THE UNIVERSITY OF CALGARY

Steady Flow in Open Channel Networks

by

Kelly Ronald Finigan

A THESIS
SUBMITTED TO THE FACULTY OF GRADUATE STUDIES
IN PARTIAL FULFILMENT OF THE REQUIREMENTS FOR THE
DEGREE OF MASTER OF SCIENCE

DEPARTMENT OF CIVIL ENGINEERING

CALGARY, ALBERTA
MARCH, 1995

THE UNIVERSITY OF CALGARY

FACULTY OF GRADUATE STUDIES

The undersigned certify that they have read, and recommend to the Faculty of Graduate Studies for acceptance a thesis entitled "Steady Flow In Open Channel Networks" submitted by Kelly Ronald Finigan in partial fulfilment of the requirements for the degree of Master of Science.

Supervisor, Dr. D.H. Manz
Department of Civil Engineering

Dr. Robert L. Day
Department of Civil Engineering

Laurence R. Bentley

Dr. Laurence Bentley
Department of Geology and Geophysics

Date March 31/95

ABSTRACT

Steady-state solution of open channel networks is reviewed. Problems presented by complex systems, particularly irrigation networks, are considered.

Constraint Rules are proposed to determine whether a given system has a solution. These rules consider the number and locations of constraints and variables available in a system. "Under-constrained" systems occur if the number of variables exceeds the number of equations. Parametric solutions result. Systems wherein the number of constraints supplied exceeds the number of variables are "over-constrained" and are generally not simply solved. Solution alternatives and examples are presented for both over-constrained and under-constrained systems.

Solution algorithms are prepared for networks. A computer model, SNAP (Steady Network Analysis Program), has been programmed to illustrate the algorithms and some of their potential capabilities. Several examples of the model's unique capabilities are provided.

ACKNOWLEDGEMENTS

The author gratefully acknowledges the guidance and advice of his supervisor and friend Dr. David Manz. Additionally, the support and patience of my co-worker Byron Buzunis is appreciated. I am grateful to Michael Schaalje for the use of one of his many computers; this resource greatly facilitated the preparation of the SNAP model and this thesis. I thank Carol Withey for her encouragement and support and for finding many of the grammatical errors present in this work.

TABLE OF CONTENTS

APPROVAL SHEET ii

ABSTRACT iii

ACKNOWLEDGEMENTS iv

TABLE OF CONTENTS v

LIST OF TABLES x

LIST OF FIGURES xii

LIST OF SYMBOLS xiii

1 INTRODUCTION 1

2 OPEN CHANNEL MODELLING 4

2.1 *CHANNEL PROPERTIES* 4

2.2 *RIVERS* 6

2.3 *DELTAS* 6

2.4 *IRRIGATION SYSTEMS* 7

2.5 *STORM SEWERS* 7

2.6 *NEED FOR AN OPEN CHANNEL NETWORK MODEL* 8

2.7 *OBJECTIVES* 11

3 FUNDAMENTALS 13

3.1 *STEADY GRADUALLY VARIED FLOW* 13

3.2 *DISTRIBUTED LATERAL INFLOWS AND OUTFLOWS* 19

3.3 *DATA AND COMPUTATIONAL ERRORS* 21

3.4 *CONTROL SECTIONS* 24

3.5 *NETWORK COMPONENTS* 25

3.5.1 Reach 26

3.5.2 Reservoir 26

3.5.3 Loop 26

3.5.4 Structures 27

3.5.4.1 *Controlled And Uncontrolled Structures* 27

3.5.4.2 *Satisfactory And Unsatisfactory Structures* 28

3.5.4.3 *Equations Of Flow At Structures* 28

3.5.5 Junctions 31

3.5.5.1 *Confluences* 33

3.5.5.2 *Bifurcations* 37

3.6 *CHAPTER SUMMARY* 43

4 STEADY FLOW MODELS 46

4.1 *CLASSICAL STEADY STATE ANALYSIS* 46

4.1.1 Analytical Methods 46

4.1.2 Graphical Methods 54

4.1.3 Numerical Integration 58

4.1.3.1 *Direct-Step Method* 59

4.1.3.2 *Standard Step Method* 60

4.1.3.3 *Prasad's Method* 61

4.1.3.4 *Newton Raphson Method* 62

4.1.3.5 *Runge-Kutta Method* 64

4.2 *STEADY STATE NETWORK SOLUTIONS* 65

4.2.1 Wylie's Computer Program 65

4.2.2 RIVER4 67

4.2.3 Washington State University Computer Program 71

4.2.4 Irrigation Conveyance System Simulation Model 73

4.3 *CHAPTER SUMMARY* 74

5 NETWORK SOLUTION TECHNIQUES 77

5.1 *ITERATIVE SOLUTION TECHNIQUES* 78

5.2 *SIMULTANEOUS SOLUTION TECHNIQUES* 79

5.3 *MATRIX SOLUTION* 83

5.3.1 Standard Matrix Solution 83

5.3.2 Sparse Matrices 84

5.3.3 Banded Matrices 85

5.3.4 Bandwidth Reduction Techniques 86

5.4 *CHAPTER SUMMARY* 89

6 SOLUTION ALGORITHM OBJECTIVES 91

6.1 *SUMMARY OF EXISTING TECHNOLOGY* 91

6.2 *OBJECTIVES OF A SOLUTION ALGORITHM* 93

7 SOLUTION ALGORITHM 96

7.1 *INDIVIDUAL NETWORK COMPONENTS* 96

7.1.1 Solution Overview 97

7.1.2 Governing Equation 97

7.1.3 Control Structures 100

7.1.4 General (Type I) Bifurcations 102

7.1.5 Type II Bifurcations 105

7.1.6 Confluences 108

7.1.7 Loops 109

7.1.8 Seepage And Precipitation 110

7.1.9 Extraordinary Conditions 112

7.1.9.1 *Exceeding System Capacity* 112

7.1.9.2 *Low Flow Conditions* 113

7.1.9.3 *Water Shortages* 114

7.2 *CONSTRAINT RULES* 117

7.2.1 Problem Situations 117

7.2.2 Development Of The Constraint Rules 123

7.2.3 Applications Of The Constraint Rules 132

7.2.4 Implications For Iterative And Simultaneous Solution 136

7.2.5 Operational Strategies 137

7.2.6 Summary Of The Constraint Rules 138

7.3 *GENERAL NETWORK SITUATIONS* 140

7.4 *SUMMARY OF SOLUTION ALGORITHM* 142

8 RESULTS 144

8.1 *MODEL VERIFICATION* 144

8.1.1 Backwater Computations For A Single Reach 144

8.1.2 Control Structures And Variations Of Channel Properties 145

8.1.3 Operable Control Structures 150

8.1.4 Distributed Lateral Flows 152

8.1.5 Summary Of Program Verification 153

8.2 *DEMONSTRATIONS* 154

8.2.1 Bifurcations 154

8.2.2 Loops 161

8.2.3 Excessive Demands 169

8.2.4 Large Network Application 175

8.3 *CHAPTER SUMMARY* 182

9 DISCUSSION 190

9.1 *SUMMARY OF RESEARCH* 190

9.2 *CONCLUSIONS* 191

9.2.1 Constraint Rules 192

9.2.2 Solution Algorithms 192

9.2.3 Steady Network Analysis Program (SNAP) 193

9.3 *FUTURE RESEARCH* 194

9.3.1 Low Discharge Situations 194

9.3.2 Deltas 195

9.3.3 Channel Junctions 195

9.3.4 Stability Requirements Of Newton Raphson Numerical Method 196

9.3.5 Computer Algorithm To Apply The Constraint Rules 196

9.4 *CHAPTER SUMMARY* 197

LITERATURE CITED 198

LIST OF TABLES

Table 8.1	Flow Routing Verification In A Rectangular Channel	146
Table 8.2	Flow Routing Verification In A Trapezoidal Channel	146
Table 8.3	Physical Data For Channel With Control Structures, Varying Bottom Slope, And Variation Of Manning Roughness (From Swain, 1988)	148
Table 8.4	Verification Of Flow Routing In The Channel Described In Table 8.3	148
Table 8.5	Modelling Results Of Type II Bifurcation	156
Table 8.6	Modelling Results Of Modified Type II Bifurcation	156
Table 8.7	Modelling Results Of Type II Bifurcation Including Seepage	158
Table 8.8	Modelling Results Of Bifurcation	160
Table 8.9	Modelling Results Of Bifurcation With Operable Control At Bifurcation	161
Table 8.10	Modelling Results Of Bifurcation With Operable Control Structure At Bottom Of System	162
Table 8.11	Modelling Results Of Loop	165
Table 8.12	Modelling Results Of Loop With Different Roughness Values Along Each Branch	166
Table 8.13	Modelling Results Of Loop With Locally Operable Control Structure	168
Table 8.14	Physical Properties Of System Portrayed In Figure 8.3	171
Table 8.15	Modelling Results For "Base Case"	172
Table 8.16	Modelling Results Of System With Supply Discharge Specified	173

Table 8.17	Modelling Results Of System With Discharge Exceeding Capacity Of Upstream Control Structure	176
Table 8.18	Physical Properties Of System Portrayed In Figure 8.4	178
Table 8.19	Control Structure Specifications For System Portrayed In Figure 8.4	180
Table 8.20	Details Of Operable Control Structures Described In Table 8.19	181
Table 8.21	Specified Conditions For System Portrayed In Figure 8.4	181
Table 8.22	Modelling Results Of System Portrayed In Figure 8.4	183
Table 8.23	Energy Conditions At Junctions For System Portrayed In Figure 8.4	186
Table 8.24	Control Structure Settings Generated By Model For System Portrayed In Figure 8.4	187

LIST OF FIGURES

Figure 3.1	Profile Of Open Channel Flow	15
Figure 3.2	Confluence Of Open Channels	34
Figure 3.3	Bifurcation Of Open Channels	39
Figure 7.1	Solution Overview	98
Figure 7.2	General Channel Bifurcation	103
Figure 7.3	Solution Algorithm For Bifurcations	104
Figure 7.4	Solution Algorithm For Type II Bifurcations	107
Figure 7.5	Solution Algorithm For Loops	111
Figure 7.6	Portion Of An Irrigation Network	115
Figure 7.7	General Irrigation Network	118
Figure 7.8	Series Of Bifurcations	121
Figure 7.9	Sketch Defining Channel Order	129
Figure 8.1	General Bifurcation	155
Figure 8.2	Loop Configuration	163
Figure 8.3	Example Network For Water Shortages	170
Figure 8.4	High Order Network	177

LIST OF SYMBOLS

A	cross-sectional area of flow
A_1	constant of integration
A_x^y	rate of change of area with respect to x with y held constant.
B	channel top width
$B(\eta)$	the varied flow function
C	Chezy roughness coefficient
C_c	contraction coefficient
C_1	lumped distributed lateral flow coefficient
cp	control parameter
d_1	depth
dfc	downstream flow condition
dsf	designer-selected flow rate
dsh_u	designer-selected upstream depth
dx	elemental length
E	total energy
F	constant value; function
Fr	Froude number
$f(\,)$	hydraulic parameter; structure rating function
fc	conditions upstream or downstream that determine flow
g	acceleration of gravity
$H_{s'}$	head loss gradient
h	head loss; distance increment in numerical analysis; head on weir
h_e	head loss from eddies
i	index; an exponent
J	lumped variable

j	index
K	channel conveyance; head loss coefficient
K_o	conveyance for uniform flow
k	constant
k_1, k_2, k_3, k_4	functions for Runge Kutta method
L	reach length; distance
l	reach length
M	an exponent; index
m	constant
N	the hydraulic exponent; index
n	Manning roughness coefficient; matrix dimension
nf	structure-dependent exponent
obf	observed flow rate
obh_u	observed upstream water depth
osf	operator selected flow rate
P	wetted perimeter
p	weir height
pcs	adjustable physical structure characteristics
Q	discharge
Q_k	branch discharge
Q_L	distributed lateral inflow or outflow
Q_{Ni}	flow added or subtracted at node
Q_{supply}	stipulated maximum supply discharge
R	hydraulic radius
Ra	rating function
S_A	estimate of friction slope
S_B	estimate of friction slope
S_f	friction slope

S_o	channel slope
s	submergence across structure
sv	state variable
U	energy and friction loss function; lateral outflow velocity
u	ratio of depth to uniform depth
ufc	upstream flow condition
v	flow velocity; a variable
v_m	mean flow velocity across section
W	a lumped variable
x	distance; station in channel
x_o	initial distance
Y_u	upstream water depth
y_o	initial depth
y	water depth
y_C	critical depth
$y()$	a function
z	channel invert with respect to datum
z_1, z_2	water surface elevations
$'$	ordinary differentiation with respect to x
∂	partial derivative
$\lvert\ \rvert$	absolute value of
α (alpha)	velocity correction coefficient
β (beta)	momentum correction coefficient; lumped variable
Δ (delta)	increment
δ (delta)	scaling factor
Φ (phi)	Bresse function; function; hydraulic parameter
ϕ (phi)	correction factor for unit system used; angle from channel axis
η (eta)	ratio of depth to uniform depth

κ (kappa)	lumped structure parameter
Π (pi)	function
π (pi)	variable
θ (theta)	angle of channel bed; junction angle
ρ (rho)	fluid density
ξ (xi)	coefficient
ΔE	correction determined by Newton-Raphson technique
ΔH_s	energy variation
Δx	distance increment
Δy	depth increment
Δz	elevation increment

1 INTRODUCTION

Preparing steady state water surface profiles for open channels is a common problem for hydraulic engineers. With such profiles, one can analyse changes in upstream water levels caused by the addition of dams, weirs or other structures. They also permit the proper sizing of canals for irrigation or water supply purposes. Other applications include the determination of water surface profiles for different stages in a river, or for a river that interacts with a tidal estuary.

These problems are so important and have recurred so frequently that many methods exist for their solution. Among these solution techniques are those which utilise high-speed computers. An example of a computer program widely used in North America is HEC-2, developed by the US Army Corps of Engineers. The model should only be used to analyse single channels (Hydrologic Engineering Center, 1982). Recently, other programs (Wylie, 1972; Schulte, 1985; Smith and Ashenhurst, 1986) have been developed to analyse divided channels such as occur around islands.

None of these programs facilitates solution of systems that include all the following: operable structures; distributed lateral inflows and outflows; and multiple branches that separate from the main channel and do not rejoin it. These restrictions prohibit general open channel networks from being simulated properly. River deltas, for instance, often assume geometries in which many branches separate from the main channel and terminate in another body of water, while irrigation systems typically include numerous structures and flow bifurcations. Some programs also require stringent numbering procedures to describe network geometry to generate efficient solutions. This requirement limits the flexibility and range of application of these programs.

Several computer programs exist for the simulation of *unsteady* flows in open channel networks. Such programs include the DWOPER network model (Mays, 1986), the modified DWOPER model (Husain et al., 1988; Husain et al., 1991), the One-Dimensional Hydrodynamic model (Environment Canada, 1988), the Irrigation Conveyance System Simulation model (Manz and Schaalje, 1992), the Utah State University model (Gichuki et al., 1990) and the Network model (Swain and Chin, 1990). Use of these programs has identified another use for steady flow profiles: as a starting point for unsteady simulation models.

Unsteady flow computations require accurate initial conditions as a starting point. Initial conditions have been approximated in several manners. These include (Mays, 1986):

- estimated stages and discharges;
- observed stages;
- computed stages saved from a previous computer run; and
- assumed steady flow conditions.

Kamphuis (1970), Husain et al. (1988, 1991) and Gichuki et al. (1990) have recognised that the accuracy of the assumed initial conditions impacts the accuracy of unsteady computations. Therefore, a procedure that provides accurate initial conditions is essential.

Algorithms are developed to compute steady flow profiles in open channel networks. The networks considered may include a wide range of structure types and have multiple downstream end locations. Additionally, conditions are established for use of the algorithms.

Descriptions of typical types of open channel networks are presented in Chapter 2, as are motivations for the development of a more general steady-state network model. Objectives of the present research are identified and summarised.

In Chapter 3, the hydraulic behaviour of network elements is reviewed from a theoretical and experimental standpoint. This includes the development of the governing equations for linear channels, methods of treating control structures, and the properties of confluences and bifurcations. Additionally, the effects of errors introduced in modelling processes are discussed.

Mathematical treatment of the flow equations introduced are reviewed in detail in Chapter 4, followed by discussion of existing steady-state and unsteady flow models that treat network situations. Chapter 5 compares simultaneous and iterative solution techniques in detail.

The limitations and restrictions of current technology are summarised in Chapter 6 and the research objectives are restated in more detail. In Chapter 7 the Constraint Rules are proposed. They provide a basis of determining whether or not a unique solution exists for a given network. Solution algorithms are developed for network situations, with an emphasis toward irrigation canal systems.

Chapter 8 includes demonstrations of SNAP (Steady Network Analysis Program), a computer-coded version of the algorithm. These samples include: single channels with wide ranges of control structures and seepage effects; junctions; loops; instances wherein systems cannot satisfy desired branch discharges; and sophisticated networks of high order.

Finally, the significance of the Constraint Rules and the solution algorithm developed are discussed in Chapter 9. Areas where further work is required are also identified.

2 OPEN CHANNEL MODELLING

The basic characteristics of open channels—both natural and man-made—are presented. Several types of open channel networks are examined and their major differences outlined. Reasons for modelling steady flow in open channel networks are then developed.

2.1 *CHANNEL PROPERTIES*

The physical properties, hydraulic behaviour and operation of open channel networks will vary considerably depending upon the channel characteristics. Open channels are either natural or man-made. Natural channels are typically found in river systems or delta situations, whereas most irrigation networks and sewer systems are man-made.

Natural channels usually flow through sand, clay, silt or grass. Vegetation is often present on the channel floor and walls, within the water, or as floating material. Channel shapes are often irregular, therefore they must be approximated for many modelling purposes. The channel roughness coefficient is typically a function of flow depth and distance along a given channel. Variations of the roughness coefficient with water depth can become significant,. especially when overbank flow (flooding) occurs. Also, the velocity across a channel section is often not uniform because of channel irregularities and vegetation effects. Velocity correction coefficients must be determined and applied to such channels for modelling purposes.

In a network of natural channels, there will typically be very few hydraulic structures. Structures that may be present include dams, weirs, sluice gates, fish ladders, and rapids. In natural systems, structures tend to be "fixed" (they cannot be adjusted or "set" to change the channel discharge or depth). Analysis of fixed control structures is relatively straightforward.

Man-made channels can be formed from wood, steel, concrete, or earth. Roughness coefficients of wooden, steel or concrete channels can generally be determined quite accurately. Variations in water depth or location along the channel tend to have little effect on these values. For earth channels, the roughness coefficient can vary with water depth and longitudinal distance. These variations are generally less than those encountered in a natural channel of similar length and discharge capacity. Most man-made channels are relatively uniform in cross-section and irregularly shaped channels are rare. Channel shapes such as trapezoidal, rectangular and circular are common. Cross-sectional velocity variations are generally less pronounced than for natural channels.

Hydraulic structures are abundant in many man-made systems. A typical irrigation network may include numerous weirs, check structures, gated orifices, culverts and other structures. A large percentage of these structures may be operable. By adjusting operable structures, the depth and/or discharge in the channel can be modified. To model such a network, analysis of a wide range of structures is necessary.

Brief summaries of the main characteristics of several types of open channel networks follow. Then the reasons for developing a new steady state network model are established.

2.2 *RIVERS*

Rivers are formed by natural processes. They originate as surface runoff and subsurface flow from the groundwater table. Rivers collect water and convey it to a lower elevation where it joins another body of water such as a lake, an ocean or another river. A river network is a collector system typically composed of varying sizes of streams and storage reservoirs. Common control structures in river systems are waterfalls, rapids and weirs. Rapids have been represented as critical flow sections described by a formula (Environment Canada, 1988) while waterfalls are essentially drop structures that can also be represented in equation form.

2.3 *DELTAS*

A delta can occur where a river enters another body of water, such as a lake or ocean. Distributary channels separate from the main channel as the river approaches the receiving water body. Bifurcations may be numerous and loops can develop as channels branch away from and rejoin the network. Typically, there will be several branches that separately enter the receiving water body. Certain channels within a delta may flow full during high discharge periods yet be dry during lower flows. Flow reversals—that is, flow progressing from downstream to upstream—may occur, particularly during periods of low flow, when the receiving body of water "encroaches" upon the delta.

2.4 *IRRIGATION SYSTEMS*

Unlike Smith and Ashenhurst's (1986) representation, irrigation systems generally consist of a main canal that delivers water to several lower orders of canal. Irrigation systems transmit water from regions where it is available to agricultural areas needing additional moisture. Generally, these systems allow delivery by gravity alone; however, pumping stations may also be encountered.

Irrigation canals are generally man-made. They may be lined or unlined and their flow resistance is not always known accurately. If the canals are unlined, seepage effects must be accounted for, hence the governing equation should be able to analyse lateral outflows. Also, the channel bed and banks may potentially be movable. Sediment within irrigation systems can complicate analysis because of possible silting problems.

Since irrigation systems distribute water, numerous bifurcations and operable control structures are typical. Often, bifurcations are regulated in an effort to control the distribution of the water. Structures common to irrigation networks are weirs, spillways, radial gates, gated orifices, culverts, and pumps. Channel confluences are uncommon, but they do occur, especially in systems that receive return flow.

2.5 *STORM SEWERS*

Storm sewer systems transport water from collection areas to discharge areas at lower elevations. These systems typically have numerous junctions where tributaries join the system. Manholes add bulk inflows to the system, so the governing flow equation should be capable of simulating these features.

The channels are usually concrete or steel, with relatively well defined flow resistance values and rigid channel boundaries. Yet, sediment transport occurs because the water carries silt, sand and debris as it enters the system. However, as established by Chang (1985), the power required to move sediment is usually small, so the effects of the sediment load should be negligible. Side-spillway weirs are typical control structures found in sewer systems Spillway weirs are often located at junctions and drop manholes.

2.6 *NEED FOR AN OPEN CHANNEL NETWORK MODEL*

Computer programs that model steady-state flows in natural channels have existed for many years. One well-known model is HEC-2 (US Army Corps of Engineers, 1988). Used extensively for its designed purpose, the model has probably been used for purposes far beyond its capabilities as well. These situations likely occurred during attempts to model networks or hydraulic structures.

Since the development of HEC-2, programs have been created to model some open channel networks (Wylie, 1972; Ashenhurst, 1981; Schulte, 1985). Other models were developed specifically to model irrigation systems complete with their relatively complicated structures. Amongst these programs is the ICSS model. Unfortunately, the programs that are capable of modelling network situations do not include the wide variety of structures encountered in irrigation systems, nor can they adequately simulate the behaviour of complex, generalised networks with highly looped geometries. Also, programs such as the ICSS model cannot treat general network situations. Hence, there is a need for a computer program that can reliably model general network situations having wide varieties of hydraulic structures.

During simulation of the St. Lawrence River, Kamphuis (1970) identified a need for accurate initial water levels for use in unsteady flow modelling.These initial conditions are available from a variety of sources (Mays, 1986):

- estimated stages and discharges;
- observed stages;
- computed stages saved from a previous computer run; and
- assumed steady flow conditions.

Data from the first two of these sources is not always accurate or of sufficient quantity to adequately define conditions throughout a network. The accuracy of a previous computer run can itself be affected by initial conditions. Even if it is not adversely affected, initial conditions of some sort will be required to perform *that* simulation. Hence, the fourth of these alternatives appears best able to provide complete and accurate starting values for an unsteady simulation.

Gunaratnam and Perkins (1970) state that initial conditions may be arbitrarily assumed. If the unsteady simulation is long enough, any inaccuracies introduced through the assumption of these conditions will lessen and results of much of the simulation will be accurate. Because of this, they maintain there is no need to determine steady water surface profiles at all. Misra et al. (1992) also used an extended unsteady simulation during which no system parameters were changed to determine initial conditions. The simulation time required to provide steady values is not mentioned by either of these sets of investigators, but Swain (1988) describes a similar method of determining initial conditions. In this instance, approximately ten hours of flow was simulated using one minute time steps. The technique of using an extended unsteady simulation to determine initial conditions is computationally inefficient. Errors may also be introduced if the unsteady flow algorithm used is non-conservative (that is, if the algorithm does not guarantee conservation of mass throughout the system).

Initial conditions are also necessary when using the Environment Canada One-Dimensional Hydrodynamic model (Environment Canada, 1988). Unless the actual conditions are known, they must be approximated or assumed.

The unsteady flow model DWOPER has been modified for application to a large network with tributaries, bifurcations and several types of structures (Husain et al., 1988). The program developers attest that the program calculates discharges and depths for use as initial conditions, given only the discharge or water level at the upstream end of the network. These values are determined "using a subroutine that considers the geometry and surface roughness of the system." The method used to determine the initial conditions has apparently been improved subsequently (Husain et al., 1991). However, the descriptions are lacking detail and are hence of little utility.

Husain et al. (1988, 1991) point out that the accuracy of the initial conditions impacts the reliability of unsteady computations. This observation seems out of place if the modified DWOPER model actually calculates initial values with any degree of accuracy. Large discrepancies have been observed between recorded and simulated water levels near the beginning of simulations, while agreement between these water levels has been good for times further into simulations. These errors have been attributed (Husain et al., 1991) to a storage effect that predominates at the initial stage. They might also be explained in part by incorrect initial values that the subsequent unsteady simulation dampens.

Another attempt to model unsteady flow in networks requires initial flow conditions consisting of either empty channels or a previously determined steady or unsteady condition (Gichuki et al., 1990). One of the sources of error encountered during the use of the program resulted from the accuracy of the approximated initial steady-state conditions.

Obviously, unsteady flow models require accurate initial conditions. The technique of running an unsteady simulation for an extended time without changing any system parameters is computationally inefficient and can not *guarantee* a result of accurate initial conditions. Additionally, many unsteady models are now used to simulate open channel networks. Hence, a method of determining steady state conditions for open channel networks is needed for use with unsteady model applications.

2.7 *OBJECTIVES*

There is a need to analyse steady flows in open channel networks for use in unsteady modelling and for other design purposes. A solution algorithm is required to model network situations including bifurcations and various types of control structures. Irrigation networks are good examples of open channel systems that contain these features. They are also often more hydraulically complex than other types of systems. Therefore, a solution algorithm should be applicable to irrigation networks in particular and should be capable of modelling:

- channels that split from the network and do not rejoin it;
- all types of junctions and energy losses associated with them;
- a wide range of control structures and transitions; and
- loops.

It is possible that no solution may be attainable for certain systems. A means of identifying such "ill-posed" problems is a necessary accompaniment to the development of a solution algorithm. Therefore, another objective of this research is to develop criteria to determine whether a given system may or may not be solved.

3 FUNDAMENTALS

Terminology describing open channel flow is reviewed as is the development of the governing equation for steady gradually-varied flow and the effects of errors commonly introduced during its solution. Components of open channel networks are introduced and methods of treating structures and channel junctions are presented.

3.1 STEADY GRADUALLY-VARIED FLOW

An open channel is a conduit that conveys liquid from one location to another with the liquid having a surface in contact with the atmosphere. The liquid predominantly encountered in open channel flows is water, hence the following discussions are restricted to systems conveying water. Common examples of open channels include rivers, irrigation canals, storm sewers not flowing full and drainage ditches.

Open channel flow may be laminar, turbulent, or transitional. Detailed descriptions of these flow types are presented in Henderson (1966) and French (1985). It is assumed that all flows dealt with in this thesis are fully turbulent. Under steady flow conditions, water depths and discharges may be different at various locations, but the discharge and depth at a given location do not vary with time. French (1985), assumes that gradually-varied flows satisfy the following:

- head loss in a reach equals head loss occurring in that reach for uniform flow having the same hydraulic radius and average velocity;
- the channel slope is small;
- there is no air entrainment; and
- the velocity distribution for the channel is fixed.

Associated with a given flow is a certain energy level. This energy consists of potential and kinetic energy components. Kinetic energy relates to the movement of the water, while potential energy is a function of the water surface elevation with respect to an established datum. Figure 3.1 depicts a flow moving from station A to station B. The total energy per unit weight at any section is (French, 1985):

$$E = z + y + \frac{\alpha v^2}{2g} \qquad \textbf{3.1}$$

where:

E = total energy per unit weight at the location;

z = channel invert elevation with respect to datum;

y = water depth;

v = velocity of flow;

g = acceleration of gravity; and

α = velocity correction coefficient.

In a channel section there are often regions with distinct flow characteristics (Henderson, 1966). A common example is a river in which part of the flow occurs in the overbank region. The problem of determining a total head

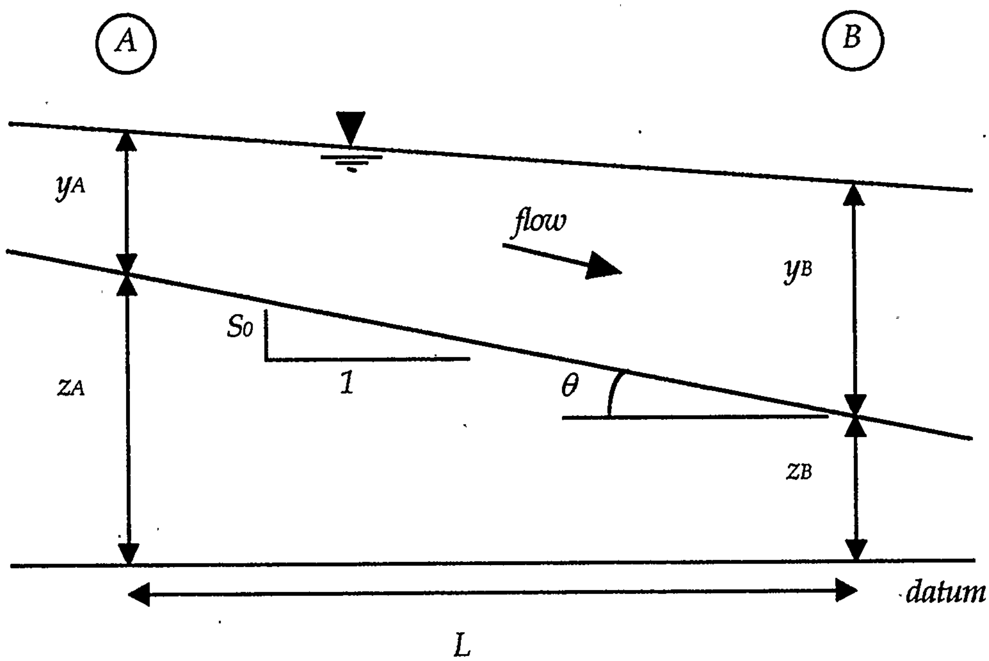

Figure 3.1 Profile Of Open Channel Flow

representative of the entire cross section arises. To determine this head, the velocity correction coefficient is defined as

$$\alpha = \frac{\Sigma(v_i^3 A_i)}{v_m^3 \Sigma A_i} = \frac{(\Sigma A_i)^2}{(\Sigma Q_i)^3} \Sigma\left(\frac{Q_i^3}{A_i^2}\right) \qquad 3.2$$

where v_m is the mean flow velocity of the section, equal to the total discharge divided by the total flow area. The velocity, area and discharge of the individual subsections are v_i, A_i and Q_i respectively. If the same friction slope is applicable to each portion of the cross section,

$$\alpha = \frac{\int v^3 dA}{v_m^3 A} \qquad 3.3$$

where v_m is as above.

Assuming that α is equal to unity and the channel slope is small (so the cosine of θ is approximately equal to one) differentiating equation 3.1 with respect to longitudinal distance x and simplifying yields the governing equation of gradually-varied flow for all forms of channel section

$$\frac{dy}{dx} = \frac{S_o - S_f}{1 - Fr^2} \qquad 3.4$$

where:

$$S_f = -\frac{dE}{dx} \qquad 3.5$$

$$S_o = -\frac{dz}{dx} \qquad 3.6$$

and

$$Fr^2 = \frac{Q^2 B}{g A^3} \qquad 3.7$$

Here,

Fr = Froude number;

Q = channel discharge;

B = channel top width; and

A = cross-sectional area of flow.

The term S_o corresponds to the channel bottom slope. The S_f term is called the friction slope and accounts for energy losses occurring within the flow. These may be broken down into:

- eddy losses;
- bend losses; and
- losses created by channel roughness.

Eddy losses occur where turbulent mixing is introduced into the flow as in locations where the channel cross section expands or contracts. These losses may be represented (Ezra, 1954a) as

$$h_e = K\left(\frac{v_A^2}{2g} - \frac{v_B^2}{2g}\right) \qquad 3.8$$

where:

h_e = head loss from eddies;

K = head loss coefficient;

v_A = velocity at upstream end of channel; and

v_B = velocity at downstream end of channel.

K is often assumed as 0.5 for abrupt expansions and contractions while for gradual expansions a value of 0.2 may be appropriate. For gradually converging sections K may vary from zero to 0.1. Bend losses are generally included in the channel roughness term (Ezra, 1954a). Typically, the friction slope is described by an empirical or semi-empirical equation of the form

$$S_f = kQ^m \tag{3.9}$$

where m and k are constants. Two forms of this equation that have been widely used are the Chezy and the Manning equations. The Manning equation will be dealt with here. It generally (Henderson, 1966; French, 1985) takes the form

$$S_f = \frac{Q^2 n^2}{\phi^2 A^2 R^{4/3}} \tag{3.10}$$

where:

Q = discharge;

ϕ = 1.49 if Imperial units are used: 1.0 if SI units are used;

R = hydraulic radius; and

n = Manning roughness coefficient.

3.2 *DISTRIBUTED LATERAL INFLOWS AND OUTFLOWS*

Flow that is continuously added to a channel from a lateral direction is defined as a distributed lateral inflow. Examples include channels receiving flow from a side-channel spillway, seepage into a channel, and precipitation. As the entering water mixes with that already flowing in the channel, appreciable energy losses may occur. The energy equation cannot analyse this situation suitably.

Following the analysis of Humpidge and Moss (1971), applying momentum considerations in the direction of flow for an element of length dx yields

$$\rho g A S_o dx - \rho g A \frac{dy}{dx} dx - \rho g A S_f dx = \frac{d(\rho Q v)}{dx} dx \qquad 3.11$$

Noting that $v = Q/A$

$$gA\left(S_o - \frac{dy}{dx} - S_f\right) = \frac{d}{dx}\left(\frac{Q^2}{A}\right) \qquad 3.12$$

and that

$$\frac{d}{dx}\left(\frac{Q^2}{A}\right) = 2\frac{Q}{A}\frac{dQ}{dx} - \frac{Q^2}{A^2}\frac{dA}{dx} \qquad 3.13$$

which may be expanded to yield

$$\frac{d}{dx}\left(\frac{Q^2}{A}\right) = 2\frac{Q}{A}\frac{dQ}{dx} - \frac{Q^2}{A^2}\frac{\partial A}{\partial x} - \frac{Q^2 B}{A^2}\frac{dy}{dx} \qquad 3.14$$

gives, upon substitution,

$$\frac{dy}{dx}=\frac{S_o-S_f-\frac{2Q}{gA^2}\frac{dQ}{dx}+\frac{Q^2}{gA^3}\frac{\partial A}{\partial x}}{1-\frac{Q^2B}{gA^3}} \qquad 3.15$$

This equation is applicable for prismatic and non-prismatic channels. For prismatic channels, the channel area will not vary with distance along the channel (at a constant depth), hence the $\partial A/\partial x$ term will be zero. When no lateral inflows are present, the discharge will not vary along the channel length, causing the dQ/dx term to vanish. Obviously, for prismatic channels with no lateral inflows, equation 3.15 reduces to equation 3.4.

The analysis presented above is not suitable for distributed lateral outflows. The flow leaving the channel possesses momentum that must be accounted for. It is simpler to examine energy considerations by assuming the energy of the flow within the channel is not affected largely by the exiting water. Assuming constant energy within the channel, French (1985) showed that for a prismatic channel the energy equation may be differentiated along the direction of flow:

$$\frac{dE}{dx}=\frac{dz}{dx}+\frac{dy}{dx}+\frac{1}{2g}\left(\frac{2Q}{A^2}\frac{dQ}{dx}-\frac{2Q^2}{A^3}\frac{dA}{dx}\right) \qquad 3.16$$

If S_f and S_o are defined as previously and

$$\frac{dA}{dy}=\frac{dA}{dy}\frac{dy}{dx}=B\frac{dy}{dx} \qquad 3.17$$

then

$$\frac{dy}{dx}=\frac{S_o - S_f - \frac{Q}{gA^2}\frac{dQ}{dx}}{1-\frac{Q^2B}{gA^3}} \qquad 3.18$$

French (1985) further shows that non-uniform velocity profiles may be analysed by including the energy correction factor in front of the terms containing the discharge, Q. Equations 3.15 and 3.18 differ only in that equation 3.18 does not account for non-prismatic channel effects and that the coefficient in front of the dQ/dx term in equation 3.15 is a 2 whereas it is a *1* in equation 3.18. Monem and Manz (1994) have rewritten equation 3.18 to account for the effects of non-prismatic channels:

$$\frac{dy}{dx}=\frac{S_o - S_f - \frac{Q}{gA^2}\frac{dQ}{dx}+\frac{Q^2}{gA^3}\frac{\partial A}{\partial x}}{1-\frac{Q^2B}{gA^3}} \qquad 3.19$$

A similar equation was presented for distributed lateral outflows (Monem and Manz, 1994).

3.3 *DATA AND COMPUTATIONAL ERRORS*

Given the n value and geometry of a channel, the discharge may be determined. Unfortunately, no technique has been developed to accurately determine the n value. In general, experienced hydraulic engineers can only *estimate* an n value that may be appropriate. Because of this, inaccuracies are introduced into most discharge computations. Error introduced in the friction slope propagates into calculations involving the velocity head and minor losses, possibly affecting the computed water surface profile significantly.

For the majority of natural channels, the n value will vary longitudinally along the channel. This presents the dilemma of determining how to best describe the roughness coefficient for a length of channel. Existing methods are based upon the roughness estimates at the ends of the channel. The arithmetic, geometric and harmonic means of these values have been used, as has an arithmetic mean of the channel conveyance at the end locations. Review of these methods (Laurenson, 1986) has determined that under most circumstances, the arithmetic mean of the end roughness values introduces the smallest possibility for computational error. The value of the friction slope obtained can be written

$$S_f = \frac{(S_A + S_B)}{2} \qquad \textbf{3.20}$$

where:

S_A = estimate of friction slope at location A; and

S_B = estimate of friction slope at location B.

Using the geometric mean value of the end friction slopes

$$S_f = (S_A S_B)^{1/2} \qquad \textbf{3.21}$$

is recommended when subcritical flow occurs in a channel reach immediately upstream of a converging channel reach. In all other subcritical flow situations, the maximum potential error is minimised if equation 3.20 is used. Errors may also be reduced if cross-sectional data is obtained for channel locations where the flow profile has a high degree of curvature (Laurenson, 1986). Such locations include converging or diverging channel sections.

Nearly one hundred data sets for subcritical flow in natural watercourses were reviewed by Burnham and Davis (1990) to determine the sensitivity of computed water surface profiles to errors in channel cross-sectional data and estimates of the Manning roughness value. Manning *n* values used were "best estimates" from experienced hydraulic engineers. It was concluded that variations in the roughness value did not significantly affect the computed water surface profile if the cross-sectional data were reliable (e.g. field survey data). If the cross-sectional data was less accurate, deviations in the Manning *n* value could cause large variations in the computed flow profile. Accordingly, caution must be exercised in the determination of at least one, and preferably both, of these groups of data. Equations have been developed (Burnham and Davis, 1990) describing the magnitude of the error introduced by combined errors in survey and roughness data. With the assistance of these relationships, error bounds for the computed profiles may be estimated.

McBean and Perkins (1975) analysed the magnitude of errors likely to be introduced by numerical solution of the governing equation. Their study was limited to the effects of round-off and truncation errors. The numerical scheme analysed was the trapezoidal method of integration (Prasad, 1970) but in principle the same method of analysis may be applied to other integration schemes. For comparative purposes, exact solutions were obtained for a wide rectangular prismatic channel.

A conservative bound was determined for the potential error introduced. Hence an upper limit for propagated error was obtained. For a given distance increment Δx the cumulative error increased with increasing distance from the computational origin. As the distance increment decreased, the magnitude of the numerical integration error first decreased then began to increase. Hence, there existed a Δx value for which the numerical error is a minimum. This reduction and subsequent growth in error occurred because truncation errors were dominant for larger Δx values while round-off errors were more significant for smaller computational steps. Generally, errors introduced were relatively small, especially when compared to those introduced by small errors in physical data. However, it *was* observed that significant numerical error was introduced for relatively large Δx values.

The following guide was presented for the selection of distance increments for wide rectangular prismatic channels

$$\Delta x < \frac{3y}{5S_o} \qquad 3.22$$

It was believed that this guideline could be followed for non-prismatic, non-rectangular channels as well, but smaller values of Δx should be used if the channel shape is very irregular (McBean and Perkins, 1975).

3.4 *CONTROL SECTIONS*

A control section with a defined relationship between depth and discharge must be selected as a starting point for a backwater calculation. Control sections are commonly taken at control structures or at locations where critical or normal depth occurs (Humpidge and Moss, 1971).

A computer program has been developed (Molinas and Yang, 1985) that automatically determines critical depth locations along channels with irregular sections. Another technique has been presented (Lopes and Shirley, 1993) to determine critical sections for channels with uniform lateral inflows. The analysis was limited to trapezoidal, triangular and rectangular channel sections. As one intention of the present research is to analyse channels of general cross-section that may receive lateral inflows, neither of these treatments is adequate on its own. Hence, locations where flow changes from supercritical to subcritical will not be identified by the algorithm developed. Therefore, the analysis performed will be restricted to subcritical flows.

Contrary to established opinion (Henderson, 1966; French, 1985), McBean and Perkins (1975) observed that flow profile calculations can be performed either upstream or downstream from a control regardless of the nature—i.e., subcritical or supercritical—of the flow. However, the results obtained were much more sensitive to the distance increment used when calculations were performed in the direction opposite to that in which the control was acting. In fact, Prasad (1970) solved the gradually-varied flow equation numerically in both directions from controls with no apparent difficulty. Fread and Harbaugh (1971) observed that if subcritical computations proceed downstream from a control, errors in the assumed starting depth were exaggerated at the other end of the reach. Conversely, if computations proceed upstream, the effects of initial errors were actually lessened. This yielded a better estimate of the correct depth at the other end of the reach. Therefore, although not required, it seems prudent to perform calculations for subcritical flow in the upstream direction.

3.5 NETWORK COMPONENTS

Open channel networks can be broken down into a number of basic "building blocks". The hydraulics and operation of several of these parts can be described simply while structures and junctions are often more difficult to accurately represent. This observation should be intuitive considering that many of the changes occurring within a system will be initiated at or will happen in the vicinity of junctions and control structures.

3.5.1 Reach

A reach is a length of channel bounded at the upstream and downstream ends by structures, transitions, junctions or cross sections where specific information is provided or known. The hydraulic characteristics—friction slope, cross-sectional area, hydraulic radius, channel invert slope—should change as little as possible within a reach to provide better accuracy. If these values vary widely, it is advisable to subdivide the reach. Each end of a reach may be represented by a node, at which a change in the hydraulic behaviour or governing equations may be introduced.

3.5.2 Reservoir

A reservoir is characterized by relatively deep, slowly moving water. The velocity head of an incoming flow is generally considered lost as the flow enters the reservoir. Flow velocity within reservoirs is often negligible. Equations may be developed to describe the relationship between inflow, outflow and storage for a particular reservoir. These equations may be treated similarly to those for structures.

3.5.3 Loop

A loop occurs where a branch separates from the flow and rejoins the network further downstream. In previous analysis (Ashenhurst, 1981; Smith and

Ashenhurst, 1986), distributary channels have had to rejoin the same channels from which they separate. Branches that divert to satisfy a demand then return to a different channel, and branches that cross and interconnect in a meandering river or a delta are examples where this condition is not met. For an algorithm to be of general use, this should not be a requirement.

3.5.4 Control Structures

A control structure is a component or section of a reach where the gradually-varied flow equations are no longer necessarily applicable. Structures may occur at either or both ends of a given reach. In general, a relationship between discharge and water depth exists that may be represented in equation form. Various classes of structures exist. They may be grouped by their method of operation as either "controlled" or "uncontrolled" and they may be classified by their hydraulic behaviour as "satisfactory" or "unsatisfactory".

3.5.4.1 Controlled and Uncontrolled Structures

Structures are identified as "controlled" if their operation is governed by human or mechanical intervention. Operation of the structure does not rely solely upon the prevailing hydraulic conditions. Instead, some physical parameter of the structure must be adjustable to ffect control of the discharge. Controlled structures include radial gates, adjustable-height weirs, and many farm offtakes. Uncontrolled structures behave of their own accord and cannot be adjusted. These include structures that operate automatically and independently such as drainage inflows, siphon spillways, and fixed-height weirs.

3.5.4.2 Satisfactory and Unsatisfactory Structures

Consider a structure separating the downstream end of one reach from the upstream end of another. Two situations may exist: the depth downstream of the structure may influence the discharge or water depth upstream of the structure; or the depth downstream of the structure will have no effect upon the upstream hydraulic behaviour. In the former case a structure is defined as "unsatisfactory", while a structure satisfying the latter is termed "satisfactory". If a structure is satisfactory, it must be *guaranteed* that the downstream flow depth will not affect the upstream flow.

It is often difficult to determine whether downstream depth will *absolutely* not influence upstream flow conditions, hence demarcation between satisfactory and unsatisfactory structures is not straightforward. Some structures behave in a satisfactory manner at times yet are unsatisfactory at others. For instance, if a gated culvert with a free outlet becomes submerged, its operation may switch from satisfactory to unsatisfactory.

Satisfactory structures can be used as control sections to begin backwater computations. Because they permit downstream flows to affect those upstream, unsatisfactory structures cannot be used as starting points. Hence, such structures should be incorporated into adjacent channel reaches if simultaneous solution of a system is desired (Manz, 1985a).

3.5.4.3 Equations of Flow at Structures

The equations of flow at a control structure must satisfy continuity and the hydraulic conditions present. If time-dependent effects are excluded from Manz's (1987) analysis, a steady state discharge relationship may be obtained for any structure:

$$Q = Ra\left(cp, dfc, dsf, dsh_u, fc, obf, obh_u, osf, pcs, Q_L, sv, ufc, L\right) \qquad \textbf{3.23}$$

where:

Ra = rating function;

cp = control parameter (e.g. operating rules);

dfc = downstream flow condition;

dsf = designer selected flow rate (imposed during design and construction phase);

dsh_u = designer selected upstream depth (imposed during design and construction phase);

fc = flow conditions upstream or downstream of the structure that determine flow through the structure;

obf = observed flow rate;

obh_u = observed upstream water depth;

osf = operator selected flow rate;

pcs = adjustable physical structure characteristics affecting its hydraulic control;

Q_L = distributed lateral inflow or outflow;

sv = state variable (factors other than *dfc* and *pfc* that directly affect flow hydraulics through the structure);

ufc = upstream flow condition; and

L = distance along reach as measured from the upstream node.

Many of these variables do not influence the operation of specific structures. For satisfactory structures, hydraulic control does not pass from downstream to upstream, hence by definition *dfc* is of no consequence. In contrast, for unsatisfactory structures both *dfc* and *ufc* influence the hydraulic behaviour. Similarly, the rating equation for uncontrolled structures will not include *osf* or *pcf* terms, while these variables will significantly affect the behaviour of controlled structures.

Flow through control structures has been represented in a network (Gichuki et al., 1990) by relationships of the form

$$Q = \kappa Y_u^{nf}(1-s)^{nf} \tag{3.24}$$

where:

κ = parameter accounting for discharge coefficient, opening width, gate setting, and physical structure constants;

Y_u = upstream water depth;

s = submergence across the structure; and

nf = structure-dependent exponent.

For unsubmerged structures, the s value is taken as zero. When more than one structure exists at a location, additional terms are added to the right side of the above equation. This technique lets all structures be represented by a single form of equation. However, this form of equation *may* not adequately describe the behaviour of all structures encountered.

In Network (Swain, 1988), the equations of flow at structures are represented by a continuity equation

$$Q_1 - Q_2 = 0 \tag{3.25}$$

and a rating curve of the form

$$f(z_1, z_2) - Q = 0 \tag{3.26}$$

where:

Q_1 = discharge upstream of the structure;

Q_2 = discharge downstream of the structure;

Q = discharge through the structure;

z_1 = upstream water stage;

z_2 = downstream water stage; and

$f(z_1, z_2)$ = structure rating function.

These equations are structure specific. However, they may all be written as functions that can be equated to zero. Therefore, they can be incorporated into matrix form. Control structureshave been treated this way for unsteady simulation of unsteady networks (Swain, 1988; Manz and Schaalje, 1992; Joliffe, 1984).

3.5.5 Junctions

Two types of junction can exist in a network. A confluence is the region in which two or more tributaries join together. Bifurcations are situations where the flow splits into two or more distributary channels. Both situations are reviewed to determine how they should be treated. At a confluence it would be desirable to determine the water depth in each incoming channel without having to specify those values. It is attempted to determine whether these depths can be obtained given only channel and confluence geometries and the downstream discharge and depth. For bifurcations, it would be desireable to determine the depth and discharge in one distributary given only junction geometry and depths and discharges for the other distributaries channels. Whether or not this is possible is also explored.

To supply boundary equations at a junction, two assumptions have traditionally been made. The first of these is satisfaction of continuity while the second is some form of compatibility equation.

A commonly assumed compatibility condition is a common water surface elevation for all branches (Stoker, 1957; Kamphuis, 1970; Gunaratnam and Perkins, 1970; Joliffe, 1984; Environment Canada, 1988; Swain, 1988). This treatment implicitly neglects any energy losses that occur at the junction itself. Measurements have verified that water levels in the two channels upstream of a confluence were nearly equal regardless of the junction angle (Taylor, 1944; Webber and Greated, 1966). However, only a few specific flow situations were considered during laboratory testing. Assuming that these observations can be applied universally would be inappropriate.

Wylie (1972) and Chaudhry and Schulte (1986) have assumed that the total energy—i.e., the water surface elevation *plus* the velocity head—was equal at all branches of a confluence or bifurcation. Ashenhurst (1981) made this same assumption for junctions, while at bifurcations equality of water levels was assumed.

Misra et al. (1992) have allowed for a head loss to be included at bifurcations, while Wylie (1972) states that his formulation will also allow such losses to be easily incorporated. Chaudhry and Schulte (1986) also maintain that head losses may be incorporated in their model, but they go on to point out that typically equality of water levels is an acceptable assumption.

Since different investigators have made different assumptions of the physical phenomena at junction locations, it is necessary to determine which assumptions are most reasonable for general situations. Also, the amount of energy lost at junctions must be reviewed to determine its effect upon the flow conditions. The earliest significant analysis of junctions (Taylor, 1944) indicated that the problems introduced by confluences were generally less complicated than those of bifurcations. Hence, the two types of junction are reviewed independently.

3.5.5.1 Confluences

Detailed analysis was performed by García-Navarro and Savirón (1992) for subcritical and supercritical flow in frictionless channels meeting at any angle θ. They concluded that the assumption of common water stages at a junction is appropriate only for low Froude numbers, and that momentum must be considered when flows have higher Froude numbers. Unfortunately, no guideline was presented to help determine at what point the Froude number is considered "low" or "high". If only subcritical flows are considered, analysis can be simplified by neglecting the effects of momentum.

Theoretical and laboratory investigation of confluences (Figure 3.2), has centred upon determining the upstream depths in the joining channels. Variables contributing to the problem include the angle the channels join at, the channel widths and the respective channel discharges (Taylor, 1944). A theory developed depended upon several assumptions:

- all channels were horizontal and had the same cross-sectional dimensions;
- the upstream water depths were equal in the joining channels;
- wall friction was assumed to be negligible; and
- flow was parallel to the channel walls immediately above and below the confluence.

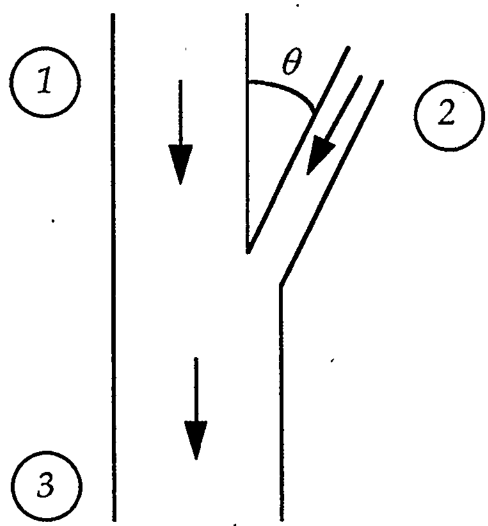

Figure 3.2 Confluence Of Open Channels

The theory was experimentally verified when the channels met at an angle θ of 45 degrees. Hence the assumptions seem correct for this particular case. Test results for a junction angle θ of 135 degrees showed little correlation with the theory. Discrepancies were believed to result because the flow did not remain parallel to the channel walls. This caused a non-uniform velocity distribution below the confluence (Taylor, 1944).

Analytical and laboratory testing (Rammamurthy et al., 1988) was done to estimate the rise in water surface immediately upstream of a confluence. This work was conducted on horizontal channels with identical cross sections joining at a 90° angle. It was assumed that:

- both upstream flows were subcritical;
- boundary friction in the junction vicinity was negligible; and
- the downstream flow was not submerged and was critical for a certain range of flow values.

When the ratio of branch discharge to main channel discharge exceeded 0.3, a hydraulic jump occurred a short distance downstream of the confluence. Depth d_1 could be determined given only Q_2 and Q_3 (or equivalently, Q_1 and Q_3) if there was critical flow shortly downstream of the confluence. However, d_1 could not be determined if the flow just downstream of cross-section 3 was submerged. When velocity v_3 was not critical, the relationship between d_1 and Q_2 and Q_3 was no longer valid. By imposing a hydraulic jump downstream of the junction, a "control" is added to the system. It follows intuitively that one of the flow variables (i.e., d_1) at the confluence no longer must be stated explicitly.

Testing was performed for intersection angles, θ, of 30°, 60° and 90° to evaluate energy losses within a confluence (Webber and Greated, 1966). Approach flows were subcritical and the channels were horizontal and of equal width. An equation and graphs were developed to relate head loss to the downstream Froude number, Fr_3, and the intersection angle. Energy loss increased with increasing intersection angle, as did the ratio of the upstream depth to the downstream depth (d_1/d_3) in the main channel. Equality of upstream water depths was again experimentally verified as being reasonable. However, it was also noted that the influence of the channel shape cannot be neglected.

Energy losses occurring in a confluence were examined by Lin and Soong (1979). They tested a rectangular tributary joining a rectangular main channel of the same width at an angle θ of ninety degrees. Turbulent mixing losses were of the same order of magnitude as boundary friction. A "discharge ratio" was defined as the tributary discharge divided by the main channel discharge upstream of the junction ($d_r = Q_2/Q_1$). Energy transfer coefficients were compared to discharge ratios for varying values of Q_1. These coefficients and the energy loss within the confluence increased with the discharge ratio. A "backwater effect" (an increase in flow depth upstream) was measured in the upstream portion of the main channel. Hence, the assumption of equal water levels upstream of the confluence does not appear to be generally applicable (Lin and Soong, 1979).

Best and Reid (1984) tested rectangular channels of equal width joining at angles, θ, of 15°, 45°, 70° and 90° for a range of discharge ratios Q_2/Q_3. Decreasing water depth was observed across the confluence (Best and Reid, 1984). The depth decreases were strongly dependent upon the discharge ratio and weakly dependent upon the junction angle.

In summary, the following are concluded:

- momentum effects do not have to be considered if the Froude numbers are low;
- backwater effects are created in the upstream section of the main channel by the joining flow; test results indicate the assumption of equal water levels may be close for certain cases, but assuming this generally is not correct;
- if supercritical flow occurs downstream of the confluence, one of the upstream depths can be deduced given the discharge distribution; and
- relationships have been developed to describe the effects of confluence angle and discharge ratio upon energy losses.

Because of these results, a general treatment of confluences for subcritical flow should not be based upon assuming equal water levels in the joining branches. Equality of energy levels is a more appropriate assumption because it allows energy losses to be incorporated. Physically, these losses often appear as drops in the water level or as backwater effects across the confluence. Provided that flow velocities are relatively low, momentum considerations may be disregarded. Unfortunately, this simplification comes at the expense of no longer being able to apply the results of Rammamurthy et al. (1988). Those results reduce the amount of information required for situations where critical flow occurs downstream of the confluence.

3.5.5.2 Bifurcations

Most bifurcation treatments are for cases where the junction is stationary. If the bifurcation moves with changes in water depth, these techniques will not be valid. A method to analyze bifurcations that shift has been presented for unsteady simulations by Li et al. (1983). However, such movement is unlikely to occur frequently. Most situations may be modelled adequately neglecting this effect, so it is assumed that all bifurcations treated herein are stationary.

A general bifurcation is illustrated in Figure 3.3. Taylor (1944) attempted to determine the flow distribution at a bifurcation. Equality of water levels was *not* assumed at the bifurcation. The analysis considered the following variables:

- the bifurcation angle;
- the three channel depths;
- two of the three discharges; and
- the velocity of the splitting stream.

Curves were developed from experimental data relating the depth ratio y_1/y_3 to the discharge ratio Q_3/Q_1 for various values of the Froude number Fr_1. Given Q_1, Q_3, Fr_1 and y_3 the depth y_1 was obtained. The value y_3 was determined by performing a backwater calculation up to the bifurcation. The corresponding value of y_2 results from the values of y_1 and y_3. However, these results were for the case of θ equal to 90 degrees for the specific channel geometry tested. Taylor (1944) notes that this treatment merely defined the data required to solve such a problem, and that rational analysis of this class of problem is unlikely.

Hager (1986) investigated channels with decreasing discharge in the flow direction. This occurs at side weirs, side-openings or bottom openings. Momentum considerations were applied in the longitudinal direction to derive the following equation for non-prismatic channels:

$$y' = \frac{\left[S_o - S_f + \frac{\beta Q^2}{gA^3}\frac{\partial A}{\partial x} - \left(1 + \beta - \frac{U\cos\phi}{v}\right)\frac{QQ'}{gA^2} - \frac{Q^2\beta'}{gA^2}\right]}{\left(1 - \frac{\beta Q^2}{gA^3}\frac{\partial A}{\partial y}\right)} \qquad 3.27$$

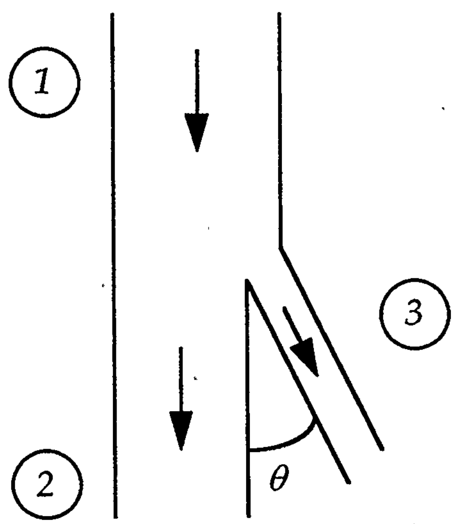

Figure 3.3 Bifurcation Of Open Channels

where:

- primes represent ordinary differentiation with respect to x;
- U = lateral outflow velocity deflected from channel axis at angle ϕ; and
- β = momentum correction coefficient.

The momentum correction coefficient can be neglected in the lowest order of approximation (Hager, 1986). The flow profile and the discharge $Q(x)$ were determined for varying side weir heights and widths given the relationship between discharge and distance. The head loss gradient across the lateral outflow was determined as

$$H_{s'} = -\left(\beta - \frac{U\cos\phi}{v}\right)\frac{QQ'}{gA^2} \qquad 3.28$$

and the average head loss gradient due to the outflow was

$$S_b = \frac{\Delta H_s}{\Delta L} = \frac{\xi v_u^2}{2g\Delta L} \qquad 3.29$$

where:

$H_{s'}$ = head loss gradient;

ΔH_s = integrated energy variation over outflow length ΔL;

Q_u = upstream discharge;

and

$$\xi = \frac{4}{5}\frac{\Delta Q}{Q_u}\left(\frac{\Delta Q}{Q_u} - \frac{1}{2}\right) \qquad 3.30$$

These results are valuable because a weir of zero height might be able to approximate conditions at a bifurcation.

Head loss occurring in a bifurcation can be determined given the upstream discharge, the change in discharge over the length of channel, the main channel flow velocity, the angle between channels and the bifurcation size (i.e., the width of the distributary channel). This accounts for the actual size of the bifurcation, but this analysis is applicable for only two distributary channels at one location. To extend this concept to situations involving more distributary channels, the concept may possibly be repeated by adding extra nodes to the system at nearby locations.

For a rectangular channel of zero slope, tests were performed and non-dimensional graphs prepared to facilitate the preparation of water surface profiles (Hager, 1986). It appeared that the surface profile depended upon:

- local Froude number;
- main channel outflow geometry;
- lateral channel outflow geometry;
- flow conditions upstream of the outflow location; and
- flow conditions downstream of the outflow location.

A bifurcation has also been compared to the splitting of flow occurring in a closed conduit fitted with a barrier (Rammamurthy and Satish, 1988). The relationship

$$\frac{Q_3}{Q_1} = \frac{1}{3\sqrt{3}} C_c \frac{L}{B} F_1^2 \left(1 + \frac{2}{F_1^2}\right)^{\frac{3}{2}} \qquad \textbf{3.31}$$

existed between the discharge in the distributary channel and the Froude number calculated downstream of the bifurcation. Here, C_c was a contraction coefficient as derived for pipe situations. Two of the assumptions inherent in the analysis were:

- the flow conditions were approximately critical in the branch channel a short distance downstream from the branch location (or, equally, the Froude number in the branch channel exceeded 0.35 at a location close to four times the branch channel width downstream from the branch location); and
- energy loss in the bifurcation itself was negligible.

The first requirement effectively introduces an additional boundary condition. If the branch discharge is to be determined only from knowledge of the downstream discharge in the main channel, a critical section must exist in the branch channel. If this does not occur the discharge in the branch must also be specified. Hence, if the flow regime is restricted to subcritical, the observed behaviour may not be utilised. The assumption of energy loss in the bifurcation being negligible may also not be reasonable if the findings of Lin and Soong (1979) are applicable to bifurcations in addition to confluences.

An open channel splitting at a right angle into two channels of width equal to the main channel has been investigated (Rammamurthy et al., 1990). For upstream flows with Froude numbers less than 0.75 the flow distribution could be determined. The water depth in the downstream section of the main channel was required, but the depth in the branch channel did not have to be measured. It was assumed that:

- the channels were horizontal;
- friction and energy losses in the bifurcation were negligible; and

- flows short distances upstream and downstream of the bifurcation were nearly uniform.

Although not explicitly stated, it was implied that a hydraulic jump will occur in one of the distributary channels. Hence, the results of this investigation are also applicable only to systems with supercritical flow conditions.

All of the relationships discussed may be useful for flat, rectangular channels, but they should not be applied to channels of arbitrary and irregular cross-section and slope without further testing. Despite this, several general findings may be applicable. Theoretically, an energy loss occurs in a bifurcation (Hager, 1986). This has been substantiated by Taylor's (1944) measurements of unequal water depth upstream and downstream of a bifurcation. Also, Rammamurthy et al. (1990) observed an increase in flow depth in the main channel downstream of the bifurcation and a decrease in depth in the branch channel. These observations indicate that the assumption of equal water surfaces at a bifurcation should not be made. However, the correct treatment of a general bifurcation has not been established. In fact, as stated by Taylor (1944), it may be necessary to treat each case individually. To accommodate the observed difference in water levels, a modelling algorithm should allow head loss terms to be incorporated as desired at bifurcations.

3.6 *CHAPTER SUMMARY*

The governing equation of steady one-dimensional flow has been derived for open channels. The equation was developed from energy considerations of the flow. Empirical formulae relating the discharge and the flow resistance (roughness) of a channel to the energy loss that occurs in the channel have been reviewed. The energy equation has been modified to allow non-uniform velocity profiles, non-prismatic channel sections and distributed lateral inflows and outflows to be treated adequately.

Error may be introduced from two sources during water surface profile computations: data will be incorrect to a certain degree, and numerical computations can introduce round-off and truncation effects. Errors arising from numerical sources are generally smaller than those due to incorrect data. The effects of errors introduced by incorrect Manning roughness coefficients can be minimised, in most cases, by using the arithmetic mean of the roughness values at each end of the channel. A guideline has been presented to select computational distance increments to minimise numerical errors.

Requirements for control sections were reviewed. Types of controls were discussed and their adequacy for modelling purposes examined. The importance of performing subcritical calculations in the upstream direction (and supercritical calculations in the downstream direction) was established.

Basic network components, or "building blocks", were described. In particular, distinction was made between the different classes of control structures. Structures can differ in their hydraulic behaviour and in their ability—or inability—to change or "control" discharge and depth. Regardless of the structure type, equations exist or can be developed to describe the relationship between discharge, upstream depth, downstream depth, and the physical structure parameters.

Laboratory and theoretical treatment of channel confluences was reviewed. Equality of energy levels is a more appropriate assumption than equality of water levels for the joining channels. This allows energy losses to be incorporated as desired. Momentum considerations may be disregarded provided flow velocities are relatively low.

Equal energy levels should also be assumed at bifurcations. Experiments have indicated that energy losses occur there too. A general method of determining the magnitude of these energy losses has not been developed. Therefore, bifurcations may have to be modelled on an individual basis.

Assuming equality of energy or water levels at a junction imposes a "compatibility" condition upon the system. A condition such as this attempts to ensure that the conditions at the joining (or separating) branches are physically compatible. The preceding literature review indicates that compatibility of energy levels should be used for bifurcations and confluences. Throughout this document the terms "compatibility" and "compatibility conditions" are used interchangeably with the phrase "compatibility of energy levels".

4 STEADY FLOW MODELS

Classical approaches for obtaining the steady-state flow characteristics for a single channel are reviewed. Analytical, graphical and numerical treatments of the governing equation of flow are examined. Previous attempts at solution of the steady-state network problem are discussed.

4.1 CLASSICAL STEADY STATE ANALYSIS

By solving either equation 3.4 or 3.19 the water surface profile may be obtained. For certain applications these relationships may be solved analytically; however, in general this is not the case. Hence, significant attention has been given to approximations of these equations. Solution attempts have included approximate solutions invoked with the assistance of simplifying assumptions, graphical methods, and numerical integration schemes.

4.1.1 Analytical Methods

An early attempt at the solution of equation 3.4 was made by Bresse

$$K = AC\sqrt{R} \tag{4.1}$$

(Bakhmeteff, 1932) who began by defining

$$\beta = \frac{S_o C^2 B}{gP} \tag{4.2}$$

and

where:

K = the conveyance of the channel;

A = the cross sectional area of flow;

C = Chezy roughness coefficient;

S_o = channel slope;

R = the hydraulic radius;

g = acceleration of gravity;

P = wetted perimeter; and

B = channel top width.

If equation 3.4 is rewritten and integrated along a channel reach of length l,

$$l = x_2 - x_1 = \frac{1}{S_o}\left[y_2 - y_1 + \int_{y_1}^{y_2} \frac{1-\beta}{\left(\frac{K}{K_o}\right)^2 - 1} dy \right] \qquad 4.3$$

corresponding to a difference in water depths, y_2-y_1, the following results: where K_o is the value of K corresponding to uniform flow conditions (i.e., for S_f equal to S_o). The integral is a function of y only. As the other terms in equation 4.3 may be easily determined, evaluation of the integral permits solution of the gradually varied flow equation. Solution of this integral has been considered by Dupuit, Ruhlmann, and Tolkmitt (Bakhmeteff, 1932). Dupuit considered an idealised flow profile, Tolkmitt assumed a parabolic profile and Bresse and Ruhlmann limited analysis to very wide rectangular channels. In all these situations, the assumed channel roughness, C, was considered constant over the entire range of water depths. This allowed tables to be prepared for the solution of the integral. The assumption of constant channel roughness despite variations in water depth introduces approximations in many situations, and the idealised

Bakhmeteff (1932) observed that over a reasonable range of depths, the conveyance closely follows the approximate relationship:

$$K^2(y) = A^2C^2R = constant \times y^N \qquad \textbf{4.4}$$

From this, he ascertained that

$$\left(\frac{K}{K_o}\right)^2 = \frac{A^2C^2R}{A_o^{\,2}C_o^{\,2}R_o} = \left(\frac{y}{y_o}\right)^N \qquad \textbf{4.5}$$

where N is defined as the hydraulic exponent and all terms with subscripts correspond to uniform flow conditions. By plotting log(K) versus log($ACR^{1/2}$) this exponent was determined for a particular channel. It was found that N varied over the range $2 \leq N \leq 5.5$. This was generally a good approximation for associated values of N_A and N_B over a water depth range $y_A < y < y_B$. Hence an average N value $(N_A+N_B)/2$ could be used. If the approximation was not adequate, the depth range could be subdivided to provide a better approximation. It was asserted (Bakhmeteff, 1932) that such action was rarely required. To use the hydraulic exponent in situations where the channel roughness varies, the C value used may be replaced by a term of the form C_oR^P.

The assumption was made that

$$\left(\frac{K}{K_o}\right)^2 = \left(\frac{y}{y_o}\right)^N \qquad \textbf{4.6}$$

and η was defined as y/y_o. Since β does not vary greatly, a channel may generally be subdivided into lengths over which b is approximately constant. This allows equation 4.3 to be easily integrated over the distance l, yielding

$$l = x_2 - x_1 = \frac{y_o}{S_o}\left[(\eta_2 - \eta_1) + (1-\beta)\int_{\eta_1}^{\eta_2}\frac{d\eta}{\eta^N - 1}\right] \qquad 4.7$$

where $\eta_1 = y_1/y_o$ and $\eta_2 = y_2/y_o$. The integral may be rewritten as a function $B(\eta)$. With the substitution $\Pi(\eta) = (1-\beta)B(\eta)$ equation 4.7 becomes

$$x_2 - x_1 = \frac{y_o}{S_o}\left[\Pi(\eta_2) - \Pi(\eta_1)\right] \qquad 4.8$$

Values of $B(\eta)$, the varied flow function, have been tabulated for a range of flow conditions, allowing the distance at which a specified flow depth occurs to be determined.

Henderson (1966) began with equation 4.4 and employed the Manning equation so that the conveyance was

$$K = \frac{\phi AR^{2/3}}{n} \qquad 4.9$$

For a constant discharge:

$$K^2 \propto \frac{1}{S} \qquad 4.10$$

Noting that $(S_o\text{-}S_f)$ may be written equivalently as

$$S_o\left(1-\frac{S_f}{S_o}\right)=S_o\left(1-\frac{K_o^2}{K^2}\right) \tag{4.11}$$

and assuming that

$$\left(AR^{2/3}\right)^2 \propto y^N \tag{4.12}$$

it is determined

$$\frac{K_o^2}{K^2}=\left(\frac{y_o}{y}\right)^N \tag{4.13}$$

Equation 4.13 is strictly correct only for very wide sections that follow the relationship

$$\frac{B}{B_S}=\left(\frac{y}{y_s}\right)^i \tag{4.14}$$

in which i is an exponent and B_S and y_S are the values of the top width and the flow depth at the station. For conditions of critical depth, $y = y_C$ and $Fr = 1$. If it is assumed that

$$\frac{A^3}{B} \propto y^M \tag{4.15}$$

the following results

$$\frac{dy}{dx} = S_o \frac{1-\left(\frac{y_0}{y}\right)^N}{1-\left(\frac{y_0}{y}\right)^M} \qquad 4.16$$

For rectangular channels $M = 3$ and for very wide channels $N = 10/3$. Bresse's analysis was shown (Henderson, 1966) to be the special case in which M = N = 3. Rewriting equation 4.16 with these values and integrating,

$$S_o x = y - y_o\left[1-\left(\frac{y_c}{y_0}\right)^3\right]\Phi \qquad 4.17$$

in which Φ is known as the Bresse function. This function is evaluated as

$$\Phi = \int \frac{du}{1-u^3} = \frac{1}{6}\log\left(\frac{u^2+u+1}{(u-1)^2}\right) - \frac{1}{3}\arctan\left(\frac{\sqrt{3}}{2u+1}\right) + A_1 \qquad 4.18$$

where A_1 is a constant of integration and $u = y/y_o$. Values of the Bresse function have been tabulated to assist in the preparation of water surface profiles.

Henderson (1966) also presented Bakhmeteff's analysis in a slightly different format. He considered the function

$$F(u,N) = \int_0^u \frac{du}{1-u^N} \qquad 4.19$$

which may be evaluated even if N is not a whole number. Tabulating this function for a range of u and N values allows water surface profiles to be obtained for various channel shapes. A difficulty occurs in the solution of equation 4.16 if N does not equal M, as integrals of the form of equation 4.19 do not result. These may be treated by setting $Fr^2 = \beta S_f / S_o$ which implies that

$$\beta = \frac{C^2 S_o}{g} \frac{B}{P} \tag{4.20}$$

For moderately wide channels the value of β is approximately constant. If this assumption is made, equation 4.16 reduces to

$$S_o \frac{dx}{dy} = \frac{1-\beta\left(y_0/y\right)^N}{1-\left(y_0/y\right)^N} \tag{4.21}$$

which allows the function to be tabulated for various values of N and β.

Ven te Chow's analysis (Henderson, 1966) begins with equation 4.16 in which N and M have different values. Substituting $u = y/y_o$, this becomes

$$dx = \frac{y_o}{S_o}\left[1 - \frac{1}{1-u^N} + \left(\frac{y_c}{y_o}\right)^M \frac{u^{N-M}}{1-u^N}\right]du \tag{4.22}$$

The further substitutions $v = u^{(N/J)}$ and $J = N/(N-M+1)$ allow a portion of the above equation to be rewritten as a form of the varied flow equation

$$\int \frac{u^{N-M}}{1-u^N} du = \frac{J}{N}\int_0^v \frac{dv}{1-v^J} \tag{4.23}$$

Integrating equation 4.23,

$$x = \frac{y_o}{S_o}\left[u - F(u,N) + \left(\frac{y_c}{y_o}\right)^M \frac{J}{N} F(v,J)\right] + A_1 \tag{4.24}$$

where A_1 is a constant of integration. This permits the preparation of an expanded set of tables to determine flow profiles for general cases. In fact, any cross section may be treated in this manner with the stipulations that

$$M = \frac{y}{A}\left(3B - \frac{A}{B}\frac{dB}{dy}\right) \qquad 4.25$$

and

$$N = \frac{2y}{3}A\left(5B - 2R\frac{dP}{dy}\right) \qquad 4.26$$

It has been shown by Pickard (1963) that the three "backwater integrals"

$$B1_{\pi}(z) = \int \frac{z^{\pi} - 1}{1 - z} dz \qquad 4.27$$

$$B2_{\pi}(z) = \int z^{\pi - 1}\, dz \qquad 4.28$$

and

$$B3_{\pi}(z) = \int \frac{z^{\pi - 1}}{1 + z} dz \qquad 4.29$$

can be combined to represent any possible flow profile if appropriate values are substituted for π. These integrals are transcendental functions, and their solution is not straightforward. Evaluation of these functions can become very involved and computationally difficult (Pickard, 1963) hence their application to real problems has been quite limited.

Besides often being applicable for only specific channel geometries, the above techniques were developed for cases not involving lateral inflows or outflows. Additionally, all these methods have a drawback for computer use: they require that tables of functions be evaluated, prepared and stored within the computer. As Henderson (1966) points out, this is generally a nuisance and a process that consumes both human and computer resources.

4.1.2 Graphical Methods

Analytical solution of the gradually varied flow equation requires that tables of integrals be prepared for various parameter values. Although these tables have been prepared by previous investigators, there is often a need to interpolate to determine flow depths for parameter values falling between those considered when the tables were prepared. Worse still, for cases in which the parameters are entirely outside the range of values previously tabulated, one must consider the basic equation again and integrate it. To reduce the work associated with the use of analytical methods and to facilitate the computation of additional backwater profiles for a single channel with slightly modified physical parameters,as is often encountered during the design process,graphical techniques were developed.

Typically, graphical techniques also eliminate the requirement of using trial and error methods as are generally used in numerical solutions of the governing equation. Graphical methods require a minimum of repetitious calculations. This feature was particularly important prior to the widespread accessibility of hand calculators and computers. Even as computers have become more common, graphical techniques have continued to be used to solve the equation of flow. For example, the RIVER4 (Ashenhurst, 1981) computer program is based upon a graphical technique.

Escoffier (1946) presented a graphical method to determine the flow profile in a channel reach. The technique used the Manning equation with the friction slope represented by h/L where h is the head loss and L is the length of the reach. Solving for h yields

$$h = WLQ^2 \qquad \textbf{4.30}$$

where,

$$W = \left(\frac{nAR^{2/3}}{\phi} \right)^2 \qquad \textbf{4.31}$$

The W value used must be representative of the entire reach. Typically the arithmetic mean of the W values for each end of the reach, W_{us} and W_{ds}, has been used. Curves representing the variation of W_{us} and W_{ds} with water surface elevation are prepared. Then, imposing a control at one section of the channel, the associated W value is located on the plot. The corresponding W value at the other end of the reach is determined by constructing a line with slope of magnitude $LQ^2/2$. This line begins at the curve for the downstream section and intersects the curve for the upstream section. The water surface level at that location is then directly read from the graph. Since only one set of curves is required for the analysis of varying discharges within a given channel (Henderson, 1966), the method is very convenient for certain applications. However, changes in channel geometry or roughness necessitate the preparation of new charts. Hence the technique appears limited for use in channel design. Results will be affected by the accuracy with which the graph is prepared and read. However, computer generation of the required curves may mitigate this problem.

The Ezra Method (Ezra, 1954a) has been used to determine water surface profiles in natural—i.e., irregular—channels. Solution of the steady flow profile begins with energy considerations. Losses due to friction along the channel bottom are represented by the arithmetic mean of the end friction slopes as recommended by Laurenson (1986) and given in equation 3.13 while eddy losses can be described by equation 3.8. If z_1 and z_2 are taken as water surface elevations at the upstream and downstream ends of a channel reach, the energy equation may be rewritten as

$$z_1 + f(z_1) = z_2 + \Phi(z_2) + K\left(\frac{v_1^2}{2g} - \frac{v_2^2}{2g}\right) \qquad \textbf{4.32}$$

Here, $f(z_1)$ and $\Phi(z_2)$ are hydraulic parameters depending upon channel discharge and section properties, and accounting for friction losses introduced within the reach. These parameters are calculated for several water depths at each channel cross section, added to the channel elevation and plotted against the water stage. If a suitable control exists at one end of the reach, the flow profile may be determined. For subcritical flow, the known water depth allows $z_2+\Phi(z_2)$ to be read from the graph for the downstream section. By subtracting the eddy loss term, the corresponding $z_1 + f(z_1)$ term can be determined for the upstream location. By locating this value on the chart, the upstream water surface elevation may be read directly. Effects of eddy losses can be incorporated into the Manning n value along with the effects of bends along the channel. This eliminates the need to perform any calculations once the charts have been prepared.

Analysis (Gray, 1954; Ezra, 1954b) indicates that the difference between the friction-head loss of the Ezra method and the term

$$\int S dx \qquad \textbf{4.33}$$

is in the order of 12%. This error was felt to be reasonable (Ezra, 1954b) considering the uncertainties inherent in the determination of the Manning n value. However, other methods of analysis, most notably numerical methods, tend to introduce much smaller error values.

The Ezra method is most appropriate if profiles are required for the same channel for various water levels at the control section. Modifications to the Ezra method (Henderson, 1966) have allowed one chart to be used for varying discharges in the same channel. Because the plotted hydraulic parameters account for friction losses within the channel reach, two different curves must be prepared for cross sections acting as the upstream boundary for one reach and the downstream boundary for another reach. Therefore, the labour involved in the preparation of the hydraulic curves can be considerable. Also, new curves must be generated if the channel geometry changes. Hence, applications involving the selection of "best" designs or modifications to channels may be very time-consuming.

Grimms' method (Henderson, 1966) may be used with minimal field data. Only stage-discharge relationships are required at cross section locations. However, application of this procedure is limited to cases where the energy slope may be assumed equal to the water surface slope. This implies that velocity head is entirely neglected. This assumption prevents the method from being applied to general situations, so this method is not discussed further.

Henderson (1966) describes a step method of determining water surface profiles. A beginning station, and a distance increment, Δx, at which the water depth is to be determined, are selected. Referring to Figure 3.1 again,

$$E_1 - E_2 = \left(S_o - \frac{1}{2}S_{f_1} - \frac{1}{2}S_{f_2}\right)\Delta x \qquad 4.34$$

If the following definition is made,

$$U = E - \frac{1}{2}S_f\,\Delta x = U(y, \Delta x) \qquad 4.35$$

then

$$U_2 - U_1 = \left(S_{f_1} - S_o\right)\Delta x \qquad 4.36$$

For a constant discharge a plot of U versus y may be prepared. If an initial water depth, y_1, and starting location, x_1, are known, U_1 may be obtained. Values of U_2 and y_2 can be subsequently obtained from the graph.

Since graphs must be tabulated for computer applications, graphical techniques are also not recommended for computer use (Henderson, 1966). Despite this, the Ezra method is the basis for the computer program RIVER4 (Ashenhurst, 1981; Smith and Ashenhurst, 1986).

4.1.3 Numerical Integration

Two forms of step method have been introduced (Henderson, 1966; French, 1985) for the numerical solution of the gradually-varied flow equation. These begin by writing the equation in finite difference form. In the simplest of these, designated the direct-step method, a water surface elevation is assumed adjacent to the selected control section. From this assumption, the location of the corresponding channel section is determined. In the standard step method, a channel distance is selected and the associated water surface elevation is determined. This latter technique is iterative in nature, however, the extra work required is often warranted because of the convenient form of the results.

Methods using more general forms of the gradually varied flow equation include the technique presented by Prasad (1970), the Newton-Raphson method (Fread and Harbaugh, 1971; Manz, 1985b) and the Runge-Kutta method (Humpidge and Moss, 1971).

4.1.3.1 Direct-Step Method

This technique is similar to the graphical step method described in Section 4.1.2. A desired water depth occurs at an unknown distance L upstream of a control section. The objective of the technique is to determine the distance L. The energy equation for a channel reach is approximated in finite difference form (Henderson, 1966) by

$$\frac{\Delta E}{\Delta x} = \frac{\Delta\left(y + \frac{v^2}{2g}\right)}{\Delta x} = S_o - S_f \qquad 4.37$$

For a uniform channel, S_o is constant, hence the change of energy throughout a reach is purely a function of discharge and the physical properties of the channel. The flow area, wetted perimeter, hydraulic radius and velocity of flow are calculated for the selected water depth at the new section. The friction loss between the control section and that at distance Δx is determined by equation 3.10. The R value used is the mean hydraulic radius of the end sections. Once the difference between the channel slope and the friction slope is known, $\Delta E/\Delta x$ can be evaluated. Since ΔE is the difference between the sums of water surface elevation and velocity head of the two stations, Δx may be determined directly.

Although this technique yields valid results, interpolation is generally required to determine water depths at specific locations along the channel profile. If results are required at such locations, accuracy may be lost in the interpolation process. It is preferable to use the standard step method in these instances or when the channel is irregular.

4.1.3.2 *Standard Step Method*

For irregular channels, the required cross sectional and roughness data is generally available at only a few locations. It is necessary to work from these locations in order to determine the water surface profile. Often the channel properties vary irregularly, making the determination of x from a given water depth y difficult. In such cases, the problem is generally "flipped about", with the x value being known and the y unknown. Solution progresses through a trial process.

A channel location is selected at which the water surface elevation is desired. Again, the basic energy equation is utilised and the calculation must begin at a location where the discharge and water depth are known. A value is assumed for the unknown water depth, allowing the flow area and velocity to be determined. These provide a total energy value for the upstream location. A value is determined for the friction slope at the upstream section, allowing the energy loss due to friction throughout the reach to be determined. From this friction loss, the energy level at the downstream end of the reach is determined and compared to the energy level known to exist there. If the two values differ by less than a specified tolerance, the assumed upstream water stage is considered correct. Computations continue upstream in a similar manner with the newly determined upstream flow conditions being used as the new downstream location. Otherwise, an adjustment is made in the assumed upstream stage and the process is repeated.

Henderson (1966) and French (1985) describe ways to adjust the upstream depth to help speed up the rate of convergence toward the solution. Also, eddy losses can be accounted for by multiplying an appropriate loss coefficient by the mean velocity head of the reach. Of course, this introduces the problem of determining a loss coefficient that adequately describes the eddy losses.

4.1.3.3 Prasad's Method

A numerical integration scheme was introduced (Prasad, 1970) based upon applying the trapezoidal method of integration (Swokowski, 1983) to the energy equation. This method is used in the hydraulic computation component of the WQRRS computer program (Hydrologic Engineering Center, 1988). The form of the energy equation used was very general, allowing both prismatic and non-prismatic channels with or without lateral inflows to be analysed. The energy equation is rewritten in the form

$$\frac{dy}{dx} = \Phi(S_o, n, Q, \alpha, geometry, y) \qquad \mathbf{4.38}$$

The technique is a variation of the other step methods. Again, solution progresses from a control section to an adjacent location where the flow depth is desired. The value of dy/dx at the new location is initially assumed to equal the value of dy/dx at the known location. The corresponding unknown flow depth is estimated by the trapezoidal technique and used in the energy equation to determine a new dy/dx value. This value is compared to the previously assumed value. Improvements are made in an iterative fashion until the difference between successive values is smaller than a stipulated maximum error.

One limitation of the procedure is that it tends not to converge quickly if $\partial\Phi/\partial y$ is large. This tends to occur in the vicinity of critical depth sections. Even though the rate of convergence decreases, the technique still converges near critical flow sections (Prasad, 1970). Using this method the solution may progress upstream *or* downstream of a control regardless of the flow regime. This has been verified for subcritical and supercritical flows and for various channel sections (Prasad, 1970).

4.1.3.4 Newton Raphson Method

The Newton Raphson technique has been used to solve equation 3.4. This method was selected because it was considered to be more straightforward and computationally more efficient than other techniques (Fread and Harbaugh, 1971). Solution began by writing a differential form of the gradually varied flow equation

$$\frac{dy}{dx} = \frac{(S_o - S_f)}{1 + \alpha \frac{d}{dy}\left(\frac{v^2}{2g}\right)} \qquad 4.39$$

and the energy gradient given by

$$S_f = \frac{n^2 v^2 C}{R^{4/3}} \qquad 4.40$$

together in the form

$$y_1 + \frac{\alpha Q^2}{2gB^2 y_1^2} - \frac{n^2 Q^2 \Delta x C^2}{B^2 y_1^2 \left[(B + 2y_1)/(By_1)\right]^{4/3}} + F = 0 \qquad 4.41$$

where

$$F = S_o \Delta x - y_2 - \frac{\alpha Q^2}{2gB^2 y_2^2} - \frac{n^2 Q^2 \Delta x C^2}{B^2 y_2^2 \left[(B + 2y_2)/(By_2)\right]^{4/3}}$$

Equation 4.41 is substituted for $f(y_1)$ in Newton's method. A trial water depth is selected for the reach end with unknown flow conditions. For the case of calculations progressing upstream, the value of y_1 is to be determined. A revised guess for the unknown water depth is obtained from

$$y_1^{k+1} = y_1^k - \frac{f\left(y_1^k\right)}{f'\left(y_1^k\right)} \qquad 4.42$$

where $f(y_1^k)$ is the value of equation 4.41 for the present value of y_1 and $f'(y_1^k)$ is the derivative of $f(y_1^k)$ evaluated for the same value of y_1. The value of y_1^{k+1} is used as the new value of y_1^k and the process is repeated. When the difference between y_1^k and y_1^{k+1} is smaller than a stipulated tolerance, the iteration ceases. The unknown water depth at the end of the next reach upstream is determined similarly. This method is flexible because the distance increment Δx may vary from reach to reach. This method has also been applied to a more general form of the gradually varied flow equation similar to equation 3.12 (Manz, 1985b). Solution of these equations then followed the method presented by Fread and Harbaugh (1971).

It has been noted (De Neufville and Hester, 1969; Wylie, 1972; Gichuki, 1988) that the Newton-Raphson technique will not always converge. Divergence may occur when certain channel elements have flow resistances greatly exceeding that of other elements within the network, or if the initial estimate of the solution is not "good". Unfortunately, guidelines regarding how to determine whether initial estimates are "good" or not are not available. Epp and Fowler (1970) have presented a means of initially allocating flows in water distribution systems to reduce the likelihood of divergence. However, no general criteria regarding the allowable resistance of these "troublesome elements" has been observed. Hence, there is always potential for divergence when this technique is used, but if the individual channel elements provide resistance to flow of the same approximate magnitude, divergence is less likely to occur (De Neufville and Hester, 1969).

4.1.3.5 *Runge-Kutta Method*

Humpidge and Moss (1971) presented a method of solving a general form of the gradually varied flow equation similar to that of equation 3.19. They employed the Runge-Kutta method (Cheney and Kincaid, 1985), a numerical technique that permits a function $y(x)$ to be evaluated given x_o and y_o at another location. With a fourth-order version of the method, the value of y at a location $x = x_o + h$ is determined as

$$y = y_o + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4) \qquad \textbf{4.43}$$

where:

$k_1 = hf(x_o, y_o);$

$k_2 = hf(x_o + h/2, y_o + k_1/2);$

$k_3 = hf(x_o + h/2, y_o + k_2/2);$ and

$k_4 = hf(x_o + h, y_o + k_3).$

This allows a function to be solved without evaluating its derivative as is required when using the Newton Raphson method. Hence, no limit of complexity need be placed upon the function if a solution is still to be obtained. The technique *does* require that several computations be made, increasing the opportunity for propagation of numerical errors.

4.2 STEADY STATE NETWORK SOLUTIONS

Eichert (1970) reviewed computer programs that calculated water surface profiles and determined that none were capable of analysing flow in divided channels. Wylie (1972) adapted a method of analysis from water distribution systems to these situations. Subsequent techniques were based on the Ezra method (Smith and Ashenhurst, 1986) or modifications of Wylie's technique (Schulte, 1985; Schulte and Chaudhry, 1987).

4.2.1 Wylie's Computer Program

A method of analysing divided channel systems was introduced (Wylie, 1972) whereby continuity was satisfied at each junction and equality of total energy—water surface elevation plus velocity head—was assumed for all branches at a confluence or bifurcation. This allowed localised energy losses to be incorporated, but these effects were not actually included in the examples provided. The form of the continuity equation used was:

$$F_i = \sum_{k=1}^{M} Q_k + Q_{N_i} = 0 \qquad \textbf{4.44}$$

where Q_k was a branch discharge and Q_{Ni} was a flow added or subtracted at a node. This allowed nodal flows to be included at a junction location. Such flows could include bulk lateral inflows from drainage culverts in irrigation or river networks, or from manholes in sewer systems.

Initial values were assumed for the unknown flows then the Newton-Raphson technique was used to solve the resulting equations simultaneously. The solution was obtained in an iterative fashion. To help avoid divergence of the

Newton-Raphson technique (De Neufville and Hester, 1969), a scaling factor was used when determining corrections for the next iteration. This scaling factor was included as follows:

$$E_i^{m+1} = E_i^{m} + \delta_i \Delta E_i^{m+1} \qquad \textbf{4.45}$$

where:

E_i^{m+1} = the next estimate of the energy level at the location;

E_i^{m} = the last estimate of the energy level at the location;

ΔE_i^{m+1} = the correction determined by the Newton-Raphson technique; and

δ_i = scaling factor.

If good initial guesses of the energy levels were made, divergence of the iteration procedure was unlikely (Wylie, 1972). For both of the examples presented, initial guesses were that energy levels at all nodes were equal to that at the known location. Although this assumption is rather unrealistic, convergence of the numerical scheme was obtained and only seven iterations were required. In the second example provided, two independent sources were introduced to the system. Both these discharges were specified as initial conditions.

Wylie also points out that although this program provides correct solutions for channels in series, other programs exist that will generate the same results much more efficiently. This technique is suitable only for prismatic and modestly non-prismatic channels. Also, it cannot be used to analyse situations with distributed lateral inflows or outflows.

4.2.2 RIVER4 Computer Model

The RIVER4 program (Ashenhurst, 1981; Smith and Ashenhurst, 1986) was developed to perform flood plain analyses and to determine overland flow conditions for subdivision design. Another purpose was to facilitate trial and error calculations arising during the design of stormwater management facilities. In addition to modelling open channels such as natural drainage courses, the program has also been used to simulate flow in closed channels. The most frequent application of this type has been sewer pipes. Analysis of bifurcated branches and associated "island" flow was also performed. Only one resistance coefficient was allowed for each section, hence changes in resistance with water depth such as those encountered when overbank regions become flooded, cannot be modelled. Since one intention of this program was to perform flood plain analyses, this appears to be a serious program limitation.

The program can be used (Smith and Ashenhurst, 1986) to model or analyse systems including:

- natural and man-made open channels with or without bridges;
- sewer systems;
- a gravity irrigation system with flow removal;
- a sewer system with diversion;
- a sewer system and overland flow; and
- multiple islands.

The illustrated "gravity irrigation system with flow removal" consisted merely of a single canal with lateral offtakes. This bears little resemblance to actual gravity irrigation systems that generally have a more complex hierarchy of canals to distribute the water. These systems also often include operable control structures. It would be beneficial if more comprehensive systems could be modelled by the program.

Channel transitions were modelled by overlaying sets of cross-sectional data at one location. Head losses were calculated by multiplying a coefficient by the difference in velocity heads between two sections. Other researchers (Ezra, 1954a; Henderson, 1966) generally recommend using the mean velocity head in the reach rather than difference across the reach. Other than channel reaches, network components that have been modelled include bridges, culverts, overhanging banks, weirs (Smith and Ashenhurst, 1986) and drop manholes (Ashenhurst, 1981). Structures were represented by contractions and expansions where the flow resistance equation was applied. This technique does not compare favourably to those that represent structures by functions describing the actual hydraulic behaviour as discussed in section 3.5.4.3. Interestingly, by comparing the RIVER4 analysis of a linear system having several structures to results obtained by HEC-2, unexplained water level differences of the order of seventeen to thirty-six percent were observed (Ashenhurst, 1981). Perhaps these discrepancies resulted from an inability to adequately model control structures.

When a system was input into the program, the nodes had to be numbered according to the following guidelines (Ashenhurst, 1981):

- all tributaries had to be numbered consecutively in the direction of flow;
- the furthest downstream section had to have the highest number representing the maximum number of sections; and
- only one tributary or branch could exist at any node.

This first requirement implies that the flow direction in all channels is known prior to the analysis. In complex looped networks or deltas it may not be possible to correctly determine the direction of flow initially. Adhering to this rule prevents the program from analysing flow reversals.

A built-in program feature renumbered the nodes when additional ones were incorporated, allowing changes to be made to the network easily. This could be of great utility to model users. However, violation of the first two numbering requirements may cause the program to "crash" or generate incorrect results. This could occur if nodes were inadvertently numbered incorrectly by the user.

One of the biggest limitations of RIVER4 results from the second numbering requirement. It must be *guaranteed* that the largest node number is situated at the furthest downstream location. Because of this, RIVER4 cannot treat situations where the flow splits and does not rejoin—i.e., where there are two or more end discharges. This prevents networks having more than one terminus from being modelled properly. Hence, the vast majority of river deltas and irrigation systems cannot be treated adequately by the program.

Water surface profiles were determined based upon the Ezra method. Although this was originally a graphical technique, the program authors converted it for computer application. As previously mentioned, adapting graphical methods for computer application is generally not recommended (Henderson, 1966).

Deviating somewhat from the Ezra method, the authors did not use the arithmetic mean of the friction slope values at the given cross sections. Rather, they utilised the geometric mean of the slope values. As discussed, Laurenson (1986) determined that using the arithmetic mean of the friction slope generally introduces less error. Hence the method of averaging friction values in RIVER4 should probably be modified.

Initial estimates of the flow depths were made and revised by an iterative scheme. Subsequent estimates of water depths were obtained by interval-halving techniques. An initial guess for the distribution of flow at a bifurcation began with the calculation of the critical energy level and discharge for the lower-numbered channel (i.e., the branch channel). The flow distribution estimate was determined as

$$Q_{main} = \frac{Q_{tot}^{\ 2}}{(Q_{tot} + Q_{branch})} \qquad 4.46$$

and

$$Q_{branch} = Q_{tot} - Q_{main} \qquad 4.47$$

where:

Q_{main} = discharge in main channel;

Q_{branch} = discharge in branching channel; and

Q_{tot} = total discharge.

Network analysis in RIVER4 began by calculating water surface profiles for hte individual channels to satisfy the discharge distribution determined for each junction by equations 4.46 and 4.47. If the energy levels at a junction were incompatible, adjustments were made based upon the principles known as Kirchoff's laws (Ashenhurst, 1981):

- the sum of the head loss around a loop is zero; and
- the sum of the inflows equals the sum of the outflows at each junction.

In fact, these "laws" are an application of continuity and a comparison of energy levels at junctions. If these conditions were not approximated closely enough by the assumed flow conditions, a discharge adjustment was computed for each junction in turn. Discharge adjustments were determined similarly to the way flow increments are obtained when the Hardy Cross method (Daugherty et al., 1985) is applied to pipe analysis problems. These adjustments were made for every channel at each branch, then the junction criteria were checked again. Strictly speaking, in dendretic or otherwise branched systems, no "loops" exist around which head loss adjustments can be made. This is one factor that limits the model's ability to treat typical distributary systems.

The program can be used to determine water surface profiles for non-prismatic channels. However, non-uniform velocity profiles cannot be accommodated because α_1 and α_2 are both assumed to equal 1.0 and their values cannot be changed by the user.

The network solution was obtained by a trial and error method. This was selected in preference to matrix analysis because it was believed that the program would be used primarily for non-branched systems (Ashenhurst, 1981). This last statement accentuates the limitations of the program and underscores the need for a more comprehensive alternative.

4.2.3 Washington State University Computer Program

An algorithm was presented (Schulte, 1985; Chaudhry and Schulte, 1986) to determine flow in parallel channels around islands. Analysis was limited to parallel channels that branched from one common location and rejoined at another common location. Because of the definition of parallel channels used, the program did not appear capable of analysing systems with multiple terminal locations such as found in irrigation systems and many deltas.

Networks were analysed simultaneously, instead of by graphical or manual trial and error methods as in RIVER4. An energy equation was written for every channel section, then the system of resulting equations was solved in an iterative fashion using the Newton-Raphson method. The equations were arranged so that a banded Jacobian (coefficient matrix) resulted. At junctions, equations were introduced to satisfy continuity and the requirement that the total energy of flow be equal for all channels present. Additional equations were supplied at the downstream boundary as a specified discharge and stage of flow.

It was initially assumed that flow depths throughout the system were equal to the specified downstream depth. In large networks having wide variations in flow depth, such an assumption may not provide an accurate enough starting point to allow convergence of the numerical method. The flow distribution at a junction was initially allocated equally among all the channels so as to satisfy continuity. Even when continuity was not initially satisfied, convergence still occurred, albeit at a slower rate (Schulte, 1985).

Schulte (1985) also presented a form of the energy equation that treated flow reversals correctly. The form of the equation used was:

$$F_{i,k} = \Delta y + \Delta z + \frac{1}{2g}\left(\frac{\alpha_i Q_{i,j+1}\left|Q_{i,j+1}\right|}{A_{i,j+1}^2} - \frac{\alpha_i Q_{i,j}\left|Q_{i,j}\right|}{A_{i,j}^2}\right) + \frac{\Delta x}{2}\left(\frac{Q_{i,j+1}\left|Q_{i,j+1}\right| n_i^2}{C_o^2 A_{i,j+1}^2 R_{i,j+1}^{4/3}} + \frac{Q_{i,j}\left|Q_{i,j}\right| n_i^2}{C_o^2 A_{i,j}^2 R_{i,j}^{4/3}}\right) \quad \textbf{4.48}$$

where:

$\Delta x = x_{i,j+1} - x_{i,j}$;

$\Delta y = y_{i,j+1} - y_{i,j}$; and

$\Delta z = z_{i,j+1} - z_{i,j}$.

Proper flow directions were accounted for by using the absolute value on some of the discharge terms. This technique should eliminate some of the potential for errors introduced by other programs. It was pointed out (Schulte, 1985) that this algorithm could be enhanced by generalising the technique to include individual features such as controls and transitions. Given the preceding comments, the author concurs.

Examination of the computer program listing indicates that significant amounts of code may have to be rewritten if different network geometries are simulated. Hence, an objective of the present research is to produce a program that accepts general network geometries without requiring modifications to the program itself.

4.2.4 Irrigation Conveyance System Simulation Model

The Irrigation Conveyance System Simulation (ICSS) model was developed to help determine the performance evaluation of irrigation conveyance systems (Manz, 1985c). Subsequently, the model was used for real time operations assistance and training purposes as well as an aid to engineers, operators and managers in the design, maintenance, operation, rehabilitation and management of irrigation conveyance systems (Manz and Schaalje, 1992). The program has also been used for various consulting and research projects in Canada and Egypt.

The model is capable of simulating steady and unsteady flows in lined or unlined channels. The hydraulics and operation of a broad range of structures may be simulated and variation of canal characteristics including shape, roughness, distributed lateral inflows or outflows and slope along the channel length may be accommodated.

Although the ICSS model was originally designed to perform unsteady flow simulations, it incorporates subroutines that perform backwater calculations (Manz and Schaalje, 1992). Steady flow is simulated using a form of the gradually varied flow equation similar to equation 3.19 (Sim-Flo Engineering Ltd., 1991; Manz and Schaalje, 1992) while the channel roughness is accounted for by equation 3.10. This formulation of the gradually varied flow equation allows non-prismatic channel sections to be simulated. A finite difference formulation similar to that of Fread and Harbaugh (1971) is used as the solution technique.

Network situations are solved by assuming a discharge distribution at a junction and calculating the water depth at the junction from one downstream location. The water depth is then calculated at the same junction by performing backwater computations from the other downstream location or locations meeting at the common upstream location. If the difference in the two depths is less than a specified tolerance, flow conditions at the next most upstream junction are obtained in the same manner. Otherwise, adjustments are made to the branch flows and the junction depths are evaluated from each downstream point again. This process repeats until the junction depths are sufficiently close.

This is an iterative technique with the network being solved piece-by-piece. No procedure has been developed to treat *general* network configurations, and it is up to the program operator to decide in which order calculations proceed throughout the network. Additionally, the model "cannot treat bifurcations where the control structures are unsatisfactory (submerged) very well" (Manz, 1994). Clearly, a steady-state model should be able to simulate bifurcations independent of the types of controls present.

4.3 *CHAPTER SUMMARY*

The gradually varied flow equation has been analysed in several manners. These include analytical treatments, graphical techniques and numerical methods. Analytical methods provide exact solutions of the governing equation; however, strict limitations are imposed upon the channel geometries that can be analysed. Also, the methods do not generally allow distributed inflows and outflows to be incorporated. Graphical techniques can treat a wider range of channel geometries. Unfortunately, these techniques are laborious and can require extensive tabular data for computer formulation. The solutions obtained are often strongly dependent upon the specific flow conditions assumed—that is, if downstream discharge is changed, then new curves must often be prepared. Numerical methods are capable of treating sophisticated forms of the flow equation, allowing general channel shapes, non-prismatic effects and distributed lateral flows to be treated. Treatment of the equation in numerical forms is also straightforward on computer. Therefore, numerical methods tend to be the most attractive solution alternative.

Various numerical methods have been used to solve the governing flow equation. These include: the direct step method; the standard step method; Prasad's technique; the Newton-Raphson method; and Runge-Kutta methods. Of these techniques, the latter two have been preferred in recent research. The Newton-Raphson method generally converges within a few iterations, although it can diverge if the original estimates for the solution values are sufficiently far from the actual solution values. In contrast, Runge-Kutta techniques are numerically stable; however, they require numerous computations. These increase the opportunity of introducing large round-off and truncation errors into the solution.

Previous attempts have been made to solve open channel network situations. These essentially began with Wylie's (1972) treatment of divided channels. Solutions were obtained using the Newton-Raphson method, but the form of flow equation used could not simulate non-prismatic channels or distributed lateral flows. The Washington State University model also used the Newton-Raphson method to determine flow situations in more complex looped systems. The form of equations used allowed flow reversals to be predicted correctly. Large sections of the computer version of the algorithm may have to be rewritten when applying the model to new situations. Neither of these models allowed control structures to be modelled.

The RIVER4 program was based upon a graphical treatment of the governing equation. Solution was iterative, wherein water surface profiles were prepared for individual channels, continuity and compatibility checked at junctions, and modifications in flow conditions made as necessary. Treatment of hydraulic structures differed from established theory, and the results obtained err significantly in comparative tests with the HEC-2 model. Numbering requirements for the program were complex. Another model which solves network situations in an iterative manner is the ICSS model. Although this model treats control structures and distributed lateral flows, the program operator must determine what order to use when computing profiles for individual channels.

5 NETWORK SOLUTION TECHNIQUES

Once the appropriate form of the governing flow equation has been selected or developed, it must be applied to each channel within a network. The flow equation is combined with a relationship that describes the roughness effects of the channel, such as the Manning roughness equation. The resulting equation can then be applied to successive locations along channel, dividing it into segments over which the relevant physical properties are assumed to be essentially constant.

After each channel has been subdivided into reaches, and relationships developed to describe how the energy varies in each reach, the boundary conditions at each end of every channel must be considered. For a channel in which the downstream depth and discharge are specified, the boundary conditions may be taken as these known quantities. If a structure is at an end of a channel, then the structure equation is a boundary equation. When a channel simply meets another channel "head on", a continuity equation may be developed. At junctions, a continuity relationship must be prepared and equations describing the energy loss between the joining or splitting channels must be developed.

Once equations have been determined for all the boundary conditions and along all the channels, a method of solution must be employed. Methods of determining flows within open channel networks may be either simultaneous or iterative. Iterative methods are based upon taking estimates of the unknown values and improving these estimates by trial techniques. The network equations are written and solved independently. When using a simultaneous method, the equations are written so that they are dependent upon equations for other portions of the network. Solution of the equations for the entire network is achieved simultaneously. Both types of technique have been used in the analysis of pipe flows, unsteady open channel flows and steady open channel flows. Each method is reviewed and their relative benefits and drawbacks are discussed.

5.1 *ITERATIVE SOLUTION TECHNIQUES*

In iterative solution schemes, assumptions are made regarding the values of the flow characteristics—that is, the discharge distribution, flow depth, and energy losses—occurring at channel junctions. Then, backwater computations are performed for each channel that meets at the junction. If the assumed conditions cannot be satisfied by the situation being modelled, as realised by inconsistent energy heads or violation of continuity, then the assumed conditions are modified and the calculations repeated. Once the flow conditions at a junction have been determined, remaining junctions are treated in the same manner. Structures are treated similarly, by assuming a discharge, upstream and downstream head. If the assumed values are incorrect, they are adjusted and computations are repeated.

The steady state network program RIVER4 employs this type of procedure to determine flows throughout networks (Ashenhurst, 1981), as does the ICSS model (Manz and Schaalje, 1992). Both of these programs have been applied successfully to open channel networks.

Numerous calculations are often required for an iterative formulation, because generally several iterations are required. Because of this same reason, it may be expected that solution times tend to be long. Additionally, strict numbering requirements may be required when entering channel geometry—as with RIVER4 (Ashenhurst, 1981). Program users may have to specify the simulation order for network channels, as is required with the ICSS model (Manz and Schaalje, 1992). For large systems this could be very time-consuming.

However, a computer system does not necessarily need a large memory capacity to run a program based upon an iterative solution scheme because relatively little data manipulation is required at any one time.

5.2 *SIMULTANEOUS SOLUTION TECHNIQUES*

Simultaneous solution techniques are facilitated by developing a set of equations that can be assembled into matrix form. The matrix equation is solved, providing the discharges and flow depths throughout the system simultaneously.

Schulte (1985) prepared equations for the N_i reaches of channel number i:

$$y_{i,1}+\alpha_{i,1}\frac{Q_{i,1}^2}{2g\,A_{i,1}^2}+z_{i,1}=y_{i,2}+\alpha_{i,2}\frac{Q_{i,2}^2}{2g\,A_{i,2}^2}+z_{i,2}+\left(\frac{x_{i,2}-x_{i,1}}{2}\right)\left(\frac{Q_{i,2}^2 n_{i,2}^2}{C_o^2\,A_{i,2}^2 R_{i,2}^{4/3}}+\frac{Q_{i,1}^2 n_{i,1}^2}{C_o^2\,A_{i,1}^2 R_{i,1}^{4/3}}\right)$$

$$y_{i,2}+\alpha_{i,2}\frac{Q_{i,2}^2}{2g\,A_{i,2}^2}+z_{i,2}=y_{i,3}+\alpha_{i,3}\frac{Q_{i,3}^2}{2g\,A_{i,3}^2}+z_{i,3}+\left(\frac{x_{i,3}-x_{i,2}}{2}\right)\left(\frac{Q_{i,3}^2 n_{i,3}^2}{C_o^2\,A_{i,3}^2 R_{i,3}^{4/3}}+\frac{Q_{i,2}^2 n_{i,2}^2}{C_o^2\,A_{i,2}^2 R_{i,2}^{4/3}}\right)$$

$$\vdots$$

$$y_{i,N_i}+\alpha_{i,N_i}\frac{Q_{i,N_i}^2}{2g\,A_{i,N_i}^2}+z_{i,N_i}=y_{i,N_i+1}+\alpha_{i,N_i+1}\frac{Q_{i,N_i+1}^2}{2g\,A_{i,N_i+1}^2}$$
$$+z_{i,N_i+1}+\left(\frac{x_{i,N_i+1}-x_{i,N_i}}{2}\right)\left(\frac{Q_{i,N_i+1}^2 n_{i,N_i+1}^2}{C_o^2\,A_{i,N_i+1}^2 R_{i,N_i+1}^{4/3}}+\frac{Q_{i,N_i}^2 n_{i,N_i}^2}{C_o^2\,A_{i,N_i}^2 R_{i,N_i}^{4/3}}\right) \quad \mathbf{5.1}$$

where section 1 is at the upstream end of the channel and section N is taken at the downstream end. These equations were rewritten as a series of functions, $F_{i,N} = 0$ as:

$$F_{i,1} = y_{i,2} - y_{i,1} + z_{i,2} - z_{i,1} + \frac{1}{2g}\left(\frac{\alpha_{i,2}Q_{i,2}^2}{A_{i,2}^2} - \frac{\alpha_{i,1}Q_{i,1}^2}{A_{i,1}^2}\right) + \frac{\Delta x_{i,1}}{2}\left(\frac{Q_{i,2}^2 n_{i,2}^2}{C_o^2 A_{i,2}^2 R_{i,2}^{4/3}} + \frac{Q_{i,1}^2 n_{i,1}^2}{C_o^2 A_{i,1}^2 R_{i,1}^{4/3}}\right) = 0$$

$$F_{i,2} = y_{i,3} - y_{i,2} + z_{i,3} - z_{i,2} + \frac{1}{2g}\left(\frac{\alpha_{i,3}Q_{i,2}^2}{A_{i,3}^2} - \frac{\alpha_{i,2}Q_{i,2}^2}{A_{i,2}^2}\right) + \frac{\Delta x_{i,2}}{2}\left(\frac{Q_{i,3}^2 n_{i,3}^2}{C_o^2 A_{i,3}^2 R_{i,3}^{4/3}} + \frac{Q_{i,2}^2 n_{i,2}^2}{C_o^2 A_{i,2}^2 R_{i,2}^{4/3}}\right) = 0$$

•
•
•

$$F_{i,N_i} = y_{i,N_i+1} - y_{i,N_i} + z_{i,N_i+1} - z_{i,N_i}$$
$$+ \frac{1}{2g}\left(\frac{\alpha_{i,N_i+1}Q_{i,N_i+1}^2}{A_{i,N_i+1}^2} - \frac{\alpha_{i,N_i}Q_{i,N_i}^2}{A_{i,N_i}^2}\right) + \left(\frac{x_{i,N_i+1} - x_{i,N_i}}{2}\right)\left(\frac{Q_{i,N_i+1}^2 n_{i,N_i+1}^2}{C_o^2 A_{i,N_i+1}^2 R_{i,N_i+1}^{4/3}} + \frac{Q_{i,N_i}^2 n_{i,N_i}^2}{C_o^2 A_{i,N_i}^2 R_{i,N_i}^{4/3}}\right) = 0 \tag{5.2}$$

Many of the terms in this series of equations vary with the depth of the water. For instance, the area and hydraulic radius of a channel change as the water level rises or falls. The Manning roughness coefficient and the velocity correction coefficient may also change with variations in the water depth along some channels. However, these F functions depend only on the water depth at the ends of the reach. Therefore, the above equations may be represented as follows:

$$F_{i,1} = f(y_{i,1}, y_{i,2}) = 0$$

$$F_{i,2} = f(y_{i,2}, y_{i,3}) = 0$$

•
•
•

$$F_{i,N_i} = f(y_{i,N_i}, y_{i,N_i+1}) = 0 \tag{5.3}$$

giving a set of N equations that may be applied to solve for the $N+1$ unknown depths. Using a specified depth condition at the downstream channel end:

$$y_{i,N_i+1} = y_d \qquad 5.4$$

or

$$F_{i,N_i+1} = h\left(y_{i,N_i+1}\right) = 0 \qquad 5.5$$

provides an $(N+1)^{st}$ equation. Next, the set of equations is solved using the Newton-Raphson method. Considering the general reach between sections j and $j+1$ of channel number i,

$$F_{i,j} = y_{i,j+1} - y_{i,j} + z_{i,j+1} - z_{i,j}$$
$$+\frac{1}{2g}\left(\frac{\alpha_{i,j+1}Q_{i,j+1}^2}{A_{i,j+1}^2} - \frac{\alpha_{i,j}Q_{i,j}^2}{A_{i,j}^2}\right) + \left(\frac{x_{i,j+1} - x_{i,j}}{2}\right)\left(\frac{Q_{i,j+1}^2 n_{i,j+1}^2}{C_o^2 A_{i,j+1}^2 R_{i,j+1}^{4/3}} + \frac{Q_{i,j}^2 n_{i,j}^2}{C_o^2 A_{i,j}^2 R_{i,j}^{4/3}}\right) = 0$$

5.6

or

$$F_{i,j} = f\left(y_{i,j}, y_{i,j+1}\right) = 0 \qquad 5.7$$

By taking the partial derivatives of all such functions with respect to $y_{i,j}$ and $y_{i,j+1}$,a series of coefficients is obtained that can be assembled into a coefficient matrix. The system can be solved by making an estimate of the solution values, solving the matrix, comparing the values obtained with those estimated, and revising the estimate. Repeating this process leads to the estimate and the calculated values differing by less than a specified tolerance.

Additional channels are incorporated into the coefficient matrix in the same manner. Boundary equations must be provided for each additional channel. Schulte (1985) took these boundary conditions as specified downstream depths for all channels. Continuity was considered at junction locations, and compatibility of water levels was assumed. Boundary equations are treated like the rest of the equations to determine partial derivatives for the coefficient matrix.

The primary advantage of using a simultaneous technique is that trial and error processes are limited to those required to solve the matrix. This may take only a few trials, whereas several trials may be necessary for *each* junction when using an iterative method. Hence, the speed of the simultaneous method may be several times that of the iterative one.

A drawback of the scheme is that a very large matrix will be created for large systems. This matrix may require excessive storage space in the computer memory, and solution of such a large matrix may be a lengthy process. Also, as previously mentioned, divergence of the Newton-Raphson scheme has been observed if the initial estimates of the solution are not close enough to the actual solution. Because of this, a program based on this scheme may fail without warning.

5.3 *MATRIX SOLUTION*

If a simultaneous solution technique is used to determine the water surface profile throughout an open channel network, a matrix equation as described in the previous section can result. When the system being simulated is large, the resulting coefficient matrix will also tend to be quite large. Therefore it is important to examine various methods of solving such a matrix. A preferred method will be computationally quick and accurate and require a minimum of computer memory for its operation.

The coefficient matrices generated for network solutions generally have special properties that distinguish them from other matrices. These matrices tend to contain a lot of zeroes—that is, the matrices are sparse. In other cases, matrices will be "banded" along the main diagonal. Because of these characteristics, several investigators have used special matrix solution techniques to increase the speed or accuracy of network solutions. Special routines can minimise the bandwidth of a matrix, while hydraulic model users have developed numbering techniques to reduce the matrix bandwidth. Therefore, matrix solution techniques previously used in network situations are reviewed.

5.3.1 Standard Matrix Solution

Gaussian elimination with partial pivoting (Cheney and Kincaid, 1985) has been effective in the analysis of pipe networks (Demuren and Ideriah, 1986). The forward step of the Gaussian elimination process transforms the coefficient matrix into upper triangular form. In so doing, all the elements below and to the left of the main diagonal are converted to zero values by multiplying and adding rows within the matrix. A "pivot" equation is multiplied by a certain coefficient and added to the other rows of the matrix to effect this. The equations are rewritten so that each variable appears as a function of variables to its right in the coefficient matrix. The equation in the remaining "unsorted" matrix with the largest leading coefficient becomes the pivot equation. This minimises round-off errors and helps ensure that division by zero does not occur. After conversion to upper triangular form, the coefficients are determined by substituting back as in naive Gaussian elimination. Results obtained by Demuren and Ideriah (1986) were within the same accuracy as those obtained using a sparse matrix solution technique

The main disadvantage of this technique is that it does not take advantage of the large number of zero elements that can originally exist in the coefficient matrix. Therefore, the time required for this procedure tands to be excessive.

5.3.2 Sparse Matrices

In a model proposed by Joliffe (1984) using the matrix routine of Gupta and Tanji (1977), arbitrary node numbering was allowed provided that nodes within a channel were numbered consecutively. The node numbers selected did not influence the solution process itself or the required computer storage. Storage requirements were proportional to the number of nodes within the network, as opposed to being a quadratic relation of the number of nodes in models using other storage modes (Joliffe, 1984). For larger networks the appeal of this approach increases.

On the other hand, special sparse matrix techniques may more often prove beneficial in comparison to simpler methods such as Gaussian elimination. Tests using the Network model indicate that a sparse matrix solution without bandwidth reduction capabilities has performed about eight times as quickly as a Gaussian elimination routine (Swain, 1988).

5.3.3 Banded Matrices

Solution of sparse, diagonally banded matrices is possible using double-sweep techniques. These methods are limited to matrices in which all the elements lie within a specified distance, or "bandwidth", of the diagonal running from the upper left corner to the lower right corner of the matrix. Strelkoff (1992) developed a method of treating a column of non-zero coefficients appearing outside the main band of coefficients. Further modification of this process "*may* [author's emphasis] allow matrices containing non-zero elements at other locations to still be solved in the double-sweep manner" (Strelkoff, 1992). It appears that little confidence is held for this technique. As described by Cunge et al. (1980), these methods are well suited for channels in series. However, complex algorithms would need to be developed and a significant increase in computer work would be required for application to looped networks.

Since irrigation systems may have loops within them (for example, major river diversions in India and return irrigation flow in North American systems), these techniques do not appear suitable to model irrigation networks. Additionally, if channels are not numbered very carefully, the banded form of the matrix may disappear entirely, thereby rendering this solution technique useless.

Network (Swain, 1988) utilises the algorithm presented by Martin and Wilkinson (1967) to solve diagonally banded matrices in which the coefficient band is not symmetric about the diagonal. This technique employs partial pivoting and results in an upper-triangular matrix that may be solved by straightforward elimination techniques. For a coefficient matrix of dimensions $n \times n$, $n-1$ major steps are required to transform the matrix into upper-triangular form. The decomposition method is numerically stable.

5.3.4 Bandwidth Reduction Techniques

Using a routine that reduces the bandwidth of the coefficient matrix for a given network can optimise the speed of banded-matrix solution routines. Several methods of reducing the bandwidth exist; some of these depend upon additional computer subroutines while others require that special numbering rules be followed by the person performing the modelling.

Poor numbering schemes will produce a Jacobian of large bandwidth. This increases computer time *and* storage requirements (Schulte, 1985). Kao (1980) introduced a channel numbering sequence for networks. It allowed channels joined in series or at junctions to have different computational distances provided the increment size does not change along individual channels. For parallel channels, each one had to be subdivided into the same number of computational sections. Node numbers along a particular channel were incremented by the number of channels with which it was in parallel, plus one. A method of converting the resulting sparse coefficient matrix into a more compact form was also presented. Together, use of these techniques reduced computation time for an unsteady flow simulation from approximately 7.7 minutes to 16 seconds even though the computational time step was four times smaller for the latter case.

Chaudhry and Schulte (1986) presented a technique to achieve the minimum possible bandwidth for the Jacobian. The energy equation was written for the first section of all channels in parallel, followed by the continuity equation for those same channels in the same sequence. Then the energy and continuity equations were written in the same order for the next downstream sections of the channels. Each channel in parallel had to be subdivided into the same number of sections. It was stated that for a system of M parallel channels the resulting coefficient matrix would have a bandwidth of $M+1$. This is not correct: the proper bandwidth should be *3M+1*. Schulte (1985) also recommended that for networks more complex than parallel channels the Jacobian be sketched for each alternative numbering sequence considered. This would determine the sequence producing the optimum coefficient matrix bandwidth. For large systems this is obviously a ridiculous proposition.

The stipulation of both Chaudhry and Schulte (1986) and Kao (1980) that each parallel channel be subdivided into the same number of sections reduces the flexibility of the numbering scheme. This may result in using exceedingly long sections on longer channels. This would introduce inaccuracies into the computed profile. Or, extremely short sections could be required on shorter channels, reducing the computational efficiency of the solution. In order to keep the computational increment more in line with the recommendations of McBean and Perkins (1975) these numbering requirements should be carefully examined before use.

The unsteady flow model Network (Swain and Chin, 1990) contained a routine to minimise the solution matrix bandwidth. The matrix was diagonally banded for a single canal or a series of canals joined end-to-end. When parallel channels were included, coefficients were introduced outside the band. This effect became more pronounced as the size of the network, and hence the matrix, increased. However, deviation from the banded configuration could be minimised.

Examination of the matrix began in the outermost bands and worked, band-by-band, toward the main diagonal. If a non-zero coefficient was encountered, the same column was searched for a zero coefficient. Searching began at the main diagonal and worked toward the non-zero coefficient to be moved. If a zero was encountered, its row and the row containing the non-zero coefficient were switched. Upon confirmation that the bandwidth had not increased, the bandwidth reduction process continued. If a zero was not present in the column containing the non-zero coefficient, the coefficients in the same row were checked. Now, if a zero is located, the corresponding columns were switched. Again, the bandwidth of the matrix was determined to ensure that it had not increased through the switching procedure. The bandwidth reduction procedure ended when no zero coefficients were identified in either the column or the row of the non-zero coefficient.

Using this procedure, processing times for unsteady simulations decreased only when the duration of the simulation was quite long. Since for steady state problems, the matrix calculations are not performed as frequently as in unsteady modelling, such a bandwidth reduction scheme may not be justified. Bandwidth reduction was recommended for long simulations and where significant reduction of the bandwidth may occur (Swain, 1988).

Newton's method has been used in the simultaneous solution of steady state water distribution problems (Epp and Fowler, 1970). The technique determined the minimum path between nodes, allowing a coefficient matrix of small bandwidth to be formed. Although the algorithm did not guarantee the matrix will have the *minimum* bandwidth, in practice it was consistently *close* to the minimum.

To determine initial flow values for the first solution iteration a computer subroutine created a "minimum spanning tree" to represent the network (Epp and Fowler, 1970). Pipes that offered great resistance to flow were represented by long "branches" and were designated as having zero or very small initial flow values. Correspondingly, pipes that offered little resistance to flow were represented by short branches. Using educated initial guesses of the actual solution values could reduce the number of iterations required and increase confidence in the convergence of the numerical solution scheme.

By using either of these computer subroutines the bandwidth of large matrices can be minimised without having to follow strict numbering guidelines or having to sketch coefficient matrices. This gives the program user greater flexibility and should allow a more efficient use of human resources.

5.4 *CHAPTER SUMMARY*

There are two basic solution methodologies for network situations. Iterative techniques solve for the conditions throughout the network in a step-bystep fashion while simultaneous formulations solve the entire network at once. For both methods, solution progresses from known boundary conditions, most commonly stipulated discharges and depths at downstream boundaries.

Simultaneous techniques require that only one "pass" be made through the system, hence they can often be faster than iterative methods. When simultaneous techniques are used, a matrix of coefficient values is invariably assembled. Coefficient matrices are typically sparse, and most frequently banded. Since solution of the system relies upon matrix inversion, techniques that lessen the time required to solve the coefficient matrix are of interest. Methods that have been used for network solutions include:

- numbering the network elements in special manners, creating as small a bandwidth as possible during matrix assembly;
- bandwidth reduction techniques that reduce the bandwidth of an assembled matrix; and

- sparse matrix solution algorithms that increase the solution efficiency as compared to standard matrix inversion techniques.

Of these techniques, the first is the least desirable because of the stringent rules imposed upon the algorithm user during simulations. Either of the other alternatives seem beneficial.

6 SOLUTION ALGORITHM OBJECTIVES

Existing technology is reviewed with respect to appropriateness for computer solution of network situations. Criteria are established to develop an effective model capable of simulating networks having the components previously identified.

6.1 *SUMMARY OF EXISTING TECHNOLOGY*

Fundamental concepts relating to physical data and errors have been presented. These must be considered when modelling open channel flows. Several methods of estimating the roughness for a channel section have been developed based upon information available only at the end locations of the channel. Of these methods, arithmetically averaging the end roughness values generally introduces the smallest amount of error. Errors introduced by computational procedures have been compared to errors introduced through estimating physical channel parameters. Effects of round-off and truncation errors are usually small relative to errors resulting from roughness coefficients or field survey information. A guideline for the selection of appropriate distance increments was presented (equation 3.22) that should minimise the effects of errors introduced from numerical sources. To lessen the opportunity for error, water surface profiles should be calculated upstream from control sections for subcritical flow conditions and downstream from controls for supercritical conditions.

Network components have been introduced and discussed. A wide range of control structures have been reviewed. Assumptions made in previous models for the two junction types—confluences and bifurcations—have been examined. Experiments indicate that energy losses should be considered at both types of junction, hence model users have been incorrect by assuming equality of water levels at junctions.

Natural and man-made open channel networks have been compared and contrasted. A notable difference is that man-made systems such as irrigation networks generally have numerous control structures while natural systems, like rivers, typically have very few. Also, natural channels are more likely to have irregular or non-prismatic sections, channel roughness that varies significantly with water depth, and non-uniform flow velocity profiles.

Numerical integration techniques are most appropriate for solution of the governing equation for channels of arbitrary section and for those with lateral inflows or outflows. Analytical methods are generally limited to certain channel geometries while graphical techniques require that function values be calculated and stored in tabular form within the computer. Storage requirements could become onerous when large numbers of computations are involved. Use of graphical methods may also necessitate significant amounts of work if a system or its individual elements are reconfigured.

Several numerical techniques have been presented to evaluate the governing equation. The Newton-Raphson method has been widely used but divergence of the technique has been reported. No criterion has yet been developed to accurately predict when convergence and divergence will occur. Still, the method has been used frequently and the reported complaints have been few. Runge-Kutta methods are numerically stable and may be preferable in certain situations, but they introduce additional computations and an associated potential for numerical error.

Computer programs have been used to determine steady state water surface profiles for open channel networks. Among these are the ICSS model (Manz and Schaalje, 1992), the RIVER4 model (Ashenhurst, 1981) and the models developed at Washington State University (Schulte, 1985) and by Wylie (Wylie, 1972). Many of the programs have built-in limitations that make their use rather inconvenient or inefficient at times. These shortcomings include:

- solution techniques that need the user to specify the computational order;
- stringent numbering requirements that may impose unreasonable computational lengths for parallel channels;
- requiring program code be rewritten when network geometry is changed;
- complicated data entry requirements;
- having to sketch Jacobian matrices to achieve optimal node numbering;
- an inability to model channels that split from the network and do not rejoin it; and
- a failure to simulate a wide range of control structures.

6.2 *OBJECTIVES OF A SOLUTION ALGORITHM*

There is a need to analyse steady flows in open channel networks for use in unsteady modelling and for other design purposes. A solution algorithm is required to solve network situations including bifurcations and various types of control structures. A fully general algorithm would be applicable to natural and man-made open channel systems and be capable of modelling:

- channels that split from the network and do not rejoin it;
- all types of junctions and associated energy losses;
- a wide range of control structures;
- distributed lateral inflows and outflows;
- channels with non-uniform velocity profiles;
- flow reversals;
- variation of channel resistance coefficient with fluctuating water depth;
- non-prismatic channel sections; and
- subcritical and supercritical flows.

Although a completely general algorithm incorporating all the features listed above *could* be developed given unlimited resources, such a model is not necessarily required here. Several of the individual components that are desirable in the "ultimate" model already exist in other steady flow models. For instance, flow profiles can be determined for channels in which both subcritical and supercritical flows are present (Molinas and Yang, 1985; Lopes and Shirley, 1993). Analysis of these methods in conjunction should permit all flow regimes to be determined for most channel types. Non-uniform velocity profiles have been modelled by a number of investigators, as have channel roughnesses that vary with water depth.

What *is* required of a solution algorithm is a method of treating the most hydraulically complex types of open channel networks, while providing a framework into which "extra" capabilities can be inserted. Prior to solving a network however, a means is required of determining whether a solution is indeed attainable given the stipulated system information. If a solution can exist, then the solution algorithm should be utilised. The algorithm must handle the following network features:

- bifurcations;
- confluences;
- multiple downstream end locations;
- loops; and
- a variety of control structures.

Irrigation networks are systems that are most likely to include all these features. In addition to these features, these systems also present unique hydraulic situations because of physical limitations of channels and/or control structures. Accordingly, the solution algorithm should be developed with a view toward irrigation systems.

The solution algorithm should also:

- solve the governing equation by numerical integration;
- have simple node and channel numbering requirements;
- use distance increments that minimise numerical errors; and
- not require changes to computer code if network geometry is changed.

Additionally, if matrix solution is used, the program should independently solve the system matrix efficiently—that is, the user should not have to perform any special checks. Finally, the program should be available for modern personal computers. This will greatly increase the likelihood of the program actually being used as a design, operations or management aid.

7 SOLUTION ALGORITHM

Solution algorithms are presented for network situations. Since networks are comprised of basic building blocks, an algorithm that properly simulates networks must simulate the individual components. An overview relating the algorithms for individual components is presented, then the component algorithms are discussed. Several special conditions that may arise in the simulation of actual open channels are addressed. Following the solution algorithm, the Constraint Rules are developed. These rules are used to determine whether or not a unique solution exists for a given network. Examples are provided to demonstrate methods of converting unsolvable systems to forms more amenable for solution. Strategies are presented to treat complex networks composed of arrangements of the basic network elements.

7.1 NETWORK ALGORITHMS

To develop algorithms to solve general network situations, it is necessary to consider the effects of individual network elements. After presenting the overall solution algorithm and organization, several of these elements are reviewed. Of particular importance are: the form of the governing equation used along channel reaches; control structures that are operable or that may become submerged; channel junctions; loops; and the effects of distributed lateral inflows and outflows.

7.1.1 Solution Overview

The various components of the solution algorithm are described in the following sections. These components are linked together as illustrated in Figure 7.1. As the diagram depicts, solution progresses from the downstream ends of a given system. Solution progresses iteratively upstream, beginning where discharge and depth conditions are specified. When depth and discharge are known at a location, it is considered "finished". Solution upstream of a location that is not "finished" is pointless, and is hence not attempted. Upon "finishing" flow routing for all locations, a check is made to determine whether specified maximum discharges have been violated at any point in the system. If such a discharge *has* been exceeded, all locations are considered "not finished", the scheduling routine is invoked and the flow is rerouted through the system.

Systems are represented as a series of "nodes" and "controls". Nodes contain geometric information for the channel reach immediately upstream, while controls hold the descriptive information for all fixed and operable control structures. This information includes gate settings and openings, structure coefficients and energy losses. Locations of nodes and controls relative to other nodes and controls are required as input.

7.1.2 Governing Equation

The form of the governing flow equation presented in equation 3.19 is suitable for distributed lateral inflows. Combining the equation with the similar one derived by Monem and Manz (1994) for distributed lateral outflows yields a composite form of gradually varied flow equation

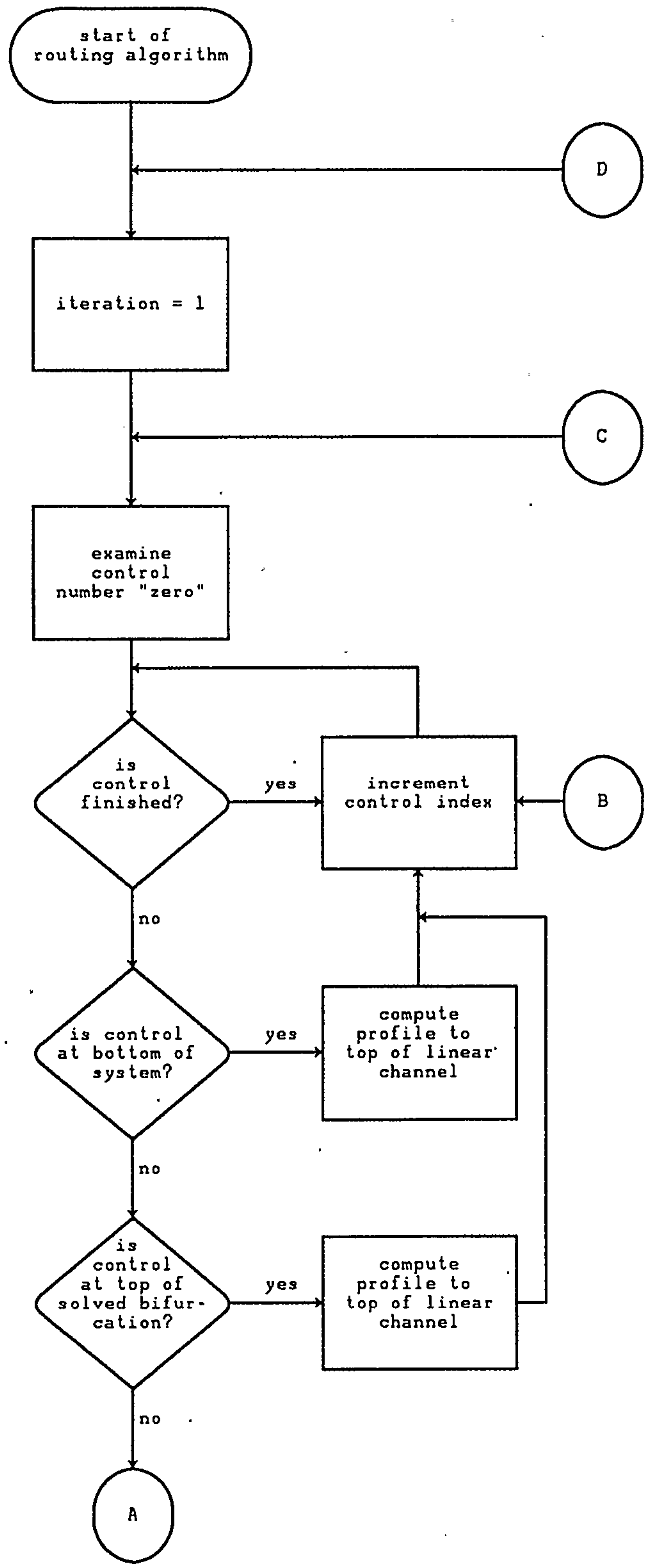

Figure 7.1 Solution Overview

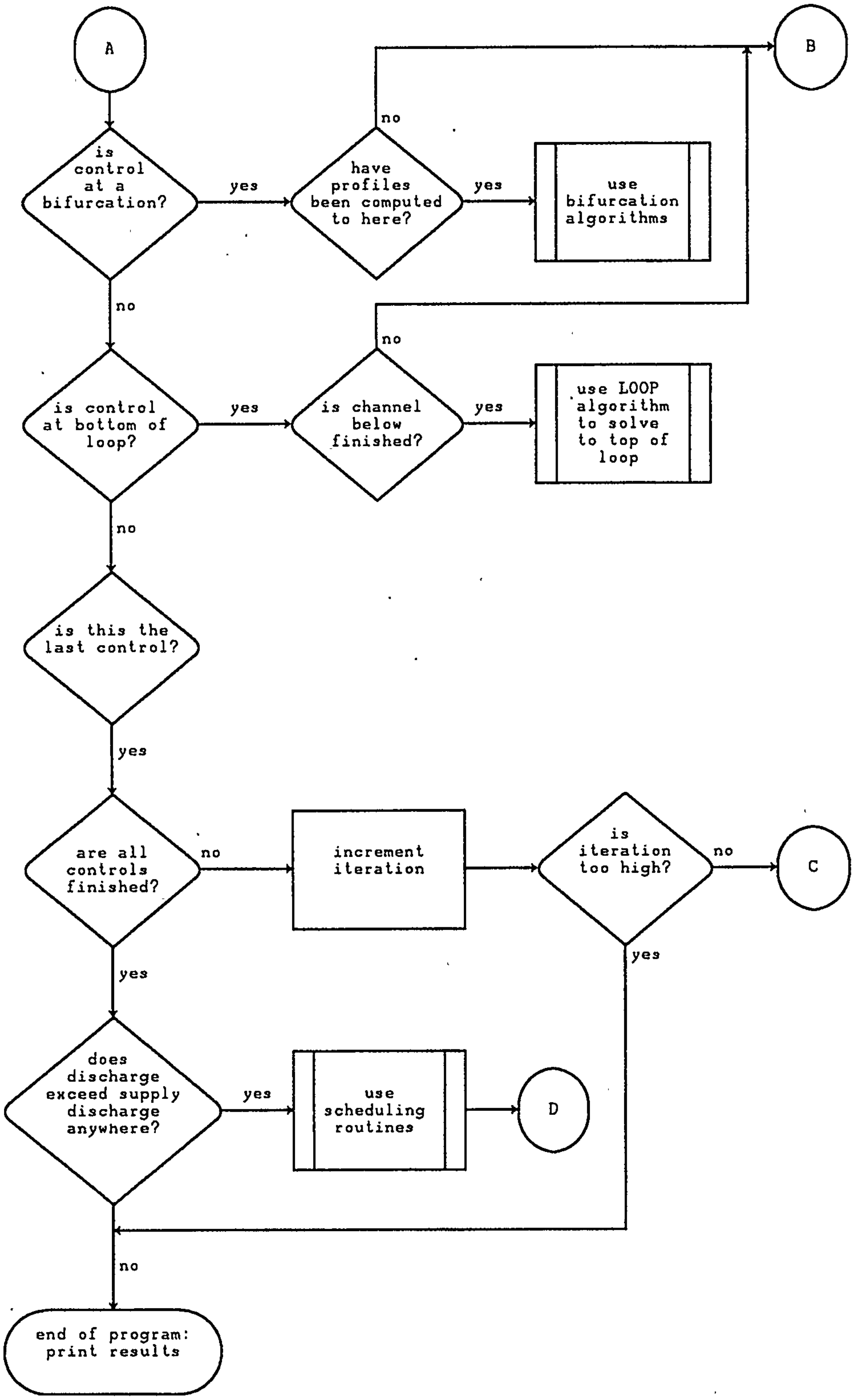

Figure 7.1 (cont.) Solution Overview

$$\frac{dy}{dx} = \frac{\left[S_o - S_f - \frac{C_1 Q}{gA^2}\frac{dQ}{dx} + \frac{1}{Ag}\left(\frac{Q^2}{A^2}\right) A_x^y \right]}{\left(1 - Fr^2\right)} \qquad 7.1$$

where:

C_1 = 1.0 for distributed outflow, 2.0 for distributed inflow;

A_x^y = rate of change of area with respect to x with y held constant; and

dQ/dx = variation of discharge with distance.

All shapes of prismatic and non-prismatic channels having distributed lateral inflows or outflows can be analysed using Equation 7.1. Since the mathematical development of the equations treating inflows and outflows are quite different, this composite equation cannot strictly be derived. However, from an engineering viewpoint, this form of presenting the two separate equations is very convenient. By using appropriate values for the C_1 coefficient, distributed lateral inflows and outflows may be lumped into an "effective" lateral flow.

7.1.3 Control Structures

A control structure may be represented as a relationship between upstream water depth, downstream water depth, discharge, and physical structure parameters. Performing backwater computations to the downstream side of a control will provide the depth and discharge. From continuity, the discharge immediately downstream of a control must equal that through the control and immediately above it. Given the hydraulic relationships for the control, the upstream depth is then easily obtained, permitting backwater computations to proceed upstream. If a control is submerged, the structure equations will reflect the tailwater effects in the calculation of the upstream depth.

Often, controls are operable. Gate openings or settings, weir heights, or other physical parameters may be adjusted to control the discharge through a structure or the water levels upstream or downstream of it. Prime examples of operable controls are adjustable check structures that may be used to provide sufficient water depth for deliveries upstream of the check. In practice, the desired delivery is generally known. From this information, the canal system operator determines the depth of water required at the check structure. Then the check is raised or lowered to provide the desired depth. In short, the control allows a desired depth to be obtained.

If backwater computations are performed up to a control where a desired depth has been specified, treatment of the control is similar. Continuity still applies through the control, so the discharge is known. Using the downstream depth and a guess at the proper structure setting, the structure equation yields an upstream depth. If the depth obtained differs significantly from that desired, the structure parameters are adjusted and the upstream depth recalculated. This process is programmed using the Newton Raphson method and continues until the calculated depth closely approximates the desired depth.

Practical issues may prevent the desired depth from being obtained. With real structures, there are upper and lower limits of operation for the adjustable parameters. Given a check structure, for instance, it cannot possibly be adjusted higher than the channel banks, nor can it be set lower than the channel bottom. In fact, its actual *useful* range of operation is likely much more restrictive than this. Hence, for a given discharge and downstream depth, it may not be physically possible to produce the desired depth upstream of the control.

Secondly, controls are typically adjustable in finite increments. Given discharge and downstream depth, each structure setting will produce a corresponding upstream depth—i.e., the adjustment increments create increments in the upstream depth. Even if the desired depth is within the available range of depths, the desired depth may not correspond to one of the possible structure settings. When the desired depth falls between two available depths, the setting that produces the deeper upstream depth is selected. This is done since it seems preferable to over-satisfy upstream deliveries than to fall short of those requirements.

7.1.4 General (Type I) Bifurcations

A general bifurcation is illustrated in Figure 7.2. The controls immediately below the bifurcation may be satisfactory or unsatisfactory, and either operable or fixed. It is assumed that the discharge is known at the bottom, or tail, of either branch 1 or branch 2. The discharge may also be known at the bottom of the other branch. If it is not, a value is assumed for that branch discharge. Solution progresses from the tail locations to the downstream sides of the controls at the bifurcation. Discharges and depths are therefore determined below both controls, allowing discharge and depth to be computed immediately upstream of the structures. Conservation of energy dictates that the energy above control 1 on branch 1 equals the energy above control 1 on branch 2 (this assumes that there are no energy losses; energy losses can be incorporated in the algorithm, but their inclusion complicates this discussion). If the energy levels above the two structures are not within a specified tolerance, the given flow conditions *cannot* exist at the bifurcation—i.e., it is not a solution for the channels considered.

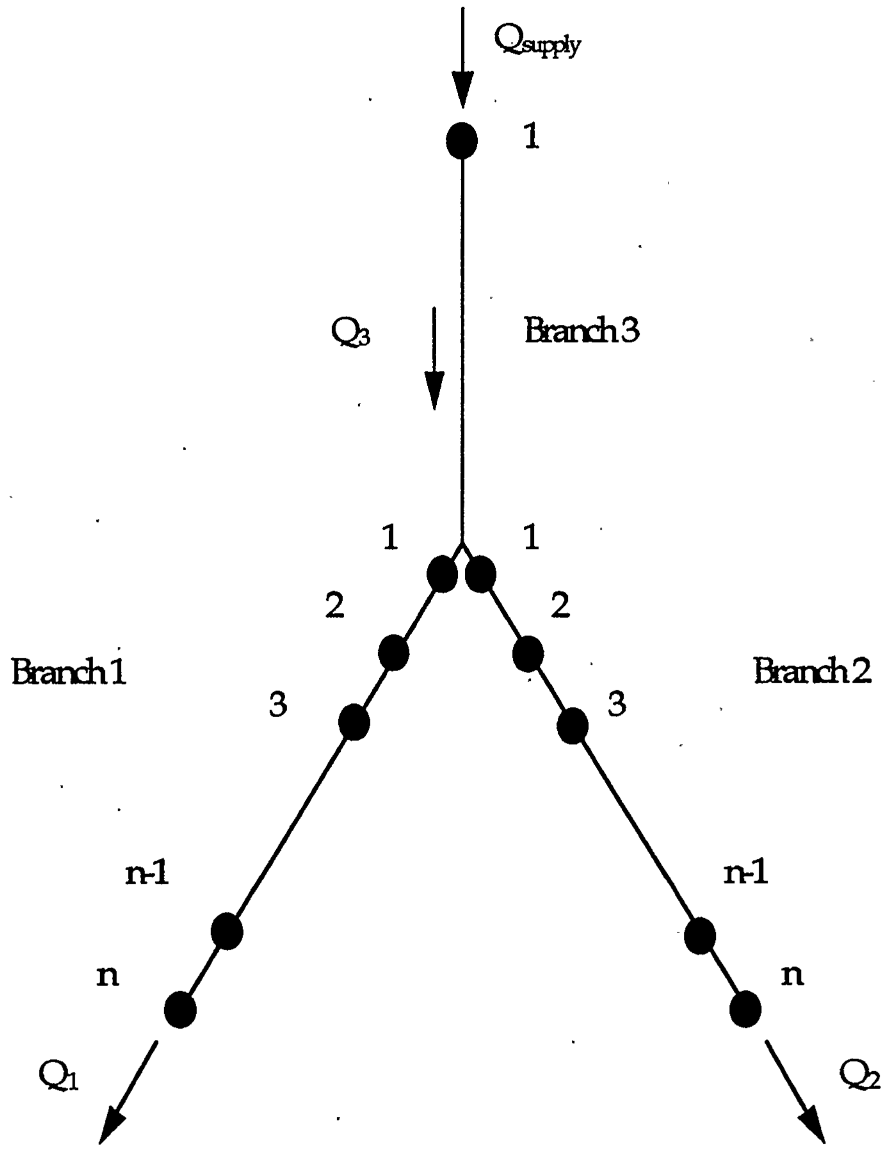

Figure 7.2 General Channel Bifurcation

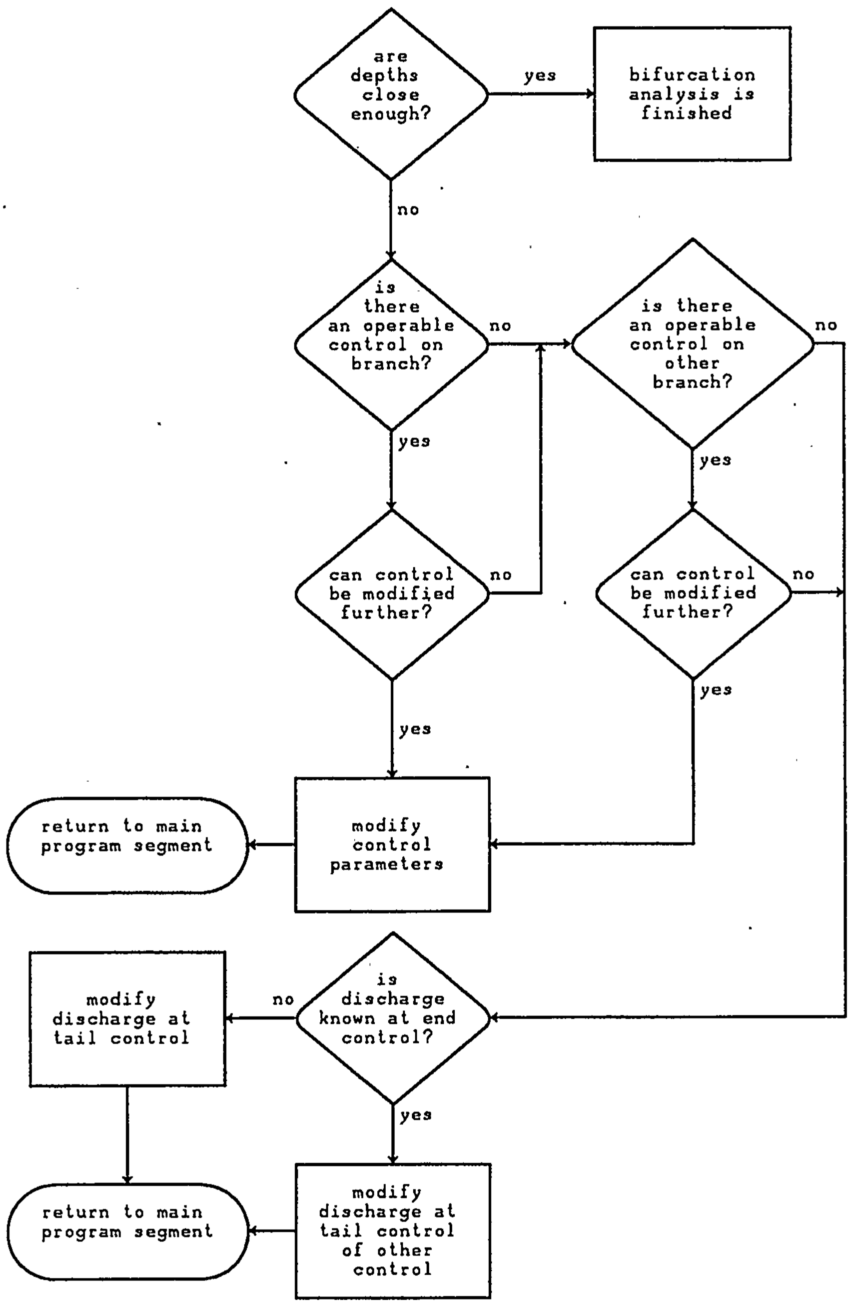

Figure 7.3 Algorithm For Bifurcations

Since the system as considered does not have a solution, at least one system parameter must be modified. An algorithm for modifying bifurcations of this type is illustrated in Figure 7.3. If only one of the downstream discharges is known, the unknown downstream discharge is adjusted. If both downstream discharges are known, the structure parameters of an operable control structure are modified. Then the flow profile throughout the modified branch must be calculated again. This provides a different upstream depth and energy level at the bifurcation. By repeatedly modifying the downstream discharge or control structure parameters, the solution (compatibility of energy levels at the bifurcation) is obtained. Computations then proceed upstream from the bifurcation. Acceleration of the convergence to the solution is achieved through use of a shooting method algorithm (Cheney and Kincaid, 1985) based on previous structure settings or downstream discharges.

7.1.5 Type II Bifurcations

A special class of bifurcation exists in which the downstream depths and discharges are all unknown, and all the control structures in the branches downstream of the bifurcation are either fixed or have specified depths if they are operable. If the upstream discharge is known (for example, as the discharge released into an irrigation system from a headwater control structure), the flow distribution at the bifurcation may still be obtained. If the upstream discharge is not known, too little information has been provided to permit solution of the problem. Bifurcations satisfying these requirements are designated as Type II bifurcations. The analysis required for this class of bifurcation is different than that required for any others. All other bifurcations are referred to as "Type I Bifurcations".

The solution algorithm for Type II bifurcations is presented in Figure 7.4. Solution begins by assuming discharges for branch 1 and branch 2 and determining the water surface profiles up to the bifurcation. If the resulting energy levels at the bifurcation are not compatible, then the discharge in one branch (say branch 2) can be adjusted (repeatedly, if necessary) to obtain compatibility. Once the energy levels are close, then the water surface profile is developed for the channel upstream. If the computed discharge at the top—that is, at control 1—of branch 3 corresponds to the stipulated discharge at the head control, Q_{supply}, then the solution has been attained. Otherwise, the discharge in branch 1 is adjusted, and the discharge in branch 2 is modified as described above to again provide compatibility of energy levels at the bifurcation. The flow profile is again computed from the bifurcation along branch 3 to the head of the system, and the computed and stipulated discharges are compared. If the difference in these values is still appreciable, the discharge in branch 1 is again modified and the process continues.

Using this solution method, an interesting side-effect occurs: the computer algorithm tends to "learn" from itself. This is manifested by a reduction in the number of iterations required when adjusting the discharges in branch 2 to generate an energy level compatible with that of branch 1. Assuming that the discharge at the tail of branch 1 is known, an arbitrary guess is made for the discharge at the bottom of branch 2. The discharge estimate may need modification. This typically occurs several times before compatibility of energy levels is achieved at the bifurcation. When the backwater profile is subsequently computed upstream of the bifurcation, it is likely (especially when there are additions to or subtractions from the discharge along the way) that the discharge at the head of the bifurcation does not equal the supply discharge dictated. The discharge in branch 1 is therefore adjusted. However, the discharge estimate for

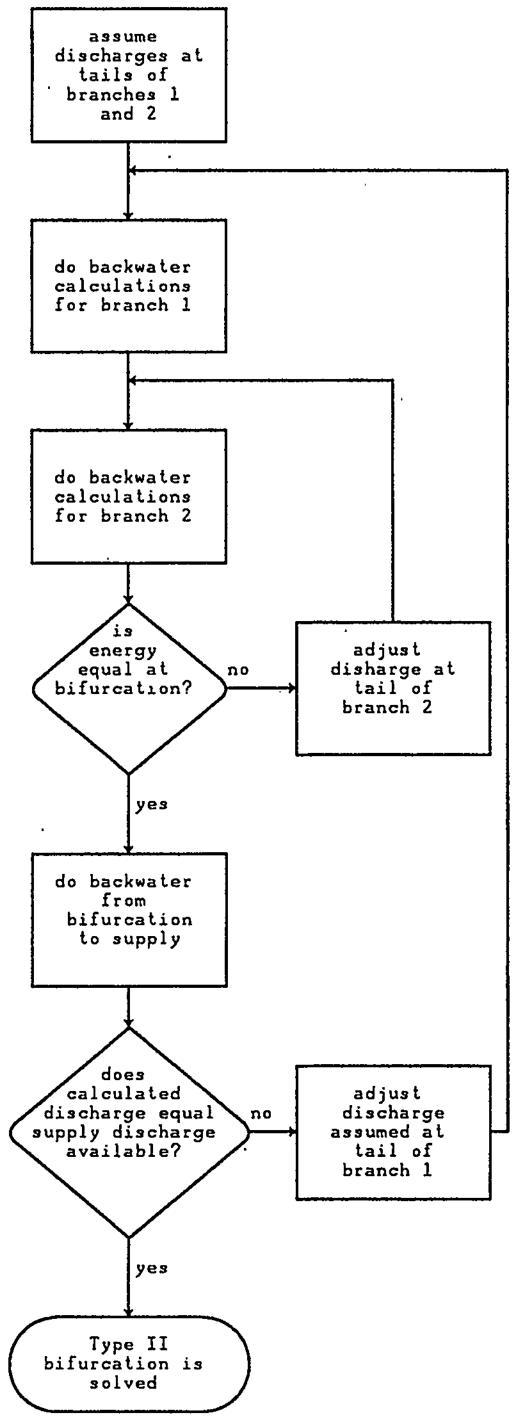

Figure 7.4 Algorithm For Type II Bifurcations

branch 2 retains the value that provided compatibility for the previous discharge in branch 1. This value becomes the first discharge estimate used when seeking compatibility with the new branch 1 discharge. In most cases, the number of adjustments to the discharge in branch 2 required to achieve compatibility is dramatically reduced because of the improvement in this first guess.

7.1.6 Confluences

Confluences occur where two channels join to form one. Common occurrences include the joining of a tributary channel to a main channel or the rejoining of two channels that split upstream to pass an island or other obstruction. At a confluence, control structures may or may not be present, and an energy loss due to the mixing of the flows is likely. However, as previously discussed, the magnitude of such a mixing loss is often unknown and may have to be determined by physically modelling the confluence. Hence, no actual energy loss values are used during program demonstrations, but provisions are made to allow energy losses to be incorporated in one of two ways. Losses can be accounted for by a user-defined number—e.g. 3.2 feet of head loss—or as a percentage of the velocity head of the entering channel.

Once the backwater calculation has been done to the control at the confluence, the discharge, water depth and energy at the confluence are known. From these values, the upstream conditions are determined. The solution technique differs for the distinct cases of loops (discussed in section 7.1.7) and tributary channels.

If the supply discharge is known for the main channel above the confluence (e.g. given from an irrigation headwork, or from stream gauge data on a monitored river) the discharge at the bottom of the tributary channel is simply the difference between the discharge downstream of the confluence and the discharge required to provide the supply discharge upstream. The energy at the bottom of the tributary and main channels equals the energy downstream of the confluence *plus* any stipulated energy losses. From these given energy values, water epths are determined, and the profiles up the tributary and main channels are computed.

When no information exists regarding the main channel discharge above the confluence, the discharge in the tributary channel *must* be stipulated if the proper flow distribution is to be determined. Once the flow distribution has been established, solution follows in a manner similar to above.

7.1.7 Loops

A simple loop in an open channel network can be represented as a bifurcation downstream of which the splitting channels rejoin. The looping channels can have lateral inflows and outflows between the bifurcation and confluence. Additionally, control structures can be present along these channels. A simple loop—as found around islands, in major irrigation projects involving great river diversions, and in return flow situations from municipalities or industrial users (e.g. a large diversion for cooling purposes)—is comprised of only two parallel channels. Compound loop situations, such as occur in some river deltas and around groups of islands, most often do not include controls.

Analysis of a single loop begins after the water surface profile has been determined up to the confluence, providing known discharge, depth and energy conditions. The flow distribution between the two joining channels is approximated. Energy losses are added as required at the downstream end of each channel. Then depths are determined for these end locations. Backwater computations are performed along both channels to the bifurcation. There, the energy levels of the splitting channels are determined. The difference in these values is calculated. If this is appreciable, the previously discussed method of treating bifurcations is used. The flow distribution at the confluence is modified, and the profiles recalculated. The algorithm used for solution of loops is illustrated in Figure 7.5.

7.1.8 Seepage and Precipitation

The most common physical occurrences of distributed lateral inflows and outflows respectively are probably precipitation and seepage. Precipitation can be represented by a number describing the amount of inflow per unit channel length. (If complicated hydrological analysis is required, precipitation can be represented by sophisticated functions and treated similarly to seepage, as described following). This number is then incorporated in the weighted distributed flow term described in section 7.1.2.

In contrast to this simple treatment of precipitation, seepage was examined in more detail. As noted by Kraatz (1977), various empirical formulae are used to determine seepage losses. Many of these seepage equations account for the properties of the channel boundaries, while the results of others depend upon the depth of the water in the channel. This complicates the backwater process.

The water depth at the upstream end of a reach is estimated, allowing the estimated seepage from the channel to be calculated. The flow profile is then determined along the reach, yielding a calculated upstream depth. Based on this value, the actual seepage from the channel can be determined. If the actual seepage and estimated seepage values are not approximately the same, the

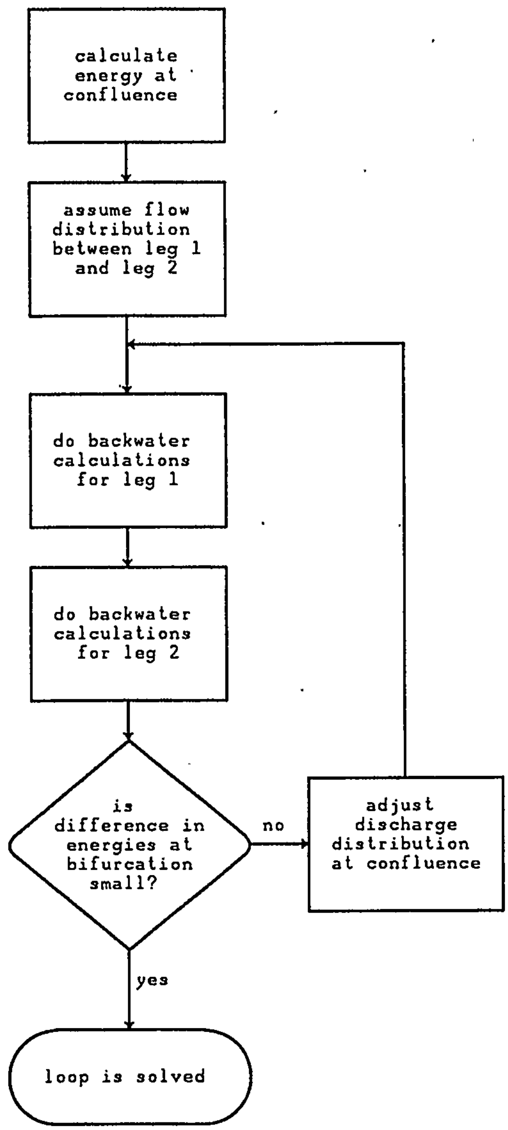

Figure 7.5 Algorithm For Loops

backwater computation for the reach is repeated. The computed upstream water depth is used as the new estimate of the actual value, the profile is again determined and the associated seepage is calculated. This is again compared to the seepage estimate and, if not close enough, this process continues.

The seepage value is simply incorporated into the weighted distributed flow factor. In the program, two types of seepage formulae are included, however, any seepage equation desired can be added. Seepage is calculated as a percentage of the downstream discharge. This primarily allows results from the model to be easily verified. The Moritz equation (Kraatz, 1977) is programmed using both SI and Imperial forms of the equation to demonstrate the simple inclusion of a widely accepted seepage relationship.

7.1.9 Extraordinary Conditions

Situations can occur that cause the solution of the governing equation to fail. Other cases may arise whereby the system's physical limitations are exceeded. These odd situations may be introduced by ill-posed boundary conditions or system parameters, by unusual trial values generated by the solution algorithm, or by a mathematical inability to treat the governing equation under certain situations. Problems that may arise include: discharges exceeding the capacity of channel reaches or control structures; low-flow conditions; and system demands that cannot be satisfied by the system supply.

7.1.9.1 Exceeding System Capacity

Special problems may be introduced during the solution of some network situations. Because the algorithm determines new trial values based on previous structure settings and discharge values, these "next guesses" should be reasonable in most cases. Occasionally, however, the new estimates may present possibilities the system cannot physically accommodate.

For instance, while solving a Type II bifurcation, the new discharge estimates may become so large that one of several alternatives may occur. Firstly, the water depth may exceed the specified bank height of the channel. Yet again, the required discharge may not be able to physically pass through a control structure (such as a gated orifice) depending upon the channel bank heights and structure parameters. Another possibility is that a satisfactory control structure may become submerged by the required discharge. This can render the programmed structure equation useless.

If the required water depth exceeds the canal bank height, the water depth is restricted to the bank height. Solution continues with the water depth taken at this value. Because imposing a ceiling on the allowable depth does not truly reflect the flow conditions that may result (actually, the discharge would overflow the channel banks, and some sort of side-spillway structure would probably form), the computer program provides a message describing the situation and the treatment performed on the system. When a satisfactory control structure becomes submerged, the program again supplies a message. But, instead of continuing the solution, the simulation terminates immediately because the results obtained may be grievously incorrect.

7.1.9.2 Low-Flow Conditions

Special situations may when the discharge (or flow depth) approaches zero. Zero discharge may develop during simulation of a reach in which seepage equals the downstream discharge. Zero depth may develop along reaches in which a distributed lateral inflow contributes most of the available discharge. This situation can also arise if the solution algorithm generates an unreasonable discharge estimate. The water in the channel may then "run out", or intercept the channel bottom at some point along the reach. As a consequence, a portion of channel will be "dry".

This dry channel condition cannot be easily modelled because there is no discharge in a portion of the reach. This in turn impacts the control immediately upstream—zero discharge below a control implies zero discharge through it, and consequently zero discharge upstream of it. Because of this problem, when a zero discharge or zero depth condition is encountered, solution of the system is aborted and a message is provided. This allows specified conditions to be changed before the model is run again.

7.1.9.3 Water Shortages

In some situations it is possible to "run out" of water before the head of the system is reached. This could occur in channels with lateral outflows or where distributary channels draw a substantial amount of water. Consider the solution progression of the network segment represented in Figure 7.6. Water surface profiles are calculated for the several large distributary channels fed by the lateral channel. It is possible that the total discharge required to supply these distributaries will exceed the maximum amount of water that can physically pass through the control structure at the upstream end of the lateral channel. As a consequence, it would appear that the demand of the distributary nearest the upstream end of the lateral channel (and possibly the next one as well) would not be satisfied. This situation cannot physically occur as described, since water *will* be available to the most upstream distributaries first; however, there will certainly be a delivery shortfall somewhere in this portion of the system.

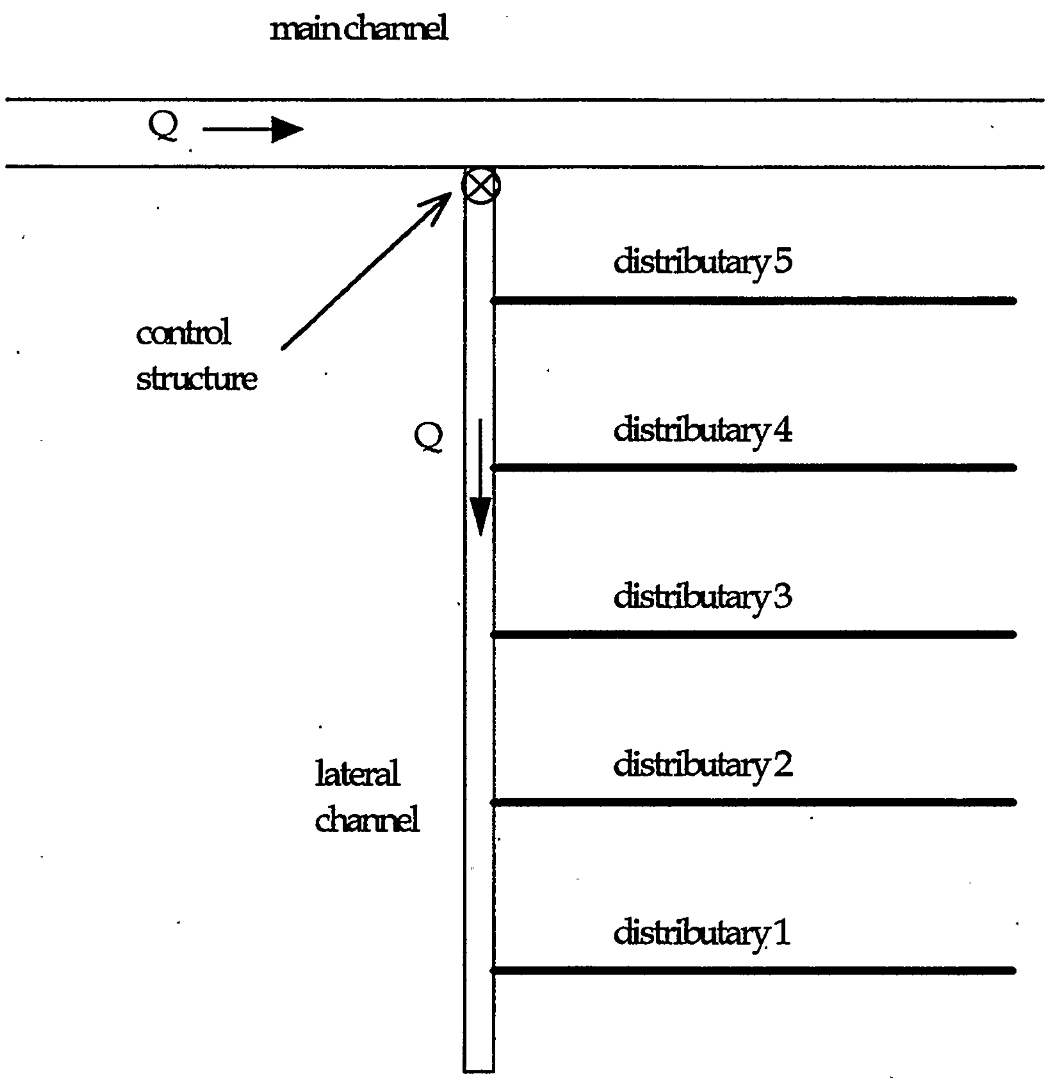

Figure 7.6 Portion Of An Irrigation Network

If such a problem develops during solution, it is generally manifested in one of two ways. The water depth may exceed the bank height of the channel. This would occur as the system attempts to provide a maximum head to "push" as much water through the structure as possible. An alternate symptom of this problem is that the program will set the control structure parameters (if the structure is operable) to allow the maximum amount of water through.

A similar problem can occur even if the control structure at the upper end of the lateral channel does have the capacity to pass the discharge required by its downstream deliveries. The main channel may not have enough available discharge to meet the requirements, either because of a limited headwater discharge, or because of other large deliveries taken from it elsewhere.

Regardless of the problem origin—a lack of structure capacity stemming from physical parameters, or a *bona fide* water shortage in the system—treatment of the situation is identical. It is evident that the deliveries cannot be satisfied as intended. Decisions must be made as to which, if any, deliveries are to be fully satisfied, and which ones have secondary importance. In short, a delivery schedule must be prepared for the system. Appropriate schedules will vary considerably according to the specific system and the management and operational objectives of the user.

This treatise is not concerned with the management objectives of networks. Therefore, no effort has been expended to develop sophisticated delivery schedules. Instead, in systems with water shortages the computer algorithm prorates all deliveries. Deliveries are reduced to ninety-five percent of their original values, and the system is analysed again. If water shortages are still present, the deliveries are factored down by the same ratio, and the solution process is repeated. Other sophisticated or specialised techniques may be added to the scheduling routine to satisfy the specific goals and needs of individual users.

7.2 CONSTRAINT RULES

Given a general network situation such as portrayed in Figure 7.7, a question arises: how much information is required to "solve" (determine discharges and depths throughout) the system? A second related question is: what form must the information take? To answer these questions a new theory based upon network characteristics is needed. In response to this, the Constraint Rules are developed. These rules are based upon the operation of control structures and solution criteria for bifurcations.

7.2.1 Problem Situations

Given an operable control structure, one must ask *why* it is allowed to operate. There are two logical reasons: there may be a requirement for a specified upstream depth in the vicinity of the control itself; or, the control may be required to provide compatibility of energy levels at some location in the system. If the objective is to provide a stipulated depth at the control, then the control *cannot* be used to adjust the system to achieve global compatibility conditions. The structure can be regarded as operable along the channel, but from the perspective of solving part of a network, the structure is perceived as fixed—it cannot be used to influence the interaction between the channel it is on and the other channels in the network. In other words, the control will be *operable* from an internal, or *local*, perspective whereas it will be *fixed* from an

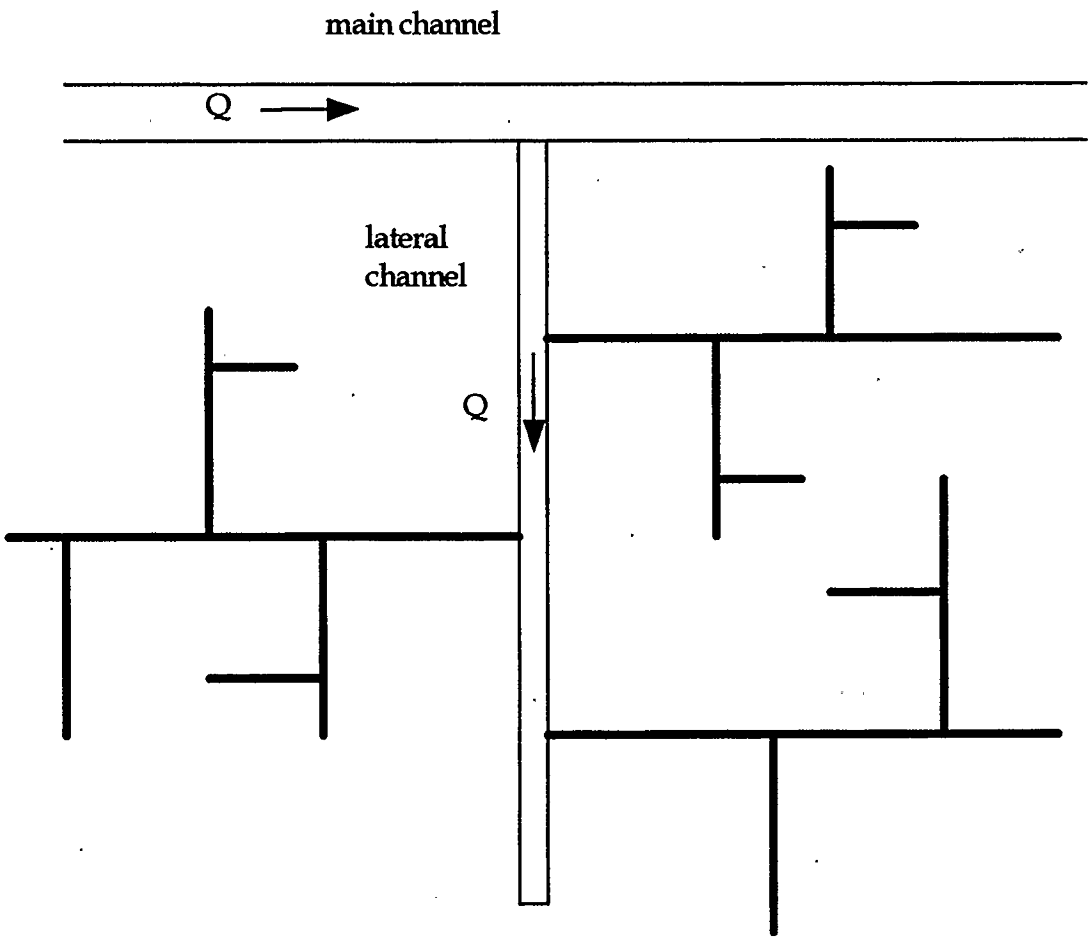

Figure 7.7 General Irrigation Network

external, or *global,* viewpoint. A globally operable control can be operated to modify the upstream energy level in an attenpt to obtain compatibility at an upstream bifurcation. For the control to influence the hydraulic behaviour (exhibited by changes in the energy value) of the upstream bifurcation, it follows that no satisfactory control structures can exist between a globally operable control and the upstream bifurcation. Several dilemmas can be introduced by the presence of globally operable control structures. These include the cases described following.

Case 1

Interesting situations can arise along a channel that has operable control structures. Provided that the channel discharge is sufficient and that channel banks are high enough, solution is relatively straightforward when all the controls are locally operable. If one or more is globally operable, however, then a unique solution does not exist. This is because a different solution exists for each different structure setting. The problem is exacerbated if more than one globally operable control is present because each structure can be set independently. By varying the relative settings of all the globally operable structures along a channel it may be possible to realise the same upstream conditions with different permutations of structure settings.

Case 2

The potential for more than one solution occurs again if a globally operable structure is present along either of the legs of a loop. Consider a loop in which half of the discharge is allocated to each channel. If the energies at the tops of the two legs are unequal, adjustment of a globally operable control on one leg may permit compatibility of energy to be realised. Now, if the discharge distribution is changed slightly, so that forty-nine percent is transmitted by one channel while the other conveys fifty-one percent, it is very likely that the same control structure could be adjusted to allow compatibility at the bifurcation again. If this is possible, then *at least two* possible solutions exist for the system.

Case 3

Multiple solutions are possible if an ordinary bifurcation is being solved. This can occur if there is a globally operable structure on a branch for which the downstream discharge has been specified. By performing the backwater computation to the top of the branch, an energy level is determined. If some discharge in the other channel will allow compatibility of energy levels at the bifurcation, then a solution exists. However, a different setting of the operable control can create a different water elevation upstream of the control. This depth will be transmitted to the top of the channel, giving a different energy value. To obtain compatibility, a different discharge will be required in the joining channel. Therefore, a set of solutions may be generated. If both the discharge in the other channel and the control structure setting can vary, a solution may be found but it will not be unique.

Using the example of the single channel with three branch flows shown in Figure 7.8, the necessary requirements for solution of bifurcations can be explored. Imagine that only one downstream discharge is provided. If this discharge is supplied at node 1, backwater computations are performed along the channel from 1 to 2. Then a discharge is assumed at node 6 and the corresponding water surface profile is determined up to node 2. Compatibility is checked at node 2 and the branch discharge at node 6 is solved for as described in section 7.1.4. Once compatibility is achieved, a discharge is

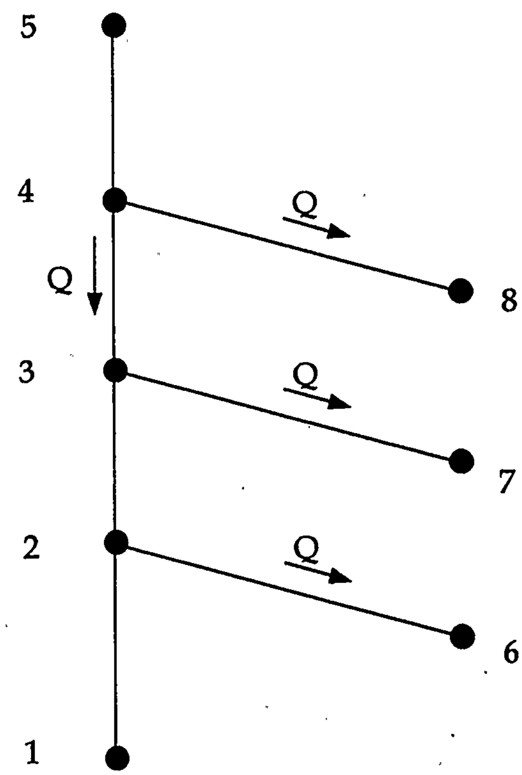

Figure 7.8 Series of Bifurcations

assumed at node 7, and this branch is solved similarly. Finally, a discharge is assumed at node 8 and the process is repeated. If compatibility cannot be achieved at any of the nodes, no solution exists for the situation as posed. If the discharge is instead known at node 6, solution proceeds by assuming a discharge at node 1 and working as described above. However, if different conditions are originally stipulated, several new problems can result as explained in cases 4 through 8.

Case 4

If the only information provided is the discharge at node 7, no solution can be obtained for the system. The water surface profile can only be computed up to the main channel at node 3. Continuing the solution below this node requires additional information.

Case 5

If the information provided is instead the demand discharge at node 8, solution can again progress to the main channel. Again, a difficulty is encountered because of a shortage of information. This prevents solution downstream of the bifurcation at node 4.

Case 6

Additional situations can arise when more than one end discharge is stipulated. If, for instance, discharges are given at nodes 1 and 6, profiles can be determined for both branches. However, a dilemma comes to light: the energy levels at node 2 are most likely (almost guaranteed) not equivalent. Without a means of altering an energy value, no solution is possible.

Case 7

On the other hand, discharge may be specified at both node 7 and node 8. Again, water surface profiles can be determined up to the main channel. However, if no more information is provided, the flow cannot be accurately routed in the remaining portions of the system. This is obvious, because no data is given regarding the flow distribution present at node 2.

Case 8

Branch discharges may be provided at one of the most downstream nodes and one other node (node 1 and node 7, for example). Again, water surface profiles can be determined up to the main channel. However, in a situation such as this, the problem encountered in case 6 will arise: at one bifurcation in the system, two different energy levels will exist. There will be a requirement to modify one of the energy levels. Because the downstream discharges yielding these values are both stipulated, the system does not have any capacity to do this.

7.2.2 Development Of The Constraint Rules

Equations available for solution of a system include control structure relationships and continuity and energy relations for channels. Inequality or limiting relations such as channel bank heights and maximum permissible discharges at certain locations are also often available. For this work, all the available equations and inequality relations for a network are considered "constraints" for the system. Cases 1, 2, and 3 presented in section 7.2.1 arise when the number of variables in the system exceed the number of equations available. Too few constraints have been imposed upon the system to permit a unique solution to be determined. Such "under-constrained" systems can generally still be solved, however the solution generated is only one of a set of possible solutions. These systems are described mathematically as having parametric solutions. A sequence of solution "curves" exist, one of which may be selected by stipulating all of the parameters (variables free to take a variety of values) but one. This potential for multiple solutions indicates that when a globally operable control structure is present in a single channel, a parametric solution exists. That is, the system solution depends not only upon the stipulated downstream discharge, but also upon the setting of the control structure.

The situations described in cases 4 and 5 are dictated by the adequacy of the number of constraints supplied to the system. A method of analysing these and other situations is required so that it can be known in general whether a solution is or is not possible. Testing the sufficiency of the constraints hinges upon observing that a portion of the overall system must be solved first. If the constraints supplied are sufficient to facilitate solution of this "core" segment of the system, then solution of the larger system *may* be possible. If the core segment cannot be solved, continuation along the larger system cannot possibly occur.

The specific situation presented in case 4 is considered to develop a solution strategy. The system equations and unknowns for this system are examined and compared to those for the straightforward case in which the discharge is stipulated at node 1.

If discharge is specified at node 1, the core segment is simply the bifurcation between nodes 1, 2 and 6. There are ten unknowns in this system: the depth and discharge at node 1; depth and discharge at node 6; depth and discharge at node 2; depth and discharge at the top of the channel between nodes 1 and 2 (immediately downstream of node 2); and depth and discharge at the top of the channel between nodes 6 and 2 (again, immediately downstream of node 2). There are ten corresponding equations: two compatibility relationships between the channels joining at the bifurcation; continuity at node 2; specified discharge and depth at node 1; a discharge-depth relationship at node 6; and energy and continuity relationships along the bifurcation branches (two equations each). The number of constraints supplied is adequate to determine the unknown discharge for the other branch of the bifurcation. Hence, the system is properly constrained.

By supplying known discharge conditions only at node 7 (case 4), the core segment expands from the previous to include the bifurcation between nodes 2, 3, and 7. The number of variables in the system is increased by eight: two each at nodes 3 and 7; and two more at the tops of the channels between nodes 7 and 3 and between nodes 2 and 3. Additional equations are available in the form of: an additional depth-discharge relationship at control 1; energy and continuity relations along the branch between nodes 7 and 3; energy and continuity relations along the branch between nodes 2 and 3; and two compatibility relations at node 3. The number of equations available is seventeen, one less than the number of variables.

Clearly, the system solution is parametric. One of the multiple solutions can be obtained by selecting values for the discharge at nodes 1 or 6. This yields a situation like case 8 in section 7.2.1. However, the possibility of specifying the available supply at node 3 is also presented. This option is often attractive from a practical viewpoint. In irrigation systems, it is likely that the magnitude of a headwater discharge will be known with more certainty than will a farm level discharge. Therefore, using the supply discharge at the upstream location may better reflect the conditions presented in actual use. If the available supply discharge is known at node 3, an additional equation is provided, giving a total of eighteen equations. The core system can therefore be solved.

Situations have been presented in cases 6, 7 and 8 in which two branch discharges have been stipulated. In two cases, too much information has been presented, while in the other too little has been given. Different analysis is required for these situations.

The system becomes "over-constrained" in case 6 where the lower two discharges are known. This is evident if the number of unknowns and equations available are examined. As before, the number of unknowns for the core segment is ten. The available equations are as before, with the addition of another discharge at the bottom of the system. This provides eleven equations in total, proving that too much information has been given. A possible solution to this problem is to introduce another variable.

A second example with more than one stipulated branch discharge, as presented in case 7, can have conditions given at node 7 and node 8. Here, there is an inability to solve the system along the main channel because the system is "under-constrained". This is due to the large number of variables—twenty-six—introduced by the larger core segment. Unfortunately, there is no way to solve this situation without adding more equations.

If a system is encountered where only one of the two bottom discharges is provided, while the other one is provided further upstream (case 8), the solution strategy is the same as for a system having both bottom discharges stipulated. Another variable must be introduced to avoid the possibility of parametric solutions.

Control structures can be used as sources of additional variables. If a globally operable control structure is added to the system, another unknown quantity—the structure setting—is included in the analysis. Physically, adding a control such as this allows energy levels to be adjusted at a bifurcation. Of course, the structure must be placed appropriately: it must be in a position where it can impact conditions at the relevant bifurcation. To do so, it *must* be on one of the branches for which the downstream discharge is known. It must also have direct access to the bifurcation—that is, the control must conform to the definition of a globally operable control.

Equations can be added to under-constrained systems in several forms. Downstream discharges can be specified for additional branches, or flow distribution equations can be provided for bifurcations. Unfortunately, when more downstream discharges are introduced into a system, it is possible to create a situation similar to case 6 or case 8. This in turn will have to be dealt with. A drawback of using specified flow distribution equations at bifurcations is that these functions are not generally known with confidence unless hydraulic models of the actual channels have been studied. This is a time-consuming and often expensive process. Because of the difficulties involved in adding equations to a situation such as case 7, it is often advisable to use different downstream discharges if at all possible.

An exception occurs when the upstream discharge is known and only Type II bifurcations exist below. The lower Type II bifurcations can be separated from the upper portion of the system. This eliminates channels and associated unknowns from the core segment to be analysed. Therefore, the number of variables will be reduced. By specifying the upstream discharge, another equation is also included. Hence, it is possible to obtain a properly constrained system.

Observations from the above cases can be extended to suit systems where more than two branch discharges are specified. For every end discharge specified in excess of the original one, another variable must be introduced. This generally corresponds to adding a globally operable control to each corresponding branch. Of course, every new stipulated discharge that must be accommodated this way removes flexibility from the system. As more and more globally operable controls are added to a system, the individual controls may eventually not be able to physically pass the required discharges or to generate the water levels necessary for compatibility. When under-constrained systems are encountered, Type II systems should be checked for in the lower portions of the network. If they are present, they can be broken off from the rest of the network and solved separately.

From the perspective of solving larger, more complex networks, new observations are required. A numbering system is required for channel systems. The following general definitions are made using the channel network in Figure 7.9 for illustrative purposes. Given the system, there is one channel of order "one". This is the channel that does not split off from any other channel. Obviously, this corresponds to the channel labelled "main channel" in Figure 7.9. Channels of order "two" are fed by the channel of order "one". With respect to the diagram, there is one channel of order "two" that splits from the main channel and goes toward the bottom of the page. Channels of order "two" feed channels of order "three". In the diagram, there are three order "three" channels. Order "four" channels receive discharge from order "three" channels. This continues as necessary to assign an order to every channel in the system. The system in Figure 7.9 is of order "five". The end of a channel is of the same order as the rest of the channel. Therefore, a channel of order "three" will have a channel of order "three" and a channel of order "four" at its most downstream bifurcation. If a channel of order "n" feeds a simple loop, the loop legs and the

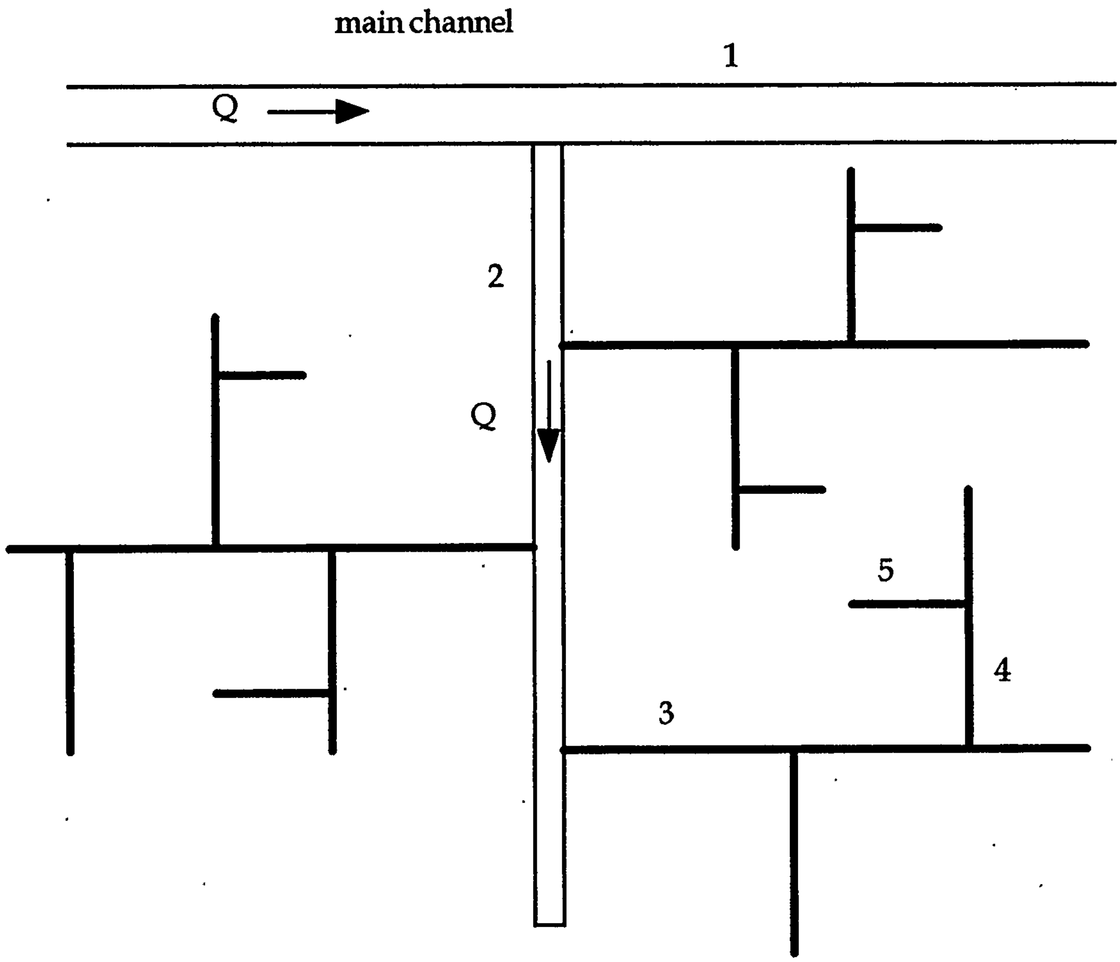

Figure 7.9 Sketch Defining Channel Order

channel downstream of the loop are of order "n" as well. If a tributary channel joins a channel of order "n", the tributary is of order "n+1".

The limitations and requirements described above are observed for all channels that supply branch channels. For example, if the three order "two" channels depicted in Figure 7.8 distribute flow to smaller sub-networks, known discharges are available where these channels join order "one" channel. Looking at this system, there are at least three stipulated discharges (four, if the discharge is known from node 1 as well). Because this presents an over-constrained system, more unknowns are needed. Globally operable controls can be included along the main channel or in the individual subsystems in locations where they can change the energy levels at the relevant bifurcations. These controls are additional to the controls required for each sub-network. This is necessary because the operable controls in the individual sub-networks are needed to force compatibility at a lower stage in the system. Obviously, one control cannot be required to perform two separate tasks.

Based upon the system order, the number of bifurcations it contains and the number and locations of its specified downstream discharges and globally operable control structures, it is possible to determine whether a system can be uniquely solved. The criteria for a single solution existing for a system of order "n" are:

1. For every channel with a bifurcation, the discharge *must* be specified at either the extreme end of the channel *or* from the branch of the most downstream bifurcation. This is the minimum solution requirement for a system, and can only be violated:

- when there are only Type II bifurcations below this location and the supply discharge is known in the channel directly upstream of the Type II bifurcation;
- if "n" is equal to one—discharge must be provided at the downstream end of the channel; or
- when tributary channels join the network, the discharge must be specified at the upstream end of the tributary *or* the supply discharge must be provided for the main channel.

2. Every additional specified branch discharge *must* be accompanied by a globally operable control structure on the same branch. The control *must* have hydraulic access to the upstream bifurcation (no satisfactory controls can be between it and the bifurcation).
3. When a channel, including its downstream end, supplies "x" separate channels that have specified branch discharges (or downstream of which, branch discharges are specified) "x-1" globally operable controls *must* be included.

These three criteria are denoted as the "Constraint Rules". If these guidelines are employed, distinct solutions will be attainable. As noted earlier, a simulation program will still generate answers when a system has a parametric solution. Such cases may result if globally operable controls in addition to the ones required are included. The solutions obtained are correct but not unique. Cases such as this may be encountered if existing systems with too many operable controls are simulated.

7.2.3 Applications Of The Constraint Rules

The problems presented in section 7.2.1 are revisited in light of the concepts and requirements developed in section 7.2.2. Difficulties encountered in solution of these situations are explained from the viewpoint of the Constraint Rules and remedies are provided as possible.

Case 1

Single channels that have globally operable controls violate the Constraint Rules because there is no reason to include the controls. By including these types of controls, parametric solutions result. Multiple solutions can be eliminated by adding constraints to the system or by removing variables. There is little new information to include in function form other than distinct values for structure settings. To provide a distinct solution, the structure parameters of all globally operable controls should be stipulated, effectively "fixing" the controls.

Case 2

The problem introduced by globally operable control structures in loop situations also violates the Constraint Rules. In a situation such as this, there is no useful purpose for globally operable control structures, hence they should not be included. Again, to solve this problem, an equation must be added to the system. This can come in the form of an explicitly stated structure setting. An alternate equation that can be provided is the actual flow distribution between the channels. Of course, if this value is known, there is little need to simulate the system behaviour.

Case 3

Having a globally operable control on the branch of a bifurcation for which the downstream discharge is stipulated also goes against the Constraint Rules. Again, there is no purpose for the globally operable control structure. This problem can be resolved in one of two manners. Firstly, as for cases 1 and 2, a specific value can be given for the control structure setting, providing an additional equation to the system. The other solution comes through employing the second Constraint Rule. The discharge at the bottom of the other leg of the bifurcation can be stipulated. If this is done, a globally operable control is required along one of the two legs. Obviously, the control is already present. In either case, the number of unknowns and system constraints will be equal, allowing a solution to be determined.

Case 4

By providing only the discharge at node 7, the first of the Constraint Rules is violated in that the minimum requirements for solution of the system are not satisfied. Since there is a lack of information, extra equations must be supplied. The additional data can come in the form of a stipulated downstream discharge at node 1 or node 6, or as a supply discharge at node 3.

Assuming that the discharge is provided at node 1 or node 6, solution is straightforward to node 2. At node 3, however, the potential exists for incompatible energy levels. Therefore, a globally operable control must be added to one of the relevant branches. This follows from principle 2 of the Constraint Rules. Solution thereafter is simple.

If this upstream discharge is known, the profile can determined down to the lower bifurcation at node 2. Solution of the remaining discharges, 1 and 2, can be achieved only if there are no freely operable controls on either branch --

that is, the bifurcation must be of Type II. Since the branch discharges are not specified for nodes 1 and 6, and because there are no globally operable controls present in the system, the Type II requirements are met. This is clearly a demonstration of the first exception listed in the first Constraint Rule.

Case 5

In this situation, the branch discharge at node 8 is the only information provided. This case also violates the first of the Constraint Rules. To generate another equation, the supply discharge at node 4 can be provided. This in turn allows the discharge at node 3 to be determined. Since there are no operable structures along the channel between nodes 2 and 3 or along the channel between nodes 7 and 3, then this section of the system can be treated as a Type II bifurcation. This bifurcation must be split from the lower portion of the network and solved to generate discharge and depth values at nodes 2 and 7. The discharge at node 2 determined this way is then used as a supply discharge for the final portion of the network, the bifurcation between nodes 6, 2, and 1. If either of the lower bifurcations do not satisfy the requirements of Type II bifurcations—that is, if there are operable controls along any of these branches—then the solution process must stop.

Solution can also occur if the discharge at either node 1 or node 6 is stipulated. Because of principle two of the Constraint Rules, a globally operable control structure must also be included along the branch upstream of node 8 or upstream of the additional specified discharge.

Case 6

In this case, the downstream discharges are provided at the two most downstream nodes in the system. Because this is all that is known, this arrangement opposes the second Constraint Rule (it may also be interpreted as a violation of rule three in that two channels with specified discharges are supplied while no globally operable control is included). Solution can be enabled in one of two ways: as per the second rule, a globally operable control can be added along either of the two branches; or one of the specified discharges can be allowed to vary, creating the simplest form of system. In either case, the net effect is the same—an additional variable is added to the system. This will yield a properly constrained system.

Case 7

In this situation, branch discharges are specified at nodes 7 and 8. This is a direct violation of all three Constraint Rules. Rule one is violated in that the minimum requirements for solution are not provided. Rules two and three are broken because there are two specified branch discharges, yet no globally operable control is present. Despite these violations, the system may be solved if: a branch discharge is supplied at node 1 or node 6, and two globally operable controls are added at appropriate locations; or if the upstream discharge is provided at node 3, allowing the lower portion of the system to be solved as a Type II bifurcation. In this latter case, a globally operable control must also be provided to allow compatibility to occur in the upper portion of the system where two discharges are specified.

Case 8

This situation is similar to case 6. The second and third Constraint Rules are violated because two branch discharges are stipulated without having a globally operable structure in the system. A globally operable control can be added to one of the branches that has a stipulated bottom discharge. This supplies the "missing" equation the system requires, enabling solution to occur.

7.2.4 Implications For Iterative And Simultaneous Solution Techniques

The Constraint Rules have implications upon the manner in which systems may be solved. If a system is properly constrained, then solution is straightforward whether iterative or simultaneous methods are used.

For under-constrained systems, a set of parametric solutions should be realised. Using an iterative technique, a system such as this can be easily solved. During a pass through the system, values can be assumed for the "extra" variables. If a matrix solution is used, a similar result can be obtained. The matrix solution can generate results if specific values are assigned to the extra variables. This creates a system with equal numbers of equations and variables, that can treated by matrix arithmetic. Under each circumstance—simultaneous or iterative solution—the solution obtained is one of a number of possible solutions. It is a solution for the remaining variables in terms of the ones for which values have been assumed.

A system in which the number of equations exceeds the number of variables is over-constrained. This corresponds, for example, to a situation in which a maximum upstream discharge is specified in addition to a demand discharge. An iterative solution can be facilitated by "ignoring" the extra equation(s) for a time, solving the system, then comparing the results to the additional constraint(s). If required, values of the original constraints can be revised, the solution recalculated and the "over-ruling" constraint(s) compared again. For simultaneous solution it is also necessary to disregard the extra equation(s) at first. This is required because the number of variables and unknowns should be equal to permit straightforward solution of the coefficient matrix. When a solution has been generated using the matrix, the results can be compared to the left-over constraint(s). If this equation(s) are not satisfied, the other equations can be modified and the solution process repeated.

In cases of under-constrained or over-constrained systems, both simultaneous and iterative solution techniques seem appropriate. However, adjustments to some equations may be required if any "extra" equations are considered. The algorithms to determine these adjustments may not be suitable for incorporation into matrix form. Rather than mix simultaneous and iterative techniques in this manner, the decision was made to use an entirely iterative approach in the preparation of solution algorithms. Hence, the algorithms presented in this section use iterative solution methods.

7.2.5 Operational Strategies

Methods of handling over-constrained and under-constrained situations must be considered for application to actual situations. Unique solutions may actually be possible in under-constrained cases, while over-constrained cases may also be solvable by examining physical system limitations.

In under-constrained systems, additional equations are required to obtain a solution. For instance, there may be a requirement that the water depth cannot fall below a specified level at a certain location. Thus, an additional equation is imposed upon the system. This in effect selects the appropriate solution from the family of available solutions. Other equations can be added by requiring maximum water levels or stipulating that structures be set at specific or "optimal" settings as defined by the user.

Over-constrained systems result when the number of equations exceeds the number of variables. In an irrigation network, for example, analysis of the system may be possible by first disregarding some of the equations. This may be appropriate in situations wherein depths are limited by channel bank heights. Suppose that by omitting the inequality equations describing these limitations, a system with equal numbers of variables and equations results. Solution of this system is straightforward. The solution obtained can then be compared to the previously neglected equations to check whether they too are satisfied. Supply restrictions and various other "inequality" equations can be treated in this manner.

7.2.6 Summary Of The Constraint Rules

In addition to information about the geometry and physical properties of the channels in a particular system, data must be provided for a minimum number of boundary conditions and control structures. Although these items vary from application to application, generally they fall into one of two categories. A known discharge is invariably required somewhere throughout a network. By stipulating the discharge at a downstream end of a system, the corresponding water depth is automatically determined using the appropriate control structure equation. Desired, or target, depth settings must also be specified for each operable control unless the control is to be globally operable.

The Constraint Rules have been developed to establish sufficient requirements needed to determine whether a unique solution exists for a given system. The rules are summarised in section 7.2.2.

At a minimum, discharge must be provided for one branch of the system. For every globally operable control introduced, another constraint must be added to the system if a unique solution is to be attained. The constraints can be in the form of specified depths at controls, specified branch discharges, or information about flow distribution at bifurcations (this in turn usually implies a branch discharge). If additional constraints are not included, a parametric set of solutions results. Solution of these systems is generally possible, but the result obtained is not *the* solution to the system—it is merely one of a set of possible solutions. Other solutions can be obtained when any of the globally operable structures has a different structure setting.

If more than one branch discharge is stipulated for the distributaries of a single channel, a system can become over-constrained. Solution will not be possible because too many equations exist. A remedy for this is to supply additional unknowns in the form of globally operable controls on every branch with a specified discharge in excess of the minimum requirements. Alternately, the "extra" specified conditions can be removed from the system.

Locations within the network of stipulated branch discharges are also important. This results because solution begins with a "core" segment. Solution of this core is performed first. It is possible to solve a system if the core includes only one bifurcation. Solution is also possible when multiple bifurcations are included in the core system if the lower bifurcations are of Type II *and* the supply discharge is known at the upper bifurcation. Solution may also be possible if more than one branch discharge is provided. One of the lowermost discharges must be supplied, and globally operable controls must be included to obtain compatibility. Situations other than these do not appear solvable by the methods explored.

Users of the solution algorithm must be aware of the possibility of parametric solutions. The algorithm does not stop when too many globally operable controls are encountered (although this feature could be incorporated). Solution of these cases is still attempted. This may be important during preliminary design stages of projects to see if certain conditions are possible with a given system.

An attempt to work through the problem of parametric solutions is made by requiring that only the control nearest the bifurcation be operated. Sometimes a control may be adjusted to its operational limits without achieving compatibility at the bifurcation. In this case, the control is considered as becoming "fixed", then the next closest globally operated control is adjusted. Admittedly, if a solution is obtained in this manner, it is not unique. It merely shows that a possible solution does exist.

Because of the physical realities presented by actual systems, it may be possible to solve systems that at first inspection appear "unsolvable". Solution may be possible through introducing additional equations or by temporarily "removing" extra equations from the analysis. The equations to remove or add may include those that describe limits of the operational capabilities of the system. In particular, this includes ones that limit or stipulate certain values for discharge, water depth or structure settings.

7.3 GENERAL NETWORK SITUATIONS

Network situations may be broadly classified according to their origin and primary use. Three major categories include: irrigation systems incorporating operable control structures; irrigation systems with fixed controls; and natural systems containing simple or complex looping situations.

Irrigation systems as used in much of North America and Europe are typically composed of a series of interconnected bifurcations, replete with operable control structures. Assuming that the systems are properly constrained as per the requirements of the Constraint Rules presented in section 7.2.2, solution is straightforward, beginning at specified downstream discharges (delivery quantities) and working upstream to the system headwork.

Large irrigation systems in India fed by major river diversions (Asawa, 1993) are prime examples of systems that include Type II bifurcations as well. Operable structures are often present in the upper canals of the system, while the majority of bifurcations and channels further downstream are fixed. Solution of these networks requires that the Type II portions be disconnected from the remainder of the system. The operable portion is solved to satisfy the deliveries and headwork discharges specified. Then the discharges at the downstream ends of this system can be used as upstream, or supply, discharges for the Type II portions below. When Type II bifurcations are "stacked" one above each other, each Type II unit must be separated and solved on its own as discussed in section 7.2.3. Solution progresses downstream from the most upstream bifurcations, using the downstream discharges from each as the supply discharges for the lower bifurcations. Minimum solution requirements and solution strategies for these situations also fall under the Constraint Rules developed in section 7.2.2.

In some rivers and irrigation networks, flow occurs in parallel channels and/or loops. Simple loops are automatically treated as ordinary network components by the programmed algorithm. Their solution is dictated by the Constraint Rules presented in section 7.2.2. In particular, the restrictions regarding globally operable controls are of interest.

If loops are complex, with interconnected channels, the system is almost certainly natural. The likelihood of an operable control structure within a system like this is very remote. Because algorithms exist for the solution of this class of problem (e.g. Schulte, 1985), it is recommended that looped network portions such as this be detached from the remainder of a network. The individual portions should be solved, then pieced together with the appropriate discharge and depth conditions. If fixed control structures are present in these loops, Schulte's algorithm will need revision. Structure equations can be prepared as F functions, included in the solution matrix and solved. Operable control structures in these loops will most likely generate parametric solutions. If more than one operable control is included, inversion of the coefficient matrix may indicate that a solution has not been achieved. Obviously, in such a case, values must be assigned to the extra variables. Unfortunately, no information will be provided by the solution matrix as to which control requires adjustment, so external decisions may be required.

7.4 SUMMARY OF SOLUTION ALGORITHM

The solution algorithms, requirements and restrictions presented above have been coded into a computer simulation model to demonstrate their behaviour. A computer program was written using the C programming language to illustrate applications of the algorithms. The governing equation as presented in equation 7.1 is solved for each channel reach throughout the network.

The Newton-Raphson method is used because many existing steady state hydraulic models are based upon this numerical technique. It is recognised that the *potential* exists for the model to unexpectedly diverge because this technique is used. Iterative solution techniques are used because they seem more straightforward for application to over-constrained systems.

Numerical errors introduced during the solution of the equations are lessened by use of the guideline presented by McBean and Perkins (1975) for determining the distance between computational nodes along a reach. After all computations are done, the program checks if this guideline has been violated. If so, warnings are issued so that the user may go through the system again. Additionally, care has been taken to minimise the amount of program reconfiguration required to simulate different networks with the program. Only the sizes of primary arrays must be altered to simulate larger networks (provided of course that the machine memory is sufficient).

Junctions are treated by comparing energy levels among the common branches. This treatment allows energy losses to be introduced and accounted for as appropriate. Channels that split from the network and do not rejoin are analysed given the flow conditions—i.e., depth and discharge—at their downstream ends. This has been discussed extensively in the treatments of bifurcations. Rectangular and trapezoidal cross sections are programmed; however, the governing equation uses general geometric terms, allowing other shapes to be easily incorporated. A wide range of structures can be simulated by representing them by structure equations. This permits irrigation networks and other sophisticated open channel systems to be modelled.

8 RESULTS

Testing of the simulation model can be broadly classified as falling into one of two areas: verification of results provided by previous researchers; and investigation of novel situations which could not be simulated by other algorithms.

8.1 MODEL VERIFICATION

Due to its novel approach, the algorithm cannot be fully verified by any single existing model or combination of models. However, several of the program's individual components have been tested against standard results or hand calculations performed by the author. These components include the backwater flow routing algorithm applied to individual channel reaches, the behaviour of the fixed and operable control structures currently programmed, the effects of seepage, and the analysis of various bifurcations, confluences and simple loops.

8.1.1 Backwater Computations For A Single Reach

The basic flow routing algorithm was tested for two different channel cross-sections in straight channel reaches. For a wide channel of rectangular cross section, the model was run and channel depths were recorded at several distances upstream from the stipulated downstream conditions. These results are presented in Table 8.1, along with comparative numbers from Example 5.2 of Henderson (1966). The model was also used to test a trapezoidal cross-section of 20 ft bottom width and side slopes of 1.5:1 (horizontal to vertical). These results were compared to those from Example 5.1 of Henderson (1966) and are presented in Table 8.2. Henderson computed both profiles using a direct step method. Henderson's results have been used for comparison since that work is one of primary reference texts for hydraulic engineers.

In both cases the model results compare favourably to the expected values. For the rectangular channel, deviations between the model results and Henderson's values range from 0.02 percent to 0.08 percent, as shown in Table 8.1. These differences are insignificant. Very close correlation is seen over the eight channel reaches shown in Table 8.2. Again, differences in water levels are negligible, ranging from zero difference to just over one percent difference. Since any errors that are introduced in backwater computations can be magnified with distance, this close correlation indicates that the flow routing algorithm used in the model produces valid results.

8.1.2 Control Structures And Variations Of Channel Properties

Several types of control structures have been programmed into the simulation model. Each permits the flow to be routed through it according to the structure properties and parameters. In addition to the control structures that the author regarded as being most common, or necessary—check structure, gated orifice, operable check-drop, operable orifice—two controls used by Swain (1988) were also programmed. By including these controls, a Parshall flume and

Table 8.1 Flow Routing Verification In A Rectangular Channel

wide channel, Q= 40 cfs, bottom slope = 0.001, n = 0.025, y_o= 10 ft

distance from end	depth(ft), Henderson	depth(ft), model	percent difference
0	10.00	10.00	n/a
770	9.375	9.377	0.02
1590	8.750	8.757	0.08
2510	8.125	8.129	0.05

Table 8.2 Flow Routing Verification In A Trapezoidal Channel

bottom width 20 ft, side slope 1.5:1 (horizontal : vertical)

Q= 1000 cfs, bottom slope = 0.001, n = 0.025, y_o= 10 ft

distance from end	depth(ft), Henderson	depth(ft), model	percent difference
0	3.85	3.85	n/a
17	4.30	4.347	1.09
54	4.60	4.630	0.65
120	4.90	4.918	0.37
230	5.20	5.211	0.21
407	5.50	5.506	0.11
641	5.75	5.753	0.05
1011	6.00	6.000	0.00
1231	6.11	6.098	0.20

a sharp-crested weir, the basic operation of the routine in charge of control structures could be examined.

Included in Swain's (1988) work was a steady state example—Verification 2—of a channel including these two controls. The bottom slope of the channel varied between reaches along the channel, as did the Manning roughness coefficient. By simulating this example it was ensured that energy levels and discharges were transmitted properly through the controls along a linear channel. Also, the model's capability to handle changes in slope and channel roughness between nodes was demonstrated. The physical properties of the channel simulated are summarised in Table 8.3, while results from the model and from Swain's testing are presented in Table 8.4.

Water levels computed by the model agree closely with Swain's results. The maximum difference is 0.43 percent. This close correspondence illustrates the model's capability to route flows through linear channels with: variable bottom slopes; variable Manning *n* values; submerged controls; and unsubmerged controls. Additionally, because the program was developed to see a system as a series of "nodes", distinct upstream and downstream depths can exist at the same location. This eliminates the need of placing two nodes to describe a control structure, as Swain did.

The structure equations used by Swain (1988) are rather exotic, and the author felt the utility of the Parshall flume was low for the examples considered. Other controls needed to be developed, programmed and tested. The most common, and therefore most useful controls, in irrigation systems are the gated orifice and the check-drop structure.

The check drop used is described by the following equation (Smith, 1985),

Table 8.3 Physical Data For Channel With Control Structures, Varying Bottom Slope, and Variation of Manning Roughness (From Swain, 1988)

bottom width 10 ft, Q= 172 cfs, y_o= 10 ft

node, model	reach length, ft	Manning Roughness, n	elevation (ft)
0	1300	0.013	13.0
1	1400	0.013	15.0
2	n/a	0.013	17.0
2 (upstream side)	5971	n/a	17.0
3	6155	0.013	19.0
4	n/a	0.013	21.0
4 (upstream side)	4500	n/a	27.0
5	5571	0.0135	28.0
6	n/a	n/a	30.0

Table 8.4 Verification Of Flow Routing in The Channel Described in Table 8.3

node, model	depth (ft), Swain	depth (ft), model	percent difference
0	2.38	2.38	n/a
1	2.82	2.832	0.43
2	2.70	2.700	0.00
2 (upstream side)	4.16	4.156	0.10
3	4.54	4.543	0.07
4	4.61	4.625	0.33
4 (upstream side)	4.94	4.943	0.06
5	5.13	5.136	0.12
6	4.77	4.766	0.08

$$Q = CbH^{3/2} \tag{8.1}$$

where:

Q = discharge through the structure;

C = discharge coefficient for the particular structure;

b = channel width; and

H = total head above the structure crest.

$$Q = CA\sqrt{2gH} \tag{8.2}$$

The gated orifice uses the equation (Brater and King, 1976) in which the variables are as above except:

A = the area of the gate opening; and

H = the difference in head across the gate opening.

The fixed check-drop structure was tested using the lower part of Swain's channel previously described. It was placed at node 2 where a Parshall flume had been. Since the water depth for the channel was known *a priori* at the downstream side of the flume, it was a simple matter to validate the operation of the new control. Hand calculations predicted that, based on discharge of 172 cfs, drop height of 6.0 ft., and a structure coefficient of 1.837, the depth upstream of the check-drop structure would be 4.18 ft. When the check drop was included in the simulation program, the depth obtained upstream of it was 4.187 ft. The difference between these values is 0.7 percent.

Similar hand calculations were done for the fixed gated orifice. This structure was tested in a rectangular channel with bottom width of 20 ft. From a previous program run, the depth at the downstream side of the structure was known to be 5.229 ft. when a discharge of 500 cfs was in the channel. Given that the orifice was 15 ft. wide, that its opening height was 5.0 ft., and that it had a structure coefficient of 0.90, an upstream depth of 6.081 ft. was predicted. The program result was an upstream depth of 6.081 ft. Obviously, this correlation could not be better.

Results for both controls were extremely good. The velocity head was included in the determination of total head for the check structure, necessitating an iterative approach to determine the upstream depths. It is possible the hand calculations may not have been performed through enough iterations to provide the required accuracy. This may account for the differences (although admittedly small) in water depths observed.

8.1.3 Operable Control Structures

Two forms of operable controls are included in the program: an operable check-drop and an operable gated orifice. The orifice uses the same equation as the fixed orifice structure, however, the hydraulics of the operable check-drop are related to the height of the weir itself. Since this is not included in equation 8.1, a different equation is needed for operable checks. The equation used (Smith, 1985) is:

$$Q=\frac{2}{3}\sqrt{2g}\left[0.605+\frac{0.001}{h}+\frac{0.08h}{p}\right]bh^{3/2} \qquad 8.3$$

In this equation

p = weir height; and

h = difference between upstream water depth and weir height.

The operation of this control was tested in the same rectangular channel used to validate the fixed orifice structure. A water depth of 8.3 ft. was desired

The operation of this control was tested in the same rectangular channel used to validate the fixed orifice structure. A water depth of 8.3 ft. was desired upstream of the check structure. The weir height was allowed to vary between 3.0 ft. and 10.0 ft. in 0.1 ft. increments. Initially, the weir height was set at 5.0 ft. For a given discharge of 500 cfs and 5.229 ft. depth downstream of the check, hand calculations indicated that the upstream depth would be obtained when the weir height was in the interval between 4.7 ft. and 4.8 ft. When the program was run, it generated a solution in which the structure setting (weir height) was 4.7 ft. This in turn created an upstream water of depth of 8.360 ft. This water depth is slightly above the requested value due to the discrete structure settings allowed, as previously discussed in section 7.1.2. Although the result does not meet the depth specification exactly, it is a superior outcome because it better reflects the operations of a real control structure. This said, the operation of the adjustable check drop is considered validated.

The operable gated orifice was tested in the same channel under the same discharge conditions. Again, the water depth downstream of the control was 5.229 ft. The orifice width was 15.0 ft. and the structure coefficient was 0.90. The orifice height was allowed to range from 0.0 ft.—that is, from the channel bed—to 10.0 ft. in 0.01 ft. increments. The desired upstream depth was 10.0 ft. and initially the orifice opening was 3.0 ft. Hand calculations indicated that an orifice height of 2.11 ft. would generate an upstream water depth slightly in excess of 10.0 ft. The model also computed a required orifice height of 2.11 ft., with an associated upstream water depth of 10.013 ft. In light of the discussion in the previous paragraph, this too demonstrates the ability of the program to handle operable control structures satisfactorily.

8.1.4 Distributed Lateral Flows

Distributed lateral outflows were tested by simulating precipitation into and seepage from a series of channels. The precipitation case considered is straightforward ,while the seepage effects are more complex. However, as mentioned in section 7.1.8, more sophisticated treatments of precipitation can be included in the model.

Both precipitation and seepage were tested in a rectangular channel composed of three reaches each 1000 ft. long. The channel had a bottom slope of 0.001, Manning roughness value of 0.025, bottom width of 20.0 ft., and downstream discharge of 500 cfs. The downstream boundary condition was a fixed check structure that created a known depth condition of 4.097 ft.

Precipitation was considered uniform over the entire surface area of the channel. One-twentieth of an inch of precipitation was assumed to enter the system at a steady rate over this area. This flow addition contributed 0.083333 cubic feet of water per foot of channel (equivalently, 83.333 cubic feet for each 1000 ft. reach length). Analysis indicated that discharge would drop by 83.888 cfs for each reach as the system was solved. The model provided discharges of 416.670 cfs, 333.340 cfs, and 250.010 cfs at the upper ends of the reaches, progressing from downstream to upstream. These values agree well with the expected results.

Two forms of calculating seepage are provided in the program, and others can be added in the seepage subroutine. Seepage can be calculated as a percentage of the downstream discharge. This is useful primarily because of the ease of verifying this. The Moritz equation (Kraatz, 1977) is included to illustrate treatment of a more accurate and widely accepted form of seepage loss.

In the most downstream reach of the test channel, the seepage loss was ten percent of the discharge at the bottom of the channel. In the next reach upstream, the Moritz equation was used with a soil coefficient of 0.66 (sandy loam), while in the upper reach a soil coefficient of 1.68 (sandy soil with rock) was used with the Moritz equation. Hand calculations predicted a discharge of 550 cfs at the top of the first reach, a discharge of 550.262 cfs at the top of the second reach and a discharge of 550.946 cfs at the upper end of the channel. Calculations for the upper two reaches were made using crude estimates of the anticipated average depths over the respective reaches. The model generated discharges of 550 cfs, 550.263 cfs and 550.947 cfs at these locations respectively.

Obviously, these results correlate well. The differences between predicted and calculated values are negligible, reflecting the capability of the model to simulate seepage effects.

8.1.5 Summary of Program Verification

The flow routing component of the program has been validated through comparative testing against established results. Rectangular and trapezoidal channel sections have been verified, as has the program's ability to route flow through satisfactory and unsatisfactory control structures. Performance of fixed and operable control structures has been highlighted. Distributed lateral inflows and outflows have been modelled by the program.

8.2 DEMONSTRATIONS

Demonstrations of the program were conducted to illustrate the features of the developed algorithm. It was desired to show the model's ability to treat bifurcations, confluences/loops, and situations wherein system demands exceed supply capability. The systems analysed are all properly constrained as defined by the Constraint Rules developed in section 7.2.2.

8.2.1 Bifurcations

Several bifurcations were analysed using the model. The bifurcations were variants of the situation illustrated in Figure 8.1. In this system, the lower channels are rectangular with bottom widths of 20 ft. and have Manning roughness values of 0.025 and bottom slopes of 0.001. The upper channel is rectangular, having a bottom width of 40.0 ft, Manning roughness value of 0.025 and bottom slope of 0.001. The distance between nodes 0 and 1 is 1000 ft., as is the distance between nodes 5 and 2 and nodes 3 and 4. Fixed check-drop structures with drop heights of 6.0 ft. are present at nodes 0, 1, 3, and 4. Two cases of Type II bifurcation were tested, then several ordinary bifurcations were examined to illustrate the solution alternatives programmed.

A Type II bifurcation was tested with an upstream discharge of 1000 cfs specified at node 2. Because the branch channels are identical, an equal flow distribution is expected. The program results are tabulated in Table 8.5. It is evident that the program routed the flow correctly in this situation. The three energy values at the bifurcation — that is, for node 5 and the upstream sides of nodes 1 and 4 — are equivalent. These energy values are total energies, hence

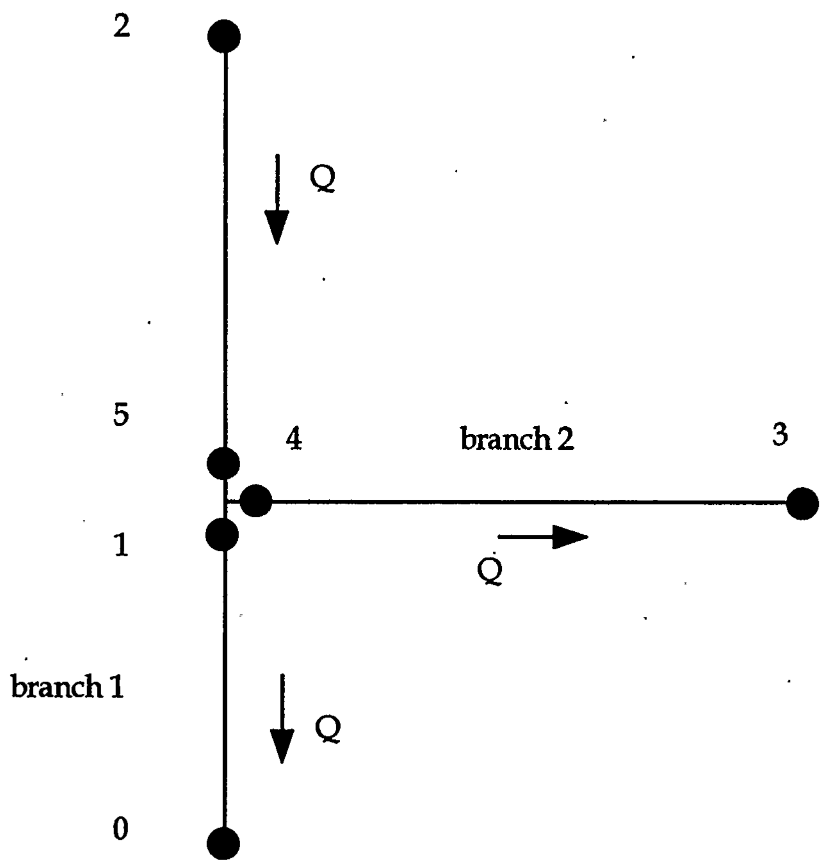

Figure 8.1 General Bifurcation

Table 8.5 Modelling Results of Type II Bifurcation

Manning roughness 0.025 for branch 1 and branch 2

location	depth (ft)	discharge (cfs)	energy/weight (ft)*
0	4.096771	500	4.675015
1 (downstream)	5.228692	500	6.583675
1 (upstream)	4.096771	500	11.675015
2	4.899931	1000	13.304148
3	4.096771	500	4.675015
4 (downstream)	5.228692	500	6.583675
4 (upstream)	4.096771	500	11.675015
5	4.096771	1000	11.675015

Table 8.6 Modelling Results of Modified Type II Bifurcation

Manning roughness: 0.025 for branch 1; 0.015 for branch 2

location	depth (ft)	discharge (cfs)	energy/weight (ft)*
0	3.819435	424.639679	4.299276
1 (downstream)	4.724633	424.639923	6.038221
1 (upstream)	4.724633	424.639923	12.038221
2	4.949797	1000.000610	13.345911
3	4.343539	575.360718	5.024696
4 (downstream)	4.363183	575.360718	6.038220
4 (upstream)	4.363183	575.360718	12.038220
5	4.574432	1000.000610	12.038221

*derived result: energy per unit weight is sum of elevation, velocity head and water depth;

variations in channel bed height could easily be accommodated at the bifurcation. Continuity is also satisfied through the bifurcation.

Another Type II bifurcation was tested, using the same parameters as before with one exception. The Manning roughness in branch 2 was changed to a value of 0.015. By reducing the flow resistance in this branch, it was expected that a larger proportion of the flow would be carried toward node 3, which was in fact observed, as shown in Table 8.6. Again, it should be noted that conservation of energy is maintained at the bifurcation. Continuity is also upheld at that location, and the discharge obtained at node 2 satisfies the stipulated available supply discharge.

The results of a third test on a Type II bifurcation are presented in Table 8.7. In this scenario, the branch channels had different roughness values as in the second example. However, there was also seepage from all three of the channels. In the previous two examples, only the portion of the algorithm responsible for allocating the discharge between the two branch channels was put to task. By incorporating seepage, it was possible to tell if the solution algorithm actually computed the "head" or supply discharge correctly. Seepage was taken as ten percent of the discharge at the bottom of each channel. The results indicate that the algorithm does treat all portions of Type II bifurcations correctly: the stipulated supply discharge was provided while maintaining continuity and energy compatibility at the bifurcation. The effect of the difference in Manning roughness values is still exhibited by the difference between the discharges in the two branch channels.

Ordinary bifurcations were tested to demonstrate the types of problems that the algorithm treats. Again, these situations are based upon the original bifurcation presented in Figure 8.1. For the first example, the downstream

Table 8.7 Modelling Results of Type II Bifurcation Including Seepage

Manning roughness: 0.025 for branch 1; 0.015 for branch 2;
seepage loss of 10.0 percent of downstream discharge along all branches

location	depth (ft)	discharge (cfs)	energy/weight (ft)*
0	3.663091	386.177582	4.094543
1 (downstream)	4.456379	386.177429	5.747895
1 (upstream)	4.456379	386.177429	11.747895
2	4.77069	999.999634	13.197103
3	4.17466	522.913147	4.783735
4 (downstream)	4.123658	522.913147	5.747892
4 (upstream)	4.123658	522.913147	11.747892
5	4.317654	909.090576	11.747895

*derived result: energy per unit weight is sum of elevation, velocity head and water depth

discharge is stipulated as 500 cfs at node 0. The controls along both the lower branches are identical and all fixed. Therefore, it is expected that the discharge at the downstream end of the other branch—that is, at node 3—will be adjusted to provide compatibility of energy levels at the bifurcation. This would require the node 3 discharge to become set to 500 cfs as well. The model results, as evidenced in Table 8.8, provide the expected outcome. Discharge at node 3 is 500 cfs (within the tolerance of the program), and the nodes at the bifurcation, namely 5, and the upstream sides of nodes 1 and 4, all have the same energy value.

In the second example, the discharge was stipulated at the downstream ends of both branch channels. At node 0, the discharge was again 500 cfs, while at node 3 the discharge was 400 cfs. A globally operable control structure was included at node 4 at the bifurcation. This control was a check-drop structure having a weir height that could be adjusted in 0.1 ft. increments from a minimum of 1.0 ft. to a maximum of 10.0 ft. This structure had a drop height of 6.0 ft. and initially had a weir setting of 5.0 ft. It was anticipated that the algorithm would adjust the control to allow the smaller discharge to create an energy condition at the bifurcation equivalent to that created by the larger discharge. Results from the model are presented in Table 8.9. The model required that the weir be set at a height of 1.2 ft. to achieve the system solution. Energy compatibility and continuity conditions are satisfied by the model output.

The final bifurcation demonstration has a stipulated downstream discharge of 200 cfs at node 0 ,while at node 3 the discharge requirement is 400 cfs. A globally operable control structure is again present, as dictated by the Constraint Rules. However, this time it is at a downstream channel end—at node 0, in particular. This operable control has the same physical capabilities

Table 8.8 Modelling Results of Bifurcation

discharge specified as 500 cfs at location 0

location	depth (ft)	discharge (cfs)	energy/weight (ft)*
0	4.096771	500	4.675015
1 (downstream)	5.228692	500	6.583675
1 (upstream)	4.096771	500	11.675015
2	4.89993	999.999634	13.304147
3	4.09677	499.999664	4.675013
4 (downstream)	5.228689	499.999664	6.583672
4 (upstream)	4.09677	499.999664	11.675013
5	4.096772	999.999634	11.675015

Table 8.9 Modelling Results of Bifurcation With Operable Control At Bifurcation

specified discharge of 500 cfs at location 0, 400 cfs at location 3; weir height = 1.2 ft.

location	depth (ft)	discharge (cfs)	energy/weight (ft)*
0	4.096771	500	4.675015
1 (downstream)	5.228692	500	6.583675
1 (upstream)	4.096771	500	11.675015
2	4.632064	900	12.998443
3	3.720577	400	4.169274
4 (downstream)	4.553786	400	5.853308
4 (upstream)	4.346200	400	11.675017
5	4.237160	900	11.675014

*derived result: energy per unit weight is sum of elevation, velocity head and water depth

and limitations as the check-drop in the previous example. In order to achieve this, the satisfactory controls at node 1 had to be removed from the system, as it prevented the operable check from having "hydraulic access" to the bifurcation. The weir at node 4 was also removed (although this was not *necessary*). Accordingly, it was again expected that the control would be used by the algorithm to allow compatibility to occur at the bifurcation. The downstream discharges should not be changed, and solution (if attainable given the system parameters) should occur merely by operating the control. Depths and discharges generated by the model are summarised in Table 8.10. This table clearly shows that the downstream branch discharges remained unaltered, while compatibility of energy was attained at the bifurcation. The weir was set to a height of 2.4 ft. to achieve the solution.

These three examples with Type I bifurcations illustrate the capability of the algorithm to solve situations in which any of a wide number of possible data combinations are provided. In fact, the algorithm and programmed model can solve any of the potential situations that may be put forth from the set of "properly constrained" problems as defined in the Constraint Rules.

8.2.2 Loops

Three loop situations were modelled using the program. These were simple loop configurations, without complex interconnected geometries. More complex examples were not presented because, as previously mentioned, their treatment has been documented elsewhere (e.g. Schulte, 1985).

Of the cases considered, the first is a straightforward test of the algorithm's ability to model a basic loop. Figure 8.2 is a schematic of the system tested. Two parallel channels flow between nodes 1 and 4. Both of the channels

Table 8.10 Modelling Results of Bifurcation With Operable Control Structure At Bottom of System

specified discharge of 200 cfs at location 0, 400 cfs at location 3; weir height = 2.3 ft.

location	depth (ft)	discharge (cfs)	energy/weight (ft)*
0	5.592620	200	5.642266
1 (downstream)	4.785503	200	5.853308
1 (upstream)	4.785503	200	11.853308
2	4.236118	600	12.430816
3	3.720577	400	4.169274
4 (downstream)	4.553786	400	5.853308
4 (upstream)	4.553786	400	11.853308
5	4.694796	600	11.853308

*derived result: energy per unit weight is sum of elevation, velocity head and water depth

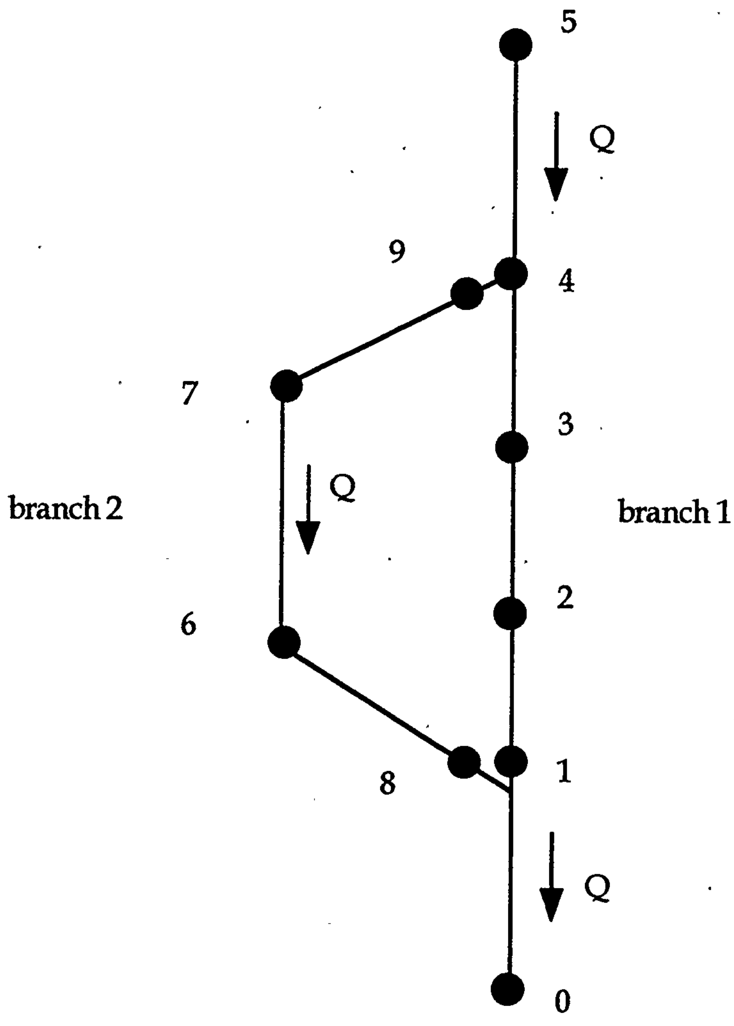

Figure 8.2 Loop Configuration

follow a slope of 0.001, and both are rectangular in section with bottom widths of 20.0 ft. The Manning roughness values for the channels are also the same, being 0.025 in each case. The channel sections upstream and downstream of the loop have the same roughness and slope. They are rectangular and both have bottom widths of 40.0 ft. A discharge of 500 cfs is introduced at the downstream end of the system (at node 0).

Because the channels are alike, an equal flow distribution is expected. Energy levels should also be the same for the nodes situated at the confluence and for the nodes at the bifurcation. In fact, the program results show these exact things. Inspection of the output in Table 8.11 shows that the discharge is split equally between the two parallel channels. Furthermore, energy equality is satisfied amongst the nodes at the confluence and amongst the nodes at the bifurcation.

The second application of loop solution capabilities considers the effects of varying the channel roughness along one of the parallel paths. The Manning n value is changed to 0.020 along branch 2 while the n value remains at 0.025 along branch 1. Reduction of flow resistance along the former channel should lead to increased discharge along that route. Continuity and energy compatibility should still be maintained throughout the system. Program output as summarised in Table 8.12 clearly shows that the flow distribution has changed, being "drawn" to the less resistant channel. Compatibility and continuity remained satisfied at the top and bottom of the loop.

The final demonstration of loop situations had the same channel roughness coefficients as in the second case. In addition, a locally operable gated orifice was placed at location 6. Target depth upstream of the control was specified at 5.5 ft. The height of the orifice opening was allowed to vary between 0.0 ft. and 10.0 ft. in 0.05 ft. increments, while the width of the opening

Table 8.11 Modelling Results of Loop

discharge specified as 500 cfs at location 0

location	depth (ft)	discharge (cfs)	energy/weight (ft)*
0	4.509118	500	4.628448
1 (downstream)	4.614848	500	5.728773
1 (upstream)	4.614848	250	5.728773
2	3.865517	250	6.027892
3	3.295053	250	6.518518
4 (downstream)	2.963344	250	7.239637
4 (upstream)	2.963344	500	7.239637
5	4.191400	500	9.329507
6	3.865517	250	6.027892
7	3.295053	250	6.518518
8	4.614848	250	5.728773
9	2.963344	250	7.239637

*derived result: energy per unit weight is sum of elevation, velocity head and water depth

Table 8.12 Modelling Results of Loop With Different Roughness Values Along Each Branch

Manning roughness: 0.025 along branch 1; 0.020 along branch 2
discharge specified as 500 cfs at location 0

location	depth (ft)	discharge (cfs)	energy/weight (ft)*
0	4.509118	500	4.628448
1 (downstream)	4.614848	500	5.728773
1 (upstream)	4.634101	228.849426	5.728773
2	3.843146	228.849426	5.980797
3	3.2167	228.849426	6.413186
4 (downstream)	2.833827	228.849426	7.086994
4 (upstream)	2.771017	500	7.086994
5	4.219929	500	9.356175
6	3.763836	271.150574	5.965308
7	3.093402	271.150574	6.391667
8	4.593508	271.150574	5.728773
9	2.693624	271.150574	7.086994

*derived result: energy per unit weight is sum of elevation, velocity head and water depth

was 10.0 ft. Initially, the orifice opening was 5.0 ft. high. Again, continuity was expected to be maintained at the upper and lower ends of the node. Energy compatibility was also expected at these locations. Because the orifice introduces additional flow resistance along branch 2, it was anticipated that discharge in the branch would be less than in the second example. Results from the model, as summarised in Table 8.13, indicate that theconditions would occur as expected. The energy values at location 8 and the upstream and downstream sides of location 1 are nearly identical. Similarly, energy is conserved between location 9 and the upstream and downstream sides of location 4. The discharge in branch 2 is reduced to approximately 178.8 cfs from the 271.2 cfs obtained in the previous example. Hence, the resistance effects of the orifice are demonstrated. Additionally, the water depth at the upstream side of the orifice, 5.53 ft., is very close to the stipulated depth of 5.5 ft. As before, this discrepancy is explained by the finite structure increments used.

Several other demonstrations of loops could be made. These include situations wherein fixed satisfactory and unsatisfactory controls are placed along one or both of the parallel channels. Locally operable controls could also be placed along these paths. These cases are all simple applications of the model, requiring only the basic flow routing algorithm used for single channels. In fact, it makes no difference to the algorithm which individual control is substituted for the orifice in the last demonstration (unless it is globally operable, of course). Therefore, there is little utility in demonstrating other "tried-and-true" elements in loop situations.

Table 8.13 Modelling Results of Loop With Locally Operable Control Structure

Manning roughness: 0.025 along branch 1; 0.020 along branch 2
discharge specified as 500 cfs at location 0; locally operable gated orifice at location 6, desired upstream depth 5.5 ft., final orifice opening 1.85 ft.

location	depth (ft)	discharge (cfs)	energy/weight (ft)*
0	4.509118	500	4.628448
1 (downstream)	4.614848	500	5.728773
1 (upstream)	4.533998	321.159271	5.728773
2	3.95704	321.159271	6.212754
3	3.580946	321.159271	6.893194
4 (downstream)	3.394508	321.159271	7.741997
4 (upstream)	3.549412	500	7.741997
5	4.252719	500	9.386872
6	5.533462	178.840729	7.574012
7	4.573382	178.840729	7.632744
8	4.67188	178.840729	5.728766
9	3.648736	178.840729	7.741997

*derived result: energy per unit weight is sum of elevation, velocity head and water depth

8.2.3 Excessive Demands

As discussed in section 7.1.8.2, it is possible for the system demands to exceed the available system supply. These situations are demonstrated using the system displayed in Figure 8.3. The channel geometries and physical data are summarised in Table 8.14. The branch discharge at location 1 is specified as 300 cfs. For a unique solution to exist, the Constraint Rules dictate that no globally operable controls can be in this system. However, fixed controls can be included: a gated orifice is at location 7. The orifice has a structure coefficient of 0.90, with an opening 5.0 ft. high and 30.0 ft. wide.

In the first case considered, no available supply discharge is stipulated. This provides a "base-level" case to compare against. The discharges and water depths produced by the model are presented in Table 8.15. As before, continuity is maintained and energy compatibility achieved at all bifurcations, including the one at the control—that is, at location 7.

To illustrate the "scheduling" capabilities of the algorithm, a finite limit was imposed upon the available discharge at location 9. This corresponds to a maximum available discharge such as may be presented at the headwork of an irrigation system, for example. Given the available supply of 1800 cfs, the downstream branch discharges clearly cannot all be satisfied. The model should reduce the stipulated discharges (in this case, the only one stipulated is at location 1), and recalculate the flow profiles throughout the system. In turn, this should reduce the required discharges throughout the entire system.

Output from the model is presented in Table 8.16. The model reduced the stipulated discharge at location 1 to 244.35 cfs. This allowed the discharge at location 9 to be less than the maximum permissible. Since the "schedule" used is

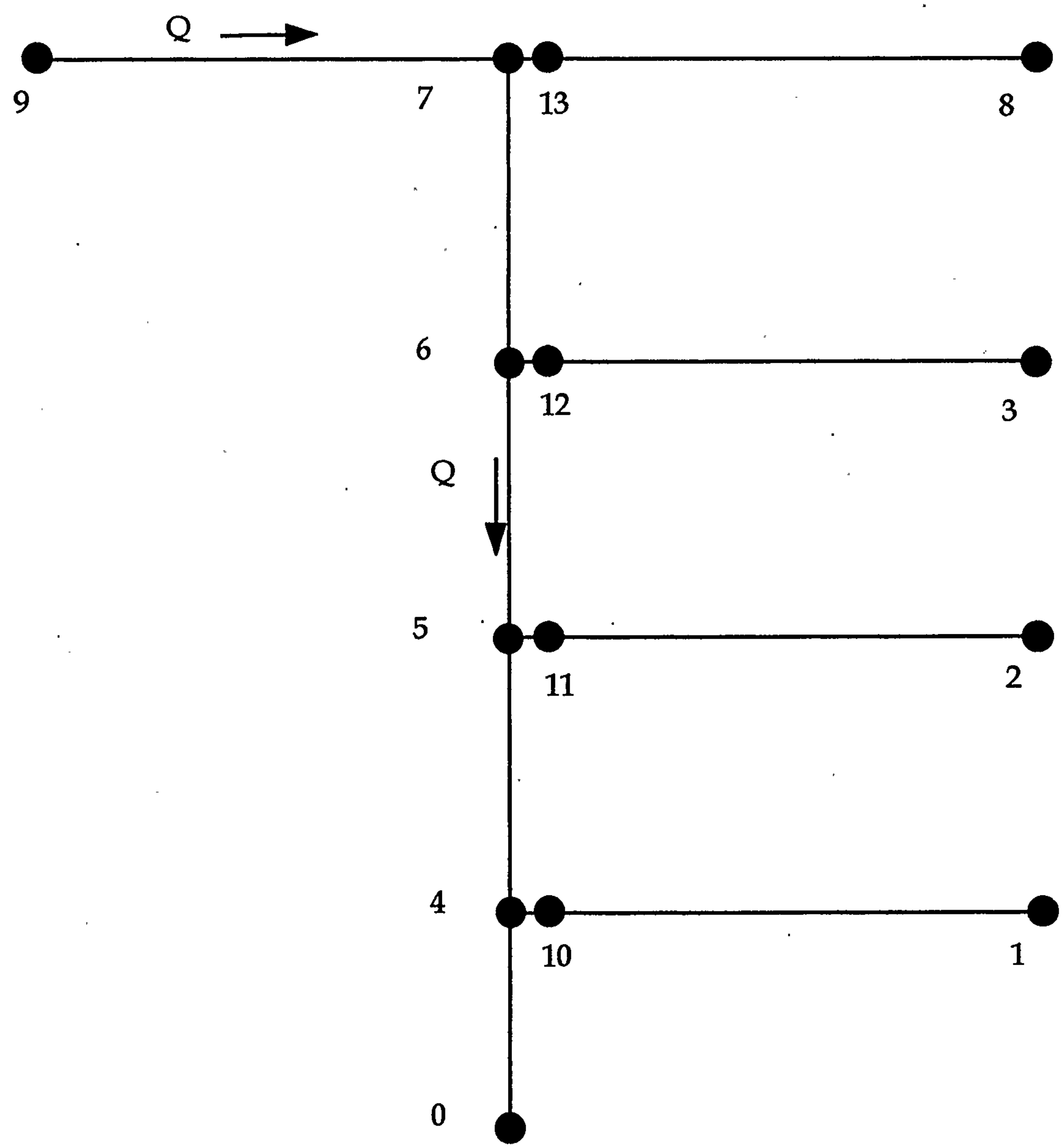

Figure 8.3 Example Network for Water Shortages

Table 8.14 Physical Properties of System Portrayed in Figure 8.3

all channels are rectangular

location	elevation, ft.	width, ft.	upstream length, ft.	upstream Manning roughness
0	0.0	20.0	1000.0	0.025
1	0.0	20.0	1000.0	0.025
2	1.0	20.0	1000.0	0.025
3	2.0	20.0	1000.0	0.025
4	1.0	40.0	1000.0	0.025
5	2.0	40.0	1000.0	0.025
6	3.0	40.0	1000.0	0.025
7	4.0	50.0	1000.0	0.025
8	3.0	20.0	1000.0	0.025
9	5.0	n/a	n/a	n/a

Table 8.15 Modelling Results for "Base Case"

discharge stipulated as 300 cfs at location 1;
no maximum supply discharge stipulated

location	depth (ft)	discharge (cfs)	energy/weight (ft)*
0	3.265701	300.000031	3.593301
1	3.265701	300	3.593301
2	3.234225	293.824646	4.554623
3	3.898067	445.050537	6.404095
4 (downstream)	3.817518	300.000031	5.057255
4 (upstream)	3.817518	600	5.057255
5 (downstream)	3.757689	600	6.005120
5 (upstream)	3.287874	893.824646	6.005121
6 (downstream)	4.860549	893.824646	8.188741
6 (upstream)	4.204738	1338.875244	8.188741
7 (downstream)	5.792729	1338.875244	10.311180
7 (upstream)	7.320038	1338.875244	11.644712
7 (top of junction)	7.070115	2150.408691	11.644712
8	4.996364	811.533386	9.020503
9	7.141208	2150.408691	12.704421
10	3.817518	300	5.057255
11	3.769221	293.824646	6.005121
12	4.863699	445.050537	8.188741
13	7.143737	811.533386	11.644712

*derived result: energy per unit weight is sum of elevation, velocity head and water depth

Table 8.16 Modelling Results of System With Supply Discharge Specified

discharge initially stipulated as 300 cfs at location 1;
maximum supply discharge stipulated as 1800 cfs at location 9

location	depth (ft)	discharge (cfs)	energy/weight (ft)*
0	2.964056	244.351868	3.227879
1	2.964056	244.351868	3.227879
2	2.925372	237.761032	4.181805
3	3.592661	369.733002	6.003809
4 (downstream)	3.366522	244.351868	4.571036
4 (upstream)	3.366522	488.703735	4.571036
5 (downstream)	3.297561	488.703735	5.510718
5 (upstream)	2.902935	726.464783	5.510718
6 (downstream)	4.350116	726.464783	7.620774
6 (upstream)	3.822738	1096.197754	7.620775
7 (downstream)	5.105453	1096.197754	9.552861
7 (upstream)	6.129275	1096.197754	10.439698
7 (top of junction)	5.913504	1721.197876	10.439698
8	4.493045	625.000122	8.244206
9	6.083267	1721.197876	11.580502
10	3.366522	244.351868	4.571036
11	3.310477	237.761032	5.510719
12	4.338889	369.733002	7.620775
13	6.021474	625.000122	10.439698

*derived result: energy per unit weight is sum of elevation, velocity head and water depth

rather crude—specified discharges are merely factored down to ninety-five percent of their previous values—in the solution obtained there is quite a large gap between the computed and permissible discharges at location 9. More sophisticated allocation schemes could be used to reduce this difference, providing a solution that better utilises the available discharge. Obviously, continuity and energy compatibility were preserved throughout the analysis.

Specified discharges may also not be fulfilled if a control structure is incapable of passing the discharge required by demands downstream of it. Taking the base case, the gated orifice must have a capacity of at least 1338.88 cfs to supply the downstream channels. By altering the orifice opening and imposing a finite channel bank height upstream of the control, a delivery shortfall can result. The orifice opening is changed to a height of 5.0 ft. and a width of 15.0 ft. The water depth at the downstream side of the orifice will be approximately 5.8 ft. if the discharge is again specified as 300 cfs at location 1. Using the same structure coefficient of 0.90, a differential head of approximately 6.11 ft. is required, as calculated using equation 8.2. This corresponds to a required water depth of about 11.91 ft. upstream of the orifice. By imposing a channel bank height less than this value—say, 10.0 ft.—it will be physically impossible to deliver the required discharge.

In this situation, the algorithm should again use the built-in scheduling routine, factoring down the specified discharge until the control structure can satisfy the branch discharges downstream. When the model was run for this situation, the specified branch discharge was factored down to 257.21 cfs, yielding a discharge of about 1152 cfs at the orifice. The control was able to satisfy this demand given the upstream height limitation. For this test, the channel upstream of location 8 was widened to 40.0 ft. to convey the increased discharge required for compatibility at the bifurcation upstream of the orifice. Results of this test are included in Table 8.17.

The tests presented in this section illustrate the model's capability to treat situations where the system discharge demands exceed the system's abilities. System abilities can be limited by either a stipulated maximum discharge at a network location or by the physical sizes and properties of the control structures or channels modelled.

8.2.4 Large Network Application

The solution algorithm can model various types of control structures and all bifurcation situations. Because of this, it is well suited for solution of large distribution systems such as irrigation networks. Such systems are generally comprised of channels of several orders. Solution may be difficult because of this "system hierarchy" and the large number of bifurcations included. To the author's knowledge, no other solution algorithm exists that can model these situations without considerable user direction (by specifying the precedence of channel computations, for instance). Therefore, the system in Figure 8.4 has been modelled to indicate the potential of the algorithm developed.

The physical properties of this system are summarised in Table 8.18. All of the channels have rectangular cross sections and fixed check structures are assumed at all downstream terminal locations. If a coefficient of 0 is included for the Moritz equation, no seepage is assumed to occur from the channel upstream of the location. Branch discharges are specified at locations 1, 14, 17, 26 and 31. Control structures are placed at locations 7, 15, 29, 24, 36, 41 and 42.

Table 8.17 Modelling Results of System With Discharge Exceeding Capacity of Upstream Control Structure

discharge initially stipulated as 300 cfs at location 1

location	depth (ft)	discharge (cfs)	energy/weight (ft)*
0	3.037598	257.212524	3.315939
1	3.037598	257.212494	3.315939
2	3.000536	250.676636	4.271483
3	3.667378	387.195862	6.100095
4 (downstream)	3.474138	257.212524	4.686925
4 (upstream)	3.474138	514.425049	4.686925
5 (downstream)	3.407074	514.425049	5.628320
5 (upstream)	2.994960	765.101685	5.628320
6 (downstream)	4.471580	765.101685	7.755706
6 (upstream)	3.914946	1152.297607	7.755706
7 (downstream)	5.267933	1152.297607	9.732280
7 (upstream)	9.793114	1152.297607	13.927478
7 (top of junction)	9.143455	3248.536377	13.927478
8	9.787856	2096.23877	13.233000
9	9.274564	3248.536377	15.036577
10	3.474138	257.212524	4.686925
11	3.419727	250.676636	5.628320
12	4.463597	387.195862	7.755706
13	9.449928	2096.23877	13.927478

*derived result: energy per unit weight is sum of elevation, velocity head and water depth

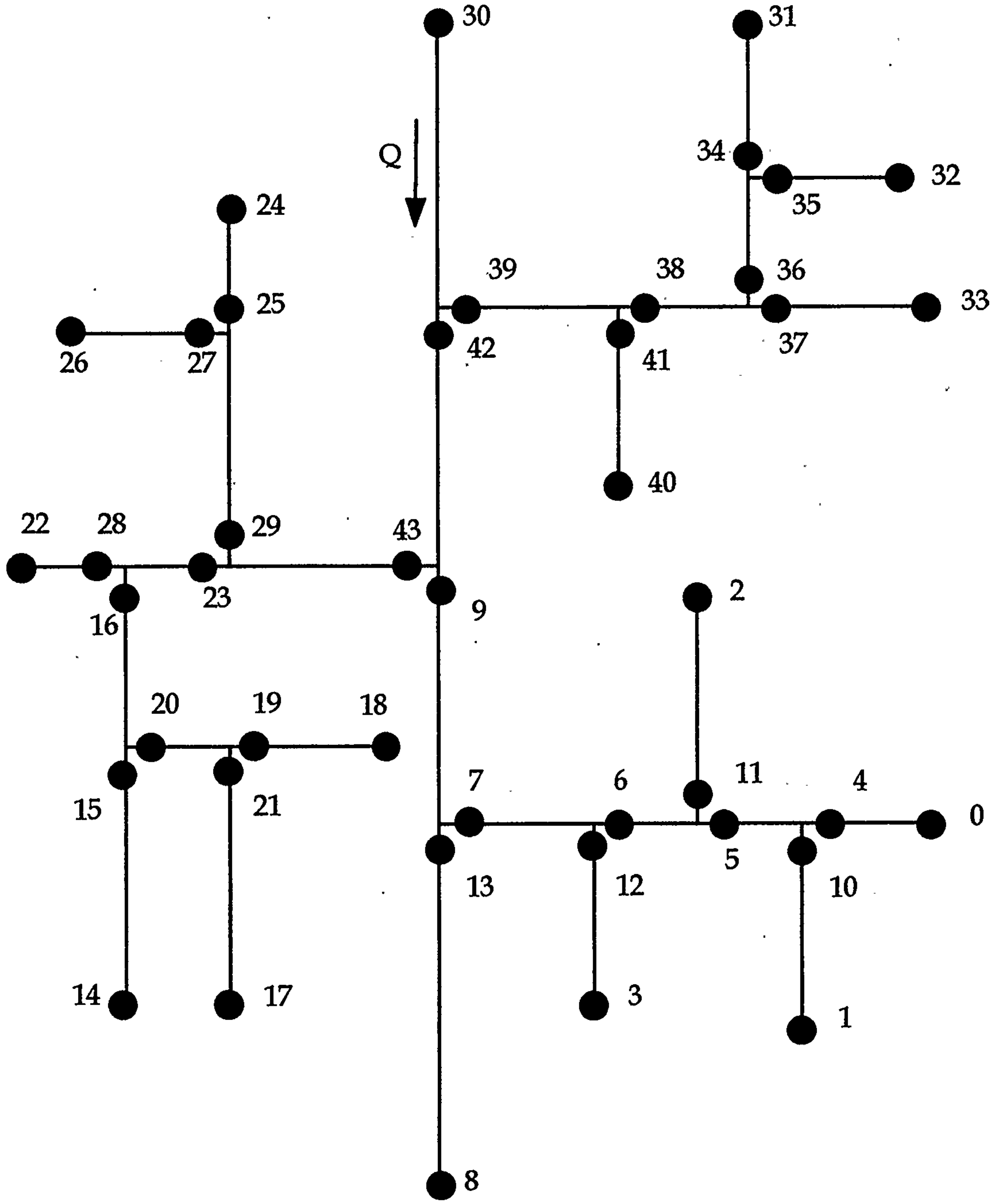

Figure 8.4 High Order Network

Table 8.18 Physical Properties of System Portrayed in Figure 8.4

all channels are rectangular

location	elevation (ft)	width (ft)	upstream length (ft)	upstream Manning *n* value	Moritz coefficient
0	0.0	20.0	1000.0	0.025	0
1	0.0	20.0	1000.0	0.025	0
2	1.0	20.0	1000.0	0.025	0
3	2.0	20.0	1000.0	0.025	0
4	1.0	40.0	1000.0	0.025	0
5	2.0	40.0	1000.0	0.025	0
6	3.0	40.0	1000.0	0.025	0
8	3.0	20.0	1000.0	0.025	0
9	5.0	90.0	1000.0	0.025	0
13	4.0	50.0	1000.0	0.025	0
14	1.5	20.0	500.0	0.027	0.66
15	2.0	40.0	1000.0	0.027	0
17	6.5	20.0	500.0	0.027	0.66
18	6.3	20.0	700.0	0.027	0
19	7.0	40.0	1000.0	0.027	0

Table 8.18 (cont.) Physical Properties of System Portrayed in Figure 8.4

all channels are rectangular

location	elevation (ft)	width (ft)	upstream length (ft)	upstream Manning n value	Moritz coefficient
22	8.5	20.0	500.0	0.020	0
23	10.0	50.0	1000.0	0.020	0
24	8.5	20.0	500.0	0.020	0
25	9.0	40.0	1000.0	0.020	0
26	8.0	20.0	1000.0	0.020	0
28	9.0	50.0	1000.0	0.020	0
30	25.0	–	–	–	–
31	14.2	20.0	800.0	0.0175	0
32	14.5	20.0	500.0	0.020	0
33	21.0	20.0	1000.0	0.020	0
34	15.0	40.0	1000.0	0.022	0
37	22.0	40.0	1000.0	0.019	0
38	23.0	40.0	1000.0	0.019	0
40	22.0	20.0	1000.0	0.025	0
42	12.0	200.0	1000.0	0.017	0

Table 8.19 Control Structure Specifications for System Portayed in Figure 8.4

location	structure type	structure coefficient	width (ft)	height (ft)
0	fixed check drop	1.837	20.0	–
1	fixed check drop	1.837	20.0	–
2	fixed check drop	1.837	20.0	–
3	fixed check drop	1.837	20.0	–
7	fixed gated orifice	0.90	30.0	5.0
8	fixed check drop	1.837	20.0	–
9	globally operable check drop	–	50.0	5.0
14	fixed check drop	1.837	20.0	–
15	globally operable check drop	–	20.0	5.0
17	fixed check drop	1.837	20.0	–
18	fixed check drop	1.837	20.0	–
22	fixed check drop	1.837	20.0	–
23	globally operable gated orifice	0.90	45.0	3.0
24	fixed check drop	1.837	20.0	–
26	fixed check drop	1.837	20.0	–
27	globally operable gated orifice	0.90	15.0	3.0
31	fixed check drop	1.837	20.0	–
32	fixed check drop	1.837	20.0	–
33	fixed check drop	1.837	20.0	–
36	fixed check drop	–	40.0	5.0
40	fixed check drop	1.837	20.0	–
41	globally operable gated orifice	0.90	15.0	3.0
42	globally operable check drop	–	90.0	5.0

Table 8.20 Details of Operable Control Structures Described in Table 8.19

location	structure type	adjustment increment (ft)	minimum setting (ft)	maximum setting (ft)
9	check drop	0.10	1.0	10.0
15	check drop	0.10	1.0	10.0
23	gated orifice	0.05	0.0	20.0
27	gated orifice	0.05	0.0	8.0
41	gated orifice	0.05	0.0	8.0
42	check drop	0.10	1.0	10.0

Table 8.21 Specified Conditions for System Portayed in Figure 8.4

note: no maximum supply discharge stipulated

location	stipulated discharge (cfs)
1	300
14	100
17	350
26	500
31	400

Control structure specifications are summarised in Table 8.19 and Table 8.20. Magnitudes of the specified discharges are presented in Table 8.21.

This system satisfies the Constraint Rules. The globally operable controls at locations 9, 23, 27 and 42 are required to provide energy compatibility at junctions. Because branch discharges are specified on both sides of the junction at 15/20, a globally operable control must also be situated at one of these two locations. The control structures at locations 7, 36 and 41 are fixed because one of the branch discharges downstream is allowed to vary. Drop heights of 6.0 ft. are assumed for all check-drop controls with the exception of the control at location 42 which has a drop height of 12.0 ft.

Results from the model are summarised in Table 8.22. Continuity and energy compatibility were achieved at all bifurcations, as shown in Table 8.23 Also, the gate openings and weir heights required for the solution were generated by the algorithm. These are presented in Table 8.24

8.3 CHAPTER SUMMARY

SNAP, the computer model based on the solution algorithm, has been tested in several situations. Some of these cases have been used to validate the basic flow routing capabilities of the model. Others have been used to demonstrate special features of the algorithm.

Results from linear channels correlate well with standard solutions from Henderson (1966). Close agreement with Henderson's results is significant because that text is a standard reference for hydraulic engineers. In effect, by generating solutions compatible with Henderson, the model has been validated

Table 8.22 Modelling Results Of System Portrayed In Figure 8.4

location	depth (ft)	discharge (cfs)	energy/weight (ft)*
0	3.265701	300.000031	3.593301
1	3.265701	300	3.593301
2	3.234225	293.824646	4.554623
3	3.898067	445.050537	6.404095
4	3.817518	300.000031	5.057255
4 (top of junction)	3.817518	600	5.057255
5	3.757689	600	6.005120
5 (top of junction)	3.287874	893.824646	6.005121
6	4.860549	893.824646	8.188741
6 (top of junction)	4.204738	1338.875244	8.188741
7 (below control)	6.044930	1338.875244	10.521023
7 (above control)	7.572239	1338.875244	11.875646
7 (top of junction)	7.321198	2187.387939	11.875647
8	5.089199	848.512756	9.168325
9 (below control)	7.322384	2187.387939	12.8766532
9 (above control)	7.445823	2187.387939	18.981867
9 (top of junction)	7.061355	4893.145508	18.981867
10	3.817518	300	5.057255
11	3.769221	293.824646	6.005121
12	4.863699	445.050537	8.188741
13	7.359636	848.512756	11.875646
14	1.891078	100	3.499629

*derived result: energy per unit weight is sum of elevation, velocity head and water depth

Table 8.22 (cont.) Modelling Results Of System Portrayed In Figure 8.4

location	depth (ft)	discharge (cfs)	energy/weight (ft)*
15 (below control)	1.980789	100.077782	4.079884
15 (above control)	4.423391	100.077782	12.443262
15 (top of junction)	4.086524	783.48584	12.443261
16	4.456111	783.48584	13.756127
16 (top of junction)	4.199915	1256.822144	13.756128
17	3.505020	350	10.392108
18	3.427981	333.299377	10.094964
19	4.087103	333.299377	11.345265
19 (top of junction)	4.071888	683.408081	11.345265
20	4.184386	683.408081	12.443261
21	4.056025	350.108704	11.345265
22	4.00258	473.336334	13.045472
23 (below control)	4.353562	1256.822144	14.871207
23 (above control)	6.620146	1256.822144	16.844011
23 (top of junction)	5.086298	2705.757324	16.844012
24	5.321204	948.935364	15.055751
25	6.292228	948.935364	16.175142
25 (top of junction)	6.724573	1448.935303	16.175143
26	4.096771	500	12.675015
27 (below control)	4.608995	500	14.065853
27 (above control)	6.9757	500	16.175143
28	4.281712	473.336334	13.756128
29	6.336574	1448.935303	16.844012

*derived result: energy per unit weight is sum of elevation, velocity head and water depth

Table 8.22 (cont.) Modelling Results Of System Portrayed In Figure 8.4

location	depth (ft)	discharge (cfs)	energy/weight (ft)*
30	7.764083	7249.050781	33.102487
31	3.720577	400	18.369274
32	3.646499	382.25531	18.573087
33	5.562305	1065.68811	27.987272
34	3.764327	400	19.202655
34 (top of junction)	3.788996	782.25531	19.202655
35	3.812382	382.25531	19.202655
36 (below control)	3.984141	782.25531	20.35827036
36 (above control)	8.144274	782.25531	30.233808
36 (top of junction)	7.670533	1847.943359	30.233808
37	7.436615	1065.688110	30.233809
38	7.262916	1847.943359	30.891190
38 (top of junction)	6.686341	2355.905518	30.891191
39	7.184741	2355.905518	32.228229
39 (top of junction)	7.901494	7249.050781	32.228231
40	4.124144	507.962067	26.713054
41 (below control)	5.280586	507.962067	28.639799
41 (above control)	7.723266	507.962067	30.891191
42 (below control)	7.538988	4893.145508	32.228231
42 (above control)	7.387115	4893.145508	32.228230
43	7.072875	2705.757320	18.981867

*derived result: energy per unit weight is sum of elevation, velocity head and water depth

Table 8.23 Energy Conditions At Junctions For System Portayed in Figure 8.4

Locations at Junction	Common Energy/Weight Value (ft)
4, 10	5.057255
5, 11	6.005121
6, 12	8.188741
7, 13	11.875646
9, 43	18.981867
19, 21	11.345265
15, 20	12.443262
16, 28	13.756127
23, 29	16.844012
25, 27	16.175143
34, 35	19.202655
36, 37	30.233808
38, 41	30.891190
39, 42	32.228231

Table 8.24 Control Structure Settings Generated By Model For System Portrayed In Figure 8.4

Control Location	Structure Setting (ft)
7	5.0
9	2.2
15	1.2
23	2.55
27	3.0
41	3.0
42	1.6

for linear channel situations. The effects of seepage and precipitation have been demonstrated. Satisfactory and unsatisfactory control structures have been tested and compared against hand calculations. Behaviour of fixed and operable structures has been established.

The examples provided in section 8.2 have shown the ability of the SNAP model to simulate several novel network situations. All types of bifurcations can be modelled, complete with a wide variety of control structures. Type II and "ordinary" bifurcations have been tested. Loop situations have been demonstrated, including a case with a locally operable control structure along one of the loop branches.

Cases in which the system cannot satisfy the specified system demands have also been modelled. These cases can result when a system is limited by a maximum supply discharge or by the physical properties of individual elements, such as control structures or channel bank heights. Both types of limitation have been dealt with, showing the model's ability to use a built-in delivery schedule to reallocate the specified demands, permitting solution after flow profiles were recomputed.

Finally, a network of order four was simulated. Operable and fixed control structures were included, as were channels with seepage effects. Incidentally, the model's numbering requirements are very flexible. This is also illustrated in the last example.

Because the example presented in section 8.2.4 requires the solution to progress through four orders of channel, a small amount of numerical error may have accumulated at the most upstream location. This could be introduced by the iterative solution. A small amount of error can be carried forward into the solution from every bifurcation (and from every backwater computation along linear channels, as well) because absolute agreement of energy levels is not enforced. However, these effects should be negligible in most cases since the permissible tolerance in energy levels at bifurcations and through control structures is either 0.000001 ft. or 0.000001 metres. These effects should be small relative to the uncertainties introduced in estimating the channel roughness and in estimating energy losses at junctions. Hence, the SNAP model should be considered sufficiently accurate for most engineering purposes.

9 DISCUSSION

The scope of the present research is summarised. Conclusions are drawn regarding the steady-state solution of open channel networks. Despite the advances that have been made, some aspects of open channel networks have not been investigated. Potential topics for future work are discussed.

9.1 SUMMARY OF RESEARCH

Development of the governing equations of open channel flow was reviewed. Basic solution techniques were reviewed and discussed. Numerical methods previously used to treat these equations were described in detail, and the effects of numerical and data errors were examined. Simultaneous and iterative solution techniques previously used for network applications were compared, and several methods of improving simultaneous solution techniques were presented. These methods centred upon improving the efficiency of solving the large sparse coefficient matrices associated with simultaneous solutions.

Open channel network situations have been analysed. These situations have included bifurcations, confluences, loops, and a wide variety of possible types of control structures. A variety of bifurcation types were identified, and their behaviour and treatment classified in respect to the control structures they "command". Solution methodologies have been developed for individual network components and then linked together.

The Constraint Rules were developed. These rules detail the necessary requirements to generate a unique system solution for a given network. Several examples of systems that do not meet the requirements of the Constraint Rules were presented. Methods of modifying these systems to satisfy the rules have been discussed.

Solution algorithms were prepared for network situations, with an emphasis upon irrigation systems. These systems were emphasised because they are generally the most complex type of network commonly encountered. This complexity arises primarily because of the large number of bifurcations and number and type of control structures they typically include.

The solution algorithms were programmed into an illustrative computer model, SNAP (Steady Network Analysis Program). The model was used to demonstrate application of the algorithms to various network situations.

9.2 CONCLUSIONS

The most significant contributions are the development of the Constraint Rules, the preparation of algorithms for the solution of general network situations, and the development of a computer model capable of illustrating the features of the solution algorithms. The first of these accomplishments is used to determine whether or not a solution or solutions exist for a given system. Solution algorithms generate these solutions when they are possible. The SNAP model based upon these algorithms has been shown to simulate a wide variety of network situations.

9.2.1 Constraint Rules

The Constraint Rules were developed to indicate whether a system in fact has a solution. The requirements necessary to generate a unique system solution for a given network are established. Several examples of systems that do not satisfy the Constraint Rules are also presented. Methods of modifying these systems to permit solution are discussed.

These rules are essential if networks are to be modelled. If systems are identified as over-constrained, solution may be possible if the extra equations are "relaxed" or removed from the system entirely. A unique solution can exist for properly constrained systems, while parametric solutions may be generated in under-constrained systems. Both simultaneous and iterative solution techniques seem capable of treating constrained and under-constrained cases. It appears that iterative methods must be used to solve over-determined systems.

9.2.2 Solution Algorithms

Solution algorithms were developed and presented in the form of flow chart diagrams. The algorithms were based upon iterative solution techniques. Methods were developed to treat:

- channel bifurcations;
- channel confluences;
- simple loops;
- fixed, operable, satisfactory and unsatisfactory control structures;

- systems having a specified maximum supply discharge;
- systems having individual elements incapable of satisfying stipulated branch demands; and
- high order distribution systems.

The solution algorithms were based upon conservation of energy at confluences and bifurcations, and upon maintaining continuity throughout all portions of a system. The individual algorithms were linked into a scheme that relies upon flow being routed properly through the downstream portions of a network before solution progresses upstream. Algorithms were developed with a focus upon networks of irrigation channels.

Additionally, strategies were presented for solution of interconnected loops containing fixed and/or operable control structures. These techniques are extensions of Schulte's (1985) work.

9.2.3 Steady Network Analysis Program (SNAP)

A computer simulation model, SNAP (Steady Network Analysis Program), was coded and used to illustrate the application of the solution algorithm to several network situations. The examples included are only a glimpse at the possible algorithm applications. The SNAP model, and hence the algorithm, has been proven to simulate the network features the algorithms were designed to simulate. Model results show that continuity and compatibility of energy levels are ensured at all bifurcations and confluences.

The model is not meant to be the "last word" in steady-state models. It *is* intended to illustrate solution of the *network* features expected in typical irrigation systems. It is believed this objective has been met.

9.3 FUTURE RESEARCH

Several directions for further work have been identified. These include analytical and laboratory methods as well as those using computer models. Specific areas include: investigation of open channel junctions; convergence of numerical methods; investigation of low-discharge conditions; analysis of delta situations; and the development of computer algorithms that determine whether systems satisfy the Constraint Rules.

9.3.1 Low Discharge Situations

In section 7.1.9.2 the problems inherent in modelling very small channel depth were discussed. Low flows can arise when the network algorithm is attempting to obtain a solution by adjusting previous discharge conditions. Obviously, this is not the only possible way in which these conditions can develop. Low discharge and low depth flows occur in nature. In delta situations, this can happen when discharge increases, causing flow to spill into a channel having a bed height slightly above the present water level. Cases of low discharge are also prevalent in irrigation networks that do not supply water continuously. This includes most properly operated canal systems, as regular "no-flow" conditions are employed during maintenance operations and to help prevent water-logging problems). During the canal filling stages, low discharge is often a requirement to minimise erosive action upon the canals.

An effective method of analysing the hydraulics of low-flow situations will permit the above cases to be modelled. The method used should alleviate the "breakdowns" that occur in the governing equations. Special numerical treatment of the equations may be necessary. Treatment may be as simple as selecting appropriate distance increments for computations. More likely, examination of the physics of the specific problem will indicate that these situations must be treated with *different* equations.

9.3.2 Deltas

Deltas are characterised by complex loops, variations in channel bed elevations at junctions, multiple confluences and bifurcations, possible flow reversals and low-flow conditions. Treatment of all these elements is currently possible with the exception of the last. Since proper methods of treating low-flow conditions are not known, it follows that adequate analysis of delta situations is not currently possible. Following the development of theory appropriate for the treatment of the problem described in 9.3.1, solution methodologies for deltas should be possible.

9.3.3 Channel Junctions

As indicated in section 3.5.5, behaviour of open channel confluences and bifurcations is strongly dependent upon the specific physical properties of the channels involved. A limited amount of laboratory testing has been conducted, but there is no adequate description of junction behaviour and associated energy (mixing) loss characteristics. Development of such a theory would enable appropriate values to be inserted as loss coefficients in the simulation model developed. Further investigation may also indicate whether additional "classes" of junction exist from mathematical or modelling perspectives. If such "classes" are discovered, solution algorithms can be prepared accordingly.

9.3.4 Stability Requirements Of Newton-Raphson Numerical Method

Divergence of the Newton-Raphson numerical technique has occasionally been observed. Divergence results because the technique evaluates the derivative of the backwater function to obtain a next guess for the function solution. To avoid potential instability, an alternative method that does not rely upon evaluation of the derivative may be possible.

With backwater computations, divergence problems generally occur where the curvature of the water surface profile is steep. This is common in the regions adjacent to control structures and where hydraulic jumps occur. If odd results *are* exhibited in a region of a system, then the system can be remodelled using shorter distance increments in the region(s) of concern. Provided that the distance increment is appropriately small in these areas, the divergence difficulties may be resolved.

Such treatment may provide a practical solution for the problem of divergence, but it will still require guesswork. Rigorous treatment of the convergence properties of the numerical method should probably be reserved for researchers in the field of applied mathematics.

9.3.5 Computer Algorithm To Apply The Constraint Rules

The Constraint Rules have been proposed to determine whether a unique solution exists for a system. If a unique system does not exist, strategies have been given to change a system to a form having a unique solution. Currently, the rules must be applied independently of the computer algorithms. A computer algorithm should be developed to determine whether the Constraint Rules are satisfied. If the rules are not satisfied, the algorithm should then select an action to modify the system to allow a unique solution. Such a computer algorithm could be linked to a version of the SNAP model, in effect acting as a "pre-processor" utility. Immediately after this computer routine is run, the flow routing algorithms could be automatically invoked.

9.4 CHAPTER SUMMARY

The scope of the present research has been highlighted. Several significant conclusions have been presented and their relevance discussed. Additionally, a number of points have been proposed as possible extensions of or supplements to this work. Theoretical, laboratory and modelling efforts will all be of benefit in these regards.

LITERATURE CITED

Asawa, G.L., (1993), Irrigation Engineering, Wiley Eastern Limited, New Delhi, India, 568pp.

Ashenhurst, P.B. (1981), "Analysis of Flow Profiles," a project report submitted to the school of graduate studies in partial fulfilment of the requirements for the degree Master of Engineering, McMaster University, 107pp.

Bakhmeteff, Boris A. (1932), Hydraulics of Open Channels, McGraw-Hill Book Company Inc., New York, NY, 329pp.

Best, James L. and Reid, Ian (1984), "Separation Zone at Open-Channel Junctions," *Journal of Hydraulic Engineering*, Vol. 110, No. 11, pp. 1588-1594.

Brater, Ernest F. and King, Horace Williams (1976), Hamdbook of Hydraulics, Sixth Edition, McGraw-Hill Book Company Inc., New York, NY.

Burnham, Michael W. and Davis, Darryl W. (1990), "Effects of Data Errors on Computed Steady-Flow Profiles," *Journal of Hydraulic Engineering*, Vol. 116, No. 7, pp. 914-929.

Chang, Howard H. (1985), "River Morphology and Thresholds," *Journal of Hydraulic Engineering*, Vol. 111, No. 3, pp. 503-519.

Chaudhry, M. Hanif and Schulte, Andreas M. (1986), "Computation of Steady-State, Gradually Varied Flows in Parallel Channels," *Canadian Journal of Civil Engineering*, Vol. 13, pp. 39-45.

Cheney, Ward and Kincaid, David (1985), Numerical Mathematics and Computing, 2nd ed., Brooks/Cole Publishing Company, Monterey, California, 562pp.

Cunge, J.A., Holly, F.M. and Verwey, A., (1980), Practical Aspects of Computational River Modelling, Pitman Advanced Publishing Program, London, 420pp.

Daugherty, Robert L., Franzini, Joseph B. and Finnemore, John E. (1985), Fluid Mechanics With Engineering Applications, 8th ed., McGraw-Hill, Inc., New York, NY, 598pp.

Demuren, Ayodeji O. and Ideriah, Felix J.K. (1986), "Pipe Network Analysis by Partial Pivoting Method," *Journal of Hydraulic Engineering*, Vol. 112, No. 5, pp. 327-334.

De Neufville, Richard and Hester, James (1969), discussion of "Water Distribution Systems Analysis" by Uri Shamir and Charles D. D. Howard, *Journal of the Hydraulics Division, Proceedings of the American Society of Civil Engineers*, Vol. 95, No. HY1, pp. 484-486.

Eichert, Bill S. (1970), "Survey of Programs for Water-Surface Profiles," *Journal of the Hydraulics Division, Proceedings of the American Society of Civil Engineers*, Vol. 96, No. HY2, pp. 547-563.

Environment Canada (1988), " Environment Canada One-Dimensional Hydrodynamic Model Computer Manual," Water Modelling Section, Water Planning and Management Branch, Inland Waters Directorate, 28pp.

Epp, Robert and Fowler, Alvin G. (1970), "Efficient Code for Steady-State Flows in Networks," *Journal of the Hydraulics Division, Proceedings of the American Society of Civil Engineers*, Vol. 96, No. HY1, pp. 43-56.

Escoffier, Francis F. (1946), "Graphical Calculation of Backwater Eliminates Solution by Trial," *Engineering News-Record*, Vol. 136, p. 71.

Ezra, Arthur A. (1954a), "A Direct Step Method for Computing Water Surface Profiles," *Transactions of the American Society of Civil Engineers*, Vol. 119, pp. 453-462.

Ezra, Arthur A. (1954b), discussion of "A Direct Step Method for Computing Water Surface Profiles" by Arthur A. Ezra, *Transactions of the American Society of Civil Engineers*, Vol. 119, pp. 477-479.

Fread, D.L. and Harbaugh, T.E. (1971), "Open-Channel Profiles by Newton's Iteration Technique," *Journal of Hydrology*, Vol. 13, pp. 70-80.

French, Richard H. (1985), Open-Channel Hydraulics, McGraw-Hill Book Company, New York, NY, 705pp.

García-Navarro, M.P. and Savirón, J.M. (1992), "Numerical Simulation of Unsteady Flow at Open Channel Junctions," *Journal of Hydraulic Research*, Vol. 30, No. 5, pp. 595-609.

Gichuki, F.N. (1988), "Development of a Branching Canal Network Hydraulic Model," Water Management Synthesis II Project, WMS Report 72, Utah State University, Agricultural and Irrigation Engineering Department, 121pp.

Gichuki, F.N., Walker, W.R. and Merkley, G.P. (1990), "Transient Hydraulic Model for Simulating Canal-Network Operation," *Journal of Irrigation and Drainage Engineering*, Vol. 116, No. 1, pp. 67-82.

Gray, C.A.M., (1954), discussion of "A Direct Step Method for Computing Water Surface Profiles" by Arthur A. Ezra, *Transactions of the American Society of Civil Engineers*, Vol. 119, pp. 469-470.

Gunaratnam, Daniel J. and Perkins, Frank E. (1970), "Numerical Solution of Unsteady Flows in Open Channels," Report No. 127, Hydrodynamics Laboratory, Department of Civil Engineering, Massachusetts Institute of Technology, 260pp.

Gupta, S.K. and Tanji, K.K. (1977), "Computer Program for Solution of Large, Sparse, Unsymmetric Systems of Linear Equations," *International Journal for Numerical Methods in Engineering*, Vol. 11, pp. 1251-1259.

Hager, Willi H. and Volkart, P.U. (1986), "Distribution Channels," *Journal of Hydraulic Engineering*, Vol. 112, No. 10, pp. 935-952.

Henderson, F.M. (1966), Open Channel Flow, Macmillan Publishing Co. Inc., New York, NY, 522pp.

Humpidge, H.B. and Moss, W.D. (1971), "The Development of a Comprehensive Computer Program for Calculation of Flow Profiles in Open Channels," *Proceedings, Institution of Civil Engineers*, Vol. 50, pp. 49-64.

Husain, Tahir, Abderrahman, Walid A., Khan, Hasin U., Khan, Suhail M., Khan, Asfahan U. and Eqnaibi, Badie S. (1988), "Flow Simulation Using Channel Network Model," *Journal of Irrigation and Drainage Engineering*, Vol. 114, No. 3, pp. 424-441.

Husain, Tahir, Khan, Hasin U. and Khan, Suhail M. (1991), "Dynamic Node-Numbering Concept in Channel Network Model," *Journal of Irrigation and Drainage Engineering*, Vol. 117, No. 1, pp. 48-63.

Hydrologic Engineering Center (1982), "HEC-2 Water Surface Profiles Users Manual," US Army Corps of Engineers, 40pp.

Hydrologic Engineering Center (1988), "Stream Hydraulics Package User's Manual," US Army Corps of Engineers.

Joliffe, Ian B. (1984), "Computation of Dynamic Waves in Channel Networks," *Journal of Hydraulic Engineering*, Vol. 110, No. 10, pp. 1358-1370.

Kamphuis, J. William (1970), "Mathematical Tidal Study of St. Lawrence River," *Journal of the Hydraulics Division, Proceedings of the American Society of Civil Engineers*, Vol. 96, No. HY3, pp. 643-664.

Kao, Kwan-Hong (1980), "Improved Implicit Procedure for Multichannel Surge Computations," *Canadian Journal of Civil Engineering*, Vol. 7, pp. 502-512.

Kraatz, D.B., (1977), Irrigation Canal Lining, FAO Land and Water Development Series, Publication No. 1, Food and Agriculture Organization of the United Nations, Rome, 199pp.

Laurenson, Eric M. (1986), "Friction Slope Averaging in Backwater Calculations," *Journal of Hydraulic Engineering*, Vol. 112, No. 12, pp. 1151-1163.

Li, Zi-Cai, Zhan, Ling-Jia, and Wang Hui-Li (1983), "Difference Methods of Flow in Branch Channel," *Journal of Hydraulic Engineering*, Vol. 109, No. 3, pp. 424-446.

Lin, J.D. and Soong, H.K. (1979), "Junction Losses in Open Channel Flows," *Water Resources Research*, Vol. 15, No. 2, pp. 414-418.

Lopes, Vincente L. and Shirley, Edward D. (1993), "Computation of Flow Transitions in Open Channels With Steady Uniform Lateral Inflow," *Journal of Irrigation and Drainage Engineering*, Vol. 119, No. 1, pp. 187-200.

Manz, David H. (1985a), "Implicit Finite Difference Solution of the One-Dimensional Equations of Unsteady Gradually Varying Open Channel Flow," Research Report No. CE85-6, Department of Civil Engineering, University of Calgary.

Manz, David H. (1985b), "Newton Iterative Solution for the One-Dimensional Steady Spatially Varied Flow Equations Expressed in Finite Difference Form," Research Report No. CE85-7, Department of Civil Engineering, University of Calgary.

Manz, David H. (1985c), Systems Analysis of Irrigation Conveyance Systems, a thesis submitted to the faculty of graduate studies and research in partial fulfilment of the requirements for the degree of Doctor of Philosophy in Water Resources Engineering, University of Alberta, Edmonton, Alberta, Canada, 435pp.

Manz, David H. (1987), "Classification of Irrigation Water Conveyance System Components," *Journal of Irrigation and Drainage Engineering*, Vol. 113, No. 4, pp. 479-496.

Manz, David H. (1994), personal communication.

Manz, David H. and Schaalje, Mike (1992), "Development and Application of the Irrigation Conveyance System Simulation Model," International Workshop on The Application of Mathematical Modelling for the Improvement of Irrigation Canal Operation, Montpellier, France, pp. 101-120.

Martin, R.S. and Wilkinson, J.H. (1967), "Solution of Symmetric and Unsymmetric Band Equations and the Calculation of Eigenvectors of Band Matrices," *Numerische Mathematik*, Vol. 9, pp. 279-301.

Mays, Larry W. (1986), "User's Manual for the Unsteady Flow Routing Model DWOPER (Network)," for the World Bank, 1818 M Street NW, Washington, DC, 20433.

McBean, Edward and Perkins, Frank (1975), "Numerical Errors in Water Profile Computation," *Journal of the Hydraulics Division*, Vol. 101, No. HY11, pp. 1389-1403.

Misra, Rajeev, Sridharan, K., and Kumar, M.S. Mohan (1992), "Transients in Canal Networks," *Journal of Irrigation and Drainage Engineering*, Vol. 118, No. 5, pp. 690-707.

Molinas, Albert and Yang, Chih Ted (1985), "Generalized Water Surface Profile Computations," *Journal of Hydraulic Engineering*, Vol. 111, No. 3, pp. 381-397.

Monem, M.J. and Manz, D.H. (1994), "Application of Simulation Techniques for Improving the Performance of Irrigation Conveyance Systems," *Iranian Journal of Water Resources Engineering* (*IJWRE*), Vol. 2, No. 1, pp. 1-22.

Pickard, William F. (1963), "Solving the Equations of Uniform Flow," *Journal of the Hydraulics Division, Proceedings of the American Society of Civil Engineers*, Vol. 89, No. HY4, pp. 23-38.

Prasad, Ramanand (1970), "Numerical Method of Computing Flow Profiles," *Journal of the Hydraulics Division, Proceedings of the American Society of Civil Engineers*, Vol. 96, No. HY1, pp. 75-86.

Rammamurthy, Amruthur S. and Satish, Mysore G. (1988), "Division of Flow in Short Open Channel Branches," *Journal of Hydraulic Engineering*, Vol. 114, No. 4, pp. 428-438.

Rammamurthy, Amruthur S., Carballada, Luis B. and Tran, Duc Minh (1988), "Combining Open Channel Flow at Right Angled Junctions," *Journal of Hydraulic Engineering*, Vol. 114, No. 12, pp. 1449-1460.

Rammamurthy, Amruthur S., Tran, Duc Minh and Carballada, Luis B. (1990), "Dividing Flow in Open Channels," *Journal of Hydraulic Engineering*, Vol. 116, No. 3, pp. 449-455.

Schulte, Andreas M. (1985), "Computation of Steady, Gradually-Varied Flows in Open-Channel Networks," a thesis submitted in partial fulfilment of the requirements for the degree of Master of Science in Civil Engineering, Washington State University, Department of Civil and Environmental Engineering, 77pp.

Schulte, A.M. and Chaudhry, M.H. (1987), "Gradually-Varied Flows in Open-Channel Networks," *Journal of Hydraulic Research*, Vol. 25, No. 3, pp. 357-371.

Sim-Flo Engineering Ltd., (1991), Eastern Irrigation District Water Delivery Management/Operation Improvement Project, Draft Report, 54pp.

Smith, Alan A. and Ashenhurst, Paul B. (1986), "RIVER4: A Program for Flow Profile Computation," *Canadian Journal of Civil Engineering*, Vol. 13, pp. 335-344.

Smith, C.D. (1985), Hydraulic Structures, University of Saskatchewan Printing Services, 364pp.

Stoker, J.J. (1957), Water Waves, Interscience Publishers Inc., New York, NY, 567pp.

Strelkoff, Theodor (1992), "EQSWP: Extended Unsteady-Flow Double Sweep Equation Solver," *Journal of Hydraulic Engineering*, Vol. 118, No. 5, pp. 735-742.

Swain, Eric D. (1988), "A Model of Flow in Regulated Open Channel Networks," a thesis submitted to the Faculty of the University of Miami in partial fulfilment of the requirements for the degree of Master of Science in Civil Engineering, 165pp.

Swain, Eric D. and Chin, David A. (1990), "Model of Flow in Regulated Open-Channel Networks," *Journal of Irrigation and Drainage Engineering*, Vol. 116, No. 4, pp. 537-556.

Swokowski, Earl W. (1983), Alternate Edition Calculus with Analytic Geometry, Prindle Weber & Schmidt, Boston, Massachusetts, 934pp.

Taylor, Edward H. (1944), "Flow Characteristics at Rectangular Open-Channel Junctions," *Transactions of the American Society of Civil Engineers*, Vol. 109, pp. 893-902.

Webber, Norman Bruton and Greated, Clive Alan (1966), "An Investigation of Flow Behaviour at the Junction of Rectangular Channels," *Proceedings, Institution of Civil Engineers*, No. 34, pp. 321-334.

Wylie, E. Benjamin (1972), "Water Surface Profiles in Divided Channels," *Journal of Hydraulic Research*, Vol. 10, pp. 325-341.