy, the characteristic load intensity is 2.4 kN/m^2. Thus, for an area, $A = (0.75)^2$, the force given by Eqn. 6.37 is 6.0 kN. This value is not too far removed from the specified concentrated force for offices of 9.0 kN. This would suggest that perhaps Eqn. 6.35 should be applied for all areas and no concentrated force considered.

Another possible change to the code specifications concerns the use of tributary areas. It has been suggested by McGuire and Cornell (1974) and others that tributary area is a much less consistent parameter than influence area for the calculation of equivalent uniformly distributed loads. (Influence area is that area over which the influence surface is significantly non-zero.) The reason for this can be seen by comparing the two load effects of axial force in a column and bending in a beam. For the former, the tributary area is one fourth of the influence area while for the latter, the tributary area is one half of the influence area. Thus, the portion of the influence surface for bending that is neglected when tributary area is used, is considerably less than the portion neglected for axial load. Clearly, if one loading intensity is to be used for all load effects, this intensity must be applied over the influence area rather than tributary area. Accordingly, reduction formulae for intensity with area must also be in terms of influence rather than tributary area.

For the purposes of comparing the implications of the statistical parameters selected with the current specifications, the reduction formula, as currently specified, is used. For all loading conditions, the tributary area for one span only is substituted into the reduction formula. This is in keeping with what the author feels is standard practice in most design offices. The reduction factor is taken to be applicable only for areas in excess of 20 m^2 and the concentrated force of 9 kN (office occupancy) is considered separately.

6.5.3 Comparison of Probabilistic and Deterministic Results

A number of proposals have been suggested for the variation of the expected value and the standard deviation of loading intensity with area (McGuire and Cornell, 1974; Ellingwood and Culver, 1977; Chalk and Corotis, 1980). As mentioned previously, there is insufficient extra-ordinary load data available at this time for an accurate determination of these relationships. However, it is reasonable to assume some variation with area. Accordingly, while recognizing that they are based partly on engineering judgement, the following relationships, proposed by McGuire and Cornell (1974) for office loading, will be adopted:

$$\bar{q}_{Ln} = 0.712 + 11.7/(A_I)^{\frac{1}{2}} \quad \text{kN/m}^2 \tag{6.38}$$

$$\sigma^2_{Ln} = 0.0259 + 3.194/A_I \quad (\text{kN/m}^2)^2 \tag{6.39}$$

(Their conversion to metric units is erroneous and has been corrected here.) As this expression for variance includes variance from all sources, σ_{Lo} is taken to be zero:

$$\sigma_{Lo} = 0 \tag{6.40}$$

For a simply supported beam, the mid-span design moment, as found using the probabilistic method, is:

$$M_{prob} = (\bar{q}_{Ln} + \beta\sigma_{Ln})\ell^2/8 \tag{6.41}$$

where 3.0 has been suggested as an appropriate value for β for all types of loading (Galambos, Ellingwood, MacGregor and Cornell, 1982). The corresponding design moment as found by the traditional deterministic design procedure and in accordance with the Canadian code is:

$$M_{det} = 1.5\, q_{LR}\ell^2/8 \tag{6.42}$$

where q_{LR} is the reduced intensity. Hence, the ratio of the moment implied by the traditional procedure to that implied by the probabilistic approach is:

$$\frac{M_{det}}{M_{prob}} = \frac{1.5\, q_{LR}}{\bar{q}_{Ln} + 3\sigma_{Ln}} \tag{6.43}$$

where $\bar{q}_{Ln}$ and σ_{Ln} are given by Eqns. 6.38 and 6.39, respectively and q_{LR} is given by Eqns. 6.35 and 6.36 using a characteristic loading intensity, $q_L = 2.4\ kN/m^2$. The ratio, (M_{det}/M_{prob}) is plotted in Fig. 6.4 as a function of influence area. It can be seen that for the simply supported beam, the procedures imply similar results for typical influence areas. For all areas greater than 40 m^2, the discrepancy between M_{prob} and M_{det} is less than 3%. Below this value, where the reduction formula (Eqn. 6.35) is no longer applicable, the discrepancy rises sharply. Also plotted in Fig. 6.4 is the ratio M_{det}/M_{prob}, when the reduction formula is applied for influence areas less than 40 m^2 (tributary areas less than 20 m^2). It can be seen that this gives much better results. While it is recognized that the relationships of Eqns. 6.38 and 6.39 may not be applicable for small areas, it is this author's opinion that in the absence of more reliable data, the conservative option should be exercised. Accordingly, when a deterministic design procedure is to be used, Eqn. 6.35, or its equivalent in terms of influence area, should be applied for all areas rather than merely for tributary areas in excess of 20 m^2.

Regardless of the reduction formula used, it is difficult to calibrate the results of the traditional deterministic design method with the probabilistic results for all beam geometries. To illustrate this point, support and mid-span moments in the spans of two further beams are considered. The first of these is the interior span BC, of the symmetrical beam in Fig. 6.2. The design moment at the supports B and C implied by the probabilistic procedure is:

$$M_{prob} = A_{S1}\ell/180\{-15\bar{q}_{Ln} - 3[225\sigma_{Lo}^2 + 153\sigma_{Ln}^2]^{\frac{1}{2}}\} \tag{6.44}$$

where A_{S1} is the area of one slab. The support moment implied by the traditional design procedure is:

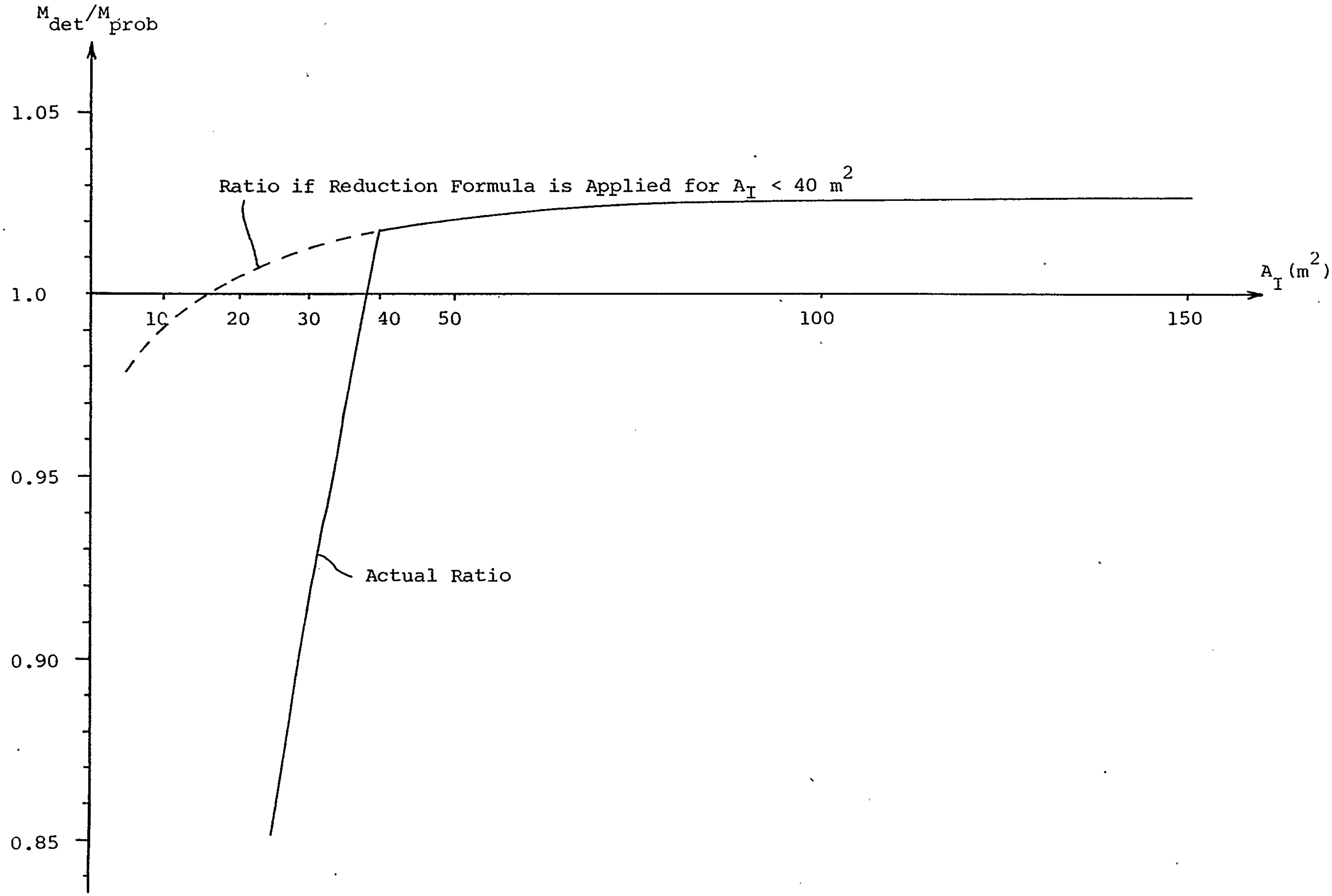


Fig. 6.4. - Variation of (M_{prob}/M_{det}) with Influence Area

$$M_{det} = A_{S1}\ell/180\{-25.5(2.4)(0.3 + [9.8/A_S]^{\frac{1}{2}})\} \qquad (6.45)$$

Taking a slab area equal to 40 m^2, $\bar{q}_{Ln}$ and σ_{Ln} are calculated using Eqns. 6.38 and 6.39 with A_I = 80 m^2. The resulting ratio is,

$$\frac{M_{det}}{M_{prob}} = 1.22 \qquad (6.46)$$

Hence, whereas the difference in the implications of the two procedures was less than 3% for a simply supported beam (at A_I = 80 m^2), the traditional procedure is conservative by 22% for the support moment in an interior beam. The deterministic procedure is even more conservative for the mid-span moment in the interior beam, the ratio of M_{det} to M_{prob} at A_I = 80 m^2, being

$$\frac{M_{det}}{M_{prob}} = 1.42 \qquad (6.47)$$

The reason for this is that the mid-span moment is proportional to,

$$-q_1 + 5q_2 - q_3$$

where q_1, q_2 and q_3 are the respective loading intensities in the three spans. Hence, the actual mean moment is proportional to $3\bar{q}_{Ln}$. In the deterministic procedure, the beneficial influence of the expected loadings in the first and third spans is neglected so that the unfactored moment is proportional to $5q_{LR}$. The significant difference between 3 and 5 (as opposed to 15 and 17 for the support moment) accentuates the inaccuracies associated with the deterministic approach.

The second beam considered is illustrated in Fig. 6.5. The influence areas are 50 m^2 and 100 m^2 for Spans 1 and 2 respectively. The implied values for the ratio (M_{det}/M_{prob}) are:

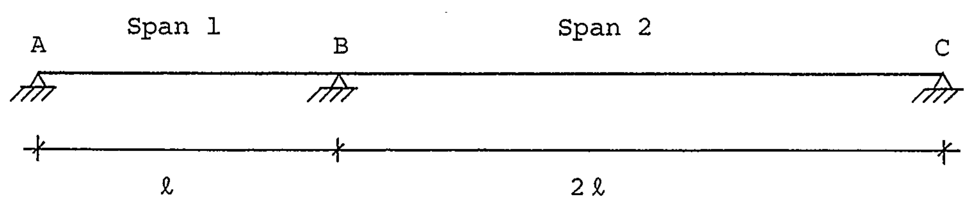


Fig. 6.5. - Unsymmetrical Example

Support: $M_{det}/M_{prob} = 1.06$ (6.48)

Mid-Point: $M_{det}/M_{prob} = 1.09$ (6.49)

The deterministic approach gives good results for both moments. Due to the large difference in the span lengths in this example, both of the moments considered are affected primarily by the loading in Span 2. Hence, as for the simply supported beam in which only one loading is involved, the agreement is relatively good.

6.6 PROBABILISTIC ELASTIC DESIGN PROCEDURE FOR SHEAR

A similar treatment to that used for moment is applied to analysis for shear. However, as will be seen, certain modifications are necessary to ensure consistent levels of safety. As in the case for moment design, the maximum-in-lifetime total loading intensity is assumed to be as given by Eqn. 6.10. Hence, the actual shear at a point in the beam AB of Fig. 6.2, is:

$$S(x_o) = \int_{A_I} W(x, y) I_{SXY}(x, y)\,dA \qquad (6.50)$$

where $I_{SXY}(x, y)$ is the influence surface for shear. This is found by differentiating the corresponding function for moment $I_{XY}(x, y)$, with respect to X_o. Assuming that $\varepsilon_1(x, y)$ is uncorrelated, the mean and variance for shear in AB are, respectively:

$$\bar{S}(X_o) = A_{S1}\,\bar{\varepsilon}_o\,m_{SI} \qquad (6.51)$$

$$\sigma_S^2(X_o) = \sigma_{\varepsilon o}^2\,m_{SI}^2\,A_{S1}^2 + \delta A \sigma_{\varepsilon 1}^2\,A_{S1} \int_6 I_{SXY}^2(X, Y)\,dA \qquad (6.52)$$

where,

$$m_{SI} = \int_6 I_{SXY}(X, Y)\,dA \qquad (6.53)$$

The shear force resulting from loadings of q_1, q_2 and q_3 in Spans 1 to 3 in Fig. 6.2, respectively, is:

$$S(X_o) = A_{S1}\{q_1 \int_{1,2} I_{SXY}(X, Y)\,dA + q_2 \int_{3,4} I_{SXY}(X, Y)\,dA$$
$$+ q_3 \int_{5,6} I_{SXY}(X, Y)\,dA\} \qquad (6.54)$$

where $\int_{i,j}$ implies integration over Slabs i and j as illustrated in the figure. If the same probabilistic design procedure is adopted as for moment, then the mean and variance which will be provided for are:

$$\text{Provided Shear Mean} = A_{S1}\, q_{Lo}\, m_{SI} \tag{6.55}$$

$$\text{Provided Shear Variance} = A_{S1}^2\{\sigma_{Lo}^2\, m_{SI}^2 + \sigma_{Ln}^2[(\int_{1,2} I_{SXY}(X, Y)dA)^2 + (\int_{3,4} I_{SXY}(X, Y)dA)^2 + (\int_{5,6} I_{SXY}(X, Y)dA)^2]\} \tag{6.56}$$

A comparison of Eqns. 6.51 and 6.55 shows that the mean provided equals that required (it is assumed that $q_{Lo} = \bar{\varepsilon}_o$). Similarly, comparing Eqns. 6.52 and 6.56, it can be seen that the first terms are equal and the second terms would be equal if,

$$A_{S1}^2\, \sigma_{Ln}^2[(\int_{1,2} I_{SXY}(X, Y)dA)^2 + (\int_{3,4} I_{SXY}(X, Y)dA)^2 + (\int_{5,6} I_{SXY}(X, Y)dA)^2] = \delta A\, \sigma_{\varepsilon 1}^2\, A_{S1} \int_6 I_{SXY}^2(X, Y)dA \tag{6.57}$$

Unfortunately, the variation of the left-hand side of Eqn. 6.57 with X_o is significantly different from the variation of the right-hand side with this variable. The difference is in the integration over Slabs 3 and 4. The function, $\int_{3,4} I_{SXY}(X, Y)dA$, is linear in X_o while the term $\int_{3,4} I_{SXY}^2 dA$, is a fourth order polynomial. The term, $\{\int_{3,4} I_{SXY}^2(X, Y)dA\}^{\frac{1}{2}}$ is plotted in Fig. 6.6 together with the term, $\int_{3,4} I_{SXY}(X, Y)dA$, and a

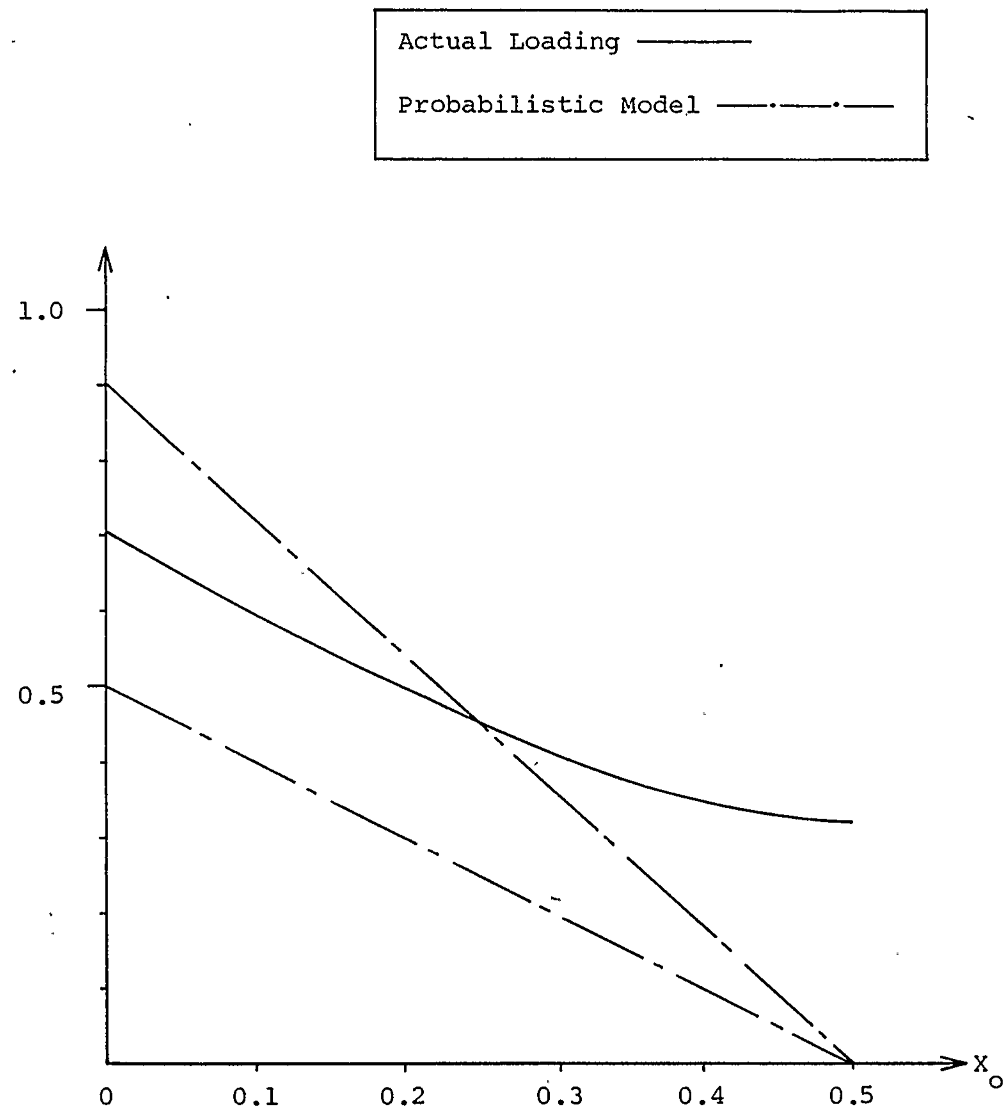


Fig. 6.6. - Variation of Standard Deviation Terms with X_o

multiple thereof. As the latter term is zero at $X_o = 0.5$, it can never provide a conservative estimate of the former. However, as the discrepancy is due to the distribution of loading assumed in Span BC only, it is not difficult to rectify the problem. This is achieved in the probabilistic design procedure by adding a correction function to the term which involves the influence surface for the span being considered. This function is,

$$F = \int_{3,4} I^2_{SXY}(X, Y)\,dA - \{\int_{3,4} I_{SXY}(X, Y)\,dA\}^2 \tag{6.58}$$

which can be shown to be:

$$F = 0.246 - X_o/7\{3 + X_o - 8X_o^2 + 4X_o^3\} \tag{6.59}$$

The application of this correction function to the standard deviation does not unduly complicate the design procedure and will give results in close agreement with the "actual" shear as determined using the assumptions outlined in this chapter.

6.7 EXAMPLES OF PROBABILISTIC ELASTIC DESIGN

6.7.1 Example 1 - Live Load Only

For the beam illustrated in Fig. 6.7a, find the design bending moment diagram and shear force diagram due to office live loading. The span lengths are 8 m, and the clear distance between spans is 6 m.

The influence area for each of these spans is:

$$A_I = 96 \text{ m}^2$$

From Eqn. 6.39,

$$\sigma^2_{Ln1} = \sigma^2_{Ln2} = 0.0592 \quad (\text{kN/m}^2)^2.$$

The mean live loadings are, from Eqn. 6.38,

$$\bar{q}_{Ln1} = \bar{q}_{Ln2} = 1.91 \quad \text{kN/m}^2.$$

These load intensities are applicable over their entire influence areas. However, the effect of applying a force per unit area, q, over the entire influence area, A_I, is the same as that of applying a force per unit length of $(q\ A_I/2\ell)$ over the complete span length. This follows from the fact that,

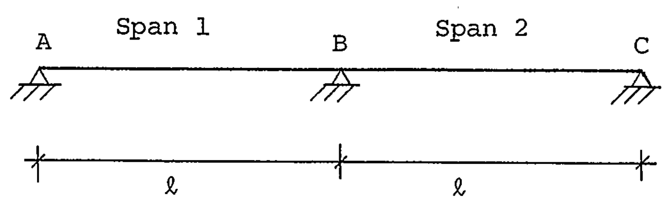
$$\int I_Y \, dY = \frac{1}{2}$$

(where I_Y is as given in Eqn. 6.8).

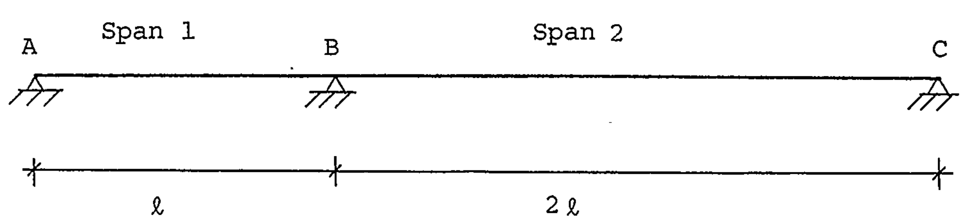
As this structure is symmetrical, only Span 1 will be considered. The moment in this span at a distance (ℓX_o) to the right of A is:

$$M(X_o) = \{q_{Ln1}(7 - 8X_o) - q_{Ln2}\}A_I \ \ell X_o/32 \tag{6.60}$$

where q_{Ln1} and q_{Ln2} are the live load intensities in Spans 1 and 2 respectively. Hence, the mean and standard deviation of moment in Span



(a) Example 1



(b) Example 2

Fig. 6.7. - Symmetrical and Unsymmetrical Design Examples

1 are, respectively,

$$\bar{M}(X_o) = \bar{q}_{Ln1}\,(6 - 8X_o)\; A_I \ell\; X_o/32 \tag{6.61}$$

and

$$\sigma_M(X_o) = \{\sigma^2_{Ln1}[(7 - 8X_o)^2 + 1]\}^{\frac{1}{2}}\; A_I\; \ell\; X_o/32 \tag{6.62}$$

The design bending moment of mean plus 3 standard deviations is plotted in Fig. 6.8. Also plotted in this figure are the bending moment diagrams which would have been found using the traditional method of analysis. The agreement between the two methods is good at the support. However, at mid-span, the moment found by the traditional method is 33% in excess of that found using the probabilistic procedure. The reason for this is that this moment is proportional to,

$$3q_1 - q_2$$

from which it can be seen that Span 2 has a significant (beneficial) effect. This effect is ignored in the traditional procedure which leads to over-conservative design. The contribution of the live loading to the length of top steel is also conservatively modelled in the traditional procedure. The difference in the cut-off points in Fig. 6.8 is substantial. However, the significance of this for the actual lengths of top steel clearly depends on the influence of dead load. An example which includes consideration of dead load is given in the next subsection.

The shear force in Span 1 is, by differentiation of Eqn. 6.60,

$$S(X_o) = \{q_{Ln1}(7 - 16X_o) - q_{Ln2}\}A_I/32 \tag{6.63}$$

Hence, the mean and standard deviation are, respectively:

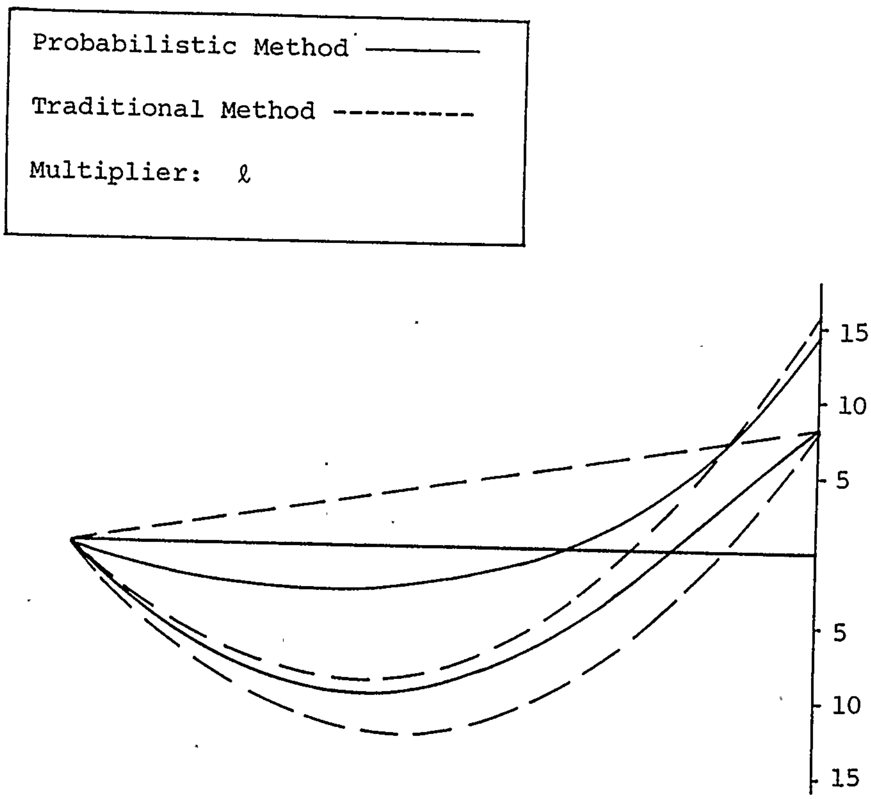


Fig. 6.8. - Bending Moment Diagrams for Example 1

$$\bar{S}(X_o) = \bar{q}_{Ln1}(6 - 16X_o)A_I/32 \tag{6.64}$$

$$\sigma_S(X_o) = \{\sigma^2_{Ln1}(7 - 16X_o)^2 + \sigma^2_{Ln2}\}^{\frac{1}{2}} A_I/32 \tag{6.65}$$

As stated in Section 6.6, it is necessary to apply a correction function to the term involving the span being considered. In "standard" form,

$$\sigma_S(X_o) = \{\sigma^2_{Ln1}(7 - 16X_o)^2/256 + \sigma^2_{Ln2}/256\}^{\frac{1}{2}} A_I/2 \tag{6.66}$$

Hence, the corrected standard deviation is:

$$\sigma_S(X_o) = \{\sigma^2_{Ln1}[(7 - 16X_o)^2/256 + F + 1/256]\}^{\frac{1}{2}} A_I/2 \tag{6.67}$$

where F is given by Eqn. 6.59. The design shear force $\bar{S}(X_o) \pm 3\ \sigma_S(X_o)$ is plotted in Fig. 6.9. The envelope derived from the traditional multiple-loadings approach is also illustrated in this figure. There is good agreement at the supports. However, there is an interesting discrepancy at the points of minimum (absolute) shear. At $X_o = 0.43$, the traditional method implies only 57% of the shear force implied by the probabilistic. This discrepancy results from the same phenomenon that required the probabilistic method to be adapted for shear, namely, that a uniformly distributed loading implies a linear shear distribution and, usually, a point of zero shear. Live loading is not, in fact, uniform. Several load distributions are possible and there is, in general, no interior point in a span at which the shear is zero for all these distributions. Thus, even when a number of loading conditions are considered with different uniform loadings in each span, the minimum implied design shear is not a reasonable reflection of the probable shear at that point.

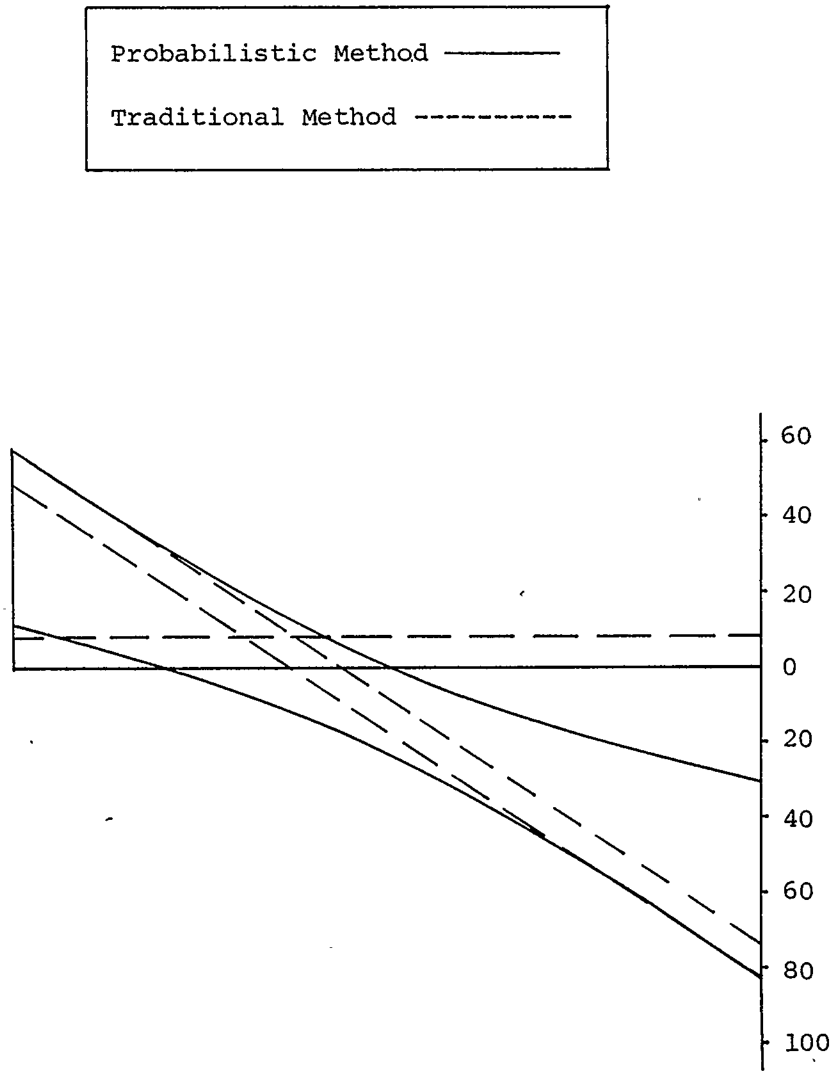


Fig. 6.9. - Shear Force Diagrams for Example 1

6.7.2 Example 2 - Dead Plus Live Load

Determine the design bending moments and shear forces for the beam illustrated in Fig. 6.7b. The span lengths are 5 m and 10 m for Spans 1 and 2 respectively and the clear distance between spans is 6 m. The dead load intensities for the two spans are:

$$q_{D1} = 3.3 \text{ kN/m}^2 \tag{6.68}$$

and

$$q_{D2} = 3.6 \text{ kN/m}^2 \tag{6.69}$$

The structure will be subjected to the live loading intensity appropriate to offices.

It would be inconvenient to use the traditional design procedure for dead loading while using the probabilistic approach for live loading. Accordingly, it is proposed to apply the probabilistic procedure to the effects of dead as well as live loading intensity. The dead loading intensities for the two spans will be assumed to consist of span-dependent and span-independent portions (for example, variation in member size may be span-dependent while variation in material density may be span-independent). The means are taken to be:

$$\bar{q}_{Do} = 0 \tag{6.70}$$

$$\bar{q}_{Dn1} = 3.3 \tag{6.71}$$

$$\bar{q}_{Dn2} = 3.6 \tag{6.72}$$

The standard deviations will be selected so that the mean plus three standard deviations equals 1.25 times the characteristic load. This is more conservative than the implications of the traditional method for the case of "uplifting" ($\gamma_D = 0.85$).

$$\bar{q}_{Dn1} + 3(\sigma^2_{Do} + \sigma^2_{Dn1})^{\frac{1}{2}} = 1.25\, q_{Dn1} \tag{6.73}$$

$$\Rightarrow \sigma^2_{Do} + \sigma^2_{Dn1} = 0.076 \tag{6.74}$$

Similarly,

$$\sigma^2_{Do} + \sigma^2_{Dn2} = 0.09 \tag{6.75}$$

Taking,

$$\sigma^2_{Do} = 0.02, \tag{6.76}$$

implies,

$$\sigma^2_{Dn1} = 0.056 \tag{6.77}$$

$$\sigma^2_{Dn2} = 0.07 \tag{6.78}$$

The statistical parameters for live loading are given by Eqns. 6.38 and 6.39:

$$\bar{q}_{Ln1} = 2.22 \text{ kN/m}^2 \tag{6.79}$$

$$\bar{q}_{Ln2} = 1.78 \text{ kN/m}^2 \tag{6.80}$$

$$\sigma^2_{Ln1} = 0.0791 \ (\text{kN/m}^2)^2 \tag{6.81}$$

$$\sigma^2_{Ln2} = 0.0525 \ (\text{kN/m}^2)^2 \tag{6.82}$$

The forces per unit length are, for both spans, six times the load intensities (in force/area). For intensities of q_1 and q_2 in Spans 1 and 2 respectively, the moment at a distance $(X_o \ell)$ from the left-hand support is, for each span,

$$\text{Span 1:}\quad M(X_o) = 25X_o/24\{(6q_1)(11 - 12X_o) - 8(6q_2)\} \tag{6.83}$$

$$\text{Span 2:}\quad M(X_o) = (1 - X_o)25/24\{-(6q_1) + 8(6q_2)(6X_o - 1)\} \tag{6.84}$$

In this example, the loading intensities are made up of three components each:

$$q_1 = q_{Do} + q_{Dn1} + q_{Ln1} \tag{6.85}$$

$$q_2 = q_{Do} + q_{Dn2} + q_{Ln2} \tag{6.86}$$

Substituting from Eqns. 6.85 and 6.86 in Eqns. 6.83 and 6.84 gives:

$$\text{Span 1:}\quad M(X_o) = 25X_o/4\{3q_{Do}(1 - 4X_o) + (q_{Dn1} + q_{Ln1})(11 - 12X_o) - 8(q_{Dn2} + q_{Ln2})\} \tag{6.87}$$

$$\text{Span 2:}\quad M(X_o) = 25(X_o - 1)/4\{3q_{Do}(3 - 16X_o) + q_{Dn1} + q_{Ln1} + 8(q_{Dn2} + q_{Ln2})(1 - 6X_o)\} \tag{6.88}$$

Hence, the means and standard deviations are as follows:

$$\text{Span 1:}\quad \bar{M}(X_o) = 25X_o/4\{5.52(11 - 12X_o) - 8(5.38)\} \tag{6.89}$$

$$\sigma_M(X_o) = 25X_o/4\{9(.02)(1 - 4X_o)^2 + (.135)(11 - 12X_o)^2 + 64(.123)\}^{\frac{1}{2}} \tag{6.90}$$

$$\text{Span 2:}\quad \bar{M}(X_o) = 25(X_o - 1)/4\{5.52 + 8(5.38)(1 - 6X_o)\} \tag{6.91}$$

$$\sigma_M(X_o) = 25(1 - X_o)/4\{9(.02)(3 - 16X_o)^2 + (.135)$$

$$+ 64(.123)(1 - 6X_o)^2\}^{\frac{1}{2}} \tag{6.92}$$

The design bending moment function $\{\bar{M}(X_o) \pm 3\,\sigma_M(X_o)\}$ is illustrated in Fig. 6.10. Also illustrated are the bending moment diagrams as found using the traditional deterministic design procedure. For these, the minimum span loading was taken to be 0.85 times the dead loading. At all locations in the beam, the bending moment envelope, as found using the traditional procedure, is conservative relative to the probabilistic moment "envelope". The greatest discrepancies are for maximum (sag) moments. In Span 1, the maximum deterministic moment is conservative by more than 150% being 33 kN•m in excess of the maximum probabilistic moment.

The equations for shear force are found by differentiation of Eqns. 6.83 and 6.84. The means and standard deviations, before application of the correction factor, can be shown to be as follows:

$$\text{Span 1:}\quad \bar{S}(X_o) = 1.25\{(\bar{q}_{Dn1} + \bar{q}_{Ln1})(11 - 24X_o) - 8(\bar{q}_{Dn2} + \bar{q}_{Ln2})\} \tag{6.93}$$

$$\sigma_S(X_o) = 1.25\{9\sigma_{Do}^2(1 - 8X_o)^2 + (\sigma_{Dn1}^2 + \sigma_{Ln1}^2)(11 - 24X_o)^2$$

$$+ 64(\sigma_{Dn2}^2 + \sigma_{Ln2}^2)\}^{\frac{1}{2}} \tag{6.94}$$

$$\text{Span 2:}\quad \bar{S}(X_o) = 0.625\{\bar{q}_{Dn1} + \bar{q}_{Ln1} + 8(\bar{q}_{Dn2} + \bar{q}_{Ln2})(7 - 12X_o)\} \tag{6.95}$$

$$\sigma_S(X_o) = 0.625\{9\sigma_{Do}^2(19 - 32X_o)^2 + \sigma_{Dn1}^2 + \sigma_{Ln1}^2$$

$$+ 64(\sigma_{Dn2}^2 + \sigma_{Ln2}^2)(7 - 12X_o)^2\}^{\frac{1}{2}} \tag{6.96}$$

Upon application of the factor F (as given in Eqn. 6.59), the standard

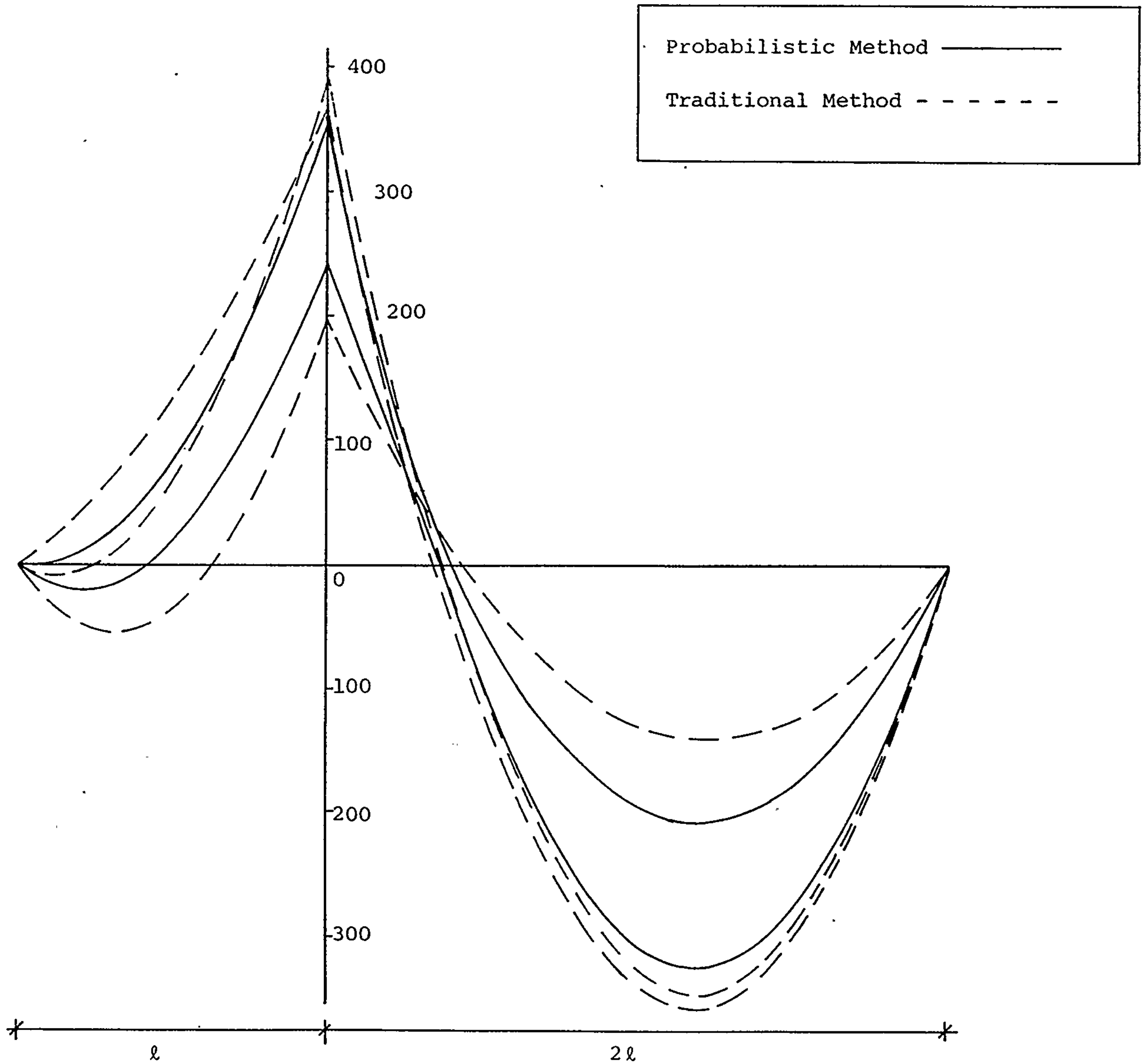


Fig. 6.10. - Bending Moment Diagrams for Example 2

deviations become:

$$\text{Span 1:} \quad \sigma_S(X_o) = 1.25\{9\sigma^2_{Do}(1 - 8X_o)^2 + (\sigma^2_{Dn1} + \sigma^2_{Ln1})[(11 - 24X_o)^2 + 576F] + 64(\sigma^2_{Dn2} + \sigma^2_{Ln2})\} \tag{6.97}$$

$$\text{Span 2:} \quad \sigma_S(X_o) = 0.625\{9\sigma^2_{Do}(19 - 32X_o)^2 + \sigma^2_{Dn1} + \sigma^2_{Ln1} + 64\sigma^2_{Dn2}(7 - 12X_o)^2 + \sigma^2_{Ln2}[64(7 - 12X_o)^2 + 2304F]\}^{\frac{1}{2}} \tag{6.98}$$

The correction factor is applied to the live loading term only for it is anticipated that dead loading will be approximately uniformly distributed across a span while live loading will not be. This is clearly a subjective assumption and designers may wish to specify portions of the dead loading variance to which the correction factor should be applied.

The design shear force diagram is illustrated in Fig. 6.11 together with the shear force diagrams as found using the traditional procedure. There is reasonable agreement in the results for the long span. However, a significant discrepancy exists at Support A.

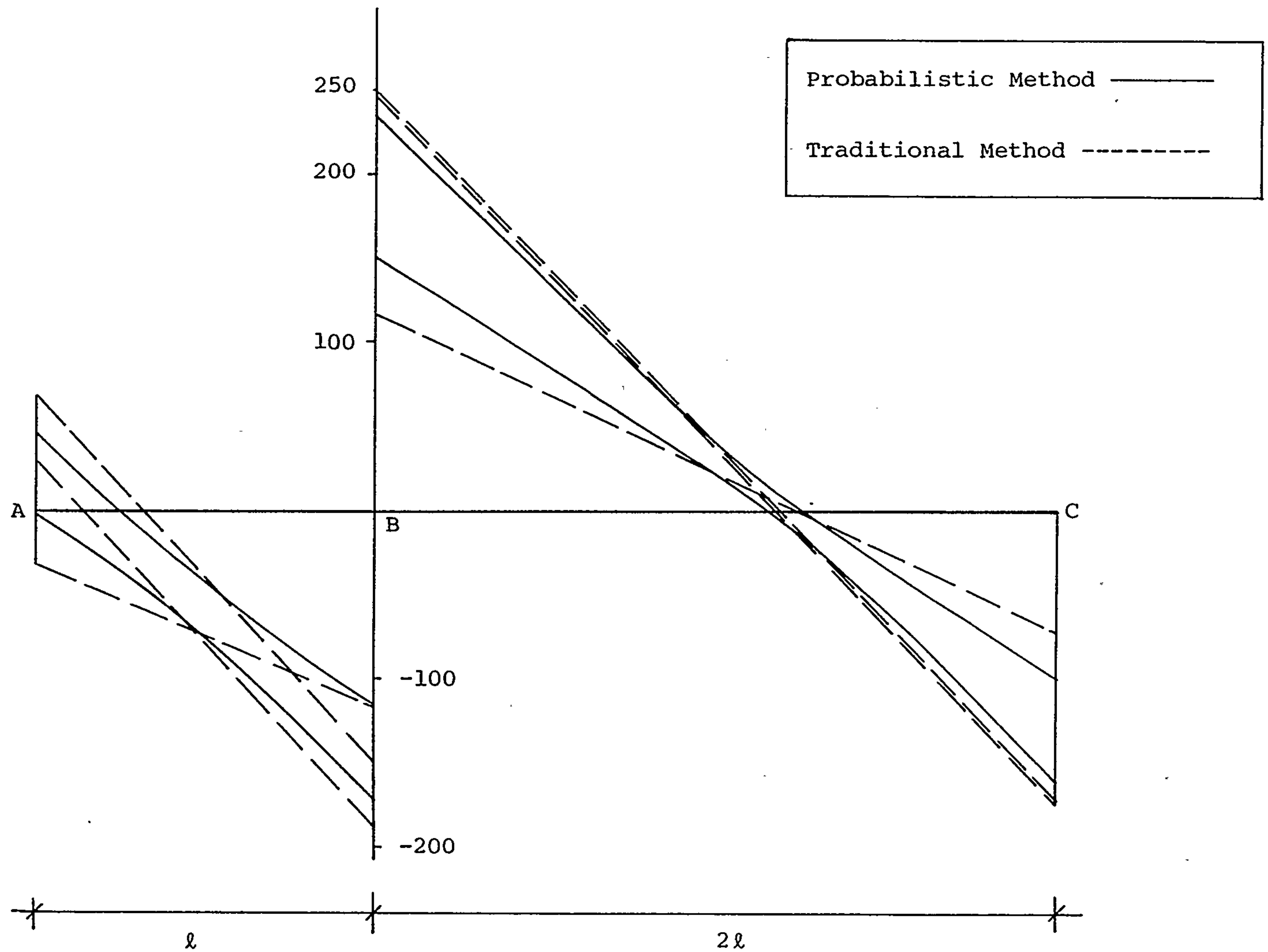


Fig. 6.11. - Shear Force Diagrams for Example 2

6.8 PLASTIC MOMENT REDISTRIBUTION

6.8.1 Introduction

It is fortunate that the development of the probabilistic model for elastic design, described in the previous sections, was possible. The method is based on sound probabilistic principles and the resulting design calculations are conveniently simple. Unfortunately, no such model could be developed for elastic-plastic design of reinforced concrete beams. The process of plastic moment redistribution, while being a simple matter when a deterministic procedure is followed, is a highly complex one when probabilistic principles are applied. The complexity stems from the interaction that takes place between moments at different points after initial yield and before plastic collapse. The result of this interaction is that the design moment at a point can no longer be considered in isolation of the design moment at other points in the beam. In this section, the statistical nature of the plastic moment redistribution problem is described. In addition, a possible approach for design is introduced that is, of necessity, only loosely based on the theory of probability. However, this is only included as an aid to future researchers. It is hoped that it will provide them with ideas that might be pursued further in the future and an indication of some of the problems that arise. Finally, a conservative approximate solution is proposed for the problem of plastic moment redistribution. It is suggested that this be used for design and it is pointed out that probabilistic elastic design combined with this simple approach to redistribution still results in cost savings over the conventional approach.

6.8.2 General

In order to consider in detail the statistical nature of plastic moment redistribution, it is first necessary to distinguish between section moments of resistance, M_{RB}, M_{RC}, M_{RD} and "applied moments", M_B, M_C, M_D, i.e., moments due to applied loads. While moments of resistance are parameters whose values are selected by the designer, the applied moments, being functions of the loads, are random variables. If the member moments of resistance are sufficiently large that no plastic hinge rotation occurs, the applied moments are simple linear functions of the loads. Hence, given the statistical properties of these loads, the extreme values of the applied moments can readily be determined. For elastic-plastic behaviour however, the applied moments are dependent on the moments of resistance as well as the loads. Even if these moments of resistance are known, the statistical distribution of applied moment at a point becomes complex. For example, in the beam of Fig. 1.1, the applied mid-point moment in Span 2 is,

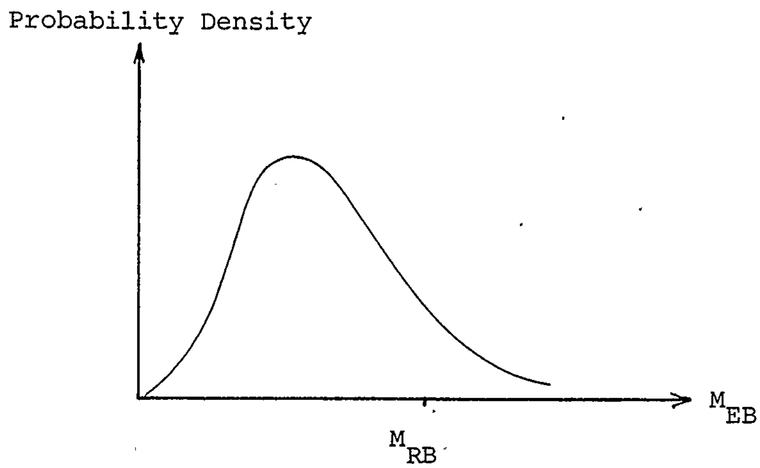
$$M_D = q_2 \ell^2/8 - (M_B + M_C)/2 \tag{6.99}$$

where, q_2, is the uniform loading intensity in Span 2. While, q_2, is a simple random variable, the applied moments, M_B and M_C, are functions of random variables and fixed constants:

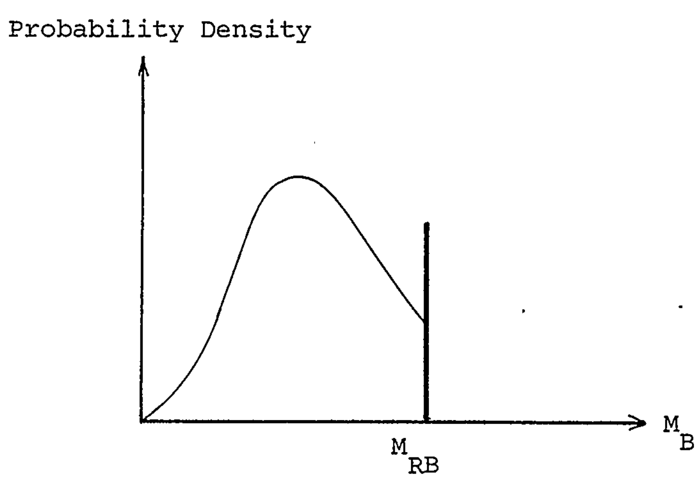
$$M_B = \min(M_{EB}, M_{RB}) \tag{6.100}$$

$$M_C = \min(M_{EC}, M_{RC}) \tag{6.101}$$

where M_{EB} and M_{EC} are the value of M_B and M_C as found by elastic analysis only. Thus, M_{EB} and M_{EC} are linear functions of random variables and their statistical properties can readily be determined



(a) Probability Density Function for M_{EB}



(b) Probability Density Function for M_B

Fig. 6.12. - Probability Density Functions for Applied Moment and Moment as Implied by Elastic Analysis

(see probability density function, Fig. 6.12a). On the other hand, M_B and M_C have probability density functions of the complex form illustrated in Fig. 6.12b.

It is useful to refer to the deterministic equivalent of the probabilistic moment redistribution problem. For the example of Fig. 1.1, the optimal distribution process may involve the increasing of M_{B1} (to reduce sag moment in Span 2) and the reduction of M_{B2} (to reduce the hog moment at B). The probabilistic bending moment "envelope" includes representation of the extreme conditions which the various deterministic loading cases are intended to model. Hence, this envelope may be viewed as a spectrum of bending moment diagrams which includes diagrams similar to those illustrated in Fig. 1.1b. The probabilistic design moment at B will range from a point around M_{B1} corresponding to a system of loading something like Loading 1, to a point around M_{B2} corresponding to a condition similar to Loading 2.

The probabilistic equivalent to the reduction of M_{B2} based on plastic considerations is the reduction of the support moments for the spectrum of loadings which result in large moments at B. However, the probabilistic procedure provides the designer with the statistical properties of moment at B only. The information necessary for redistribution, namely, the combinations of loading which result in the large moment, is not available. These combinations will include cases of high loading intensity in both spans and high loading intensity in Span 1 only, each weighted by the probability of its occurrence. Similarly, the minimum extreme of moment at B will correspond to loadings such as Loading 1, again weighted by the probability of their occurrence.

The requirements of the (Static) theorem of plastic collapse for Span 2 are:

$$M_{RD} - (M_{RB} + M_{RC})/2 \geq q_2 \ell^2/8 \tag{6.102}$$

where M_{RB}, M_{RC} and M_{RD} are the moments of resistance at these points and q_2 is an appropriate uniform loading intensity in Span 2. In accordance with the code specifications, any combination of M_{RB}, M_{RC} and M_{RD} may be selected provided that Ineq. 6.102 and the requirements of ductility and concrete cracking are satisfied. Thus, if the support moments of resistance, M_{RB} and M_{RC}, have been selected, the minimum required value for M_{RD} can be found from Ineq. 6.102. However, an "appropriate" value for q_2 would first have to be determined and, as will be seen this is no simple matter.

6.8.3 Possible Future Approach to Design

The case of plastic collapse of one span of a continuous beam is considered. In order for collapse to occur, the applied moments must equal the moments of resistance at both supports and at the point of maximum sag moment. Thus, the probability of failure is the probability of the applied moments at these three points simultaneously equalling the moments of resistance. The problem is complicated by the fact that the applied moments at one of the points may be a function of the moments of resistance at one or both of the others, depending on the load combination which leads to collapse.

A comprehensive non-linear analysis procedure might be as follows. First, a probabilistic elastic analysis is carried out and the section where elastic yield is most likely to first occur is identified.

Then, further analysis is carried out on the assumption that the applied moment at the point of yield equals the moment of resistance. Two factors must be kept in mind at this stage. First, for elastic yield to occur at any point, is an unlikely event because in order for this to happen, the moment implied by elastic analysis must be greater than or equal to 80% of its extreme value (assuming 20% maximum redistribution). Thus, qualitatively, part of the desired safety factor has been attained and it will not be necessary, in the subsequent stages of the analysis, to strive for a safety level of $\beta = 3$, (where β is the number of standard deviations from the mean). The second factor to be considered at this stage is that the original statistical parameters (mean and standard deviation) for loading intensity, are no longer valid. This follows from the fact that the second stage of analysis is based on the premise that elastic failure has occurred. In order for this to happen, the moment, as implied by elastic analysis, had to be large. If moment is positively correlated with loading intensity then, the fact that moment is known to be large, increases the expected value (mean) of the loading. In fact, the expected value of each loading intensity is the conditional mean given that a linear function of loading intensities exceeds a given parameter. Similarly, the variance is the conditional variance given this situation. Referring to the example of Fig. 1.1, if it is assumed that the moment implied by elastic analysis at B, M_{EB}, equals the moment of resistance., M_{RB}, then the expected value of, q_1, is:

$$E\{q_1 \mid M_{EB}(q_1, q_2) = M_{RB}\} = \int q_1 \, f(q_1 \mid M_{RB}) \, dq_1 \qquad (6.103)$$

where, $f(q_1 \mid M_{RB})$, is the probability density function of q_1, given that M_{EB} equals M_{RB}. This can be found by dividing the joint probability density function of q_1 and M_{EB} by the probability density function of M_{EB}. The mean of q_1, given that, $M_{EB}(q_1, q_2) \geqq M_{RB}$, can be found by integrating the expression of Eqn. 6.103 with respect to M_{EB} in the interval between M_{RB} and infinity.

If the conditional mean and standard deviation could be determined given that plastic hinge rotation was occurring at one point, another linear analysis could be carried out to determine where the next plastic hinge was most likely to occur and the process repeated until sufficient hinges were identified for plastic collapse. This procedure neglects the possibility of hinges occurring in any other order. At any rate, it is clearly too complex for design office use.

A heuristic procedure is now introduced which is considerably more suitable for design office use. As before, a probabilistic elastic analysis is carried out to determine where the first plastic hinge is most likely to occur. Given that this hinge has occurred, the conditional means and standard deviations are estimated on the basis of the correlation between the loading intensities and the elastic moment at the point of failure. This correlation between loading, q and moment, M is reflected in the correlation coefficient, r_{qM}, which can readily be calculated given the moment-loading relationship. Clearly, if correlation between moment and loading intensity is large and positive, the expected value of the intensity given that the moment is high, will be larger than before. On the other hand, if the correlation is low, the expected value will be largely unchanged. This logic is reflected in Eq. 6.104:

$$E\{q \mid M_E = M_R\} = \bar{q} + \frac{r_{qM}\,\sigma_q\,(M_R - \bar{M}_E)}{\sigma_{ME}} \qquad (6.104)$$

where $\bar{q}$ and σ_q are the (original) mean and standard deviation for q and $\bar{M}_E$ and σ_{ME} are the mean and standard deviation for M_E. It can be seen that deviation of the conditional mean loading intensity from its original mean value depends on the deviation of the elastic moment from its mean and on the relative standard deviations of q and M. This equation is, in fact, the exact relationship for the case when q and M are joint-Normally distributed. The corresponding expression for standard deviation is,

$$\sigma\{q \mid M_E = M_R\} = \sigma_q(1 - r_{qn}^2)^{\frac{1}{2}} \tag{6.105}$$

This reflects the fact that, if q if highly correlated with M_E, and M_E is constrained to equal a fixed value, then the standard deviation of q is small.

In the plastic collapse problem, of course, the elastic moment at the point of initial yield is required to exceed rather than to equal the moment of resistance. Rather than derive expressions for means and standard deviations conditional on moment exceeding given values and to derive expressions to allow for the portion of the "safety factor" that has already been attained, it is proposed here to cater for both complexities at once. Even when the elastic moment at a point, M_E, has reached or exceeds the moment of resistance, there remains a certain statistical variability in this moment, i.e. it has a non-zero conditional standard deviation. If load intensity, q, is highly correlated with M_E, then the conditional standard deviation of q will approach a level consistent with that of M_E. While the conditional variance of M_E is not known, the level of M_E corresponding to an acceptable level of safety is known, namely, $\bar{M}_E + 3\sigma_M$. It is proposed to select a "factored deviation" of q, s_q, corresponding to, $\beta\sigma_q$, that would imply the desired level of safety at the yield point. This is achieved by specifying a factored deviation in M_E, s_M, equal to the acceptable deviation of this moment from M_R, Hence,

$$s_M = (\bar{M}_E + 3\sigma_{ME}) - M_R \tag{6.106}$$

Then, the corresponding acceptable deviation in q will depend on s_M and the correlation between M_E and q:

$$s_q = 3\sigma_q\{1 + r_{qM}^2[(s_M/3\sigma_{ME})^2 - 1]\}^{\frac{1}{2}} \tag{6.107}$$

It can be seen that, if q and M are totally uncorrelated, then the conditional deviation in q is unchanged at $3\sigma_q$. On the other hand, if there is full correlation, the allowable deviation, s_q, becomes $s_M\sigma_q/\sigma_{ME}$. If M_R equals the extreme value for elastic moment, $s_M = 0$, and accordingly, $s_q = 0$. Thus, the expected value of q given that $M_E = M_R$ is adequate for design. If, on the other hand, M_R is, say, one standard deviation from $\bar{M}_E$, then, $s_M = 2\sigma_{ME}$ and the acceptable deviation of q is two standard deviations (i.e., $s_q = 2\sigma_q$).

Example: Example 1 of Section 6.7 is used to illustrate the new procedure. For this beam, the support moment is,

$$M_B = (q_{Ln1} + q_{Ln2})\ A_I\ \ell/32 \tag{6.108}$$

with mean and standard deviation given by,

$$\bar{M}_B = (\bar{q}_{Ln1} + \bar{q}_{Ln2})\ A_I\ \ell/32 \tag{6.109}$$

$$\sigma_{MB} = \sqrt{2}\ \sigma_{Ln1}\ A_I\ \ell/32 \tag{6.110}$$

The covariance of q_{Ln1} and M_B is, by definition, the expected value of their product less the product of their expected values. Hence,

$$\mathrm{cov}(q_{LN1}, M_B) = E\{(q_{Ln1}^2 + q_{Ln1}\, q_{Ln2})\, A_I\, \ell/32\} - \bar{M}_B\, \bar{q}_{Ln1} \tag{6.111}$$

from which it can readily be shown that,

$$\mathrm{cov}(q_{Ln1}, M_B) = \sigma_{Ln1}^2\, A_I\, \ell/32 \tag{6.112}$$

The correlation co-efficient of q_{Ln1} and M_B is the ratio of their covariance to the product of their standard deviations. Thus,

$$r(q_{Ln1}, M_B) = 1/\sqrt{2}$$

The extreme value of elastic moment at the support, is,

$$\bar{M}_B + 3\sigma_{MB} = 4.85\, A_I\, \ell/32$$

If the selected moment of resistance is, $4A_I\, \ell/32$, Eqn. 6.107 gives:

$$s_{Ln1} = 3\sigma_{Ln1}(0.92)$$

Thus, the deviation of q_{Ln1} to be used for plastic distribution is slightly less than that used for elastic design. This accounts for the fact that the elastic moment is restricted to a deviation of 0.85 as opposed to a previous deviation of, $3\sigma_{MB} = 1.032$. This is a reduction of 18%. As the moment is only partly dependent on q_1, the allowed deviation in this parameter is reduced by the lower value of 8%. The conditional expected value of q_1 given that support moment equals, $4A_I\, \ell/32$, is, from Eqn. 6.104,

$$E\{q_{Ln1} \mid q_{Ln1} + q_{Ln2} = 4\} = \bar{q}_{Ln1} + \frac{(1/\sqrt{2})\ \sigma_{Ln1}(0.18)}{(\sqrt{2}\ \sigma_{Ln1})} = \bar{q}_{Ln1} + 0.09$$

Thus, the conditional expected loading intensity exceeds the original expected value by one half of the difference between the selected moment of resistance and the expected moment at the support. Knowing the conditional mean and factored deviation, the design loading intensity in Span 1 appropriate to the selected support moment of resistance is,

$$E\{q_{Ln1} \mid q_{Ln1} + q_{Ln2} = 4\} + s_{Ln1}$$

$$= \bar{q}_{Ln1} + 0.09 + 3\sigma_{Ln1}(0.92)$$

$$= 2.67$$

The corresponding maximum sag moment is, 10.6 ℓ. This suggests (see Fig. 6.8) that with this amount of reduction in the elastic support moment (17.5%), little or no increase in the area of bottom steel is required.

This development is presented as one possible approach to probabilistic plastic moment distribution. However, it has not been pursued further for it is felt that the resulting design procedure, despite the many assumptions, tends to be somewhat complex. It is included here as a possible starting point for future researchers. It should be noted that additional equations are required for interior spans of continuous beams in which three plastic hinges must occur before collapse.

6.8.4 Proposed Design Procedure

Clearly, there are many cases in which different deterministic loading conditions will be critical for support moment and for maximum moment in the interior of a span (see, for example, Span 1 in Fig. 6.10). In such cases, it is possible that some plastic moment reduction of the probabilistic design moment at the support could be safely carried out without increasing the design moment for sag in the span.

However, there are also some examples (such as Span 2 in Fig. 6.10) in which very little reduction can be made in the design moment at the support without simultaneously increasing the design moment for sag. As the probabilistic procedure tends to attach less importance to the extreme loading situations used in the traditional approach, it is likely that, in general, there is less scope for redistribution than in the traditional deterministic approach. It is recommended here that *any reduction in an elastic probabilistic design moment, must be accompanied by a linear shift in the complete design envelope*. Thus, any reduction in a support design moment must be accompanied by a corresponding increase in design moments for sag and vice versa. Clearly, this approach is conservative. It is emphasized that additional savings may be available in a more comprehensive treatment of this problem. However, the design procedure using the rule described above is conveniently simple and has been found to result in savings over the traditional method for examples considered.

The problem of optimum plastic moment redistribution now degenerates to one of selecting the design moment at each support. High values will imply savings in bottom steel but increases in area and length of top steel. On the other hand, values below the elastic will result in steel area savings on top but increases in the bottom. Clearly, a trade-off will exist. As only one variable exists for each beam support, the optimization problem becomes conveniently simple. In view of the serial nature of the problem and the fact that steel area is a discrete variable (with a very small number of possible values for any given elastic solution), the optimization problem is particularly well suited for dynamic programming.

6.9 CONCLUSIONS

A probabilistic approach to the design of beams is developed which is simple enough for design office use but provides more consistent results than the multiple loadings approach. Use of the method requires knowledge of only the most basic of statistical principles and is hardly more complex than the traditional deterministic procedure. The results of the method are compared to the effects of "actual" live loading as indicated by the more complex statistical models available. Comparison shows the statistical model to give excellent agreement while the results of the traditional approach are relatively inconsistent. A special correction procedure is applicable for shear design to ensure consistent accuracy. This is required as no system of uniformly loaded spans can accurately reflect the shear distribution due to non-uniform loadings.

Statistical parameters for the live loading model are adopted from the literature. These indicate that the current load/area reduction formula specified in the National Building Code (1980) gives inconsistent results and that the allowance for using no such relationship for tributary areas of less than 20 m^2, is especially so. The reduction formula appears to be particularly inadequate for the case of multi-span beams where the deviations from the results of the probabilistic model are relatively large.

Examples demonstrate the use of the probabilistic procedure to derive bending moment and shear force envelopes. An example also indicates how the probabilistic approach can be readily extended to include dead loading. While only a limited number of examples were considered, the traditional multiple loadings approach appears to be conservative everywhere but particularly so for maximum sag moment.

The probabilistic equivalent to plastic moment distribution is explored and a convenient approximation proposed. The optimum probabilistic design process with plastic moment distribution is described and it is pointed out that the probabilistic method is particularly well suited for optimum design.

CHAPTER 7

CONCLUSIONS AND RECOMMENDATIONS

7.1 SUMMARY AND CONCLUSIONS

7.1.1 Design of Reinforcement

The optimum design of reinforced concrete continuous beams has been considered. A major portion of the optimum design problem consists of the design of reinforcement which is the subject of Chapter 4. The design of reinforcement involves those calculations which are required after specification of the concrete geometry and the stress envelopes. Detailed descriptions are given of algorithms for the calculation of the areas, lengths and locations of longitudinal reinforcement. The calculations required for the provision of compression reinforcement when required, are also outlined. Similarly, an algorithm is described for the calculation of the areas and spacings of transverse reinforcement for shear.

All of the algorithms described here have been included as subroutines in an optimum design program. The design details are considered in much greater depth in these subroutines than is usually afforded them when the design is being done "by hand". It is felt that considerable savings will ensue from this.

7.1.2 Optimum Deterministic Design

The determination of the design bending moment and shear force diagrams is the subject of Chapters 5 and 6. Two approaches are described, deterministic (Chapter 5) and probabilistic (Chapter 6). The deterministic approach is the traditional method of design for multiple loadings as currently specified in the Canadian code of practice. Accurate optimum design in accordance with these specifications was not previously feasible due to the excessively large amount of computer time required. To overcome this problem, a method of optimization called sequential decomposition (SD) has been developed. A number of variations on the basic SD approach were tested by example and the most efficient form for the beam design problem determined. Application of this form of SD to a typical example resulted in an 83% reduction in computer time. Clearly, the method has considerable potential for large optimization problems of a serial nature.

A further considerable reduction in the computer time required to determine the optimum solution resulted from the degeneration of the optimization subproblems. It has been shown that at the optimum solution, most of the subproblem variables are in fact dependent on two parameters. This reduction in the number of independent variables greatly facilitates the optimization process. The application of SD combined with this subproblem degeneration reduced the time required to solve a typical problem to less than 1% of what was formerly required. This remarkable improvement in the efficiency of the optimization procedure is a striking example of how dramatic the improvements can be when knowledge of structural behaviour is combined with knowledge of optimization for the solution of structural optimization problems.

Using unit costs obtained from a major Canadian construction firm, a small series of beam design examples were run to test the sensitivity of the optimum beam design to a number of problem parameters. This study has indicated that accurate near-optimal solutions can readily be found by the following two-step procedure:

(1) Find the optimum dimensions while allowing no optimization of the quantities of plastic moment redistribution.

(2) With the section dimensions fixed at these values, find the optimum quantities of redistribution.

Also evident from the results is that the optimum designs are particularly sensitive to the applied depth penalty, that is, the cost term used to reflect the additional overall building costs which result from deep beams.

7.1.3 Probabilistic Analysis and Design

A probabilistic design procedure is developed for use in place of the conventional deterministic approach. For elastic analysis, the method is based on simple probabilistic concepts and is conveniently easy to use. For elastic-plastic analysis, the statistical properties of the system are highly complex but a conservative assumption provides a ready solution to this. Using statistical parameters suggested by McGuire and Cornell (1974), the results of design by the new procedure were compared to conventional design results. The conventional approach was found, in general, to be conservative. Even when the simplifying assumption for plastic redistribution were adopted, the results of the new method were less conservative than the conventional procedure. Hence, the probabilistic approach using these statistical parameters, can result in cost savings while providing more consistent levels of safety.

The assumption regarding probabilistic plastic moment redistribution is particularly convenient for optimum design. The problem is suitable for solution by dynamic programming, a procedure which is particularly well suited to discrete problems such as this one.

7.2 RECOMMENDATIONS FOR FUTURE RESEARCHERS

7.2.1 Optimum Deterministic Design

Despite the great quantity of research on structural optimization, there remains much to be done. What follows are some suggestions of what the author feels should and should not be done.

1. The relatively small savings which often result from optimization and the relatively large round-offs which result from discretization of the final result has led the author to believe that efficient discrete programming methods are particularly desirable. However, very few efficient general procedures appear to be available. In addition, it is felt that it would be worthwhile to develop such methods specifically for structural problems. More general methods may be unnecessarily rigorous and are prohibitively inefficient for large problems.
2. There is considerable scope for the incorporation of forms of artificial intelligence into optimum design programs. This subject would appear from the literature to still be in its infancy (Rooney and Smith, 1982; 1983). As relative unit costs tend to change slowly with time, a program could develop a "feel" for what section dimensions are likely to be close to the optimum. A short discrete search could then verify the solution.
3. The success of the method of sequential decomposition would suggest that it could be useful if applied to other structural problems as well. Two avenues of research are possible here. First, work could be done on the improvement of the method, both in terms of efficiency and "robustness". Second, new applications could be sought where the method could be applied to advantage.

7.2.2 Probabilistic Approach to Design

The development of a probabilistic design procedure that is simple enough for design office use is a tremendous breakthrough. It is anticipated that other similar approaches to structural problems will quickly follow. Some developments which would be useful are as follows:

1. Most statistical theory is mathematically complex and, as such, is unsuitable for design. Thus, most design is based on simplistic deterministic approaches. Yet, even the most rudimentary probabilistic analysis is likely to be more rational than a complex deterministic one. It is essential that tools be developed for simple approximate probabilistic analysis. It is felt by the author that the concepts of mean, variance and covariance are more useful for this than failure probabilities. A great deal of work needs to be done on the development of approximate relationships and on the testing of such relationships for accuracy.
2. There is clearly a great problem in the lack of statistical data on live loading. In particular, there appears to be very little information available on extraordinary loading events.
3. For optimum beam design, a detailed study should be made on the problem of plastic moment redistribution. It is necessary to determine how conservative is the suggestion made herein. If considerable savings can be made, a method of design should be developed and provided as an option in the codes of practice.
4. The problem of optimum probabilistic design should be programmed. In view of the suitability of the problem for solution by dynamic programming, the algorithm would be relatively efficient and could properly treat the variables as discrete.

REFERENCES

Allen, D.E., (1975), "Limit States Design - A Probabilistic Study", Canadian Journal of Civil Engineering, Vol. 2, 1975, pp. 36-49.

Andam, K.A. and Knapton, J. (1980), "Optimum Cost Design of Precast Concrete Framed Structures", Engineering Optimization, Vol. 5, 1980, pp. 41-50.

Ang, Alfredo H.S., and Tang, Wilson, H. (1975), "Probability Concepts in Engineering Planning and Design", Vol. I - Basic Principles, Wiley and Sons, 1975.

Arora, J.S. and Haug, E.J. (1978), "Efficient Hybrid Methods of Optimal Structural Design", A.S.C.E., Journal of the Engineering Mechanics Division, June 1978, pp. 663-680.

Arora, J.S., Haug, E.J. Jr. and Rim, K. (1975), "Optimal Design of Plane Frames", A.S.C.E., Journal of the Structural Division, Oct. 1975, pp. 2063-2078.

Ayer, F.N. and Cornell, C.A. (1968), "Grid Moment Maximization by Mathematical Programming", A.S.C.E., Journal of the Structural Division, Feb. 1968, pp. 529-549.

Beaufait, F.W. and Gerlein, M.A. (1979), "Optimum Preliminary Design of Moment Resisting Reinforced Concrete Frames", Conference on Electronic Computation, 7th, Washington University, 1979, pp. 150-167.

Beeby, A.W. (1981), "Are Our Code Provisions for Slabs Safe?", The Structural Engineer, Vol. 59A, No. 11, Nov. 1981, pp. 351-356.

Bellman, R.E. and Zadeh, L.A. (1970), "Decision Making in a Fuzzy Environment", Management Science, Vol. 17, No. 4, Dec. 1970, pp. B141-164.

Birkeland, H.W. (1974), "How to Design Prestressed Concrete Beams of Minimum Cross Section", Journal of The American Concrete Institute, Vol. 71, No. 12, Dec. 1974, pp 634-641.

Bradley, J., Brown, L.H. and Feeney, M. (1974), "Cost Optimization in Relation to Factory Structures", Engineering Optimization, Vol. 1, No. 2, 1974, pp. 125-138.

Brondum-Nielson, T. (1985), "Optimization of Reinforcement in Shells, Folded Plates, Walls and Slabs", Engineering Optimization, Vol. 82, No. 3, May-June, 1985, pp. 304-309.

Brown, C.B. (1960), "Concepts of Structural Safety", A.S.C.E. Proceedings, Vol. 86, Dec. 1960, pp. 39-57.

Brown, R.H. (1975), "Minimum Cost Selection of One-Way Slab Thickness", A.S.C.E., Journal of the Structural Division, Dec. 1975, pp. 2585-2589.

Canadian Portland Cement Association (1985), "Metric Design Handbook for Reinforced Concrete Elements", Draft 1985 Edition, C.P.C.A., Royal Trust Building, 116 Albert Street, Ottawa, Ont., K1P 5G3.

Canadian Standards Association (1984), "Design of Concrete Structures for Buildings", CAN3-A23.3-M84, 1984.

Carmichael, D.G. (1980), "Computation of Pareto Optima in Structural Design", International Journal for Numerical Methods in Engineering, Vol. 15, 1980, pp. 925-952.

Cauvin, A. (1979), "Nonlinear Elastic Design and Optimization of Reinforced Concrete Frames", CSCE-ASCE-ACI-CEB International Symposium, Nonlinear Design of Concrete Structures, University of Waterloo, 1979.

Chalk, P.L. and Corotis, R.B. (1980), "Probability Model for Design Live Loads", A.S.C.E., Journal of the Structural Division, Oct. 1980, pp. 2017-2034.

Chou, T. (1977), "Optimum Reinforced Concrete T-Beam Sections", A.S.C.E., Journal of the Structural Division, Aug. 1977, pp. 1605-1617.

Chuang, P-H. and Munro, J. (1983), "Linear Programming with Imprecise Data", Civil Engineering Systems, Sept. 1983, pp. 37-41.

Cohn, M.Z. and MacRae, R.J. (1984), "Optimization of Structural Concrete Beams", Journal of Structural Engineering, A.S.C.E., July 1984, pp. 1573-1588.

Cornell, C.A. (1969), "A Probability Based Structural Code", Proceedings, American Concrete Institute, Vol. 66, No. 12, Dec. 1969, pp. 974-985.

Corotis, R.B. and Tsay, W-Y. (1983), "Probabilistic Load Duration Model for Live Loads", Journal of Structural Engineering, A.S.C.E., April 1983, pp. 859-874.

Culver, C.G. (1976), "Live-Load Survey Results for Office Buildings", A.S.C.E., Journal of the Structural Division, Dec. 1976, pp. 2269-2284.

Dilger, W.H. (1981), Comments on "Shear and Torsion Design of Prestressed and Non-Prestressed Concrete Beams", by M.P. Collins and D. Mitchell, Prestressed Concrete Institute Journal, Nov.-Dec. 1981, pp. 97-98.

Dlesk, D.C. and Liebman, J.S. (1983), "Multiple Objective Engineering Design", Engineering Optimization, Vol. 6, No. 3, March 1983, pp. 161-175.

Douty, R.T. (1976), "Structural Design by Conversational Solution to the Nonlinear Programming Problem", Computers and Structures, Vol. 6, No. 4-5, 1976, pp. 325-331.

Dunham, J.W. (1947), "Design Live Loads in Buildings", Transactions, A.S.C.E., Paper No. 2311, Vol. 112, 1947, pp. 725-744.

Ellingwood, B. and Culver, C. (1977), "Analysis of Live Loads in Office Buildings", A.S.C.E., Journal of the Structural Division, Aug. 1977, pp. 1551-1560.

Ellingwood, B., McGregor, J.G., Galambos, T.V. and Cornell, C.A. (1982), "Probability Based Load Criteria: Load Factors and Load Combinations", A.S.C.E., Journal of the Structural Division, May 1982, pp. 978-997.

Elms, D.G. (1982), "Fuzzy Set Determination of Code Criteria", Proceedings of the Eighth Australasian Conference on the Mechanics of Structures and Materials, The University of Newcastle, New South Wales, Aug. 1982.

Fairweather, V. (1985), "Building in Space", Civil Engineering, June 1985, pp. 66-69.

Firmin, A., Gilmor, M.I, and Collins, R.A. (1972), "Computer Programs for Structural Analysis and Design - A Canadian View", Canadian Journal of Civil Engineering, Vol. 4, 1972, pp. 72-85.

Friel, L.L.. (1974), "Optimum Singly Reinforced Sections", Journal of the American Concrete Institute, Vol. 71, No. 11, Nov. 1974, pp. 556-558.

Frind, E.O. and Wright, P.M. (1975), "Gradient Methods in Optimum Structural Design", A.S.C.E., Journal of the Structural Division, April 1975, pp. 939-956.

Galambos, T.V., Ellingwood, B., McGregor, J.G. and Cornell, C.A. (1982), "Probability Based Load Criteria: Assessment of Current Design Practice", A.S.C.E., Journal of the Structural Division, May 1982, pp. 959-977.

Gallagher, R.H. and Zienkiewicz, D.C. (1973), "Optimum Structural Design, Theory and Applications", 1973, John Wiley and Sons.

Gellatly, R.A. and Berke, L. (1971), "Optimal Structural Design", Air Force Flight Dynamics Lab (FBR), Wright-Patterson AFB, Ohio, 45433, April 1971.

Gallatly, R.A. and Berke, L. (1973), "Optimality-Criterion-Based Algorithms", Chapter 4, Gallagher, R.H. and Zienkiewicz, O.C., "Optimum Structural Design, Theory and Applications", 1973, John Wiley and Sons.

Gerlein, M.A. and Beaufait, F.W. (1980), "Optimum Preliminary Strength Design of Reinforced Concrete Frames", Computers and Structures, Vol. 11, 1980, pp. 515-524.

Ghali, A. and Neville, A.M. (1978), "Structural Analysis, A Unified Classical and Matrix Approach", 2nd Edition, 1978, Chapman and Hall.

Gill, P.E., Murray, W. and Wright, M.H. (1981), "Practical Optimization", Academic Press, A Subsidiary of Harcourt Brace Jovanovich, Publishers, 1981.

Goble, G.G. and Moses, F. (1975), "Practical Applications of Structural Optimization", A.S.C.E., Journal of the Structural Division, Apr. 1975, pp. 635-648.

Gorzynski, J.W. and Thornton, W.A. (1975), "Variable Energy Ratio Method for Structural Design", A.S.C.E., Journal of the Structural Division, April 1975, pp. 975-990.

Gumbel, E.J. (1958), "Statistics of Extremes", Columbia University Press, New York and London, 1958.

Harris, M.E., Corotis, R.B. and Bova, C.J. (1981), "Area-Dependent Processes for Structural Live Loads", A.S.C.E., Journal of the Structural Division, May 1981, pp. 857-872.

Haftka, R.T. and Kamat, M.P., "Elements of Structural Optimization", Kluwer Academic Publishers, MTP Press Ltd., Falcon House, Queen Square, Lancaster LA1 1RN, U.K.

Hakkala, L. and Hirvonen, J. (1978), "Co-ordination Strategies for the Interaction Prediction Principle Using Gradient and Multiplier Methods", International Journal of Systems Science, Vol. 9, No. 10, 1978, pp. 1179-1195.

Hill, L.A. Jr., (1966), "Automated Optimum Cost Building Design", A.S.C.E., Journal of the Structural Division, Dec. 1966, pp. 247-263.

Higgins and Hollington (1973), "Designed and Detailed (CP110:1972)", Cement and Concrete Association, 52 Grosvenor Gardens, London SW1 0AQ, U.K., 1973.

Horne, M.R. (1951), "The Variation of Mean Floor Loads with Area", Engineering, Vol. 171, Feb. 16, 1951, pp. 179-182.

Imai, K. and Schmit, L.A. Jr. (1981), "Configurational Optimization of Trusses", A.S.C.E., Journal of the Structural Division, May 1981, pp. 745-756.

IMSL (1982), Library of Computer Programs, Edition 9, IMSL, Customer Relations, Sixth Floor, NBC Building, 7500 Bellaire Blvd., Houston, Texas 77036-5085, U.S.A.

Johnson, R.B. (1984), "Concrete Column Cost Optimization", Concrete Technology Today, Portland Cement Association, Vol. 5, No. 3, Sept. 1984.

Kaliszky, S. (1965), "Economic Design by Ultimate Load Method I, II, and III", Concrete and Construction Engineering, Vol. 60, No. 10, Oct. 1965, pp. 365-372, No. 11, Nov. 1965, pp. 424-433, No. 12, Dec. 1965, pp. 462-467.

Kaneko, I. and Ha, C.D. (1983), "Decomposition Procedure for Large Scale Optimum Plastic Design Problems", International Journal for Numerical Methods in Engineering, Vol. 19, 1983, pp. 873-889.

Khalil, A.B.I. (1977), "Minimum Cost Design of Concrete Slabs", Thesis, Department of Civil Engineering, The University of Calgary, 1977.

Khot, N.S. and Berke, L. (1981), "Achievements in Structural Optimization Using Optimality Criteria Methods", Proceedings of the International Symposium on Optimum Structural Design, The 11th ONR Naval Structural Mechanics Symposium, Oct. 1981, University of Arizona, Tucson, Arizona, pp. 1.1-1.3.

Kirsch, U. (1975), "Multilevel Approach to Optimum Structural Design", A.S.C.E., Journal of the Structural Division, April 1975, pp. 957-974.

Kirsch, U. (1981), "Optimum Structural Design", McGraw Hill, 1981.

Kirsch, U. (1983), "Multilevel Optimal Design of Reinforced Concrete Structures", Engineering Optimization, Vol. 6, No. 4, July 1983, pp. 207-212.

Kirsch, U. and Moses, F, (1977), "Optimization of Structures with Control Forces and Displacements", Engineering Optimization, Vol. 3, No. 1, 1977, pp. 37-44.

Kirsch, U. and Moses, F. (1979), "Decomposition in Optimum Structural Design", A.S.C.E., Journal of the Structural Division, Jan. 1979, pp. 85-100.

Kirsch, U., Reiss, M. and Shamir, U. (1972), "Optimum Design by Partitioning into Substructures", A.S.C.E., Journal of the Structural Division, Jan. 1972, pp. 249-267.

Koo, D. (1977), "Elements of Optimization", Springer, 1977.

Lasdon, L.S. (1968), "Duality and Decomposition in Mathematical Programming", IEEE Transactions on Systems, Science and Cybernetics Group, Vol. SSC-4, No. 2, July 1968, pp. 86-100.

Lasdon, L.S. (1970), "Optimization Theory for Large Systems", The MacMillan Co., London, 1970.

Lev, O.E. (1978), "Optimum Truss Geometry for Two Loading Conditions", A.S.C.E., Journal of the Structural Division, Aug. 1978, pp. 1203-1210.

Lipson, S.L., Gwin, L.B. (1977), "Discrete Sizing of Trusses for Optimal Geometry", A.S.C.E., Journal of the Structural Division, May 1977, pp. 1031-1046.

Loov, R.E. (1984), "Optimum Lengths of Link Beams for Cantilevered Roof Construction", 2nd Canadian Seminar on Systems Theory for the Civil Engineer, May 17-18, 1984, Dept. of Civil Engineering, University of Calgary.

Loov, R.E. and Bhatia, S. (1978), "Minimum Cost Design of Reinforced Concrete Columns", Paper presented at a seminar organized by Compa International Pte. Ltd., Singapore, 1978, pp. 25-27.

Loov, R.E. and Khalil, A.B. (1980), "Optimum Design of Reinforced Concrete and Prestressed Concrete Slabs", Advances in Concrete Slab Technology, Pergamon Press, Oxford and New York, 1980.

Luenberger, D.G. (1973), "Introduction to Linear and Non-Linear Programming", Addison Wesley, 1973.

McAdam, P.S. (1983), "Designing to Aid Construction, Optimizing Formwork", Eleventh Biennial Conference, Concrete Institute of Australia, Sydney, 1983.

MacGregor, J.G. (1976), "Safety and Limit States Design for Reinforced Concrete", Canadian Journal of Civil Engineering, Vol. 3, No. 4, Dec. 1976, pp. 484-513.

McGuire, R.K. and Cornell, C.A. (1974), "Live Load Effects in Office Buildings", A.S.C.E., Journal of the Structural Division, July 1974, pp. 1351-1366.

McNeely, D.K., Sneep, J.S. and Smith, K.N. (1985), "Approximate Analysis and Optimization of Stiffened Tied Arch Bridges", Canadian Journal of Civil Engineering, Vol. 12, No. 2, June 1985, pp. 307-315.

Majid, K.I. (1974), "Optimum Design of Structures", A Halsted Press Book, Wiley, 1974.

Mesarovic, M.D., Macko, D. and Takahara, Y. (1969), "Two Coordination Principles and Their Application in Large Scale Systems Control", Proceedings, IFAC Congress, Warsaw, Poland, 1969.

Mitchell, G.R. and Woodgate, R.W. (1971), "Floor Loadings in Office Buildings - The Results of a Survey", Building Research Station, Current Paper 3/71, Jan. 1971.

Morris, D. (1978), "Prestressed Concrete Design by Linear Programming", A.S.C.E., Journal of the Structural Division, March 1978, pp. 439-452.

Munro, J. (1984), "Some Applications of Fuzzy Programming to Civil Engineering", 2nd Canadian Seminar on Systems Theory for the Civil Engineer, May 17-18, 1984, Department of Civil Engineering, The University of Calgary.

Naamen, A.E. (1976), "Minimum Cost Versus Minimum Weight of Prestressed Slabs", A.S.C.E., Journal of the Structural Division, July 1976, pp. 1493-1505.

Naamen, A.E. (1982), "Optimum Design of Prestressed Concrete Tension Members", A.S.C.E., Journal of the Structural Division, Aug. 1982, pp. 1722-1738.

National Research Council of Canada (1980), "National Building Code of Canada", 1980, NRCC No. 17303.

O'Brien, E.J. and O'Keeffe, J.D. (1985), "Optimal Plastic Design of Multi-Bay Single-Storey Frames", Accepted for publication in Institution of Civil Engineers, Proceedings, Part 2, Research and Theory, 1985.

Osyczka, A. (1984), "Multicriterion Optimization in Engineering", Ellis Horwood, Chichester, 1984.

Peir, J.C. and Cornell, C.A. (1973), "Spatial and Temporal Variability of Live Loads", A.S.C.E., Journal of the Structural Division, May 1973, pp. 903-922.

Powell, M.J.D. (1964), "An Efficient Method of Finding the Minimum of a Function of Several Variables Without Calculating Derivatives", Computer Journal, Vol. 7, 1964, pp. 155-162.

Raj, P.P. and Durrant, S.O. (1976), "Optimum Structural Design by Dynamic Programming", A.S.C.E., Journal of the Structural Division, Aug. 1976, pp. 1575-1589.

Rao, S.S. (1978), "Optmization Theory and Applications", Wiley Eastern Limited, New Delhi, India, 1978.

Reinschmidt, K.F. and Norabhoompipat, T. (1975), "Structural Optimization by Equilibrium Linear Programming", A.S.C.E., Journal of the Structural Division, April 1975, pp. 921-938.

Rooney, M.J. and Smith, S.E. (1982), "Artificial Intelligence in Simple Beam Design", A.S.C.E., Journal of the Structural Division, Oct. 1982, pp. 2344-2348.

Rooney, M.F. and Smith, S.W. (1983), "Artificial Intelligence in Engineering Design", Computers and Structures, Vol. 16, No. 1-4, 1983, pp. 279-288.

Rozvany, G.I.N. (1964), "Optimum Synthesis of Prestressed Structures", A.S.C.E., Journal of the Structural Division, Dec. 1964, pp. 189-211.

Russell, A.D. and Choudhary, K.T. (1980), "Cost Optimization of Buildings", A.S.C.E., Journal of the Structural Division, Jan. 1980, pp. 238-300.

Salinas, J.J. (1974), "Minimum Cost Design of Concrete Beams", Thesis, Department of Civil Engineering, The University of Calgary, 1974.

Salinas, J.J. (1980), "Simplified Design of Reinforced Concrete Structural Elements for Minimum Cost", CSCE Annual Conference, Winnipeg, Manitoba, May 29-30, 1980, Published by University of Manitoba, 1980, Paper S16, 10 p.

Sargious, M. and Badawy, H. (1976), Mathematical Modelling for Optimizing Bridge Costs", Canadian Journal of Civil Engineering, Vol. 3, No. 3, Sept. 1976, pp. 461-466.

Schmit, L.A. (1969), "Symposium on Structural Optimization", Proceedings No. 36, Advisory Group for Aerospace Research and Development Conference, Structural Engineering Applications of Mathematical Programming Tecniques, 1969.

Schmit, L.A. (1981), "Structural Optimization, Some Key Ideas and Insights", Engineering Optimization, Oct. 1981, pp. 1-7.

Soyster, A.L. (1973), "Convec Programming with Set-Inclusive Constraints and Applications to Inexact Linear Programming", Operations Research, Vol. 21, 1973, pp. 1154-1157.

Spendley, W., Hext, G.R. and Himsworth, F.R. (1962), "Sequential Application of Simplex Designs in Optimization and Evolutionary Operation", Technometrics, Vol. 4, No. 4, Nov. 1962, pp. 441-461.

Spillers, W.R. and Kountouris, G.E. (1980), "Geometric Optimization Using Simple Code Representation", A.S.C.E., Journal of the Structural Division, May 1980, pp. 959-971.

Spunt, L. (1971), "Optimum Structural Design", Prentice-Hall Inc., 1971.

Stelzer, J.F. (1981), "Two Applications of Optimum Design in the Field of Nuclear Technique", Engineering Optimization, Oct. 1981, pp. 1.23-1.30.

Surtees, J.O. and Tordoff, D. (1977), "Optimum Design of Composite Box Girder Bridge Structures", Institution of Civil Engineers, Vol. 63, Mar. 1977, pp. 181-194.

Templeman, A.B. (1983), "Optimization Methods in Structural Design Practice", Journal of Structural Engineering, A.S.C.E., Oct. 1983, pp. 2420-2432.

Timleck, A.J. (1972), "The Optimum Design of Rectangular Reinforced Concrete Beams and Slabs", Thesis, Department of Civil Engineering, The University of Calgary, 1972.

Twisdale, L.A. and Khachaturian, N. (1975), "Multistage Optimization of Structures", A.S.C.E., Journal of the Structural Division, May 1975, pp. 1005-1020.

Turkstra, C. and Daly, M.J. (1978), "Two Moment Structural Safety Analysis", Canadian Journal of Civil Engineering, 1978, pp. 414-426.

Woolson, I.H., (1923), "A Study of Office-Building Live-Loads", Engineering New-Record, Vol. 90, No. 13, March 1923, p. 584.

Wright, P.M. and Feng, C.C. (1971), "Optimization Design of Plane Frames Using a Multi-Mode Scheme", Transactions of the Engineering Institute of Canada, Vol. 14, No. A-5, April 1971, pp. I-VI.

Zadeh, L.A. (1965), "Fuzzy Sets", Information and Control, 8, 1965, pp. 338-353.

Zahradnik, R.L. (1971), "Theory and Techniques of Optimization for Practising Engineers", Barnes and Noble, New York, 1971.

Zimmermann, H-J. (1978), "Fuzzy Programming and Linear Programming with Several Objective Functions", Fuzzy Sets and Systems, Vol. 1, Issue 1, 1978, pp. 45-55.

Zimmermann, H-J. (1976), "Description and Optimization of Fuzzy Systems", International Journal of General Systems, Vol. 2, 1976, pp. 209-215.

Note 1: The cost information was obtained with the assistance of a professional engineer employed by a major Canadian construction firm. They were based on recent (Aug. 1985) tenders made by that firm. It is the policy of this company not to make cost information available to their competitors and their anonymity has been respected here.

Appendix A - Description of Program BOD

BOD (Beam Optimization Design) is a program for the minimum cost design of continuous beams. The logic used in the program is described here starting with the subroutines at the most basic levels of logic and working up to the main program. At one of the most basic levels of calculation is the subroutine CONANA (CONstrained ANAlysis). In this subroutine (see flowchart, Fig. A.1), the areas, locations and extents of steel for a one- or two-span substructure is calculated. The section geometries have been specified a priori. The special degeneration of subproblems described in Subsection 5.3.3 is employed for the calculation of the support moments in terms of the design moments for sag. The subroutine, JCRIT, is called for the determination of the most critical loadings for the steel areas and cut-off points (see flowchart, Fig. A.2). The subroutine AREAS (Fig. A.3) is called for the calculation of the areas of steel at the critical sections. These calculations are as outlined in Section 4.2. As described in Sections 4.3 - 4.5, the locations of all cut-off points are calculated in subroutine EXTENT (Figs. A.4 and A.5). In subroutine COMP (Fig. A.6), the extents of the bottom steels meeting at the supports are increased to provide compression reinforcement if this is required for ductility. These calculations are outlined in Section 5.6. Finally, subroutine SCOST is called for the determination of the total steel cost and the cost penalties for constraint violations. It is in this subroutine also that the quantities of transverse reinforcement are calculated.

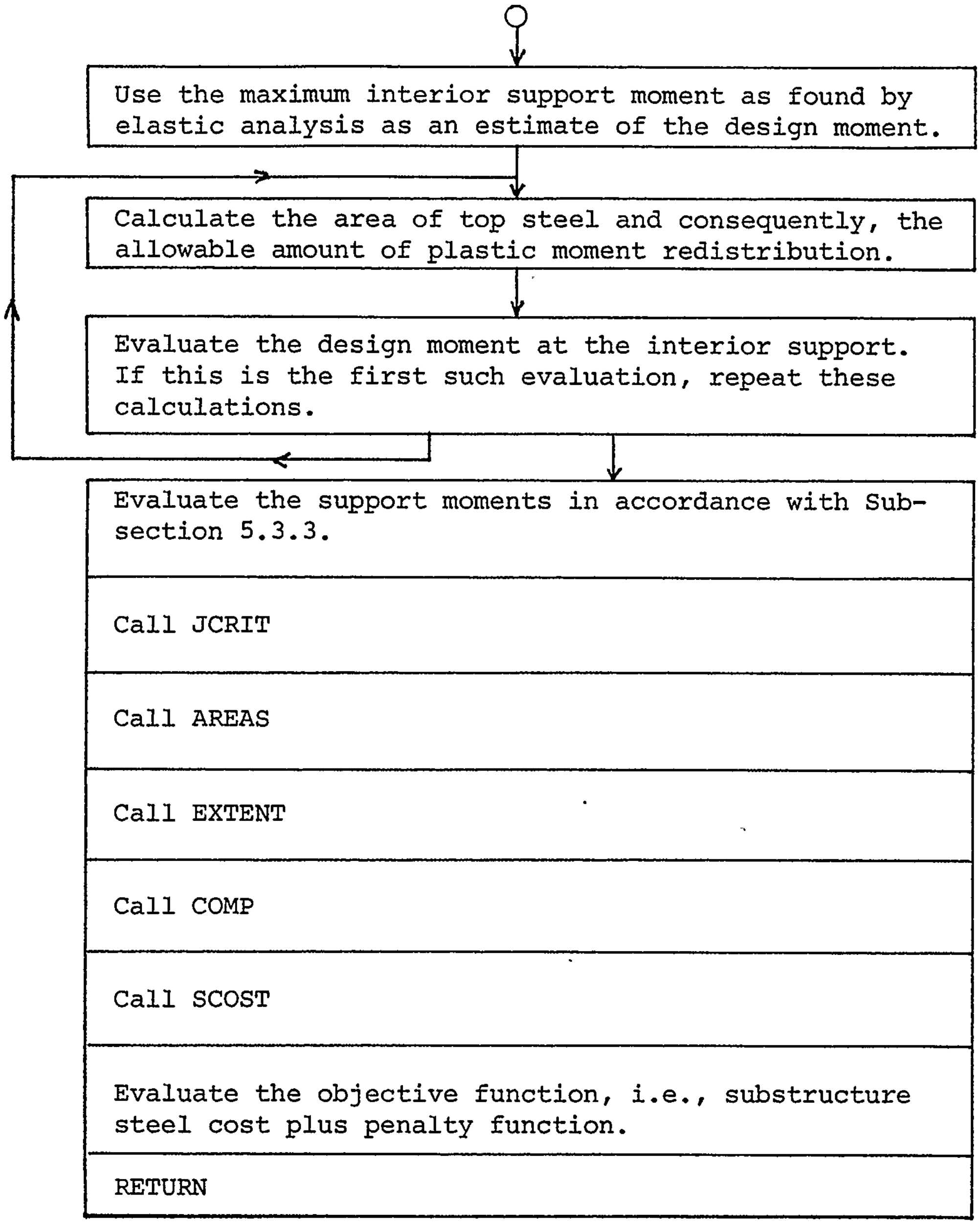


Fig. A.1 - Flowchart for the Subroutine CONANA

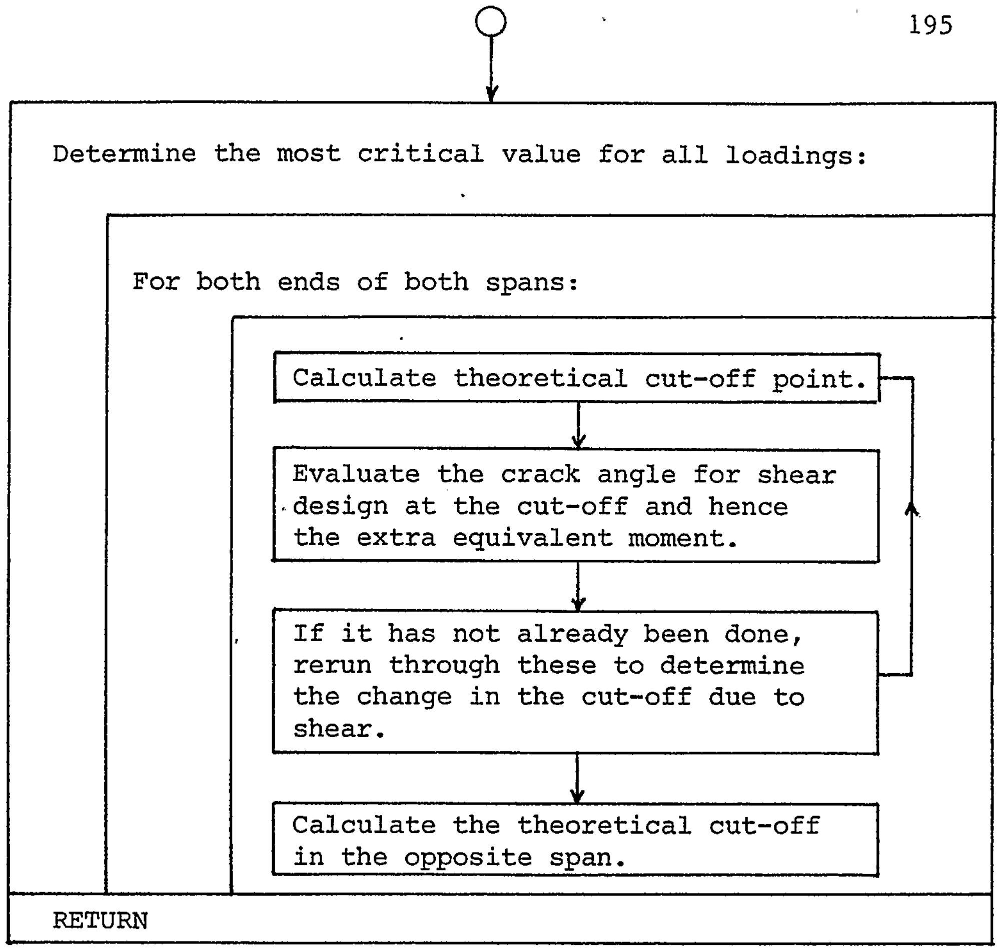


Fig. A.2. - Flowchart for the Subroutine JCRIT

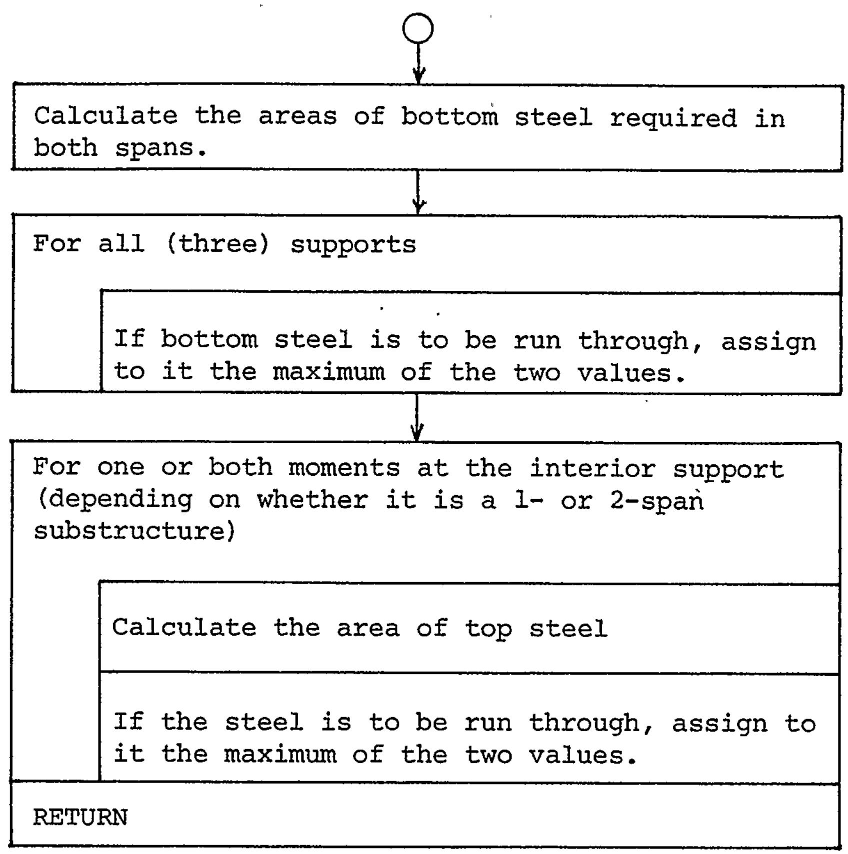


Fig. A.3. - Flowchart for the Subroutine AREAS

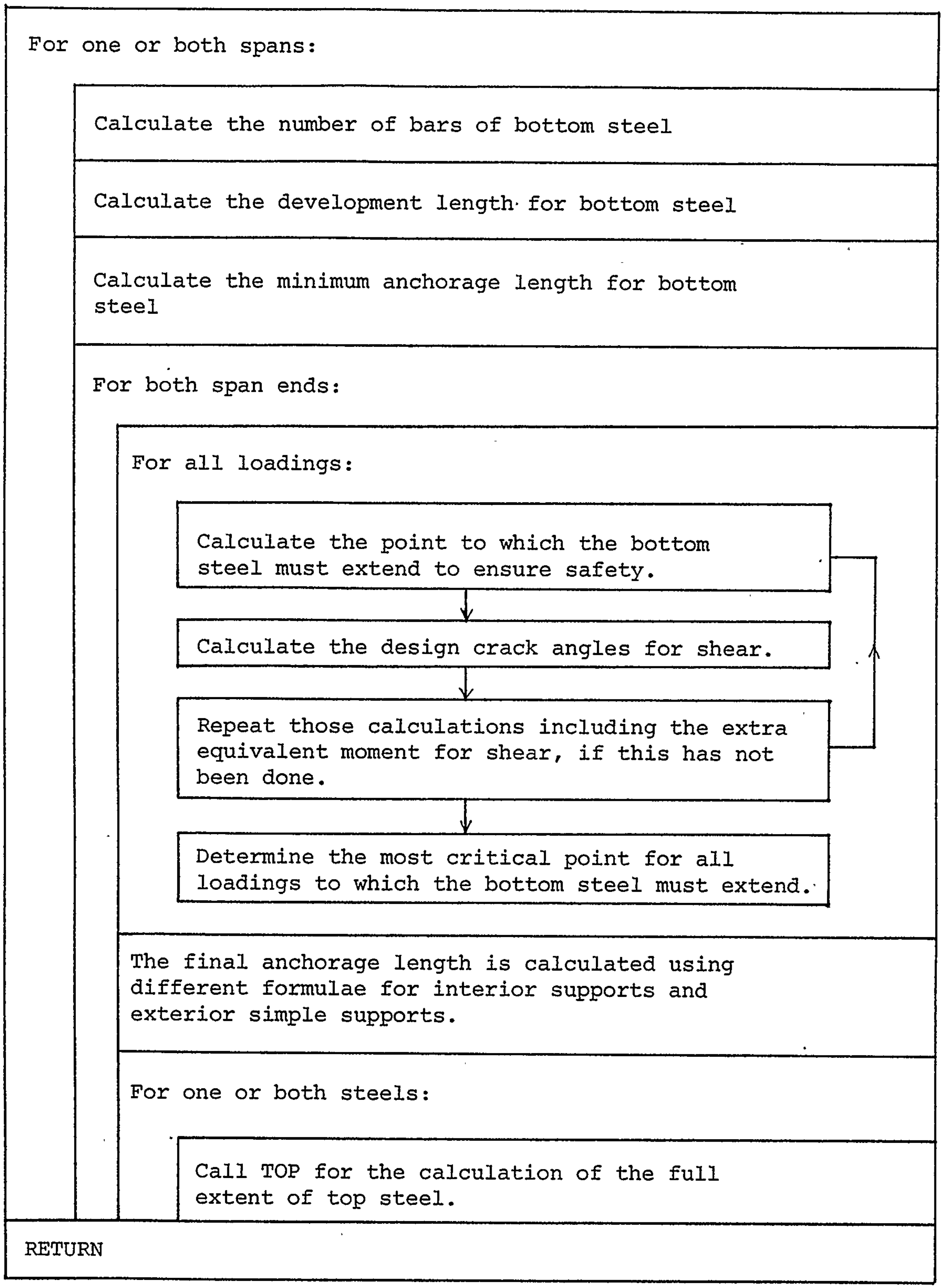


Fig. A.4. - Flowchart for the Subroutine EXTENT

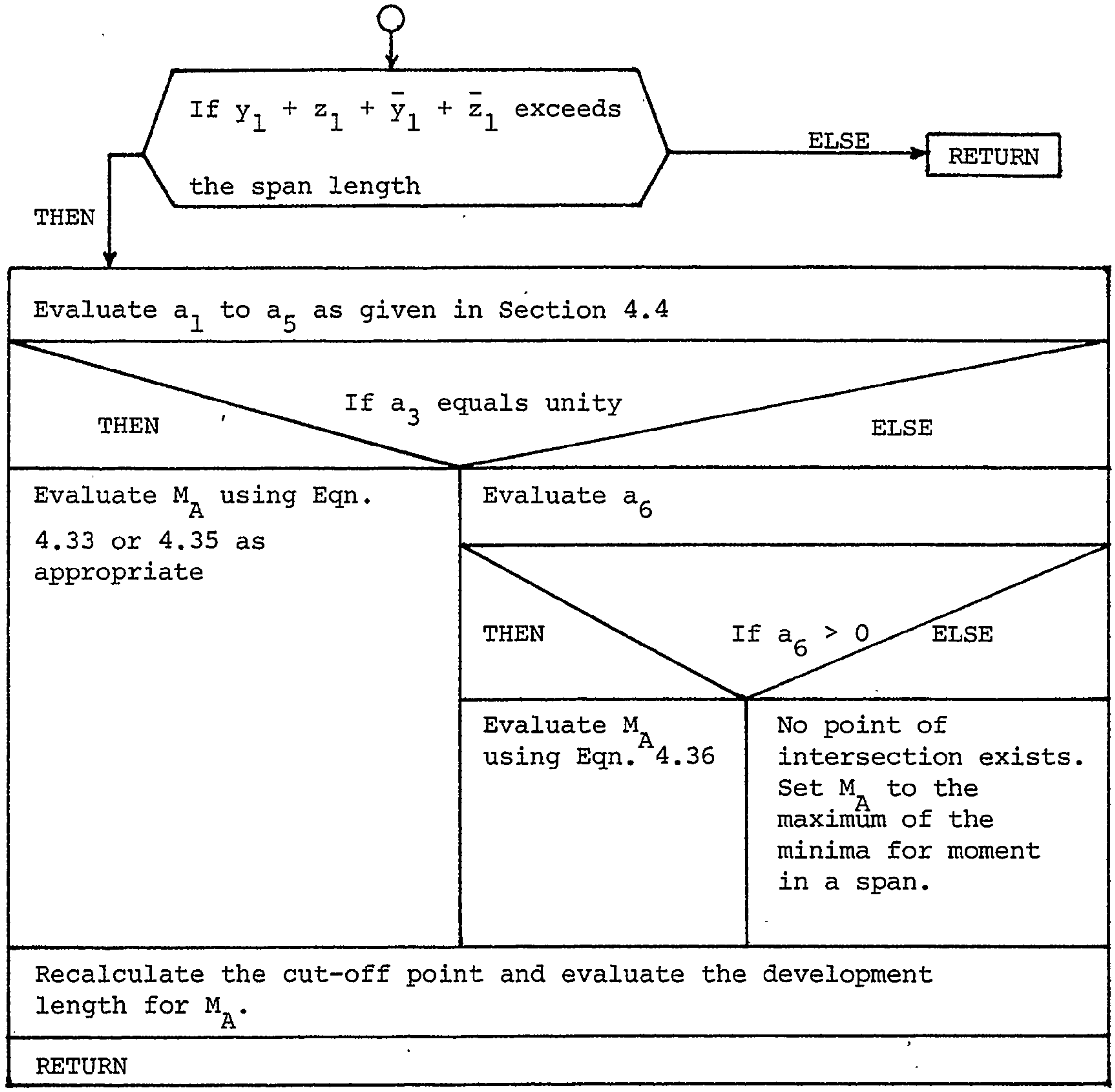


Fig. A.5. - Flowchart for TOP, a Subprogram of EXTENT

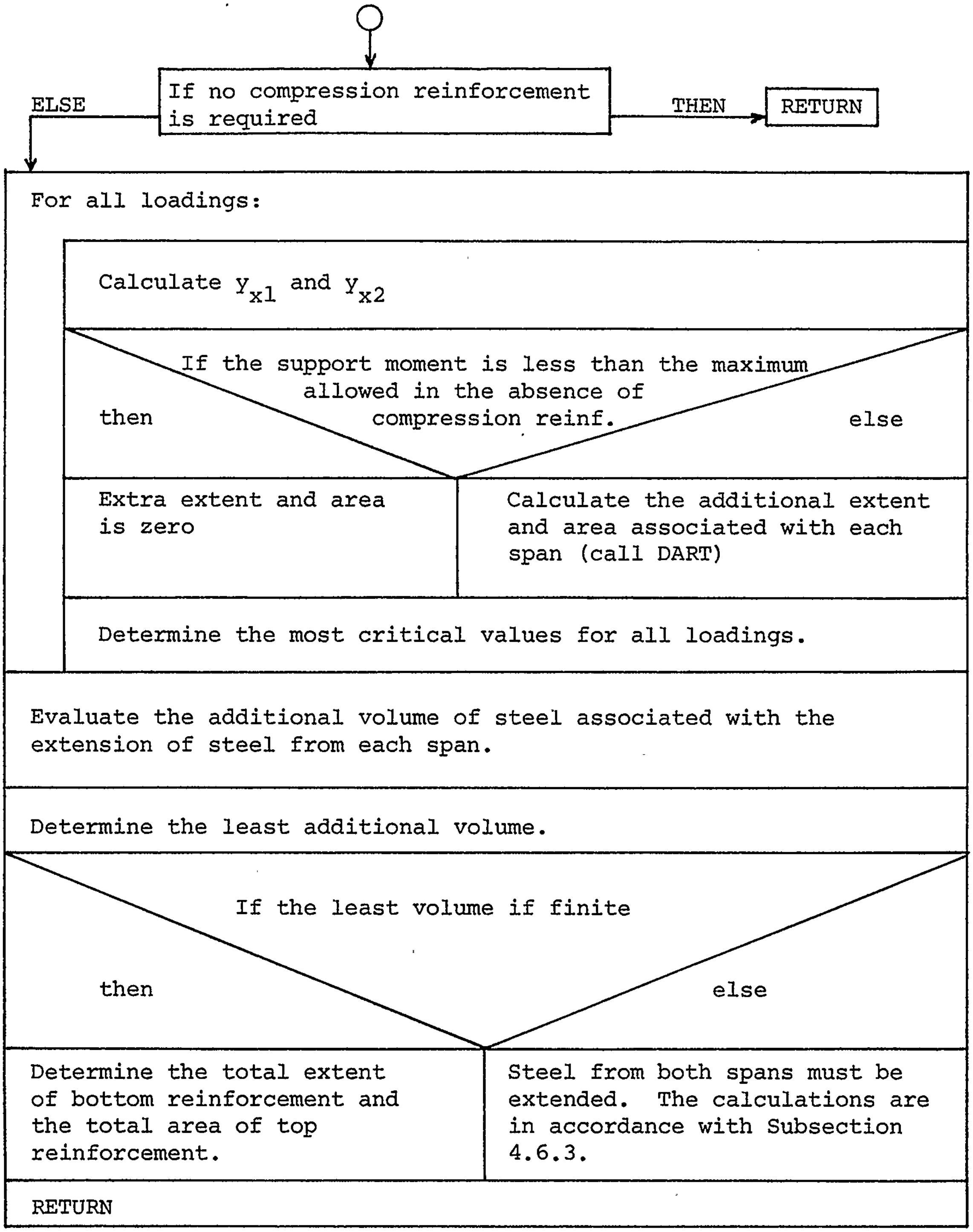


Fig. A.6. - Flowchart for the Subroutine COMP

The subroutine CONANA involves basic analysis of a one- or two-span substructure. Calling on CONANA for each objective function calculation, the subroutine, CDM, (see flowcharts, Figs. A.7 and A.8), directs a conjugate direction unconstrained search for the least cost values of the substructure design moments for sag. A description of this method of optimization is given in Subsection 3.2.2. The subroutine, OPT, calls CDM but also includes the calculation of the parameters necessary for quadratic projection (Subsection 5.2.4). The subroutine SUBOPT (Fig. A.9) directs the total substructure optimization. Initial values are given to the variables to be optimized. Subroutine OPT is called and is recalled with higher penalty constants if the optimization results in an infeasible design.

At a higher level is the subroutine ST (Fig. A.10) which directs the complete (optimal) determination of steel areas, locations and extents throughout the beam for given section geometries. Optimization is by sequential decomposition with quadratic projection. Each cycle of substructure optimizations is carried out by a series of calls of SUBOPT. Subroutine QP (Fig. A.11) is called for the execution of quadratic projection. Three tiers of accuracy are used for the solution of the subproblems, the termination criterion not being tested until after the third.

When none of the section dimensions are variable, ST is called from the main program (see flowchart, Fig. A.12). If however, some geometric variables do exist, the model coordination method is used to decompose the problem into optimization at two levels. At the first level, FAN (Fig. A.13) directs the complete steel design process for

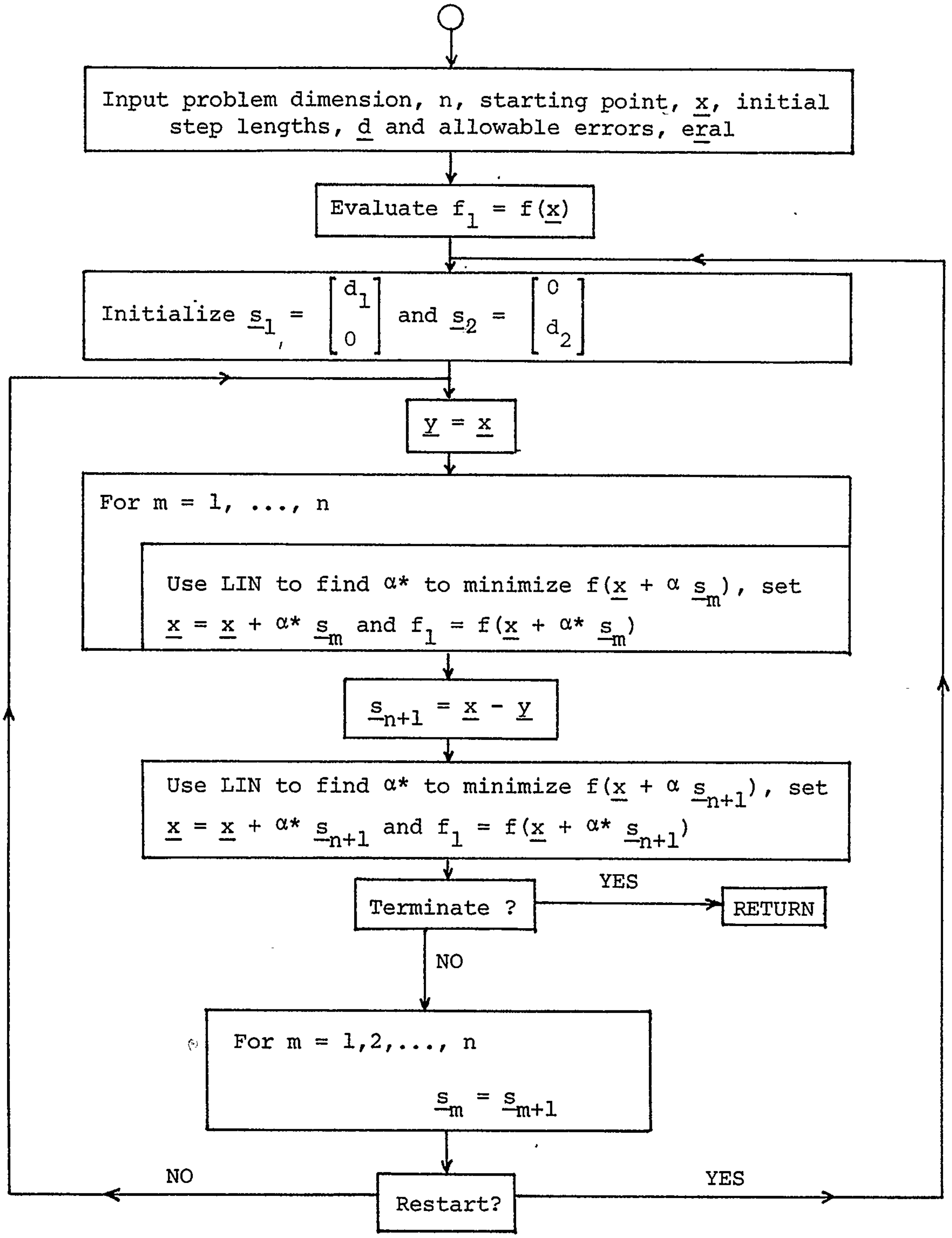


Fig. A.7. - Flowchart for Subroutines CDM and GCDM

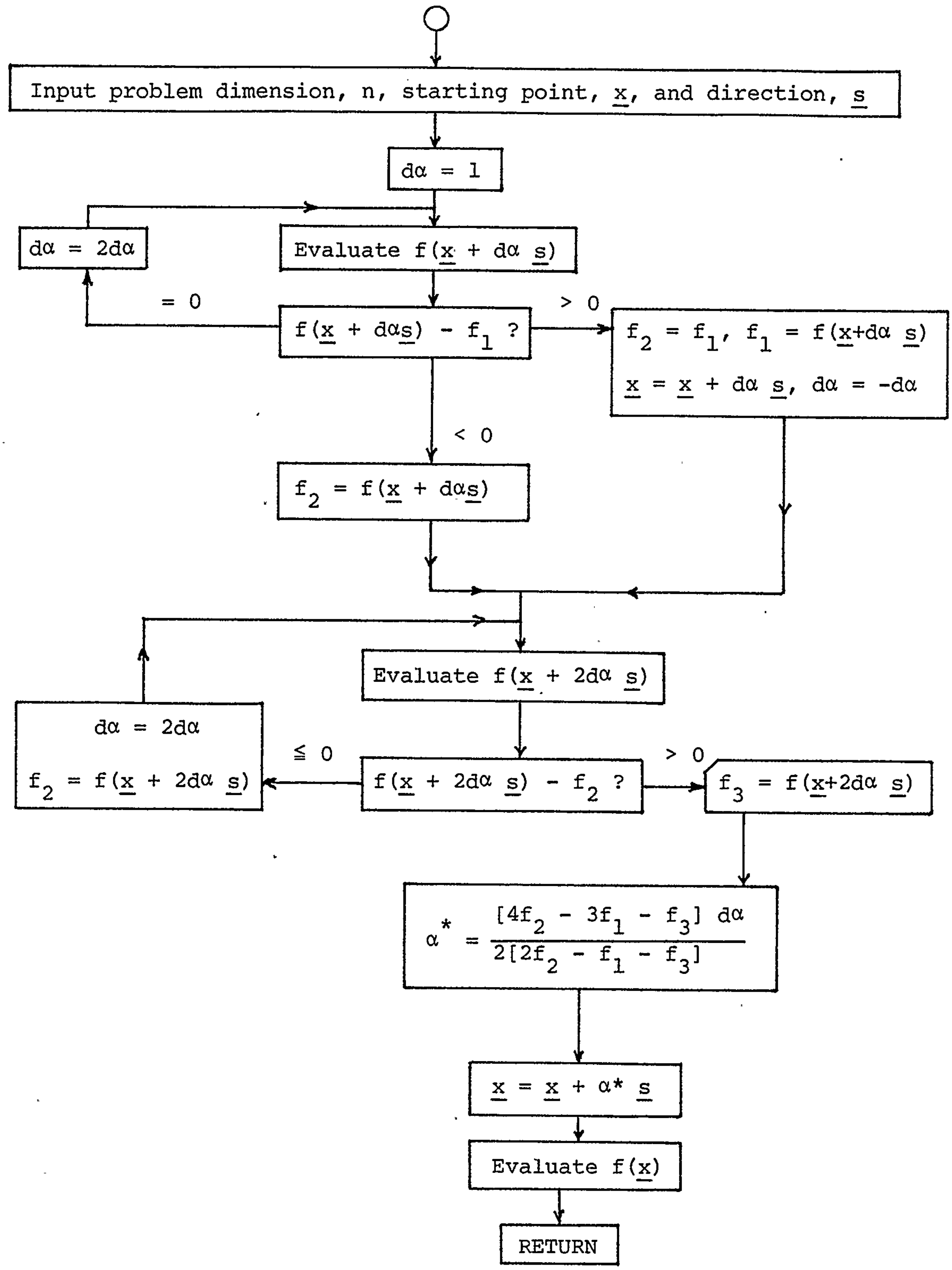


Fig. A.8. - Flowchart for Subroutine LIN, a Subprogram of CDM and GCDM

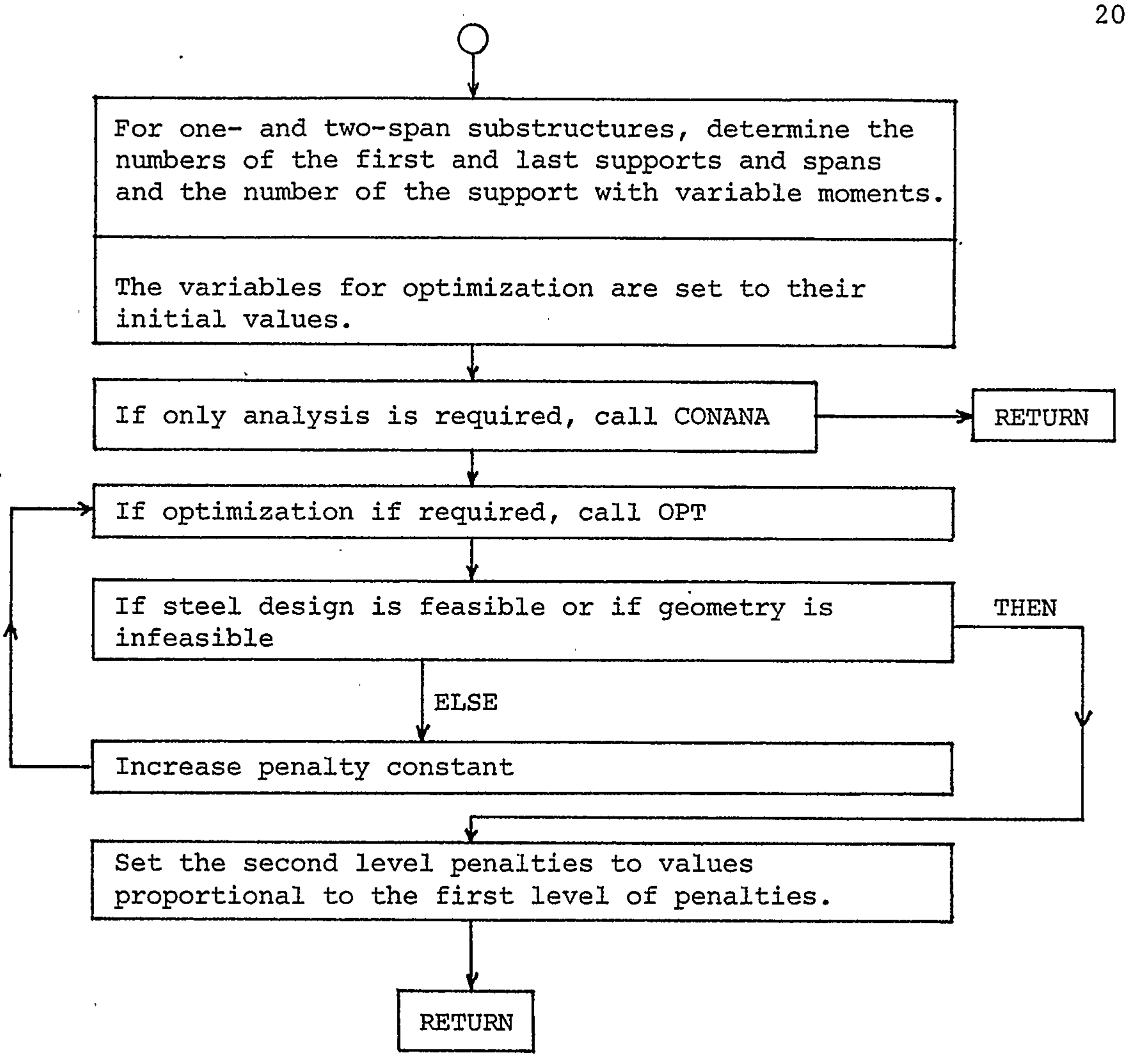


Fig. A.9. - Flowchart for the Subroutine SUBOPT

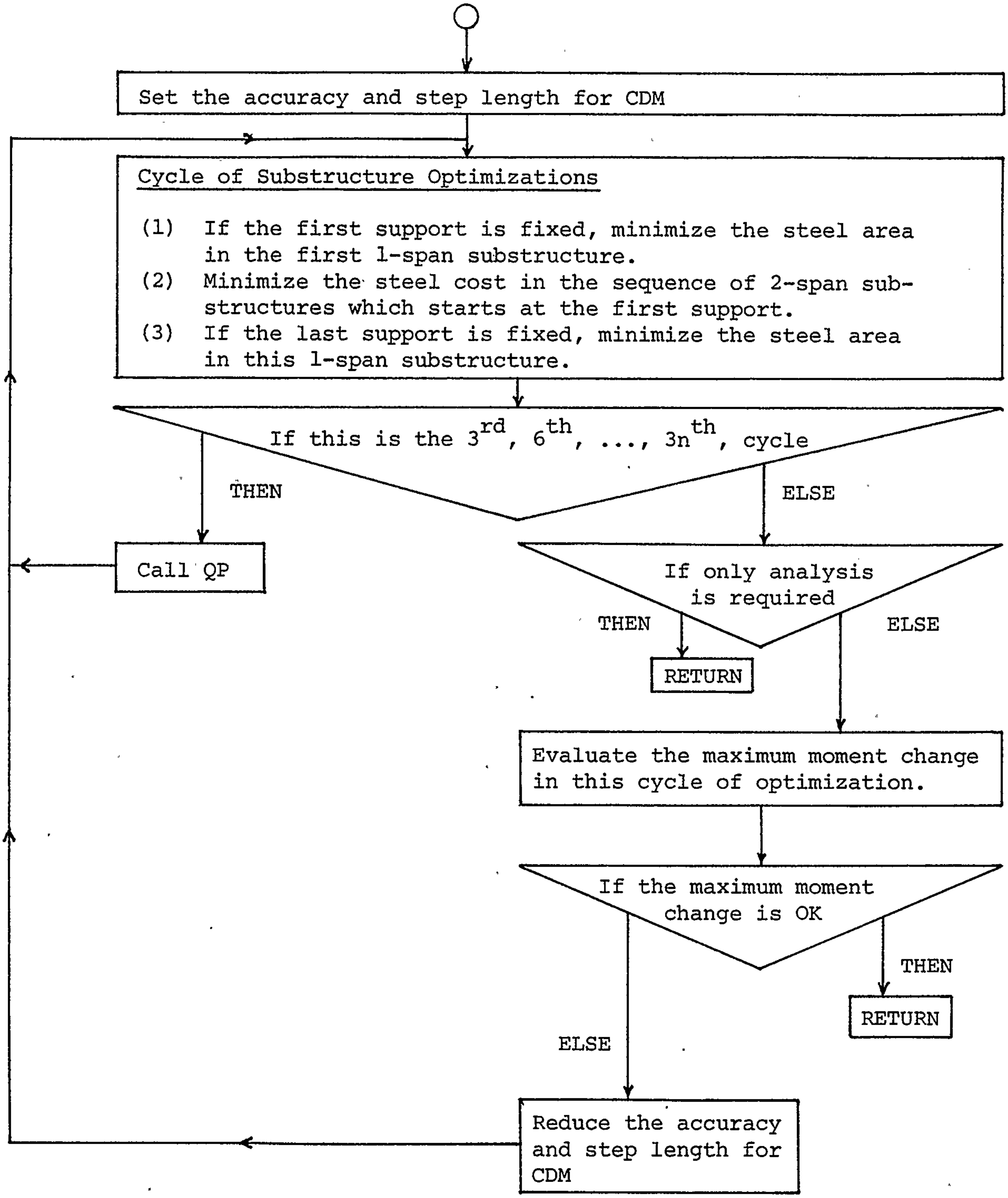


Fig. A.10. - Flowchart for the Subroutine ST

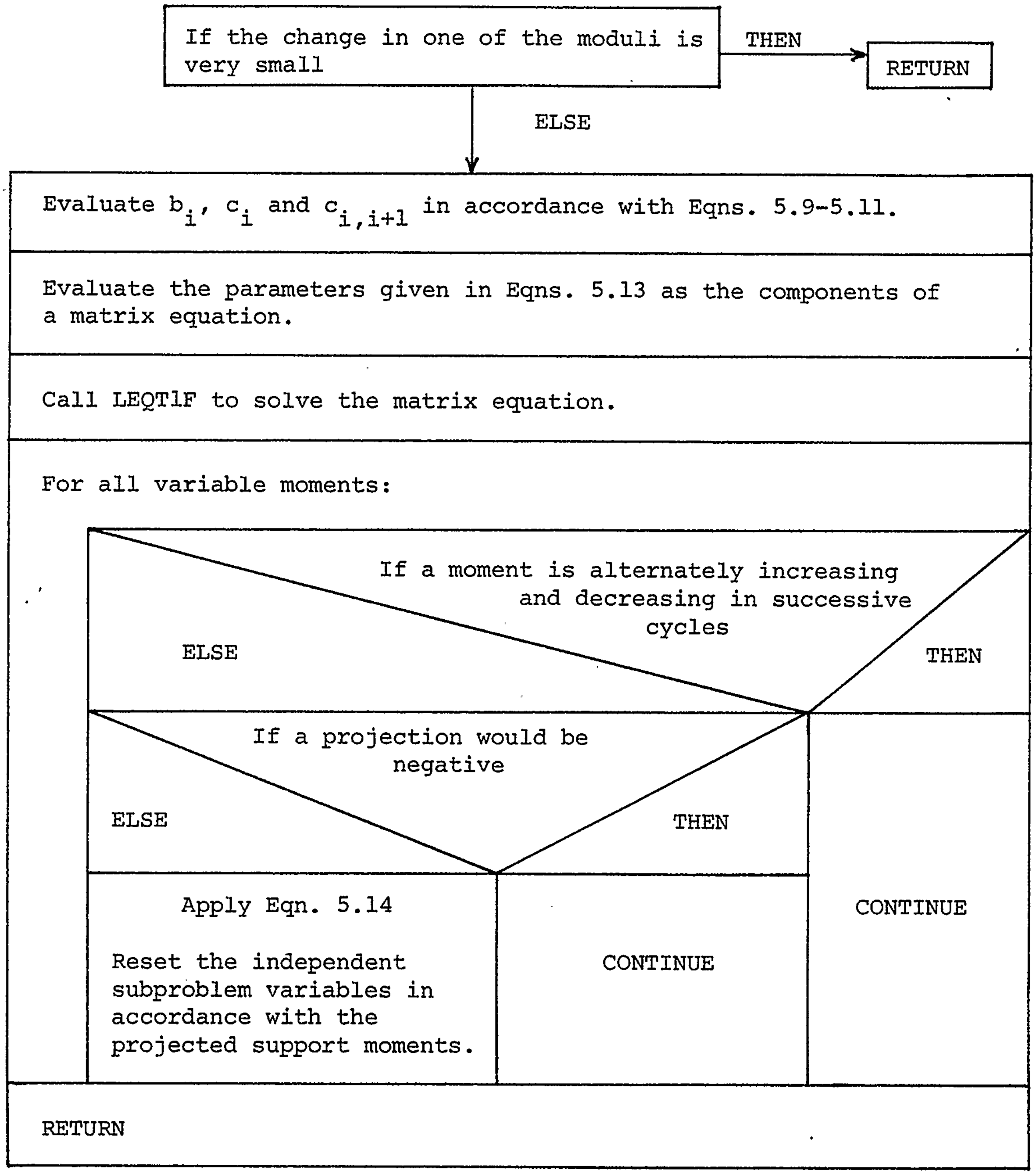
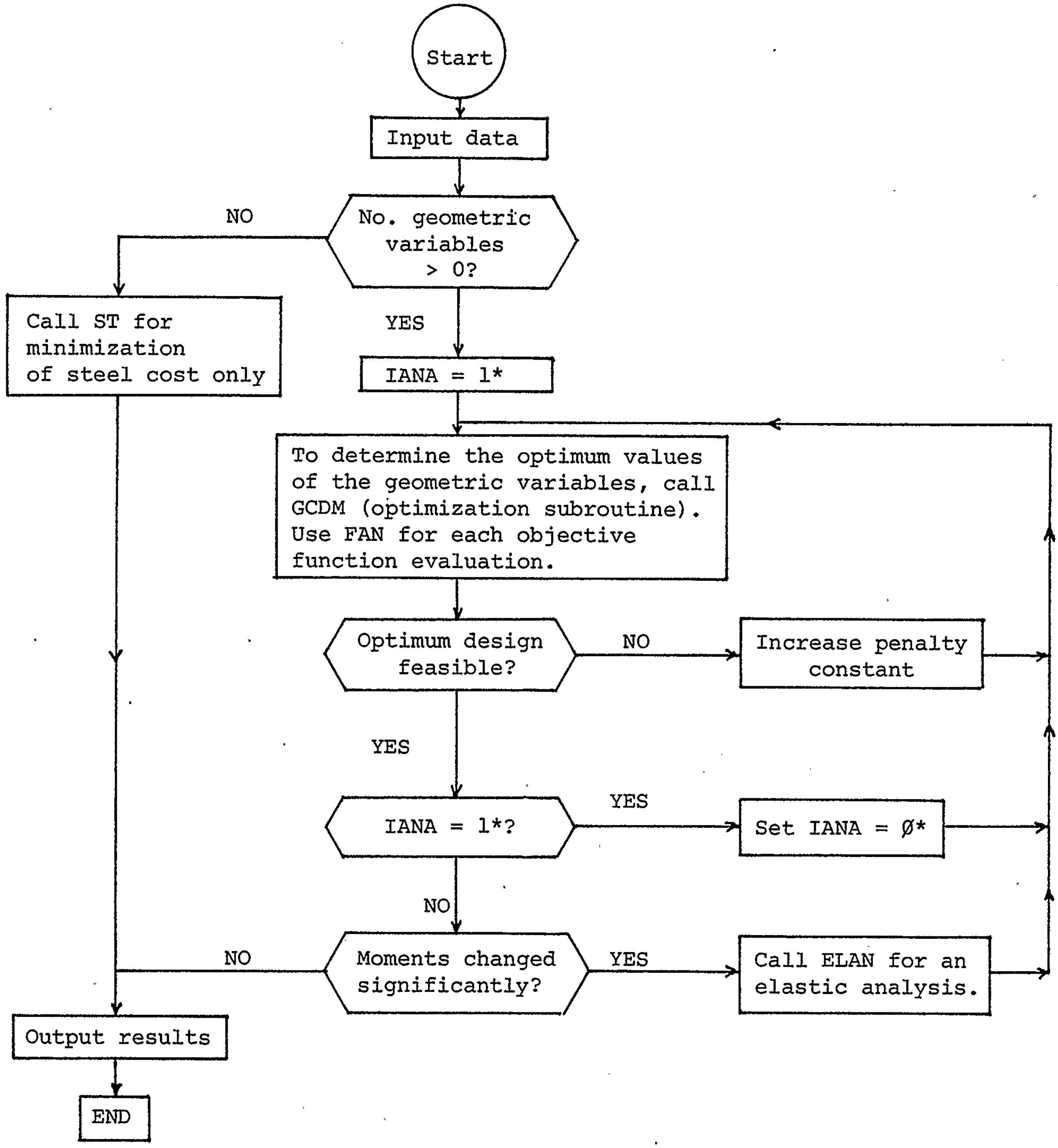


Fig. A.11. - Flowchart for the Subroutine QP



* IANA is a flag variable. For IANA = 1, no optimization of plastic redistribution is done. For IANA = 0, the optimum amount is found.

Fig. A.12. - Flowchart for Main Program BOD

Call steel optimization subroutine ST

Call DEFL for the calculation of the service load deflections and the equivalent cost for excessive values.

Calculate the penalty function for negative geometric variables.

Evaluate the total objective.

RETURN

Fig. A.13. - Flowchart for the Subroutine FAN

The maximum negative and positive moments are determined.

The effective moment of inertia is calculated using Branson's formula.

The centre point deflections are determined.

The penalties for deflections in excess of the maxima are calculated (fuzzy constraints).

RETURN

Fig. A.14. - Flowchart for the Subroutine DEFL fixed values of the geometric variables. Included in FAN is the calling of ST and the calling of DEFL (Fig. A.14) for the calculation of deflections and the associated equivalent costs. Also included is the calculation of the total cost of steel and concrete and the penalties for constraint violations. At the second level of optimization in the model coordination process, GCDM (Figs. A.7 and A.8) directs an unconstrained search, by the conjugate directions method, for the optimum values of the geometric variables. This subroutine is called from the main program when the number of geometric variables is non-zero. If constraint violations exist after optimization, the penalty function constant is increased and GCDM is called again. This process is repeated until a feasible optimal solution is found for the geometric variables. The feasible optimal solution is first found using an elastic analysis with no plastic moment redistribution. When a solution is found using this simplifying approximation, GCDM is called again and this time the amount of plastic moment redistribution that minimized steel cost is carried out. When an optimum feasible design is found again, the values of the geometric variables are compared to their values before optimization. If there is significant change, an elastic analysis is carried out (ELAN, Fig. A.15) and the optimization process is repeated. This time, the approximation for the amount of plastic moment redistribution is not applied. It has been found that two elastic analyses are usually sufficient for the determination of the solution with a third to verify that the solution has been reached.

The gross moment of inertia is calculated.

The components of the matrices [f] and {D} are evaluated. Fixed or pinned supports are allowed at each end.

Subroutine LEQT1F is called to solve the matrix equation [f]{F} = -{D} and find the elastic support moments.

RETURN

Fig. A.15. - Subroutine for Elastic Analysis ELAN

Appendix B - Sequential Simplex Method

The Sequential Simplex Method, for an n-variable problem, consists of establishing a "simplex" of (n + 1) points, $\underline{x}_j$, j = 1,2,..., (n + 1), each with components, x_{jk}, k = 1,2,..., n. The objective function, f, is evaluated at each point and the points with highest and lowest values, $\underline{x}_h$ and $\underline{x}_\ell$, are determined. The centroid of the simplex, $\underline{x}_b$ is found and the process of reflection is carried out to determine a new point, $\underline{x}_n$, with objective, f_n. Depending on the relative value of, f_n, the process of expansion is done to find, $\underline{x}_m$, and, f_m, or contraction to determine, $\underline{x}_c$ and f_c. The complete flowchart for the program is given in Fig. B.1.

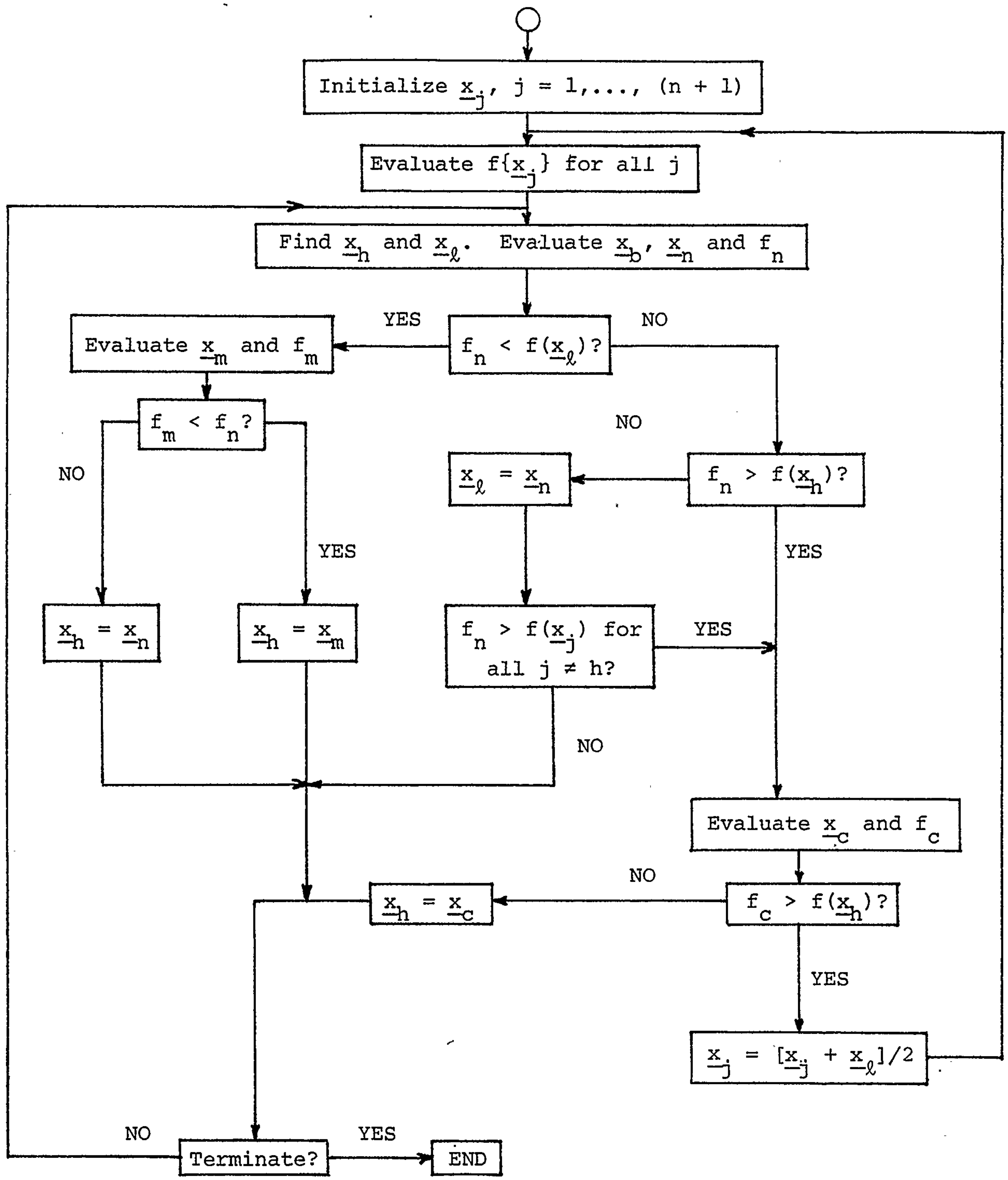


Fig. B.1. - Flowchart for Optimization Program, SSM

Appendix C - Smoothing Function for Gradient Discontinuities

The IMSL subroutine, ZXMIN, (IMSL, 1982) was found to be sensitive to discontinuities of the gradients of the objective function. Such discontinuities occur when the objective is, in some way, dependent on the maximum of two functions. For example,

$$G = \max(g_1, g_2)$$

has discontinuous first derivatives at, $g_1 = g_2$. The problem can be solved by replacing the "maximum" operation with a "smooth maximum" subprogram. A small band is defined on either side of the curve (in general, hyperspace) defined by, $g_1 = g_2$. Then, a smooth maximum function is given by:

$$G_S = g_1 \quad , \quad g_1 \geqq g_2 + b_w$$

$$G_S = \frac{(g_1 - g_2)^2 + 2b_w(g_1 + g_2) + b_w^2}{4b_w} \quad , \quad g_2 - b_w < g_1 < g_2 + b_w$$

$$G_S = g_2 \quad , \quad g_1 \leqq g_2 - b_w$$

This function satisfies the requirements of continuous first derivatives with respect to g_1 and g_2 at all points. On the curve, $g_1 = g_2$, the function, G_S, exceeds G by $b_w/4$. Thus a value of b_w equal to four times an acceptable level of accuracy, is appropriate.

A function with continuous first and second derivatives is given by:

$$G_S' = g_1 \quad , \quad g_1 \geqq g_2 + b_w$$

$$G'_S = \frac{(g_1 - g_2)^2(4g_1g_2 + 6b_w^2) - (g_1^2 - g_2^2)^2 + 8b_w^3(g_1 + g_2) + 3b_w^4}{16b_w^3} ,$$

$$g_2 - b_w < g_1 < g_2 + b_w$$

$$G'_S = g_2 \quad , \quad g_1 \leqq g_2 - b_w$$

However, the latter function involves double curvature and is therefore not suitable for programs in which convexity is a requirement.