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Real Options, Sequential Bargaining Game and Investment Decisions with Network Effects

by

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Abstract

This thesis addresses a common problem in the capital budgeting, the optimal time and size of capital investment, using techniques of real options in a cooperative game setting. In addition, it reflects a combination of real option theory to invest, coupled with competitive game between a first mover and a second mover in the development of a common-use asset and cooperative game theory between a first mover and a second mover to capture a network effect.

In the model, two firms in the same industry have similar and interacting capital investment opportunities, such as to build or purchase a production facility. There is a real option for both firms to delay the investment until they have suitable price and production conditions. There are advantages to a first mover who can build a facility to its own specifications and locational or functional preference. This first mover advantage encourages early investment. There is also a cooperative bargaining game to be played between these two firms because the launch of one firm's investment influences the payoffs and therefore the launch of the other firm's investment. Also, there is a beneficial network effect from operating synergy if the first mover successfully encourages the second mover to start production immediately by sharing the production facility.

Thus, the first mover has to decide when to build, what capacity to build and what the optimal economic rent is for using the facility. The second mover has to decide whether to use the first mover's facility or build its own facility, and if it decides to build its owns, what the optimal time and size are.

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Dedication

I wish to dedicate this thesis to my parents, Hongsheng Li and Ruidi Huang, who taught me to persevere and prepared me to face challenges with faith. It is also dedicated to my wife, Hong Ding, who offered me unconditional love and support throughout the course of this thesis, and to my son, Mido XinKun Li, who came into our life in the last year of my thesis program and accompanied me with your tireless voice.

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Chapter 1

INTRODUCTION

Corporate investment decisions sharing the characteristics of irreversibility, uncertainty and timing are often analyzed in the traditional real options literature which asserts that investments should be delayed until uncertainty is resolved, by waiting for an optimal threshold. However, in the oil and gas industry, or the real estate industry, firms are often observed to compete to become the first mover in investment by building significant excess production capacity. They do this even when the commodity price is low, and and knowing that there is a real-option value to wait. These strategic firms not only choose the optimal investment time, but also make decisions about the optimal investment size, whether to cooperate with the competitor by sharing the production facility, and how much to charge the competitor for using the facility. Making the correct decisions on these investment issues can either create or destroy significant value, which is of interest to senior management in the firms. Such investment opportunities share similar characteristics and can be analyzed using real option theory and cooperative game theory, which makes them also interesting to academic research. Moreover, from a social value perspective, it is important to determine whether having firms cooperate in the investment and share a production facility is more valuable than encouraging firms to compete for the investment opportunity and build their own facilities separately.

This thesis studies the effect of interaction between firms' flexible investment decisions — the size (capacity choice) and timing of investment, recognizing firms' capability of making strategic capacity choice, extracting rents from competitors, and gaining from network synergies with competitors. In fact, this thesis demonstrates an equilibrium real options exercise game in which the investment cash flows are not purely exogenous to the firm, but endogenous in the sense that its competitors react to the first-mover's capacity choice and timing decisions.

One application of this is the investment decisions of two adjacent gas producers. Their decisions come in two stages. In the first stage, the natural gas price and the the estimates of initial reserves will determine who develops the land (resource property) first. The first mover then has to decide on the optimal size and timing of construction depending on whether it plans to be cooperative or non-cooperative.[1] In the second stage, the first mover and the second mover play a sequential bargaining game to decide the optimal economic rent paid by the second mover to the first mover for use of the common facility. The second mover has to decide whether to use the first mover's facility or build its own facility, and if it decides to build its own, the optimal scale and timing of construction. By analyzing firms' behavior under a general setting of a sequential bargaining game of incomplete information in the presence of the positive externality, this thesis demonstrates that firms sometimes invest earlier than optimal and build excess production capacity not only for the preemptive effect of a first mover advantage, but also for being able to extract rent from the follower. The leader can improve its enterprise value by being cooperative and building excess production capacity to lease to the follower.

Furthermore, the simulation results provide several testable implications for understanding firms' bargaining behavior in these investment projects. Firstly, the relationship between firms reservation lease rate and commodity price is not monotonic. The leader may set the lease rate high if the commodity price is below the follower's exercise threshold. But as the leader observes the price rising to the follower's exercise threshold, it may reduce the lease rate to to avoid rejection of lease, justifying a larger joint capacity. The

[1]The cooperative producer recognizes the economies of scale and the positive network effect and thus will construct a larger production facility for sharing, whereas the non-cooperative producer will construct a smaller facility optimal for its own reserves.

follower tends to reject the lease contract if the commodity price and the initial reserve quantity are either low (where it doesn't invest at all) or high (where it is profitable to build its own plant). It tends to accept the lease for some medium range of price and quantity.

Secondly, the relationship between firms reservation lease rate and the network effect is also non-monotonic. The network effect is positively associated with the firms' lease rate before the peak, but becomes negatively associated after the peak.

This thesis also provides insight into industry regulation for policy makers. When several firms compete to develop a resource, it may help the regulator to decide whether firms are optimally developing the resource by allowing future joint utilization of the production facility, considering the price and quantity level and the magnitude of the network effect. If the price is relatively low or high, the regulator should allow firms to develop separately to ensure the resource is developed efficiently in time, whereas if the price is varying in some medium range, it is probably more socially optimal to encourage the cooperation among firms who wants to develop the resource. Moreover, by providing a basis for the negotiation of the rent charges, it may help the to resolve the disputes between the owner and other users of a production facility, which helps to capture the cooperative network effect.

The recent article, Novy-Marx (2007) shows that opportunity costs and supply side heterogeneity reduce the competition effect and leads to an investment threshold even later than the standard real option threshold.

1.1 Literature Review

Classic real option literature[2] showed that firms should optimally delay investment until a suitable threshold for price, demand or other stochastic variable is met. Myopic firms simply apply the classical real option techniques to decide the optimal time of investment without contemplating future ramifications of their current investment decisions. Nevertheless, the real options of different firms sometimes interact. For example, the firms may have substitutable or complementary inputs or outputs. There may be market power, patents, proprietary expertise or location that cause these interactions. In such settings, one firm's investment decision may influence the other firm's investment decision through various factors such as the first mover advantage and the network effect. The preemptive real options literature[3] documents a tradeoff between the real option to delay (and resolve uncertainty) against the first-mover advantage. They use the intersection of real options and industrial organization theory to analyze firms' strategic preemptive investment decisions. Not surprisingly, they find that competition reduces the real option value and the investment delay. Most of these articles develop a Bertrand, a Cournot, or a Stackelberg equilibrium depending on the type of competition assumed. The recent article, Novy-Marx (2007) shows that opportunity costs and supply side heterogeneity reduce the competition effect and leads to an investment threshold even later than the standard real option threshold.

However, despite the substantial development of this literature, little attention has been paid to the effects of positive externality on firms investment decision. The network effect is also offset by the first-mover advantage that encourages early investment. The

[2]This includes Brennan and Schwartz (1985); Dixit and Pindyck (1994); Dixit (1995); Capozza and Sick (1991); Sick (1995); Trigeorgis (1996)

[3]This includes Fudenberg and Tirole (1985); Smit and Ankum (1993); Grenadier (1996, 2002); Mason and Weeds (2005); Garlappi (2001); Boyer et al. (2001); Murto and Keppo (2002); Lambrecht and Perraudin (2003); Murto et al. (2003); Huisman and Kort (2004); Thijssen et al. (2006); Smit and Trigeorgis (2004)

first mover advantage accrues to the first firm that builds or purchases a production facility, because it can build or purchase the facility based on its own specifications, and locational or functional preference. Moreover, once the facility is built, it can engage in a bargaining game with later movers in which it offers to lease access to its facility. The first mover has a tradeoff between the rents it can earn on a high lease rate and the opportunity to capture network benefits by having the second mover enter early. Therefore, the strategic firms not only choose the optimal investment time, but also make decisions about the optimal investment size, whether to cooperate with the competitor by sharing the facility, and how much to charge the competitor for using the facility. Decisions on these investment issues can either create or destroy significant value, which makes them important for management. Such investment opportunities share similar characteristics and can be analyzed using real option theory and cooperative game theory.

The network effect[4] arises from cooperation that can yield operating synergy. The operating synergy may come in the form of lower cost structure. A single firm may not have enough production volume to make the construction or the purchase of the production facility economically viable. If it can induce others to participate, the unit costs will fall and it will face a lower cost structure including the saved fixed cost of repetitive construction of common production facilities, lower unit production costs, lower marketing costs, or lower transportation costs paid to a third party. Alternatively, the operating synergy may come in the form of higher overall revenue because the cooperative investment may generate larger market demand or improve the quality of goods.

[4]The industrial organization literature, such as Katz and Shapiro (1985, 1986); Chou and Shy (1990); Church and Gandal (1993); Church and Ware (1996); Economides (1996); Bakos and Nault (1997); Fudenberg and Tirole (2000); Mason and Weeds (2005); Farrell and Klemperer (2005), discuss the network effect in a game theory context but not in the real option context.

1.2 Some Real Options and Sequential Bargaining Game Models

In the oil and gas industry, the airline industry, the real estate industry and the software industry, investments usually require a large amount of capital to build or purchase a production facility, which may be a plant, an equipment, a jet aircraft, an R&D patent or some infrastructure. Investment decisions in these four industries involve a two stage game. In the first stage, firms (trying to capture the first mover advantage) will play a Betrand game in the case of differentiated product or a Cournot game in the case of homogeneous product. If the two firms have the same cost structure and payoff functions, there will be a simultaneous investment. If one firm has significant competitive advantage over the other, it will invest first and become the leader who gets a more favorable price in the Bertrand equilibrium or larger production quantity in the Cournot equilibrium. The follower's strategy would be either to sell products at a less favorable price in the Bertrand equilibrium or to produce less in the Cournot equilibrium. This could be the final equilibrium providing the price or demand is deterministic.

However, when the price or demand is stochastic, the follower has a real option to delay its investment until more favorable price or demand comes which makes both firms proceed to the second stage where the leader and the follower play a sequential bargaining game. In the second stage, the leader wants to encourage the follower to start production earlier by offering to lease part of the production facility to the follower; therefore the leader needs to determine the optimal economic rent and optimal investment scale. The follower needs to decide whether to accept the leader's offer or to wait to build its own facility. Since these investments share common characteristics and exhibit similar comparative statics, I will discuss the components of this real option bargaining game for each industry first. Then I will formally construct and analyze the real option bargaining model by solely focusing on the oil and gas industry in the rest of this thesis.

1.2.1 The airline industry

Airlines face stochastic demand for flights between city pairs. This gives them a real option to decide when to start a route between two cities, and how much capacity to put on the route.

Suppose two airlines, Air France and Lufthansa, have adjacent air transportation markets between central Europe and North America. Air France prefers a Paris hub whereas Lufthansa prefers a Frankfurt hub, because there are two advantages in locating the hub in the airlines' home countries. Building the hub at home allows the airport to be built on the airline's specification. It may also bring in potential future air travel demand. The property right is clearly defined because neither of them has the route authority in the other country. Since the air travel demand is uncertain, this is a real option to develop a new route. The production facility is the aircraft and the terminal facilities. The exercise price is the capital costs (mainly aircraft purchases) and the number of airplanes purchased determines the production capacity.

In the first stage, two airlines will play a real option exercise game in which the first mover (the leader) will develop the route and locate the airport in its home country (either Paris or Frankfurt), and the second mover (the follower) will choose to wait until more demand comes. In the second stage, to capture the network effect, the leader may encourage the follower to start selling the similar flights between Frankfurt (or Paris) and North American cities by offering a code-sharing program to the follower. The code sharing program can reduce the number of empty seats on each flight and thereby boost the revenue. The increased number of flights to the hub (the larger transportation volume) may help bring down the unit airport service fee and reduce both firms' operating costs per seat. The leader's decisions include how many planes to order, and how much to charge the follower for code sharing. The follower's decisions are to accept the leader's offer and how to bargain with the leader in terms of the code-sharing, or to delay its plane purchase until more uncertainty about the demand is revealed.

1.2.2 The real estate industry

Real estate developers often make decisions on whether and when to develop adjacent undeveloped properties. Suppose there are two real estate developers who own adjacent undeveloped properties that can be developed into a residential area. There is a network effect arising from shared infrastructure (roads, schools, shopping centers). The demand for houses in that area is uncertain and so is the selling price. There is a real option to develop for both companies. In the first stage real option exercise game, the leader and the follower will be determined depending on the house price and number of houses to be built. The leader becomes the main developer and the follower is the home builder. In the second stage, the leader can offer lots in its developed area, upon which the second mover can build. The leader and follower can capture the network effect if they can induce third parties to build schools, shopping centers and upgrading roads. This is more likely to happen if they cooperate and build more houses. The leader has to decide the size of the neighborhood, the construction scale of these infrastructure, how much to charge the follower for sharing the infrastructure. The follower has to decide whether to accept the leader's offer and start to develop immediately, or wait to build its own infrastructure in another neighborhood and develop in the future.

1.2.3 The software industry

Software companies often have to decide whether to develop multiple software packages with related functionality. Software packages can share file standards or inter-operability (plug-ins). In the first stage, the two companies will have a real option patent game to develop new software as discussed by Miltersen and Schwartz (2004). In the second stage, the leader may offer the follower a license contract which allows the follower to use the leader's patented software to develop related applications. The leader's decisions include the optimal software capability — the number of functions provided by the software as well as the optimal license fee. The follower's decision is whether to use the leader's patent by paying the license fee, or to delay and perhaps develop its own software later, depending on the evolution of demand. The network effect may result from the avoidance of repetitive R&D investment, or from the increased software value due to improved compatibility and a larger customer pool.

1.2.4 The oil and gas industry

In the oil and gas industry, producers often own adjacent lands from which they may produce in the future. This provides for an opportunity for joint use of infrastructure to exploit the resource. Two such types of infrastructure typically have different ownership structure.

1. Gas processing plants remove liquids and hydrogen sulfide from the gas at the field before it can be safely shipped by pipeline. Gas plants are typically owned and operated by the first company to drill in a particular field, and they may build excess capacity and lease out that capacity to other producers in the same area.

2. Pipeline gathering systems are needed to ship the gas to central hubs, where they join the main line pipelines that distribute gas to consuming areas. These are typically owned by a company that specializes in pipelines, and it usually isn't a producer.

There are fixed costs in both of these types of infrastructure, which generates a network effect. A single gas producer may not have enough reserves to make a gas plant or pipeline connection economically viable. Also, if it can induce others to participate in the infrastructure, the unit costs will fall and it will tend to face a lower overall cost structure to produce its reserves. The first mover advantage accrues to the first company (the leader) that builds a gas processing plant to serve the field. The advantage arises because the leader can locate the plant near its part of the field and can customize the construction of the plant to be most efficient with the type of gas it owns. In the first stage, firms having similar size of initial reserves will invest simultaneously whereas if one firm has larger initial reserve, it will develop first and becomes the leader. In the second stage, once the plant is built, the leader can extract rents from the follower because of the fixed costs of building a competitive plant. However, the leader's efforts to extract rents are offset by its desire to have the follower agree to produce, thereby enabling the pipeline to be built or reducing the toll charges it has to pay the pipeline owner to induce it to build the pipeline. Also, there is a tradeoff between the first-mover advantage for building the gas plant and the real options incentive to delay construction until more uncertainty about volumes and prices can be resolved. The leader decides the optimal plant capacity and the leasing fee. The follower decides whether to accept the leader's offer or wait to build its own processing plant.

This thesis is a collection of one theoretical paper analyzing dynamic real option sequential bargaining game between firms with large capital investment decisions, one paper using the least squares Monte-Carlo method to simulate the equilibrium of the game, and an empirical paper describing the application of the theoretical model to the oil and gas industry. The objective of this thesis is to analyze real option exercise games by allowing size and timing decisions, as well as by incorporating the network effect into a dynamic bargaining game of incomplete information. This thesis will identify and characterize the set of equilibria of the associated game and empirically test the dynamics between output price and firm's choice regarding cooperation given network effects, real options and incentives for preemption.

The remainder of the thesis is organized as follows. Chapter 2 develops a theoretical real options bargaining model for firms having mutually affecting capital investment projects. As in Grenadier (1996); Garlappi (2001); Murto et al. (2003); Imai and Watanabe (2005), I extend the backward induction solution for a real option in this game theory setting to provides a simple computation of a subgame-perfect Nash equilibrium. Firms' prior and posterior beliefs are explicitly laid out and analyzed. Those non-credible threats and promises are ruled out. Finally, a perfect Bayesian equilibrium is characterized for this dynamic bargaining game under incomplete information using Coasian Dynamics as discussed in Fudenberg and Tirole (1991, Ch10). Chapter 3 applies the Longstaff and Schwartz (2001) least squares Monte-Carlo method to simulate and optimize the real options values which leads to an efficient computational procedure to determine optimal investment time and size, when and whether firms should be cooperative, what the optimal economic rents are under the assumptions of stochastic prices and production quantities. Chapter 4 then empirically tests the hypotheses developed in previous chapters.

Chapter 2

A MODEL OF THE REAL OPTIONS AND SEQUENTIAL BARGAINING GAME

This chapter develops a real option exercise game model using the oil and gas industry as an example. However, the derived perfect Bayesian equilibrium of firms' non-cooperative or cooperative investment decisions would be of general instructive value to other aforementioned industries as well.

2.1 Model Assumptions

Suppose there are two gas explorers, A and B, who have adjacent properties for gas exploration and production. There are two kinds of uncertainty.

Production uncertainty

The first is the technical uncertainty of the estimated quantity of reserves on the property. Let $Q_i(t)$ be producer i's expected remaining reserves conditional on information gathered to time t and production up to time t.

$$dQ_i = \mu_i(Q_i)dt + \sigma_i(Q_i)dz_i, \qquad i \in \{A, B\}$$

where the correlation $\rho_Q = \text{corr}(dz_A, dz_B)$.

Production at rate q_i does two things:

1. It depletes the reservoir at rate q_i;

2. It provides information that causes revised information about total reserves. So $\sigma_i(q_i)$ is non-decreasing in q_i.

$$dQ_i = -q_i dt + \sigma_i(q_i) dz_i.$$

One can assume exponentially declining production volume:

$$q_i = \alpha_i Q_i$$

where α_i is the production rate. But, this doesn't usually happen, because there are two constraints on production. One is a regulatory or technical upper bound on the production rate [1] $\overline{q_i} = \overline{\alpha}_i Q_i$, for some fixed $\overline{\alpha}_i$.

The other is the capacity of the processing plant, q_i^c. Therefore, the production rate q_i must satisfy the following constraint if there is one producer and one plant only:

$$q_i \leq \min\{q_i^c, \overline{\alpha}_i Q_i\}. \tag{2.1}$$

Initially, the capacity constraint is binding. But there is a production switch when $\overline{\alpha}_i Q_i$ falls below q_i^c for $i \in \{A, B\}$. After this switch, the regulatory and technical constraint is binding.

Otherwise, if each producer has a plant (and is willing to lease production capacity to the other), then the constraints are:

$$q_i \leq \overline{\alpha}_i Q_i \qquad i \in \{A, B\}$$

$$q_A + q_B \leq q_A^c + q_B^c.$$

[1] Regulators often restrict the production rate to avoid damaging the rock formation and having water floods, which could reduce the ultimate production from the field. Also, there is a natural maximum flow rate for the field depending on the porosity of the rock.

At the start of production, when plant capacity is binding:

$$\frac{dQ_i(t)}{dt} = -q_i^c \Longrightarrow Q_i(t) = Q_i(\tau_i) - q_i^c\, t \qquad (\tau_i \le t \le \theta_{i,\text{trans}})$$

where τ_i is player i's production starting time. If the gas used in the production process is significant, this differential equation would need to be adjusted. $\theta_{i,\text{trans}}$ is defined as the transition time from the capacity constraint to the technology/regulatory constraint:

$$\overline{\alpha}_i Q_i(\theta_{i,\text{trans}}) = q_i^c \tag{2.2}$$

$$\Longrightarrow \overline{\alpha}_i \left[Q_i(\tau_i) - q_i^c \theta_{i,\text{trans}}\right] = q_i^c$$

$$\Longrightarrow \theta_{i,\text{trans}} = \frac{Q_i(\tau_i)}{q_i^c} - \frac{1}{\overline{\alpha}_i}. \tag{2.3}$$

After $\theta_{i,\text{trans}}$, the reserve quantity is binding, so the actual production rate is $\overline{\alpha}_i$.

$$\frac{dQ_i(t)}{dt} = -\overline{\alpha}_i Q_i(t) \Longrightarrow Q_i(t) = Q_i(\theta_{i,\text{trans}}) e^{-\overline{\alpha}_i (t-\theta_{i,\text{trans}})} \qquad (t \ge \theta_{i,\text{trans}}).$$

Thus, producer i's production function is

$$q_i(t) = \begin{cases} q_i^c & t \in [\tau_i, \theta_{i,\text{trans}}] \\ \overline{\alpha}_i Q_i(\theta_{i,\text{trans}}) e^{-\overline{\alpha}_i (t-\theta_{i,\text{trans}})} & t \in [\theta_{i,\text{trans}}, \theta_i] \end{cases} \tag{2.4}$$

where θ_i is producer i's maximum production time of its property.[2]

[2]The remaining reserves continue drop once the production starts. After producing for certain period of time, the remaining reserves will drop below a critical level at which it may be optimal to shut down the production because the profit may not be able to cover the variable production cost then.

Price uncertainty

The price of gas P is a source of economic uncertainty. Assume it follows the diffusion

$$dP = \mu(P)dt + \sigma(P)dz_P$$

where the correlation between technical and economic uncertainty is zero: $\mathrm{corr}(dz_P, dz_A) = \mathrm{corr}(dz_P, dz_B) = 0$.

More specifically, to simplify the numerical simulation of the investment game in later sections, the gas price is assumed to follow a Geometric Brownian motion, i.e.,

$$dP = \mu P dt + \sigma_P P dz_P$$

The general standard deviation $\sigma(P)$ becomes a functional form $\sigma(P) = \sigma_P P$. The drift rate $\mu(P) = \mu P$.

Construction cost

The cost of constructing a gas plant with capacity of q_i^c has fixed and variable components:

$$K(q_i^c) = a + bq_i^c \qquad i \in \{A, B\}$$

where the producers have the same construction parameters $a, b > 0$.

2.2 The Players' Investment Decisions

Let r be the risk free rate of return and $\tilde{f}$ be the systematic risk factor. Suppose the underlying asset is priced by the capital asset pricing model. The investment asset is expected to earn a risk premium in proportion to the covariance between asset price changes and the risk factor, which suggests the following relationship:

$$\mu P + \delta P = rP + \lambda_P \beta_P P$$
$$\Rightarrow \mu + \delta = r + \lambda_P \beta_P$$

where $\beta(P) = \frac{\text{cov}(dP, d\tilde{f})}{\sqrt{\text{var}(dP)\text{var}(d\tilde{f})}}$, λ_P is the risk premium for the systematic risk factor $\tilde{f}$, and $\delta(P, t)$ is the rate of convenience yield of the underlying assets. The risk-neutral drift of price, $\hat{\mu}$ becomes $\mu - \lambda_P \beta_P = r - \delta$. Since the price is assumed to follow the GBM, the futures price P_t follows

$$P_t = P_{\tau_i} e^{(\hat{\mu} + \sigma_P^2/2)(t - \tau_i)}.$$

Similarly, the risk-neutral drift of Q is: $\mu(Q) - \lambda_Q \beta(Q) = -q_t$, where $\beta(Q) = 0$ because the production rate $q_t = 0$ is zero before the initial investment. After the production starts, the producers are price takers and their level of reserves is unrelated to market prices.

2.2.1 Investment decisions with isolated players

Suppose that neither producer initially has a gas processing facility. If the producers' properties are not adjacent, the problem for each producer will be a classic two dimensional real option problem. The real option decisions are those that would be made by a monopolist owner of the project, without any consideration of interaction with the other producer. The optimal development option for producer $i \in \{A, B\}$ has a threshold $\{(P^*(Q_i), Q_i) \,|\, Q_i \in \mathbb{R}^+\}$ where $P_i^* : \mathbb{R}^+ \to \mathbb{R}^+$ is the threshold development price if the estimated reserves are Q_i. That is, producer i develops the first time $(P_t, Q_{i,t})$ are such that $P_t \geq P_i^*(Q_{i,t})$.

The cash flow for producer i at time t is $\pi_{i,t} : \mathbb{R}^+ \times \mathbb{R}^+ \rightarrow \mathbb{R}$ given by

$$\pi_{i,t} = (P_t - C)q_{i,t} \tag{2.5}$$

where C is the variable production cost. The risk neutral expected payoff from an investment made by player i at time τ_i, is:

$$W_i(P, Q_i, \tau_i) = \widehat{E}_{\tau_i} \int_{\tau_i}^{\theta_i} e^{-r(t-\tau_i)}\pi_{i,t}\, dt - K(q_i^c). \tag{2.6}$$

This evolves as a diffusion process which may have a simplified threshold level[3] of cash flow π^*. But this is not necessarily the case, since the uncertainty and risk neutral growth rates in Q and P may not be the same, so that the level of profit may vary over the threshold boundary. These isolated producers are non-cooperative in the sense that they do not have to consider the strategic effect from the investments by the competitors. As P and Q are assumed uncorrelated, generally, these non-cooperative firms' value of the investment opportunity(real option values) $V\big(W(P,Q),t\big)$ must satisfy the valuation PDE:[4]

$$\begin{aligned} \frac{1}{2}\left[\sigma^2(Q)V_{QQ}(P,Q) + \sigma^2(P)V_{PP}(P,Q)\right] + V_Q(P,Q)\mu(Q) + V_P(P,Q)(r-\delta) \\ + V_t = rV(P,Q) \end{aligned} \tag{2.7}$$

and the value-matching and smooth pasting boundary conditions:[5]

[3]Lambrecht and Perraudin (2003) discuss the possibility of a sufficient statistic to determine the threshold.

[4]This is an two dimensional extension of the classic model of operating real options by Brennan and Schwartz (1985) and Sick (1989) to finite reserves using Ito's lemma, assuming no risk premium for the quantity variable and no correlation between price and quantity.

[5]See appendix A for a detailed derivation of the two smooth-pasting conditions.

$$
\begin{aligned}
V(P^*, Q^*) &= W(P^*, Q^*) \\
V_P(P^*, Q^*) &= \frac{\partial W_i}{\partial P_{\tau_i}} = \widehat{E}_{\tau_i}\left[\int_{\tau_i}^{\theta_{i,\text{trans}}} e^{(\hat{\mu}-r)(t-\tau_i)} q_i^c dt \right. \\
&\qquad \left. + \int_{\theta_{i,\text{trans}}}^{\theta_i} e^{(\hat{\mu}-r)(t-\tau_i)-\overline{\alpha}_i(t-\theta_{i,\text{trans}})} q_i^c dt \right] \\
V_Q(P^*, Q^*) &= \frac{\partial W_i}{\partial Q_i} = \widehat{E}_{\tau_i}\left[\int_{\theta_{i,\text{trans}}}^{\theta_i} e^{-r(t-\tau_i)}(P_t - C)\overline{\alpha}_i e^{-\overline{\alpha}_i(t-\theta_{i,\text{trans}})} dt\right].
\end{aligned} \tag{2.8}
$$

Equation (2.7) with the boundary conditions, equation (2.8) can be easily solved numerically as Section 3.2 will demonstrate.

2.2.2 Adjacent players' investment decisions

Cooperative producers will follow a symmetric, subgame perfect equilibrium entry strategy in which each producer's exercise strategy maximizes value conditional upon the other's exercise strategy, as in Kreps and Scheinkman (1983); Kulatilaka and Perotti (1994); Mason and Weeds (2005); Garlappi (2001); Thijssen et al. (2002); Huisman et al. (2003); Imai and Watanabe (2005). The solutions have two different exercise models: simultaneous and sequential exercise.

Equilibrium with simultaneous exercise

Suppose both producers have the same expectations of initial reserves on their own property, after the exploration. Denote F as the follower, and L as the leader, $F, L \in \{A, B\}$. In this case, $P_A^*(Q_A) = P_B^*(Q_B) = P_F^*(Q_F) = P_L^*(Q_L)$, and both producers have the same trigger price. Once the price hits the trigger, they both want to exercise the real option and build their own plant immediately. Whoever moves faster becomes the natural leader. However, given that the prices P and quantities Q_A, Q_B are continuously distributed and not correlated, this is a knife-edge condition that only occurs with probability zero if the producers do not interact. In other words, they do not move together, i.e., $Q_A \neq Q_B$ at the development threshold.

However, when their properties are adjacent, they can interact. The leader can build a plant large enough to process both producers' gas and offer a processing lease rate to the follower to induce the follower to cooperate and generate a network effect. The follower can accept the offer and process its gas in the leader's plant, or build its own processing plant right away or later. If they are cooperative, they will exercise simultaneously and play a bargaining game at that time to determine the lease rate l and plant capacity q_L^c. I define the follower in this simultaneous exercise case as a *big follower*, denoted as F_b. For simplicity, I assume that they both commit not to renegotiate the lease later.

Equilibrium with sequential exercise

Suppose the leader has a larger initial reserve and therefore lower optimal trigger price $P^*(Q_L)$. In this case, $P_L^*(Q_L) < P_F^*(Q_F)$ for $L, F \in \{A, B\}, L \neq F$. The leader will enter alone, building a gas processing plant to cover its own production only. Once its production volumes decline, it will offer excess capacity to the follower at a lease rate l to be negotiated, bearing in mind the follower's reservation cost of building its own plant.

Thus, there is a bargaining game played at and after the time the leader decides to build the plant. This game determines whether the follower starts production at the same time or delays. If the follower accepts the lease, both producers start production simultaneously and the game ends. If the follower rejects the lease, they play the same sequential bargaining game at subsequent dates, where the leader offers a lease rate and capacity, and the follower decides whether to accept the offer, build its own plant or delay further. I define the follower in this sequential exercise case as a *small follower*, denoted as F_s.

2.3 The Sequential Bargaining Game under Incomplete Information for Adjacent Players

One significant difference between this thesis and other option exercise game papers is that I model the expected payoff $W(P, Q, q_i^c, l, t; N)$ as a result of a lease vs. build (exercise the real option of investment) bargaining game when the two producers have adjacent properties in the presence of the network effect N. This bargaining game is a dynamic game of incomplete information as the leader does not have the information about the follower's payoff function.

Denote τ_L as the first time (P, Q_L) hits the threshold $\big(P^*(Q_L), Q_L\big)$. The follower also solves for a threshold trigger price $P^*(Q_F)$ that determines the optimal condition under which it would build its own plant and start production. Denote the first hitting time to the threshold $\big(P^*(Q_F), Q_F\big)$ by the stopping time $\tau_F \in [\tau_L, \infty)$. Hence the big follower exercises at τ_{F_b} and $\tau_{F_b} = \tau_L$ because the big follower's initial reserve is of the same size as the leader's. The small follower exercises at $\tau_{F_s} > \tau_{F_b}$ because the small follower's initial reserve is smaller than the big follower. The lease will start at $\tau_{\text{lease}} \in [\tau_L, \tau_{F_s}]$. The leader's maximum production time is θ_L. The big or small follower's maximum production time is θ_{F_b} or θ_{F_s} respectively.

The timing of the game is illustrated in Figure 2.1.

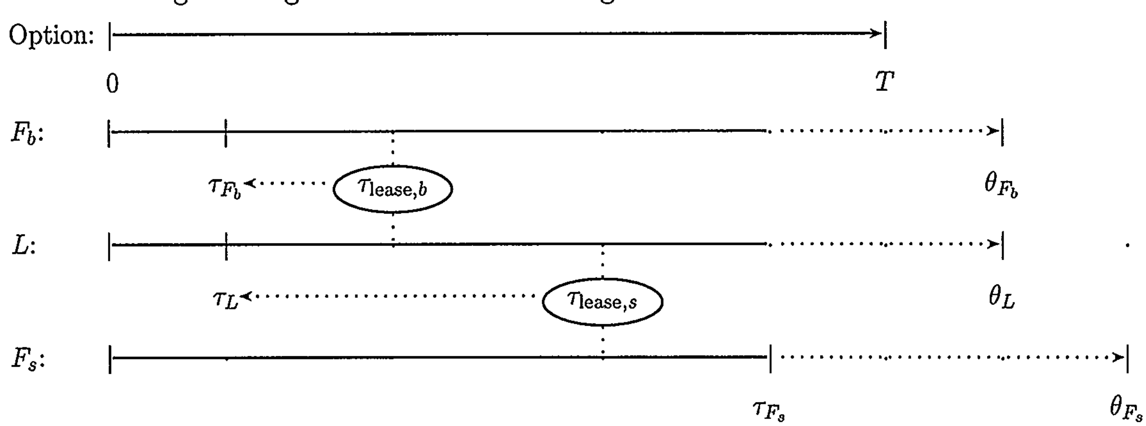


Figure 2.1: **The leader and the follower's timeline**

I now formally construct this sequential bargaining game under incomplete information. There are two players in the game, the leader and the follower. The product to be traded is the leader's excess processing capacity, where the leader sells capacity to the follower. The quantity of product to be traded is the contracted fixed lease production capacity per unit of time q_{FL}. The network effect is the gain from cooperation. The transfer is the leasing fee l from the follower to the leader. The leader knows its cost of providing the excess capacity $K(\cdot)$. The follower has private information about its valuation $l_F \in \{\underline{l}_F, \overline{l}_F\}$, which will be defined in Section 2.3.3. The benefit from bargaining with the leader is smaller for the big follower than for the small follower. Hence, there are two types of buyers, the low type buyer (the big follower, F_b) who values the lease at $\underline{l}_F$ and the high type buyer (the small follower, F_s) who values the lease at $\overline{l}_F$. The leader does not know what type of buyer the follower is. Therefore, there is a conflict between efficiency (the realization of the gain from cooperation) and rent extraction in mechanism design. The leader's strategy space is to offer the lease at either $\underline{l}_F$, or $\overline{l}_F$.[6] The follower's strategy space is to either accept or reject the leader's offer. If the follower accepts, the game ends. If the follower rejects, the leader will make another offer in the next period. The decision variables are the leasing rate l, the cooperative and non-cooperative plant capacity choices q_L^{Ω}, or q_L^{c} and q_F^{c}, which determine the construction costs $K(q_L^{\Omega})$, or $K(q_L^{c})$, $K(q_F^{c})$ and production volumes q_L and q_F. The players' expected payoff functions will be discussed in detail in Section 2.4. The exogenous variables are the stochastic gas price P, the expected reserve quantities at the time of construction, Q_L and Q_F as assumed in Section 2.1, and the network effect N.

[6] I decide to analyze the mechanism bargaining on the lease rate l only, in which $q_{F_B L} = q_{F_S L} = q_{\mathrm{FL}}$, which leads to $\int_{\tau_{F_b}}^{\theta_{F_b}} t dt > \int_{\tau_{F_s}}^{\theta_{F_s}} t dt$ because the F_b has larger initial reserve. There is another way of designing the bargaining mechanism. The leader can provide two types of contracts, $\{\underline{l}_F, q_{F_B L}\}$ and $\{\overline{l}_F, q_{F_S L}\}$, where $\underline{l}_F < \overline{l}_F$ and $q_{F_B L} > q_{F_S L}$. This is a bargaining game on both the lease rate and the lease quantity.

2.3.1 Two-stage optimization for adjacent players

Each player i has three decision variables over which it must optimize. One is a function $P(Q_i)$, rather than just a single variable. Player i must select a manifold of prices and quantities that describe the trigger threshold for exercise. It is the same for both players, such that the player i develops as soon as the random variables (P, Q_i) are such that $P \geq P(Q_i)$. The second variable, $q^c_{i,t}$ is the capacity chosen by the players. The third variable, l_t is the reservation lease rate for the players, i.e., the highest lease rate the follower would accept or the lowest rate the leader would accept respectively. Let

$U_{i,\mathbf{coop}}(P, Q_i, q^{\Omega}_{i,\mathbf{coop}}, l_i; N_{\mathbf{coop}})$ be the total enterprise value for player i when it is playing cooperatively, and

$U_{i,\mathrm{nc}}(P, Q_i, q^c_{i,\mathrm{nc}}; N_{\mathrm{nc}})$ be the total enterprise value for player i when it plays non-cooperatively, or in isolation from the other player.

The optimization of player i's non-cooperative enterprise value, $U_{i,\mathrm{nc}}$ is done in two stages.

- Stage 1: For each feasible (P, Q_i), assume that the firm develops the field at that pair. Solve for the optimal capacity as

$$q_i^{c*} = \underset{q_i^c}{\operatorname{argmax}}\, U_{i,\mathrm{nc}}(P, Q_i, q_i^c; N_{\mathrm{nc}}).$$

  The solution is $q_i^{c*}(P, Q_i)$ and the value is $U_{i,\mathrm{nc}}$.

- Stage 2: Given $U_{i,\mathrm{nc}}\big(P, Q_i, q_i^{c*}(P, Q_i); N_{\tau_i}\big)$ as a function of (P, Q_i) solve for the optimal development threshold $P^*_{\tau_i}(Q_i)$.

Use this two-stage process to solve the problem of the follower, as a function of (P, Q_i) and the capacity the leader offers and the lease rate it offers for that capacity.

This will give a reservation lease rate for each capacity and price-quantity pair such that the follower is just indifferent between accepting the lease and taking the non-cooperative value.

Finally, the two-stage solution will determine the leaders optimal capacity to build, capacity to offer for lease and lease rate associated with that capacity. It optimizes, knowing the follower's reaction to the lease rates and capacity offered. Then, it optimizes the threshold for development in the stochastic price-quantity space.

The equilibrium of the game affects the expected payoff $W(P, Q_i, q_i^c, l_i; N)$, which affects the optimal exercise trigger of the option. Conversely, the exercise of the option determines the value of P^* and Q_i^*, which will affect the expected payoff $W(P^*, Q_i^*, q_i^{c*}, l_i^*; N)$ which further affects the refinement of the players' strategy space and hence the equilibrium of the game.

In this game, the adjacent players will maximize their own total enterprise values by optimally controlling their respective capacity choices q_L^c, q_F^c and the lease rate l, given the two stochastic variables P and Q_i that evolve over time, and the exogenous network effect N.

2.3.2 The network effect — gains from cooperation

The network effect N is modeled as the reduction in pipeline tolls, one component of the production cost that affects players' cash flow. Economy of scale and network effect of pipeline arise because the average cost of transporting oil or gas in a pipeline decreases as total throughput increases. As discussed in Church and Ware (1999), there are two categories of costs that generate network effects. In the context of the joint pipeline, they are:

1. Long-run fixed operating costs: The cost of monitoring workers is a long-run fixed cost due to the indivisibility of workers – a minimum number of monitoring workers

is required. This cost is fixed as it is independent of throughput in the pipeline.

2. Capital investment cost

   - *Setup costs:* The expenses associated with the planning, design and installation of the pipeline are fixed setup costs, which results in economies of scale.

   - *Volumetric returns to scale:* The costs of steel are proportionate to the surface area. The capacity of the pipeline depends on its volume and the amount of horsepower required. The amount of horsepower required is determined by resistance to flow, which is decreasing in the diameter of the pipeline.

Among these two cost categories, if the total throughput increases, the long-run fixed operating costs per unit of throughput capacity will decrease, which generates the category 1 network effect N^1. N^1 is monotonic increasing when the total throughput increasing. Hence, producers will get N^1 only when they both produce. In addition, setup costs and volumetric return to scale will generate the category 2 network effect N^2 if the pipeline company is strategic and can anticipate the future exercise of both players. If the pipeline company observes a higher probability that players will be producing together for a certain period of time, it may build a larger pipeline to accommodate both of them. Thus, the producers will get N^2 if the producers can make a commitment to a larger throughput volume.

The pipeline company has to decide whether to build and, if it builds, what the capacity and toll rate should be. For simplicity, I will assume that, based on the information about both producers' initial reserve Q_L, Q_F and production rate q_L, q_F, the pipeline company can estimate and build a pipeline to accommodate the non-cooperative total transportation throughput, $(q_{L,\text{nc}} + q_{F,\text{nc}})$, for the leader and the follower.

$$\text{The actual non-cooperative pipeline throughput} \\ = \begin{cases} q_{L,\text{nc}}(t) & \text{when } t < \tau_F; \\ q_{L,\text{nc}}(t) + q_{F,\text{nc}}(t) & \text{when } t \geq \tau_F. \end{cases}$$

This results in a higher pipeline toll rate for the leader before τ_F,[7] and a lower pipeline toll rate (category 1 network effect N^1) for both producers after τ_F as the total throughput transported increases. If the lease contract is negotiated successfully at $\tau_{\text{lease}} < \tau_F$ or even simultaneously at τ_L, the pipeline company sees the producers' commitment, and it will construct a larger pipeline to accommodate this larger cooperative total throughput, $q_{L,\text{coop}}(\tau_{\text{lease}}) + q_{F,\text{coop}}$, which will generate the category 2 network effect, N^2.

$$\text{The actual cooperative pipeline throughput} \\ = \begin{cases} q_{L,\text{coop}}(t) & \text{when } t < \tau_{\text{lease}}; \\ q_{L,\text{coop}}(t) + q_{F,\text{coop}}(t) & \text{when } t \geq \tau_{\text{lease}}. \end{cases}$$

2.3.3 The follower's individual rationality constraint

Small follower F_s's IR

The small follower can either lease the capacity from the leader at τ_{lease}, or delay further until τ_{F_s} to build its own plant. The small follower gets the network effect in both cases. The difference is that if it chooses to build its own plant, the benefit of the network effect comes only after τ_{F_s} and will end at θ_L when the leader's production ends. Denote this network benefit for a small follower that builds its own plant as $N_{\tau_{F_s}}^{\theta_L} = N \cdot \int_{\tau_{F_s}}^{\theta_L} q_{Lt}\, dt$. If it chooses to lease, the lease contract may allow the small follower to start production earlier

[7] I will suppress the subscript B and S for F if I are not differentiating the F_b from the F_s in the context.

than τ_{F_s} and small follower will get the network effect in the interval $[\tau_{\text{lease}}, \theta_L]$. Denote this network benefit for the small follower who leases the plant as $N^{\theta_L}_{\tau_{\text{lease}}} = N \cdot \int_{\tau_{\text{lease}}}^{\theta_L} q_{Lt}\, dt$. Clearly, $N^{\theta_L}_{\tau_{\text{lease}}} > N^{\theta_L}_{\tau_{F_s}}$ as $\tau_{\text{lease}} < \tau_{F_s}$. Therefore, for small follower, the lease contract not only saves capital investment,[8] but also increases the total amount of network effect received. The small follower will make the comparison of $U_{F_s,\text{nc}}$ and $U_{F_s,\text{coop}}$ at the date after τ_L whenever the leader offers a lease at rate l. This gives the small follower's *individual rationality constraint*:

$$U_{F_s,\text{coop}}(P, Q_{F_s}, \bar{l}_F; N^{\theta_L}_{\tau_{\text{lease}}}) \geq U_{F_s,\text{nc}}(P, Q_{F_s}, q^{c*}_{F_s}; N^{\theta_L}_{\tau_{F_s}}) \tag{2.9}$$

which defines the high type buyer's valuation of the lease:

$$\bar{l}_F \equiv \sup\left\{ l_F \in \mathbb{R}^+ : U_{F_s,\text{coop}} \geq U_{F_s,\text{nc}}|_{q^c_{F_s} = q^{c*}_{F_s}} \right\}. \tag{2.10}$$

Big follower F_b's IR

The big follower develops the field at the same time as the leader. The big follower's *individual rationality constraint* is:

$$U_{F_b,\text{coop}}(P, Q_{F_b}, \underline{l}_F; N^{\theta_L}_{\tau_{\text{lease}} = \tau_{F_b}}) \geq U_{F_b,\text{nc}}(P, Q_{F_b}, q^{c*}_{F_b}; N^{\theta_L}_{\tau_{F_b}}). \tag{2.11}$$

For the big follower, $N^{\theta_L}_{\tau_{\text{lease}}} = N^{\theta_L}_{\tau_{F_b}}$, the lease does not increase its total amount of network effect received, but reduces its capital cost. Hence, the low type buyer's valuation of the lease:

$$\underline{l}_F \equiv \sup\left\{ l_F \in \mathbb{R}^+ : U_{F_b,\text{coop}} \geq U_{F_b,\text{nc}}|_{q^c_{F_b} = q^{c*}_{F_b}} \right\}. \tag{2.12}$$

[8] The annual cost of owning an asset over the its entire life is calculated as $\text{EAC}(K(q^c_F)) = \frac{K(q^c_F)r}{1-(1+r)^{-n}}$.

Notice the right hand sides of equation (2.10) and (2.12) are optimized over q_F^c, which means $U_{F,\text{coop}}$ has to be greater than $U_{F,\text{nc}}$ when the follower builds the optimal capacity for itself. Since $U_{F,\text{coop}}$ is decreasing in l, when equation (2.10) and (2.12) are binding, they determine a reservation lease rate $\bar{l}_F$ or $\underline{l}_F$ for the small follower or the big follower respectively.

2.3.4 The leader's individual rationality constraints

At τ_L, the leader has a non-cooperative optimal capacity q_L^c which maximizes its total non-cooperative enterprise value $U_{L,\text{nc}}(P, Q_L, q_L^c; N_{\tau_F}^{\theta_L})$, where $N_{\tau_F}^{\theta_L} = N \cdot \int_{\tau_F}^{\theta_L} q_{Lt}\, dt$.

$$q_L^{c*} = \underset{q_L^c}{\operatorname{argmax}}\, U_{L,\text{nc}}(P, Q_L, q_L^c; N_{\tau_F}^{\theta_L}).$$

A non-cooperative leader is a leader who does not consider the possibility of leasing excess capacity to the follower in the future. Thus, the $U_{L,\text{nc}}$ function does not involve a lease rate l. The network effect $N_{\tau_F}^{\theta_L}$ occurs when the follower's production starts at τ_F and ends at θ_L. This is different from the leader's cooperative enterprise value $U_{L,\text{coop}}(P, Q_L, q_L^\Omega, l; N_{\tau_\text{lease}}^{\theta_L})$ as defined in subsection 2.3.1, where $N_{\tau_\text{lease}}^{\theta_L} = N \cdot \int_{\tau_\text{lease}}^{\theta_L} q_{Lt}\, dt$. This early network effect $N_{\tau_\text{lease}}^{\theta_L}$ occurs when the follower's production starts at τ_lease and ends at θ_L. I now define the leader's cooperative optimal capacity as:

$$\begin{aligned} q_L^{\Omega*} &= \underset{q_L^\Omega}{\operatorname{argmax}}\, U_{L,\text{coop}}(P, Q_L, q_L^\Omega, l; N_{\tau_\text{lease}}^{\theta_L}) \\ \text{st.} &\quad \tau_\text{lease} \le \tau_F. \end{aligned} \tag{2.13}$$

The leader will build cooperative capacity if the following *individual rationality or participation constraint* $I(\text{IR}_\text{I})$ is satisfied:

$$U_{L,\text{coop}}(P, Q_L, q_L^{\Omega*}, l; N_{\tau_\text{lease}}^{\theta_L}) \ge U_{L,\text{nc}}(P, Q_L, q_L^{c*}; N_{\tau_F}^{\theta_L}). \tag{2.14}$$

Moreover, the leader's additional cost of building extra capacity $(q_L^\Omega - q_L^c)$ has to be compensated by the present value of all future leasing fees, plus the benefit difference between $N^{\theta_L}_{\tau_{\text{lease}}}$ and $N^{\theta_L}_{\tau_F}$, i.e., the leader's *participation constraint II* (IR_{II}):

$$
\begin{aligned}
&\int_{\tau_{\text{lease}}}^{\theta_F} e^{-rt}(q_{\text{FL}} \cdot l)\ dt + \left(N^{\theta_L}_{\tau_{\text{lease}}} - N^{\theta_L}_{\tau_F}\right) \geq b \cdot (q_L^\Omega - q_L^c) \\
\Rightarrow &\int_{\tau_{\text{lease}}}^{\theta_F} e^{-rt}(q_{\text{FL}} \cdot l)\ dt + N \cdot \int_{\tau_{\text{lease}}}^{\tau_F} q_{Lt}\ dt \geq b(q_L^\Omega - q_L^c).
\end{aligned}
\tag{2.15}
$$

If inequalities (2.14) and (2.15) are binding, they determine the leader's cooperative capacity q_L^Ω and the lease rate l. Otherwise, they set the upper bound for q_L^Ω and lower bound for l.

If the follower is the high type F_s, the leader obtains an increase in network effect. Equation (2.15) then becomes:

$$
\int_{\tau_{\text{lease}}}^{\theta_{F_s}} e^{-rt}(q_{\text{FL}} \cdot \bar{l}_F)\ dt + N \cdot \int_{\tau_{\text{lease}}}^{\tau_{F_s}} q_{Lt}\ dt \geq b(q_L^\Omega - q_L^c). \tag{2.16}
$$

If the follower is the low type F_b, the leader obtains no increase in network effect by encouraging F_b to lease because $\tau_{F_b} = \tau_L$, and $\tau_{\text{lease}} = \tau_L \Longrightarrow N^{\theta_L}_{\tau_{\text{lease}}} = N^{\theta_L}_{\tau_F}$. If F_b accepts the lease, it saves the capital cost of $K(q^c_{F_b})$. Equation (2.15) then becomes:

$$
\int_{\tau_{\text{lease}}=\tau_L=\tau_{F_b}}^{\theta_{F_b}} e^{-rt}(q_{\text{FL}} \cdot \underline{l}_F)\ dt \geq b(q_L^\Omega - q_L^c). \tag{2.17}
$$

In other words, $\bar{l}_F$ and $\underline{l}_F$ defined in equation (2.10) and (2.12) have to satisfy equation (2.16) and (2.17) respectively, in order to give the leader enough motivation to build extra capacity.

Also, it makes no sense for the leader to build cooperative capacity that cannot be used when production is at a maximum, so by (2.1), $q_L^\Omega \leq \overline{q}_{L,\tau_L} + \overline{q}_{F,\tau_L} = \overline{\alpha}_L Q_{L,\tau_L} + \overline{\alpha}_F Q_{F,\tau_L}$. If this inequality is strict, the joint production is constrained until the leader and follower have produced enough so that their combined maximum production rate is below the plant capacity. The leader's cooperative capacity has to be at least as large as its own maximum production rate, i.e., $q_L^{\Omega} \geq q_{L,\tau_L}$.

2.3.5 The leader's control set $\{q_L^{\Omega}, l\}$

Recall that q_L and q_F[9] are defined as the leader's and the follower's production volume respectively, q_L^c is the leader's non-cooperative capacity and $\overline{\alpha}_L$ and $\overline{\alpha}_F$ are the maximum production rates that are set by a regulator or technological constraints. From equation (2.4), the non-cooperative leader and follower's production functions are:

$$q_{L,\text{nc}}(t) = \begin{cases} q_L^c & t \in [\tau_L, \theta_{L,\text{trans}}]; \\ \overline{\alpha}_L Q_L(\theta_{L,\text{trans}}) e^{-\overline{\alpha}_L(t-\theta_{L,\text{trans}})} & t \in [\theta_{L,\text{trans}}, \theta_L], \end{cases} \tag{2.18}$$

and

$$q_{F,\text{nc}}(t) = \begin{cases} q_F^c & t \in [\tau_L, \theta_{F,\text{trans}}]; \\ \overline{\alpha}_F Q_F(\theta_{F,\text{trans}}) e^{-\overline{\alpha}_F(t-\theta_{F,\text{trans}})} & t \in [\theta_{F,\text{trans}}, \theta_F]. \end{cases} \tag{2.19}$$

After $\theta_{L,\text{trans}}$, the non-cooperative leader's capacity is not binding, and it can offer the follower its excess processing capacity $q_L^c - q_L$ providing the follower has not built its own plant yet.

This gives the cooperative follower's production volume under leasing:

$$q_{F,\text{coop}} = \min\{q_L^c - q_L,\ \overline{\alpha}_F Q_F\}.$$

Suppose that there is asymmetric information about the leader's and follower's initial reserves. The leader can only make an estimation about the follower's expected initial

[9]For notation simplicity, I suppress the subscripts S and B for F in this subsection as F_b and F_s's production functions share the same functional form.

reserve quantity Q_F and maximum production rate and $\overline{\alpha}_F$. Based on this estimation, the leader builds a gas plant which can process the amount $q_L^{\Omega} \geq q_{L,\text{coop}} + q_{F,\text{coop}}$ per unit of time. The results of the bargaining game depend on the amount of information available to the leader and the follower. The cooperative leader will estimate both producers' needs and build a gas plant with capacity $q_L^{\Omega} \geq q_L^c$. Therefore, the above production function becomes:[10]

$$\begin{gathered} q_{F,\text{coop}} = \min\{q_L^{\Omega} - q_{L,\text{coop}},\ \overline{\alpha}_F Q_F\}; \\ 0 \leq q_{L,\text{coop}} \leq \min\{q_L^{\Omega}, \overline{\alpha}_L Q_L\}. \end{gathered} \tag{2.20}$$

The cooperative leader has an excess capacity of $q_L^{\Omega} - q_{L,\text{coop}}$, which will increase as the leader's production volume $q_{L,\text{coop}}$ falls over time. Assume that the cooperative follower will use all the capacity offered in the lease until reserves drop to constrain the production rate. That is, $q_{F,\text{coop}} = \min\{q_{\text{FL}}, \overline{\alpha}_F Q_F\}$. Once excess capacity reaches the contracted leasing capacity q_{FL} at τ_{lease}, the lease can start. The cooperative production function is:

$$q_{L,\text{coop}}(t) = \begin{cases} q_L^{\Omega} & t \in [\tau_L, \theta_{L,\text{trans}}]; \\ \overline{\alpha}_L Q_L(\theta_{L,\text{trans}}) e^{-\overline{\alpha}_L(t-\theta_{L,\text{trans}})} & t \in [\theta_{L,\text{trans}}, \theta_L], \end{cases} \tag{2.21}$$

and

$$q_{F,\text{coop}}(t) = \begin{cases} q_{\text{FL}} & t \in [\tau_{\text{lease}}, \theta_{F,\text{trans}}]; \\ \overline{\alpha}_F Q_F(\theta_{F,\text{trans}}) e^{-\overline{\alpha}_F(t-\theta_{F,\text{trans}})} & t \in [\theta_{F,\text{trans}}, \theta_F]. \end{cases} \tag{2.22}$$

The cooperative leader's choices about q_L^{Ω} and l will have opposite effects on τ_{lease}. On one hand, the cooperative leader can control an early or late τ_{lease} by controlling

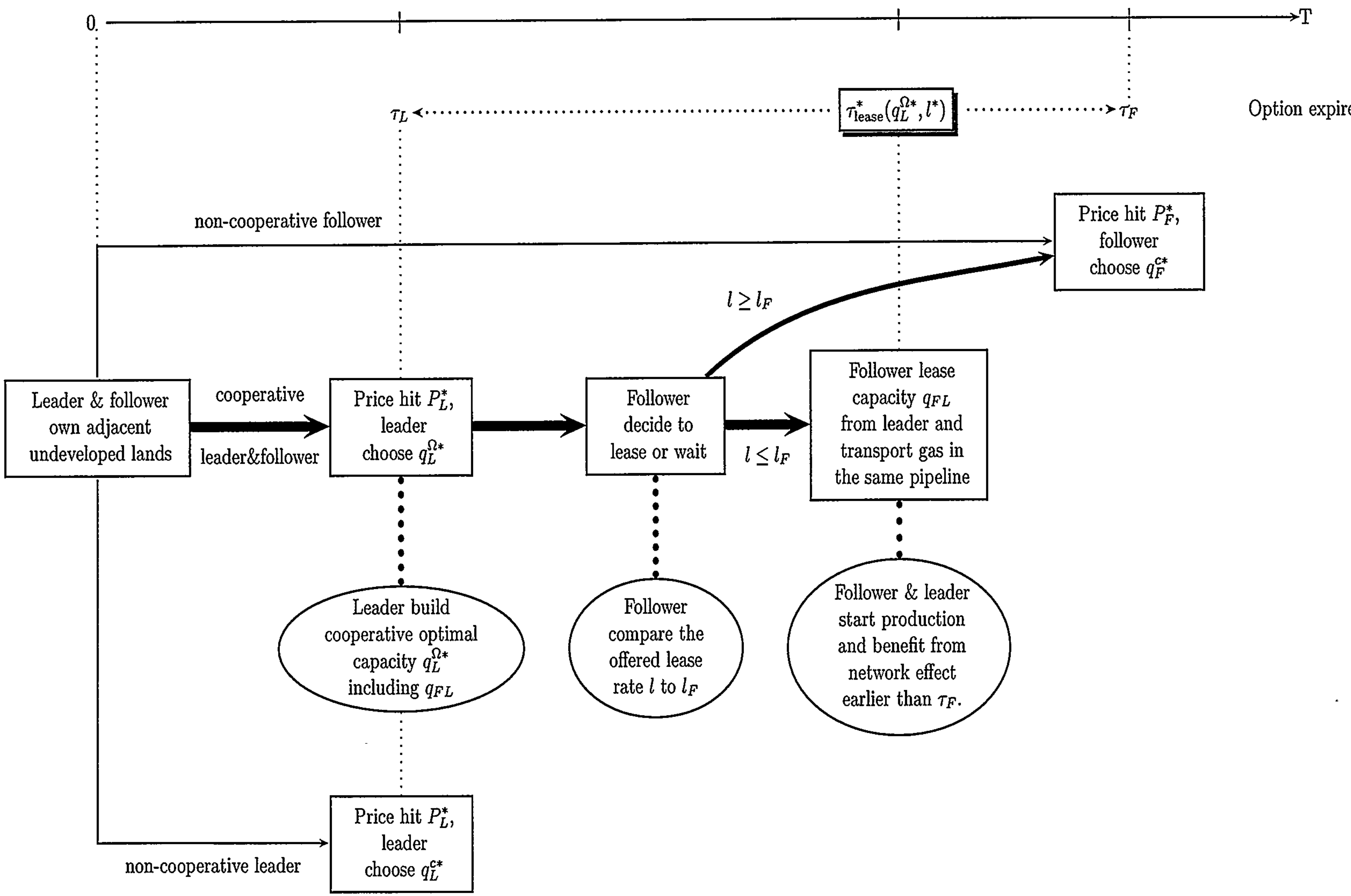
[10] The production volume for the leader might be set at the upper constraint in equation (2.1), but it is also possible that the leader will constrain production to induce the follower to enter, so it may also negotiate with the follower on the time-profile of gas plant capacity offered, as well as the lease rate.

the size of its cooperative capacity q_L^Ω. When q_L^Ω is larger, the lease can happen earlier. The earlier lease will allow the cooperative leader to benefit from the network effect earlier than τ_F. The incremental benefit of this earlier network effect is calculated as $N\big(\int_{\tau_{\text{lease}}}^{\theta_L} q_{Lt}\ dt - \int_{\tau_F}^{\theta_L} q_{Lt}\ dt\big)$ in equation (2.15). On the other hand, the cooperative leader wants to charge the follower the highest leasing rate up to $\bar{l}_F$ for a small follower or $\underline{l}_F$ for a big follower as defined by equation (2.10) and (2.12).[11] Thus, the lease offer is inversely related to the time the lease is accepted. The cooperative leader's objective is to find a balance among the incremental network effect benefit, the earlier leasing fee, and the extra construction costs of $q_L^\Omega - q_L^c$, bearing in mind the fact that a high lease rate will cause the follower to delay. Denote this equilibrium leader cooperative capacity as $q_L^{\Omega *}$, which gives the leader the largest total enterprise value and also ensures $\tau_{\text{lease}}^* \leq \tau_F$, as defined in equation (2.13).

In addition, both the leader and the follower will have to consider how much pipeline space to request and the term of the request. If the producer(s) commit(s) to a larger volume or longer-term contract, the pipeline toll rates will be even smaller, generating a category 2 network effect as discussed in Section 2.3.2. The leader and the follower's strategy map is shown in Figure 2.2.

[11] In fact, this is the standard way of extracting rents through price discrimination without losing the efficiency.

Figure 2.2: **The leader and the follower's strategy map with timeline**



2.4 The Leader's and the follower's Cash Flows and Expected Payoff

Let C be the variable production cost for both the leader and the follower, including the pipeline tolls. The network effect N is the toll reduction that arises from transporting a larger amount of oil and gas with smaller unit breakeven toll rates.

2.4.1 Non-cooperative leader and small follower

In this case, the leader and the small follower each build up a gas plant to process their own gas separately. The leader builds a plant only large enough to process its own gas. The small follower enters later and builds its own plant. The leader will not get the network effect until the small follower also starts producing. The leader builds at the stopping time $\tau_L \geq 0$ and the small follower builds at $\tau_{F_s} > \tau_L$.

Stage 1: $t \in (\tau_L, \tau_{F_s})$, only the leader produces

The leader has started production but the small follower is still waiting. The network effect does not exist at this stage because the pipeline can only charge the leader. The operating profit is

$$\pi^{S1}_{L,\mathrm{nc},t|F_s} = (P_t - C)q_{L,\mathrm{nc},t}, \quad t \in (\tau_L, \tau_{F_s})$$

where $q_{L,\mathrm{nc},t}$ is defined in equation (2.18). The risk-neutral expected payoff to the leader is

$$W^{S1}_{L,\mathrm{nc},\tau_L|F_s} = \widehat{E}_{\tau_L} \int_{\tau_L}^{\tau_{F_s}} e^{-rt} \pi^{S1}_{L,\mathrm{nc},t|F_s}\, dt$$

where $\widehat{E}_t$ is the risk-neutral expectation conditional on information available at time t. The small follower has not built yet in this stage and therefore its cashflow is zero.

Stage 2: $t \in (\tau_{F_s}, \theta_L)$, the leader and the small follower both produce

The small follower enters at τ_{F_s}, but can only ship gas in the residual space on the pipeline, which was built to accommodate non-cooperative total throughput. The leader and the small follower will get the network effect in this stage, and their cash flows will be:

$$\pi^{S2}_{L,\text{nc},t|F_s} = (P_t - C + N)q_{L,\text{nc},t}, \quad t \in (\tau_{F_s}, \theta_L)$$
$$\pi^{S2}_{F_s,\text{nc},t} = (P_t - C + N)q_{F_s,\text{nc},t}, \quad t \in (\tau_{F_s}, \theta_L)$$

where $q_{F_s,\text{nc},t}$ is defined in equation (2.19) by replacing F with F_s. The expected payoffs to the leader and the small follower are, respectively:

$$W^{S2}_{L,\text{nc},\tau_{F_s}|F_s} = \widehat{E}_{\tau_{F_s}} \int_{\tau_{F_s}}^{\theta_L} e^{-rt}\pi^{S2}_{L,\text{nc},t|F_s}dt$$
$$\text{and} \quad W^{S2}_{F_s,\text{nc},\tau_{F_s}} = \widehat{E}_{\tau_{F_s}} \int_{\tau_{F_s}}^{\theta_L} e^{-rt}\pi^{S2}_{F_s,\text{nc},t}dt.$$

Stage 3: $t \in (\theta_L, \theta_{F_s})$, the leader's production ends and only the small follower remains in production

The leader's production ends at θ_L the small follower's production ends at θ_{F_s}. I assume that the leader and follower take the same amount of time to deplete their fields. Thus $\theta_L - \tau_L = \theta_{F_s} - \tau_{F_s}$. As the leader's production starts earlier, I have $\theta_L < \theta_{F_s}$. The follower's cash flow and expected payoff are:

$$\pi^{S3}_{F_s,\text{nc},t} = (P_t - C)q_{F_s,\text{nc},t}, \quad t \in (\theta_L, \theta_{F_s})$$
$$W^{S3}_{F_s,\text{nc},\theta_L} = \widehat{E}_{\theta_L} \int_{\theta_L}^{\theta_{F_s}} e^{-rt}\pi^{S3}_{F_s,\text{nc},t}dt.$$

To sum up, the non-cooperative leader and small follower's total expected payoff from all three stages are:

$$W_{L,\mathrm{nc},\tau_L|F_s} = \widehat{E}_0\Bigg(\widehat{E}_{\tau_L}\int_{\tau_L}^{\tau_{F_s}} e^{-rt}\pi^{S1}_{L,\mathrm{nc},t|F_s}\,dt + \\ e^{-r(\tau_{F_s}-\tau_L)}\widehat{E}_{\tau_{F_s}}\int_{\tau_{F_s}}^{\theta_L} e^{-rt}\pi^{S2}_{L,\mathrm{nc},t|F_s}dt - K(q^c_L)\Bigg)$$

and

$$W_{F_s,\mathrm{nc},\tau_L} = \widehat{E}_0\Bigg(e^{-r(\tau_{F_s}-\tau_L)}\widehat{E}_{\tau_{F_s}}\int_{\tau_{F_s}}^{\theta_L} e^{-rt}\pi^{S2}_{F_s,\mathrm{nc},t}dt + \\ e^{-r(\theta_L-\tau_L)}\widehat{E}_{\theta_L}\int_{\theta_L}^{\theta_{F_s}} e^{-rt}\pi^{S3}_{F_s,\mathrm{nc},t}dt - K(q^c_{F_s})\Bigg).$$

2.4.2 Non-cooperative leader and big follower

In this case, the leader and the big follower exercise their real option to invest simultaneously at $\tau_L = \tau_{F_b}$. They each build up a gas plant to process their own gas separately. They will get the network effect during the whole production life, and their cash flows will be:

$$\pi_{L,\mathrm{nc},t|F_b} = (P_t - C + N)q_{L,\mathrm{nc},t}, \quad t \in (\tau_L, \theta_L)$$
$$\pi_{F_b,\mathrm{nc},t} = (P_t - C + N)q_{F_b,\mathrm{nc},t}, \quad t \in (\tau_{F_b}, \theta_L)$$

where $q_{F_b,\mathrm{nc},t}$ is defined in equation (2.19) if substituting F with F_b. The expected payoff to the leader and the big follower are:

$$W_{L,\mathrm{nc},\tau_L|F_b} = \widehat{E}_0\Bigg(\widehat{E}_{\tau_L}\int_{\tau_L}^{\theta_L} e^{-rt}\pi_{L,\mathrm{nc},t|F_b}dt - K(q^c_L)\Bigg) \tag{2.23}$$

and $$W_{F_b,\mathrm{nc},\tau_{F_b}} = \widehat{E}_0\Bigg(\widehat{E}_{\tau_{F_b}}\int_{\tau_{F_b}}^{\theta_L} e^{-rt}\pi_{F_b,\mathrm{nc},t}dt - K(q^c_{F_b})\Bigg). \tag{2.24}$$

2.4.3 Cooperative leader and small follower

Stage 1: $t \in (\tau_L, \tau_{\text{lease}})$, only the leader produces

As discussed in Section 2.3.5, the leader may want to build a bigger gas plant of cooperative capacity q_L^{Ω} with construction costs $K(q_L^{\Omega})$. It then offers to lease the excess processing capacity to the small follower at a processing rate l. The leader's cash flow and risk-neutral expected payoff:

$$
\begin{aligned}
\pi^{S1}_{L,\text{coop},t|F_s} &= (P_t - C)q_{L,\text{coop},t}, \quad t \in (\tau_L, \tau_{\text{lease}}) \\
W^{S1}_{L,\text{coop},\tau_L|F_s} &= \widehat{E}_{\tau_L} \int_{\tau_L}^{\tau_{\text{lease}}} e^{-rt} \pi^{S1}_{L,\text{coop},t|F_s} dt
\end{aligned}
$$

where $q_{L,\text{coop},t}$ is defined in equation (2.21). The lease has not started and the small follower is waiting in this stage.

Stage 2: $t \in (\tau_{\text{lease}}, \theta_L)$, the lease starts, the leader and the small follower both produce

In this stage, the small follower agrees to lease the plant capacity from the leader. They both produce and receive the network effect. The cash flows to the leader and the small follower are:

$$
\begin{aligned}
\pi^{S2}_{L,\text{coop},t|F_s} &= (P_t - C + N)q_{L,\text{coop},t} + q_{\text{FL}}l, \quad t \in (\tau_{\text{lease}}, \theta_L) \\
\pi^{S2}_{F_s,\text{coop},t} &= (P_t - C + N)q_{F_s,\text{coop},t} - q_{\text{FL}}l, \quad t \in (\tau_{\text{lease}}, \theta_L)
\end{aligned}
$$

where $q_{F_s,\text{coop},t}$ is defined in equation (2.22) if substituting F with F_s. Their expected payoffs are:

$$W^{S2}_{L,\text{coop},\eta_{\text{case}}|F_s} = \widehat{E}_{\eta_{\text{case}}} \int_{\eta_{\text{case}}}^{\theta_L} e^{-rt}\pi^{S2}_{L,\text{coop},t|F_s} dt$$

$$\text{and} \quad W^{S2}_{F_s,\text{coop},\eta_{\text{case}}} = \widehat{E}_{\eta_{\text{case}}} \int_{\eta_{\text{case}}}^{\theta_L} e^{-rt}\pi^{S2}_{F_s,\text{coop},t} dt.$$

Stage 3: $t \in (\theta_L, \theta_{F_s})$, the leader's production ends and only the small follower produces

Similarly, the leader's production ends at θ_L, and the small follower continues until θ_{F_s}. The do not receive the network effect. The leader still receives the leasing fee. The leader's cash flow and expected payoff are:

$$\pi^{S3}_{L,\text{coop},t|F_s} = q_{\text{FL}} l$$

$$W^{S3}_{L,\text{coop},\theta_L|F_s} = \int_{\theta_L}^{\theta_{F_s}} e^{-rt}\pi^{S3}_{L,\text{coop},t|F_s} dt = \int_{\theta_L}^{\theta_{F_s}} e^{-rt} q_{\text{FL}} l dt.$$

The small follower's cash flow and expected payoff are:

$$\pi^{S3}_{F_s,\text{coop},t} = (P_t - C) q_{F_s,\text{coop},t} - q_{\text{FL}} l \quad , t \in (\theta_L, \theta_{F_s})$$

$$W^{S3}_{F_s,\text{coop},\theta_L} = \widehat{E}_{\theta_L} \int_{\theta_L}^{\theta_{F_s}} e^{-rt}\pi^{S3}_{F_s,\text{coop},t} dt.$$

To sum up, the cooperative leader and small follower's total expected payoff from all three stages are:

$$\begin{aligned} W_{L,\text{coop},\tau_L|F_s} = \widehat{E}_0 \Bigg(&\widehat{E}_{\tau_L} \int_{\tau_L}^{\eta_{\text{case}}} e^{-rt}\pi^{S1}_{L,\text{coop},t}\, dt \\ &+ e^{-r(\eta_{\text{case}}-\tau_L)} \widehat{E}_{\eta_{\text{case}}} \int_{\eta_{\text{case}}}^{\theta_L} e^{-rt}\pi^{S2}_{L,\text{coop},t} dt + e^{-r(\theta_L-\tau_L)} \int_{\theta_L}^{\theta_{F_s}} e^{-rt} q_{\text{FL}} l dt - K(q_L^{\Omega}) \Bigg) \end{aligned}$$

and

$$W_{F_s,\text{coop},\tau_L} = \widehat{E}_0\Bigg(e^{-r(\tau_{\text{lease}}-\tau_L)}\widehat{E}_{\tau_{\text{lease}}}\int_{\tau_{\text{lease}}}^{\theta_L} e^{-rt}\pi^{S2}_{F_s,\text{coop},t}dt+$$
$$e^{-r(\theta_L-\tau_L)}\widehat{E}_{\theta_L}\int_{\theta_L}^{\theta_{F_s}} e^{-rt}\pi^{S3}_{F_s,\text{coop},t}dt\Bigg).$$

2.4.4 Cooperative leader and big follower

The big follower's IR constraint ensures $\tau_{\text{lease}} \leq \tau_L$. So stage (τ_L, θ_L) converges to stage $(\tau_{\text{lease}}, \theta_L)$ in equilibrium. In this stage, the big follower agrees to lease the plant capacity from the leader. They both produce and receive the network effect. The cash flows to the leader and the big follower are:

$$\pi_{L,\text{coop},t|F_b} = (P_t - C + N)q_{L,\text{coop},t} + q_{\text{FL}}l, \quad t \in (\tau_{\text{lease}}, \theta_L)$$
$$\pi_{F_b,\text{coop},t} = (P_t - C + N)q_{F_b,\text{coop},t} - q_{\text{FL}}l, \quad t \in (\tau_{\text{lease}}, \theta_L)$$

where $q_{F_b,\text{coop},t}$ is defined in equation (2.22) if substituting F with F_b. Their expected payoffs are:

$$W_{L,\text{coop},\tau_{\text{lease}}|F_b} = \widehat{E}_{\tau_{\text{lease}}}\int_{\tau_{\text{lease}}}^{\theta_L} e^{-rt}\pi_{L,\text{coop},t|F_b}dt$$
$$\text{and} \quad W_{F_b,\text{coop},\tau_{\text{lease}}} = \widehat{E}_{\tau_{\text{lease}}}\int_{\tau_{\text{lease}}}^{\theta_L} e^{-rt}\pi_{F_b,\text{coop},t}dt.$$

2.5 The Perfect Bayesian Equilibrium

Now, we extend the backward induction solution to a real option to this game theory setting as in Grenadier (1996); Garlappi (2001); Murto et al. (2003); Imai and Watanabe (2005). This provides a simple computation of a subgame-perfect Nash equilibrium. After explicitly analyzing the player's beliefs, i.e., ruling out non-credible threats and promises, I develop a perfect Bayesian equilibrium for this dynamic bargaining game under incomplete information using Coasian Dynamics as discussed in Fudenberg and Tirole (1991, Ch 10). I assume the leader is chosen exogenously, because one of the two companies has a comparative advantage for entering early (e.g. has a larger reserve[12] or a reserve that has lower drilling costs), and that it naturally moves first.

The enterprise value of the leader (plant lessor or "seller") is common knowledge. The incomplete information aspect of the sequential bargaining is limited to the uncertainty the leader faces about the reservation lease rate of the follower (buyer). As defined in equation (2.10) and (2.12), the high type buyer F_s has a reservation lease rate of $\bar{l}_F$ and the low type buyer F_b has a reservation lease rate of $\underline{l}_F$. If the high type buyer tells the truth, its total enterprise value is $U_{F_s}(P, Q_{F_s}, \bar{l}_F; N^{\theta_L}_{\eta_{\text{lease}}})$. If the high type buyer lies successfully, its total enterprise value is $U_{F_s}(P, Q_{F_s}, \underline{l}_F; N^{\theta_L}_{\eta_{\text{lease}}})$. Since $\bar{l}_F > \underline{l}_F$ and U_{F_s} decreases on l I have

$$U_{F_s}(P, Q_{F_s}, \bar{l}_F; N^{\theta_L}_{\eta_{\text{lease}}}) < U_{F_s}(P, Q_{F_s}, \underline{l}_F; N^{\theta_L}_{\eta_{\text{lease}}}). \tag{2.25}$$

Thus, the high type buyer F_s is motivated to pretend to be the low type buyer F_b. In addition, notice that the follower's valuation is correlated with the leader's cost. A larger plant will allow the lease to start earlier, because of the extra capacity as noted in the discussion after equation (2.20). This makes a more valuable network effect, which increases the follower reservation lease rates $\bar{l}_F$ and $\underline{l}_F$. But a larger plant also incurs larger construction costs. The leader's objective is to extract maximum rents through price discrimination without losing efficiency. The leader wants the follower to accept the lease offer so that the network effect is larger.

I now consider the equilibrium of this game in a two period case. Let $t \in \{t, t+1\}$. The

[12]In Section 3.2, I shall see that larger reserve quantity will subsidize the trigger price, which gives a smaller trigger value $P^*(Q_i)$ and $i \in \{A, B\}$.

ex ante unconditional probability that the follower is high type (F_s) is $\overline{p}$, and $\underline{p} = 1 - \overline{p}$ is the probability that the follower is low type (F_b).

The leader offers lease rates l_t and l_{t+1} at time t and time $t+1$, respectively. Let $\overline{\eta}(l_t)$ denote the leader's posterior probability belief that the follower is high type (F_s) conditional on the rejection of offer l_t in period t, and define $\underline{\eta}(l_t) \equiv 1 - \overline{\eta}(l_t)$. The extensive form representation of this sequential bargaining game is shown in Figure B.1 in Appendix B.

Definition 1. *Define the leader's critical belief as* $\chi \equiv \frac{U_L(\underline{l}_F)}{U_L(\overline{l}_F)}$.

In the last period $t+1$, the leader with probability belief $\overline{\eta}(l_t)$ makes a "take it or leave it" offer l_{t+1}. The follower will accept if and only if this l_{t+1} is not greater than its reservation lease rate.

Theorem 1. *The followers' optimal strategies at date* $t+1$ *are given by:*

$$\text{If } l_{t+1} = \begin{cases} \underline{l}_F, & \text{then } F_s, F_b \text{ both accept.} \\ \overline{l}_F, & \text{then } F_s \text{ accepts}, F_b \text{ rejects.} \\ \text{Random}[\underline{l}_F, \overline{l}_F], & \text{then } F_s \text{ accepts}, F_b \text{ rejects.} \end{cases} \tag{2.26}$$

If the leader offers $l_{t+1} = \underline{l}_F$, both type followers will accept, the leader obtains the enterprise value of $U_{L,\text{coop}}(P, Q_L, q_L^{\Omega}, \underline{l}_F; N_{\eta_{\text{lease}}}^{\theta_L})$, simplified as $U_L(\underline{l}_F)$. If the leader offers $l_{t+1} = \overline{l}_F$, only the high type follower accepts, so the leader has second period enterprise value of $\overline{\eta} \cdot U_{L,\text{coop}}(P, Q_L, q_L^{\Omega}, \overline{l}_F; N_{\eta_{\text{lease}}}^{\theta_L})$, simplified as $\overline{\eta} \cdot U_L(\overline{l}_F)$.[13]

[13]Since all other variables are the same, I shall simplify the cooperative leader and follower's total enterprise value function as $U_L(\underline{l}_F)$, $U_L(\overline{l}_F)$ and $U_F(\underline{l}_F)$ and $U_F(\overline{l}_F)$ throughout this subsection. The non-cooperative leader and follower do not participate in this game and their total enterprise values only helps to define the reservation lease rate.

Theorem 2. *The leader's optimal strategy at date $t+1$ is given by:*

$$l_{t+1} = \begin{cases} \underline{l}_F, & \text{if } \overline{\eta} < \chi. \\ \overline{l}_F, & \text{if } \overline{\eta} > \chi. \\ \text{Random}[\underline{l}_F, \overline{l}_F], & \text{if } \overline{\eta} = \chi. \end{cases} \tag{2.27}$$

At time t, if the leader offers a lease rate at $l_t = \underline{l}_F$, both type followers will accept. If the leader offers a lease rate at $l_t > \underline{l}_F$, the followers' decisions are more complex.

Definition 2. *Let $y(l_t)$ be the probability that a high type follower F_s accepts l_t. According to the Bayes rule, the leader's posterior probability belief that the follower is high type conditional on the rejection is given by:*

$$\overline{\eta}(l_t) = \frac{\overline{p}(1 - y(l_t))}{\overline{p}(1 - y(l_t)) + \underline{p}}.$$

If the leader offers a lease rate at $l_t > \underline{l}_F$, the high type follower F_s should not reject this l_t with probability 1, because that will make the leader's posterior probability belief $\overline{\eta}(l_t)$ greater than χ and the leader will offer a higher second period lease rate at $l_{t+1} = \overline{l}_F$, so the high type F_s would be better off accepting l_t. On the other hand, the high type follower F_s should not accept l_t with probability 1 either, because that will make the leader's posterior probability belief $\overline{\eta}(l_t)$ less than χ and the leader will offer a lower second period lease rate at $l_{t+1} = \underline{l}_F$, so the high type F_s would be better off rejecting l_t.

Lemma 1. *In equilibrium, when $l_t > \underline{l}_F$, the high type follower has a mixed strategy of randomizing between accept and reject in order to make the leader's posterior belief satisfy $\overline{\eta}(l_t) = \chi$. The leader will offer the second period price l_{t+1} to be any randomization between $\underline{l}_F$ and $\overline{l}_F$. Let $y^*(l_t)$ denote the equilibrium probability with which the high type*

F_s accepts l_t. Then

$$y^*(l_t) = 1 + \frac{\chi \underline{p}}{\overline{p}(\chi - 1)} \in [0, 1] \tag{2.28}$$

which satisfies the equilibrium condition $\overline{\eta}(l_t) = \chi$.

Since the equilibrium has to be Pareto efficient, in order for the high type follower F_s to be indifferent between accepting and rejecting l_t, I need

Definition 3. *Let $x(l_t)$ to be the conditional probability that the high type follower receives the lowest price $\underline{l}_F$ at time $t+1$ if it rejects l_t. Then*

$$x(l_t) = \frac{U_{F_s}(l_t) - U_{F_s}(\bar{l}_F)}{e^{-r}\big(U_{F_s}(\underline{l}_F) - U_{F_s}(\bar{l}_F)\big)}. \tag{2.29}$$

Definition 4. *Let $\tilde{l}_F$ be the lease rate at which the high type follower is indifferent between accepting l_t and rejecting l_t in order to wait for $l_{t+1} = \underline{l}_F$ at time $t+1$. It is defined implicitly by*

$$U_{F_s}(l_t) = U_{F_s}(\tilde{l}_F) = (1 - e^{-r})U_{F_s}(\bar{l}_F) + e^{-r}U_{F_s}(\underline{l}_F).$$

Since the follower's enterprise value function, $U_F(l)$ decreases in l, I now summarize the optimal strategy for the follower at time t.

Theorem 3. *The low type follower only accepts $\underline{l}_F$. The high type follower always accepts an offer $l_t \in [\underline{l}_F, \tilde{l}_F]$, and accepts an offer $l_t \in [\tilde{l}_F, \bar{l}_F]$ with probability y^*.*

Suppose the leader's one period discount factor is e^{-r}. The next theorem provides the equilibrium strategy for the leader at time t.

Theorem 4. *If there is a preponderance of low type followers, defined as $\overline{p} < \chi$, then the leader is pessimistic and its optimal strategy is one of the following:*

$$l_t = \begin{cases} \underline{l}_F, & if \frac{U_L(\tilde{l}_F)}{U_L(\underline{l}_F)} < \dfrac{1 - e^{-r}\underline{p}}{\overline{p}}. \\ \tilde{l}_F, & if \frac{U_L(\tilde{l}_F)}{U_L(\underline{l}_F)} > \dfrac{1 - e^{-r}\underline{p}}{\overline{p}}. \end{cases} \tag{2.30}$$

If $\overline{p} > \chi$, the leader is optimistic and the leader's first period optimal strategy is given by one of the following.

$$l_t = \begin{cases} \underline{l}_F, & if \frac{U_L(\tilde{l}_F)}{U_L(\underline{l}_F)} < \frac{1-e^{-r}\underline{p}}{\overline{p}}, \ and \ \frac{U_L(\bar{l}_F)}{U_L(\underline{l}_F)} < \frac{1-A}{B}. \\ \tilde{l}_F, & if \frac{U_L(\tilde{l}_F)}{U_L(\underline{l}_F)} > \frac{1-e^{-r}\underline{p}}{\overline{p}}, \ and \ BU_L(\bar{l}_F) + (A - e^{-r}\underline{p})U_L(\underline{l}_F) < \overline{p}U_L(\tilde{l}_F). \\ \bar{l}_F, & if \frac{U_L(\bar{l}_F)}{U_L(\underline{l}_F)} > \frac{1-A}{B}, \ and \ BU_L(\bar{l}_F) + (A - e^{-r}\underline{p})U_L(\underline{l}_F) > \overline{p}U_L(\tilde{l}_F). \end{cases} \tag{2.31}$$

where

$$A = e^{-r}\overline{p}(1-y)x + e^{-r}x\underline{p} > 0,$$

$$B = \overline{p}y + e^{-r}\overline{p}(1-y)(1-x) > 0.$$

The proof of Theorems 1 to 4 are given in Appendix C.

The conclusion is thus that there exists a unique perfect Bayesian equilibrium, and that this equilibrium exhibits Coasian dynamics — that is, $\overline{\eta}(l_t) \leq \overline{p}$ for all l_t, so the leader becomes more pessimistic over time, and $l_{t+1} \leq l_t$, so the leader's lease rate offer decreases over time.

Chapter 3

THE SIMULATION OF THE REAL OPTIONS BARGAINING GAME

The whole game now has three decision variables: gas plant size (i.e. processing capacity), the production volume and gas plant processing leasing rate.[1] The gas price is assumed to be purely competitive, and the reserve quantity is random.[2] But they make strategic decisions on the gas plant size and the annual allocation of production volume.

3.1 Summary of the Investment Game

3.1.1 The leader's production decision

Consider a simple situation in which the follower has signed a binding contract with the leader. In this contract, the leader and the follower agree that as long as the follower wants to produce, he has to rent the leader's extra capacity, but the leasing rate will be negotiated when the production volume q_L and q_F is allocated between the two producers. The decision variables are the leasing rate l, the joint and solo plant size choices q_L^{Ω} and q_L^{c}, which determine the construction costs $K_L(q_L^{\Omega})$, $K_L(q_L^{c})$, and the production volumes q_L and q_F. The parameters are the gas price P and expected reserve quantities at the time of construction, Q_L and Q_F.

The leader's production volume is defined as $q_L(q_L^c, \overline{\alpha}_L, Q_{L,t}, P)$. Recall that q_L^c is the

[1]To simplify the analysis of the bargaining game, the production volume for the leader is assumed to be at the upper constraint in equation (2.1), which implies that the leader's all excess production capacity will be leased to the follower. It is also possible that the leader will constrain production to induce the follower to enter, so it may also negotiate with the follower on the time-profile of gas plant capacity offered, as well as the lease rate. This possibility may worth an investigation in the future.

[2]If there exists asymmetric information about the reserve quantity, then the leader and the follower can play a strategic game on this factor, but I won't consider this possibility.

leader's capacity, which is determined by the leader's plant size, and $\overline{\alpha}_L$ is the leader's maximum production rate that is set by a regulator and technological constraints. The leader offers the follower its excess processing capacity $q_L^c - q_L$. Thus, the leader's and the follower's production volume are presented as:

$$q_F = \min\{q_L^c - q_L,\ \overline{\alpha}_F Q_F\}$$
$$0 \leq q_L \leq \overline{\alpha}_L Q_L$$
$$0 \leq q_L \leq q_L^c.$$

There is a production switch when q_i falls below $\overline{\alpha}_i Q_i$ for $i \in \{A, B\}$. The leader will estimate both producers' needs and builds a gas plant with capacity $q_L^{\Omega} \geq q_L^c$. Therefore, the above relationship can be simplified as:

$$q_{\mathrm{FL}} = q_L^{\Omega} - q_L$$
$$0 \leq q_L \leq \min\{\overline{\alpha}_L Q_L, q_L^{\Omega}\}.$$

In addition, both the leader and the follower will have to consider the effect of leasing rate, l charged by the leader and the toll rate reduction (network effect, N) charged by the pipeline company.

3.1.2 The follower's production decision

Assume that the follower will use all the capacity offered in the lease until reserves drop to constrain the production rate. That is, $q_F = \min\{q_{FL}, \overline{\alpha}_F Q_F\}$.

The optimal real option trigger price for the follower can be solved in terms of q_F, denoted as $P_F^*(Q_F, l)$. The leader's objective will be to optimize value by trading off the leasing rate l against the network effect N, bearing in mind that too high a leasing rate will cause the follower to delay. The follower chooses between leasing the processing capacity from the leader and starting production later. Therefore, the leader wants to charge the follower the highest leasing rate up to the point where the follower will start to delay its production. In other words, the leader's task is to find the largest lease rate l that it can charge, such that it induces the follower to start production at the same time as the leader starts production. That is, the leader needs to find the lease rate such that $P_F^*(Q_F, l) = P_L^*(Q_L)$. This will allow it to take advantage of the network effect N. This will give an upper bound for the leasing rate, denoted as $\widehat{l}$.

Suppose the construction cost $K(q^\Omega)$ is a concave increasing function. If the leader chooses not to extract rent from the follower,[3] it will set the leasing rate at

$$\frac{K(q^\Omega)}{\text{PV of production volume with capacity of } q^\Omega}.$$

Suppose there is symmetric information about the leader's and follower's reserves expectations: the leader knows the follower's expected reserve quantity Q_F and maximum production rate and $\overline{\alpha}_F$. The leader builds a gas plant which can process the amount $q_L^\Omega \geq q_L + q_F$ per unit of time. The follower's development or exercise cost is the present value of the leasing fee plus some drilling cost:

$$K = PV(q_F l) + \text{Drilling cost}.$$

For the lease to be successfully negotiated at the same time the leader starts production, the gas price P_t must exceed the follower's exercise hurdle: $P_t \geq P_F^*(Q_F, l)$. For any certain level of l, I can calculate the critical price or exercise hurdle $P_F^*(q_F, l)$. If $P_F^*(Q_F, l) > P_L^*$, then the follower will delay and the network effect is lost; if $P_F^*(Q_F, l) <$

[3] Alternatively, it may be regulated so that it cannot extract rent from the follower.

P_L^*, then the follower earns economic rents. Thus, the equation $P_F^*(Q_F, l) = P_L^*$ implicitly determines the optimal lease rate $\widehat{l}$ for the leader.

The value of the follower's option is the net proceeds from exercising:

$$W(P_F^*, t) = P_{F,t}^* - K$$
$$W_P(P_F^*, t) = 1$$

which are the value matching and smooth-pasting conditions. The solution of the critical value for the follower is given as:[4]

$$P_F^* = \frac{\gamma_+}{\gamma_+ - 1} K$$

where

$$\gamma_{+,-} = \frac{1}{2} + \frac{\delta - r}{\sigma^2} \pm \sqrt{\left(\frac{1}{2} + \frac{\delta - r}{\sigma^2}\right)^2 + \frac{2r}{\sigma^2}}.$$

To make the discussion clearer, I list the functional relationship between variables as

[4]See, for example Pindyck (1978); McDonald and Siegel (1986); Sick (1989); Dixit and Pindyck (1994); Sick (1995); Trigeorgis (1996).

follows (the superscripts refer to scenarios 1 and 2 in Section 2.4:

$$
\begin{aligned}
W_{L,\mathrm{nc}|F_s} &= W_{L,\mathrm{nc}|F_s}(P, q_L, q_L^c; N^{\theta_L}_{\tau_{F_s}}) \\
W_{F_s,\mathrm{nc}} &= W_{F_s,\mathrm{nc}}(P, q_{F_s}, q_{F_s}^c; N^{\theta_L}_{\tau_{F_s}}) \\
W_{L,\mathrm{nc}|F_b} &= W_{L,\mathrm{nc}|F_b}(P, q_L, q_L^c; N^{\theta_L}_{\tau_{F_b}}) \\
W_{F_b,\mathrm{nc}} &= W_{F_b,\mathrm{nc}}(P, q_{F_b}, q_{F_b}^c; N^{\theta_L}_{\tau_{F_b}}) \\
W_{L,\mathrm{coop}|F_s} &= W_{L,\mathrm{coop}|F_s}(P, q_L, q_L^\Omega, q_{\mathrm{FL}}, l; N^{\theta_L}_{\eta_{\mathrm{cease}}}) \\
W_{F_s,\mathrm{coop}} &= W_{F_s,\mathrm{coop}}(P, q_{F_s}, q_{\mathrm{FL}}, l; N^{\theta_L}_{\eta_{\mathrm{cease}}}) \\
W_{L,\mathrm{coop}|F_b} &= W_{L,\mathrm{coop}|F_b}(P, q_L, q_L^\Omega, q_{\mathrm{FL}}, l; N^{\theta_L}_{\eta_{\mathrm{cease}}}) \\
W_{F_b,\mathrm{coop}} &= W_{F_b,\mathrm{coop}}(P, q_{F_b}, q_{\mathrm{FL}}, l; N^{\theta_L}_{\eta_{\mathrm{cease}}}).
\end{aligned}
$$

The key step is the leader's decision about the optimal levels of q_L^Ω, and l. The results of the bargaining game depend on the amount information available to the leader and the follower.

3.1.3 Player's objective functions

Based on the discussions about the leader and the follower's production functions, cash flows, expected payoffs, and total enterprise values in previous sections. The whole game can be summarized as the following.

The leader's objective function system is:

$$
\begin{aligned}
&\max_{\{P_{\tau_L}, Q_{\tau_L}, q_L^\Omega, l\}} U_{L,\mathrm{coop}}(P, Q_L, q_L^\Omega, l; N) \\
&= \int_t^T \int^{\Phi_t} V_{L,\mathrm{coop},t}(P_t, Q_{L,t}, q_{L,t}^\Omega, l_t; N)\, d\Phi_t(P_t, Q_{L,t}; N | q_{L,t-1}^\Omega, l_{t-1})\, dt \\
&\quad + E_{L,t}\Big[W_{L,\mathrm{coop},t}(P_t, Q_{L,t}, q_{L,t}^\Omega, l_t; N)\Big].
\end{aligned} \tag{3.1}
$$

The follower's objective function systems are:

$$
\begin{aligned}
&\max_{\{P_{\tau_F}, Q_{\tau_F}, l\}} U_{F,\text{coop}}(P, Q_F, l; N) \\
&= \int_t^T \int^{\Phi_t} V_{F,\text{coop},t}(P_t, Q_{F,t}, l_t; N)\ d\Phi_t(P_t, Q_{F,t}; N|l_{t-1})\ dt \\
&\quad + E_{F,t}\Big[W_{F,\text{coop},t}(P_t, Q_{F,t}, l_t; N)\Big]
\end{aligned}
\tag{3.2}
$$

and

$$
\begin{aligned}
&\max_{\{P_{\tau_F}, Q_{\tau_F}, q_F^c\}} U_{F,\text{nc}}(P, Q_F, q_F^c; N) \\
&= \int_t^T \int^{\Phi_t} V_{F,\text{nc},t}(P_t, Q_{F,t}, q_{F,t}^c; N)\ d\Phi_t(P_t, Q_{F,t}; N|q_{F,t-1}^c)\ dt \\
&\quad + E_{F,t}\Big[W_{F,\text{nc},t}(P_t, Q_{F,t}, q_{F,t}^c; N)\Big].
\end{aligned}
\tag{3.3}
$$

The equilibrium of this game will be reached when the leader and the follower both reach their own maximum total enterprise values, while obeying the leader's individual rational (IR) constraint I, equation (2.14), the leader's individual rationality constraint II, equation (2.15) and the follower's individual rationality constraints (IR), equation (2.9) and equation (2.11).

3.2 Numerically Solving the Game

This real option game problem has three stochastic variables: commodity price P and expected reserves for the two producers, Q_A, Q_B. Such a three-dimensional problem is not well-suited to numerical solution of the fundamental differential equations, so I will use the least-squares Monte Carlo method to determine the optimal policy. It has been implemented in a real options settings by Broadie and Glasserman (1997); Longstaff and Schwartz (2001); Murto et al. (2003); Gamba (2003). The essence of the technique is to replace the conditional risk-neutral one-step expectation of a binomial lattice model with a conditional expectation formed by regressing realized simulation values on observable variables (price and quantity) known at the start of the time step. With the conditional expectation, one can use the Bellman equation to determine the (approximately) optimal policy at each step. Then, given the optimal policy, the simulation can be run again (or recycled) to calculate the risk-neutral expected values arising from the policy.

The model also generates a sequential game between the two players. Sequential games often generate a large number of equilibria that have to be distinguished by a variety of refinements. However, in this setting, I can impose sequential play by the two players, except at the point where they may develop simultaneously. Even at this point, one of the players will be a natural leader, because one will have larger reserves expectation than the other. Thus, I can reduce the sequential game with simultaneous moves to one with sequential moves. Choosing the Nash Bargaining equilibrium at each point (typically a dominant strategy) will result in a unique solution with subgame-perfect strategies. This point has been established by Garlappi (2001); Murto et al. (2003); Imai and Watanabe (2005).

With the solution to the game, I propose to explore the sensitivity of the threshold boundary manifolds to the parameters faced by the players, compare the results to those of an isolated monopolist making a real options decision and assess the probability of the various game scenarios that can unfold.

The relevant variables may be categorized as:

1. Game-related variables, which players can control and optimize, including leasing rate, l, leasing quantity, q_{FL}, the leader's cooperative plant process capacities, q_L^{Ω}, and the leader and the follower's non-cooperative capacities, q_F^c and q_L^c.

2. Option related variables, which are purely exogenous and not controlled by the

players, including price P, initial reserve quantities, Q_F and Q_L, the network savings effect N, and the limiting regulatory or technical production rate $\bar{\alpha}$.

3. Other variables I am not interested, including all the parameters in the cost function (fixed, variable cost coefficients, drilling cost)[5] and maximum production life.

In order to get a clear idea about the comparative statics of these variables, I need to allow them to vary in my model, i.e., set them as a vector instead of a fixed number. Each vector will add one more dimension to my model. I have already have price P, initial reserve quantities Q_F or Q_L, network effect N and lease rate l as vectors. And the dependent variable, the enterprise value is a function of those five vectors. The best thing I can do in a 3D graph is to graph the value function against any two of those vectors every time.

3.3 The Comparative Statics and the Equilibrium Region Of the Game

3.3.1 The effect of lease contract and network effect on the follower's decisions

The leasing contract is specified by quantity and lease rate, (q_{FL}, l). For the simplicity at this stage, I set the leasing quantity q_{FL} equal to the leader's excess capacity. That is, the leader will build a total capacity q_L^{Ω}, and use q_L itself. The excess capacity is leased to the follower on a "take-or-pay" basis. That is, the follower pays for $q_{\mathrm{FL}} = q_L^{\Omega} - q_L$ whether it can use it or not.[6]

[5]The drilling cost can be set as a linear function of construction cost, but here I take it simply as a fixed number.

[6]I set q_{FL} equal to the government mandated maximum production rate, $\overline{\alpha}$ multiplied the follower's initial reserve Q_F. Over time, as the reserve drops, the follower's production volume could drop below q_{FL}. In fact, the capacity q_{FL} should be an optimized variable.

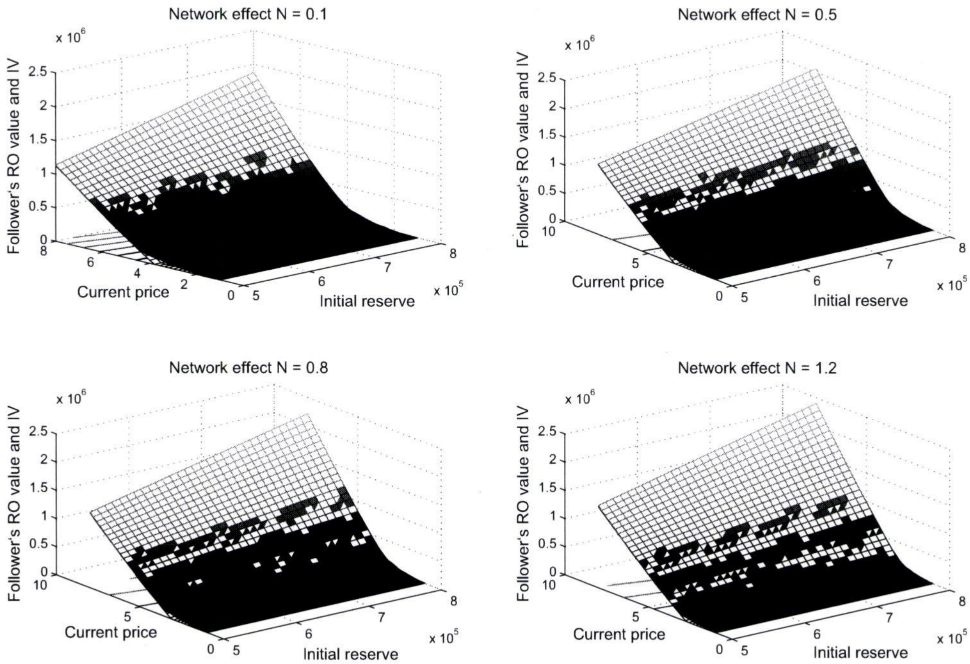


Figure 3.1: The exercise of the follower's real option and the smooth-pasting condition for different network effect levels. The dark shading manifold is the real option value and the light shading manifold is the exercise proceeds. The optimal exercise threshold is at the transition from dark to light.

Figure 3.1 is the graph showing the exercise of the follower's real option.[7] There is a substantial premium associated with the right to develop early. The follower's initial reserve has very little effect on the real option value for very low commodity prices since the option never gets exercised for such low prices. When the commodity prices are higher, the probability of exercising the option is higher, the option value and sensitivity to reserves are higher. Also, as the network effect N gets larger (moving to the right and down), both the follower's real option value and exercise proceeds become larger. But as the network effect increases, the transition from the dark manifold (the real option value) to the light manifold (the exercise proceeds) falls from around commodity price of 6 to around the price of 4, especially for the larger initial reserve. This means, the exercise of real option becomes more sensitive to the larger initial reserves as network effect increases.

Figure 3.2 indicates that the network effect has a positive effect on the follower's maximum[8] non-cooperative enterprise value, $U^*_{F,\text{nc}}$ and its optimal capacity choice q_F^{c*}. Also notice that the q_F^{c*} manifold is not smooth. This means that a small amount of increase in commodity price and initial reserve will not affect the follower's optimal capacity choice. Only for a large enough increase in commodity price and initial reserve, the follower should build a larger optimal capacity.

Figure 3.3 shows a sequence of manifolds of the follower's non-cooperative enterprise values $U_{F,\text{nc}}$ and cooperative enterprise values $U_{F,\text{coop}}$. The $U_{F,\text{nc}}$ manifold has more curvature and stays constant whereas the $U_{F,\text{coop}}$ manifold falls as l increase from the minimum of 0.1 to the maximum of 2.5. By comparing the $U_{F,\text{nc}}$ manifold with the

[7]In the least square Monte-Carlo simulation for estimating option value, I assume the follower's real option to build its own plant has a life of 20 years with quarterly decisions. To conserve the consistency and convergence of results, I choose to divide that 20 year option life into 80 time steps and 100 price paths (simulated 50 and another 50 antithetic paths). At any time step of every price path, the follower can exercise the option if it is optimal.

[8]Recall that $U^*_{F,\text{nc}}$ is achieved by the non-cooperative follower exercising its real option to invest at optimal threshold $P^*(Q_F)$ and choosing the optimal capacity q_F^{c*}

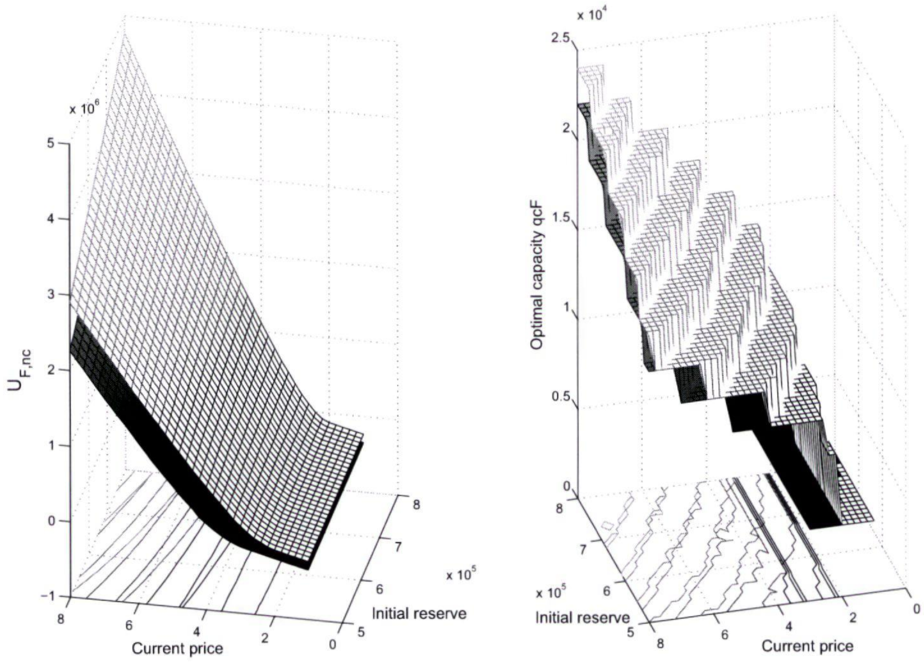


Figure 3.2: The left panel is the follower's maximum non-cooperative enterprise value $U^*_{F,\mathrm{nc}}$ for minimum and maximum network effect level. The right panel is the follower's optimal capacity choice q_F^{c*} for minimum and maximum network effect level. The dark manifold is for minimum network effect level and the light manifold is for maximum network effect level.

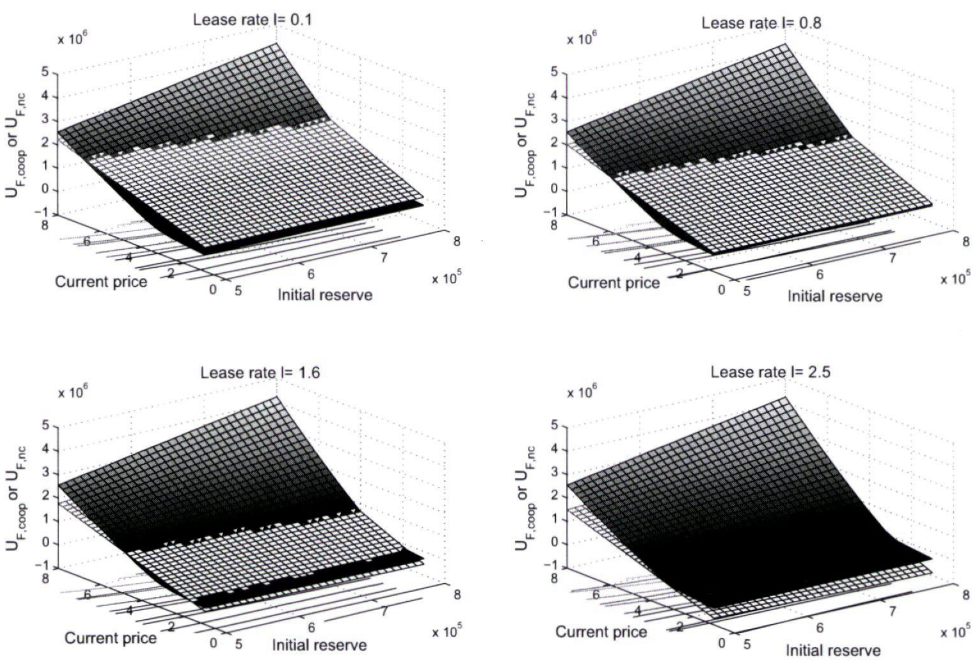


Figure 3.3: Manifolds of the follower's non-cooperative enterprise values $U_{F,\mathrm{nc}}$ and cooperative enterprise values $U_{F,\mathrm{coop}}$ as a function of initial reserves Q_F and commodity price P for successively larger lease rates l. The network effect is at its mean value. The dark manifold is $U_{F,\mathrm{nc}}$. The light manifold is $U_{F,\mathrm{coop}}$.

$U_{F,\text{coop}}$ manifold, the follower can decide whether to accept the lease offer for various commodity price levels and initial reserve levels. In the top two sub-graphs, where the lease rate is low, the $U_{F,\text{nc}}$ manifold is below the $U_{F,\text{coop}}$ manifold when the commodity price is low. This indicates that for lower commodity price and smaller initial reserve, it is better for the follower to choose lease the plant from the leader. For higher commodity prices and initial reserves, the $U_{F,\text{nc}}$ manifold is above the $U_{F,\text{coop}}$ manifold, so the follower is better off building. In the bottom-left sub-graph ($l = 1.6$), the $U_{F,\text{coop}}$ manifold moves farther below the $U_{F,\text{nc}}$ manifold and they cross on two separate curves, which shows for extreme low and extreme high commodity price, building-own-plant is better for the follower,[9] only for some middle range of commodity price, accepting the lease is better. As I move to the highest lease rates in the bottom-right sub-graph of Figures 3.3, the $U_{F,\text{coop}}$ manifold is completely below the $U_{F,\text{nc}}$ manifold, which shows that leasing is infeasible, even for high commodity prices and high initial reserves. If I look at the sub-graphs in Figure 3.3 individually, I find that $U_{F,\text{coop}}$ increases linearly in P, holding other variables fixed. The $U_{F,\text{nc}}$ grows non-linearly (convex upward) because it contains the follower's real option value which increases as commodity price increasing. When the commodity price is below the trigger threshold, $U_{F,\text{coop}}$ grows faster. After the trigger, $U_{F,\text{nc}}$ grows faster. Therefore, as commodity price get higher, $U_{F,\text{nc}}$ will finally exceed $U_{F,\text{coop}}$. Hence, the follower's benefit from lease decreases in increasing commodity price P because it loses the real option to delay if it leases.

Figure 3.4 is similar to Figure 3.3 except that now the network effect level (not the lease rate) increases as I move to the right and down. The region where the $U_{F,\text{coop}}$ manifold is above the $U_{F,\text{nc}}$ manifold becomes larger as the network effect level increases. This means there is a larger probability for the follower to accept the lease if the network effect level is higher. Also, the intersection of $U_{F,\text{coop}}$ manifold and $U_{F,\text{nc}}$ manifold shifts

[9]Notice the lower cross line in the bottom-left sub-graph is actually below zero, which means the follower should not build or lease for extremely low commodity price.

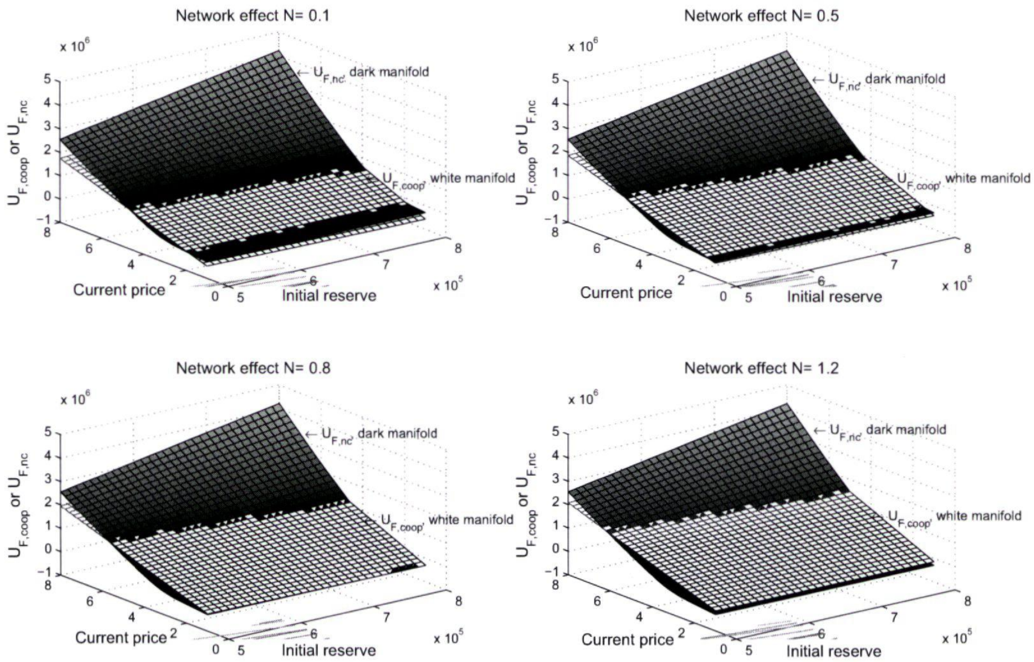


Figure 3.4: Manifolds of the follower's non-cooperative enterprise values $U_{F,\mathrm{nc}}$ and cooperative enterprise values $U_{F,\mathrm{coop}}$ as a function of initial reserves Q_F and the commodity price P for successively larger network effect levels. The lease rate is at its mean value. The dark manifold is $U_{F,\mathrm{nc}}$. The light manifold is $U_{F,\mathrm{coop}}$.

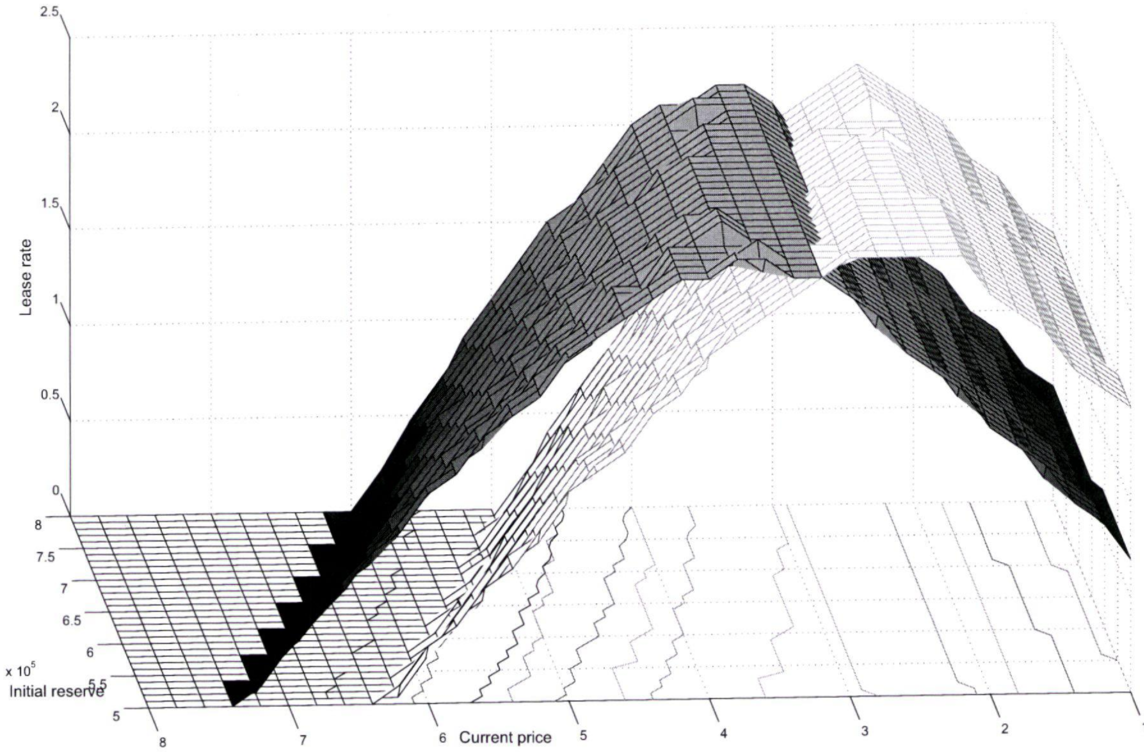


Figure 3.5: The follower's reservation lease rate for minimum (the dark shading manifold) and maximum network effects (the light shading manifold).

up as the network effect level increases. This shows that higher network effect level increases the follower's benefit from leasing, and thus $U_{F,\text{nc}}$ needs a higher commodity price level to exceed $U_{F,\text{coop}}$. In addition, the intersection curve of the $U_{F,\text{coop}}$ and $U_{F,\text{nc}}$ manifolds moves toward to the vertical axis as the initial reserve increases. This means larger initial reserve will make $U_{F,\text{nc}}$ exceed $U_{F,\text{coop}}$ at lower commodity price level. This is because, in the case of follower building-own-plant, the construction cost is relatively fixed as a function of the initial reserves. But the total leasing fee charged by the leader is proportional to the size of initial reserves Q_F. Hence, larger initial reserves makes the follower pay a larger total leasing fee while leaving the construction cost relatively constant, which reduces the relative lease benefit.

Figure 3.5 shows the reservation lease rates the follower is willing to accept corresponding to different initial reserves and current gas prices.

First, I analyze how the follower's reservation lease rate changes as the commodity price and initial reserve change. Since the two manifolds have similar shape, I can focus on one of them. As commodity price increases, the follower's reservation lease rate first increases then decreases. To understand this, recall that the follower's reservation lease rate is defined as $\bar{l}_F \equiv \sup\{l_F \in \mathbb{R}^+ : U_{F,\text{coop}} \geq U_{F,\text{nc}}\}$. In other words, it is equivalent to the distance between $U_{F,\text{coop}}$ and $U_{F,\text{nc}}$. As I observe in Figure 3.3 and Figure 3.4, initially $U_{F,\text{coop}} > U_{F,\text{nc}}$, as commodity price increases, both $U_{F,\text{coop}}$ and $U_{F,\text{nc}}$ increase, and $U_{F,\text{coop}}$ increases faster than $U_{F,\text{nc}}$. The distance gets larger. However, above the follower's trigger threshold, $U_{F,\text{nc}}$ increases faster than $U_{F,\text{coop}}$, and the distance becomes smaller, eventually, $U_{F,\text{nc}}$ will catch up (intersect) with and then exceed $U_{F,\text{coop}}$. That is why the follower's reservation lease rate first increases and then decreases. Furthermore, one can infer that the follower's peak reservation lease rate is achieved when the distance between $U_{F,\text{coop}}$ and $U_{F,\text{nc}}$ is largest, i.e., the neighbor area below the trigger threshold.

Second, the contours of the manifolds show that the reservation lease rate manifold is not very sensitive to initial reserve for low commodity price, but it becomes more sensitive when the commodity price is high. In fact, for high commodity prices, the reservation lease rate decreases as the initial reserve increases. This is because larger initial reserve helps $U_{F,\text{nc}}$ exceed $U_{F,\text{coop}}$ faster, i.e., at lower commodity price, which verifies my discussion for Figure 3.4.

Third, by comparing the dark manifold (minimum network effect level) with the light manifold (maximum network effect level), I find that before the peak,[10] the high network effect gives the follower a larger reservation lease rate if holding the price level fixed. But after the peak, the high network effect gives the follower a smaller reservation lease rate if holding the price level fixed. This is because before real option being exercised, the network effect is favoring $U_{F,\text{coop}}$ more (making $U_{F,\text{coop}}$ increase faster), whereas after real

[10]Notice the peak is reached below the commodity price of 3, and Figure 3.1 shows that the optimal option exercise region is between commodity price 4 and 6.

option being exercised, the network effect is favoring $U_{F,\mathrm{nc}}$ more (making $U_{F,\mathrm{nc}}$ increase faster). The implication for the leader from this observation is that for extremely low commodity price, larger network effect increases the follower's willingness to pay for the lease. For a high commodity price, larger network effect decreases the follower's willingness to pay for the lease, certeris paribus.

3.3.2 The effect of the lease contract and network effect on the leader's decisions

The leader has two options:

1. Build a plant with optimal non-cooperative capacity q_L^{c*} to process its own gas only for a construction cost $K(q_L^c)$. The effect of building this small plant on the optimal exercise point is mixed: it could be earlier or later than if a large plant is built.

2. Build a plant with optimal cooperative capacity ($q_L^{\Omega*}$) to process his (q_L) and the follower's gas q_{FL}. The larger plant has a construction cost $K(q_L^\Omega) > K(q_L^c)$. The cash flow from this decision is also larger because

    (a) leasing gives a lower toll rate (network effect)

    (b) the leasing fee is a cash inflow to the leader.

As discussed in Section 2.3.5, the leader wants to find a balance among the incremental network effect benefit, the earlier leasing fee, and the extra construction costs, $K(q_L^\Omega) - K(q_L^c)$, bearing in mind the fact that a high leasing rate will cause the follower to delay.

Figure 3.6 shows the exercise of the leader's real option. Similar to the follower's real option value, the leader's initial reserve has very little effect on its real option value for very low commodity prices since the option never gets exercised for such low prices. When the commodity prices are higher, the probability of exercising the option is higher,

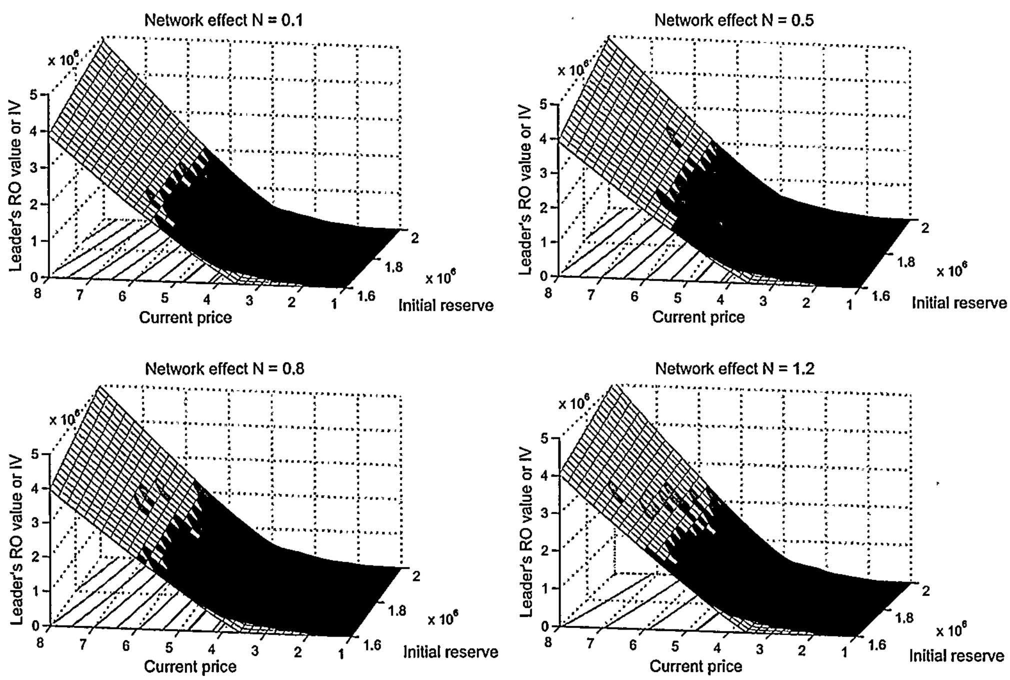


Figure 3.6: The exercise of leader's real option and the smooth-pasting condition for different network effect levels. The lease rate is at the its mean value. The dark shading manifold is the real option value and the light shading manifold is the exercise proceeds. The optimal exercise threshold is at the transition from dark to light.

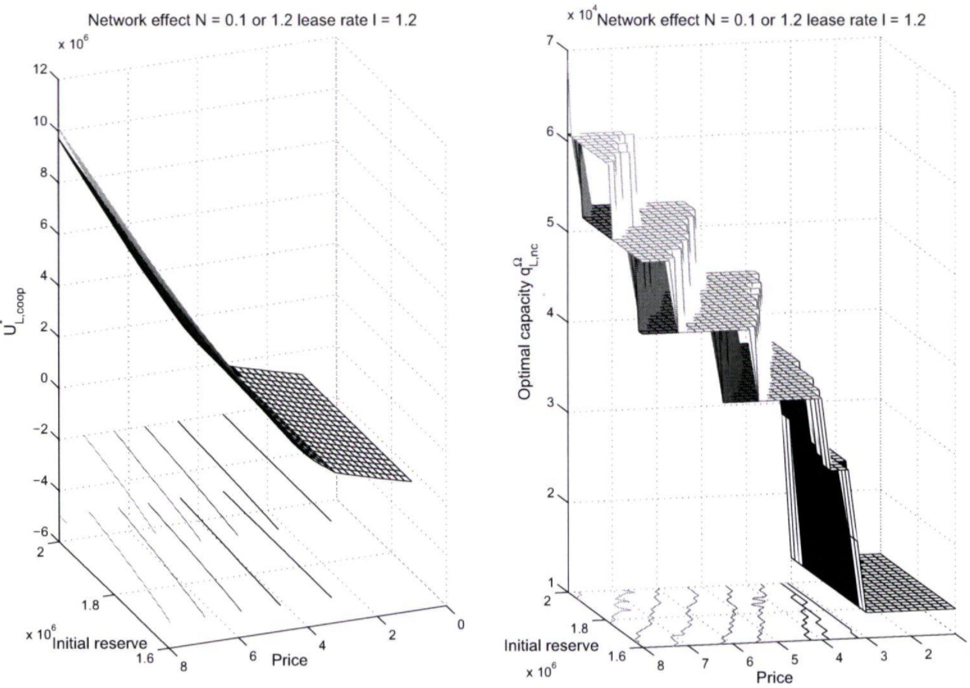


Figure 3.7: The left panel is the leader's maximum cooperative enterprise value $U^*_{L,\text{coop}}$ for minimum and maximum network effect level. The right panel is the leader's optimal capacity choice $q_F^{\Omega *}$ for minimum and maximum network effect level. The dark manifold is for minimum network effect level and the light manifold is for maximum network effect level. The lease rate stays at its mean value.

the option value and sensitivity to reserves are higher. Also, as the network effect N gets larger (moving to the right and down), both the leader's real option value and exercise proceeds become larger. However, unlike the follower's real option exercise threshold which falls from a price of 6 to 5, the leader's optimal exercise of threshold (the transition from dark to light) does not fall significantly (stays between 6 and 5) as the network effect level increases. The leader and the follower's optimal exercise thresholds are around the same range because the leader's is also choosing the optimal capacity so that it can exercise right before the follower in order to be able to offer the lease.

Figure 3.7 plots the leader's maximum cooperative enterprise value and optimal capacity for different levels of the network effect. In the left panel, the two value manifolds are very close to each other, which shows that the increase in network effect has a very minor effect on the leader's cooperative enterprise value. The right panel shows that the leader's cooperative optimal capacity $q_L^{\Omega*}$ manifold is not smoothly increasing with the increase of commodity price and initial reserves, which is similar to the follower's optimal capacity q_F^{c*}. The right panel also shows that the leader should build larger cooperative capacity if the network effect level is higher.

The distance between the two manifolds increases as the current commodity price increases.[11] Therefore, although the construction cost of the small plant is less than that of the large plant, i.e.$(K_1 < K_2)$, the leader may still decide to build the large plant, and lease the excess capacity to the follower in order to encourage the follower to start production early. The leasing fee and the cheaper toll rate will compensate for the leader's higher construction cost. Of course, there is an upper bound for the lease rate which makes the leader's cooperative enterprise value equal to its non-cooperative enterprise value.

Figure 3.8 compares the leader's optimal cooperative capacity $q_L^{\Omega*}$ with its optimal non-cooperative capacity q_F^{c*} when the network effect level is changing. From the left panel to the right panel, the difference between $q_L^{\Omega*}$ and q_F^{c*} does not increase significantly as the network effect increases. Figure 3.9 compares the leader's optimal cooperative capacity $q_L^{\Omega*}$ with its optimal non-cooperative capacity q_F^{c*} when the lease rate is changing. From the left panel to the right panel, the difference between $q_L^{\Omega*}$ and q_F^{c*} increases significantly as the lease rate increases. A comparison of Figures 3.8 and 3.9 shows that the lease rate has a larger positive effect on the leader's capacity choice than the network effect has.

Figure 3.10 graphs the leader's reservation lease rates against the commodity price and initial reserve. For low commodity prices between 1 and 3.5, the leader's reservation lease rate is zero, meaning the leader has not exercised the real option yet and hence

[11]This may not be very clear if only looking at the manifold. But the contours on the floor of the graph in Figure 3.7 indicate that this distance gets larger for higher commodity prices.

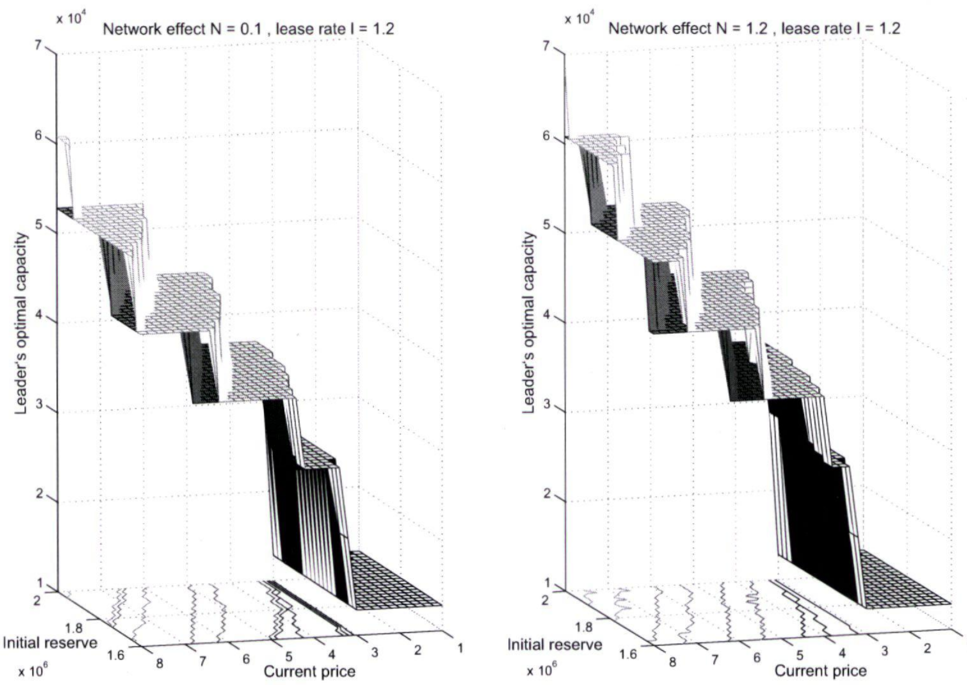


Figure 3.8: The leader's optimal cooperative capacity (the light manifold) and its optimal non-cooperative capacity (the dark manifold). The left panel is for minimum network effect level and the right panel is for maximum network effect level. The lease rate stays at its mean value.

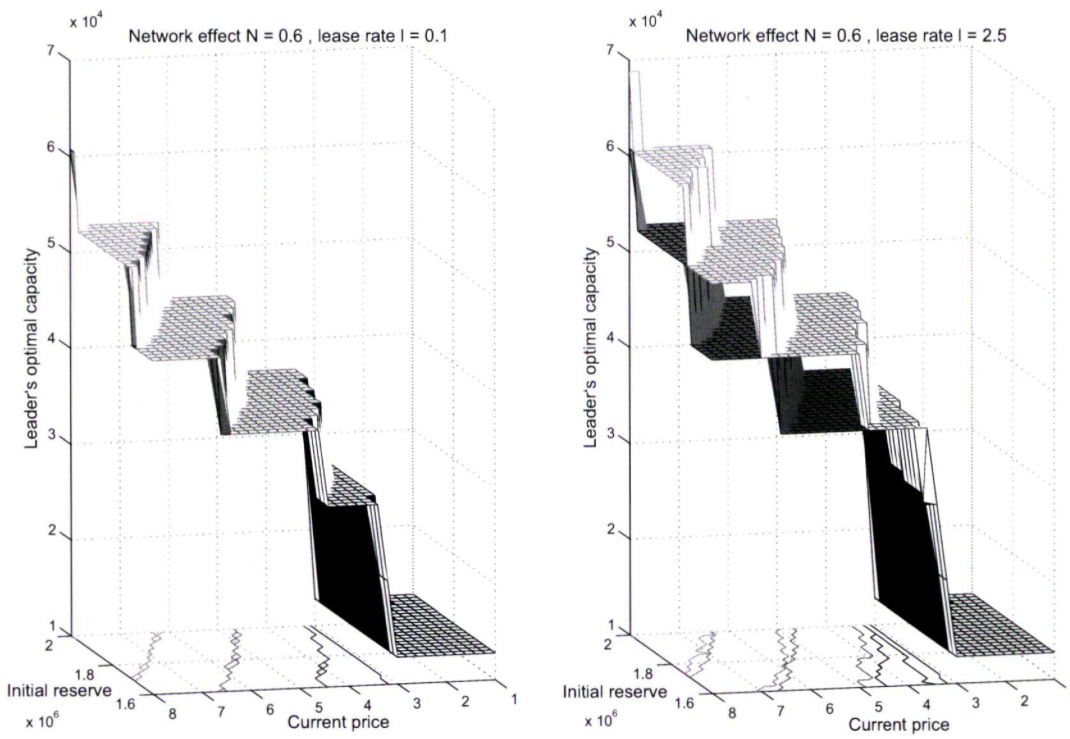


Figure 3.9: The leader's optimal cooperative capacity (the light manifold) and its optimal non-cooperative capacity (the dark manifold). The left panel is for minimum lease rate and the right panel is for maximum lease rate. The network effect level stays at its mean value.

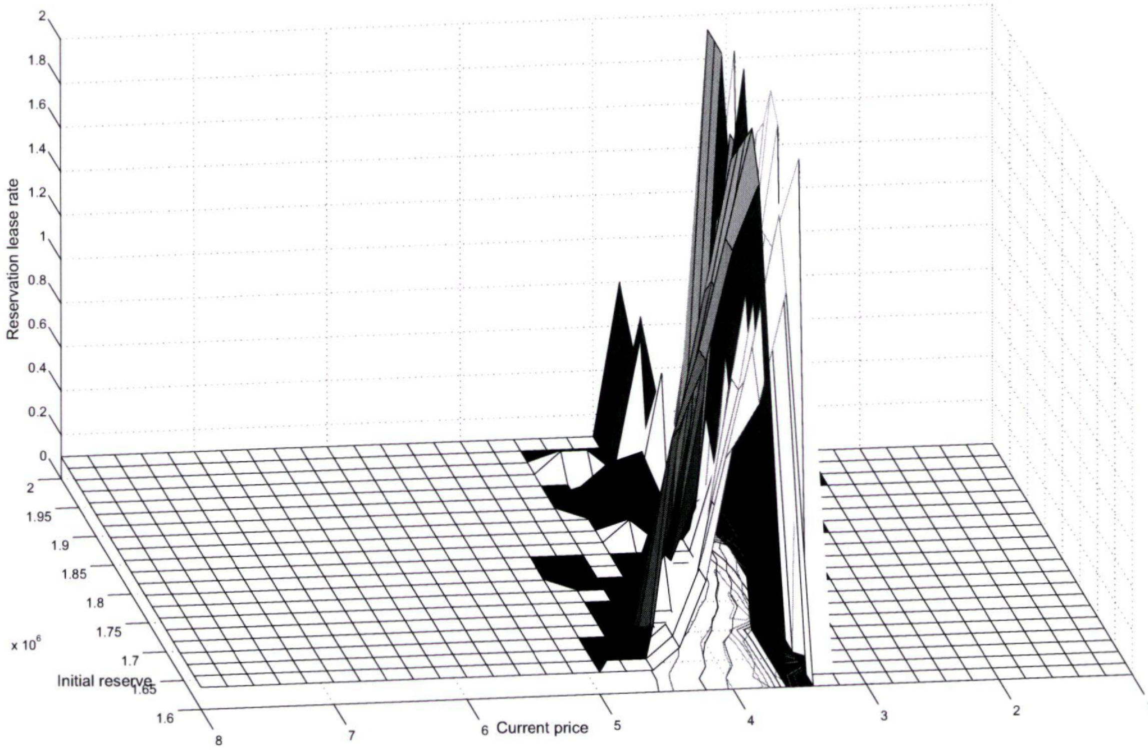


Figure 3.10: Manifolds of the leader's reservation lease rates for minimum (the dark manifold) and maximum (the light manifold) network effect level.

can not provide the lease. Above the commodity price of 4, the leader's reservation lease rate quickly drops to the lowest lease rate of 0.1 before the commodity price hits 5. This means that, when the commodity price gets very close to the follower's exercise threshold (between the commodity price of 5 and 6), the leader is willing to accept the lowest lease rate in order to avoid the follower's rejection of lease. The contours of the manifolds indicate that as initial reserve increases, the leader's reservation lease rate decreases. Moreover, by comparing the dark manifold (minimum network effect level) with the light manifold (maximum network effect level), I find that before the peak, the high network effect gives the leader a larger reservation lease rate if holding the price level fixed. But after the peak, the high network effect gives the leader a smaller reservation lease rate if holding the price level fixed. The implication for the leader from this observation is that, once it exercises the real option, a larger network effect will reduce the leader's reservation lease rate even further, i.e., the leader is willing to set a lower lease rate to capture the larger network effect, ceteris paribus.

3.3.3 The possible equilibrium region for the lease rate

Figure 3.11 is a combination of Figure 3.5 and Figure 3.10. It indicates the possible region for bargaining an equilibrium lease. In the left panel, the leader's reservation lease rate exceeds the follower's reservation lease rate for the commodity price range of 3 to 4, which means the leader and the follower cannot negotiate a deal for those low commodity prices. However, there is an feasible region (in the commodity price range from 4.6 to 5.6) below the lowest thick red line and above the highest dotted black line. The distance between those graphs is the gain from cooperation for which the leader and the follower will bargain. Also notice that as the commodity price gets higher than 5.7, the follower's reservation lease rate drops zero because at that commodity price level, the follower's

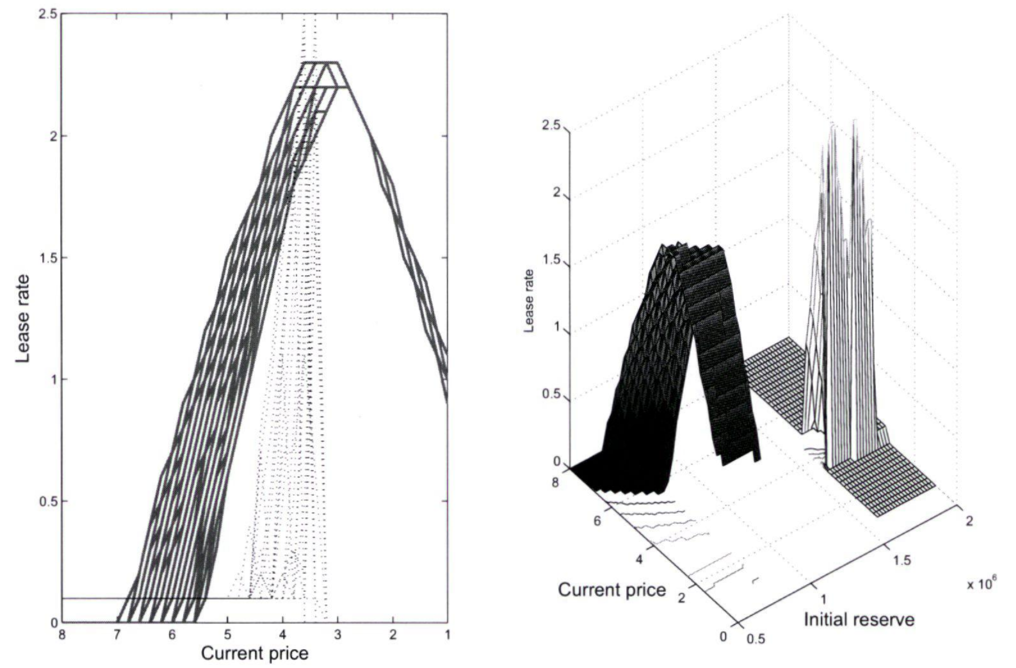


Figure 3.11: The leader's reservation lease rate crossing with the follower's reservation lease rate. The left panel is a 2-D projection of the manifolds in the right panel. The network effect is at its mean value. In the left panel, the thick red line is the follower's reservation lease rate and the dotted black line is the leader's reservation lease rate. In the right panel, the manifold corresponding to the low initial reserve is the follower's reservation lease rate, and the manifold corresponding to the high initial reserve is the leader's reservation lease rate.

optimal option exercise threshold[12] has already been hit, therefore the follower would rather build its own plant.

3.4 Conclusions

In this chapter I have developed a model to analyze the real option exercise game with two asymmetric players, the leader and the follower. In this game, two players have to decide when to explore and develop their adjacent oil or gas lands. The game is a dynamic sequential bargaining game of one-sided incomplete information. Players are bargaining over the lease rate which is going to be specified by the leasing contract.

My simulation model illustrates the region of the equilibrium lease rate while treating other variables such as leasing quantity, and government regulated maximum production rate as fixed variables. In the three dimensional space spanned by enterprise value, current commodity price, initial reserve, the equilibrium lease rate may be located inside the 3-D space bounded by the follower's and the leader's reservation lease rate.

I observe that the follower tends to accept the lease contract if the commodity price and the initial reserves are low, and rejects the lease contract if the commodity price and the initial reserves are high. With a high commodity price and initial reserves, the follower has more bargaining power, so the leader should charge a relatively low lease rate to encourage the follower's immediate start of production. If the commodity price and initial reserve are high, the leader should lower the lease rate, which coincides with the behavior of its reservation lease rate. Furthermore, the network effect positively affects the follower's reservation lease rate, which creases larger space for bargaining.

On the other hand, when considering whether to be cooperative or non-cooperative, the leader is always better off being cooperative as long as the incremental construction

[12] As shown in Figure 3.1, the follower's optimal option exercise threshold is definitely below commodity price of 6 no matter how much initial reserve it has.

cost of the excess capacity can be covered by the present value of the leasing fee and increase in total network effect.

One possible extension of my model would be to change the underlying process from GBM to mean reverting process with/without jump and then start to solve, analyze the characteristics of the equilibrium. Alternatively, from the game theory perspective, one can change my one-sided incomplete information setting to two-sided incomplete information, or allow the leader and follower to provide alternating offers, or extend the two type followers assumption to continuous type followers, or extend the game from multi-period finite time horizon to infinite time horizon.

Chapter 4

EMPIRICAL EVIDENCE

4.1 Introduction

Firms' optimal investment decisions under uncertainty has been a controversial topic for a long time due to the observed deviation from zero NPV threshold. The standard real options literature, including Brennan and Schwartz (1985); Dixit and Pindyck (1994); Dixit (1995); Capozza and Sick (1991); Sick (1995); Trigeorgis (1996), asserts that investments should be delayed until uncertainty is resolved or wait for the optimal threshold. However, the competitive real options literature, including Fudenberg and Tirole (1985); Grenadier (1996, 2002); Mason and Weeds (2005); Garlappi (2001); Boyer et al. (2001); Murto and Keppo (2002); Lambrecht and Perraudin (2003); Huisman and Kort (2004); Thijssen et al. (2006); Smit and Trigeorgis (2004), argues that competition diminishes the real option values and mitigates investment delays, thus, with sufficient competition, firms' investment threshold may be pushed back to zero net prsent value (NPV). The recent article, Novy-Marx (2007) shows that opportunity costs and supply side heterogeneity reduce the competition effect and leads to an investment threshold even later than the standard real option threshold.

Empirical work in testing the competitive real option theory is rare probably due to the shortage of firms' capital budgeting data in irreversible investment at a project level . In a non-competitive setting, Favero et al. (1994) develop and test a duration model to explain the appraisal development lag for investment in oil fields. Hurn and Wright (1994) use a discrete time hazard regression models to analyze the appraisal lag and the production start-up lag using North Sea Oil Data. Bulan (2005) test the real options behavior in capital budgeting decisions using a firm-level panel data set of U.S. companies in the manufacturing sector by looking at the relationship between the firm's investment to capital ratio and total firm uncertainty, measured as the volatility of the firm's equity returns. They find that increased industry uncertainty negatively affects firm investment, and increased firm-specific uncertainty also depresses firm investment. Bulan et al. (2002) examine condominium developments in Vancouver in an competitive setting. They find that risk increase leads to delay of new real estate investments, and increases in competition negates the negative effect of risk on investments. All these previous empirical results support that (1) Uncertainty defers the investment because firms want to keep the real option value; (2) Competition accelerates the investment because it erodes option values.

This chapter tests whether firms will consider the possibility of cooperating with their competitors when the competition becomes too fierce using the project level data from Alberta natural gas exploration and processing industry. As discussed in Sick and Li (2007), in industries with economies of scale or network effects, firms may benefit from cooperation by avoiding the erosion effect of competition on real option value.

4.2 The Data

4.2.1 The background of natural gas production industry

The development of a natural gas field can be a long-term process. First, in the exploration stage, firms need to collect geological survey data, seismic data and gravitational data in order to examine the surface structure of the earth, and determine the possible locations of gas reservoir. Second, in the drilling stage, firms need to drill several discovery wells to determine the approximate depth and quantity of the gas reservoir. If the discovery wells find that the underground gas reserve is large enough to merit the production, then firms officially have a real option to invest. To start the production, firms need to drill more production wells in order to extract gas from underground reservoir at an optimal scale. Depending on the natural gas commodity price and the estimated reserve quantity, firms may or may not wait for years before they start the actual production. This waiting period between the registration dates of discovery well and the production well is defined as the investment lag in this thesis.

Raw gas needs to be processed in a gas processing plant before it can be sold in the market. The natural gas sold in the market consists mainly of methane. The raw gas extracted from the production wells is a mixture of methane and other heavier hydrocarbons - such as ethane, propane, butane and pentane - as well as water vapour, hydrogen sulphide,[1] carbon dioxide, nitrogen and other gases. If the natural gas at the wellhead contains more than 1% of hydrogen sulphide, it is called sour gas and has to be processed at sour gas recovery plants to extract sulphur for sale to fertilizer manufacturers and other industries. According to the Energy Resources Conservation Board (ERCB), about 30% of Canada's total natural gas production is sour, most of it found in Alberta and northeast British Columbia. The average rate of sulphur recovery rate at Alberta's sulphur recovery plants has improved from 97.5% in 1980 to 98.8% in 2000. Because of the potential environment issues of H_2S, the construction and production of sour gas plants is stringently regulated by the ERCB.

Different types of gas plants may engage in different processes, and their construction costs and operating costs may vary in a wide range. Once the raw gas reaches the surface at the production wellhead, it is transported through the gathering systems from

[1]As described by the Energy Resources Conservation Board (ERCB), an independent quasi-judicial agency of the Government of Alberta, Canada regulating Alberta's provincial energy industries such as oil, natural gas, oil sands, coal, and pipelines, hydrogen sulphide (H_2S) is a colourless substance that is poisonous to humans and animals. Also known as hydrosulfuric acid, sewer gas, and stink damp, it is recognizable by its rotten egg smell at very low concentrations (0.01 - 0.3 parts per million). Exposure at higher concentrations of H_2S affects a person's sense of smell and, as a result, there is no perceptible odor. Exposure to high concentrations of H_2S (150 - 750 parts per million) can cause a loss of consciousness and possible death.

individual wells to centralized processing plants, where most non-methane substances are to be removed from the gas stream. A gas processing plant may undertake four main general processes which include oil and condensate removal, water removal, separation of natural gas liquids (NGLs) and sulfur and carbon dioxide removal. In Alberta, the ERCB categorizes those centralized processing plants into four types, sweet gas plant, acid gas flaring gas plant, acid gas injection gas plant and sulphur recovery gas plant. The actual processes taking place in these four types gas plants include absorption, adsorption, carbon dioxide removal, refrigeration, Turbo expander, and the Claus process to recover sulfur. After all the processes done at these centralized processing plants, the natural gas is transported to the NGL fractionation plants where the mixed stream of different natural gas liquids are separated out[2] and then to the mainline straddle plants located on major pipeline systems. To avoid unnecessary heterogeneity in gas plants, only those four types of centralized processing plants are included in the sample.

Natural gas fields are also differ by the type, depth, age and location of the underground deposit and the geology of the area. Normally, natural gas is extracted from pure gas wells and from condensate wells where there is little or no crude oil. Such gas is called non-associated gas. Sometimes, natural gas are also found in oil wells where it could be either separate from, or dissolved in the crude oil in the underground formation. These gas are called associated gas. To avoid the potential heterogeneity problem in the gas field reservoirs, only those non-associated gas fields are included in the sample.

4.2.2 The data collection process

The data collecting work was a gradual learning process which involved extensive literature reading, data organizing and interview with industry practitioners.[3] I visited

[2]The NGLs are sold separately for use as diluent in heavy oil processing, and as feedstock for petrochemical plants or as fuel.

[3]For the data collection work, I am grateful for the tremendous help and comments from: (i) Mr. Grant Ireland, senior technologist and senior compliance advisor at Corporate Compliance Group I

various data sources and institutions including University of Calgary library, ERCB library, ERCB technology and compliance office, ISEEE (Institute for Sustainable Energy, Environment and Economy) at University of Calgary and McDaniel & Associates Consultants Ltd.

Originally, I planned to do a case study on Shells development of the Caroline sour gas field and plant in early 1990*s* since it seemed to be a good fit with my theoretical model. According to press reports and ERCB publications, the Caroline gas field was discovered in 1986. No major production was initiated until 1990 when Shell Canada and Husky competed to become the leading developer of Carolin. Shell proposed to build a new gas plant near Caroline. Husky wanted Shell to transport the gas 35 miles through pipeline to Husky's existing Ram River plant for processing. ERCB approved Shell's proposal eventually. I would like to examine how Shells and Huskys strategies and decisions fit with my theoretical model if the following data were available.

- The development cost of a gas property and the construction cost of the processing plant, which has to be tied to the capacity.
- The construction costs of pipelines that are needed to transport the gas from wells to the plants.
- Any reports and analysis about the negotiation between Shell and Husky.

The first two items were documented in both Shell's and Husky's proposal files, which were buried in ERCB old filings. However, there was very few information regarding the negotiation process between Shell and Husky.

I realized that focusing on one single project may not be a good choice. I started to consider collecting a sample of oil and gas investment projects in competitive setting and

Audit & Compliance Coordination Section at ERCB. (ii) Mr. Doug Sick at Talisman Energy. (iii) Dr. Wayne Patton, Director of Energy Management Programs, ISEEE. (iv) Mr. Philip Arthur Welch, P.Eng. President & Managing Director of McDaniel & Associates Consultants Ltd.

using that to empirically test the predictions from my theoretical model. U.S. energy information administration (EIA) was the first place I went to since they provided macro and micro level data for petroleum industry. After downloaded and reviewed hundreds of spreadsheets from EIA website, I found Kansas State EIA keep very clear field level data such as the discovery year, the initial reserves, location and annual production volume etc. After running a few tests using this data, I suddenly realized that one important variable, the network effect or the competition level was not provided. Without this variable, I won't be able the test my competitive real option exercise model. I contacted Kansas State EIA office. Their response was also negative about it. I was a little discouraged after that.

One day, my supervisor, Dr. Gordon Sick reminded me that ERCB may have more detailed field level data and operating data for gas processing plant. I took a closer look at ERCB website this time and found some of their reports were related to natural gas fields and processing plants. However, most of ERCB reports were not available online but could be purchased in CDs. To avoiding paying the expensive price of these data CDs, I went to ERCB library and discovered four relevant reports, ST50, ST98, ST102 and gas pool reserve files, which were buried in their hundreds of reports and publications. Since the data was scattered in several reports, I spent more than three months to summarize and cross tabulate them together. It was especially time consuming for me to manually matching the field data with the plant data because all ST50 before 1999 were recorded in microfiche.

4.2.3 The variables

To empirically test the real option exercise game model, I collected data for more than 1,100 natural gas fields and 1,200 natural gas processing plants located in the province of Alberta. The data are from various ERCB annual publications and the registration files of individual gas processing plant and natural gas field. A citation list of these various publications is provided in appendix F.

The discovery time starts from year 1904 and ends at year 2006. It is indicated by the registration year of the earliest discovery well within each field. ST98 report provides the name, code, location and initial established reserves for every field. It also provides the discovery year of wells and mean formation depth for all reserve pools within every field. Pool level data are grouped and summarized to form the field level data including the initial reserve, discovery year and average depth. The total number of discovery wells within every fields is calculated from ST98 too.

The production startup time extends from year 1954 to 2007 as indicated by the registration year of the earliest registered gas processing plant of each field. ST102 lists the facility ID, location, subtype code for all active and inactive natural gas production facilities. By choosing subtype 401, 402, 403, 404 and 405, four types of centralized processing plants — sweet gas plant, acid gas flaring gas plant, acid gas injection gas plant and sulphur recovery gas plant are included in the sample. ST50 reports the plant ID and location, plant operator name and code, plant licensee, plant process, registered plant capacity, registration year, ERCB approval code, fields and pools that are registered to serve. ST50 are used in conjunction with ST102 to determine the plant capacity, production startup time, and to build up the association between the plants and the fields. ST50 are recored in excel file since 1999, but all previous years ST50 are recoded in the microfiche collection. In order to find the association between the plants and fields, I have to manually search through the historical ST50 in order to find the earliest plant that were registered to process gas from particular field.

The historical natural gas price from 1922 to 2007 is collected from Energy Information Administration of U.S. government. Once the field discovery time and production startup time are determined, the price at discovery and price at production are then determined by matching the year variable. Ultimately, a sample of around 500 observations was formed at the investment project level where each observation associates one plant with one fields, or several fields sometimes.

A cooperative gas processing plant is defined as one plant serving, or historically having served multiple natural gas reservoir fields that are operated by multiple field operators. A non-cooperative gas processing plant is a plant serving one field or multiple fields operated by one field operator, or historically never served multiple fields operated by multiple field operators. The variable, COOP, indicates whether the gas processing plant is cooperative. If a plant is registered to process gas from multiple fields, it is a cooperative plant and COOP is equal to one. Otherwise, it is a non-cooperative plant and COOP has a value of zero. The explanatory variables are listed in this vector,

$$\begin{aligned}\{&\text{PRICEDIS}, \text{PRICEPROD}, \text{RESERVE}, \text{WELLSDISC},\\ &\qquad\qquad \text{DURATION}, \text{DEPTH}, \text{CAPACITY}\}.\end{aligned}$$

PRICEDIS is the natural gas price at the discovery time.

PRICEPROD is the natural gas price at the time of production.

RESERVE is the initial reserve quantity of the field.

DEPTH is the average depth of all production wells within particular field, representing the drilling costs.

CAPACITY is the plant's daily processing capacity, proxying the construction cost of the plant.

DURATION measures the investment lag between discovery time and production time.

WELLSDISC is the total number discovery wells within a certain field, representing the level of network effect. More discovery wells suggest more reserves and greater future production flows. As discussed in Section 2.3.2, these greater production flows would need a larger pipeline throughput volume which generates a higher level of the network effect. This variable can also be viewed as a rough proxy of the level of competition. If one field has more discovery wells drilled, it is likely that they are owned by more firms which all have the potentials and motivations to become the first mover.

The main feature of the data and variables are summarized in Table 4.1. Out of 513 gas fields, 393 of them have been developed and producing gas as of year 2007. The mean of the variable COOP is 0.61, greater than 0.5, suggesting that more cooperative plants have been built since 1950*s*. The average of start production price (PRICEPROD) is more than three times higher than the price at discovery (PRICEDIS). The average investment lag is around 31 years. This indicates that on average, firms did wait for higher real option exercise price to start their production. The average depth of these production wells is $1,356$ MKB (Meters below Kelly Bushing). The average plant capacity is $1,182$ thousand cubic meters per day. The average gas reserve size is $11,268$ million cubic meters. The network effect or the competition effect (WELLSDISC) varies from one well per field to $1,068$ wells per field with an average of 105 wells per field.

4.3 The Empirical Models and Results

4.3.1 The logit model of cooperative investment

To analyze firms' strategic real option investment decision under competition, I develop a logit model to test whether firms may consider the option of cooperating with their competitors when facing severe competition, or simply be forced to invest as soon as NPV equals zero. The decision of cooperation is the result of a sequential bargaining

Variable	Obs	Mean	Std Dev.	Min	Max
coop	393	0.61	0.49	0.00	1.00
pricedis	442	0.48	0.78	0.05	7.33
priceprod	393	1.85	1.54	0.10	7.33
reserve	513	11267.93	19914.34	1.00	77780.00
wellsdisc	513	104.94	179.39	1.00	1068.00
duration	513	31.35	25.39	0.00	103.00
depth	511	1356.32	804.83	244.81	4187.00
capacity	387	1181.97	2156.66	11.90	11941.00

Table 4.1: **Summary Statistics.** The variable, COOP indicates whether the gas processing plant is cooperative. PRICEDIS is the natural gas price at the discovery time. PRICEPROD is the natural gas price at the time of production. RESERVE is the initial reserve quantity of the field. DEPTH is the average depth of all production wells within particular field, representing the drilling costs. CAPACITY is the plant's daily processing capacity, proxying the construction cost of the plant. WELLSDISC is the total number discovery wells within certain field, representing the level of network effect. DURATION measures the waiting period (investment lag) between discovery year and start production year.

game as discussed in Chapter 2. If firms decide to cooperate, they build a cooperative gas plant with larger capacity to process gas from multiple fields. If firms were not able to agree on the lease rate (gas processing fee), the leader would start the investment and production along, the follower would wait until its own threshold reaches.

Section 3.3.1 predicts the following:

1. Firms' reservation lease rates are concave in the commodity price (for both the leader and the follower), and the equilibrium cooperation range is decreasing in commodity price once the real option to invest is exercised.

2. Firms' reservation lease rates are not very sensitive to the initial reserve level within the non-exercising region. However, they decrease as initial reserve quantity increases, and the equilibrium cooperation range is decreasing in commodity price within the exercising region. Figure 3.11 also demonstrates the same prediction about the equilibrium range.

3. Figure 3.5 and Figure 3.10 indicate that, within the exercising region, a larger

network effect decreases the leader's and the follower's reservation lease rate. Thus, as network effect increases, the leader is willing to accept a lower lease rate but the follower is also willing to pay a lower lease rate so that the effect of network effect on equilibrium range is mixed.

These predictions yield three testable implications for the logit model of cooperation.

Hypothesis 1 The gas price has a non-monotonic effect on the probability of cooperation. It increases the cooperation probability within the non-exercising region, which is unobservable in the data sample. All the investment projects registered in ERCB have already been exercised, so they do appear in the data. As a result of this, increased gas prices are expected to have a negative effect on the cooperation probability within the exercising region.

Hypothesis 2 Initial reserve quantity is expected to have a negative effect on the probability of cooperation within the exercising region.

Hypothesis 3 The effect of network effect or competition effect on the probability of cooperation is mixed. If the competition effect dominates the network effect (economies of scale), firms are more likely to build non-cooperative plant. If the network effect dominates the competition effect, firms are more likely to build cooperative plant.

The logit regression equation is presented as the log of the odds ratio in favor of cooperation — the ratio of the probability that firms act cooperatively to the probability that firms act non-cooperatively:

$$
\begin{aligned}
\ln\left(\frac{P}{1-P}\right) &= \alpha + \beta^{\mathrm{pd}}\mathrm{PRICEDIS} + \beta^{\mathrm{pp}}\mathrm{PRICEPROD} + \beta^{\mathrm{rs}}\mathrm{RESERVE} \\
&\quad + \beta^{\mathrm{d}}\mathrm{DEPTH} + \beta^{\mathrm{c}}\mathrm{CAPACITY} + \beta^{\mathrm{w}}\mathrm{WELLSDISC} \\
&\quad + \beta^{\mathrm{dn}}\mathrm{DURATION} + \varepsilon \\
\text{where}\quad P &= E(\mathrm{COOP} = 1|\mathbf{X}) \\
\text{and}\quad \mathbf{X} &= \{\mathrm{PRICEDIS}, \mathrm{PRICEPROD}, \mathrm{RESERVE}, \mathrm{DEPTH}, \\
&\qquad \mathrm{CAPACITY}, \mathrm{WELLSDISC}, \mathrm{DURATION}\}.
\end{aligned}
$$

To control for the potential endogeneity of the explanatory variable, CAPACITY, a 2-stage logit model is also estimated. In the first stage linear regression, the exogenous variable DEPTH is used as an instrumental variable to estimate a proxy variable $\widehat{\mathrm{CAPTY}}$, resembling the original CAPACITY. In the second stage logit model, $\widehat{\mathrm{CAPTY}}$ is included as a regressor to replace CAPACITY. The 2-stage logit model is expressed as:

$$
\begin{aligned}
\text{Stage 1:}\quad & \mathrm{CAPACITY} = \alpha_1 + \delta\mathrm{DEPTH} + \varepsilon_1 \\
\text{Stage 2:}\quad & \ln\left(\frac{P}{1-P}\right) = \alpha_2 + \beta^{\mathrm{pd}}\mathrm{PRICEDIS} + \beta^{\mathrm{pp}}\mathrm{PRICEPROD} \\
&\qquad + \beta^{\mathrm{rs}}\mathrm{RESERVE} + \beta^{\mathrm{c}}\widehat{\mathrm{CAPTY}} \\
&\qquad + \beta^{\mathrm{w}}\mathrm{WELLSDISC} + \beta^{\mathrm{dn}}\mathrm{DURATION} + \varepsilon_2 \\
&\text{where}\quad P = E(\mathrm{COOP} = 1|\mathbf{X}) \\
&\text{and}\quad \mathbf{X} = \{\mathrm{PRICEDIS}, \mathrm{PRICEPROD}, \mathrm{RESERVE}, \widehat{\mathrm{CAPTY}}, \\
&\qquad \mathrm{WELLSDISC}, \mathrm{DURATION}\}.
\end{aligned}
$$

Table 4.2 reports the results from the logit regression of cooperation. In both simple logit and 2-stage logit model, the negative coefficients of PRICEPROD are consistent

Variables	Simple logit			Variables	2-stage logit		
					Stage 1		
				capacity	Coef.	Std. Err	t
				depth	0.8627	0.1348	6.40
				constant	-107.9803	226.9679	-0.48
					Stage 2		
Coop	Coef.	Std. Err.	z	Coop	Coef.	Std. Err.	z
pricedis	0.5556	0.4259	1.30	pricedis	0.4590	0.4037	1.14
priceprod	-0.5397	0.1328	-4.06	priceprod	-0.6294	0.1412	-4.46
reserve	-1.51E-05	0.0000	-1.25	reserve	-1.08E-05	0.0000	-1.02
wellsdisc	-0.0020	0.0022	-0.91	wellsdisc	0.0006	0.0003	2.51
duration	0.0070	0.0144	0.48	duration	-0.0024	0.0020	-1.22
depth	0.0004	0.0002	1.65	depth	-	-	-
capacity	0.0004	0.0002	2.75	captyhat	0.0004	0.0139	0.03
constant	0.65637	0.4608	1.42	constant	1.114297	0.415305	2.68
N	323			N	327		
Chi-square	69.80			Chi-square	58.05		
Adj. R^2	0.1669			Adj. R^2	0.1361		

Table 4.2: **Logit models of cooperation.** It provides the logit model estimates for cooperation. The dependent variable is COOP, indicates whether the gas processing plant is cooperative. PRICEDIS is the natural gas price at the discovery time. PRICEPROD is the natural gas price at the time of production. RESERVE is the initial reserve quantity of the field. DEPTH is the average depth of all production wells within particular field, representing the drilling costs. CAPACITY is the plant's daily processing capacity, proxying the construction cost of the plant. WELLSDISC is the total number discovery wells within certain field, representing the level of network effect. DURATION measures the waiting period (investment lag) between discovery year and start production year.

with Hypothesis 1. The real option exercise price has a negative effect on the probability of cooperation. For one cent increase in the initial gas price, the probability that firms build a cooperative gas plant decreases by a factor of 0.53 in the 2-stage logit model, or 0.58 in the simple logit model.[4] As explained in Section 3.3.1, this is because as gas prices increase, the follower's real option exercise hurdle price is easier to reach. Therefore the follower's willingness to play cooperatively decreases, since it has a more viable chance of building its own plant. If the leader does not lower the lease rate accordingly, the bargaining game may end in a non-cooperative equilibrium, which reduces the probability of cooperation.

The coefficient of WELLSDISC is positive (0.06), which confirms Hypothesis 3. The competition effect is dominated by the network effect and the number of discovery wells has a positive effect on cooperation. For every unit increase in WELLSDISC, the probability of building a cooperative plant increases by a factor of 1.0006. In a more crowded field, whichever firm builds the plant first will become the leader and have the ability of extract rents from the followers. All firms possessing a similar reserve size will share a similar exercise price and want to seize this opportunity. The competition level rises. Meanwhile, firms are also aware of the beneficial network effect that becomes stronger because more firms may start producing together. As the network effect dominates the competition effect, firms are more willing to cooperate rather compete with each other when more there are more discovery wells.

Hypothesis 2 is not strongly confirmed, since the coefficient of reserves is not statistically significant in either of these two models. However, the negative sign of reserve coefficient does indicate the right direction predicted by Hypothesis 2. The extremely small coefficients, -0.0000151 in simple logit and -0.0000108 in 2-stage logit are caused

[4]Here, 0.5329 is calculated as $\exp^{-0.6294}$ and 0.5829 is calculated as $\exp^{-0.5397}$. As with many other papers interpreting logit results, instead of interpreting the log odds of the dependent variable, we exponentiate the coefficients and interpret them as odds-ratios.

by the large magnitude of reserves. Capacity is found to have positive effect on the probability of cooperation in the simple logit model only. Once the heterogeneity problem is purged in the 2-stage logit model using DEPTH as the instrumental variable, the effect of capacity ceases to be statistically significant. This shows that larger plant capacity may not be the cause but the outcome of firms' cooperative investment, i.e., cooperative plants are normally larger in order to accommodate natural gas from multiple fields.

4.3.2 The duration model of investment timing for building a gas processing plant

To analyze the effect of gas price, price volatility, quantity of gas reserves and the network effect (competition effect) on the investment time lag, I use the framework of duration analysis. Denote the dependent variable t as the observed investment time lag between the reserve discovery time t_1 and the production startup time t_2. The lag t is measured as $t = t_2 - t_1$, and has a probability density of $f(t)$, and associated cumulative distribution function of $F(t)$. The survival function is $S(t) = 1 - F(t)$. The hazard function is $h(t) = \frac{f(t)}{S(t)} = \frac{f(t)}{1-F(t)}$. The functional form of $h(t)$ depends on the distribution of t. Based on this hazard function, $h(t)$, a proportional hazard (PH) model conditional on time-invariant covariates $\mathbf{x}$ can be defined as

$$h(t, \alpha; \mathbf{x}, \lambda) = h_0(t, \alpha)\theta(\mathbf{x}) \tag{4.1}$$

where $h_0(t, \alpha)$ is the baseline hazard function with parameter α and is common to all units in the population. The individual hazard functions, $h(t, \alpha; \mathbf{x}, \lambda)$ differ proportionately from the baseline hazard by a nonnegative factor $\theta(\mathbf{x})$. The function $h_0(t, \alpha)$ may either be left unspecified which gives the Cox proportional hazard, or be assumed to follow a specific distribution such as the exponential, Weibull[5] or Gompertz distribution as described in Lee and Wang (2003).

Using the Wooldridge (2002) formulation, $\theta(\mathrm{x})$ is normally parameterized as $\theta(\mathrm{x}) = \exp(\lambda\mathrm{x})$, where λ is a vector of parameters and $\mathbf{x}$ is the vector of explanatory variables. To interpret the estimates of λ, the hazard function needs be represented in the regression form

$$\ln h(t, \mathrm{x}) = \lambda\mathrm{x} + \ln h_0(t).$$

The explanatory variable vector $\mathbf{x}$ is defined as:

$$\{\mathrm{COOP}, \mathrm{PRICEDIS}, \mathrm{PRICEPROD}, \mathrm{RESERVE}, \mathrm{DEPTH}, \\ \mathrm{CAPACITY}, \mathrm{WELLSDISC}\}$$

which includes the discovery price, production price, initial reserve quantity, average well depth, plant capacity and the number of discovery wells within individual field.

An alternative assumption here would be the accelerated failure time (AFT) model which requires the choice of hazard function distribution. The AFT model is expressed as a linear function of the explanatory covariates,

$$\ln t = \lambda\mathrm{x} + \nu$$

where $\mathbf{x}$ is the covariate vector same as in PH model, λ is the regression coefficient vector,

[5]The Weibull distribution is a generalization of the exponential distribution of a Poisson Process, which has a constant hazard rate over time. A full specification of the Weibull distribution hazard function includes: $f(t) = \alpha\theta t^{\alpha-1}\exp(-\theta t^{\alpha}); S(t) = \exp(-\theta t^{\alpha}); F(t) = 1-\exp(-\theta t^{\alpha}); h(t) = -\frac{d\ln S(t)}{dt} = \alpha\theta t^{\alpha-1}$ where, $\theta = \exp(\mathrm{x}\lambda)$ and α is a parameter. If $\alpha > 1$, the hazard is monotonically increasing (positive duration dependence); if $0 < \alpha < 1$, the hazard is monotonically decreasing (negative duration dependence). The baseline hazard for the Weibull distribution is $h_0(t, \alpha) = \alpha t^{\alpha-1}$, where α is a nonnegative parameter.

and ν is the residual term. The distribution of the residual determines the regression model. If ν is assumed normal, the lognormal AFT model is obtained[6]. If ν is assumed to follow logistic distribution, the log-logistic AFT model is obtained. Similarly, assuming ν follows extreme-value theory yields the exponential AFT model or the Weibull AFT model. Assuming an incomplete gamma function for ν gives the generalized gamma AFT model.

At this stage, it is premature to determine whether the investment lag can be characterized by positive or negative duration dependence. Simply assuming an arbitrary distribution that displays certain characteristic may lead to estimation bias. Therefore, both proportional model and accelerated failure time model are estimated using all available distributions, which closely resembles Favero et al. (1994), Kiefer (1988) and Hurn and Wright (1994). Under PH model, four models are estimated including the semi-parametric Cox PH model, exponential PH model, Weibull PH model and Gompertz PH model. Under AFT model, five models are estimated including exponential AFT model, Weibull AFT model, lognormal AFT model, log-logistic AFT model and generalized gamma AFT model.

The following two hypotheses will be tested with the duration model.

Hypothesis 4 The duration of the investment lag is expected to depend negatively on the gas price and initial reserves. As discussed in Section 3.3.1, gas price and initial reserve positively related to firm's profit. When the gas price rises or firms have increased estimation reserve estimates, the expected profit would rise and firms are more likely to exercise the real option to invest.

Hypothesis 5 The competition effect and the network effect may decrease or increase the investment lag depending on which effect dominates. Grenadier (2002), Leahy

[6]Log-normal distribution leads to a non-monotonic hazard function, whose hazard rate initially increases and then decreases with time.

(1993) and Kogan (2001) argue that competition erodes real option values and reduces the development delay to zero-NPV rule, which means competition should have negative effect on the investment lag. Conversely, Novy-Marx (2007) argues that in industries with significant opportunity cost or supply side heterogeneity, this erosion effect of competition is not strong enough to offset the real option value, and firms may delay even further than the optimal investment threshold derived from the standard real option model. In the case of developing natural gas fields, the opportunity costs exist because, once the field is developed, it cannot be developed second time.[7] The firms are also heterogeneous because the sizes of their reserves are quite different. Another important factor is the network effect which tends to increase the investment lag. It is thus interesting to test which one is the dominating factor in the gas production industry. The estimation of the duration analysis shall be able to verify either one of these two results.

[7]This differs from the opportunity costs defined in Novy-Marx model, which assumes the firm can reinvest again, because the resource is renewable

Panel A. Proportional hazard

Duration	Cox Proportional Hazard			Exponential Hazard			Weibull Hazard			Gompertz Hazard		
	Haz. Ratio	Coefficient	z	Haz. Ratio	Coefficient	z	Haz. Ratio	Coefficient	z	Haz. Ratio	Coefficient	z
coop	0.9290	-0.0736	-0.58	1.0012	0.0012	0.01	1.0022	0.0022	0.02	0.9266	-0.0763	-0.61
pricedis	4.8226	1.5733	8.13	2.0697	0.7274	4.10	4.0523	1.3993	7.34	5.1257	1.6343	8.43
priceprod	0.5812	-0.5426	-7.56	0.7909	-0.2346	-4.36	0.6246	-0.4707	-6.92	0.5729	-0.5570	-7.69
reserve	1.0000	-2.82E-06	-0.61	1.0000	-1.49E-06	-0.32	1.0000	-1.67E-06	-0.36	1.0000	-2.32E-06	-0.50
wellsdisc	0.9983	-0.0017	-1.88	0.9988	-0.0012	-1.31	0.9979	-0.0021	-2.29	0.9982	-0.0018	-1.93
depth	1.0002	0.0002	2.16	1.0001	0.0001	0.93	1.0002	0.0002	1.70	1.0002	0.0002	2.09
capacity	1.0001	1.16E-04	3.65	1.0001	8.11E-05	2.63	1.0001	1.39E-04	4.34	1.0001	0.0001	3.92
Constant		-		0.0564	-2.8751	-15.10	0.0014	-6.5471	-17.13	0.0143	-4.2497	-18.27
p		-			-		-	2.1856	0.10		-	
sigma		-			-			-			-	
gamma		-			-			-		-	0.0927	15.96
N	298			298			298			298		
Log-likelihood	-1336.57			-344.59			-245.70			-218.26		
AIC*	2687.14			705.18			509.39			454.52		

Panel B. Accelerated failure-time hazard

Duration	Exponential AFT		Weibull AFT		Lognormal AFT		Log-logistic AFT		Generalized gamma AFT	
	Coefficient	z	Coefficient	z	Coefficient	z	Coefficient	z	Coefficient	z
coop	-0.0012	-0.01	-0.0010	-0.02	0.0028	0.04	0.0142	0.20	0.0085	0.18
pricedis	-0.7274	-4.10	-0.6402	-7.67	-0.8663	-7.91	-0.8549	-7.02	-0.6064	-8.04
priceprod	0.2346	4.36	0.2153	7.24	0.2660	9.65	0.2265	8.47	0.2126	6.24
reserve	1.49E-06	0.32	7.65E-07	0.36	2.11E-06	0.68	2.37E-06	0.87	1.34E-07	0.08
wellsdisc	0.0012	1.31	0.0009	2.29	0.0015	2.68	0.0011	2.15	0.0006	1.61
depth	-0.0001	-0.93	-0.0001	-1.70	-0.0001	-1.58	-0.0001	-2.28	-0.0001	-1.15
capacity	-8.11E-05	-2.63	-6.37E-05	-4.35	-1.01E-04	-5.22	-9.52E-05	-4.75	-4.12E-05	-2.64
Constant	2.8751	15.10	2.9955	32.13	2.6893	23.95	2.8944	27.30	3.0844	33.73
p	-		2.1856	1.99	-		-		-	
sigma	-		-		0.6137	0.57	-		0.3747	0.32
gamma	-		-		-		0.3343	0.30	-	
kappa	-		-		-		-		1.6589	6.74
N	298		298		298		298		298	
Log-likelihood	-344.59		-245.70		-277.36		-272.68		-240.62	
AIC*	705.18		509.39		572.73		563.36		501.25	

* AIC refers to Akaike information criterion.

Table 4.3: **Duration models of investment lag.** It presents the duration model analysis for investment lag assuming four different Hazard distribution function. The dependent variable is DURATION which measures the waiting period (investment lag) between discovery year and start production year. COOP indicates whether the gas processing plant is cooperative. PRICEDIS is the natural gas price at the discovery time. PRICEPROD is the natural gas price at the time of production. RESERVE is the initial reserve quantity of the field. DEPTH is the average depth of all production wells within particular field, representing the drilling costs. CAPACITY is the plant's daily processing capacity, proxying the construction cost of the plant. WELLSDISC is the total number discovery wells within certain field, representing the level of network effect.

Table 4.3 presents the results from aforementioned four PH models in panel A and five AFT models in panel B. To determine which model is a better fit, the log-likelihood and Akaike information criterion (AIC) need to be calculated.[8] Among four proportional hazard models, the Gompertz model is the best as it has the largest log-likelihood of −218.26 and smallest AIC of 454.52. Among five accelerated failure time models, the generalized gamma model is the best as it has the largest log-likelihood of −240.62 and smallest AIC of 501.25. Overall, Gompertz PH model is a better fit than generalized gamma AFT model since it has larger log-likelihood and smaller AIC. Therefore, the focus of the analysis will be put on Gompertz model. In Gompertz model, gamma equals to 0.0927, greater than 0, which means the hazard rate of failure rises with time. As firms wait longer, they are more likely to stop waiting and start the investment.

The discovery price and production price have opposite effects on the investment lag. PRICEDIS has a hazard ratio of 5.13 in Gompertz model, which suggests that if the discovery price increases by one cent, the firms are roughly five times more likely to start the investment. This is consistent with hypothesis 4 and standard real option theory: as commodity price rises, firms are more likely to exercise the real option to invest. The negative coefficient of PRICEDIS in generalized gamma model also verifies that the price at discovery has negative effect on investment lag.[9] One cent increase in the price at discovery would reduce the investment lag by 0.61 year. PRICEPROD has a hazard ratio of 0.57 indicating that firms are about 40% less likely to start the investment if the optimal real option exercise price increases by one cent. Generalized gamma model

[8]When parametric hazard models are not nested in the data, model comparison using likelihood ratio or Wald test may not be appropriate. As such, Akaike (1974) proposed penalizing the log-likelihood to reflect the number of parameters being estimated in a particular model and comparing them. In generalized gamma model, I also test the hypothesis that $\kappa = 0$ (test the appropriateness of the lognormal), and the hypothesis that $\kappa = 1$ (test for the appropriateness of the Weibull). The z value for $\kappa = 0$ is 6.74, suggesting suggesting that the lognormal model is not an appropriate model for analyzing investment lag. The p value for $\kappa = 1$ is 0.74%, which provides some support for the Weibull model.

[9]In Gompertz PH model and generalized gamma AFT model, the sign of coefficients are opposite because AFT model is expressed in terms of $\ln t$, the survival time (investment lag), whereas PH model is expressed as the hazard rate, the probability of the failure (investment)

gives PRICEPROD a positive coefficient of 0.21 suggesting the price at production has a positive effect on investment lag. One cent increase in the price at production would increase the investment lag by 0.21 year. The intuition is that the price at the time of production represents the threshold or hurdle price for development, which increases with development costs. Normally, higher development cost entails longer waiting period.

Hypothesis 4 regarding the initial reserve is not strongly confirmed. The hazard ratio for reserves is one and insignificant. However, the covariate RESERVE has a negative coefficient, -2.32×10^6 which indicates the correct direction — negative effect of initial reserves on firms' investment lag. The extreme small coefficient is again caused by the large magnitude of reserves. WELLSDISC has a hazard ratio of 99.82% indicating that if the competition or network effect increases by one unit, firms are 0.18% less likely to start the investment. Although this negative effect of the network effect on investment lag is quite small, it supports Novy-Marx (2007) argument that in industries with opportunity cost and heterogeneity, competition is not enough to fully erode the real option value. Other factors such as the network effect may cause the investment lag to be even longer. The hazard ratios of average well depth, DEPTH and plant capacity, CAPACITY are slightly greater than one and statistically significant. Their effects on the firms' probability of investing are marginally positive.

4.4 Conclusion

The competitive real option literature has developed various equilibrium models (Bertrand, Cournot, or Stackelberg) for firms' investment decisions under competition. This research provides another equilibrium possibility — cooperative equilibrium in which firms share common production facility and both benefit from the network effect. Strong evidence is provided to show that firms investment decisions are strategic at least in the natural gas industry. Sometimes they compete with each other by investing earlier to preempt others. Sometimes, they may cooperate with each other in order to take the advantage of network effect. The choice between competition and cooperation may depend on two factors, real option exercise price (the price at which the production starts) and the level of competition or the network effect. Higher option exercise price will decrease the possibility of cooperation, whereas higher network effect will increase the possibility of cooperation.

This research also provides the empirical evidence in favor of Novy-Marx (2007) argument. That is, in industries with significant opportunity costs such as oil and gas industry, supplier heterogeneity or network effect may offset the erosion effect of competition on real option value to delay the investment. Firms in these industries will not start investment once the NPV rises to zero. Instead, their investments are typically delayed. The duration the investment delay is affected by commodity price and the level of network effect. Commodity prices have a negative effect on the duration of investment lag. The network effect has an positive effect on the duration of investment lag.

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Appendix A

The smooth-pasting conditions for a non-cooperative player's investment decision

In order to derive the smooth-pasting conditions for a non-cooperative player's investment decision, we partially differentiate the expected payoff function W_i with respect to P_{τ_i} and Q_i. This requires differentiating a definite integral with respect to a parameter that appears in the integrand and in the limits of the integral. The following formula is discovered by Gottfried Wilhelm von Leibniz. Let f be a differentiable function of two variables, let a and b be differentiable functions of a single variable, and define the function F by

$$F(t) = \int_{a(t)}^{b(t)} f(t,x)dx \qquad \forall\ t.$$

Then

$$F'(t) = f\big(t, b(t)\big)b'(t) - f\big(t, a(t)\big)a'(t) + \int_{a(t)}^{b(t)} f_t(t,x)dx$$

I now apply this Leibniz formula to differentiate W_i with respect to P_{τ_i} and Q_i separately, where τ_i is the first time the manifold (P(Q),Q) hits the threshold (P*(Q),Q). Producer i's cash flow function is

$$\pi_{i,t} = (P_t - C)q_{i,t}$$

Producer i's production function is

$$q_i(t) = \begin{cases} q_i^c & t \in [\tau_i, \theta_{i,\text{trans}}] \\ \overline{\alpha}_i Q_i(\theta_{i,\text{trans}}) e^{-\overline{\alpha}_i (t-\theta_{i,\text{trans}})} & t \in [\theta_{i,\text{trans}}, \theta_i] \end{cases}$$

The production transition time is defined as

$$\theta_{i,\text{trans}} = \frac{Q_i(\tau_i)}{q_i^c} - \frac{1}{\overline{\alpha}_i} \implies \frac{\partial \theta_{i,\text{trans}}}{\partial P_{\tau_i}} = 0 \quad \text{and} \quad \frac{\partial \theta_{i,\text{trans}}}{\partial Q_i} = \frac{1}{q_i^c}$$

$$\overline{\alpha}_i Q_i(\theta_{i,\text{trans}}) = q_i^c$$

Therefore, we have

$$W_i \equiv \widehat{E}_{\tau_i}\left[\int_{\tau_i}^{\theta_{i,\text{trans}}} e^{-r(t-\tau_i)} (P_t - C) q_i^c dt + \int_{\theta_{i,\text{trans}}}^{\theta_i} e^{-r(t-\tau_i)} (P_t - C) q_i^c e^{-\overline{\alpha}_i (t-\theta_{i,\text{trans}})} dt \right] - K(q_i^c)$$

Since the price is assumed to follow the GBM, the futures price P_t follows

$$P_t = P_{\tau_i} e^{(\hat{\mu} + \sigma_P^2/2)(t-\tau_i)}$$

Therefore, the first smooth-pasting condition at the development time τ_i is:

$$
\begin{aligned}
V_P(P^*, Q^*) &= \frac{\partial W_i}{\partial P_{\tau_i}} \\
&= \widehat{E}_{\tau_i}\left[\int_{\tau_i}^{\theta_{i,\mathrm{trans}}} e^{-r(t-\tau_i)} e^{\hat{\mu}(t-\tau_i)} q_i^c dt \right. \\
&\quad \left. + \int_{\theta_{i,\mathrm{trans}}}^{\theta_i} e^{-r(t-\tau_i)} e^{\hat{\mu}(t-\tau_i)} q_i^c e^{-\overline{\alpha}_i(t-\theta_{i,\mathrm{trans}})} dt \right] \\
&= \widehat{E}_{\tau_i}\left[\int_{\tau_i}^{\theta_{i,\mathrm{trans}}} e^{(\hat{\mu}-r)(t-\tau_i)} q_i^c dt \right. \\
&\quad \left. + \int_{\theta_{i,\mathrm{trans}}}^{\theta_i} e^{(\hat{\mu}-r)(t-\tau_i)-\overline{\alpha}_i(t-\theta_{i,\mathrm{trans}})} q_i^c dt \right]
\end{aligned}
$$

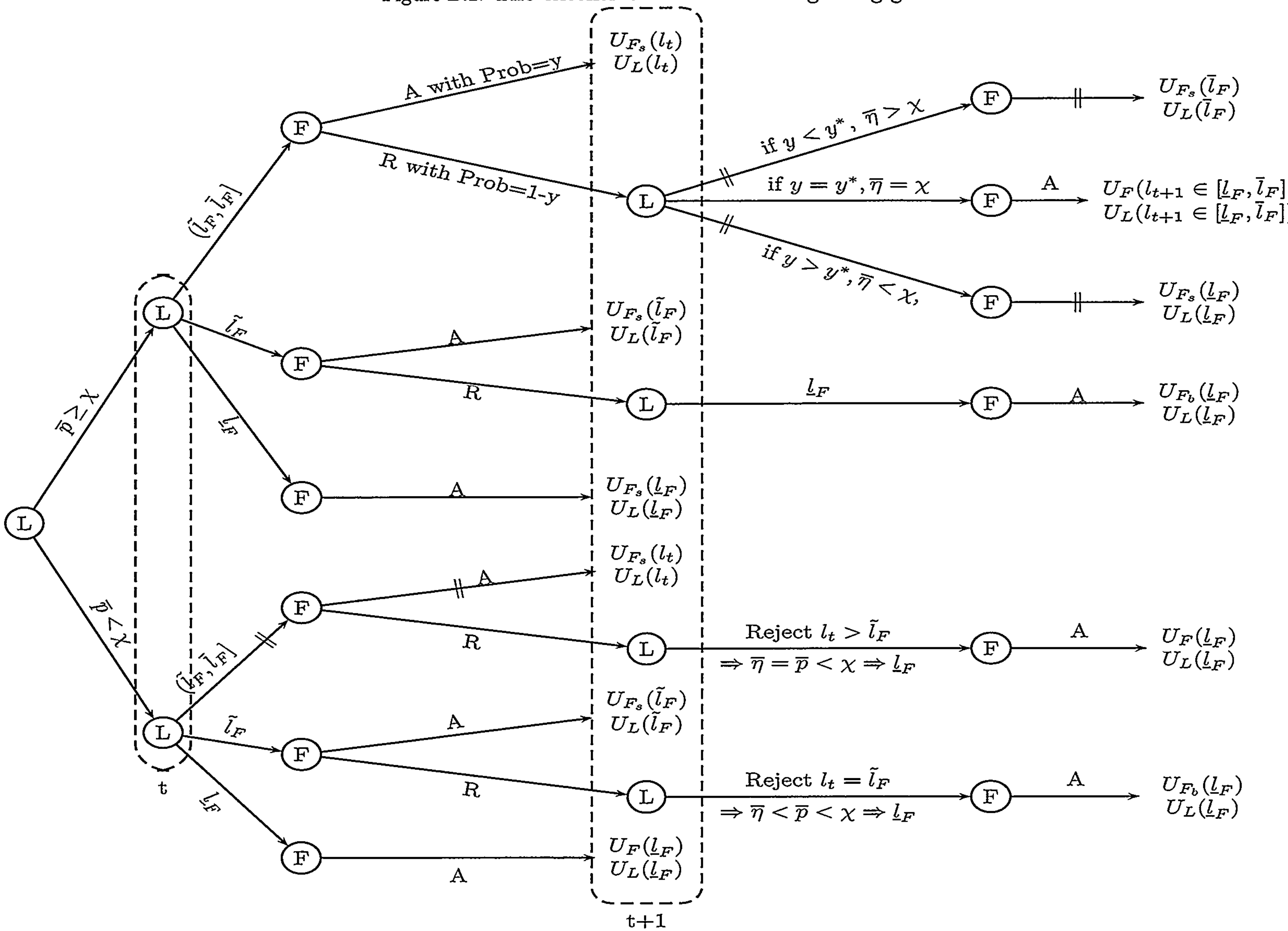
The second smooth-pasting condition follows from Leibniz' formula as:

$$
\begin{aligned}
V_Q(P^*, Q^*) &= \frac{\partial W_i}{\partial Q_i} \\
&= \widehat{E}_{\tau_i}\left[0 - \frac{1}{q_i^c} e^{-r(\theta_{i,\mathrm{trans}}-\tau_i)} (P_{\theta_{i,\mathrm{trans}}} - C) q_i^c + 0 \right. \\
&\quad + \frac{1}{q_i^c} e^{-r(\theta_{i,\mathrm{trans}}-\tau_i)} (P_{\theta_{i,\mathrm{trans}}} - C) q_i^c e^{-\overline{\alpha}_i(\theta_{i,\mathrm{trans}}-\theta_{i,\mathrm{trans}})} \\
&\quad \left. + \int_{\theta_{i,\mathrm{trans}}}^{\theta_i} e^{-r(t-\tau_i)} (P_t - C) q_i^c \frac{\overline{\alpha}_i}{q_i^c} e^{-\overline{\alpha}_i(t-\theta_{i,\mathrm{trans}})} dt \right] \\
&= \widehat{E}_{\tau_i}\left[-e^{-r(\theta_{i,\mathrm{trans}}-\tau_i)} (P_{\theta_{i,\mathrm{trans}}} - C) \right. \\
&\quad + e^{-r(\theta_{i,\mathrm{trans}}-\tau_i)} (P_{\theta_{i,\mathrm{trans}}} - C) \\
&\quad \left. + \int_{\theta_{i,\mathrm{trans}}}^{\theta_i} e^{-r(t-\tau_i)} (P_t - C) \overline{\alpha}_i e^{-\overline{\alpha}_i(t-\theta_{i,\mathrm{trans}})} dt \right] \\
&= \widehat{E}_{\tau_i}\left[\int_{\theta_{i,\mathrm{trans}}}^{\theta_i} e^{-r(t-\tau_i)} (P_t - C) \overline{\alpha}_i e^{-\overline{\alpha}_i(t-\theta_{i,\mathrm{trans}})} dt \right]
\end{aligned}
$$

Appendix B

The extensive form of bargaining game

Figure B.1: **The extensive form of the bargaining game**



Appendix C

The perfect Bayesian equilibrium for the leader and follower bargaining game.

The extensive form representation of this sequential bargaining game is shown in Figure B.1 of Appendix B, which will be used throughout the discussion of the perfect Bayesian equilibrium.

C.1 The leader and follower optimal strategies at time $t+1$

In period $t+1$, the leader with beliefs $\overline{\eta}(l_t)$ makes a "take it or leave it" offer l_{t+1} so as to maximize that period's profit. Because period $t+1$ is the last period, the leader's threat of offering no other contract in the future is credible, so the follower will accept if and only if his reservation is at least l_{t+1}. The follower's optimal strategy at date $t+1$ is defined as: [1]

$$\text{If } l_{t+1} = \begin{cases} \underline{l}_F, & F_s, F_b \text{ both accept} \\ \overline{l}_F, & F_s \text{ accepts}, F_b \text{ rejects} \\ \text{Random}[\underline{l}_F, \overline{l}_F], & F_s \text{ accepts}, F_b \text{ rejects} \end{cases} \tag{C.1}$$

The leader's offer l_{t+1} ranges from $\underline{l}_F$ to $\overline{l}_F$. If offering $l_{t+1} = \underline{l}_F$, the leader sells for sure and obtains the enterprise value of $U_{L,\text{coop}}(P, Q_L, q_L^{\Omega}, \underline{l}_F; N_{\eta_{\text{lease}}}^{\theta_L})$, simplified as $U_L(\underline{l}_F)$. If offering $l_{t+1} = \overline{l}_F$, the leader sells with probability $\overline{\eta}$ and has second period enterprise

[1]Each type follower is actually indifferent between accepting and rejecting a lease rate of l_{t+1} that exactly equals that type's reservation rate. However, as long as the supremum of the leader's total enterprise value is achieved in the limit of lease rate $l_{t+1} = l - |\varepsilon|$ as $\varepsilon \to 0$, I could assume, without loss of generality, the existence of an equilibrium given the leader's beliefs requires that type l accept $l_{t+1} = l$, and whether the other type accepts a lease rate equal to its reservation rate is irrelevant.

value of $\overline{\eta} \cdot U_{L,\text{coop}}(P, Q_L, q_L^{\Omega}, \bar{l}_F; N^{\theta_L}_{\eta_{\text{case}}})$, simplified as $\overline{\eta} \cdot U_L(\bar{l}_F)$. Therefore, there exists a unique critical probability $\chi \equiv \frac{U_L(\underline{l}_F)}{U_L(\bar{l}_F)}$, and the leader's optimal strategy at date $t+1$ is defined as:

$$l_{t+1} = \begin{cases} \underline{l}_F, & \text{if } \overline{\eta} < \chi \\ \bar{l}_F, & \text{if } \overline{\eta} > \chi \\ \text{Random}[\underline{l}_F, \bar{l}_F], & \text{if } \overline{\eta} = \chi \end{cases} \tag{C.2}$$

C.2 The leader and the follower's optimal strategy at time t

At time t, the leader and the follower's decisions are more complex. Ideally, the leader would want to offer the high type follower at $\bar{l}_F$ and the low type follower at $\underline{l}_F$. But I have already shown that the high type follower is motivated to lie. Therefore, the leader's task is to differentiate the high type follower from the low type follower by testing them with different lease rate. At time t, the low type follower F_b will accept if and only if $l_t = \underline{l}_F$ since it will never obtains a surplus at next period. Of course, the high type follower F_s accepts $l_t = \underline{l}_F$ too. The high type follower, however, if offered $l_t > \underline{l}_F$, has to consider how its rejection might affect the leader's posterior belief about the follower's type. High type follower F_s obtains a surplus only if the leader is sufficiently convinced that it is the low type follower, i.e., $\overline{\eta} < \chi$.

C.2.1 The consequence of the follower's rejection on the leader's posterior belief

I now discuss how the follower's rejection might affect the leader's posterior belief.

1. Choice of mixed and pure strategy

   Suppose the rejection of $l_t > \underline{l}_F$ generates "optimistic posterior beliefs": $\overline{\eta} > \chi$. From equation (C.2) the leader charges $l_{t+1} = \bar{l}_F$. High type F_s has no second period surplus from rejecting (continue lying) that $l_t > \underline{l}_F$. Therefore, the high type F_s is better off accepting $l_t > \underline{l}_F$. And since l_t is rejected by the low type F_b, Bayes' rule yields $\overline{\eta}(l_t) = \frac{\overline{p}\cdot 0}{\overline{p}\cdot 0+\underline{p}} = 0$, a contradiction. Thus neither of the pure strategies, accept or reject, is optimal here. In the following subsections, I will develop a mixed strategy for the follower and the leader in the case of the rejection generating optimistic posterior, and I will also elaborate the leader and follower's pure strategy in the case of the rejection generating "pessimistic posterior beliefs". Let $y(l_t)$ denote the probability that the high type F_s accepts l_t. Then the high type follower consider how its probability of rejection will affect the leader's posterior according to the following formula:

$$\overline{\eta}(l_t) = \frac{\overline{p}\big(1 - y(l_t)\big)}{\overline{p}\big(1 - y(l_t)\big) + \underline{p}}$$

(a) If F_s accept with probability of 1, then $y = 1 \Rightarrow 1 - y = 0$, then $\overline{\eta}(l_t) = \frac{\overline{p}\cdot 0}{\overline{p}\cdot 0+\underline{p}\cdot 1} = 0 < \chi$. According to equation (C.2), the leader with posterior $\overline{\eta} < \chi$ will offer $l_{t+1} = \underline{l}_F$. So F_s who anticipates this lower second period price l_{t+1} should not accept l_t with probability of 1. A contradictory.

(b) If F_s reject with probability of 1, then $y = 0 \Rightarrow 1 - y = 1$, then $\overline{\eta}(l_t) = \frac{\overline{p}\cdot 1}{\overline{p}\cdot 1+\underline{p}\cdot 1} = \overline{p}$. Now since in the top branch of the extensive form of game, I have $\overline{p} > \chi$. Therefore, $\overline{\eta} = \overline{p} > \chi$. According to equation (C.2), the leader with posterior $\overline{\eta} > \chi$ will offer $l_{t+1} = \overline{l}_F$. So F_s who anticipates this higher second period price l_{t+1} should not reject l_t with probability of 1. A contradictory.

In equilibrium the high type F_s should not reject l_t with probability 1, because in that case I would have $\overline{\eta}(l_t) = \overline{p} > \chi$ and the leader charging $l_{t+1} = \overline{l}_F$, so the high type F_s would be better off accepting l_t. But I already saw that the high type F_s cannot accept such an l_t with probability 1 either. Hence, the high type follower needs a mixed strategy here by randomizing between accept and reject, i.e., controlling the y so that the leader's posterior is $\overline{\eta}(l_t) = \chi$.

2. Rejection deteriorates the leader's ex ante belief.

   According to the Bayes rule, for any rejection of $l_t > \underline{l}_F$, the leader's posterior belief is calculated as:

$$\begin{aligned}\overline{\eta}(l_t) &= \frac{\text{Prob}(\text{type} = F_s \ \& \ \text{reject } l_t > \underline{l}_F)}{\text{Prob}(\text{reject } l_t > \underline{l}_F)} = \frac{\overline{p} \cdot \text{Prob}(l_t > \tilde{l}_F)}{\overline{p} \cdot \text{Prob}(l_t > \tilde{l}_F) + \underline{p}} \\ &= \frac{\overline{p}}{\overline{p} + \dfrac{\underline{p}}{\text{Prob}(l_t > \tilde{l}_F)}} \leq \overline{p}\end{aligned} \tag{C.3}$$

which means the posterior is always less than or equal to the prior conditional on the rejection of $l_t > \underline{l}_F$.

C.2.2 The follower's indifference lease rate $\tilde{l}_F$

To analyze the high type follower's behavior at t when offered price $l_t \in (\underline{l}_F, \overline{l}_F]$, I have to define a critical indifference least rate $\tilde{l}_F$. The high type follower F_s should accept l_t only if

$$\begin{aligned} U_{F_s}(l_t) - U_{F_s}(\overline{l}_F) &\geq e^{-r}\left(U_{F_s}(\underline{l}_F) - U_{F_s}(\overline{l}_F)\right) \\ \Rightarrow \qquad U_{F_s}(l_t) &\geq (1 - e^{-r})U_{F_s}(\overline{l}_F) + e^{-r}U_{F_s}(\underline{l}_F) \end{aligned} \tag{C.4}$$

To see this, note that $U_{F_s}(l_t) - U_{F_s}(\overline{l}_F)$ is the realized gain from lying at time t and $U_{F_s,\text{coop}}(\underline{l}_F) - U_{F_s,\text{coop}}(\overline{l}_F)$ is the maximum possible gain from continuing lying at time $t+1$. Denote $\tilde{l}_F$ as the l_t which makes the above inequality equal. That is

$$U_{F_s}(\tilde{l}_F) = (1 - e^{-r})U_{F_s}(\overline{l}_F) + e^{-r}U_{F_s}(\underline{l}_F)$$

Obviously, when $l_t = \tilde{l}_F$, the high type follower F_s is indifferent between accepting this l_t and getting $l_{t+1} = \underline{l}_F$ at time $t+1$ by rejecting this l_t. As the high type follower's enterprise value function, $U_{F_s}(l)$ decreases in l, I have the optimal strategy for the high type follower when facing the lease offer at $l_t > \underline{l}_F$.

- If $\underline{l}_F < l_t \leq \tilde{l}_F \Rightarrow U_{F_s}(l_t) \geq U_{F_s}(\tilde{l}_F) = (1-e^{-r})U_{F_s}(\bar{l}_F)+e^{-r}U_{F_s}(\underline{l}_F)$. Equation (C.4) is satisfied. High type F_s accepts this $l_t \in (\underline{l}_F, \tilde{l}_F]$.

- If $l_t > \tilde{l}_F$, rejecting l_t is optimal for the high type F_s as it is for the low type F_b, and therefore Bayes' rule yields

$$\overline{\eta}(l_t > \tilde{l}_F) = \frac{\overline{p} \cdot 1}{\overline{p} \cdot 1 + \underline{p} \cdot 1} = \overline{p}$$

  which means the posterior beliefs coincide with the prior beliefs. In other words, the follower is safe to reject any offer $l_t > \tilde{l}_F$ at time t without improving the leader's information about the follower's type.

C.2.3 The strategy of the pessimistic leader $\overline{p} < \chi$

Equation (C.3) shows $\overline{\eta} \leq \overline{p}$, combined with $\overline{p} < \chi$, I have $\overline{\eta} < \chi$. This means no matter what the first period offer is, the follower's rejection always makes the leader pessimistic. Therefore the leader's second period strategy is limited to $l_{t+1} = \underline{l}_F$ whenever it observes a rejection at time t. I now compare the leader's expected total enterprise values from three different first period strategies, as illustrated in the bottom branch of Figure B.1.

1. Bottom-Bottom strategy (BB): $l_t = \underline{l}_F$

   Both type followers will accept this l_t as they knows this is the most favorable price. BB therefore leads to a pooling equilibrium. The leader has an enterprise value of $U_L(\underline{l}_F)$.

2. Bottom-Middle strategy (BM): $l_t = \tilde{l}_F$

   The high type F_s would accept this l_t because it is indifferent as discussed in Section C.2.2. The low type F_b rejects this offer because leasing would give him a negative surplus, i.e., $U_{F_b,\text{coop}}(\tilde{l}_F) < U_{F_b,\text{nc}}$ according to equation (2.12). Thus if the leader observes a rejection, it knows the follower is low type and will set $l_{t+1} = \underline{l}_F$. BM therefore leads to a separating equilibrium. The leader's expected enterprise value from BM strategy is: $\overline{p} \cdot U_L(\tilde{l}_F) + e^{-r}\underline{p} \cdot U_L(\underline{l}_F)$.

3. Bottom-Top strategy (BT): $l_t \in (\tilde{l}_F, \overline{l}_F]$

   Again, the low type follower F_b rejects this offer because $U_{F_b,\text{coop}}(\tilde{l}_F+) < U_{F_b,\text{nc}}$. The high type follower F_s would rather reject this l_t since it knows that the consequence of rejecting the leader's offer is $\overline{\eta} = \overline{p} < \chi$ and the leader will offer a lower lease rate next period, $l_{t+1} = \underline{l}_F$. BT therefore leads to a pooling equilibrium as both type followers reject. BT strategy will give the leader a total enterprise value of $e^{-r} \cdot U_L(\underline{l}_F)$.

Clearly, BB is better than BT and BM is better than BT.[2] Either BB or BM can give the leader higher value depending on the generic values of parameters. Thus, I summarize the pessimistic leader's optimal strategy as:

$$l_t = \begin{cases} \underline{l}_F, & \text{if } \frac{U_L(\tilde{l}_F)}{U_L(\underline{l}_F)} < \dfrac{1-e^{-r}\underline{p}}{\overline{p}} \\ \tilde{l}_F, & \text{if } \frac{U_L(\tilde{l}_F)}{U_L(\underline{l}_F)} > \dfrac{1-e^{-r}\underline{p}}{\overline{p}} \end{cases} \tag{C.5}$$

[2]The leader's valuation function $U_L(l)$ increases at l. Hence, $\overline{p}U_L(\tilde{l}_F) > \overline{p}U_L(\underline{l}_F)$. Therefore, $\overline{p}U_L(\tilde{l}_F) + e^{-r}\underline{p}U_L(\underline{l}_F) > \overline{p}U_L(\underline{l}_F) + e^{-r}\underline{p}U_L(\underline{l}_F) > \overline{p}e^{-r}U_L(\underline{l}_F) + e^{-r}\underline{p}U_L(\underline{l}_F) = e^{-r}U_L(\underline{l}_F)$.

C.2.4 The strategy of the optimistic leader $\overline{p} > \chi$

1. Top-Bottom strategy (TB): $l_t = \underline{l}_F$

   The TB strategy is same as the BB strategy. Both type followers accept the lease and the leader's enterprise value is $U_L(\underline{l}_F)$, a pooling equilibrium.

2. Top-Middle strategy (TM): $l_t = \tilde{l}_F$

   This is also similar to BM strategy. The high type F_s accepts whereas the low type F_b rejects this offer, a separating equilibrium. The leader's expected enterprise value from TM strategy is: $\overline{p} \cdot U_L(\tilde{l}_F) + e^{-r}\underline{p} \cdot U_L(\underline{l}_F)$.

3. Top-Top strategy (TT): $l_t \in (\tilde{l}_F, \overline{l}_F]$

   The low type follower F_b rejects this offer. The high type follower F_s has a more complex decision because it has to consider the consequence of rejecting the leader's offer, i.e., whether the leader is going to charge a higher or lower l_{t+1}. In equilibrium the high type F_s cannot reject l_t with probability 1, because in that case I would have $\overline{\eta}(l_t) = \overline{p} > \chi$ and the leader charging $l_{t+1} = \overline{l}_F$, so the high type F_s would be better off accepting l_t. But I already saw that the high type F_s cannot accept such an l_t with probability 1 either. In fact, the offer of $l_t \in (\tilde{l}_F, \overline{l}_F]$ is a dilemma for the high type because if it rejects, the leader will charge an even higher $l_{t+1} = \overline{l}_F$; if it accepts, it gets the smallest expected enterprise value.

   Hence, the high type follower needs a mixed strategy here by randomizing between accept and reject. In equilibrium the high type F_s must randomize in order to make the leader's posterior belief satisfy $\overline{\eta}(l_t) = \chi$ so that the leader will offer the price l_{t+1} to be any randomization between $\underline{l}_F$ and $\overline{l}_F$. Let $y(l_t)$ denote the probability that the high type F_s accepts l_t. Then $\overline{\eta}(l_t) = \chi$ will give:

$$\overline{\eta}(l_t) = \frac{\overline{p}(1 - y^*(l_t))}{\overline{p}(1 - y^*(l_t)) + \underline{p}} = \chi \ \Rightarrow \ y^*(l_t) = 1 + \frac{\chi \underline{p}}{\chi \overline{p} - \overline{p}}$$

which defines a unique $y^*(l_t) = y^* \in [0, 1]$. Note that $y^*(l_t)$ is independent of l_t. Any $y < y^*$ will make the leader's posterior belief $\overline{\eta} > \chi$, which leads to $l_{t+1} = \bar{l}_F$. Any $y > y^*$ will make the leader's posterior belief $\overline{\eta} < \chi$, which leads to $l_{t+1} = \underline{l}_F$. Since the equilibrium has to be Pareto efficient, in order for the high type F_s to be indifferent between accepting and rejecting l_t, I need to define another probability $x(l_t)$ for the high type follower to realize its maximum second period gain.

$$U_{F_s}(l_t) - U_{F_s}(\bar{l}_F) = e^{-r} x(l_t) \big(U_{F_s}(\underline{l}_F) - U_{F_s}(\bar{l}_F) \big)$$

which defines a unique probability $x(l_t)$ for $l_{t+1} = \underline{l}_F$. The leader's expected enterprise value can be calculated as:

$$\overline{p} y U_L(\bar{l}_F) + e^{-r} \big[\overline{p}(1 - y)(1 - x) U_L(\bar{l}_F) + \overline{p}(1 - y) x U_L(\underline{l}_F) + x \underline{p} U_L(\underline{l}_F) \big] \qquad \text{(C.6)}$$

Any of those strategies, TT, TM and TB can generates the highest total enterprise value for the leader depending on the parameter values. I summarize the optimistic leader's optimal strategy and expected enterprise value in the first period as one the following:

$$l_t = \begin{cases} \underline{l}_F, & \text{which generates value } U_L(\underline{l}_F); \\ \tilde{l}_F, & \text{which generates value } \overline{p} \cdot U_L(\tilde{l}_F) + e^{-r} \underline{p} \cdot U_L(\underline{l}_F); \\ \bar{l}_F, & \text{which generates value } \overline{p} y \cdot U_L(\bar{l}_F) + e^{-r} \big(\overline{p}(1 - y) + \underline{p} \big) U_L(\underline{l}_F). \end{cases}$$

where the third enterprise value is computed using the fact that, for posterior beliefs $\overline{\eta} = \chi$, $l_{t+1} = \underline{l}_F$ is an optimal price in the second period for the seller as $x(\underline{l}_F) = 1$. Note that if the third value is highest, the leader never sells to the low type F_b as $x(\bar{l}_F) = 0$.

The conclusion is thus that there exists a unique perfect Bayesian equilibrium, and that this equilibrium exhibits Coasian dynamics — that is, $\overline{\eta}(l_t) \leq \overline{p}$ for all l_t, so the leader becomes more pessimistic over time, and $l_{t+1} \leq l_t$, so the leader's offer decreases over time.

Appendix D

The numerical comparison: comparison of finite difference method and least square Monte-Carlo method

D.1 Single player model for the leader

The leader chooses the optimal time point to invest and start producing, suspend or resume the operation depending on the market price P and estimated remaining reserve quantity Q.[1] Suppose $K(q)$ is the cost of developing to capacity q, as before, and K_s is the cost of suspending, and K_r is the cost of resuming suspended production. Define

$$
\begin{aligned}
V(P,Q,m) &= \text{the leader's firm value} \\
\text{where} \quad m &= 0, \text{if undeveloped} \\
m &= 1, \text{if producing} \\
m &= 2, \text{if developed, but production is suspending}
\end{aligned}
$$

The boundary triggers are $\{P_o^*, Q_o^*\}$ for developing, $\{P_{oc}^*, Q_{oc}^*\}$ for operating to close, $\{P_{co}^*, Q_{co}^*\}$ for close to operating. Then the trigger values must satisfy value-matching

[1] This is an extension of the classic model of operating real options by Brennan and Schwartz (1985) to finite reserves.

and smooth-pasting conditions:

$$V(P_o^*, Q_o^*, 0) = V(P_o^*, Q_o^*, 1) - K(q)$$

$$V(P_{oc}^*, Q_{oc}^*, 1) = VP_{oc}^*, Q_{oc}^*, 2) - K_s$$

$$V(P_{co}^*, Q_{co}^*, 2) = V(P_{co}^*, Q_{co}^*, 1) - K_r$$

$$V_P(P_o^*, Q_o^*, 0) = V_P(P_o^*, Q_o^*, 1)$$

$$V_Q(P_o^*, Q_o^*, 0) = V_Q(P_o^*, Q_o^*, 1)$$

$$V_P(P_{oc}^*, Q_{oc}^*, 1) = V_P(P_{oc}^*, Q_{oc}^*, 2)$$

$$V_Q(P_{oc}^*, Q_{oc}^*, 1) = V_Q(P_{oc}^*, Q_{oc}^*, 2)$$

$$V_P(P_{co}^*, Q_{co}^*, 2) = V_P(P_{co}^*, Q_{co}^*, 1)$$

$$V_Q(P_{co}^*, Q_{co}^*, 2) = V_Q(P_{co}^*, Q_{co}^*, 1)$$

Assume the convenience yield dividend yield on the underlying asset (petroleum) is $\delta(P, t)$. The risk-neutral drift is

$$\mu(P, t) - \lambda\beta(P) = rP - \delta(P, t)$$

where

$$\beta(P)\frac{\text{cov}(dP, df)}{\sqrt{\text{var}(dP)\text{var}(df)}}$$

and r is the risk free interest rate. Similarly, the risk-neutral drift of Q is:

$$\mu(Q) - q(m) - \lambda_Q\beta(Q) = \mu(Q) - q(m)$$

where

If I assuming the P and Q are not correlated, the firm value must also satisfy the following two dimensional PDEs:

When $m = 0$

$$\frac{1}{2}\left[\sigma^2(Q)V_{QQ}(P,Q,0) + \sigma^2(P)V_{PP}(P,Q,0)\right]$$

$$+V_Q(P,Q,0)\left[\mu(Q) - q(0) - \lambda_Q\beta(Q)\right]$$

$$+V_P(P,Q,0)\left[\mu_P(P) - \lambda_P\beta_P(P)\right] + V_t = rV(P,Q,0)$$

$$\Downarrow$$

$$\frac{1}{2}\left[\sigma^2(Q)V_{QQ}(P,Q,0) + \sigma^2(P)V_{PP}(P,Q,0)\right] + V_Q(P,Q,0)\mu(Q)$$
$$+ V_P(P,Q,0)\left[\mu_P(P) - \lambda_P\beta_P(P)\right] + V_t = rV(P,Q,0) \tag{D.1}$$

When $m = 1$

$$\frac{1}{2}\left[\sigma^2(Q)V_{QQ}(P,Q,1) + \sigma^2(P)V_{PP}(P,Q,1)\right] + V_Q(P,Q,1)\left[\mu(Q) - q(1)\right]$$
$$+ V_P(P,Q,1)\left[\mu_P(P) - \lambda_P\beta_P(P)\right] + V_t + \pi = rV^1(P,Q,1) \tag{D.2}$$

$$\Downarrow$$

$$\frac{1}{2}\left[\sigma^2(Q)V_{QQ}(P,Q,1) + \sigma^2(P)V_{PP}(P,Q,1)\right] + V_Q(P,Q,1)\left[\mu(Q) - q(1)\right]$$
$$+ V_P(P,Q,1)\left[\mu_P(P) - \lambda_P\beta_P(P)\right] + V_t + \pi(l) = rV^2(P,Q,1) \tag{D.3}$$

$$\Downarrow$$

$$\frac{1}{2}\left[\sigma^2(Q)V_{QQ}(P,Q,1) + \sigma^2(P)V_{PP}(P,Q,1)\right] + V_Q(P,Q,1)\left[\mu(Q) - q(1)\right]$$
$$+ V_P(P,Q,1)\left[\mu_P(P) - \lambda_P\beta_P(P)\right] + V_t + \pi^3 = rV^3(P,Q,1) \tag{D.4}$$

When $m = 2$

$$\frac{1}{2}\left[\sigma^2(Q)V_{QQ}(P,Q,2)+\sigma^2(P)V_{PP}(P,Q,2)\right]$$

$$+V_Q(P,Q,2)\left[\mu(Q)-q(2)-\lambda_Q\beta(Q)\right]$$

$$+V_P(P,Q,2)\left[\mu_P(P)-\lambda_P\beta_P(P)\right]+V_t=rV(P,Q,2)$$

$$\Downarrow$$

$$\frac{1}{2}\left[\sigma^2(Q)V_{QQ}(P,Q,2)+\sigma^2(P)V_{PP}(P,Q,2)\right]+V_Q(P,Q,2)\left[\mu(Q)-q(2)\right]$$

$$+V_P(P,Q,2)\left[\mu_P(P)-\lambda_P\beta_P(P)\right]+V_t=rV(P,Q,2) \tag{D.5}$$

D.2 Solve the one player model with finite difference method

I use explicit finite difference method, Euler methods and symmetric difference. I assume simple log-normal process for both P and Q, which is:

$$\begin{aligned} dP &= \mu_P dt+\sigma_P P dz_P \qquad &(D.6)\\ dQ &= \mu_Q dt+\sigma_Q Q dz_Q \qquad &(D.7)\end{aligned}$$

Also, assume a constant dividend yield rate δ_0 which makes $\delta(P,t)=\delta_0 P$. Thus,

$$\begin{aligned} \mu(P)-\lambda\beta(P) &= \mu_P P-\delta_0 P = rP-\delta_0 P \qquad &(D.8)\\ \mu(Q) &= \mu_Q Q \qquad &(D.9)\\ \sigma^2(P) &= \sigma_P^2 P^2 \qquad &(D.10)\\ \sigma^2(Q) &= \sigma_Q^2 Q^2 \qquad &(D.11)\end{aligned}$$

Discretize

$$
\begin{aligned}
V_t &= \frac{V(P_i, Q_j, t_k) - V(P_i, Q_j, t_{k-1})}{\Delta t} &\text{(D.12)}\\
V_P &= \frac{V(P_{i+1}, Q_j, t_k) - V(P_{i-1}, Q_j, t_k)}{2\Delta P} &\text{(D.13)}\\
V_Q &= \frac{V(P_i, Q_{j+1}, t_k) - V(P_i, Q_{j-1}, t_k)}{2\Delta Q} &\text{(D.14)}\\
V_{PP} &= \frac{V(P_{i+1}, Q_j, t_k) + V(P_{i-1}, Q_j, t_k) - 2V(P_i, Q_j, t_k)}{(\Delta P)^2} &\text{(D.15)}\\
V_{QQ} &= \frac{V(P_i, Q_{j+1}, t_k) + V(P_i, Q_{j-1}, t_k) - 2V(P_i, Q_j, t_k)}{(\Delta Q)^2} &\text{(D.16)}
\end{aligned}
$$

Then, equation D.1 becomes

$$
\begin{gathered}
\frac{1}{2}\sigma_Q^2 j^2 (\Delta Q)^2 \frac{V(P_i, Q_{j+1}, t_k) + V(P_i, Q_{j-1}, t_k) - 2V(P_i, Q_j, t_k)}{(\Delta Q)^2} \\
+\frac{1}{2}\sigma_P^2 i^2 (\Delta P)^2 \frac{V(P_{i+1}, Q_j, t_k) + V(P_{i-1}, Q_j, t_k) - 2V(P_i, Q_j, t_k)}{(\Delta P)^2} \\
+\mu_Q j \Delta Q \frac{V(P_i, Q_{j+1}, t_k) - V(P_i, Q_{j-1}, t_k)}{2\Delta Q} \\
+(r - \delta_0) i \Delta P \frac{V(P_{i+1}, Q_j, t_k) + V(P_{i-1}, Q_j, t_k)}{2\Delta P} \\
+\frac{V(P_i, Q_j, t_k) - V(P_i, Q_j, t_{k-1})}{\Delta t} = rV(P_i, Q_j, t_k) \qquad \text{(D.17)}
\end{gathered}
$$

$$\Downarrow$$

$$
\begin{gathered}
\frac{1}{2}\sigma_Q^2 j^2 \left[V(P_i, Q_{j+1}, t_k) + V(P_i, Q_{j-1}, t_k) - 2V(P_i, Q_j, t_k)\right] \\
+\frac{1}{2}\sigma_P^2 i^2 \left[V(P_{i+1}, Q_j, t_k) + V(P_{i-1}, Q_j, t_k) - 2V(P_i, Q_j, t_k)\right] \\
+\frac{1}{2}\mu_Q j \left[V(P_i, Q_{j+1}, t_k) - V(P_i, Q_{j-1}, t_k)\right] \\
+\frac{1}{2}(ri - \delta_0 i) \left[V(P_{i+1}, Q_j, t_k) + V(P_{i-1}, Q_j, t_k)\right]
\end{gathered}
$$

$$+\frac{V(P_i,Q_j,t_k)-V(P_i,Q_j,t_{k-1})}{\Delta t}=rV(P_i,Q_j,t_k) \tag{D.18}$$

$$\Downarrow$$

Finally, I get the recursive formula:

$$\begin{aligned}
&(1-\sigma_Q^2 j^2\Delta t-\sigma_P^2 i^2\Delta t-r\Delta t)V(P_i,Q_j,t_k)+(\frac{1}{2}\sigma_Q^2 j^2\Delta t+\frac{1}{2}\mu_Q j\Delta t)V(P_i,Q_{j+1},t_k)\\
&(\frac{1}{2}\sigma_Q^2 j^2\Delta t-\frac{1}{2}\mu_Q j\Delta t)V(P_i,Q_{j-1},t_k)+\left(\frac{1}{2}\sigma_P^2 i^2\Delta t+\frac{1}{2}(ri-\delta_0 i)\Delta t\right)V(P_{i+1},Q_j,t_k)\\
&\left(\frac{1}{2}\sigma_P^2 i^2\Delta t-\frac{1}{2}(ri-\delta_0 i)\Delta t\right)V(P_{i-1},Q_j,t_k)=rV(P_i,Q_j,t_{k-1})
\end{aligned} \tag{D.19}$$

There are three conditions for the input variables to ensure the stability of the solution to the PDEs, for i and j within the recursive formula

$$\begin{aligned}
(1-\sigma_Q^2 j^2\Delta t-\sigma_P^2 i^2\Delta t-r\Delta t) &\geq 0\\
(\frac{1}{2}\sigma_Q^2 j^2\Delta t-\frac{1}{2}\mu_Q j\Delta t) &\geq 0\\
\frac{1}{2}\sigma_P^2 i^2\Delta t+\frac{1}{2}(ri-\delta_0 i)\Delta t &\geq 0\\
\frac{1}{2}\sigma_P^2 i^2\Delta t-\frac{1}{2}(ri-\delta_0 i)\Delta t &\geq 0
\end{aligned}$$

$$\Downarrow$$

$$\frac{1}{\Delta t}-r \geq \sigma_P^2 i_{max}^2+\sigma_Q^2 j_{max}^2 \tag{D.20}$$

$$j_{min} \geq \frac{\mu_Q}{\sigma_Q^2} \tag{D.21}$$

$$r \geq \delta_0 \tag{D.22}$$

$$i_{min} \geq \frac{r-\delta_0}{\sigma_P^2} \tag{D.23}$$

Appendix E

Matlab program routines and base case parameters

E.1 The routine map for the game

E.2 The description of base case parameters of simulation

Figure E.1: The program routine map of LSM simulation of leader-follower investment game

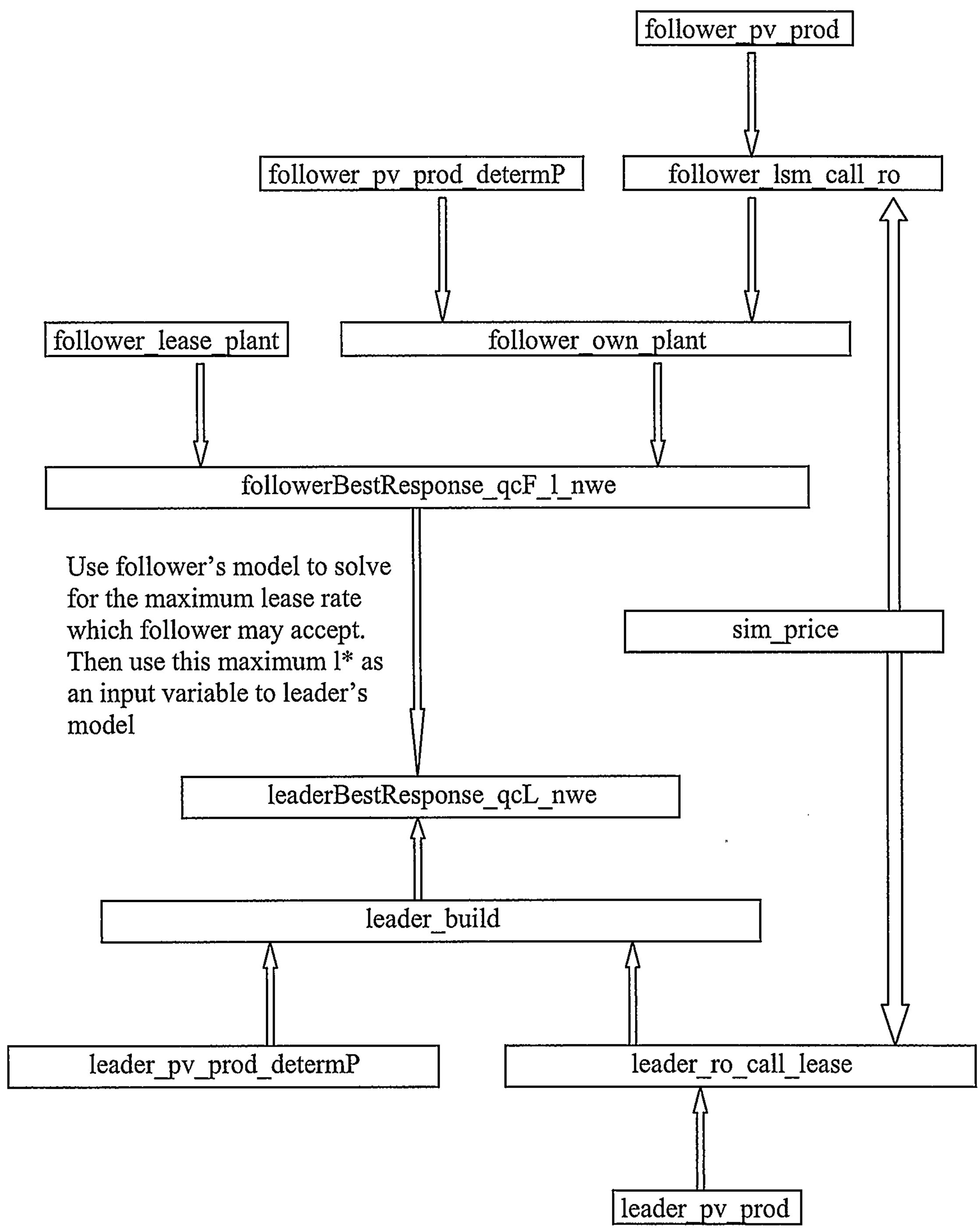


Figure E.2: **Base Case Parameters for Least Square Monte-Carlo Simulation in Matlab**

The parameter value for the follower's model

P = 1:0.5:8;	Current commodity's price vector, same for the leader and the follower.
QF=500000:5000:800000;	The follower's initial reserve quantity, 1-by-61 row vector
l = 1:0.5:20;	The lease rate vector, 1-by-81
qFL = 8000;	The production capacity which the follwer leases from the leader. Initially it was set to the follower's own plant size and will be varied later.
a = 20000;	Fixed cost of construction of gas plant
b = 100;	Variable cost of construction, bigger plant costs more to construct
qcF=8000;	The follower's plant capacity decision, i.e., production constraint II, Initially, it was set equal to maximum QF/100 now. 100 is the maximum production year. It will be allowed to vary later.
drill_c = 80000;	Drilling cost
construct_c = a+b*qcF;	The vector of construction cost, 1-by-2
K=construct_c + drill_c;	The exercise price
alpha_bar=0.08;	Government regulated daily production rate
C = 1.2;	Assume same variable production cost for the leader and the follower
nwe = 0.1*C;	The network effect as a percentage of production cost
maxT_prod = 100;	Assume the production will be terminated by government at the end of 100 years.
rf=0.08;	Risk free rate

The parameter values for the leader's model

P = 1:0.5:8;	Current commodity's price vector, same for the leader and the follower.
QL=1600000:10000:2000000;	The leader's initial reserve quantity, 1-by-41 row vector, bigger than QF
l = 10;	Assume this is the optimal lease rate calculated from follower's model
a = 20000;	Fixed cost of construction of gas plant, same for the leader and the follower.
b = 100;	Variable cost of construction, same for the leader and the follower.
qcL=[20000,28000];	The leader's cooperative plant capacity decision, i.e., production constraint II, It was set as the sum of maximum (QL/100)+qFL;
construct_c = a+b*qcL;	The vector of construction cost, 1-by-2
drill_c = 80000;	Drilling cost
K=construct_c + drill_c;	The exercise price
C = 1.2;	Assume same variable production cost for the leader and the follower
nwe = 0.1*C;	The network effect as a percentage of production cost
qFL = 8000;	The production capacity which the follwer leases from the leader. Initially it was set to the follower's own plant size and will be varied later. It is normally specified by the leasing contract. It needs to be synchonized with
rf=0.08;	Risk free rate
alpha_bar=0.08;	Government regulated daily production rate
maxT_prod = 100;	Assume the production will be terminated by government at the end of 100 years.

The parameters for the underlying process

steps=80;	Set the unit time to be quarter
paths=50;	More paths will make the option value curve smoother
alpha=rf;	The risk-neutral drift rate of the underlying asset --- natural gas price
sigma=0.15;	Volatility of the natural gas price
delta=0.06;	Convenience (dividend) yield
T=20;	The life of the real option. If T increasing, while steps unchanged, the option value curve will deviate further from its lower bound.

Appendix F

The list of ERCB publications used for data collection

ST98: Alberta's Energy Reserves & Supply/Demand Outlook Includes[1]:

- estimates of reserves for crude bitumen, crude oil, natural gas, natural gas liquids, coal, and sulphur for the province of Alberta
- supply and demand forecasts for Alberta's energy resources
- information on energy prices and economic performance
- terminology and abbreviations
- detailed tables showing established reserves, other data on a field and pool basis for crude bitumen, crude oil, and natural gas. (formerly Alberta's Reserves)

Gas Pool Reserves File Basic reservoir parameters and total reserves of all nonconfidential gas pools in the province. Separate records for each pool: approximately 24,000 pools. Used primarily for Established Reserves of Gas and Appropriate Basic Data, published as part of Statistical Series ST-98: Alberta's Reserves of Crude Bitumen, Crude Oil, Natural Gas, Ethane and other Natural Gas Liquids, Coal and Sulphur.

ST102: Facility List (formerly Guide 41 and Guide 42) A complete list of batteries, gas plants, meter stations, and other facilities in the province. As well, the

[1] The description of these publications are extracted from ERCB's 2007 Catalogue: Publications, Maps, and Services.

list has been upgraded to include additional information frequently requested by customers, such as operator name and facility sub-type description. Due to the size of the list, it will be in two parts:

- Part A List of New and Active Facilities
- Part B List of Other Facilities

ST50: Gas Processing Plants in Alberta Includes

- list of gas plants in Alberta, identifying location and fields served
- plant operator and design capacities
- sulphur recovery efficiency and maximum daily emission rates