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OPTIMAL PARAMETERS FOR URBAN RAIL PLANNING

by

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ABSTRACT

An optimization model to determine the optimal rail line length and rail termini with the objective of minimizing the sum of user time cost, bus and rail operating costs, rail line cost, bus and rail fleet costs and passenger transfer penalty cost is presented. The effects of both uniform and non-uniform rail line costs as well as the passenger transfer penalty cost are examined. A model to determine the optimal location of ring rail lines with the aim of minimizing user access and rail line costs for both uniform and variable demand densities is presented as well.

The analyses considered daily passenger many to many demand at peak and off-peak periods. The optimal parameters are obtained using calculus. Sensitivity analyses are conducted to test the robustness of the proposed models. The validity and applicability of the proposed models are tested using Calgary, Alberta as a case study.

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INTRODUCTION AND LITERATURE REVIEW

1.1 INTRODUCTION

The application of optimization methods to determine transit network parameters is becoming a prominent feature in contemporary public transit planning. Over the past years, the determination of transit network parameters such as rail line length, location of termini of a rail line and location of a ring rail line are not based on any comprehensive analysis but rather on some criteria not considered from the view point of economic and optimization concepts. Such adhoc rail planning practices generated some serious problems in the rail world. Among others, it resulted in increase in passenger travel time and related cost, loss of patronage of rail systems by passengers and loss of interest in rail ridership by private automobile owners. Other resulting problems are loss of revenue by transit operators, loss of revenue to society as a whole, abandonment of rail lines in some transportation corridors and generation of non-optimization of landuse development.

It is against this background that this research is undertaken with the aim of providing very important rail planning tools which will help solve some of the existing transit problems in the rail world. More particularly, the determination of optimal rail line length, optimal location of termini of a rail line and optimal location of a ring rail line are very important rail network parameters required for constructive rail planning, and analytical models developed to obtain these parameters are presented in this research.

Commenting on the need for construction of a rail line to serve a high travel density, Seneviratne et al (1986) emphasized that the rail line length is a critical parameter that will determine global transportation costs. They explain that the rail line length influences the initial capital investment cost, rail line maintenance cost and systems operating cost. It also affects some transit parameters such as passenger travel demand, transit system headway, transit station spacing and location, transportation corridor width as well as service area coverage. It is essential to highlight that other parameters such as optimal location of termini of a rail line and optimal location of a ring rail line are also of great importance so far as the provision of a rail line is concerned, and should also be given the utmost attention when planning for provision of rail line in an urban transportation corridor is desired.

1.2 RESEARCH PROBLEM

This research involves the determination of optimal rail line length connecting the Central Business District and Suburban Region. The objective is to minimize the sum of user time cost, bus and rail operating costs, rail line cost, bus and rail fleet costs and passenger transfer penalty cost. An analysis to determine the optimal location of the termini of a cross-town rail line will be presented as well. The effects of both uniform and non-uniform rail line costs as well as the passenger transfer penalty cost on the rail line length and termini will be discussed.

The research will seek to explore the optimal location of a ring rail in a large metropolitan region. Basically, the analysis will involve the determination of optimal radii of ring rail lines that will minimize only user access cost as well as user access and rail line costs for both uniform and variable trip demand densities. In particular, the analytical design will consider daily passenger many to many travel demand travel pattern at both peak and off-peak periods. Sensitivity analysis will be conducted to test the robustness of the proposed models. The validity and applicability of the proposed models will be tested using the existing transit line haul in Calgary, Alberta, as a case study.

1.3 TRANSPORTATION COSTS

The provision of a rail line in an urban transportation corridor will require the consideration of relevant cost parameters consistent with the desired planning objective. The relevant cost parameters are user time cost, bus and rail operating costs, rail line cost, bus and rail fleet costs and passenger transfer penalty cost.

Direct fixed capital costs associated with public transportation services are rail line cost and fleet costs. The main rail line cost components are land acquisition cost, design cost, rail track cost, rail line construction cost, station construction cost, garage construction cost, parking lots construction cost, utility relocation cost as well as the maintenance costs of these rail facilities. Fleet cost is the cost associated with the acquisition of transit vehicles. It includes the cost of transit vehicles, shipping cost, insurance cost and overhead cost. With regard to modelling for public transportation services, the fleet size cost is formulated based on the number of transit vehicles in operation, which is basically a function of the number of seats (seating and standing passenger spaces), the round trip time and operational headway of the vehicles.

The direct variable costs are user access and egress cost, user time cost, passenger waiting time cost at transit stops, user time cost, passenger transfer penalty cost, delay to passengers due to stops by transit system at transit stops and stations to allow for boarding and alighting of passengers and systems operating costs. User access and egress cost with regard to public transportation route and mode choices is the cost incurred by passengers to walk to the nearest available transit stop or station, and is basically evaluated from the point of view of time or distance.

Passengers being transported have to contribute their own time inputs into the transportation services. User time inputs involve passengers' waiting time for a transit system at bus stops or train station, and in-vehicle riding time in a transit system to destination. Delay due to stops by transit system at transit stops or stations to allow passengers to board and alight from the vehicle include time lost by transit system due to deceleration and acceleration, door opening and closing times, passenger unloading and loading times as well as the time interval between closing of door and the instant the transit system begins to move. Passenger transfer times include time taken by passengers to exit from a transit system, walk to a platform, wait for another transit system of similar or dissimilar mode and finally enters an arriving transit system for departure to destination. Associated with this transfer is loss of time and transfer penalty cost. Bus and rail operating costs, among others, include labour cost, fuel cost, maintenance cost, vehicle depreciation cost and overhead cost.

User access and egress costs, passenger waiting time at transit stops or stations, passenger delay cost due to stops by transit system at transit stops and stations are irrelevant to the determination of rail line length as well as radius of the ring rail line and hence will not be considered in this analysis. The time taken or distance travelled by passengers to access or egress the rail line in order to reach their destination points are independent of the rail line length. Hence the associated access and egress costs are neglected. Moreover, the delay to stops by transit systems at transit stops or stations is dependent on the number of boarding and alighting passengers, but independent of the number of stops if boarding and alighting at a stop and edge effects are neglected (Wirasinghe, 1992). More essentially, the delay is independent on the rail line length as well as radius of the ring rail line and is also not considered in the formulation of the total cost function. The "out of pocket" cost of passengers (i.e. transit fares) will not be included in the user time costs since the fares basically involves payments made directly by users to operators. Hence, this cost component will not be considered in the analysis.

Indirect transportation costs are the external costs generated by transportation users and inflicted on the non-travelling passenger without any payment or compensation being made to the affected non-travelling passengers. Basically, the external costs of transportation consist of noise, atmospheric pollution and vehicle congestion. Others are accidents, visual intrusion, vibration and community severance costs. For convenience, indirect or external costs will not be considered in this research.

1.4 LITERATURE REVIEW

1.4.1 RAIL LINE LENGTH

Research aimed at determining the optimal rail line length started in the mid-sixties. Creighton et al (1964) investigated the rail line length in a corridor where the demand for travel to the Central Business District (CBD) decreased exponentially with increasing distance from the CBD and obtained a single expression for the line-length that minimized the sum of the rail line construction and user time costs. Also Werner et al (1968) investigated a similar problem. They, however, considered a gamma distribution demand per unit area density, and obtained a set of equations for station spacings and line length but did not solve them.

Black (1975) considered a network of radial line to which all commuters walked, and using an iterative technique, he obtained uniform line lengths, station spacings, headways and number of lines that minimized user time, operating and construction costs for a demand per unit area that decreased exponentially. Lam (1979) extended the model of Creighton et al (1964) to include operating cost,and obtained numerical solution for the line lengths when the linear density is represented by a normal distribution function.

In 1980, the City of Calgary proposed a 9.650km radial LRT line in the North - West corridor and later decided to shorten it to about 6.400km. However, the decision regarding the "cut-back" on the rail line length was not based on any comprehensive economic analysis. It is against this background that Babalola et al (1982) presented a paper that evaluated, from an economic point of view, the length of the existing LRT line along a given centre line in North-West Calgary, Alberta. Their proposed model is aimed at determining the length of track that minimizes the sum of the rail line cost, user travel time cost, bus and rail operating costs and rail fleet cost. They disclosed that an existing analytical model was modified and employed to facilitate their investigation. The results of their analysis show that the City's proposal is 1.500km short of the optimal track length obtained in their analysis. They remarked that the length proposed by the Transit Planners at the City of Calgary could be justified by assuming a low value for a unit of riding time per passenger.

More perhaps, a paper closely related to this research is presented by Senevirante et al (1986). They developed an analytical model to investigate the optimal rail line length that minimizes the sum of user time cost, bus and rail operating costs, rail line costs and rail fleet cost as well as the threshold demand necessary to ensure that the resulting rail line length is positive. In their analysis, however, they considered both linear and area demand travel pattern from Suburban to CBD during the morning commute peak period. They obtained optimal rail line lengths for both uniform and non-uniform line costs. They found that when the cost per unit length is uniform, a minimum transport cost rail line of nonzero length exists only if the net gain in travel time and operating cost transporting the total ridership a unit distance by rail, when compared to bus, exceeds the marginal line and fleet cost per unit length. The effect of shifts in passenger demand on the optimal rail line length is discussed in their paper. Their model is tested using Calgary, Alberta as a Case Study.

In their paper entitled "Optimal Bus Route Length", Taylor et al (1989) developed some continuous transportation models to determine the optimal bus route length. Their models are derived using steps which are assumptions of continuous spatial to define trip distribution in transportation corridor, derivation of operation cost as a function of line length, estimation of travel demand, derivation of total revenue under a flat and graduated system as a function of travel demand and line length and determination of optimal line length for the operator. In particular, they developed a computer program to calculate the optimal bus route length.

They identified the factors that cause the changes in optimal bus route length. These include variation in population gradient, employment density gradient and impedance factor. Others are walking distance, unit bus operating cost and average passenger bus carried at maximum load. They presented a case to examine the model parameters, and carried out a sensitivity analysis to investigate changes in the proposed parameters with regard to travel demand and optimal bus route length.

Schonfeld et al (1993) developed an analytical model to determine the optimal transit route length, spacing, headway and stop locations subject to minimisation of the sum of operator and user time costs. They explained that their proposed equations are incorporated within an efficient algorithm which calculates the optimal values of the decision variables for a more realistic model with vehicle capacity constraints. They considered many-to-one demand travel pattern with uniform passenger trip density along transit routes emerging radially from the CBD into the low density suburbs. Their research findings include the fact that stop spacing increases along the route in the direction of passenger accumulation towards the CBD, and the spacing first decreases and then increases along the route towards the CBD. They also carried out sensitivity analysis and numerical example to test the validity and applicability of their model.

They remarked that is it rather surprising that the determination of optimal rail line length is not given more attention in literature related to rail planning given the significant impact of route length on cost. They cited the paper presented by Seneviratne et al (1986) as one that optimized radial length of a transit route in an urban transportation corridor. They indicated that Seneviratne et al did not consider stations along the line and related cost, and based the minimum rail fleet size on the peak period passenger capacity requirements, implying that the optimum route length would operate at the maximum allowable headway. They emphasized that this assumption may be unpermitted even for the peak periods since the optimal headway may be heavily influenced by user waiting time. This explains why they (i.e. Schonfeld et al) developed a model that jointly optimizes the headway, route length and stop spacing.

However, the fact that they used the same model to determine the optimal rail line length and operational characteristics of transit systems such as headway, creates an opportunity for criticizing their model. The rail line length is identified as one of the major parameters controlling the implementation of rail transit projects and therefore considered as a long-term planning tool. Headway, on the other hand, is considered as an operational characteristic of transit systems and therefore used for short-term planning policies. An attempt to develop a model to determine both long-term and short-term planning parameters under the same sets of assumptions and constraints, therefore, subjects their proposed model to criticisms.

1.4.2 RAIL LINE TERMINI

To date, no literature on optimum location of rail termini along a cross-town transportation corridor is documented. Over the past years, decisions regarding the location of termini of a rail line have not been based on any comprehensive, economic and optimization analyses. Location has an effect in the sense that cost per unit distance from the origin and destination points of passengers to the rail termini is influenced by the distance it has to be located. However, papers that discuss the optimal location of some public facilities such as police stations, sewage treatment plants, warehouses, distribution centres, communication centres are available. It is imperative to say that insight into these literature will generate some very important concepts which, to a very large extent, can assist in developing models to optimally locate the termini of a rail line.

A model developed to determine the optimum location of a "switching centre" in a communication network and to locate the best place to build a police station in a highway system is developed by Hakimi (1964). His analysis is based on the principle that the concepts of the "centre" of a graph are generalized to the "absolute centre" of a weighted graph. He defined a weighted graph as a graph with weights attached to its vertices and branches. He presented procedures for finding these locations. His analysis shows that the optimal location of a switching centre is always at a vertex of the communication network, whilst the best location for the "police station" is not necessarily at the intersection.

Goldman (1969) developed a model to solve the problem of locating centres of processing facilities in a network, with the aim of minimizing the total transportation cost associated with their use. He assumed that all movements occur between a vertex and a centre nearest to it. He then established an objective function that minimizes the total transportation costs and obtained expressions for coordinates of the centres. He concluded that only the vertex location of the centres needs to be considered.

The problem of optimally locating the central facility in a network with the aim of minimizing the sum of its distance from sources of flow is investigated by Goldman et al (1971). They assumed that each distance is approximately weighted to reflect the associated flow volume and/or cost. They obtained a simple one-pass solution algorithm for two classes of topologically simple networks which are either acyclic or contain exactly one cycle. The centres are determined in terms of coordinates.

A model that solves a dynamic-transportation-allocation problem when the number of destinations and sources are fixed is developed by Tapiero (1971). He formulated his problem as follows: given the location of each destination, the requirement of each destination, possible source capacity limitation and a set of shipping costs, it is required to determine the optimum location of each source, the allocation of destination to each source and amount to be supplied to each source. His analysis is based on the assumption that transportation costs are linear and proportional to the euclidian distance between sources and destinations.

He obtained an analytical solution from a set of conditions for optimality. He disclosed that although his model is focused on location and allocation of transshipment, it is equally applicable to optimal location of warehouses, distribution centres, communication centres or production facilities.

As discussed in his paper entitled "Minimax Location of a Facility in a Network", Goldman (1978) presented a model to solve the problem of locating a facility in a network so as to minimize the largest of its distance from the vertices of the network. He remarked that his proposed model either solves the location problem or reduces it to an analogous problem for a single "cyclic component" of network. However, for an "acyclic component" of network, an efficient algorithm solution is obtained. He presented a partial analog of these results for a "weighted distance" of the problem. Ironically, an application of his proposed model to real life issues is not discussed in his paper.

A paper that addresses the problem of locating the absolute and vertex centres of an undirected tree graph using minimax criterion is presented by Handler (1973). Based upon a convexity property of the criterion function, he developed a very simple but efficient algorithm that locates the minimax point by locating first a maximax point. He located the vertex centre using simultaneous technique. He remarked that the minimax is at the mid-point of the maximum path from the maximax point.

It is imperative to comment that the optimal location of the above-mentioned facilities is determined in terms of coordinates and not in terms of distance measured from a reference (zero) point. In this analysis, the optimal location of the rail termini (i.e. the starting and ending points of a rail line) will be determined in terms of distance measured from a reference (zero) point. In particular, the difference in length between the termini is the optimal rail line length.

1.4.3 RING RAIL LINE

Very few, if any, analytical models are developed to determine the optimal ring rail line location connecting cross-town corridor systems. Available literature are focused on location of one or more ring roads in an urban transportation corridor. Blumenfeld et al (1970) developed a model to investigate the routing of two ring roads in a circular city. In particular, they examined a situation in which a circumferential road exists at the boundary of a city, and it is desired to build a second ring road within the city. Their analysis is based on certain simplifying assumptions, which include the city being circular in shape, all origins lie outside the city and destinations within the city have angular symmetry. Others are that origins and destinations are uncorrelated except at the entry points, average speed at any point is a function of radius alone and is an increasing differential function, and drivers choose routes so as to minimize their travel time.

They emphasized that one measure of benefits of adding a second ring road to an existing ring road is purposely to relieve traffic congestion and thus to minimize average travel time. They however remarked that their model is limited in the sense that it does not consider the effect of correlation between origins and destinations, effects of a finite number of radial roads, effects of anisotropic origin and destination distributions, and redistribution of traffic load when various measures like the addition of ring roads are put into practice.

In his paper entitled "Locating Concentric Ring Roads in a City", Pearce (1974) discusses the development of a mathematical model to investigate the optimal location of concentric ring roads in a city so as to minimize the average distance travelled off the rings. He considered road networks with several concentric ring roads. His analysis is based on the assumption that travel within the city can be approximated by ring-radial routing and drivers select routes which minimize the total travel time. He also assumed that trip end lie within the city, and are distributed independently of one another and have areal densities. Furthermore, travel speeds are constant on all road types except the ring roads.

He found that although ring roads are more effective if they are characterized by a higher speed of travel than the rest of the city, their optimal location, to a very large extent, is independent of such speeds. He explained that ring roads are usually designed to reduce interactions between vehicles, so that traffic congestion in a city arises from the traffic off the ring roads. He commented that the provision of several ring roads cannot by itself reduce travel on radial roads, and therefore is not an economic and cost-effective method of reducing travel time and related cost on radial roads. Such a remark is not highly acceptable. His paper concludes with a suggestion for provision of one or two well-placed ring roads with the aim of realising the optimum travelling benefits.

A theoretical model which seeks to explore the location of two ring roads with the objective of minimizing the total radial travel in a circular town of unit radius is presented by Smith (1975). He assumed that drivers choose least time paths and that vehicle speeds are controlled so as to minimize the radial travel time. His analysis suggests that the radial travel volume can be reduced substantially provided the following are available: a high capacity-high speed outer ring road close to the town; a low capacity-low speed inner ring road around the central area; linked traffic lights to guarantee that average speeds are slow within the town, particularly on the inner road; and radial roads are connected at the town centre.

His analysis revealed that whatever the distribution of origins and destinations, if an internal ring road minimizes the total radial travel, then the radial flow inside the ring equals the radial flow just outside the ring. He also found that if origins and destinations are uniformly and independently distributed over that part of the radials within the town, and a ring road already exists, then the optimal radius of a single inner road is $\sqrt{2}-1$.

Jha (1977) developed a model to determine an optimal combination of radial and ring roads considering a radiocentric grid network serving an idealized circular city, with the objective of minimizing the sum of construction and travel costs. His analysis is based on the assumptions that originating work trips are uniformly distributed and all jobs are located in the central core of the city. More particularly, his research involved cases where all ring roads are equally spaced and ring road spacing is a function of the distance from city centre.

He conducted a sensitivity test and identified the variables that are sensitive to the total cost and speed parameters. These variables are number of radial roads, number of ring roads, percentage of ring roads and the optimal ring spacing. His findings include the fact that a road network with a varying ring road spacing is more economical than a network with constant ring spacing. He emphasised that a road network can operate at its optimum provided it consists of about eighty-percent of ring roads. This assertion seems to be in sharp contrast with the findings of Pearce (1974). In my opinion, for a road network to be described as optimal on the basis of provision of approximately eighty-percent of ring roads is not economically sound. Provision of road and other supporting facilities require a high initial capital investment, and public funds needed to construct the roads are very scarce. Perhaps, inclusion of economic analysis in his research will disclose some very important facts which are missing in the paper.

A piece of literature that discusses the development of a model to determine the optimal location of a single road to connect a radial road network is presented by Smith (1979). He assumed that radial roads have no connection other than a single ring road, and furthermore the radial roads do not meet at the centre of the town. He also assumed a fixed origin-destination distribution, and ignored trips which originate and terminate on the same radial road. His research revealed very interesting findings. He found that for any fixed origin-destination distribution, there is allocation of the ring road which minimizes the impact of radial traffic flow, for almost any criterion used to assess the impact. He remarked that an optimal ring has as many relevant trip-ends inside as on the outside.

As discussed by Porter (1992) in his paper entitled "A circumferential light rail transit line" being planned for the Stockholm City, Sweden, the proposed ring rail line was specifically designed to serve the ever-increasing transit demand not oriented towards the central business district corridors, but rather connecting the suburban areas. He explained that Stockhlom Transportation Planners recommended the deployment of circumferential light rail line in order to reduce dependence on private automobile ownership and usage. He enumerated the benefits of the provision of a circumferential LRT line, which are savings in vehicle operating costs, reduction in waiting time because of improved regularity, improved traffic safety and environmental benefits.But regrettably, he failed to present an analysis aimed at determining the optimal ring rail location in a transportation corridor.

1.4.4 LANDUSE-LRT INTERACTION

The evaluation of light rail transit-landuse interaction with the aim of improving public transit services over the past few years is a subject of great interest to public transit planners especially in North America and European states. Taber et al (1978) presented a paper that examines the potential for light rail transit operations in streets with mixed traffic. They hypothesized that street operation of light rail transit (LRT) is possible and desirable in order to achieve the reduction of capital cost in rail construction projects and improvement of transportation services. In particular, their paper attempts to establish a systematic framework for investigating the potential for a shared street environment and to stimulate a discussion among LRT planners about the role of street operations with regard to operations of LRT systems.

Their methodology involves the identification and investigation of transportation problems and the analysis of various design elements and strategies. They discussed several methods of effective street operation. Their discussion is based on reduction of street delays and prevention of accident to pedestrians, automobiles and LRT systems. They commented that the effective operations of LRT systems and automobiles on streets will reduce capital cost of construction of several rail lines, provide faster construction time, result in less environmental disturbance, and attract a large volume of transit passengers. Their discussion is based on existing traffic and transit data obtained from Toronto.

A paper that explains that LRT can work and does work in a variety of situations is presented by Tennyson (1982). His paper also analyzes the condition necessary to support the successful implementation of LRT systems. He explained that for the LRT system to be effective, it must satisfy the requirements of a substantial number of tripmakers along its route. Besides, travel time must be shortened in order to attract riders who have the option of travelling by private automobile. Alternatively, travel volume must be so high to the extent that a low transit modal split will still yield high ridership. He emphasized that LRT operation is effective only when travel demand is sufficient to justify LRT applications on its productive efficiency rather than its speed.

He disclosed that the provision of exclusive right-of-way will help minimize LRT trip time and relieve highway congestion. He further revealed that the maximum efficiency of operations of LRT can be attained by effective integration of LRT services with local bus services. However, on long fast radial lines serving the CBD, integration, although desirable, may not be a necessity. He concluded with the observation that the conditions under which LRT works best range from a long, fast, low-density suburban lines to short, slow, high density inner-city lines.

In his paper entitled "Light Rail-Technology or Way of Life", Ridley (1992) described LRT systems in terms of its technology and way of life. He defined the LRT system as a tracked, electrically driven local means of transport which can be developed step by step from a modern tramway to a means of transport running in tunnels or at above ground level. He commented that every stage of development of the LRT can be a final stage in itself, and more importantly, it permits development to the next higher stage. Hence he remarked that LRT systems are flexible and expandable as well.

He disclosed that the quality of an LRT system is basically determined by the nature and extent of separation of its track from the carriageway for private transportation. Giving priority to LRT systems at crossings with private traffic also raises the standard of the system. The common aim of these measures is to reduce delays and increase the regularity of the services. He enumerated the advantages of provision of LRT lines and other supporting facilities, which include the fact that they are relatively cheap to build and require little maintenance effort, they provide safety to waiting passengers at stations, and are punctual and reliable. Others are LRT improves accessibility to residential, business, shopping and other activity centres, provides park-and-ride facilities, and reduces vehicular congestion, air and noise pollution.

A brochure on LRT systems presented by Transit Gloria Mundi (1992) highlighted the advantages of LRT in the areas of transportation, development and environment. Commenting on the transportation aspect, it disclosed that LRT has several advantages which include winning riders away from their cars, providing good riding quality in terms of comfort and convenience, exhibiting fast and efficient operating characteristics, generating low operating cost and attracting economic opportunities. In the area of land development, the advantages of LRT include the fact that it stimulates investments and redevelopment, produces residential growth, protects residential neighbours and provides less need for highway construction. On the subject of the environment, LRT is found to reduce air and noise pollution, saves energy, reduces congestion and provides a more efficient use of land for transportation than any form of highway vehicle.

A paper that discusses the development of LRT systems in the San Diego metropolitan area, from a simple LRT to a maturing and expanding rail system is presented by Larwin et al (1992). Their paper presents the key decision made in the development of the LRT network in San Diego, the operating performance of the LRT over the past ten years and a prognosis for its future. They discussed that the desired criteria considered for the selection of the LRT are the need for a corridor that extends a relatively long distance and provides opportunity for high speed operation, a line primarily at-grade and primarily in exclusive right-of-way and a system with low operating cost and high probability of meeting operating costs with revenue.

They indicated the benefits of LRT operations. These include forcing transit planners to keep up with the state-of-the-art, producing enthusiasm for the operating personnel by giving them new challenges to look forward to, providing on-going free publicity to the transit system through routine new coverage resulting in stimulation of public enthusiasm. Others are allowing the transit system to grow intelligently with personnel and provision of a learning atmosphere where mistakes and failures are relatively small and corrective measures are easily taken to improve knowledge.

Arrington (1992) presented a paper describing how the effective integration of landuse and LRT systems resulted in great success in Portland, Oregon. He explained that a working partnership between transit planners, politicians and the community as a whole resulted in the development of successful landuse and transit strategy, which is applied to enhance the operations of public transit systems. Most essentially, he indicated that the effective and efficient operations of LRT in Portland, and more particularly in the suburban areas, resulted in the construction of several residential buildings, provision of park and ride facility, reduction of traffic congestion and related problems, reduction of air and noise pollution, promotion and enhancement of business and other economic ventures, and provision of more socio-economic activities.

Others are maximization of development around stations, increasing public transit patronage and ridership as well as improvement of accessibility to residential locations, shopping, business and other activity centres. Although this research will not give prominence to discussion on LRT-landuse interaction, it is imperative to comment that the generation of optimal passenger demand that will warrant the provision of LRT systems and other supporting facilities is largely dependent on efficient and effective interaction between LRT systems and landuse.

1.4.5 ORIGIN-DESTINATION TRIPS

A fundamental requirement in public transit planning is the estimation of passenger travel demand from a point of travel origin to a point of travel destination. It is against this background that Colangelo et al (1977) presented a paper that discusses passenger estimation in short range transit planning policies using cross-classification matrix methods. The values of model parameters used in their analysis are derived from detailed household-transit-landuse data. Various socio-economic characteristics used as indicators in the estimation procedures for travel demand include household income, household size, household density, automobile ownership, trip purpose and user's age. Level of service parameters considered in their analysis are frequency of service, condition of transit system, and automobile travel time and related cost versus transit travel time and cost.

They enumerated some refinement to their model which include changes in propensity of transit trips generation over time, refinement of the effect of level of service on transit trip making and effect of discrepancy between the service area trip rate and study area trip rate. Others are examination of details required in recommendation procedures, applicability of the model in large urban areas and applicability of the procedure when planning is done for transit systems other than conventional bus service. They disclosed that although improvement to their model can be realized with greater expenditure of money and effort, it provides a sound planning tool for many applications. The procedure is successfully used to describe the actual demand for several existing transit systems, as well as to estimate future demand reflecting different policy decisions.

Their model is successfully tested in several cites, and it is found to give reasonable and realistic results.

Hendrickson et al (1984) developed a model to estimate origin-destination travel matrices using constrained least square (CGLS) regression and quadratic methods. Besides, their proposed method is used to evaluate variances of matrix entry estimates. Their method, which does not require general origin-destination surveys, however allows available and relevant information, including uncertain information and judgement. They explained that the form of their proposed objective function is flexible, and the resulting estimates are best described as linear unbiased estimates. They commented that the estimated variances of the entry estimates represents a measure of the uncertainty associated with each entry estimates, and may be used to evaluate the estimates as well as to suggest sampling strategy approaches.

They enumerated the advantages of their regression function which include its flexibility in terms of information that can be included as constraints, ability to provide a measure of reliability of the entry estimates in terms of the variance and ability to obtain high accuracy when compared with chi-square formulation. Others are ability to account for errors in the constraints, ability to provide estimates that are consistent with available data and ability to estimate unknown parameters in linear distribution function. However, their methods exhibit some disadvantages. These are computational burdens associated with calculating the estimates and generation of unreasonable estimates if the formulation of regression function does not include non-negative constraints. They remarked that their proposed model, when compared with other estimation methods especially regarding accuracy, computational effort and use of uncertainty measures, is found to possess a better degree of origin-destination matrices from aggregate data.

As discussed in their paper entitled " Generating a Bus Route Origin-Destination Matrix from On-Off Data", Simon et al (1985) highlighted a method to generate a bus route origin-destination matrix from passenger boarding and alighting counts. They checked the route origin-destination estimates obtained from passenger boarding-alighting data against actual origin-destination data considering both simple and complex bus lines, and observed that estimates of trip length distributions and origin-destination matrices did not statistically differ from the actual data. They elaborated on some practical methods of collecting passenger origin-destination data. They commented that their method, which they recognized as inexpensive but accurate, is suitable for estimating route origin-destination matrices for existing transit lines. The advantages of their model include the fact that it is simple and can be used to test and adjust the origin-destination matrix.

Niham et al (1987) developed an algorithm to solve the problem of estimating origin-destination patterns from passenger input-output counts using recursive prediction error methods. They formulated their origin-destination matrix estimation problem considering a traffic count problem and developed a recursive prediction error (RPE) method to estimate the origin-destination matrices. They explained that the origin-destination matrix estimation problem is simplified when route choice of origins and destinations is unimportant, and available count gives the total exists from each origin and total arrivals at each destination. They disclosed that such situations occur in estimating, among others, the distribution of passengers on single bus routes and equilibrium flows through a subarea. They commented that recursive methods allow engineers and planners to track time-varying origin-destination patterns. Besides, it can be integrated with other existing input count forecasting models to forecast future trip patterns of passengers.

A model which estimates the pattern of passenger origin-destination travel along a transit route is developed by Kikuchi et al (1992). They discussed that the input to their model is boarding and alighting counts at stations and the output is the estimated passenger volume for each station pair. They used linear programming methods to estimate the origin-destination volume with the objective of minimising the expected error by locating each estimate with as close to the centre of the feasible solution as possible. They also presented numeric examples for the case when the non-directional boarding and alighting counts are available. The uses of their model include estimation of origin-destination table of a transit line, distribution of duration of stay at parking lots and estimation of vehicle travel pattern along highways and estimation of characteristics of by-pass traffic.

In this research, a 1991-92 transit demand data obtained from the Transit Operations Management at City of Calgary will be used to test the models. Passenger demand defined in terms of number of passengers, passenger-kilometre and seat-kilometre will be obtained from the data, and used for the validation of the proposed models.

1.4.6 MANY TO MANY DEMAND

Only few studies on analytical optimization for public transportation systems have given due consideration to many to many travel demand patterns. Holroyd (1965) analyzed a grid bus network having passenger trip demand distributed uniformly over an infinite plane to determine the optimal route spacing and headway with the objective of minimizing total systems costs. Newell (1979) extended Holroyd's research by investigating convexity-related difficulties in bus routes considering a many to many travel demand pattern.

In his paper entitled "The Effect of the Design of Road Networks on the Intensity of Traffic Movement in Different Parts of a Town with Special Reference to the Effects of Ring Roads", Smeed (1971) considered passenger many to many travel demand in his model formulation. He described many to many by trip density, which he assumed as a function of the positions of homes and workplaces.

Ghoneim et al (1981) optimised stop spacing for a many to many demand distribution along one route by minimizing the sum of operator and user time cost with a given fleet size. They assumed that their proposed daily travel demand function varies slowly between bus stops. They also considered non-uniformly distributed travel demand by using cumulative originating and destinating trip functions along their route. Stochastic effects due to irregular stops of buses were also considered in their analytical model.

Vaughan (1986) developed an analytical optimization model bus network consisting of a radial and ring routes to determine the optimal route spacing and headway by minimizing user travel time subject to fleet size constraints for a many to many demand travel pattern. More particularly, he described the many to many demand travel as a continuous function of the positions of a commuter's home and workplace, as also suggested by Smeed (1971). Under the assumption that buses travel at a constant speed subject to a fleet size constraint, he found that both the optimal spacing between buses are inversely proportional to the cube root of the proportion of commuters joining and leaving the routes. He presented a numerical example based on the assumption that commuters' many to many demand is uniformly distributed along the bus routes.

Wirasinghe (1990) developed an optimization model to re-examine Newell's dispatching policy for a public transportation route with time varying many to many demand. Accordingly, he proposed a dispatching policy for a bus route with many to many time-varying demand and a variable maximum load point with a capacity constraint based on seats per unit time formulation of demand. He commented that a many to many demand is characterized by boarding and alighting of passengers from buses at bus stops located on the bus routes. Under such circumstances, a seat (seating and standing passenger-space) can be used by several passengers in series, and the associated demand for travel can be measured in terms of seat per unit time.

An analytical model developed to determine the optimal route angle, headway and station spacing for a radial bus network in a heterogenous geographical environment considering a passenger many to many travel demand at both peak and off-peak periods is presented by Chang (1991). His model's objective is to minimize the sum of passenger waiting time cost, access cost, in-vehicle cost and operator cost. More importantly, his study attempts to analyze radial bus networks in which the actual irregular demand distributions in the form of step functions or origin-destination matrices are used to reflect the spatial and temporary heterogeneity of public transportation systems and their environments. Using approximation theory, he obtained closed-form solutions for the optimal route angle, headway for different time periods and stop spacing for various locations. A numerical example is illustrated in his literature. Besides, he presented an application of his model considering irregular demand patterns that are directionally imbalanced at both peak and off-peak periods.

Available literature on determination of optimal rail line length are focused only on passenger many to one demand travel pattern at the morning commute period. More specifically, this research will be based on passenger many to many demand travel pattern at both peak and off-peak periods. In particular, the demand will be described by a continuous function of the difference between the cumulative number of boarding and alighting passengers at every point on the line haul.

1.4.7 FLEET SIZE AND COST

The determination and selection of the most economic and efficient fleet of transit vehicles considering passenger many to many travel demand at both peak and off-peak periods, to serve an entire network of public transit routes, is of great importance to transit planners. It is against this background that Salzborn (1970) developed a model to determine the minimum fleet size for a suburban railway system with the objective of minimizing the number of railcars required for peak operations by cutting railcars back from station along the haul rail line as well as to minimize total driver time. He assumed that the departure times of the trains are given and also two trains departing within any given cycle will experience a minimum possible delay after each other.

His model formulation is based on the principle that the total number of railcars simultaneously in operation is equal to the maximum number of railcars, considering systems operations at both peak and off-peak periods. He derived a mathematical expression to determine the number of railcars required for peak operations, and found that the outcome of his analysis is insensitive to small changes in departure times. He commented that a certain number of railcars and a certain amount of driver's time are required in order to obtain an optimum fleet, and that not too much can be done about it by changing the departure time. His paper concludes with a demonstration of the practical usefulness of his model by applying it to an existing railway system in Aldelaide, Australia.

A fleet selection model designed for a single route under stationary, inelastic travel demand is presented by Hauzer (1971). His analysis is based on two simplifying assumptions: travel demand on the route does not depend on the quality of service; the conditions of vehicle flow on the route are affected significantly by changes in the number of public transportation vehicles serving the routes. He formulated the total cost of the provision of service on the route per unit time as a function of the number of vehicles serving the route and the capacity of vehicles that serve the route. In particular, the total cost is given by the sum of yearly operating costs, wages, amortization, insurance, and administration costs.

Based on the findings of his analysis, he remarked that for the same total cost, a public transit route can be served by few large vehicles or small ones. He explained that for a fixed budget a range of vehicle size can be selected from within which the optimal route fleet is to be selected. The upper limit of this range is the route fleet for a given budget that minimizes the expected waiting time at the critical point of the route. However, the lower limit of the range is a fleet composed of the smallest vehicles for the same budget that still can survive the entire travel demand. He disclosed that the identified parametric variables sensitive to the total cost are variation of optimal route fleet, travel demand, route length, available budget, stop density and traffic friction.

In his paper entitled "Optimum Bus Scheduling", Salzborn (1972) discussed the development of a model to determine the optimum bus scheduling with the objective of minimizing the number of buses required for operations as well as to minimize the passenger waiting time. For analytical purposes, he defined fleet size as the maximum of the difference between the vehicle capacity that has departed from any point before any given time and vehicle capacity that has arrived at any point before any given point. He remarked that although the minimum fleet size formula is derived considering a single bus route, it is also applicable to many transportation systems with more than one bus route.

A paper that discusses the development of a model to investigate the optimum railcar fleet sizing in the Northeast Transportation corridors in the United States is presented by Fourer et al (1977). Moreover, their paper is aimed at exploring the effects of fleet management strategies. Their model objective is to minimize the sum of capital and rail operating costs. A linear programming model that determines fleet requirements for several different formulations of the objective function is developed, and a minimum vehicle fleet size expressed in vehicle-kilometres per day and maximum load factor, are then determined. They remarked that passenger demand is a sensitive parameter so far as the determination of optimum fleet size and related cost is concerned. On the subject of fleet management strategies, they explained that effective implementation of fleet management policies will result in obtaining optimum vehicle fleet size, lower operating cost and increased ridership.

Their model is tested using transit data.on rail corridors in Northeast portion of United States. Their analysis indicated that the most heavily travelled portion of the corridor, Philadelphia to New York, might be better served by adding trains between these two cities. The disadvantage of their model lies in the fact that it is not capable of handling a more complex express-feeder transit network.Accordingly, they suggested that a suitable integer programming formulation might give a better results in such a case.

Gertsbach et al (1977) presented a paper that treats the problems which usually arise in constructing transportation schedules, considering the selection of minimum vehicle fleet size for a given schedule. They imagined that a transit operating agency has to carry out a given set of passages, and each passage consists of names of departure and arriving terminals as well as departure and arrival times at the terminal.The set of passages consist of a certain time period, and that each transit vehicle can carry out passages of the given set.

The problem is then to determine the minimal fleet size for the given trip and the optimal routes for each vehicle. Their analysis is based on the assumption that every vehicle, after arriving in any terminal at any time can perform every passage departing from the terminal at a particular time period. Their formulated objective function is the difference between the number of departures and arrivals occurring at any terminal for a given time interval, and proved that the required fleet size is the maximum of the formulated objective function. Furthermore, they studied a special case of periodic schedules and found that a periodic schedule can be decomposed into an optimal periodic fleet. Application of their model to practical scheduling when passages have tolerances for departure times is discussed as well.

In this research, the formulation of vehicle fleet cost will be based on passenger trips occurring in the afternoon peak period. More particularly, the fleet cost will be derived considering the number of seat (seating and standing passenger spaces) required to be dispatched during the afternoon peak period at round trip time with the maximum travel demand. A parabolic demand function will be assumed in order to obtain an expression for the fleet size and related cost.

CHAPTER TWO

RAIL LINE LENGTH: CBD-SUBURBAN CORRIDOR ANALYSIS

2.1 INTRODUCTION

A CBD-Suburban corridor is a route that provides passenger travel from CBD to Suburban Region and vice versa. The Canadian Transit Handbook (1980) described CBD-Suburban corridor as a type of transportation corridor that does not easily accommodate most trips destined to places other than the CBD regardless of the configuration of the roadway network it is superimposed. Furthermore, the Handbook reported that the CBD-Suburban corridor, while serving the CBD well, attracts few other trips. Besides, the corridor provides improved services to multiple destination but requires a great use of transfer. The CBD-Suburban corridor is characterized by short, frequent headways. It is also characterized by high impedance for trips using crosstown routes. The CBD-Suburban corridor can be modified by the addition of crosstown or circumferential routes.

Vuchic et al (1988) described a radial line as an alignment that radiates outwardly from the city centre into the suburban region. They remarked that the radial line usually follows directions of heavy passenger demand, which gradually decreases towards the suburban regions. The decreasing demand can be matched either by turning some trains back at an intermediate station, or by branching the line into several directions and thus distributing its capacity and increasing area coverage in the suburban regions.

They indicated that the main advantage of radial lines and the dominant reason for their extensive use in many cities is that they tend to serve the heaviest travel corridors in the city. Their disadvantages are that they often have limited distribution in the centre of the city and that their inner terminals may be constrained in space as a result of expensive construction, making their operations difficult.

This chapter discusses the development of an analytical optimization model to determine the optimal rail line length considering a Central Business District (CBD) to Suburban transportation corridor. The objective is to minimize the sum of user time costs, rail and bus operating costs, rail line construction and maintenance costs, fleet costs and passenger penalty costs. A case study is demonstrated to assess the practical usefulness of the proposed analytical model by applying it to the existing North-West light rail transit corridor in Calgary, Alberta. Sensitivity analysis is presented to test the robustness of the proposed model. Numerical examples are demonstrated as well.

2.2 TRANSIT NETWORK

Two transit network systems are considered in this analysis. One type of the transit network will be discussed in this section. The other type of network will be discussed in Section 2.17. An idealized metropolitan region with a dense rectangular grid road network is considered (Figure 2.1). The network is assumed to consist of two distinct sets of parallel curvilinear roads (x and y). It is planned to provide an efficient and reliable transit service by the construction of a rail line $T_C T_R$ along the transportation corridor $T_C T_S$. The proposed railway is to emanate from the heart of the CBD, T_C, to a location T_R in the suburban region but not necessary to its end T_S. Bus service will be provided in the corridor section which originates at T_R and terminates at the end of the suburban region T_S. The service is assumed to be provided by special line-haul buses operating along the corridor. The proposed rail system and line-haul buses are "fed" by feeder buses

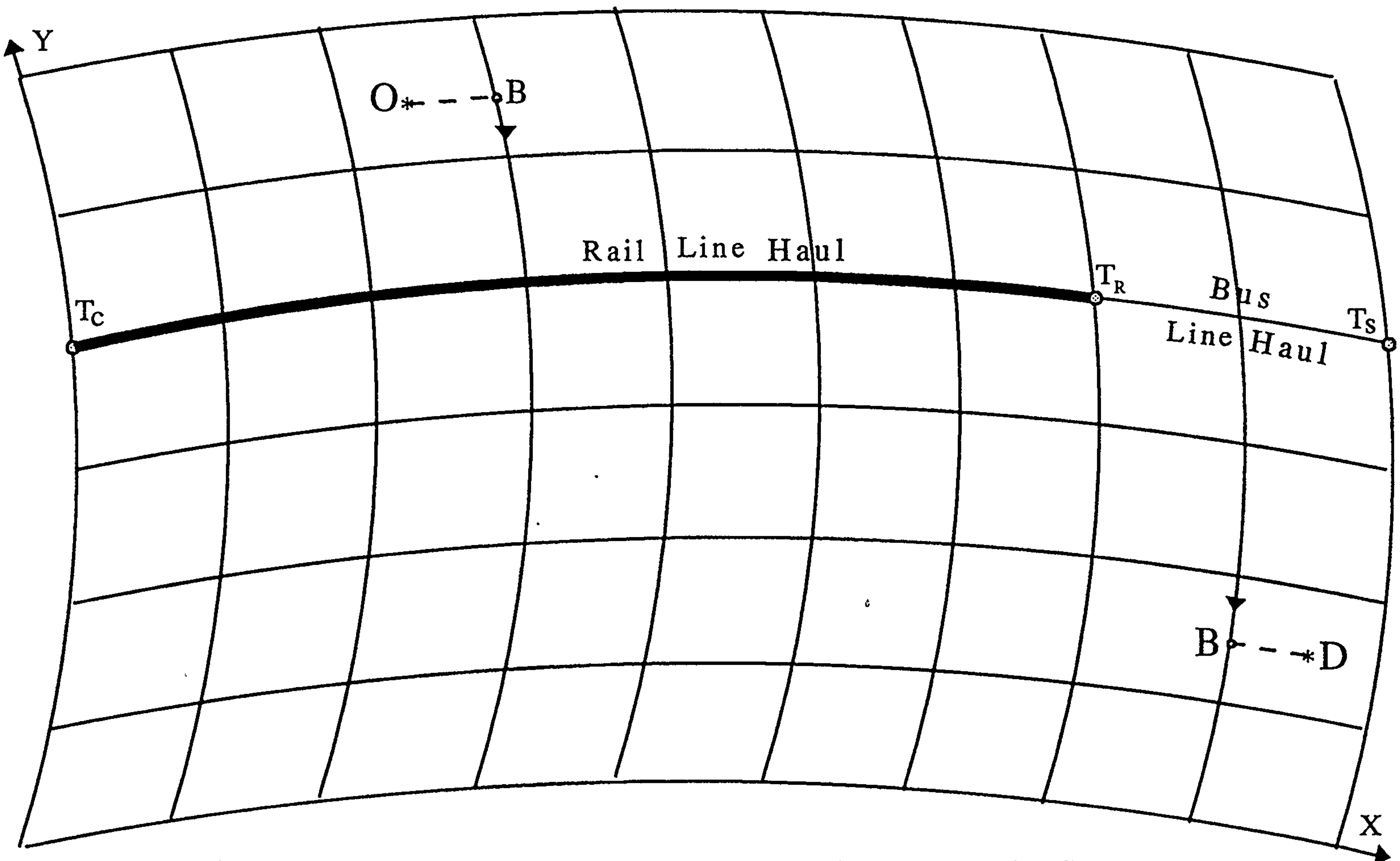

Figure 2.1 Proposed Bus and Rail Network System

operating at both the rail region T_CT_R and bus region T_RT_S respectively.

It is proposed to operate a light rail transit (LRT) system on the railway. Comparatively, a LRT system is known to provide a fast, comfortable, safe, secure and reliable services to passengers than bus transit systems (Vuchic et al, 1985). Moreover, the LRT system has a relatively low operating cost per passenger-kilometre or seat-kilometre. Others advantages include low maintenance cost, low noise and environmental pollution effects, significant improvement on land-use and reduction in private automobile usage (TRRL, 1980).

It is assumed that the trains departing from a central terminus T_C and running towards the terminal T_R will stop at stations located along the rail line to enable passengers to board and alight. Upon reaching T_R, continuing passengers will transfer into the line-haul buses. The train then makes a return trip to T_C. The buses departing from T_R and running to bus terminus T_S located at the end of the suburban region, will stop at bus stops located along the corridor to allow for boarding and alighting of passengers. At T_S, all passengers in the bus will alight from the bus, which will then make a return trip to terminus T_R. Passengers originating from bus section T_RT_S and destined for the rail section T_CT_R will transfer from the line-haul buses into the trains at T_R. The converse is also true.

Feeder bus services will be provided to the corridor from all residential zones located in the suburban regions. The feeder buses will stop at bus stops B (Figure 2.1) located in the zones to allow for boarding and alighting of passengers from the buses. It is postulated that each residential zone is served exclusively by feeder buses. It is particularly assumed that passenger trips originating at any point O and destinating at any point D in the residential zones are served by feeder buses to the nearest rail station or line haul bus stops from which a corridor line haul is available.

It is also assumed that passengers residing in the suburban region as well as those not residing at a reasonable walking distance to the nearest train station in rail section $T_C T_S$ will access the rail transit system by walking from their origins to their nearest bus stop, wait for the arrival of the first feeder bus at half the bus headway, and finally enter and ride in the bus to the nearest rail station or line-haul bus stop. However passengers residing at an appreciable walking distance to the corridor will access the LRT system and line-haul buses by walking.

2.3 THE MODEL

Consider a CBD-Suburban transportation corridor $T_C T_R T_S$ of length L where T_C and T_R represent the CBD and boundary of suburban regions respectively (Figure 2.2), and T_R is the train terminus in the suburban region. T_c and T_R are respectively the start and end points of the proposed rail line of length X_R. T_R is considered to be a major transfer point. For analytical purposes, the rail line is measured from the heart of CBD (i.e. T_C) to the terminus T_R.

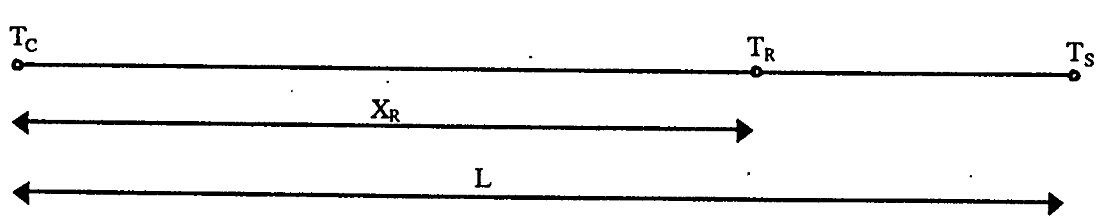

Figure 2.2 Dimensions of Proposed Transit Line Haul

The analysis will seek to determine the optimal rail line length with the objective of minimizing the sum of user time costs, rail and bus operating costs, rail line costs, fleet costs and passenger transfer penalty costs. The analysis is based on normal daily demand travel patterns during both peak and off-peak periods.

2.4 MANY TO MANY TRAVEL DEMAND FUNCTION

In this analysis, a many to many travel demand pattern between the CBD and suburban region is considered. This is the type of normal daily travel demand pattern characterized by multiple origins and destinations of passengers. With this type of demand, there exist boarding and alighting of passengers at transit stations or stops located on the transit routes at both peak and off peak periods.

The demand is assumed to be distributed along the line haul corridor. Let the daily number of boarding and alighting passengers at station or stop location i in the corridor be $b(x_i)$ and $a(x_i)$ respectively (Figure 2.3.1). In particular, $b(x_i)$ and $a(x_i)$ are respectively defined as the daily number of passengers that board and alight at locations x_i in the corridor for travel in the direction $T_C T_S$. The daily cumulative number of passengers that board $[B(x_i)]$ and alight $[A(x_i)]$ from the train (Figure 2.3.2) up to locations x_i on the line haul for travel in the direction $T_C T_S$ are:

$$B(x_i) = \sum_i b(x_i) \qquad (2.1.1)$$

$$A(x_i) = \sum_i a(x_i) \qquad (2.1.2)$$

The cumulative daily number of boarding $B(x_i)$ and alighting $A(x_i)$ passengers at

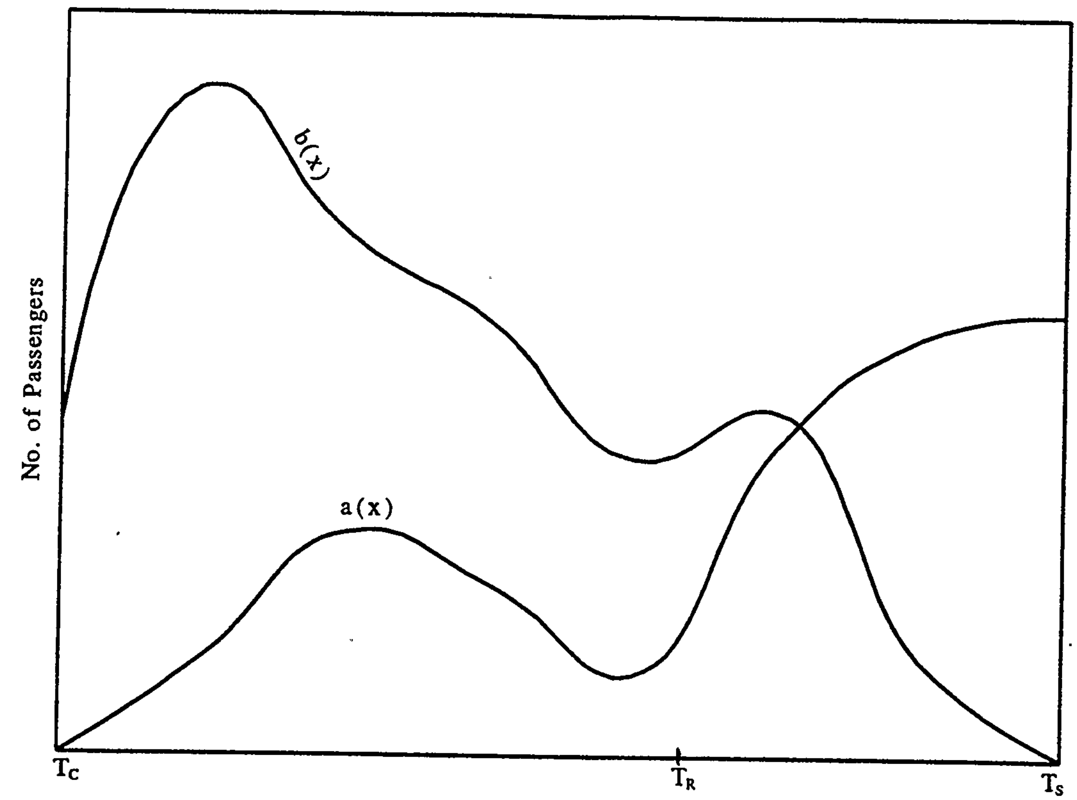

Figure 2.3.1 Typical Daily Number of Boarding and Alighting Passengers: CBD-Suburban Transportation Corridor

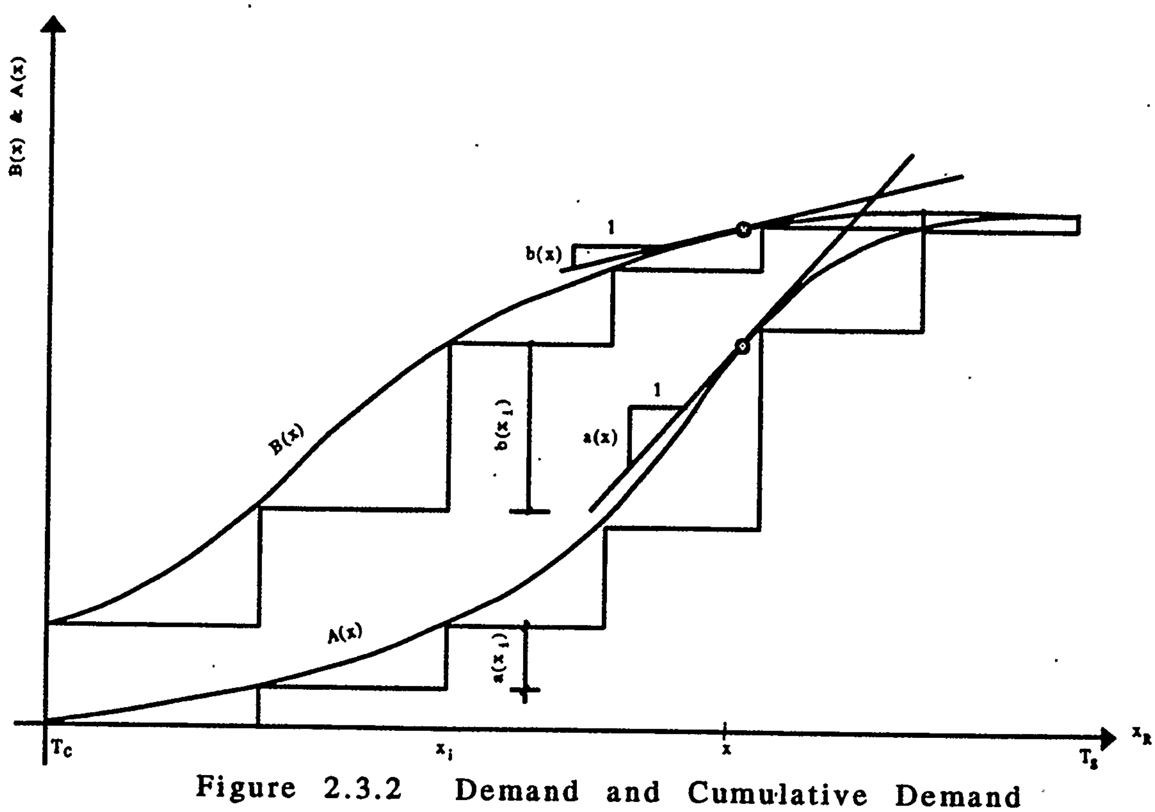

Figure 2.3.2 Demand and Cumulative Demand

locations on the line haul are plotted as step functions against the distance of locations on the line measured from T_C, and smoothed out to obtain B(x) and A(x) (Figure 2.3.2), where B(x) and A(x) respectively denote the cumulative daily number of boarding and alighting passengers up to point x on the rail line for the travel in the direction T_CT_S.

The slopes b(x) and a(x) at any point x on the B(x) and A(x) curves (Figure 2.3.2) represent respectively the daily number of boarding and alighting passengers per unit distance in the vicinity of x. The daily through passenger load M(x) at any point x for travel in the direction T_CT_S on the line haul is the difference between B(x) and A(x). Thus:

$$M(x) = B(x) - A(x) \tag{2.1.3}$$

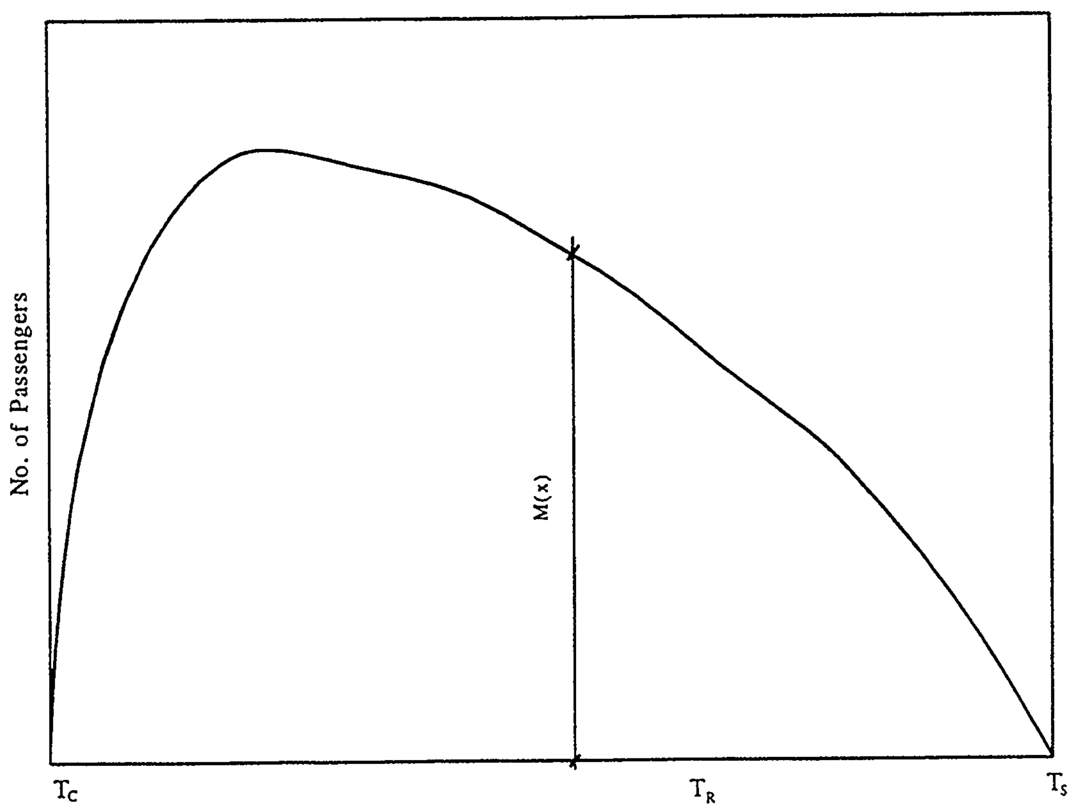

Figure 2.3.3 Typical Daily Through Passenger Load: CBD-Suburban Transportation Corridor

Figure 2.3.3 depict the graph of daily through passenger load M(x) against x plotted as a continuous function.

Essentially, the demand density pattern at any point x on the transit route will not necessarily be a function of transit parameters such as population density; residential density and land development density at x but related to the passenger travel demand densities along the feeder bus route network and line haul as well.The through passenger load M(x) of passengers is the basic data for analysing the demand for a transit line. More importantly, it provides the basic information on the number of passengers travelling past transit stations, and for that matter, every point on the line haul.

2.5 USER TIME COST

In this analysis, user time is defined as the travel time of a transit user riding by train and/or bus in the regions T_CT_R and T_RT_S respectively. The travel time of feeder buses on routes perpendicular to the rail centre line (Figure 2.1) is not related to the rail line length. It is therefore considered as independent of the length of rail line. The associated cost is irrelevant and hence not considered in the analysis. Furthermore, the travel time of feeder buses on all routes with directions parallel to the rail centre line, though dependent on the rail station locations, are independent of the rail line length and are therefore ignored in the analysis.

Consider M(x) number of passengers travelling an elemental distance dx in the direction T_CT_R (Figure 2.4). The total passenger travel (measured in passenger-kilometres) over the entire length X_R of the rail line is:

$$\int_0^{X_R} M(x)\,dx$$

Hence the daily cost of travel time by train in the direction $T_C T_R$ is:

$$\gamma_R \int_0^{X_R} M(x)\,dx \tag{2.2}$$

where γ_R is the average cost of travel time by train per passenger per kilometre. Also the daily user time cost for passengers travelling in the bus section in the direction $T_R T_S$ is given by:

$$\gamma_B \int_{X_R}^{L} M(x)\,dx \tag{2.3}$$

where γ_B is the average cost of travel time by feeder bus per passenger per kilometre.

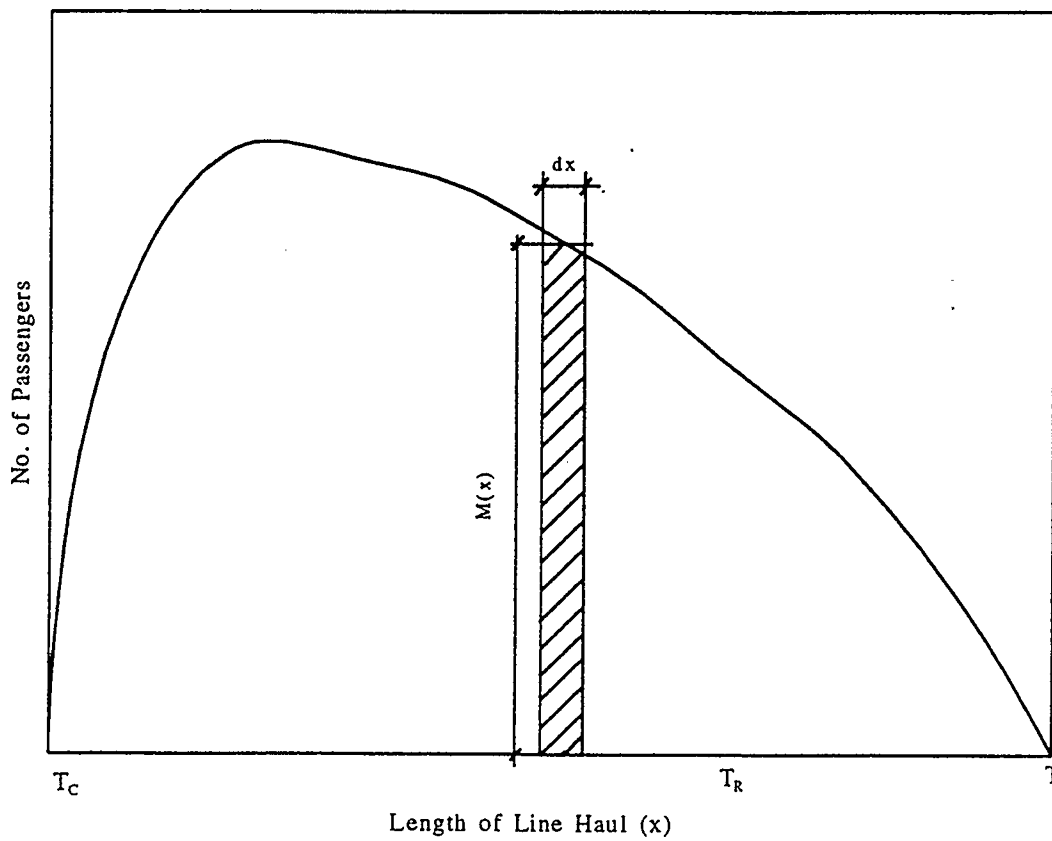

Figure 2.4 An Element of Demand: CBD-Suburban Transportation Corridor

Assuming that trips are returned by the same transit modes by retracing paths, then a symmetric trip is said to be generated. For a returned trip from T_S to T_C through T_R, the related user time cost by rail and bus is respectively given by Equations 2.2 and 2.3. Hence the total user time cost for a two directional travel is equal to twice the sum of Equations 2.2 and 2.3:

$$C_U = 2\gamma_R \int_0^{X_R} M(x)\,dx + 2\gamma_B \int_{X_R}^{L} M(x)\,dx \qquad (2.4)$$

The convenience of formulating demand by M(x) is that travel time costs can be formulated independently of individual origins and destinations, thus making the data collection exercise manageable.

2.6 RAIL AND BUS OPERATING COSTS

The purpose of operating any transit service is to transport passengers over some distance as quickly and safely as possible at a reasonable cost. Two types of operating cost units are suggested by planners (e.g. Stratton et al, 1960). These are operating cost per seat-kilometre and operating cost per passenger-kilometre. The unit of operating cost per seat-kilometre probably better reflects the real cost associated with operations of transit systems. Buses and trains are fully loaded during peak periods. However, they are lightly loaded at off-peak periods. Besides, the directional operation of vehicles during peek period is uneven; buses and trains are crowded in one direction, and are lightly loaded in the opposite direction. Trains exhibit degree of adjustment of seats to passenger usage since different train lengths are used in response to passenger loads at both peak and off-peak periods.

2.6.1 RAIL OPERATING COST CONSIDERING SEAT-KILOMETRES

During the morning and afternoon peak periods, more trains and buses are required to meet the high passenger demand. The associated passenger travel demand, expressed in seat-kilometres, is therefore high (maximum) in this case. However, during the off-peak periods, less trains and buses are in operation, giving a relatively low minimum passenger demand expressed in seat-kilometre. Besides, the passenger demand varies from train to train or bus to bus at distance X_R along the rail line (Figure 2.5) as well as the time t at peak and off-peak periods.

In planning exercises, it is undesirable to design considering only the travel demand at peak periods. However, it is reasonable to use an average travel demand expressed in seats by considering the daily maximum demand [$M_M(R)$] and minimum demand [$M_m(R)$] for train operations on the haul rail line at peak and off-peak respectively (Figure 2.5), and the daily maximum demand [$M_M(B)$] and minimum demand [$M_m(B)$] for bus operations on the haul bus line at peak and off-peak periods respectively as well.

Let $M_A(R)$ and $M_A(B)$ be the average daily number of passengers at a point on the haul rail line and haul bus line respectively. For a rail line length of X_R kilometres, the average travel demand in seat-kilometres, is $X_RM_A(R)$, and the associated daily rail operating cost of owing and operating a train a train per seat-kilometres is $\lambda_RX_RM_A(R)$ where λ_R is the average cost of operating a train per seat per kilometre. For a bus line of length (L-X_R) kilometres, the passenger travel demand expressed in seat-kilometres is (L-X_R)$M_A(B)$. The related daily bus operating cost is $\lambda_B(L-X_R)M_A(B)$ where λ_B is the average cost of owing and operating a bus per seat per kilometre. The daily rail and bus operating costs is given by the expression:

$$\lambda_R X_R M_A(R) + \lambda_B (L - X_R) M_A(B) \qquad (2.5.1)$$

Hence, for a two directional travel, the total daily rail and bus operating cost is given by twice the sum of Equation (2.5.1):

$$2[\lambda_R X_R M_A(R) + \lambda_B (L - X_R) M_A(B)] \qquad (2.5.2)$$

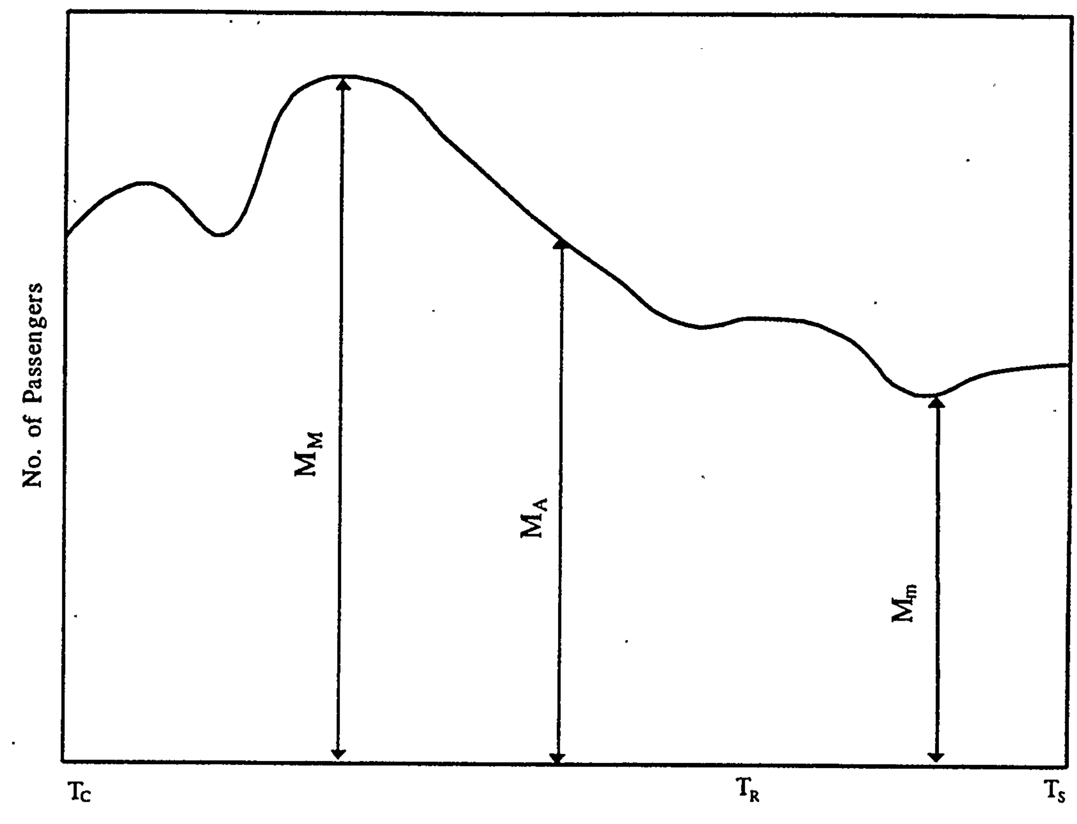

Figure 2.5 Variation of Passenger Load with Line Length

2.6.2 RAIL OPERATING COST CONSIDERING PASSENGER-KILOMETRES

It is possible to obtain the total daily travel demand and related rail and bus operating costs expressed in terms of passenger-kilometres. The operating cost of trains in rail section T_CT_R is obtained by replacing γ_R as presented in Equation 2.2.2 with λ_R, which is defined as the cost of operating a train per passenger-kilometre. Hence the rail operating cost becomes:

$$\lambda_R \int_0^{X_R} M(x)\,dx \tag{2.6.1}$$

Given the bus routing strategy discussed in Section 2.1, the cost of operating buses in a direction perpendicular to the rail centre line is considered independent of the rail line length and hence neglected. However, the relevant cost of operating buses along the corridor in the section T_RT_S is considered, and is expressed as:

$$\lambda_B \int_{X_R}^{L} M(x)\,dx \tag{2.6.2}$$

where λ_B is the average cost of operating a bus per passenger-kilometre. Considering two directional travel from T_C to T_S through T_R and back to T_C, the total rail and bus operating cost is expressed as:

$$2\lambda_R \int_0^{X_R} M(x)\,dx + 2\lambda_B \int_{X_R}^{L} M(x)\,dx \tag{2.7}$$

2.6.3 CHOICE OF APPROPRIATE OPERATING COSTS

The consideration of total rail and bus operating costs (Equation 2.5.2) developed from the concept of passenger travel demand expressed in seat-kilometre generates much complexities in the planning exercise. For that matter, the total rail and bus operating costs (Equation 2.7) based on passenger travel demand expressed in terms of passenger-kilometres is considered in the analysis. The units of passenger demand is a much more meaningful measure of demand for public transit systems in the sense that both the number of passengers and the distance of travel by passengers influences the cost of meeting the demand (Tyson, 1974).

It is imperative to remark that although trip production measured in seat-kilometres is greater than the related passenger demand expressed in passenger-kilometres for a given trip, there is an insignificant difference between the total daily rail and bus operating costs measured in terms of seat-kilometres and passenger-kilometres (Miller et al, 1973).

2.7 RAIL LINE COST

All capital and maintenance costs directly related to the rail line length such as land acquisition cost, design cost, rail track acquisition cost, rail track construction cost, station construction cost and utility relocation cost as well as maintenance cost of rail facilities are termed rail line cost. The total rail line cost is:

$$\int_{0}^{x_R} \gamma_L(x)\, dx \qquad (2.8)$$

where $\gamma_L(x)$ is the discounted rail line cost per kilometre per day at x. It should be explained that by formulating $\gamma_L(x)$ to be a function of x, allowance is made for variable land costs as well as costs of various line section scenarios (e.g. underground, at grade, elevated etc.) to be included in the analysis.

2.8 RAIL FLEET COST

Rail fleet costs are categorized as fixed capital costs. In this analysis, fleet size is defined as the number of seats (seating and standing passengers spaces) required to be dispatched during the peak period in the round trip time (r) with the maximum travel demand. The determination of rail fleet cost is based on the assumption that all trains will depart from and return to terminal T_C. The round trip time is thus defined as the time elapsed from the time of departure to the time when the train is ready for another trip. It is further assumed that, due to variation of passenger travel demand at both peak and off-peak periods, the number of trains required to carry passengers will vary. More vehicles will be used during the peak periods, with less vehicles used during off-peak operations. Peak period operations will be separated by short headways. However, the headways will be longer at off-peak periods.

The formulation of fleet cost is based on afternoon peak operations. This is due to the fact that the maximum passenger demand is most likely to occur at afternoon peak hours. The reason is that most passengers who have made a trip during morning peak period and day off peak period will be making a home journey at the afternoon peak. However, this analysis can also be applied to morning peak period with some variations. It is also assumed that the chosen vehicle headway is such that the estimated load on a train at the maximum load point does not exceed the vehicle capacity (Wirasinghe, 1990). It is given that buses will be provided to service the line haul in T_RT_S. Since the buses will be operated as single units and can also be used in other routes, the cost of owning and operating the buses is included in the bus operating cost. Also, it is assumed that LRT systems will be acquired and operated on the rail line T_CT_R. Invariably, the acquisition of the LRT vehicles requires a high initial capital upfront. Hence, prominence is given to the determination of rail fleet cost in this analysis.

Consider a situation where passengers can board and alight from the LRT system at any station location x_i on the line haul. Assume that demand for travel is expressed in terms of seats per unit time, and a seat is used in series by more than one passenger. Let the cumulative number of boarding and alighting passengers at station location i in a train j dispatched at time t_j during the afternoon peak period be $B_i(t_j)$ and $A_i(t_j)$ respectively. Then the maximum load or the demand for seats in the vehicle is:

$$s(t_j) = Max_i \; [B_i(t_j) - A_i(t_j)] \qquad (2.9.1)$$

The cumulative demand $S(t_j)$, expressed in terms of seats, for all vehicles up to time t_j during the afternoon peak period is given as:

$$S(t_j) = \sum_j s(t_j) \qquad (2.9.2)$$

Figure 2.6.1 shows a graph of cumulative demand $S(t_j)$ against t_j plotted as a step function, and smoothened out to obtain the cumulative demand S(t) at any time t on the line haul. The slope s(t) at any time t, which is the demand in terms of seats per unit time, is given by:

$$s(t) = dS(t)/dt \qquad (2.9.3)$$

In particular, the slopes s(t) at times t plotted against time t will, in all likelihood, assume a parabolic shape as shown in Figure 2.6.2 (Seneviratne et al, 1986). The maximum fleet size M(r) for the afternoon peak is the number of seats required to be dispatched during the round trip time r which includes the maximum demand per unit time M^* of travel where M^* is defined as the maximum demand per unit time during the afternoon peak

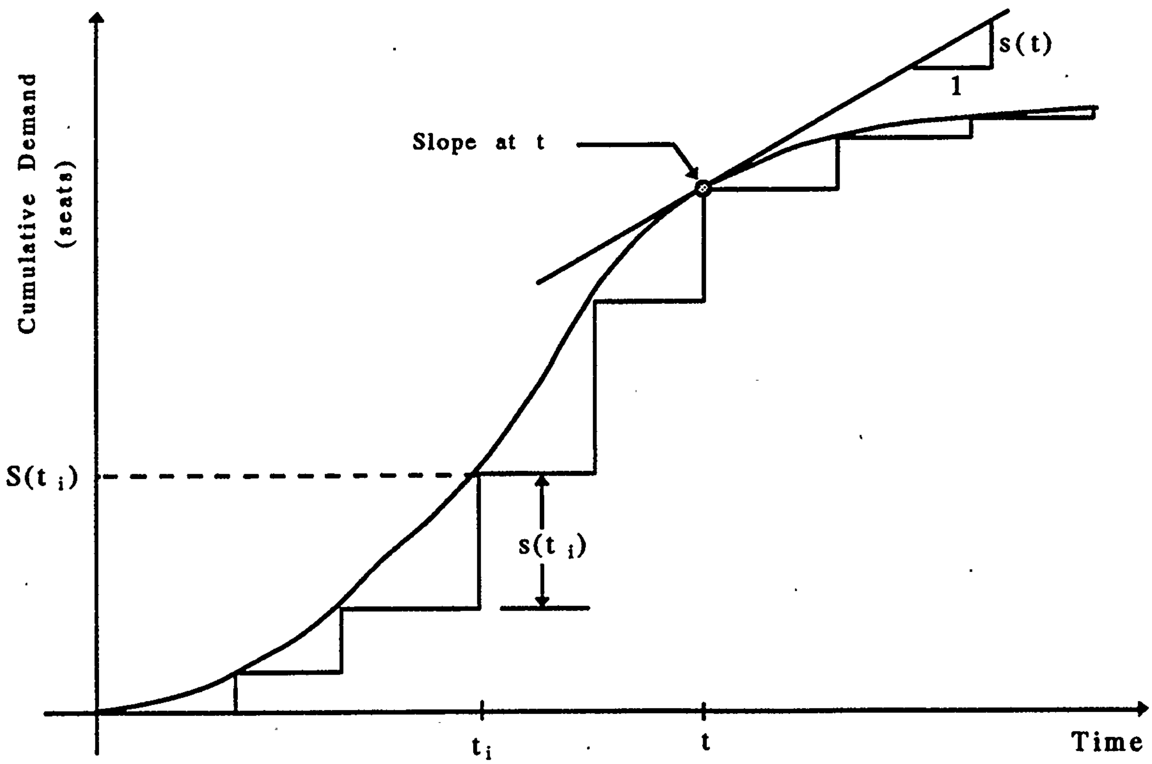

Figure 2.6.1 Variation of Many to Many Demand with Travel Time

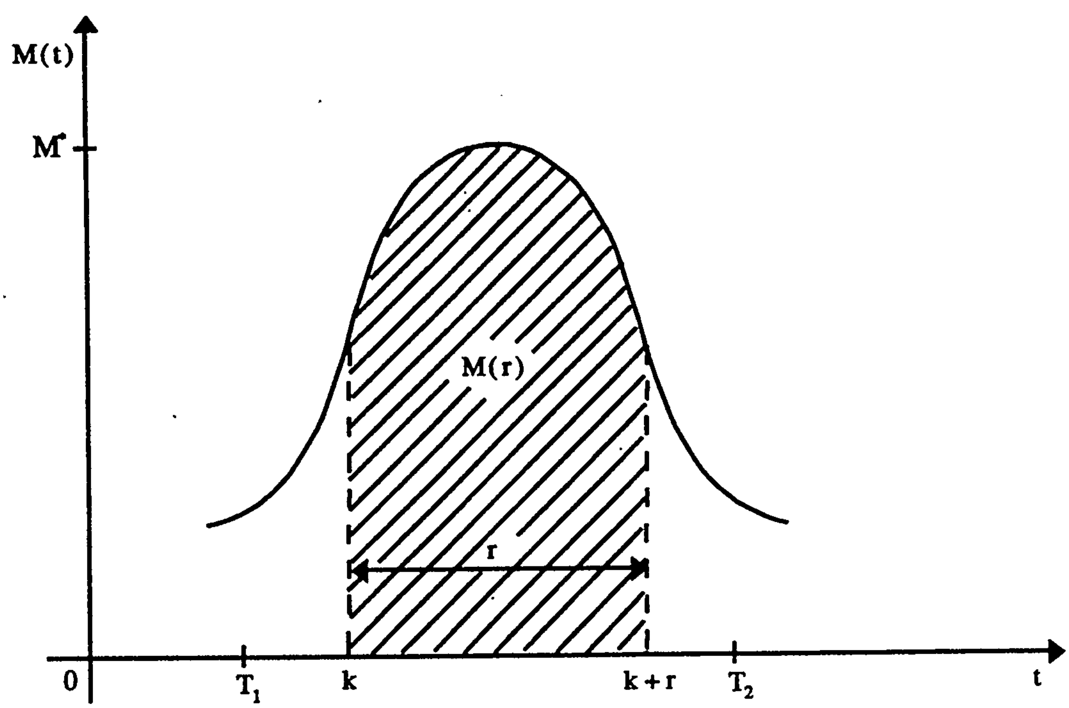

Figure 2.6.2 Variation of Many to Many Demand with Travel Time

period.

2.8.1 PARABOLIC VARIATION OF DEMAND WITH TIME

Senevirante et al (1986) reported that the demand per unit time p(t) for a many to one travel demand pattern during the morning commute period is parabolic in shape. They formulated the total corridor demand P during the morning commute period as:

$$P = \int_{T_1}^{T_2} p(t)\,dt \qquad (2.10)$$

and concluded that the minimum rail fleet size M(r) for commute period is:

$$M(r) = \int_{k}^{k+r} p(t)\,dt \qquad for \quad r < T_2 - T_1 \qquad (2.11.1)$$

$$M(r) = P \qquad for \quad r \geq T_2 - T_1 \qquad (2.11.2)$$

They also disclosed that in the absence of independent estimates, the maximum demand per unit time during commute period P_m is approximated by:

$$P_m = 6P/5(T_2 - T_1) \qquad (2.12)$$

provided the shape of the p(t) curve is parabolic.

The shape of travel demand per unit time M(t) as a function of time t for many to many demand during the afternoon peak hour period (Figure 2.6.3) is found to be similar to the shape of p(t) for a many to one demand during the morning commute period as reported by Senevirante et al (1986). Transit data on maximum passenger ridership demand in the train during the afternoon peak period, at the maximum load point, for a many to many demand travel pattern on the N-W LRT line in Calgary, plotted

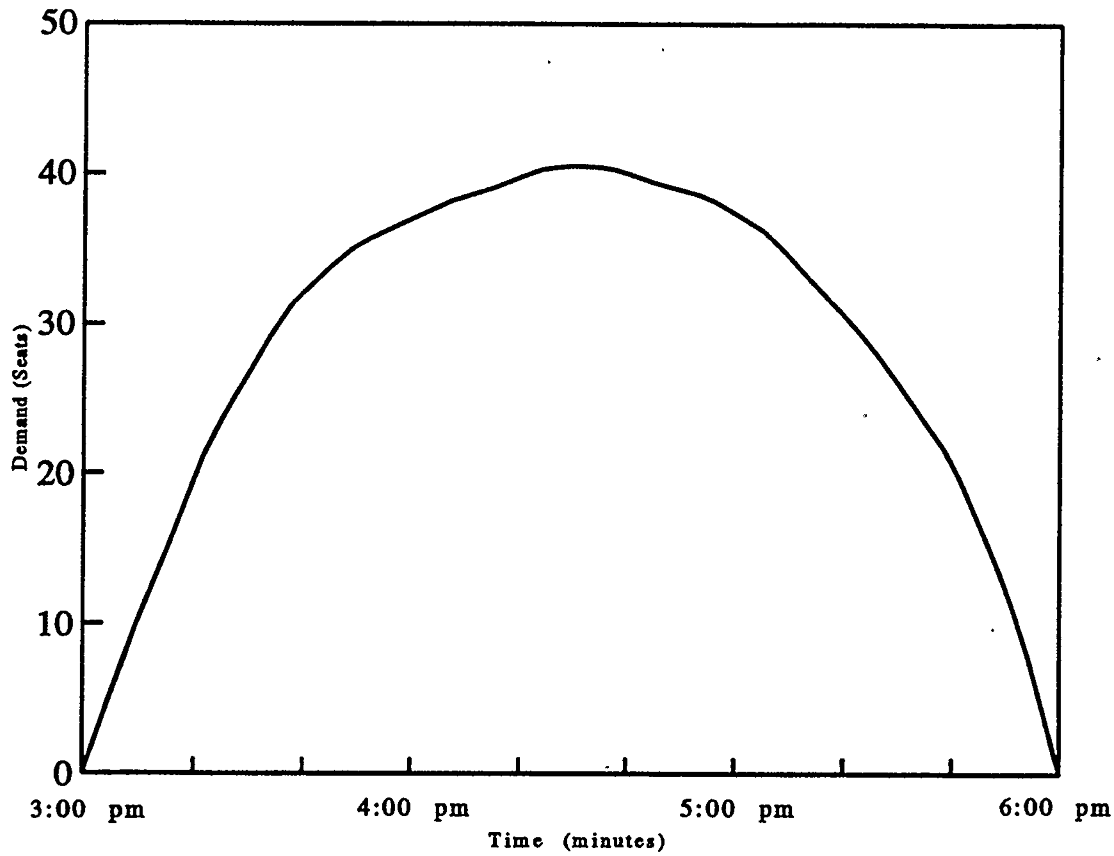

Figure 2.6.3 Afternoon Peak-Period Travel Demand Profile: CBD-Brentwood LRT Line in Calgary, Alberta

as a function of passenger demand per unit time M(t) against travel time t of trains confirm this finding (Figure 2.6.3).

Consider a case of passenger many to many travel demand pattern during the afternoon peak hour period. The graph of passenger many to many demand per unit time M(t) against time t is shown in Figure 2.6.2. In urban transit planning, the choice of appropriate round trip time r for transit vehicles operating on CBD-Suburban haul rail line (Figure 2.1) is selected so as not to exceed the duration of afternoon peak period T_2-T_1, where T_1 and T_2 respectively define the starting and ending times of the round trip time.

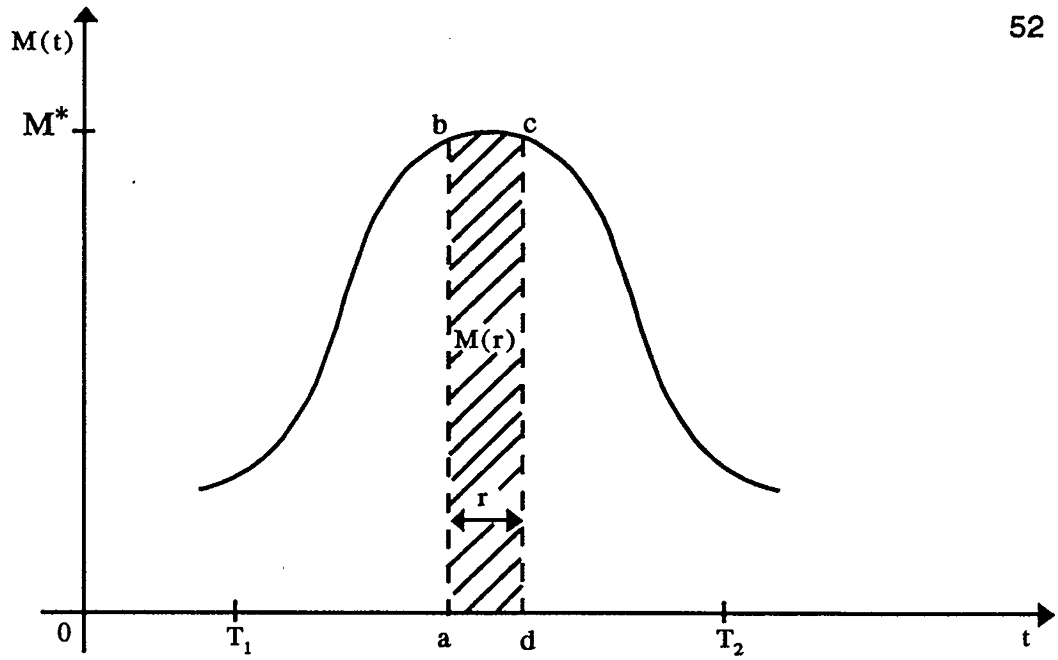

Figure 2.6.4 Variation of Many to Many Demand with Travel Time for Small r

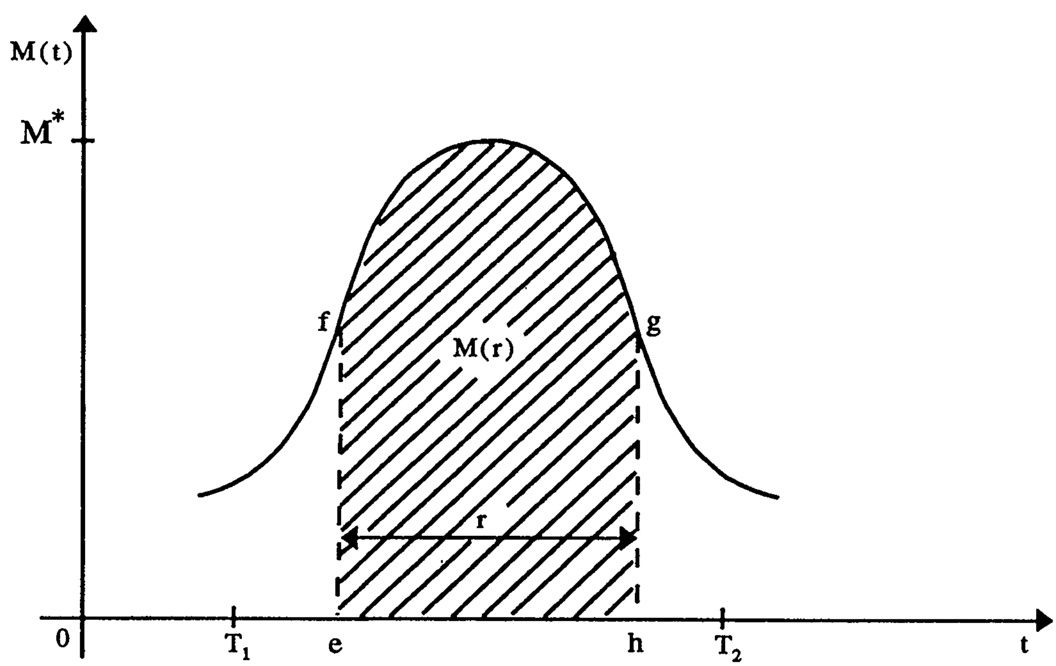

Figure 2.6.5 Variation of Many to Many Demand with Travel Time for Large r

It is therefore assumed in the ensuing analysis that the determination of fleet size M(r) for transit vehicles is based on condition that $r<T_2-T_1$. To this end, the determination of fleet size under the condition that $r>T_2-T_1$ is not considered.

If the round trip time r is small, the resulting fleet size M(r) approximates to a rectangular shape abcd (Figure 2.6.4) with value as rM^*. However if r is large, the associated M(r) is represented by the parabolic region efgh (Figure 2.6.5) having a value of $2rM^*/3$. Particularly, the average fleet sizes ($5rM^*/6$) give a reasonable fleet size estimates required for actual planning exercise and design. Allowing for fixed size of transit vehicles as well as stand-by vehicles, the average fleet size is increased by about 20% (Seneviratne et al, 1986), giving an approximate designed fleet size of value rM^*.

2.8.2 DETERMINATION OF RAIL FLEET COST

Let the round trip time r be:

$$r = 2X_R\Lambda_R + \tau_R \qquad (2.13)$$

where X_R is the length of a line, Λ_R is the inverse of train operating speed and τ_R is the sum of the turn around and average lay over times at terminal T_C and T_R. Let the discounted cost of a rail-vehicle per passenger space per day be λ_F. Then the total rail fleet cost per day C_F is given by the expression:

$$C_F = \lambda_F r M^* \qquad (2.14)$$

Substitution of Equation 2.12 into Equation 2.13 gives:

$$C_F = \lambda_F(2X_R\Lambda_R+\tau_R)M^* \qquad (2.15)$$

2.9 PASSENGER TRANSFER PENALTY COST

The decline in public transit travel in North America over the past few years has generated great concern, especially in the areas of transit planning. Factors identified as causes of decrease in public transit demand include high user riding time on transit systems user discomfort and inconvenience, lack of passengers safety and personal security and high transit fares. Others are the unreliable nature of public transit systems, generally poor level of transit services and lack of accessibility of transit system at some residential areas in suburban regions (TRRL, 1980). More importantly, the problems associated with passenger transfers from a mode of transit system to a similar or different transit modes are identified as a major factor influencing public transit demand (Doornenbal, 1985).

Passenger transfer is inherent in public transportation services, and it takes the form of change from a transit mode to a similar or dissimilar transit mode. Passenger transfer times include time taken by a passenger to exit from a transit system, walk to boarding area, wait for another transit system of similar or different mode and finally enter another transit system departure.

Results of the passenger transfer survey by Doornenbal (1985) shown that transit riders perceive public transit trips as significantly worse when the trip requires a transfer even if transfer time is negligible. Passenger transfer is identified by public transit planners (e.g. Bates, 1978) as one of the major factors that affects public transit ridership, and to some extent, influences the overall public transportation cost and benefits for a given period of time. Therein lie the reason for consideration of passenger transfer cost in this analysis.

First we consider passengers who are not within walking distance of a station or line haul bus stop either at the origin or destination of the trip. Figure 2.7 shows a typical passenger transfer phenomenon for a public transit system considering a CBD-Suburban transportation corridor. A passenger from O_1 to D_1 or O_2 to D_2 will experience one type of transfer from bus to bus at T_1 or T_2, or no transfer depending on the feeder bus route configuration. However, a passenger travelling from O_3 to D_3 will be subjected to two types of transfer. These are transfer from bus to bus at T_S followed by transfer from bus to bus at T_R. Also, passenger travelling from O_4 to D_4 will experience two types of transfer. These consist of transfer from bus to train at T_4 and then transfer from train to bus at T_C. A passenger travelling from O_5 to D_5 will experience three types of transfers. These are transfer from bus to bus at T_1, bus to train at T_R and train to bus at T_5. In much the same way, a passenger travelling from O_6 to D_6 will experience three types of transfer; transfer from bus to train at T_6, train to bus at T_R and finally bus to bus at T_S.

From the above description of passenger trip characteristics, it is observed that at most two types of transfers are recorded for passenger trips originating and terminating at the bus or rail region. However, three types of transfers are obtained for passenger trips originating at the bus region and terminating at the rail region, or vice versa. This is due to the fact that passengers travelling from bus region to rail region, or from rail region to bus region, are "forced" to transfer from one type of transit mode to a different type of transit mode at T_R. Thus an additional transfer is needed at T_R for all passengers crossing from bus region T_ST_R to rail region T_RT_C and vice versa. This penalty exists for

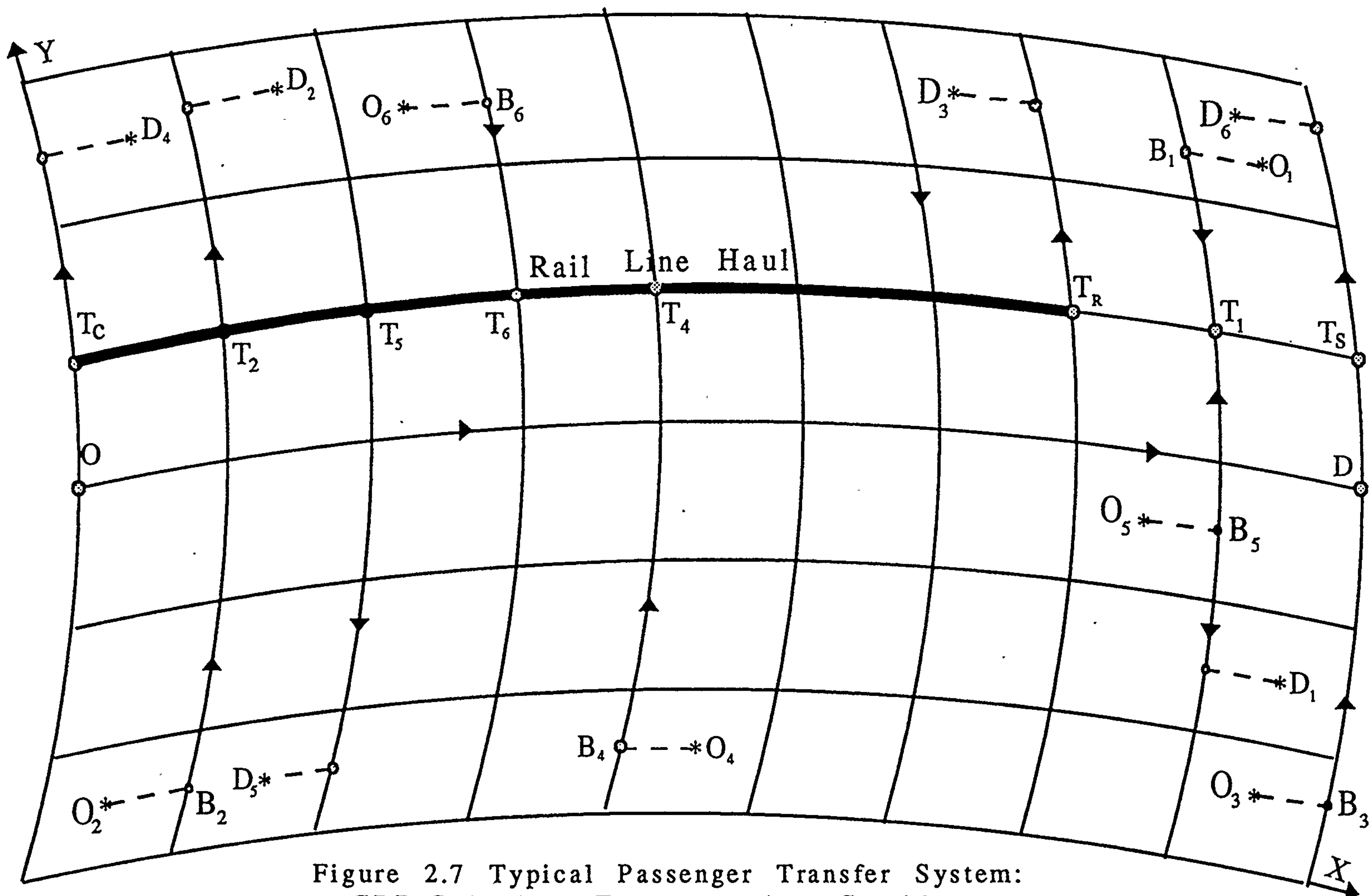

Figure 2.7 Typical Passenger Transfer System:
CBD-Suburban Transportation Corridor

passengers who do not need to use a feeder bus at the origin and/or destination, as long as they travel from $T_C T_R$ to $T_R T_S$.

Considering the passengers who walk at one end of the trip to a line haul stop or station, it is clear that only one form of transfer is needed unless they have to travel past the point T_R. No transfers are needed for passengers who can walk at both ends of their trips unless they have to cross T_R. It is therefore observed that all passengers who travel past T_R are subjected to an extra transfer in comparison to those whose travel is confined to either the $T_C T_R$ or $T_R T_S$ regions. The inconvenience associated with the transfer process is termed "passenger transfer penalty cost". The effects of this penalty is to cause the rail line length to be longer.

2.9.1 ANALYSIS FOR PASSENGER TRANSFER PENALTY COST

It is assumed in this research that the maximum number of transfers by a passenger for a given trip is two. This is in agreement with the research finding on public transfer studies undertaken by Hunt (1990). He disclosed that potential public transit riders will not patronise transit trips characterized by three or more transfers. Thus passengers who use the system described here to travel from the rail region $T_C T_R$ to the bus region $T_R T_S$ are those who need to use a feeder bus at only one end of the trip. This is due to the fact that the number of passengers transferring from train to bus at T_R is directly dependent on the rail line length, and the associated passenger transfer penalty cost is therefore used in the analysis.

Consider a passenger trip from T_C to T_S, with transfer of passengers from train to bus occurring at terminal T_R, a distance X_R from the central terminus T_C. The passenger transfer penalty cost associated with this trip is expressed as :

$$\gamma_P M(X_R) \tag{2.16}$$

where γ_P is the average cost of transferring per passenger and $M(X_R)$ is the number of transfer passengers at X_R. For a returned trip from T_S to T_C, the number of passengers transferring from train to bus at T_R is $M(X_R)$ and the related passenger transfer penalty cost is also :

$$\gamma_P M(X_R) \tag{2.17}$$

Hence the total passenger transfer cost is given by the sum of Equations 2.16 and 2.17:

$$2\gamma_P M(X_R) \tag{2.18}$$

2.10 OPTIMIZATION

The analysis will seek to determine the optimal rail line length with the objective of minimizing the sum of user time cost, rail and bus operating costs, rail line cost, rail fleet cost and passenger transfer penalty cost. The global transportation cost $[Z(X_R)]$ is obtained by summing Equations 2.4, 2.7, 2.8, 2.15, and 2.18. Thus:

$$
\begin{aligned}
Z(X_R) &= 2\gamma_R\int_0^{X_R} M(x)\,dx + 2\gamma_B\int_{X_R}^{L} M(x)\,dx \\
&+ 2\lambda_R\int_0^{X_R} M(x)\,dx + 2\lambda_B\int_{X_R}^{L} M(x)\,dx \\
&+ \int_0^{X_R}\gamma_L(x)\,dx + (2X_R\Lambda_R+\tau_R)\lambda_R M^* \\
&+ 2\gamma_P M(X_R)
\end{aligned}
\tag{2.19}
$$

The optimal rail line length which minimizes the total transportation cost is obtained by taking the derivative of Equation 2.19 and setting the resulting expression to zero: gives:

$$
\begin{aligned}
Z'(X_R) &= 2[(\gamma_R-\gamma_B) + (\lambda_R-\lambda_B)]M(X_R) + \gamma_L(X_R) \\
&+ 2\Lambda_R\lambda_F M^* + 2\gamma_P M'(X_R)
\end{aligned}
\tag{2.20.1}
$$

and therefore:

$$
\begin{aligned}
2[(\gamma_B-\gamma_R)+(\lambda_B-\lambda_R)]M(X_R) &= \gamma_L(X_R) + 2\Lambda_R\lambda_F M^* \\
&+ 2\gamma_P M'(X_R)
\end{aligned}
\tag{2.20.2}
$$

For a minimum total transportation cost to be obtained, the second derivative of Equation 2.19 with respect to X_R should be positive. Thus:

$$
\begin{aligned}
Z''(X_R) &= 2[(\gamma_R-\gamma_B)+(\lambda_R-\lambda_B)]M'(X_R) \\
&+ \gamma_L'(X_R) + 2\gamma_P M''(X_R) > 0
\end{aligned}
\tag{2.21}
$$

2.11 GRAPHICAL ANALYSIS

More insight regarding the determination of the optimal rail line length is gained by exploration of the total cost function (Equation 2.19) and its first and second derivatives (Equations 2.20.1 and 2.21 respectively) under some simplifying assumptions. Firstly, for the sake of convenience and simplicity, the passenger transfer cost expression (Equation 2.18) and it resulting first and second derivative are neglected in the foregoing analysis. Furthermore, the substitution of $\Delta\gamma = 2[(\gamma_B-\gamma_R)+(\lambda_B-\lambda_R)]$ and $\theta = 2\Lambda_R\lambda_R M^*$ into Equations 2.20 and 2.21 respectively gives:

$$Z'(X_R) = -\Delta\gamma M(X_R) + \gamma_L(X_R) + \theta \qquad (2.22)$$

$$Z''(X_R) = -\Delta\gamma M'(X_R) + \gamma_L'(X_R) \qquad (2.23)$$

2.12 UNIFORM RAIL LINE COST

It is possible to assume that the discounted rail line cost per kilometre per day $[\gamma_L(X_R)]$ is approximately constant under certain conditions. For instance, the rail line might be a continuous subway or it may run throughout along a highway median or a main rail line right-of-way. If the rail line cost is uniform, then the first derivative of the daily discounted rail line cost per kilometre per day $[\gamma_L'(X_R)]$ is zero. Equations 2.22 and 2.23 are respectively reduced to:

$$Z'(X_R) = -\Delta\gamma M(X_R) + \gamma_L + \theta \qquad (2.24.1)$$

$$Z'(X_R) = -\Delta\gamma M'(X_R) \qquad (2.24.2)$$

Figure 2.8.1 depict the graph of through passenger load $M(X_R)$ against length of line haul X_R plotted as a continuous function. Typically, $M(X_R)$ is zero at T_C where X_R=0. $M(X_R)$ then increases till it attains its maximum load M_L at the maximum load point T_L where X_R=X_L. From T_L, $M(X_R)$ decreases till it is zero at T_S where X_R=L. In particular,

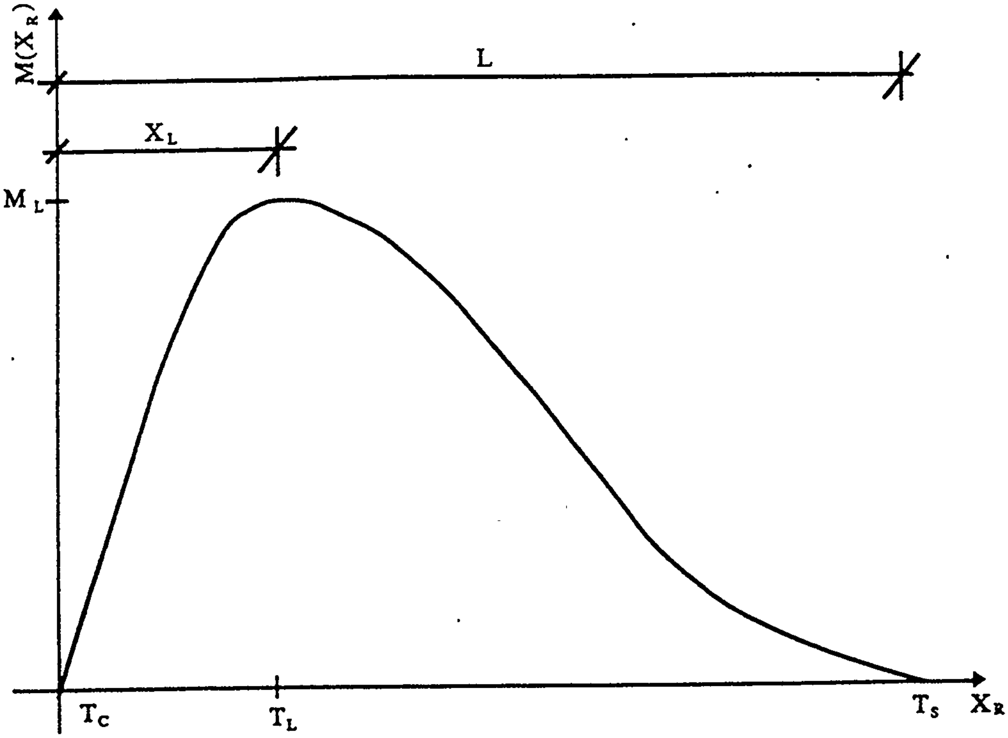

Figure 2.8.1 Variation of Daily Through Passenger Load with Line Length

$Z(X_R)$ is minimized if $Z'(X_R)=0$ and $Z''(X_R)>0$. From Equation 2.24.2, $Z''(X_R)>0$ only if $M(X_R)<0$, i.e. $M(X_R)$ is decreasing. This occurs in the region $X_L \leq X_R \leq L$. Conversely, $Z(X_R)$ is maximized when $Z'(X_R)=0$ and $Z''(X_R)<0$. $Z''(X_R)<0$ provided $M'(X_R)>0$, i.e. $M(X_R)$ is increasing. This is found to occur in the region $0 \leq X_R \leq X_L$. The solutions to Equation 2.24.1 depend on whether $\gamma_L+\theta>\Delta\gamma M_L$ or $\gamma_L+\theta<\Delta\gamma M_L$ as explained below.

2.13 CASE STUDIES

2.13.1 CASE 1: $\gamma_L + \theta > \Delta\gamma M_L$

It is possible that no solution will exists in this case. Hence the optimal $X_R=0$ (Figure 2.8.2). This possibility increases with the above inequality. Furthermore, since

$$\gamma_L + \theta > \Delta\gamma M_L \qquad (2.25.1)$$

then

$$M_L < \frac{\gamma_L + \theta}{\Delta\gamma} \qquad (2.25.2)$$

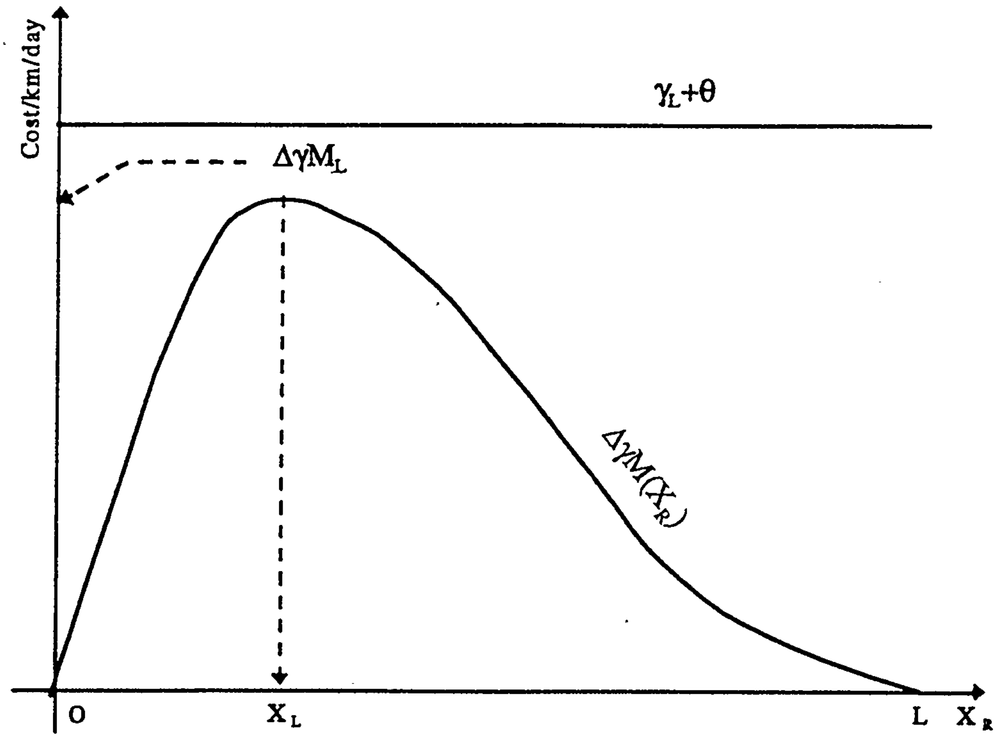

Figure 2.8.2 Optimal Line Length (Uniform Line Cost)

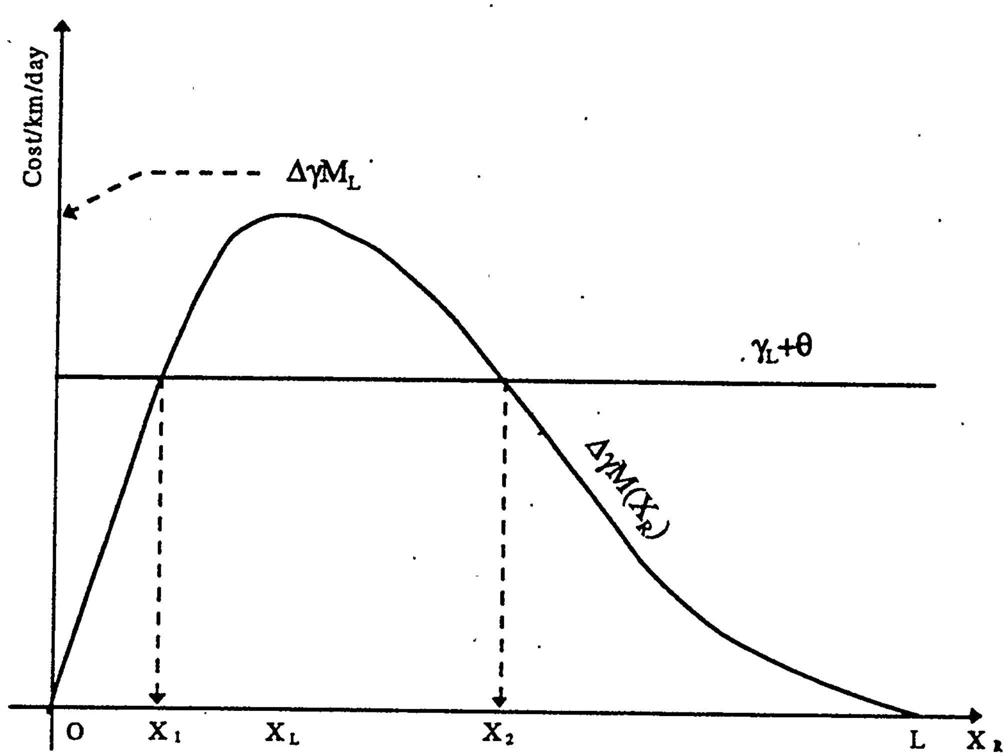

Figure 2.8.3 Optimal Line Length (Uniform Line Cost)

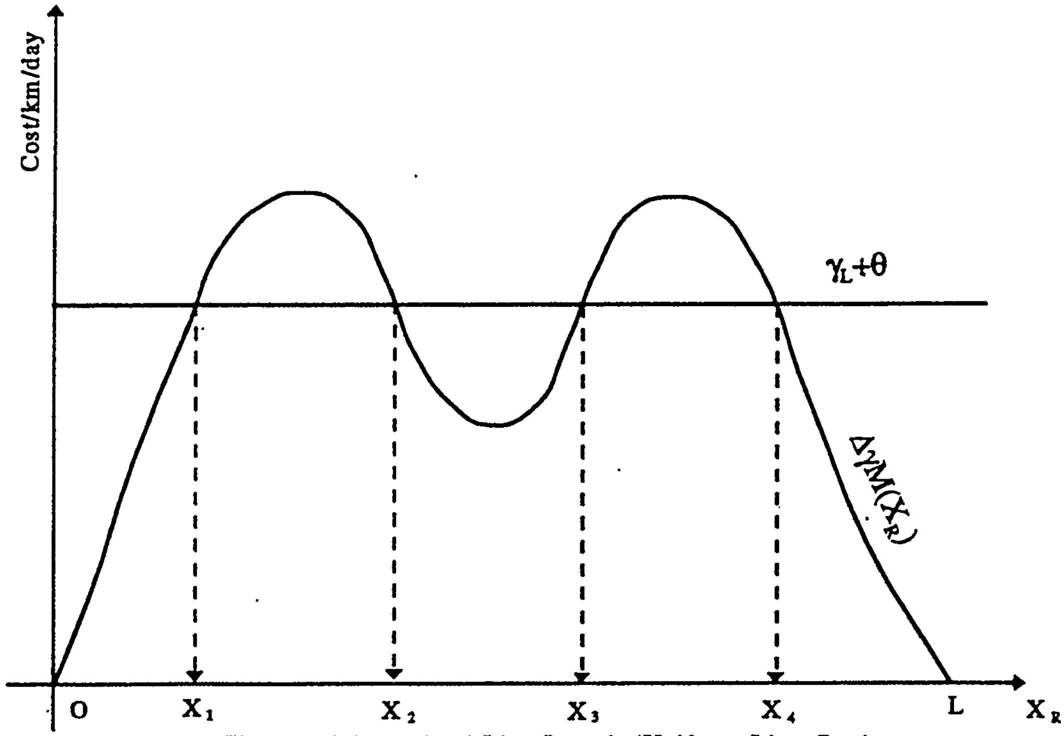

Figure 2.8.4 Optimal Line Length (Uniform Line Cost)

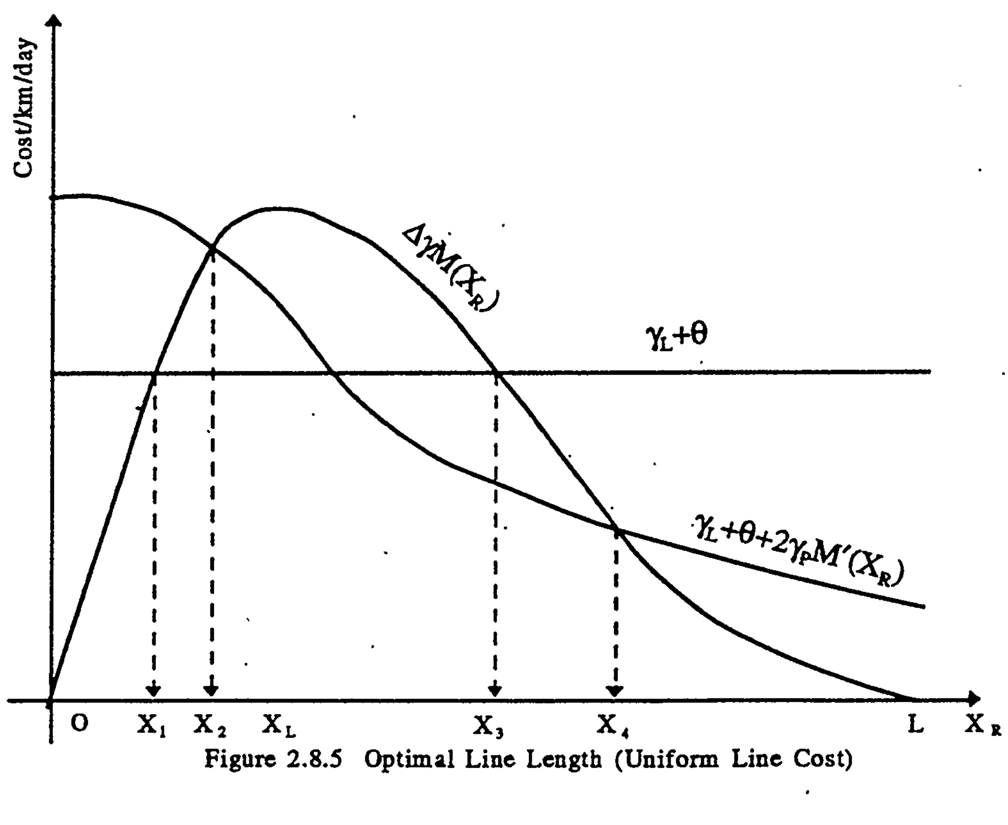

Figure 2.8.5 Optimal Line Length (Uniform Line Cost)

where θ is a function of M^*.

2.13.2 CASE 2: $\gamma_L + \theta < \Delta\gamma M_L$

In this case, there exist the possibility of obtaining two solutions of X_R (Figure 2.8.3). The total cost is locally minimized at X_2 since $Z''(X_2)>0$ at X_2. The optimal $X_R=X_2$. However, the cost is maximized locally at X_1 since $Z''(X_1)<0$.

2.13.3 SPECIAL CASE

Figure 2.8.4 describe the possibility of obtaining four solutions of X_R. Obviously, the total cost is minimized locally at X_2 or X_4. Under this condition, the total cost at X_2 and X_4, i.e. $Z(X_2)$ and $Z(X_4)$, are determined and compared. The length that gives the overall minimum cost is the global optimal.

2.13.4 CASE 4: GENERAL CASE

An insight regarding the effect of passenger transfer penalty cost (Equation 2.18) on the optimal rail line length is explored by graphical analysis (Figure 2.8.5). Consideration of passenger transfer penalty cost changes Equations 2.24.1 and 2.24.2 respectively to:

$$Z'(X_R) = -\Delta\gamma M(X_R) + \gamma_L + \theta + 2\gamma_P M'(X_R) \qquad (2.26.1)$$

and

$$Z''(X_R) = -\Delta\gamma M'(X_R) + 2\gamma_P M''(X_R) \qquad (2.26.2)$$

It is observed from Equation 2.26.2 that $Z''(X_R)>0$ if $M'(X_R)<0$ and $M''(X_R)<0$. However, $Z''(X_R)<0$ provided $M'(X_R)>0$ and $M''(X_R)<0$. Two solutions of X_R (i.e. X_2 and X_4) are obtained under this condition, and the total cost is found to be minimized locally at X_4

(Figure 2.8.5). It is observed from Figure 2.8.5 that if transfer penalty cost is not considered, the optimal line length is X_3, which is less than X_4. Hence, the effect of the transfer penalty cost is to cause the rail line length to be longer.

2.14 NON-UNIFORM RAIL LINE COST

Under some conditions, the rail line is constructed to run partly along a highway median, subway, elevated structures and main rail line right-of-way. The rail line is therefore described to be non-uniform. The discounted rail line cost per unit length per day [$\gamma_L(X_R)$] will vary with line length. Generally, one expect $\gamma_L(X_R)$ to decrease as X_R increases, i.e. as the line moves away from the CBD to the Suburban Region (Figure 2.9.1).

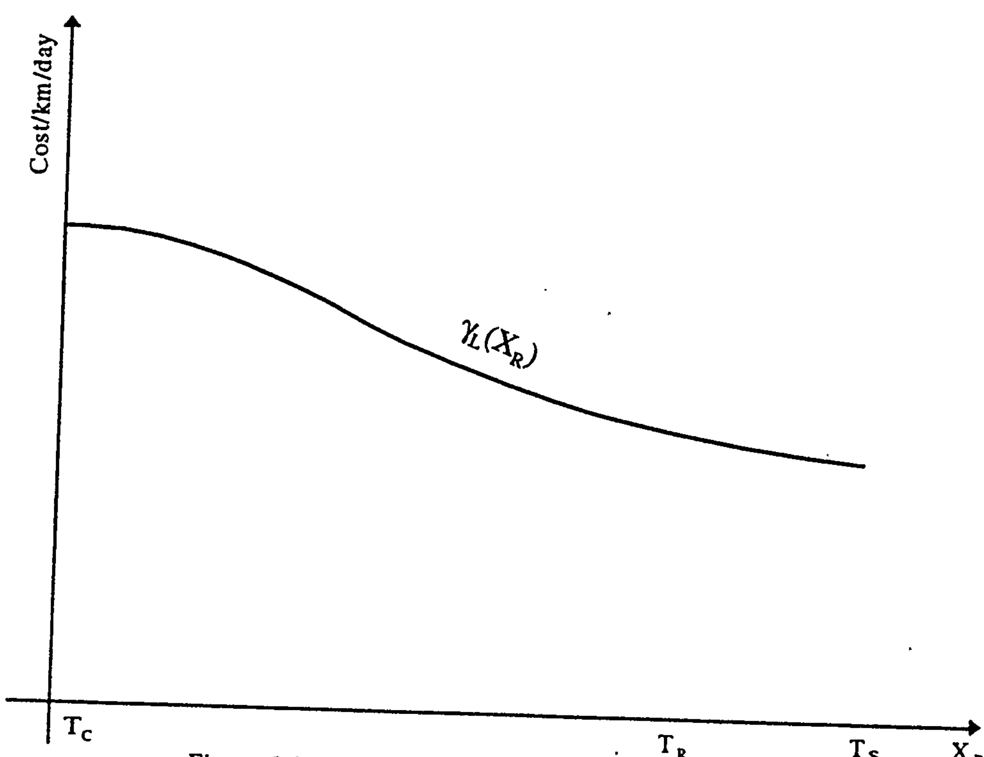

Figure 2.9.1 Variation of Rail line Cost with Line Length

In this case, $Z'(X_R)$ and $Z''(X_R)$ given by Equations 2.22 and 2.23 will apply in this case. In particular, $Z(X_R)$ is minimized if $Z'(X_R)=0$ and $Z''(X_R)>0$. From Equation 2.23, it is found that $Z''(X_R)>0$ provided $M'(X_R)<0$ and $\gamma_L'(X_R)>0$. This occurs in the region $X_L \leq X_R \leq L$. $Z(X_R)$ is maximized when $Z'(X_R)=0$ and $Z''(X_R)<0$. $Z''(X_R)<0$ if $M'(X_R)>0$ and $\gamma_L'(X_R)<0$. This occur in the region $0 \leq X_R \leq X_L$. The solutions to Equation 2.23 depend on whether $\gamma_L(X_R)+\theta>\Delta\gamma M_L$ or $\gamma_L(X_R)+\theta<\Delta\gamma M_L$ as in the following analysis.

2.14.1 CASE 1: $\gamma_L(X_R) + \theta > \Delta\gamma M_L$

Under the above condition, it is possible that no solution will exist (Figure 2.9.2). Hence, the optimal $X_R=0$. This possibility increases with the above inequality. For

$$\gamma_L(X_R) + \theta > \Delta\gamma M_L \tag{2.27.1}$$

then

$$M_L < \frac{\gamma_L(X_R) + \theta}{\Delta\gamma} \tag{2.27.2}$$

2.14.2 CASE 2: $\gamma_L(X_R) + \theta < \Delta\gamma M_L$

It is possible to obtain two solutions of X_R under this condition (Figure 2.9.3). The total cost is locally minimized at X_2 since $Z''(X_2)>0$ at X_2. Hence the optimal $X_R=X_2$. However, the total cost is maximized locally at X_1 since $Z''(X_1)<0$ at X_1.

2.14.3 SPECIAL CASE

The chances of obtaining four solutions of X_R is graphically illustrated in Figure 2.9.4. In this case, the total cost is minimized locally at X_2 or X_4. The total cost at X_2 and

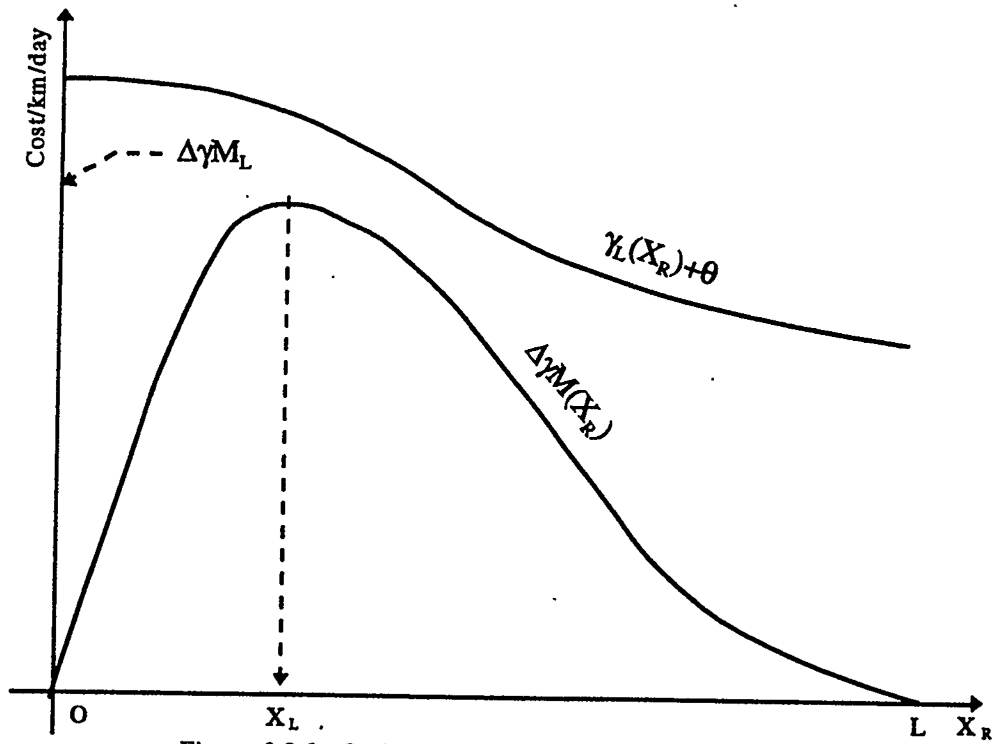

Figure 2.9.2 Optimal Line Length (Non-Uniform Line Cost)

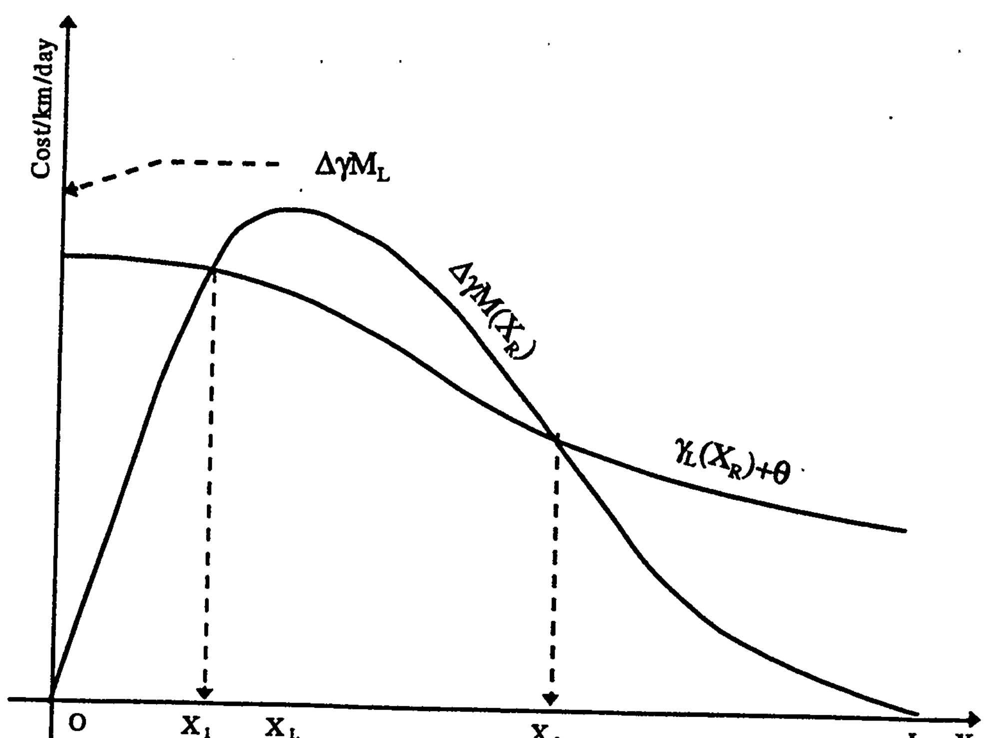

Figure 2.9.3 Optimal Line Length (Non-Uniform Line Cost)

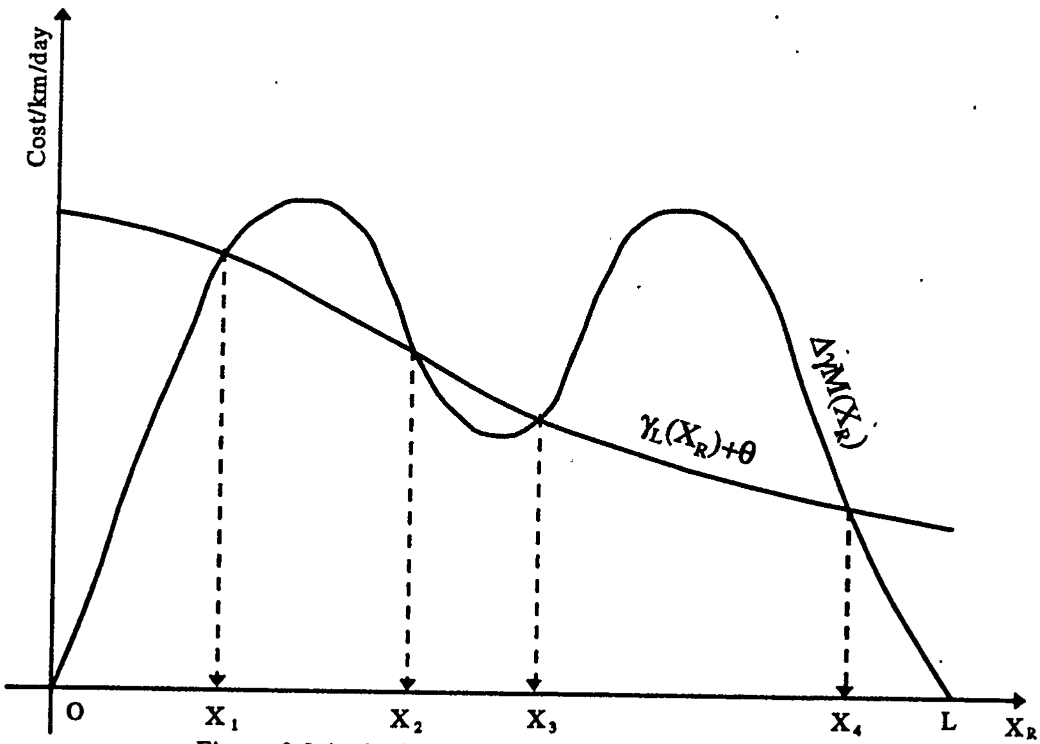

Figure 2.9.4 Optimal Line Length (Non-Uniform Line Cost)

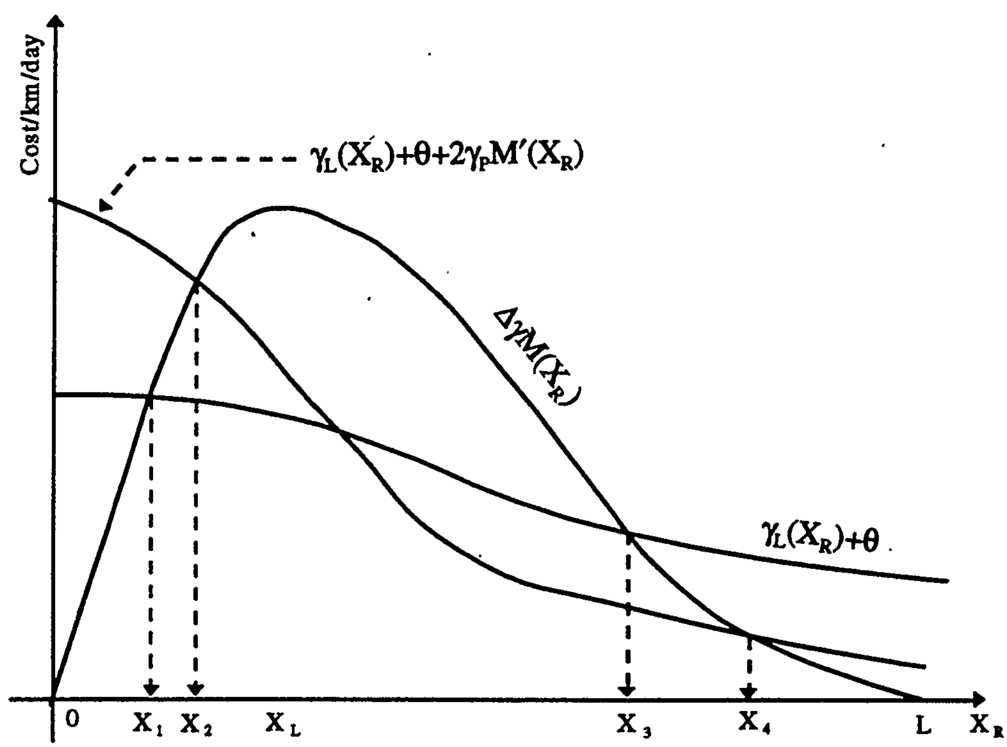

Figure 2.9.5 Optimal Line Length (Non-Uniform Line Cost)

X_4, i.e. $Z(X_2)$ and $Z(X_4)$, are enumerated and compared. The length that gives the overall minimum cost is the global optimal.

2.14.4 CASE 4: GENERAL CASE

The effect of passenger transfer penalty cost (Equation 2.18) on the optimal rail line length X_R is graphically explained using Figure 2.9.5. By considering passenger transfer penalty cost, Equations 2.22 and 2.23 respectively become:

$$Z'(X_R) = -\Delta\gamma M(X_R) + \gamma_L(X_R) + \theta + 2\gamma_P M'(X_R) \qquad (2.28.1)$$

and

$$Z''(X_R) = -\Delta\gamma M'(X_R) + \gamma_L'(X_R) + 2\gamma_P M''(X_R) \qquad (2.28.2)$$

It is found from Equation 2.28.2 that $Z''(X_R)>0$ if $M'(X_R)<0$, $\gamma_L'(X_R)>0$ and $M''(X_R)>0$. However, $Z''(X_R)<0$ provided $M'(X_R)>0$, $\gamma_L(X_R)<0$ and $M''(X_R)<0$. Two solutions of X_R (i.e. X_2 and X_4) are obtained under this condition. The total cost is minimized locally at X_4 (Figure 2.9.5). From Figure 2.9.5, it is observed that if transfer penalty cost is neglected, the optimal X_R is X_3, which is less than X_4. Therefore, the effect of the transfer penalty cost is to increase the rail line length.

2.15 MODEL APPLICATION

This section of the analysis discusses the application of the proposed analytical model to the existing North-West LRT corridor in Calgary, Alberta. More particularly, an attempt to determine if the North-West LRT line of length 8.30km is optimal and economic is discussed. Over the past years, rail lines are built along transportation corridors considering factors such as high population density, high residential density and high land development activities within the corridors (Yeates et al, 1980). Usually, comprehensive studies are not undertaken to investigate if the proposed rail line length to be constructed is optimal and economic. Essentially, analytical optimization models can be developed and used to obtain some reasonable, practical and optimal rail line length.

According to the North-West Calgary Functional Study Report (1980), the proposed overall length of the LRT line to be constructed is approximately 9.65km. However, due to the problem of scarcity of public funds needed to construct this high capital intensive project, the LRT line is subjected to a stage construction process.To date, a line length of 8.30km radiating from the heart of the CBD to Brentwood Station is constructed (Figure 2.10). The line consists of seven stations.

The proposed analytical model will be used to obtain the optimal rail line length. A 16.75km corridor emanating from Downtown Calgary and terminating at Crowfoot Station (Figure 2.10) is assumed to exist. The Dalhousie and Crowfoot Stations are assumed to be existing as well, and are located at positions indicated in Figure 2.10. All passengers originating from the northern part of Calgary (including such areas as Silver Springs, Dalhousie, Crowfoot, Varsity acres, Hawkwood, Ranchland, Edgemont, Market Mall, Northland and Brentwood) and accessing the LRT system at Brentwood Station are assumed to use feeder buses. In particular, the daily total number of feeder bus riders are distributed linearly along the Brentwood-Crowfoot section of the bus line haul. It is worth mentioning that the cost due to people using their private automobile to access the train

Table 2.2 **Values of Transit Parameters**

Symbol	Definition	Units	Value +
γ_B	Average cost of travel by bus per passenger per kilometre	\$/pass/km	0.27
γ_L	Average daily rail line cost per kilometre	\$/km/day	3148
γ_P	Average cost of transferring into bus per passenger per kilometre	\$/pass/km	0.72
γ_R	Average cost of travel by train per passenger per kilometre	\$/pass/km	0.13
λ_B	Average bus operating cost per passenger per kilometre	\$/pass/km	0.23
λ_F	Average train fleet cost per seat per day	\$/seat/day	2.84
λ_R	Average rail operating cost per passenger per kilometre	\$/pass/km	0.01
Λ_R	Average tardity of train	hr/km	0.0314
τ_R	Average layover plus turn around time at train terminal	hr	0.083
M^*	Maximum hourly passenger demand	pass	4800
r	Period of construction of rail project	year	2
I	Interest (discount) rate	%	7
N	Design life span of rail project	year	50

+ Determination of Unit Cost Estimates is Presented in Appendix I

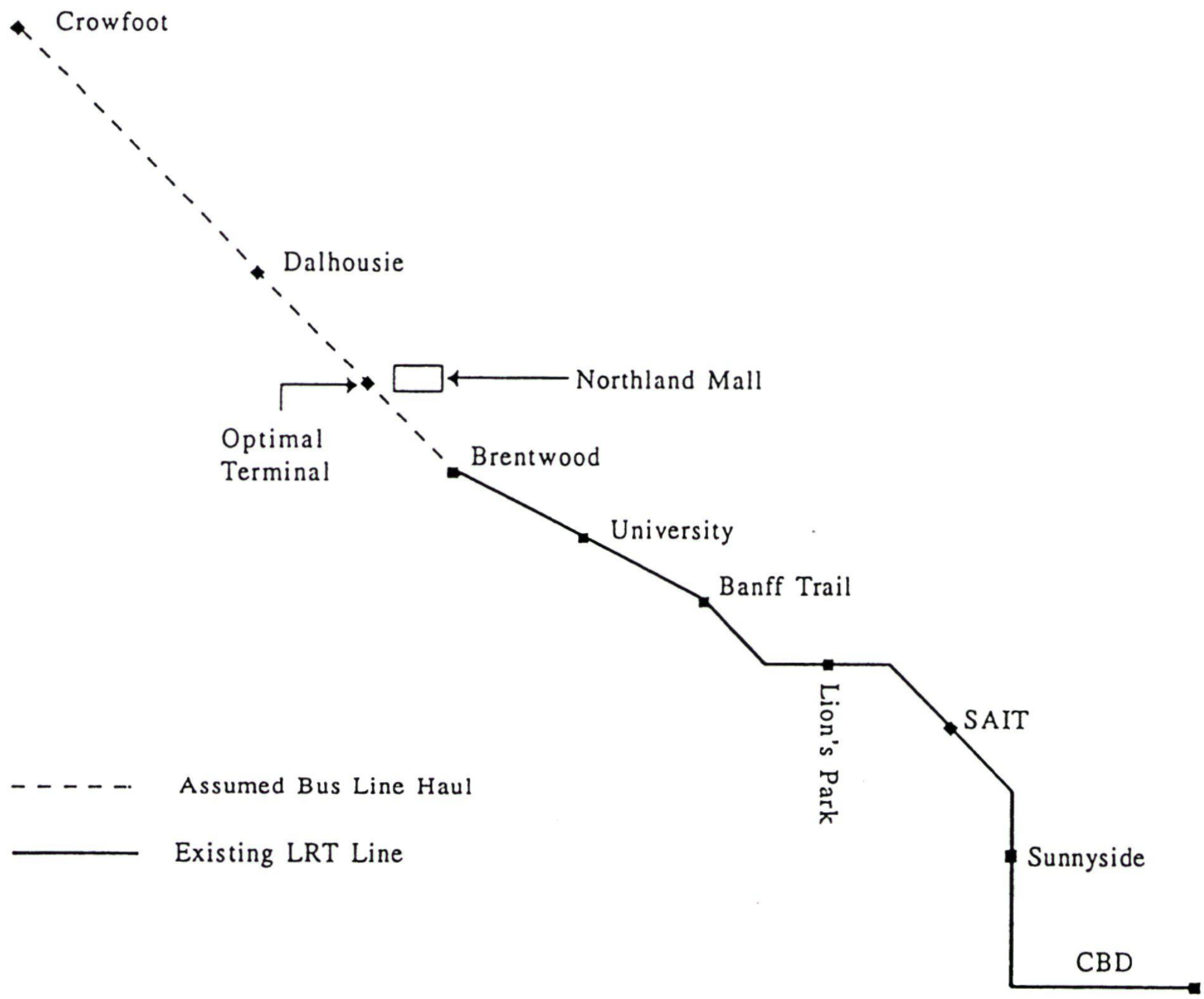

Figure 2.10 North-West Transit Line Haul in Calgary, Alberta

Table 2.1 **1991-92 Daily Passenger Demand on N-W Line Haul in Calgary, Alberta**

Sta. No.	Sta. Name	Dist. from Sta. No.1 (km)	$b(X_R)$ (pass)	$a(X_R)$ (pass)	$B(X_R)$ (pass-km)	$A(X_R)$ (pass-km)	$M(X_R)$ (pass-km)
1	CBD	0.00	0	0	0	0	0
2	CBD	2.00	10729	0	10729	0	10729
3	Sunnyside	3.05	650	790	11379	790	10589
4	SAIT	4.02	540	1800	11919	2590	9329
5	Lion's Park	5.11	350	1370	12269	3960	8309
6	Banff Trail	5.91	140	660	12409	4620	7789
7	University	7.27	740	2500	13149	7120	6029
8	Brentwood	8.30	200	2947	13349	10067	3900
9	Dalhausie	12.01	100	3382	13449	13449	1800
10	Crowfoot	16.75	0	0	13449	13449	0

Table 2.2 **Values of Transit Parameters**

Symbol	Definition	Units	Value +
γ_B	Average cost of travel by bus per passenger per kilometre	$/pass/km	0.27
γ_L	Average daily rail line cost per kilometre	$/km/day	3148
γ_P	Average cost of transferring into bus per passenger per kilometre	$/pass/km	0.72
γ_R	Average cost of travel by train per passenger per kilometre	$/pass/km	0.13
λ_B	Average bus operating cost per passenger per kilometre	$/pass/km	0.23
λ_F	Average train fleet cost per seat per day	$/seat/day	2.84
λ_R	Average rail operating cost per passenger per kilometre	$/pass/km	0.01
Λ_R	Average tardity of train	hr/km	0.0314
τ_R	Average layover plus turn around time at train terminal	hr	0.083
M^*	Maximum hourly passenger demand	pass	4800
r	Period of construction of rail project	year	2
I	Interest (discount) rate	%	7
N	Design life span of rail project	year	50

+ Determination of Unit Cost Estimates is Presented in Appendix I

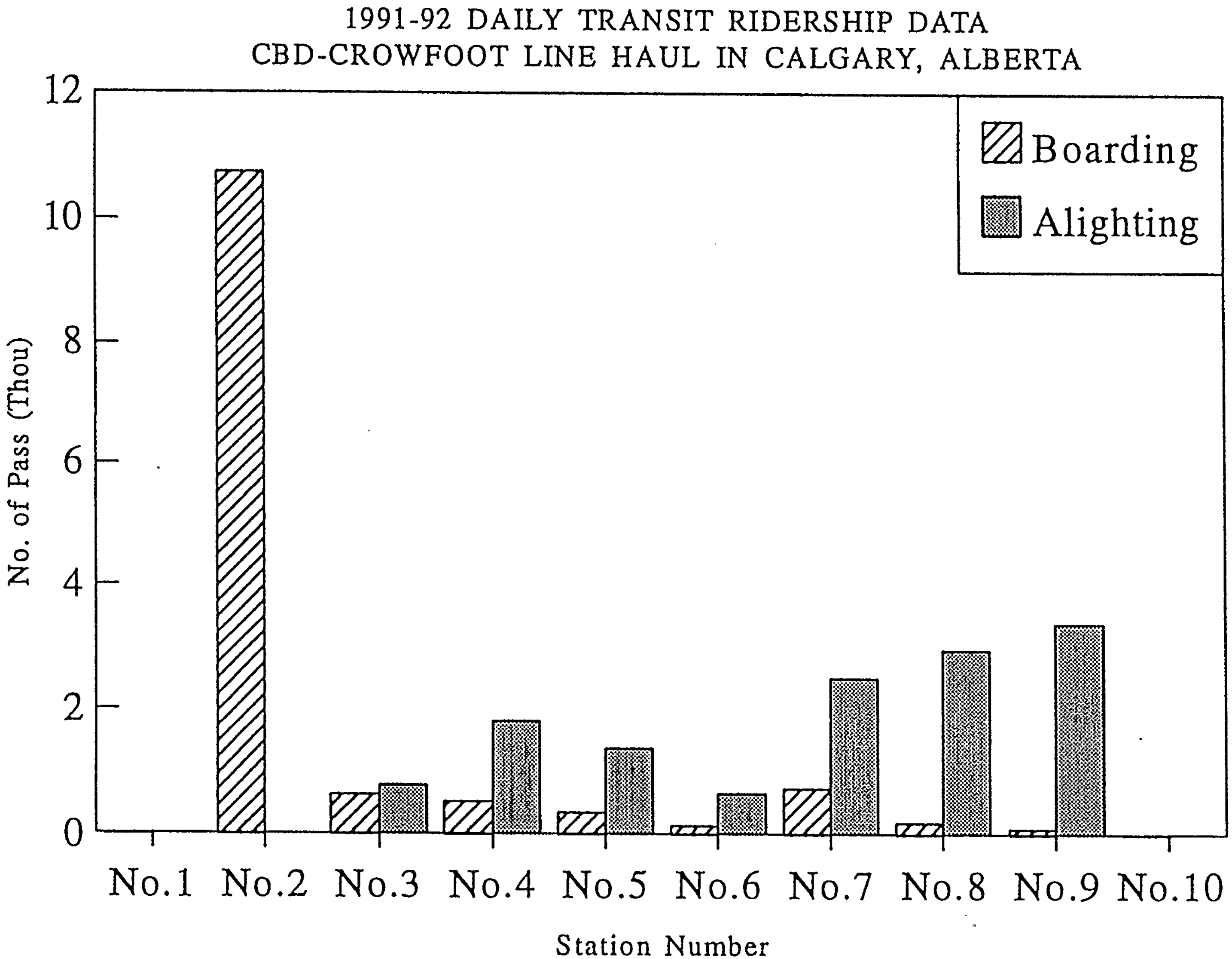

Figure 2.11 Daily Number of Boarding and Alighting Passengers

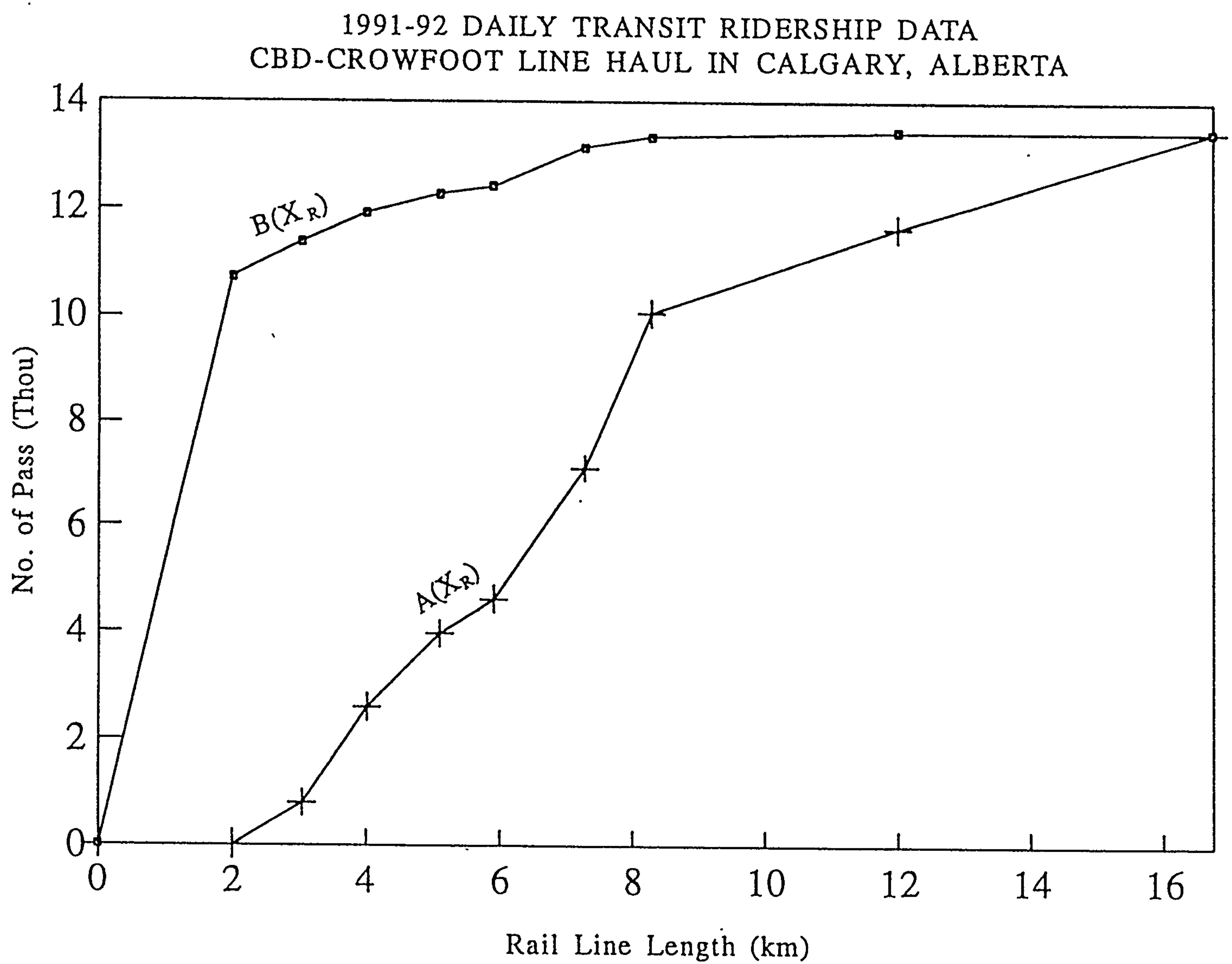

Figure 2.12 Cumulative of Number of Boarding and Alighting Passengers

1991-92 DAILY THROUGH PASSENGER LOAD
CBD-CROWFOOT LINE HAUL IN CALGARY, ALBERTA

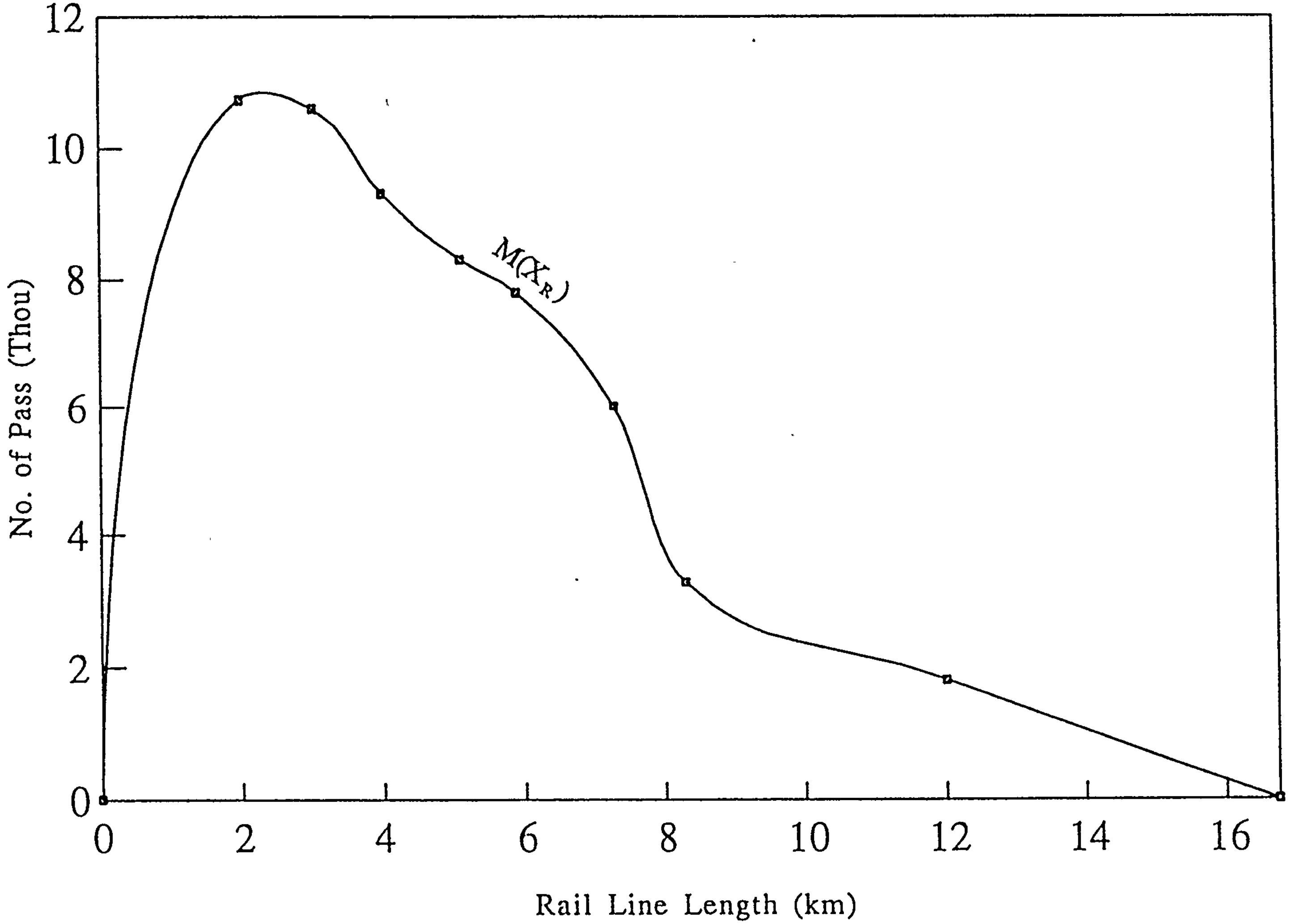

Figure 2.13 Daily Through Passenger Load

Table 2.3 **Determination of Optimal Rail Line Length**

Sta. No.	Station Name	Dist. from Sta. No. 1 (km)	$M(X_R)$ (pass/day)	$M'(X_R)$ (pass/km/day)	$\Delta\gamma M(X_R)$ + ($/km/day)	$\gamma_L+2\gamma_P M'(X_R)+\theta$ * ($/km/day)
1	CBD	0.00	0	0	0	4004
2	CBD	2.00	10729	5365	7295	6217
3	Sunnyside	3.05	10589	-472	7200	3664
4	SAIT	4.02	9329	-1106	6344	3208
5	Lions Park	5.11	8309	-1720	5650	2766
6	Banff Trail	5.91	7789	-1420	5296	2981
7	University	7.27	6029	-1308	4099	3062
8	Brentwood	8.30	3900	-2010	2652	2557
9	Dalhousie	12.01	1800	-3540	1224	1455
10	Crowfoot	16.75	0	-4765	0	573

+ LHS of Equation 2.20.2
* RHS of Equation 2.20.2

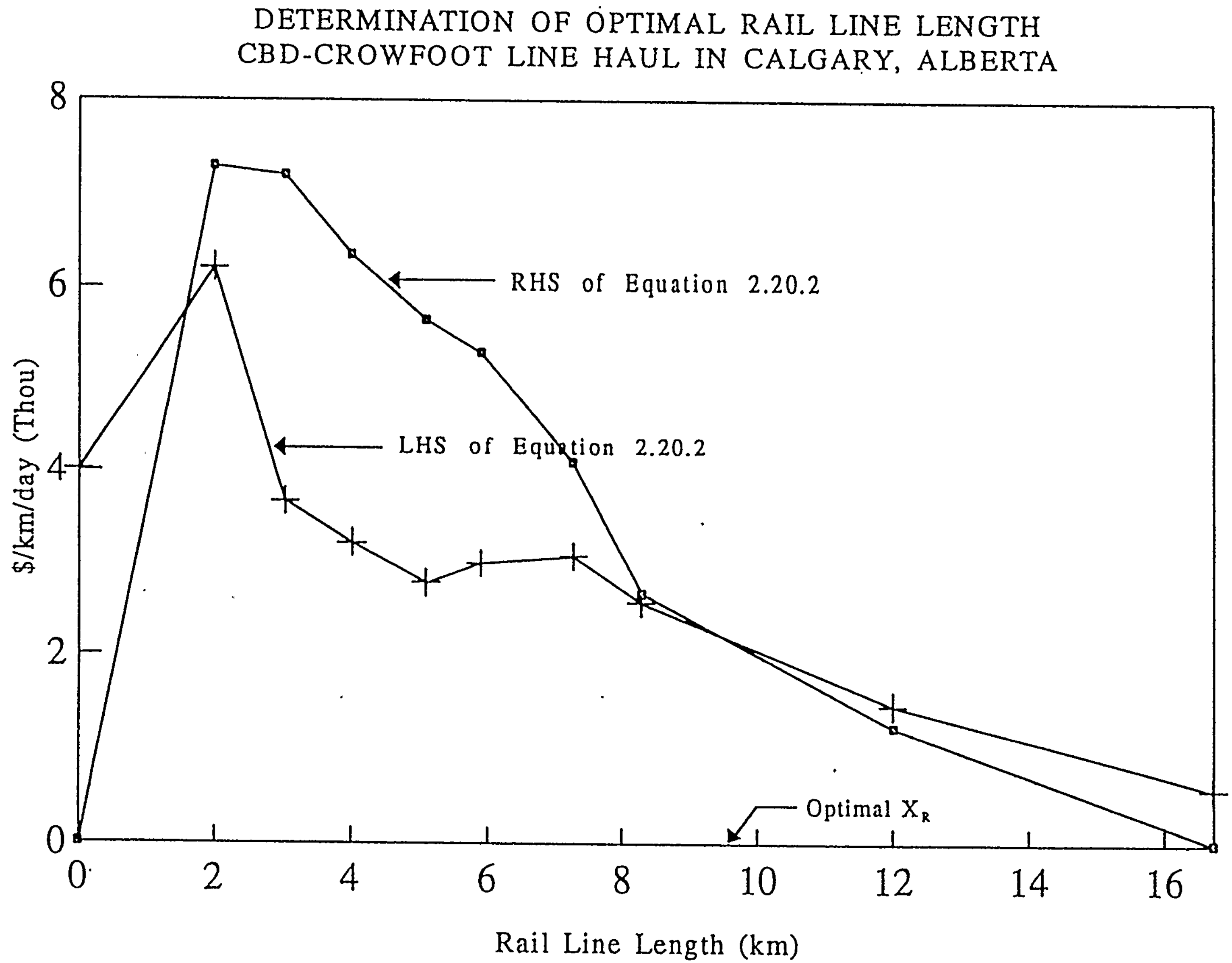

Figure 2.14 Determination of Optimal Rail Line Length

at Brentwood Station is neglected. The reason being that the line cannot be extended to reduce the private cost of people who choose to use their private automobiles.

Table 2.1 depicts the 1991-92 daily passenger (boarding and alighting) ridership on the North-West link of Calgary LRT system (Calgary Transit). Figure 2.11 shows the daily number of passengers boarding and alighting from the LRT at any point on the line haul. Figure 2.12 shows the cumulative of the daily number of boarding and alighting passengers at any point on the line haul. Figure 2.13 however depicts the difference between the cumulative of the daily number of boarding and alighting passengers at any point on the line haul.

The values of the unit cost parameters are shown in Table 2.2. The determination of the estimates of unit cost parameters is presented in Appendix I. Table 2.3 depicts the evaluation of the left hand side (LHS) and right hand side (RHS) of Equation 2.20.2 using the demand data (Table 2.1) and parameters (Table 2.2). Data shown at Columns 4 and 5 of Table 2.3 are plotted against the rail line length X_R (Figure 2.14). The optimal rail line length is found to be 9.41km. A computer program (Appendix II) developed using Equation 2.20.2 also gives the value of the optimal rail line length as 9.41km. This brings the optimal terminal close to the Northland Mall. The optimal terminal is approximately 1.11km away from the existing terminal located at Brentwood Station. It is found that an optimal line length of 8.30km (i.e. at Brentwood Station) is obtained if γ_B=0.25, γ_R=0.15, λ_B=0.21, λ_R=0.05, λ_F=3.10, γ_P=0.70 and γ_L=3200 are separately used in the analysis.

2.16 SENSITIVITY ANALYSIS

Uncertainty has emerged over the past decades as one of the major factors which transportation systems analysts, planners and decision-makers have to contend with in making constructive decisions with regard to the provision of a rail line, be it for the short or long run. In view of this, it is appropriate that sensitivity analysis be discussed in this research in order to check the robustness of the proposed analytical model as well as to provide a better understanding of critical parameters on which the model's outcome depends. Most importantly, sensitivity analysis will indicate the critical unit cost parameters that may require close attention at pre-construction, construction and post-construction stages in order to ensure that the most economic return is realised.

The optimal rail line length X_R is found to exhibit some degree of sensitivity to the unit cost parameters used in the analysis (Seneviratne et al, 1986). It should be explained that reasonable and realistic estimates of the relevant unit cost parameters required for determination of optimal rail line length is of great importance in order to obtain a reliable solution of the optimal rail line length. In this research, sensitivity analysis will be conducted on all unit cost parameters considered in the analysis. The unit cost parameters are listed in Table 2.2. Exactly ± 25% test about the central value for each parameter is considered. In particular, the sensitivity of each parameter is explored whilst keeping all other parameters at their current estimates.

Tables 2.4 depict the global summary of the sensitivity test results. Sensitivity ratings of the various unit cost parameters with respect to the optimal rail line length is shown in Column 9 of Table 2.4. It is found that the optimal rail line length X_R is

Table 2.4 **Summary of Sensitivity Test Results on Optimal Rail Line Length (X_R)**

Unit Cost Parameters	Original Optimal Rail Line Length (km)	At -25% Sensitivity Test			At +25% Sensitivity Test			Sensitivity Rating of Unit Cost Parameters on Optimal Rail Line Length
Symbols		Optimal Line Length (km)	% Change in length		Optimal Line Length (km)	% Change in length		
			Inc.	Dec.		Inc.	Dec.	
γ_B	9.41	7.43		21.04	12.01	27.63		Very Sensitive
γ_R	9.41	10.95	16.36		7.90		16.05	Sensitive
λ_B	9.41	7.61		19.12	11.43	21.47		Sensitive
λ_R	9.41	9.43	0.21		9.32		0.10	Highly Insensitive
γ_P	9.41	7.90		16.05	16.75	78.0		Very Sensitive
λ_F	9.41	9.94	5.63		9.02		4.09	Insensitive
γ_L	9.41	16.75	78.00		7.43		21.04	Very Sensitive
$M(X_R)$	9.41	7.56		12.0	12.0		17.43	Sensitive

sensitive to the parameters γ_B, γ_R, γ_L, γ_P, λ_B and $M(X_R)$. However, the optimal rail line length is insensitive to the parameters λ_F and λ_R.

It is found that no optimal line exists if γ_B, λ_B, λ_F and γ_L are separately neglected. High optimal line lengths are obtained if γ_R and λ_R are not considered individually. However, high values of optimal line length are obtained if unreasonably high values of γ_B, λ_B and γ_P are separately used. No optimal line length exists if high values of γ_R, λ_R, λ_F and γ_L are individually employed.

The result of the analysis shows that the optimal X_R increases with increasing values of $M(X_R)$, γ_B, λ_B and γ_P, and vice versa. A high value of $M(X_R)$ implies high demand for rail systems. Hence, a longer rail line length is to be provided in order to meet the high demand. High value of γ_B is due to higher user travel times, excessive delays to riders, low level of reliability of buses, low level of safety and protection of passengers, poor level of comfort and convenience as well as high levels of air and noise pollution. This situation causes the rail demand to increase, therefore causing the line length to be longer.

A very high value of λ_B is attributed to factors which include excessive traffic delays, high cost of fuel and high labour costs. Consequently, bus operators may require passengers to pay high fares for their trips in order to meet the high bus operating cost. This situation may cause a decline in demand for bus services, thereby causing the demand for rail systems to increase. A longer rail line length is to be provided to meet the increasing demand. The converse is also true. With regard to transfer of passengers from train to bus, a high value of γ_P is due to factors such as longer waiting times for arrival of buses, low level of reliability of buses, lack of safety and protection to transfer passengers at transfer points, and lack of convenience and comfort to transfer passengers at bus stops. These situations will warrant the provision of longer length of rail line haul and a corresponding shorter length of bus line haul. The effect is that potential rail riders may not be discouraged from using the rail systems. Besides, users of non-transit modes may be attracted to the rail system.

It is also found that the optimal X_R decreases with increasing γ_R and γ_L, and vice versa. High value of γ_R is caused by factors which include levels of comfort, convenience and safety. The demand for rail systems will decline under such conditions. A shorter line length is to be provided in this case. The converse is true. A high value of γ_L is attributed to factors including high cost of demolition and relocation of existing facilities as well as high land cost at CBD region. Ironically, huge funds are required to construct the a rail line with longer length. The required funds might not be available. A shorter line length is most likely to be constructed under such situation. The opposite is also true.

The analysis indicated that the optimal X_R is insensitive to λ_R and λ_F. Practically, the cost of operating and maintaining rail systems is low in comparison to that of buses. Generally, a high initial capital upfront is required to purchase railcars. However, discounting this cost over the entire life span of the rail vehicles tends to make the rail fleet cost somewhat insignificant. These may explain why the optimal X_R is insensitive to λ_R and λ_F. However, under some conditions, it is possible for the values of λ_R and λ_F to have a significant effect on optimal X_R. This occurs when λ_R and λ_F are unrealistically high. Factors which give high λ_R include high labour and administrative costs as well as high cost of owing the railcars. The demand for rail systems will therefore decline, causing the provision of shorter line length along the rail corridor.

2.17 MODEL EXTENSION

The model discussed above can be extended to determine the optimal rail line length by considering a different network scenario. A rectangular local road network consisting of two distinct sets of parallel curvilinear roads(x and y) is considered. An existing curvilinear transportation corridor with a haul bus line CS emanating from CBD towards a suburban region and parallel to x-roads (Figure 2.15) is to be replaced by a haul rail line (Figure 2.16). The bus line haul, which is a major road, is to be converted into a feeder bus route. The rail line $T_C T_R$, to be located closely parallel to the existing bus line haul, will emanate from the heart of the CBD, T_C, into the suburban region at T_R, but not necessary to its end at T_S. However, feeder bus service will be provided to the rail line from all areas, including those beyond the end of the rail line.

All assumptions pertaining to operations of trains and feeder buses as well as nature of passenger accessibility to the transit systems discussed in Section 2.2 will apply in this case. Furthermore, it is assumed that feeder buses will use the haul bus line $T_S T_R$ to access the rail terminal T_R. Moreover, all assumptions considered in the formulation of user time cost, rail and bus operating costs, rail line cost and rail fleet cost will be considered under this network scenario. Accordingly, the expressions for user time cost, rail and bus operating costs, rail line cost and rail fleet cost given by Equations 2.4, 2.7, 2.8 and 2.15 respectively, are also applicable in this case. It is imperative to remark that the demand functions given by equations 2.1.1, 2.1.2, and 2.1.3 are applicable in this case.

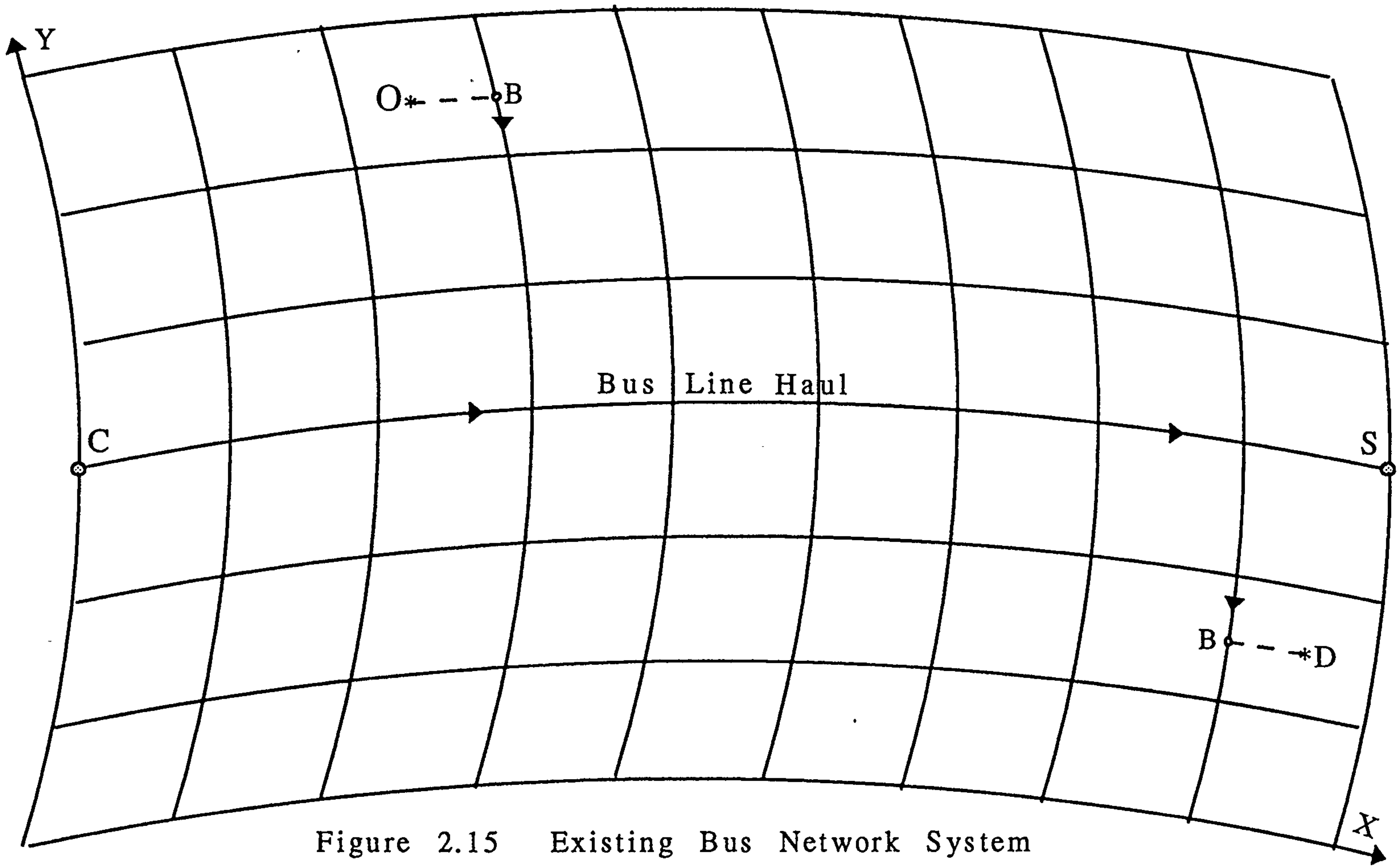

Figure 2.15 Existing Bus Network System

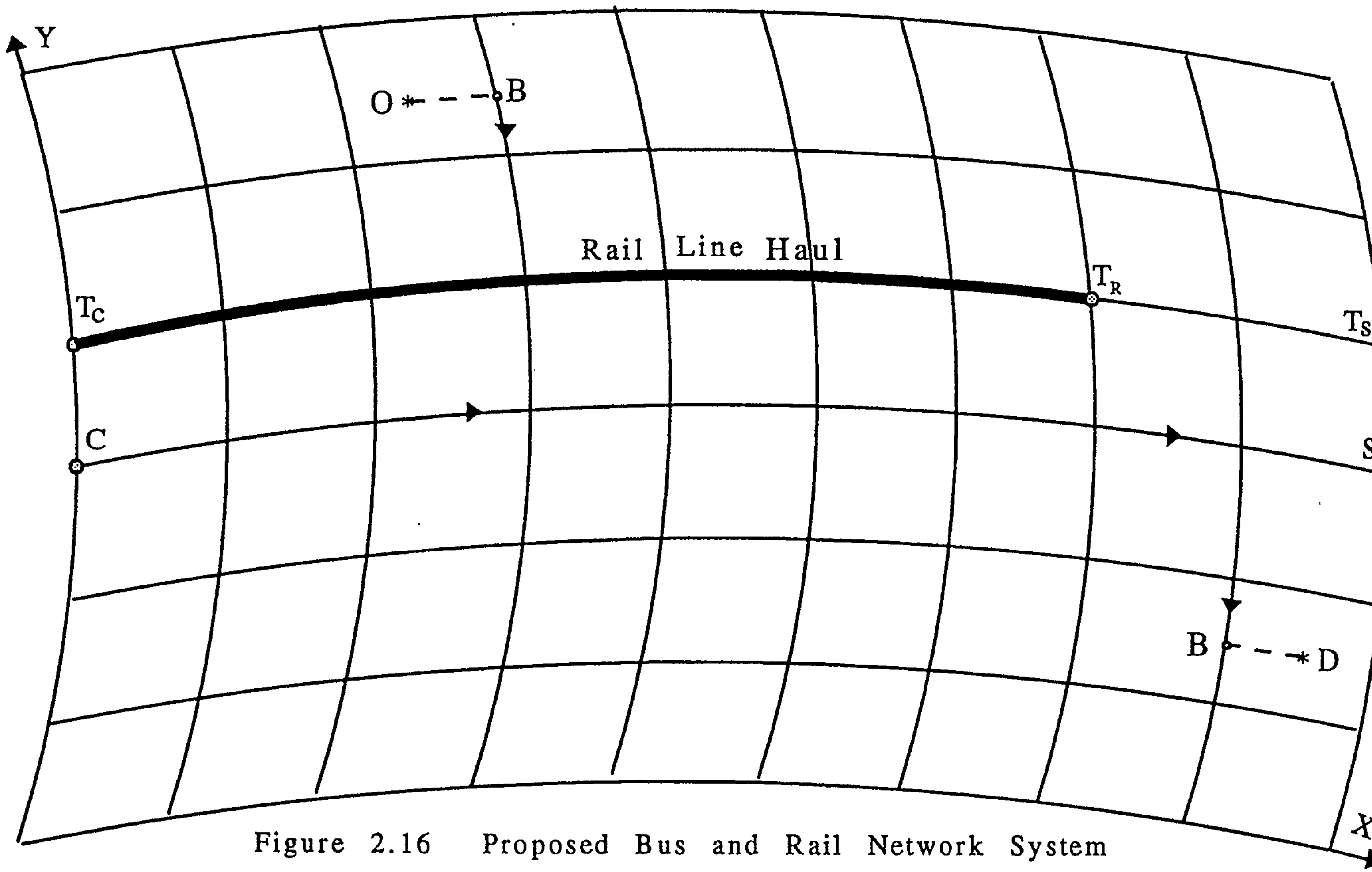

Figure 2.16 Proposed Bus and Rail Network System

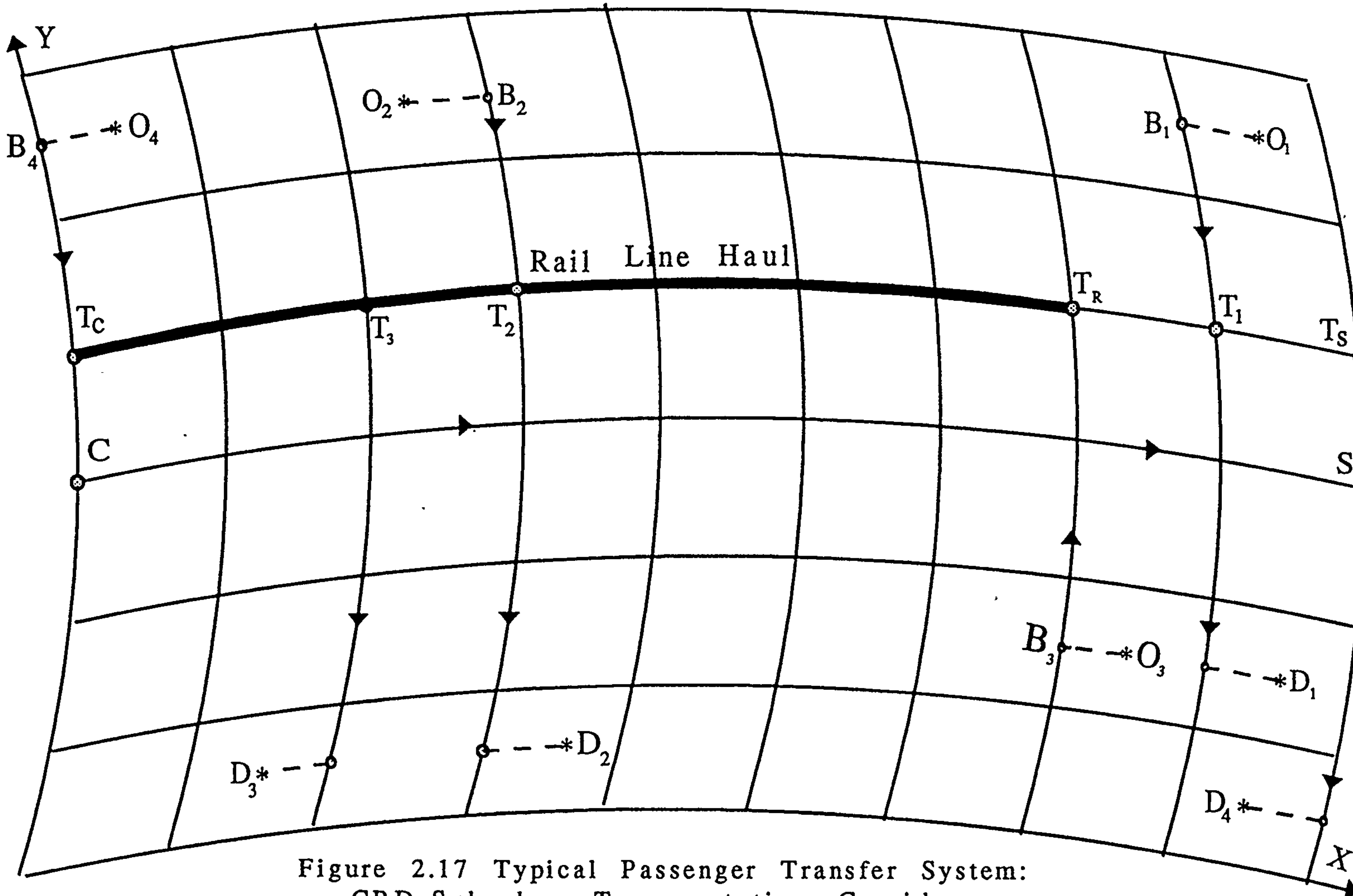

Figure 2.17 Typical Passenger Transfer System: CBD-Suburban Transportation Corridor

However, the passenger transfer penalty cost (Equation 2.18) will not hold for this particular model. The discrepancy is attributed to the different operating strategies of feeder buses assumed in the formulation of the proposed model. A careful assessment of the model indicates that the introduction of rail line will generate an additional transfer for all passengers transferring from bus to train. Accordingly, the total number of passengers boarding the train is considered in formulating the related transfer penalty cost. The formulation of passenger transfer penalty cost consistent with this network scenario is presented below.

It is observed from Figure 2.17 that a passenger travelling from O_1 to D_1 will experience one type of transfer from bus to bus at T_1 or no transfer depending on the configuration of feeder bus routes. Moreover, a passenger travelling from O_2 to D_2 will experience one type of transfer from bus to bus at T_2. However, a passenger travelling from O_3 to D_3 will be subjected to two types of transfers. These are transfer from bus to train at T_R and then transfer from train to bus at T_3. Also, a passenger travelling from O_4 to D_4 will experience two types of transfers; transfer from bus to train at T_C followed by transfer from train to bus at T_R. It is therefore observed that with bus network only (Figure 2.15) at most one transfer is needed. Once a rail line is introduced (Figure 2.16) an additional transfer is needed for all passengers boarding the train. A disutility of travel is therefore associated with the provision of the rail line. The relevant total transfer penalty cost is given by:

$$2\gamma_P \int_0^{X_R} B(x)\,dx \tag{2.29}$$

where γ_P is the average cost of transferring per passenger per kilometre and B(x) is the cumulative of the daily number of boarding passengers at point x on the rail line haul.

2.18 OPTIMIZATION

The optimal rail line length which will minimize the sum of user time cost, rail and bus operating costs, rail line cost, rail fleet cost and passenger transfer penalty cost will be explored. The overall transportation cost [$Z(X_R)$] is given by the sum of Equations 2.4, 2.7, 2.8, 2.15, and 2.29. Thus:

$$\begin{aligned} Z(X_R) &= 2\gamma_R \int_0^{X_R} M(x)\,dx + 2\gamma_B \int_{X_R}^{L} M(x)\,dx \\ &+ 2\lambda_R \int_0^{X_R} M(x)\,dx + 2\lambda_B \int_{X_R}^{L} M(x)\,dx \\ &+ \int_0^{X_R} \gamma_L(x)\,dx + (2X_R\Lambda_R + \tau_R)\lambda_R M^* \\ &+ 2\gamma_P \int_0^{X_R} B(x)\,dx \end{aligned} \tag{2.30}$$

Taking the derivative of Equation 2.30 and setting the resulting expression to zero gives:

$$2[(\gamma_B-\gamma_R)+(\lambda_B-\lambda_R)]M(X_R)=\gamma_L(X_R)+2\Lambda_R\lambda_F M^*+2\gamma_P B(X_R) \tag{2.31}$$

The minimum total transportation cost is obtained by setting the second derivative of Equation 2.30 with respect to X_R to be positive. This gives:

$$Z''(X_R) = 2[(\gamma_R-\gamma_B)+(\lambda_R-\lambda_B)]M'(X_R)+\gamma_L'(X_R)+2\gamma_P B'(X_R) > 0 \qquad (2.32)$$

2.19 OPTIMUM DEMAND

The estimation of the most economic or optimum passenger demand that warrants the provision of an appreciable length of a rail line along a transportation corridor is long recognised in rail transportation planning (Seneviratne, 1986). Transit planners identified passenger demand as the most significant determinant that will justify the provision of a rail line. Yeates et al (1980) discussed the impact of passenger demand with regard to provision of a rail line. An important fact that should be emphasised is that the benefits and savings to be realised by provision of a rail line as well as the maximization of profits by transit operators largely depends on passenger demand.

The important role played by passenger demand in rail planning has necessitated the determination of the most economic and optimal passenger demand that will warrant or justify the provision of a rail line in this research. To this end, an analysis carried out to obtain the optimum passenger demand is discussed. The optimal demand $M_O(X_R)$ that will ensure that the resulting rail line length is positive is obtained by setting Equation 2.31 to zero, and solving for $M_O(X_R)$. Accordingly, the required optimum demand is given by the expression:

$$M_O(X_R) = [\gamma_L(X_R)+2\Lambda_R\lambda_F M^*+2\gamma_P B(X_R)]/2[(\gamma_B-\gamma_R)+(\lambda_B-\lambda_R)] \qquad (2.33)$$

2.20 MODEL APPLICATION

The proposed model is tested using the existing North-West LRT line in Calgary, Alberta. It should be remarked that the current operating strategies of the public transit

Table 2.5 **Values of Transit Parameters**

Symbol	Definition	Units	Value
γ_B	Average cost of travel by bus per passenger per kilometre	\$/pass/km	0.27
γ_L	Average daily rail line cost per kilometre	\$/km/day	3148
γ_P	Average cost of transferring into train per passenger per kilometre	\$/pass/km	0.48
γ_R	Average cost of travel by train per passenger per kilometre	\$/pass/km	0.13
λ_B	Average bus operating cost per passenger per kilometre	\$/pass/km	0.23
λ_F	Average train fleet cost per seat per day	\$/seat/day	2.18
λ_R	Average rail operating cost per passenger per kilometre	\$/pass/km	0.03
Λ_R	Average tardity of train	hr/km	0.0314
τ_R	Average layover plus turn around time at train terminal	hr	0.083
M^*	Maximum hourly passenger demand	pass	4800
r	Period of construction of rail project	year	2
I	Interest (discount) rate	%	7
N	Design life span of rail project	year	50

Table 2.6 **Determination of Optimal Rail Line Length**

Sta. No.	Station Name	Dist. from Sta. No. 1 (km)	$M(X_R)$ (pass/day)	$B(X_R)$ (pass/day)	$\Delta\gamma M(X_R)$ + ($/km/day)	$\gamma_L+2\gamma_P B(X_R)+\theta$ * ($/km/day)
1	CBD	0.00	0	0	3648	4004
2	CBD	2.00	10729	10729	7295	14303
3	Sunnyside	3.05	10589	11379	7200	14928
4	SAIT	4.02	9329	11919	6344	15446
5	Lions Park	5.11	8309	12269	5650	15782
6	Banff Trail	5.91	7789	12409	5296	15917
7	University	7.27	6029	13149	4099	16627
8	Brentwood	8.30	3900	13349	2652	16819
9	Dalhousie	12.01	1800	13449	1224	16915
10	Crowfoot	16.75	0	13449	0	16915

+ LHS of Equation 2.31

* RHS of Equation 2.31

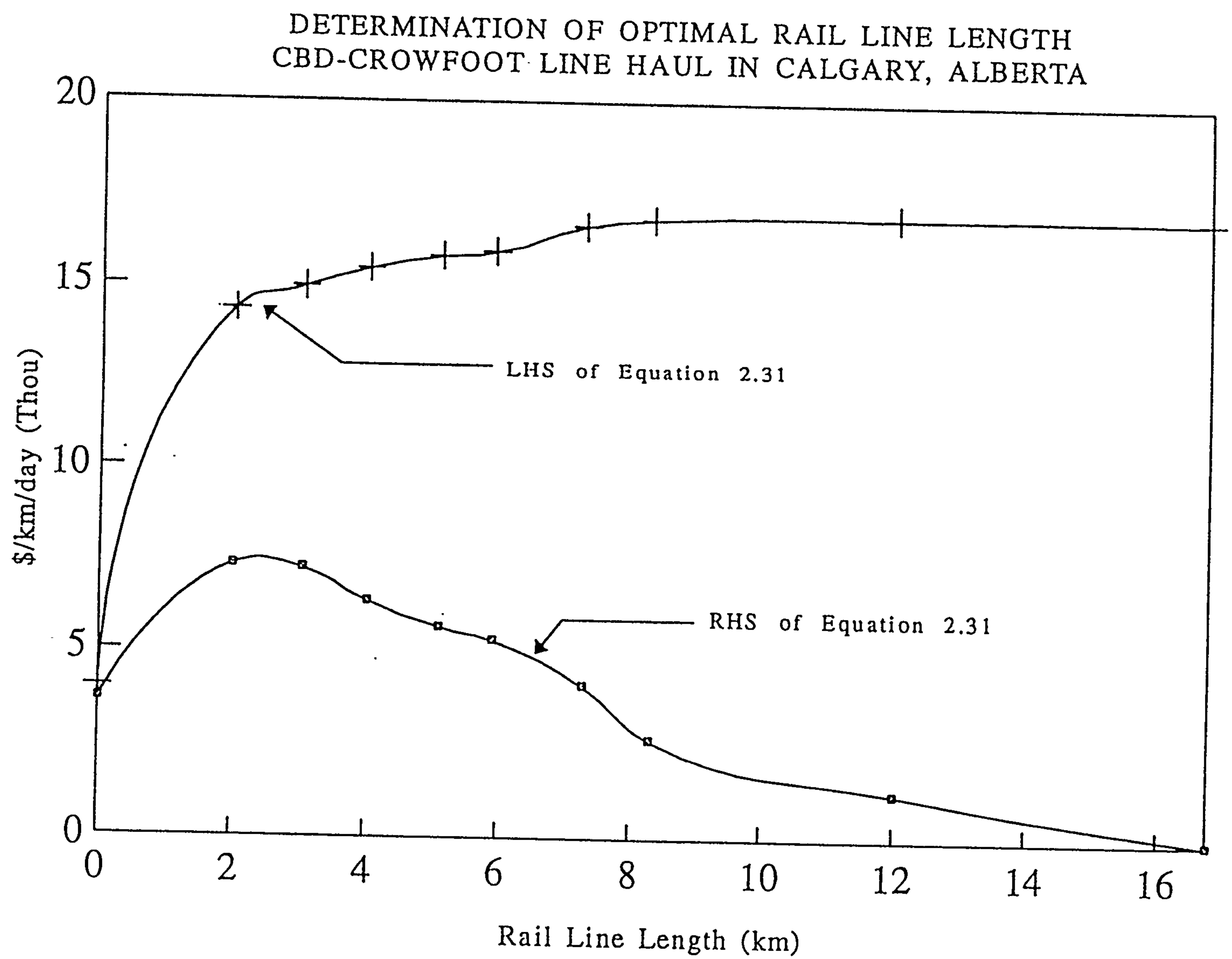

Figure 2.18 Determination of Optimal Rail Line Length

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systems in Calgary perfectly fit this scenario. The transit ridership data and the values of the unit cost parameters given respectively in Tables 2.1 and 2.5 are employed in the test. The computation of the LHS and RHS of Equation 2.31 are shown respectively at columns 6 and 7 of Table 2.6. These values are plotted against the rail line length (Figure 2.18). No optimal rail line length is obtained. Hence, the optimal rail line length is zero. A computer program (Appendix III) developed using Equation 2.31 gives a zero optimal line length.

The model extension revealed that no optimal line length exists by using the North-West transit line in Calgary, Alberta, as a case study. Nonetheless, using Calgary as an example, it is possible to obtain positive values of optimal line length under some conditions. For instance, by using γ_B=2.43, γ_R=2.30 and γ_P=0.01 separately, optimal line lengths of 9.20km, 9.01km and 7.35km are respectively obtained. Also by increasing the values of $M(X_R)$ as given in Column 8 of Table 2.1 by 1000%, an optimal line length of 10.80km is obtained.

The determination of economic or optimal demand that will warrant the provision of a rail line and other supporting facilities is essential. Generally, reasonable high demand is required to justify the implementation of LRT system. For a city like Calgary, a high bus riding unit cost or low rail riding unit cost might not necessary result in increase in passenger demand for the LRT system. Hence, effective methods of increasing passenger demand should be sought by city authorities and planners. One such method is implementation of effective park-and-ride policies. Park-and-ride facilities should be developed extensively to enhance attraction of private automobiles users to buses and rails.

However, it will be incomplete to address this measures without due consideration of the very important issue of quality of rail service. Methods of attracting people from autos to rail systems must be clearly identified. People must know that trains have such qualities as speed, safety, reliability, convenience, affordabilty, fuel economy and sustainability. An auto to rail transfer is not likely to be brought about by persuasion only; it would almost certainly require some form of state intervention, exercised through such measures as high vehicle and fuel taxation, congestion tolling and direct restriction on private vehicles.

In reality, a high demand for rail systems may require the provision of a longer rail line length. This situation results in high construction and maintenance costs of providing the rail line. The optimal decision for constructing the rail line is based on the fact that the socio-economic gains and benefit associated with the provision of the rail line, at short and long runs, exceeds the cost of providing the line.

CHAPTER THREE

RAIL LINE TERMINI:CROSSTOWN CORRIDOR ANALYSIS

3.1 INTRODUCTION

A crosstown transportation corridor is basically a route that provides passenger travel from one Suburban Region to another Suburban Region through the CBD. Most essentially, the corridor accommodates trips destined to places other than the CBD. The Canadian Transit Handbook (1980) identified some characteristics of a cross-town corridor. The book reported that in comparison to a CBD-Suburban corridor, cross-town corridors attracts more trips, serves multiple destinations and therefore provides high level of connectivity. Besides, it accommodates trips destined to places other than the CBD and provides low level of passenger transfer. The crosstown corridor is however characterized by infrequent transit headways in comparison to a CBD-Suburban corridor.

Vuchic et al (1988) described a crosstown line as a line that connects two suburban regions and pass through city centre. They explained that since the lines are connected their terminal operations take place in Suburban Regions. They emphasized that crosstown lines should be planned with two major considerations. First, their two parts from centre of the city should have maximum passenger volumes to ensure good utilization of offered capacity. Secondly, they should connect Suburban Regions between which there is demand for travel.

This Chapter presents an analytical model which determines the optimal location of the termini of a rail line. The relevant transportation costs considered in the analysis are user time costs, bus and rail operating costs, rail line costs, rail fleet costs and

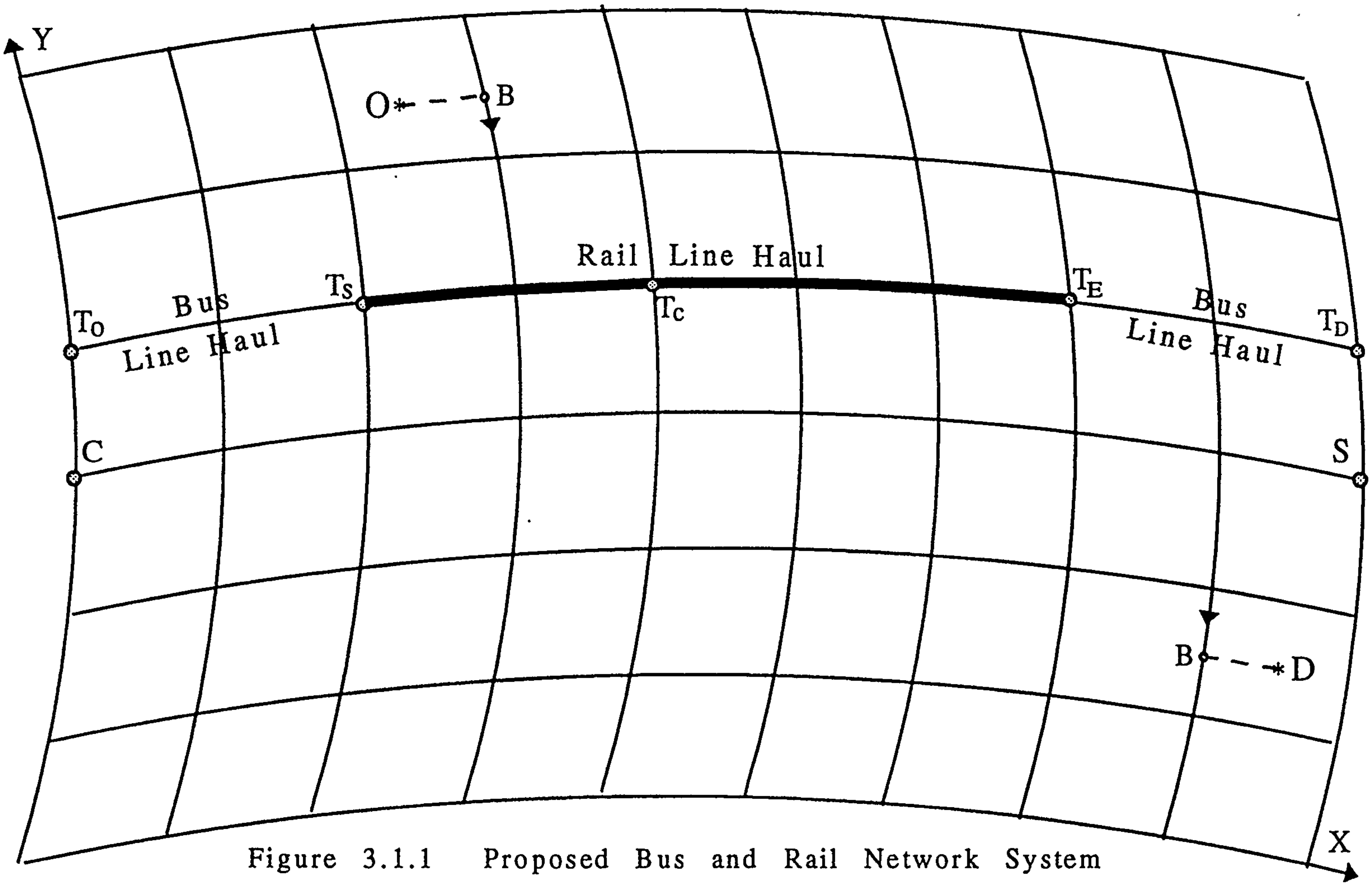

Figure 3.1.1 Proposed Bus and Rail Network System

passenger transfer penalty costs. The objective is to minimize the sum of the relevant transportation costs associated with the provision of a rail line along a cross-town transportation corridor. More particularly, the analysis will consider passenger many to many demand travel pattern at both peak and off-peak periods.

Methods used in the analysis are basically calculus with graphical and numerical illustrations. The validity and applicability of the proposed analytical model is investigated using the Northwest-South crosstown corridor in Calgary as a case study. Sensitivity test is demonstrated to explore the robustness of the proposed model.

3.2 TRANSIT NETWORK

Consider a rectangular local road network consisting of two district sets of parallel curvilinear roads (x and y) which permits orthogonal passenger movements (i.e. parallel and perpendicular to the routes. A LRT line (Figure 3.1.1) is planned to be constructed along the transportation corridor from point T_S located in a suburban region to point T_E located in another suburban region through T_C (i.e. CBD). Bus services will be provided in the corridor sections T_OT_S and T_ET_D. The service is assumed to be provided by special line haul buses operating along the corridor. The LRT systems and line haul buses will be "fed" by feeder buses operating in the rail region T_ST_E and bus regions T_OT_S and T_ET_D.

It is assumed that the line haul buses departing from T_O and running towards T_S will stop at bus stops located along the corridor to enable passengers to board and alight. Upon reaching T_S, continuing passengers will transfer into the train. The train running from T_S to T_E will stop at stations located on the haul rail line to allow for boarding and alighting of passengers. At T_E, continuing passengers will transfer into the line-haul buses.

Trips are returned by same transit modes by retracing paths. It should be stressed that all assumptions pertaining to operations of feeder buses as well as the nature of passengers' accessibility to the transit systems discussed in Section 2.2 will apply in this case.

3.3 THE MODEL

Figure 3.1.2 depicts a cross-town transportation corridor $T_O T_S T_C T_E T_D$ of length L where T_O and T_D represents the ends of two suburban regions. T_S and T_E are, respectively, the start and end points of the proposed LRT line of length X_R. In particular, T_O is assumed to be the starting chainage, i.e. zero reference point. T_C is the centroidal location of the CBD. X_S and X_E are, respectively, the distances of T_S and T_E measured from T_O. The points T_S and T_E are considered as major transfer points. It is required to determine analytically the optimal locations of the rail termini T_S and T_E.

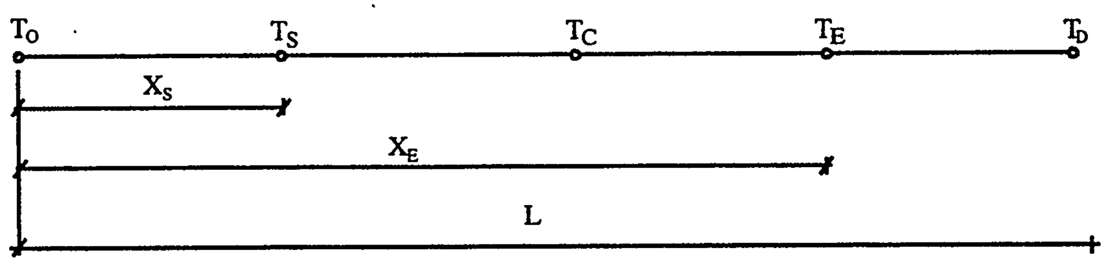

Figure. 3.1.2 Dimensions of Proposed Transit Line Haul

3.4 MANY TO MANY DEMAND FUNCTION

A detailed discussion on many to many travel demand is presented in Section 2.4. All conditions and assumptions employed in the formulation of passenger demand for the CBD-Suburban corridor analysis in Chapter 2 are considered for cross-town corridor analysis. More importantly, it is necessary to emphasize that like the formulation of many to many demand discussed under CBD-Suburban corridor analysis, the many to many demand for a cross-town corridor trips is similarly described by a continuous function defined in terms of the difference between the cumulative number of boarding and alighting passengers at every point on the haul rail line, i.e. the passenger load crossing a point on the line. A typical M(x) profile pertaining to passenger travel along a cross-town corridor is shown in Figure 3.2.1.

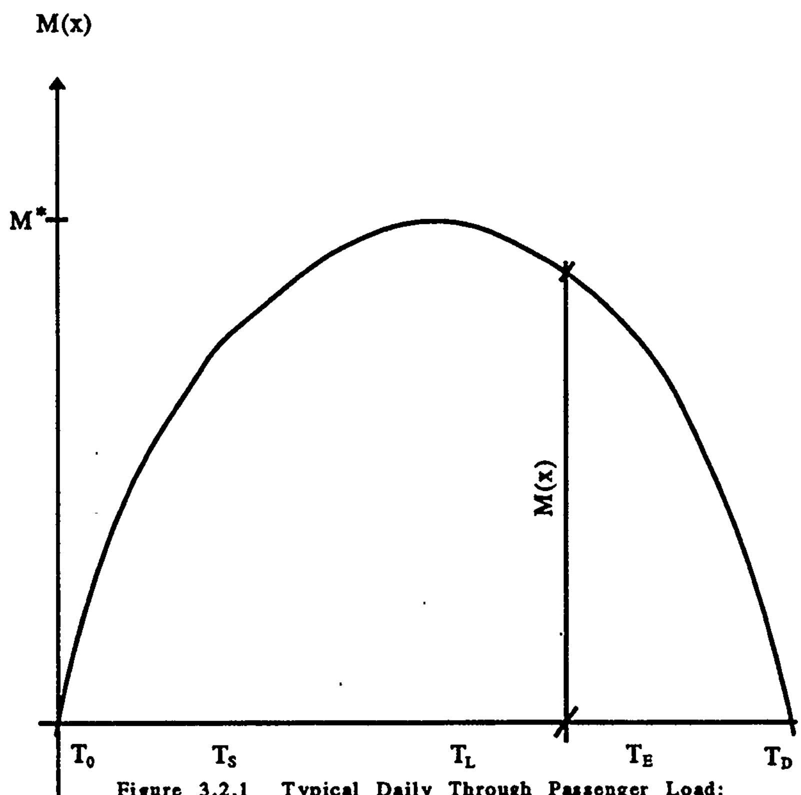

Figure 3.2.1 Typical Daily Through Passenger Load: Crosstown Transportation Corridor

Although the analysis is based on the assumption that M(x) function is continuous, it is worth mentioning the possibility of discontinuity in M(x) at CBD. Figure 3.2.2 shows three cross-town rail lines (i.e. T_ST_E, T_BT_F and T_GT_H) crossing each other at the CBD, T_C.

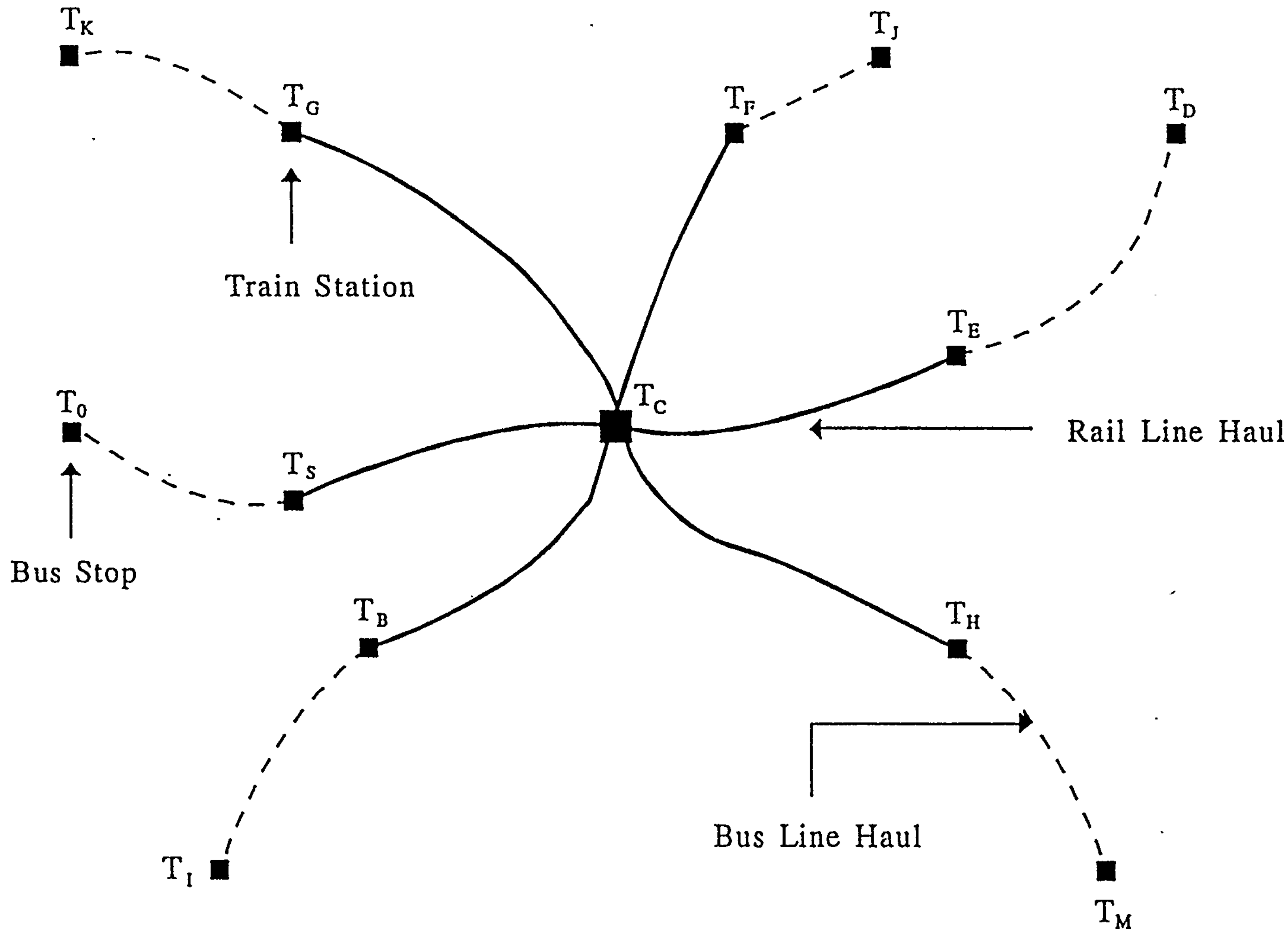

Figure 3.2.2 Location of Three Transit Haul Lines in an Urban Corridor

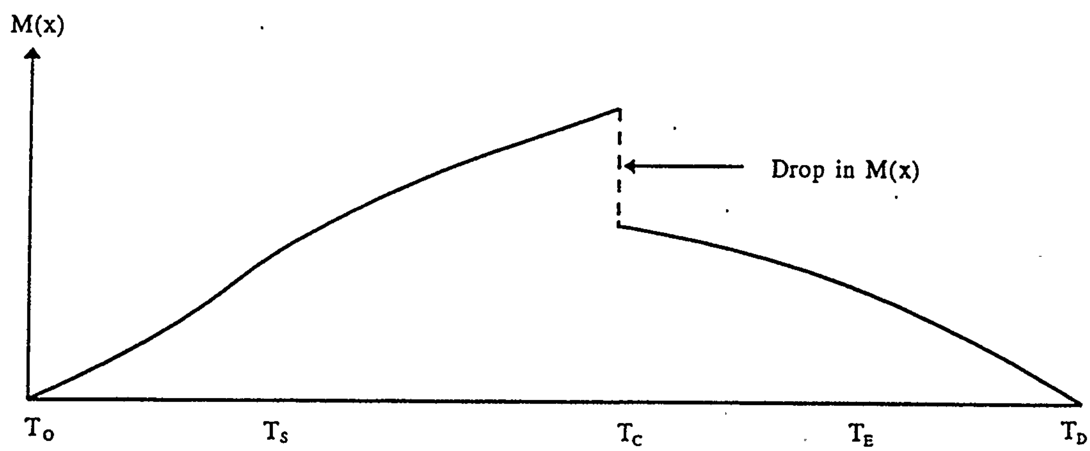

Figure 3.2.3 Drop in Daily Through Passenger Load at CBD

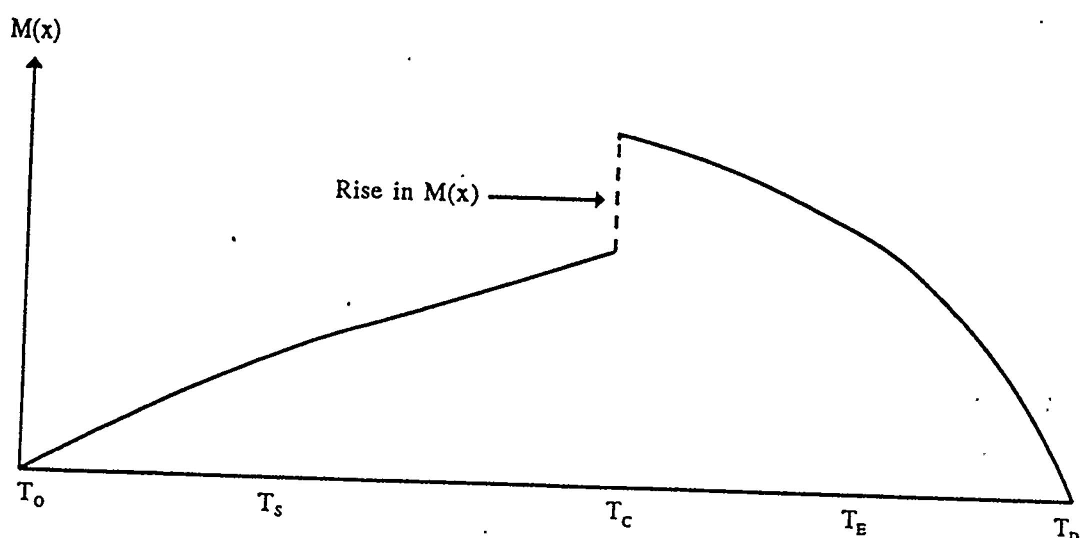

Figure 3.2.4 Rise in Daily Through Passenger Load at CBD

Consider a train running from T_S to T_E through T_C. Passengers whose trips are destined to T_C as well as places not located along the rail section T_CT_E will alight from train at T_C. This situation will cause an instant drop in M(x) at T_C (Figure 3.2.3). One can consider the situation where more passengers riding on trains operating on the rail lines T_BT_F and T_GT_H will transfer at T_C for continuation of their trips along the rail section T_CT_E, with few passengers alighting from train operating along the rail line T_ST_E at T_C. This scenario will instantly increase M(x) at T_C (Figure 3.2.4). It should be disclosed that the effect of discontinuity in M(x) at T_C will not be considered in the analysis.

3.5 USER TIME COST

In this analysis, user time cost is defined as the travel time of passengers riding in buses in the regions T_OT_S and T_ET_D as well as in train in the region T_ST_E. Consideration is given to travel time of feeder buses on routes parallel to the rail centre line. However, the travel time of feeder buses on routes perpendicular to the rail centre line is neglected since it is generally independent of the rail line length in most cases. Also travel time of feeder buses on all routes parallel to the rail centre line, though dependent on the rail station locations, is generally independent of the rail line length except perhaps at the two rail termini and hence will not be considered in the analysis. Besides, the travel time of feeder buses on routes perpendicular to the rail centre line in the rail region is neglected as well since that distance is independent of the rail line length.

The bus riding time cost due to passengers travelling by bus in Section T_OT_S in the direction T_O to T_S is given as:

$$\gamma_B \int_0^{X_S} M(x)\, dx \qquad (3.1)$$

where M(x) is the daily through passenger load at any point x for travel in the direction $T_O T_D$ and γ_B is the average cost of travel time by feeder bus per passenger per kilometre. The travel time cost by train for passenger trips in Section $T_S T_E$ is:

$$\gamma_R \int_{X_S}^{X_E} M(x)\, dx \qquad (3.2)$$

where γ_R is the average cost of travel time by train per passenger pert kilometre. The bus riding time cost by passengers travelling in section $T_E T_A$ is given by the relation:

$$\gamma_B \int_{X_E}^{L} M(x)\, dx \qquad (3.3)$$

If it is assumed that the returned trip traverses back the same route, then the total user time cost is given by twice the sum of Equations 3.1, 3.2 and 3.3:

$$2\gamma_B \int_0^{X_S} M(x)\, dx + 2\gamma_R \int_{X_S}^{X_E} M(x)\, dx + 2\gamma_B \int_{X_E}^{L} M(x)\, dx \qquad (3.4)$$

3.6 RAIL AND BUS OPERATING COSTS

As in the case of the determination of rail and bus operating costs for a CBD-Suburban corridor where the passenger travel demand is expressed in terms of passenger-kilometres (Section 2.6), the rail and bus operating costs considering a crosstown corridor are also determined such that the passenger travel demand is defined in terms of passenger-kilometres.

The total rail and bus operating costs is obtained by replacing γ_B and γ_R as given by Equation 3.4 by λ_B and λ_R respectively, where λ_B is the average cost of operating a bus per passenger per kilometre and λ_R is the average cost of operating a train per passenger per kilometre. Hence for a return trip, the total rail and bus operating cost is:

$$2\lambda_B \int_0^{X_S} M(x)\,dx + 2\lambda_R \int_{X_S}^{X_E} M(x)\,dx + 2\lambda_B \int_{X_E}^{L} M(x)\,dx \qquad (3.5)$$

3.7 RAIL LINE COST

The daily rail line cost considering a cross-town haul rail line with X_S and X_E as the start and end points of the rail line respectively, is given as:

$$\int_{X_S}^{X_E} \gamma_L(x)\,dx \qquad (3.6)$$

3.8 PASSENGER TRANSFER PENALTY COST

Figure 3.3 shows a typical passenger transfer phenomenon associated with passenger trips along a cross-town line haul. A passenger travelling from O_1 to D_1 will experience one form of transfer from bus to bus at T_1 or no transfer depending on the configuration of the feeder bus routes. A passengers travelling from O_2 to D_2 will be subjected to two forms of transfers. These are transfer from bus to train at T_S and transfer

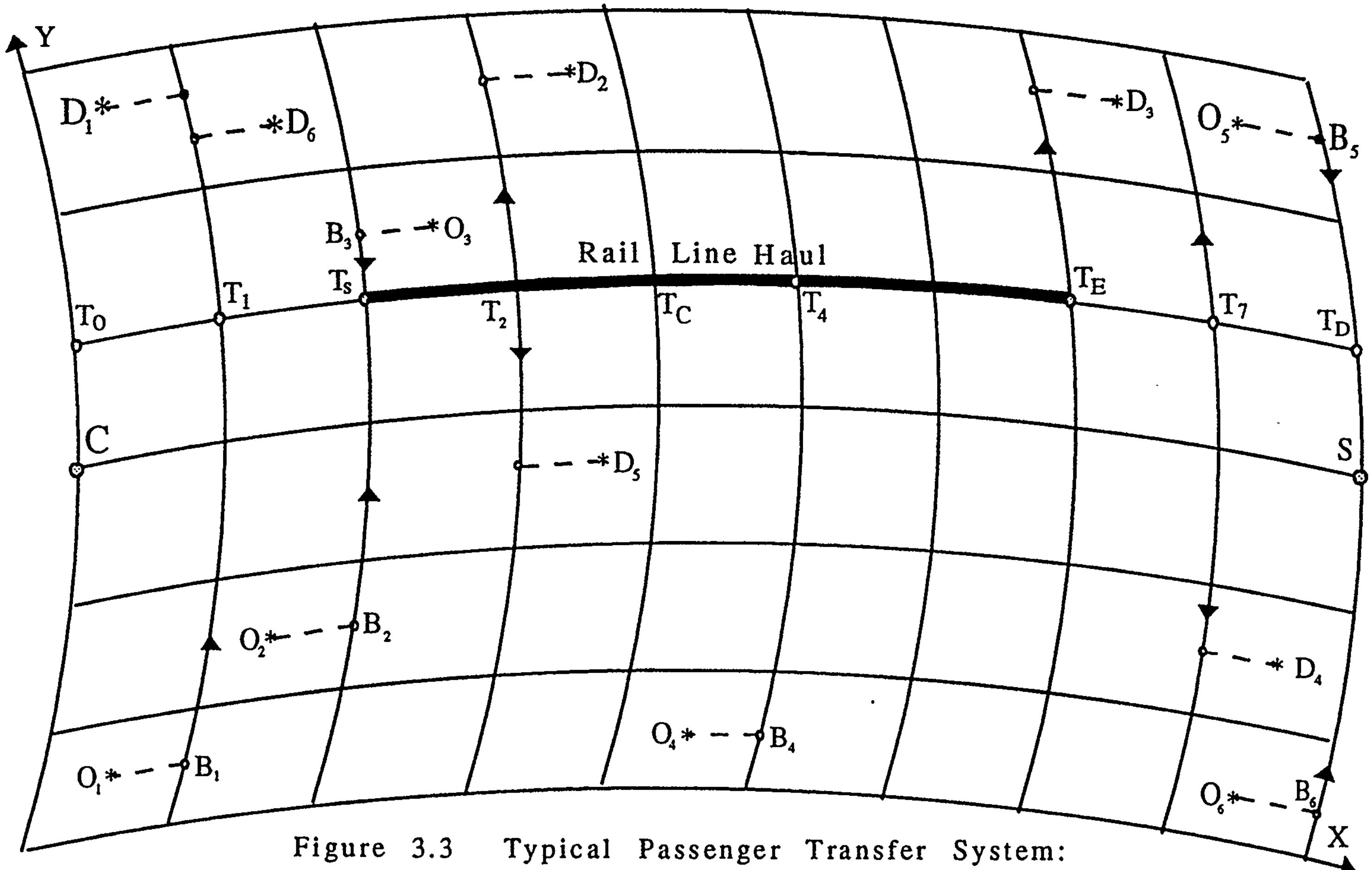

Figure 3.3 Typical Passenger Transfer System: Crosstown Transportation Corridor

from train to bus at T_2. Also, a passenger travelling from O_3 to D_3 will experience two form of transfers: transfer from bus to train at T_S followed by transfer from train to bus at T_E. A passenger travelling from O_4 and D_4 will experience two forms of transfers. These are transfer from bus to train at T_4, followed by another transfer form train to bus at T_7.

However, a passenger travelling from O_5 to D_5 will experience three types of transfers: transfer from bus to bus at T_D, bus to train at T_E and train to bus at T_2. A passenger travelling from O_6 to D_6 will be subjected to four types of transfers. These are transfer from bus to bus at T_D, bus to train at T_E, train to bus at T_S and bus to bus at T_1. As explained in Section 2.9, the maximum number of transfer by a passenger for a given trip is two. Particularly, the first and second forms of passenger transfers at termini T_S and T_E are considered. Three or more forms of transfers for a given passenger trip are unlikely to occur, i.e they will not use the system. Since T_S and T_E are assumed to be major transfer points, only the transfer of passengers from bus to train and train to bus at T_S and T_E is considered. The analysis thus pertain to determination of optimal rail line length between two adjacent major transfer points T_S and T_E.

Consider passenger trips from bus section T_OT_S passing through to rail section T_ST_E with transfer from bus to train occurring at T_S. Since the number of passengers transferring at T_S is $M(X_S)$, then the related passenger penalty cost is:

$$\gamma_P M(X_S) \tag{3.7}$$

For trips continuing from rail section T_ET_S to bus section T_ET_D with transfer of passengers occurring at T_E, the associated transfer penalty cost is:

$$\gamma_P M(X_E) \tag{3.8}$$

We note that the above includes the passenger going from section $T_O T_S$ to section $T_E T_D$. Assuming that trips are returned by the same modes by retracing paths, the total passenger transfer penalty cost is therefore given by:

$$2[\gamma_P M(X_E) + \gamma_P M(X_S)] \tag{3.9}$$

3.9 RAIL FLEET COST

Detailed discussion on the formulation of rail fleet cost for a CBD-Suburban Corridor considering a passenger many to many demand at afternoon peak period is presented in Section 2.8. In particular, the formulation of rail fleet cost for passenger travel on a train along a crosstown haul rail line haul is similar to that presented for the case of passenger travel in a train along a CBD-Suburban corridor. To this end, all assumptions and conditions considered in Section 2.8 are assumed to be applicable to the formulation of rail fleet cost for crosstown corridor analysis. Transit ridership data on the Brentwood-Anderson LRT line in Calgary during the afternoon peak period is plotted, and found to be parabolic in shape (Figure 3.4).

Consider the case of a train running from T_S to T_E through T_C over a rail line of length X_R. Assume that the return trip of the train traverses back the same route. Then with reference to Equation 2.15, the required daily rail operating cost is given as:

$$[2(X_E - X_S)\Lambda_R + \tau_R]\lambda_F M^* \tag{3.10}$$

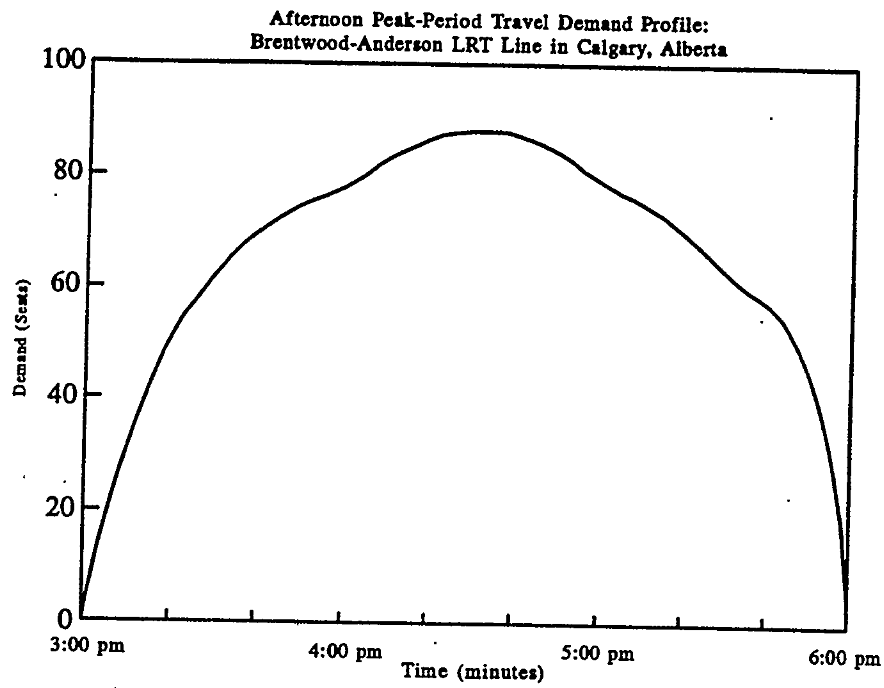

Figure 3.4 Variation of Passenger Many to Many Demand with Travel Time

3.10 OPTIMIZATION

The total transportation cost $Z(X_S,X_E)$, obtained by summing Equations 3.4, 3.5, 3.6, 3.9 and 3.10, is thus:

$$\begin{aligned} Z(X_S,X_E) &= 2\gamma_B\int_0^{X_S} M(x)\,dx + 2\gamma_R\int_{X_S}^{X_E} M(x)\,dx + 2\gamma_B\int_{X_E}^{L} M(x)\,dx \\ &+ 2\lambda_B\int_0^{X_S} M(x)\,dx + 2\lambda_R\int_{X_S}^{X_E} M(x)\,dx + 2\lambda_B\int_{X_E}^{L} M(x)\,dx \\ &+ \int_{X_S}^{X_E}\gamma_L(x)\,dx + 2[\gamma_P M(X_E) + \gamma_P M(X_S)] \\ &+ [2(X_E-X_S)\Lambda_R+\tau_R]\lambda_F M^* \end{aligned} \tag{3.11}$$

Keeping X_E constant and differentiating Equation 3.11 with respect to X_S gives:

$$\frac{\partial Z}{\partial X_S} = 2\gamma_B M(X_S) - 2\gamma_R M(X_S) + 2\lambda_B M(X_S) - 2\gamma_R M(X_S) - \gamma_L(X_S) + 2\gamma_P M'(X_S) - 2\Lambda_R \lambda_F M^* \quad (3.12)$$

Setting equation 3.11 to zero gives:

$$2[(\gamma_B - \gamma_R) + (\lambda_B - \lambda_R)]M(X_S) = \gamma_L(X_S) - 2\gamma_P M'(X_S) + 2\Lambda_R \lambda_F M^* \quad (3.13)$$

For minimum global transportation cost to be realised, the second derivative of equation 3.11 with respect X_S should be positive:

$$\frac{\partial^2 Z}{\partial X_S^2} = 2[(\gamma_B - \gamma_R) + (\lambda_B - \lambda_R)]M'(X_S) - \gamma_L'(X_S) + 2\gamma_P M''(X_S) > 0 \quad (3.14)$$

Keeping X_S constant and differentiating Equation 3.11 with respect to X_E gives:

$$\frac{\partial Z}{\partial X_E} = 2\gamma_R M(X_E) - 2\gamma_B M(X_E) + 2\lambda_R M(X_E) - 2\lambda_B M(X_E) + \gamma_L(X_E) + 2\gamma_P M'(X_E) + 2\Lambda_R \lambda_F M^* \quad (3.15)$$

for which at the optimum point X_E :

$$2[(\gamma_B - \gamma_R) + (\lambda_B - \lambda_R)]M(X_E) = \gamma_L(X_E) + 2\gamma_P M'(X_E) + 2\Lambda_R \lambda_F M^* \quad (3.16)$$

Setting the second derivative of Equation 3.11 with respect to X_E to be greater than zero gives:

$$\frac{\partial^2 Z}{\partial X_E^2} = 2[(\gamma_B - \gamma_R) + (\lambda_B - \lambda_R)]M'(X_E) + \gamma_L'(X_E) + 2\gamma_P M''(X_E) > 0 \quad (3.17)$$

3.11 GRAPHICAL ANALYSIS

This section of the report discusses the determination of the optimal rail line length X_R using graphical analysis. The central issue is to determine the optimal location of the starting point T_S and ending point T_E of the rail line (i.e. the optimal X_S and X_E) using Equations 3.12 and 3.15 respectively. The difference between X_S and X_E is the optimum rail line length X_R.

3.11.1 DETERMINATION OF X_S

Substitution of $\Delta\gamma = 2[(\gamma_B-\gamma_R)+(\lambda_B-\lambda_R)]$ and $\theta = 2\Lambda_R\lambda_R M^*$ into Equations 3.12 and 3.14, and the neglect of the first and second derivatives of passenger transfer cost, modifies Equations 3.12 and 3.14 respectively to:

$$Z'(X_S) = \Delta\gamma M(X_S) - \gamma_L(X_S) - \theta \qquad (3.18)$$

$$Z''(X_S) = \Delta\gamma M'(X_S) - \gamma_L'(X_S) \qquad (3.19)$$

3.12 UNIFORM RAIL LINE COST

If a uniform rail line cost is assumed, then $\gamma_L'(X_S)=0$. Equations 3.18 and 3.19 therefore becomes:

$$Z'(X_S) = \Delta\gamma M(X_S) - \gamma_L \qquad (3.20.1)$$

$$Z''(X_S) = \Delta\gamma M'(X_S) \qquad (3.20.2)$$

The graph of through passenger load $M(X_S)$ against length of line haul X_S is shown in Figure 3.5. $M(X_S)$ is zero at T_O where $X_S=0$. $M(X_S)$ then increases till it attains its maximum load M_L at the maximum load point T_C where $X_S=X_L$. From T_C, $M(X_S)$ decreases till it is zero at T_D where $X_S=L$. In particular, $Z(X_S)$ is minimized if $Z'(X_S)=0$ and $Z''(X_S)>0$. From Equation 3.20.2, $Z''(X_S)>0$ only if $M'(X_S)>0$, i.e. $M(X_S)$ is increasing.

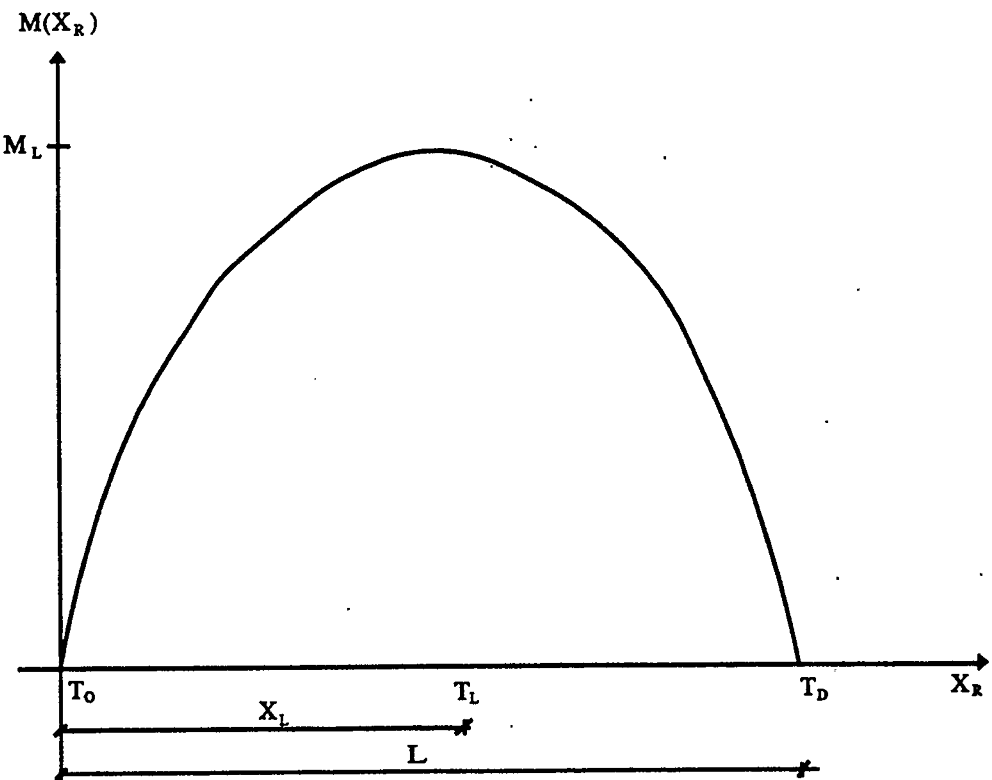

Figure 3.5 Variation of Daily Through Passenger Load with Line Length

This occurs in the region $0\leq X_S\leq X_L$. Conversely, $Z(X_S)$ is maximized when $Z'(X_S)=0$ and $Z''(X_S)<0$. $Z''(X_S)<0$ provided $M'(X_S)<0$, i.e. $M(X_S)$ is decreasing. This is found to occur in the region $X_L\leq X_S\leq L$. In particular, the solutions to Equation 3.20.1 depend on whether $\gamma_L+\theta>\Delta\gamma M_L$ or $\gamma_L+\theta<\Delta\gamma M_L$ as explained below.

3.12.1 CASE 1: $\gamma_L + \theta > \Delta\gamma M_L$

It is possible that no solution will exist in this case. Accordingly, the optimal $X_S=0$ (Figure 3.6.1). This possibility increases with the above inequality. Furthermore, since

$$\gamma_L + \theta > \Delta\gamma M_L \tag{3.21.1}$$

then

$$M_L < \frac{\gamma_L + \theta}{\Delta\gamma} \tag{3.21.2}$$

3.12.2 CASE 2: $\gamma_L + \theta < \Delta\gamma M_L$

In this case, there exist the possibility of obtaining two solutions of X_S (Figure 3.6.2). The total cost is locally minimized at X_1 since $Z''(X_1)>0$ at X_1. The optimal $X_S=X_1$. However, the cost is maximized locally at X_2 since $Z''(X_2)<0$.

3.12.3 SPECIAL CASE

Figure 3.6.3 describes the possibility of obtaining four solutions of X_S. Obviously, the total cost is minimized locally at X_1 or X_3. Under this condition, the total cost at X_1 and X_3, i.e. $Z(X_1)$ and $Z(X_3)$, are determined and compared. The length that gives the overall minimum cost is the global optimal.

3.12.4 CASE 4: GENERAL CASE

An insight regarding the effect of passenger transfer penalty cost (Equation 3.9) on optimal X_S is explored by graphical analysis (Figure 3.6.4). Consideration of passenger

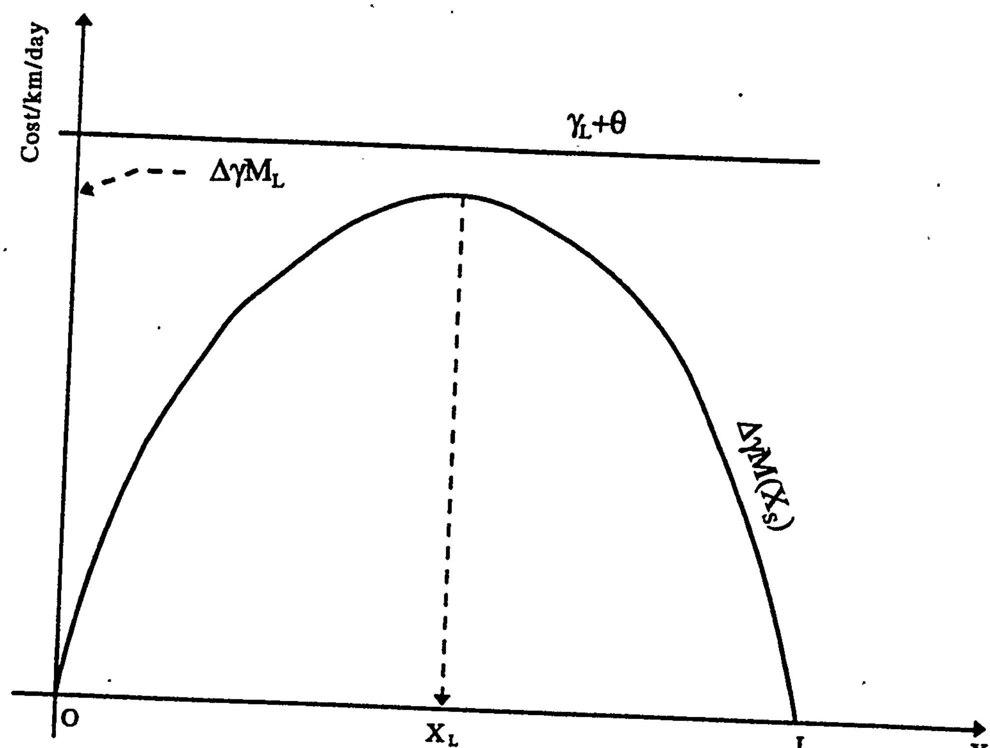

Figure 3.6.1 Optimal Line Length (Uniform Line Cost)

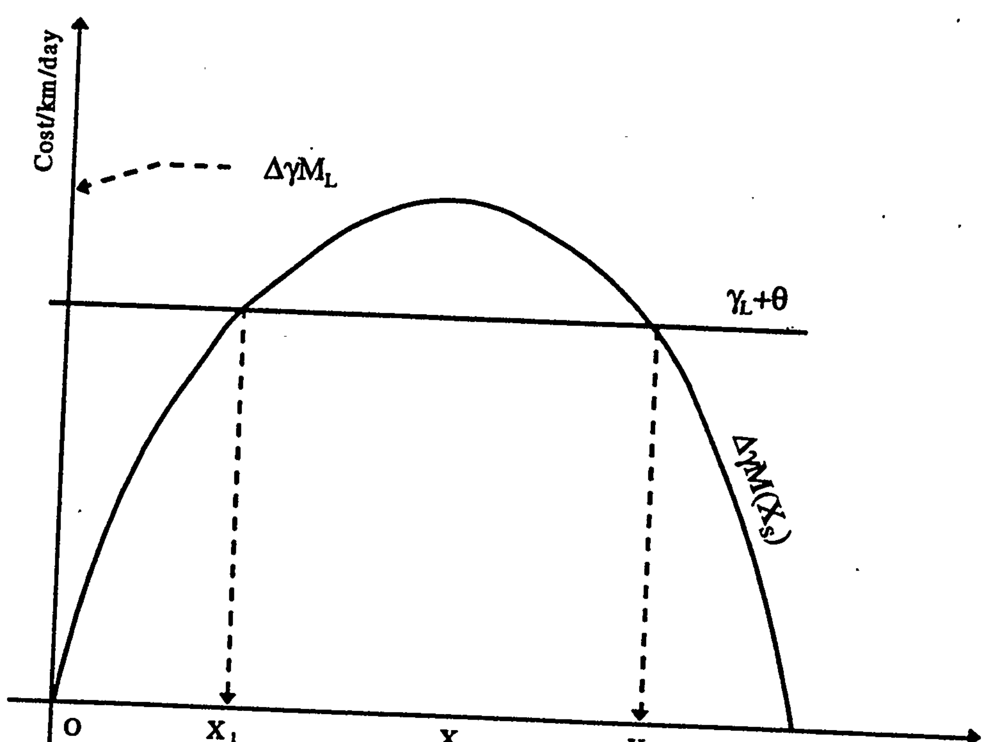

Figure 3.6.2 Optimal Line Length (Uniform Line Cost)

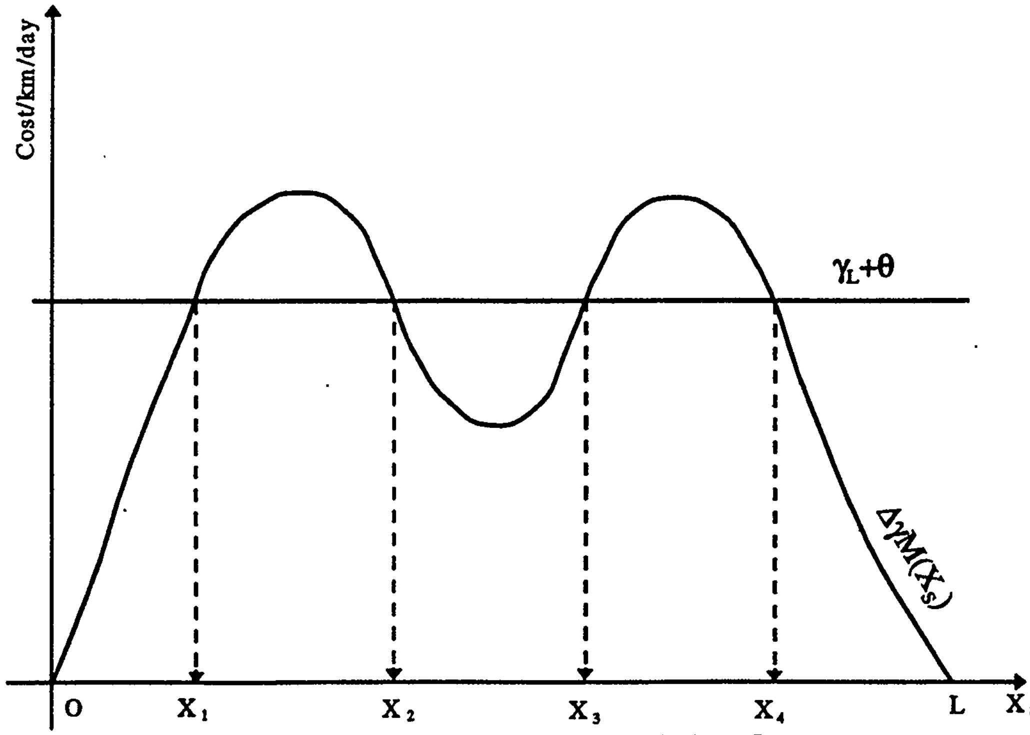

Figure 3.6.3 Optimal Line Length (Uniform Line Cost)

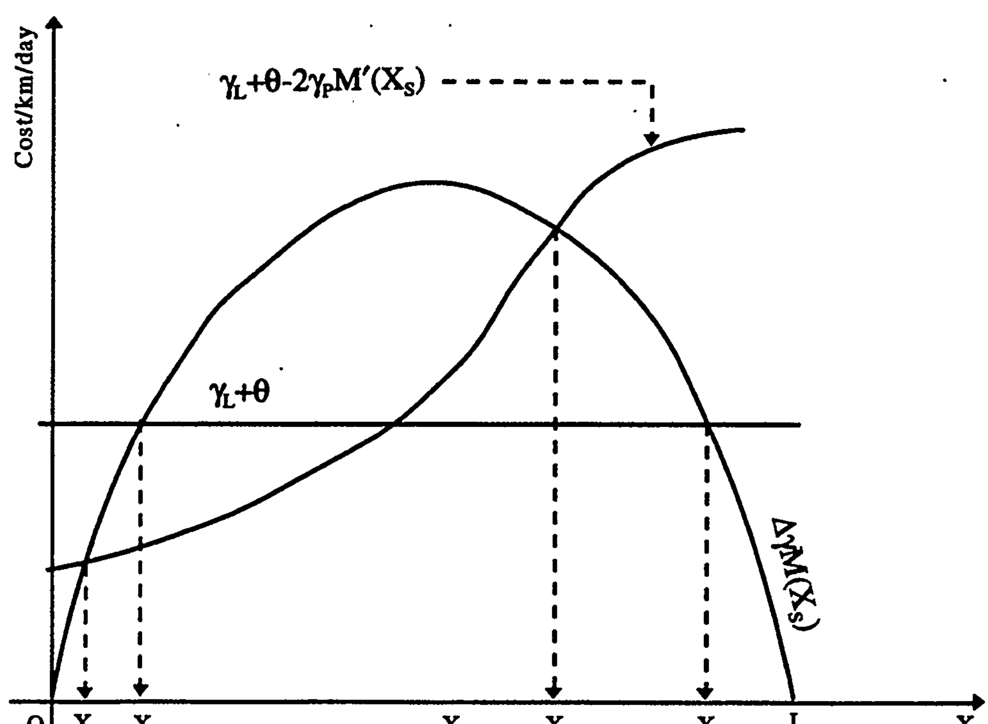

Figure 3.6.4 Optimal Line Length (Uniform Line Cost)

transfer penalty cost changes Equations 3.20.1 and 3.20.2 respectively to:

$$Z'(X_S) = \Delta\gamma M(X_S) - \gamma_L - \theta + 2\gamma_P M'(X_S) \qquad (3.22.1)$$

and

$$Z''(X_S) = \Delta\gamma M'(X_S) + 2\gamma_P M''(X_S) \qquad (3.22.2)$$

It is found that $Z''(X_S)>0$ if $M'(X_R)>0$ and $M''(X_R)<0$. However, $Z''(X_R)<0$ provided $M'(X_R)<0$ and $M''(X_R)>0$. Two solutions of X_S (i.e. X_2 and X_4) are obtained under this condition, and the total cost is found to be minimized locally at X_2 (Figure 3.6.4). It is observed from Figure 3.6.4 that if transfer penalty cost is not considered, the optimal X_S is X_1, which is less than X_2. Hence, the effect of transfer penalty cost is to decrease the optimal X_S.

3.13 NON-UNIFORM RAIL LINE COST

More generally, $\gamma_L(X_S)$ tends to increase as the line moves from the Suburban Region T_O to the CBD, T_C. $\gamma_L(X_S)$ then decrease as the line moves away from the CBD to Suburban Region T_D (Figure 3.7). The expressions for $Z'(X_S)$ and $Z''(X_S)$ given by Equations 3.18 and 3.19 will apply in this case. $Z(X_S)$ is minimized if $Z'(X_S)=0$ and $Z''(X_S)>0$. From Equation 3.19, it is observed that $Z''(X_S)>0$ provided $M'(X_S)>0$ and $\gamma_L'(X_S)<0$. This occur in the region $0\leq X_S\leq X_L$. $Z(X_S)$ is maximized when $Z'(X_S)=0$ and $Z''(X_S)<0$. $Z''(X_S)<0$ if $M'(X_S)<0$ and $\gamma_L'(X_S)>0$. This is found to occur in the region $X_L\leq X_S\leq L$. It is imperative to remark that the solutions to Equation 3.19 depend on whether $\gamma_L(X_S)+\theta>\Delta\gamma M_L$ or $\gamma_L(X_S)+\theta<\Delta\gamma M_L$ as in the following analysis.

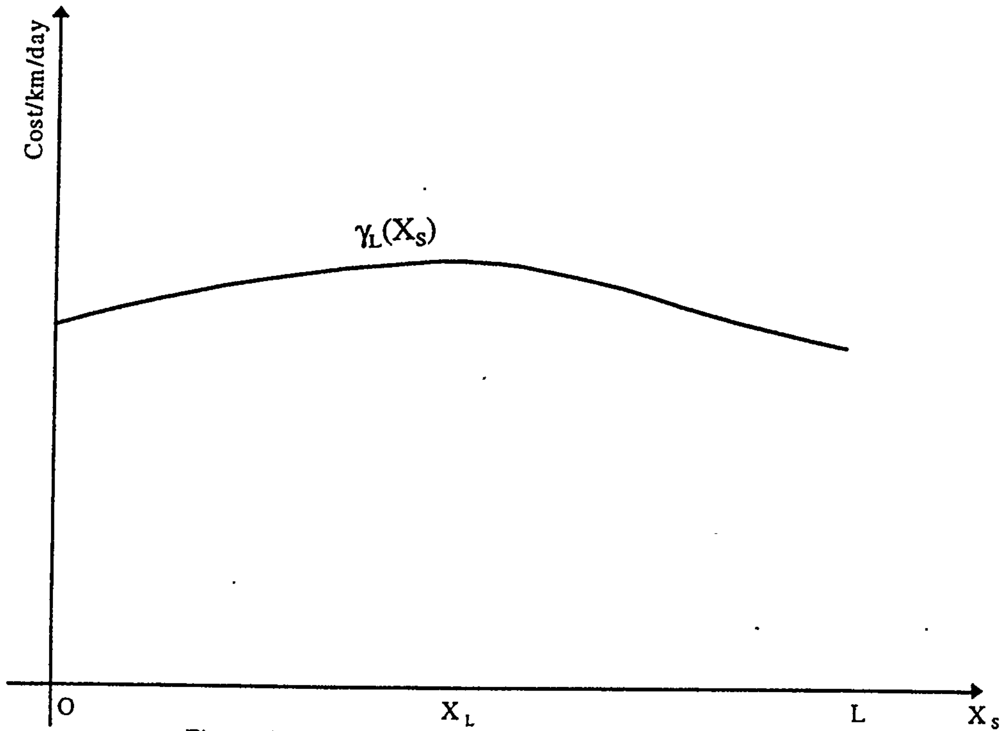

Figure 3.7 Variation of Rail Line Cost with Line Length

3.13.1 CASE 1: $\gamma_L(X_S) + \theta > \Delta\gamma M_L$

Under the above condition, it is possible that no solution will exist (Figure 3.8.1). Hence, the optimal X_S=0. This possibility increases with the above inequality. For

$$\gamma_L(X_S) + \theta > \Delta\gamma M_L \tag{3.23.1}$$

then:

$$M_L < \frac{\gamma_L(X_S) + \theta}{\Delta\gamma} \tag{3.23.2}$$

3.13.2 CASE 2: $\gamma_L(X_S) + \theta < \Delta\gamma M_L$

It is possible to obtain two solutions of X_S in this case (Figure 3.8.2). The total cost is locally minimized at X_1 since $Z''(X_1)>0$ at X_1. Hence the optimal $X_S=X_1$. However,

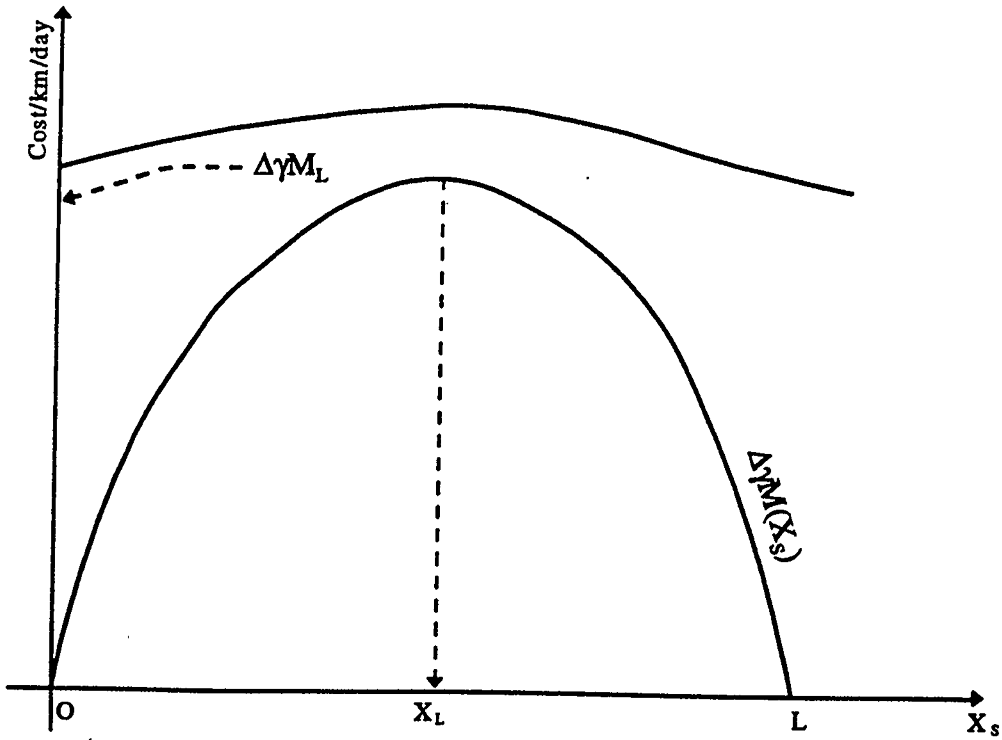

Figure 3.8.1 Optimal Line Length (Non-Uniform Line Cost)

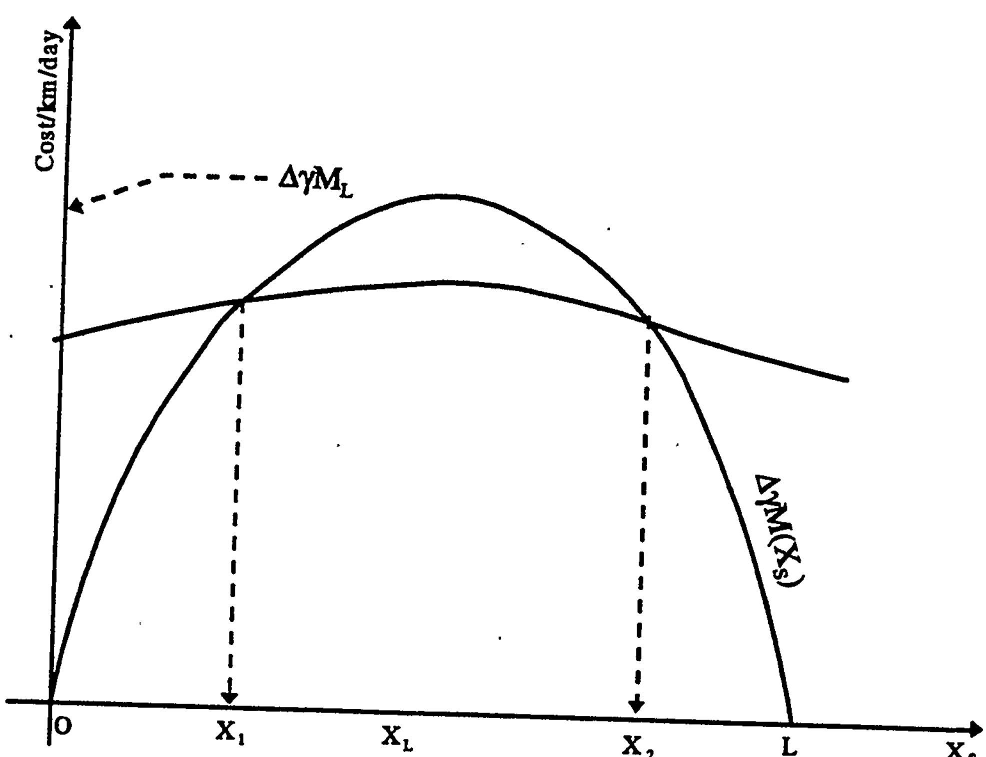

Figure 3.8.2 Optimal Line Length (Non-Uniform Line Cost)

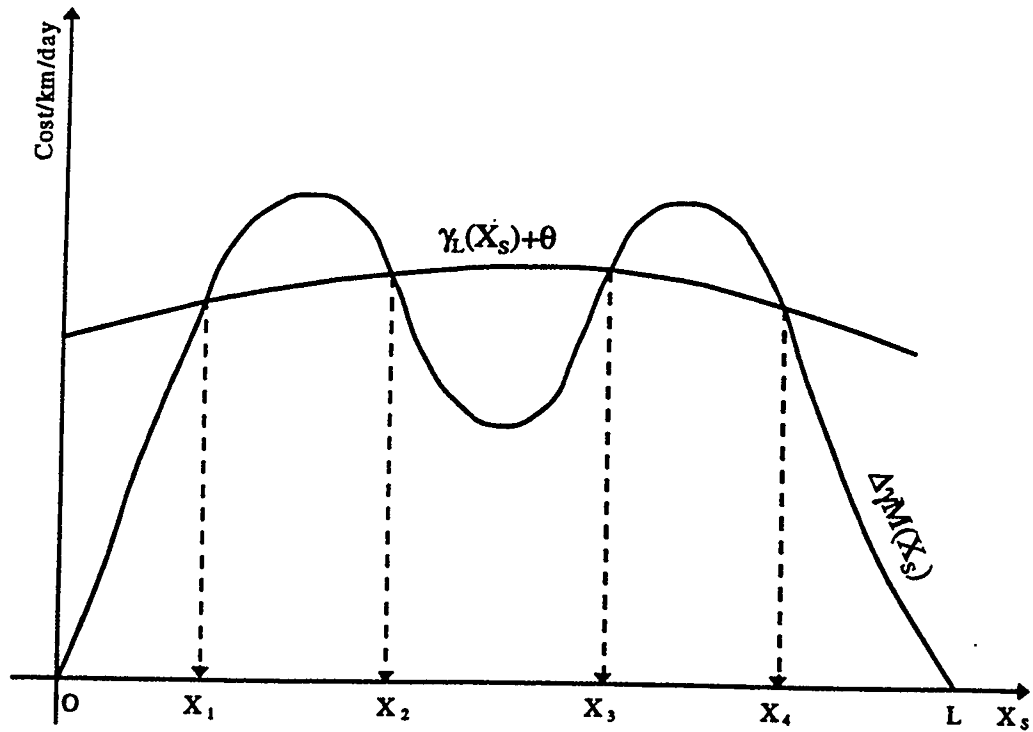

Figure 3.8.3 Optimal Line Length (Non-Uniform Line Cost)

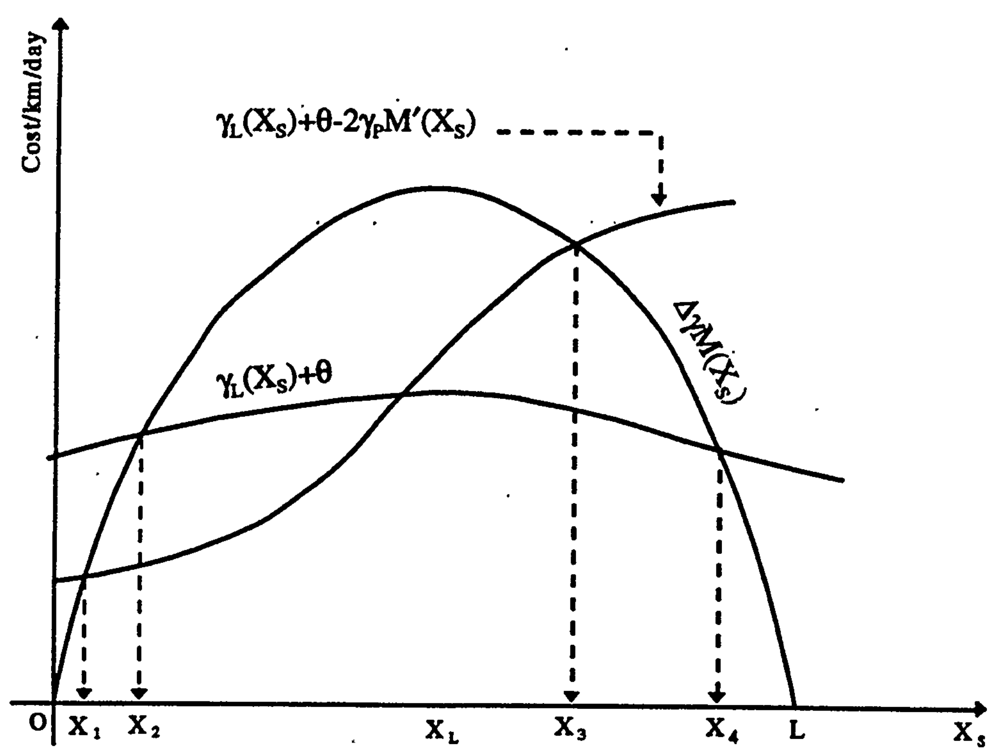

Figure 3.8.4 Optimal Line Length (Non-Uniform Line Cost)

the total cost is maximized locally at X_2 since $Z''(X_2)<0$.

3.13.3 SPECIAL CASE

The chances of obtaining four solutions of X_S is illustrated in Figure 3.8.3. In this case, the total cost is minimized locally at X_1 or X_3. The total cost at X_1 and X_3, i.e. $Z(X_1)$ and $Z(X_3)$, are enumerated and compared. The length that gives the overall minimum cost is the global optimal.

3.13.4 CASE 4: GENERAL CASE

The effect of passenger transfer penalty cost (Equation 3.9) on the optimal rail line length is graphically explained using Figure 3.8.4. By considering passenger transfer penalty cost, Equations 3.18 and 3.19 respectively become:

$$Z'(X_S) = \Delta\gamma M(X_S) - \gamma_L(X_S) - \theta + 2\gamma_P M'(X_S) \qquad (3.24.1)$$

and

$$Z''(X_S) = \Delta\gamma M'(X_S) - \gamma_L'(X_S) + 2\gamma_P M''(X_S) \qquad (3.24.2)$$

It is observed from Equation 3.34.2 that $Z''(X_S)>0$ if $M'(X_S)>0$, $\gamma_L'(X_S)<0$ and $M''(X_S)<0$. However, $Z''(X_S)<0$ provided $M''(X_S)<0$, $\gamma_L(X_S)>0$ and $M''(X_S)>0$. Two solutions of X_S (i.e. X_2 and X_4) are obtained under this condition. The total cost is minimized locally at X_2 (Figure 3.8.4). From Figure 3.8.4, it is observed that if transfer penalty cost is neglected, the optimal X_S is X_1, which is less than X_2. Thus, the effect of transfer penalty cost is to decrease the optimal X_S.

3.14 DETERMINATION OF X_E

Substitution of $\Delta\gamma = 2[(\gamma_B\text{-}\gamma_R)+(\lambda_B\text{-}\lambda_R)]$ and $\theta = 2\Lambda_R\lambda_R M^*$ into Equations 3.15 and 3.17, and the neglect of the first and second derivatives of passenger transfer cost gives:

$$Z'(X_E) = -\Delta\gamma M(X_E) + \gamma_L(X_E) + \theta \qquad (3.25)$$

$$Z''(X_E) = -\Delta\gamma M'(X_E) + \gamma_L'(X_E) \qquad (3.26)$$

3.15 UNIFORM RAIL LINE COST

Under the assumption that the discounted rail line cost per unit length is uniform, $\gamma_L'(X_E)$=0. Hence Equations 3.25 and 3.25 respectively become:

$$Z'(X_E) = -\Delta\gamma M(X_E) + \gamma_L \qquad (3.27.1)$$

$$Z''(X_E) = -\Delta\gamma M'(X_E) \qquad (3.27.2)$$

$Z(X_E)$ is minimized if $Z'(X_E)=0$ and $Z''(X_E)>0$. From Equation 3.27.2, $Z''(X_E)>0$ only if $M'(X_E)<0$, i.e. $M(X_E)$ is decreasing. This occur in the region $X_L \le X_E \le L$. Conversely, $Z(X_E)$ is maximized when $Z'(X_E)=0$ and $Z''(X_E)<0$. $Z''(X_E)<0$ provided $M'(X_E)>0$, i.e. $M(X_E)$ is increasing. This is occur in the region $0 \le X_E \le X_L$. The solutions to Equation 3.27.1 depend on whether $\gamma_L+\theta>\Delta\gamma M_L$ or $\gamma_L+\theta<\Delta\gamma M_L$ as explained as follows.

3.15.1 CASE 1: $\gamma_L + \theta \geq \Delta\gamma M_L$

It is possible that no solution will exist under the above condition. Hence the optimal X_E=0 (Figure 3.9.1). This possibility increases with the above inequality. Furthermore, since:

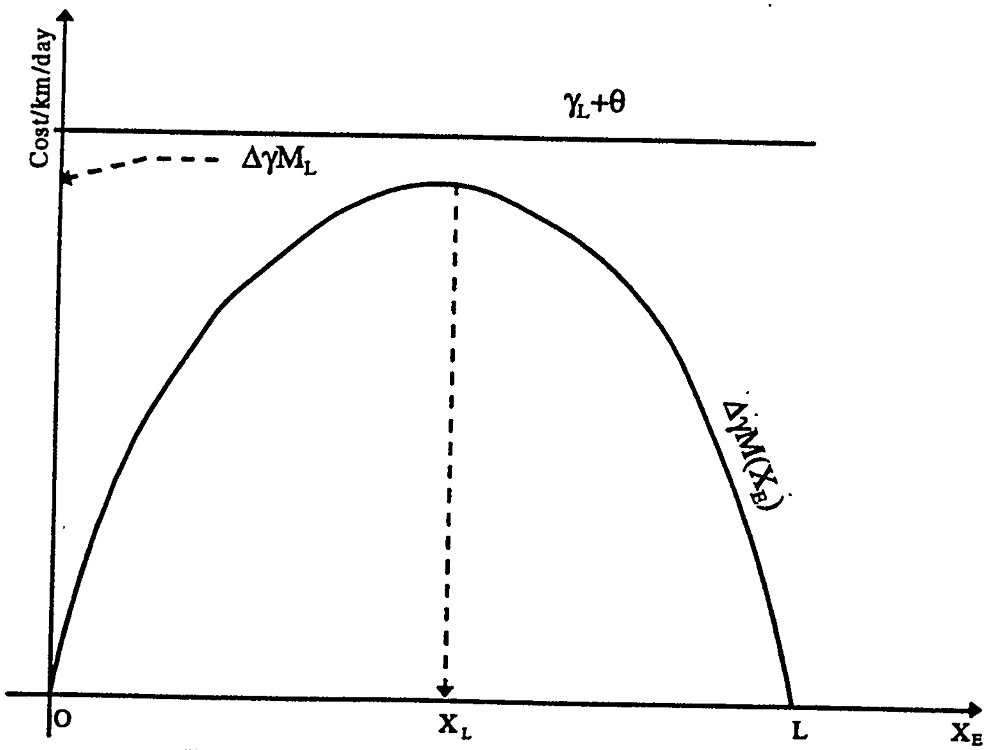

Figure 3.9.1 Optimal Line Length (Uniform Line Cost)

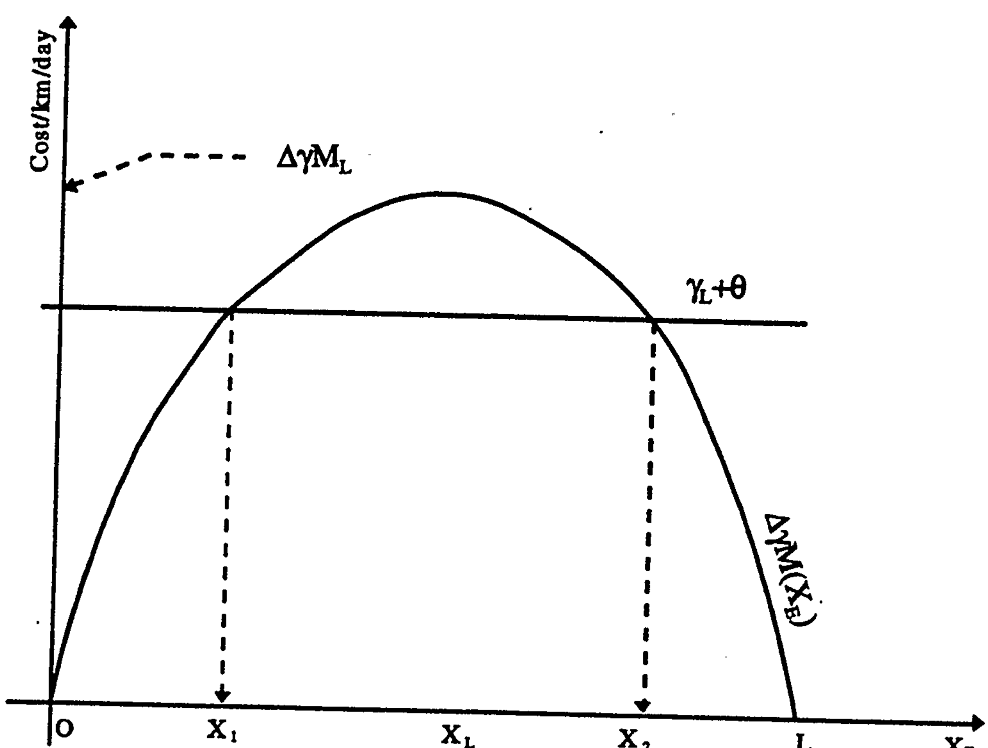

Figure 3.9.2 Optimal Line Length (Uniform Line Cost)

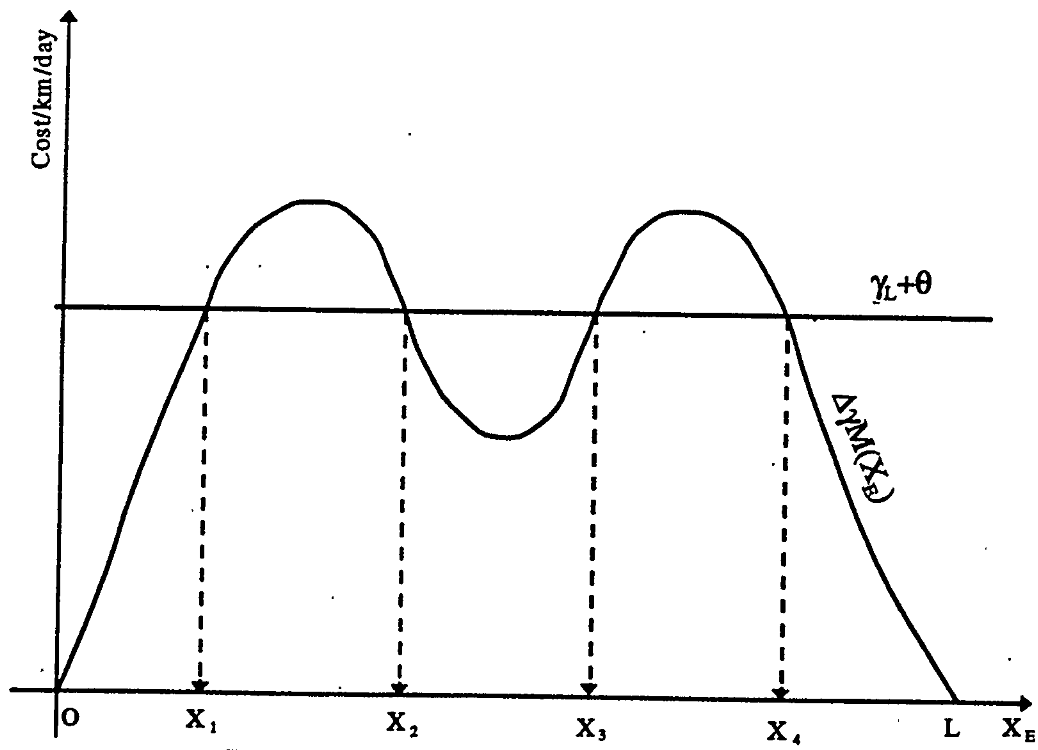

Figure 3.9.3 Optimal Line Length (Uniform Line Cost)

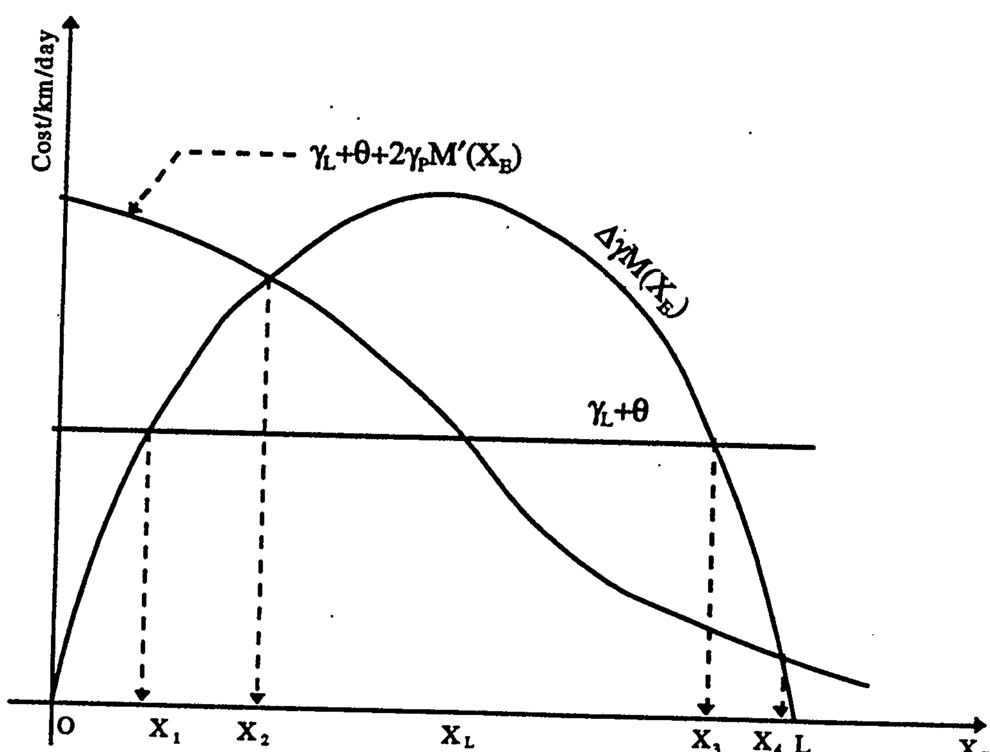

Figure 3.9.4 Optimal Line Length (Uniform Line Cost)

$$\gamma_L + \theta > \Delta\gamma M_L \qquad (3.28.1)$$

then:

$$M_L < \frac{\gamma_L + \theta}{\Delta\gamma} \qquad (3.28.2)$$

3.15.2 CASE 2: $\gamma_L + \theta < \Delta\gamma M_L$

There exist the possibility of obtaining two solutions of X_E (Figure 3.9.2). The total cost is locally minimized at X_2 since $Z''(X_2)>0$ at X_2. The optimal $X_E=X_2$. However, the cost is maximized locally at X_1 since $Z''(X_1)<0$.

3.15.3 SPECIAL CASE

Figure 3.9.3 describes the possibility of obtaining four solutions of X_E. The total cost is minimized locally at X_2 or X_4. Under this condition, the total cost at X_2 and X_4, i.e. $Z(X_2)$ and $Z(X_4)$, are determined and compared. The length that gives the overall minimum cost is the global optimal.

3.15.4 CASE 4: GENERAL CASE

The effect of passenger transfer penalty cost (Equation 3.9) on optimal X_E is explored by graphical analysis (Figure 3.9.4). Consideration of passenger transfer penalty cost changes Equations 3.27.1 and 3.27.2 respectively to:

$$Z'(X_E) = -\Delta\gamma M(X_E) + \gamma_L + \theta + 2\gamma_P M'(X_E) \qquad (3.29.1)$$

and

$$Z''(X_E) = -\Delta\gamma M'(X_E) + 2\gamma_P M''(X_E) \qquad (3.22.2)$$

$Z''(X_E)>0$ if $M'(X_E)<0$ and $M''(X_E)>0$. However, $Z''(X_E)<0$ provided $M'(X_E)>0$ and $M''(X_E)<0$. Two solutions of X_E (i.e. X_2 and X_4) are obtained under this condition, and the total cost is found to be minimized locally at X_4 (Figure 3.9.4). It is observed from Figure 3.9.4 that if transfer penalty cost is not considered, the optimal X_E is X_3, which is less than X_4. Hence, the effect of the transfer penalty cost is to increase the optimal X_E.

3.16 NON-UNIFORM RAIL LINE COST

The expressions for $Z'(X_E)$ and $Z''(X_E)$ given by Equations 3.25 and 3.26 will apply in this case. $Z(X_E)$ is minimized if $Z'(X_E)=0$ and $Z''(X_E)>0$. From Equation 3.26, it is found that $Z''(X_E)>0$ provided $M'(X_E)<0$ and $\gamma_L'(X_E)>0$. This occurs in the region $X_L \leq X_E \leq L$. $Z(X_E)$ is maximized when $Z'(X_E)=0$ and $Z''(X_E)<0$. $Z''(X_E)<0$ if $M'(X_E)<0$ and $\gamma_L'(X_E)>0$. This is occur in the region $0 \leq X_E \leq X_L$. The solutions to Equation 3.25 depend on whether $\gamma_L(X_E)+\theta>\Delta\gamma M_L$ or $\gamma_L(X_E)+\theta<\Delta\gamma M_L$ as explained below.

3.16.1 CASE 1: $\gamma_L(X_S) + \theta > \Delta\gamma M_L$

In this case, it is possible that no solution will exist (Figure 3.10.1). Hence, the optimal $X_E=0$. This possibility increases with the above inequality. For

$$\gamma_L(X_E) + \theta > \Delta\gamma M_L \qquad (3.30.1)$$

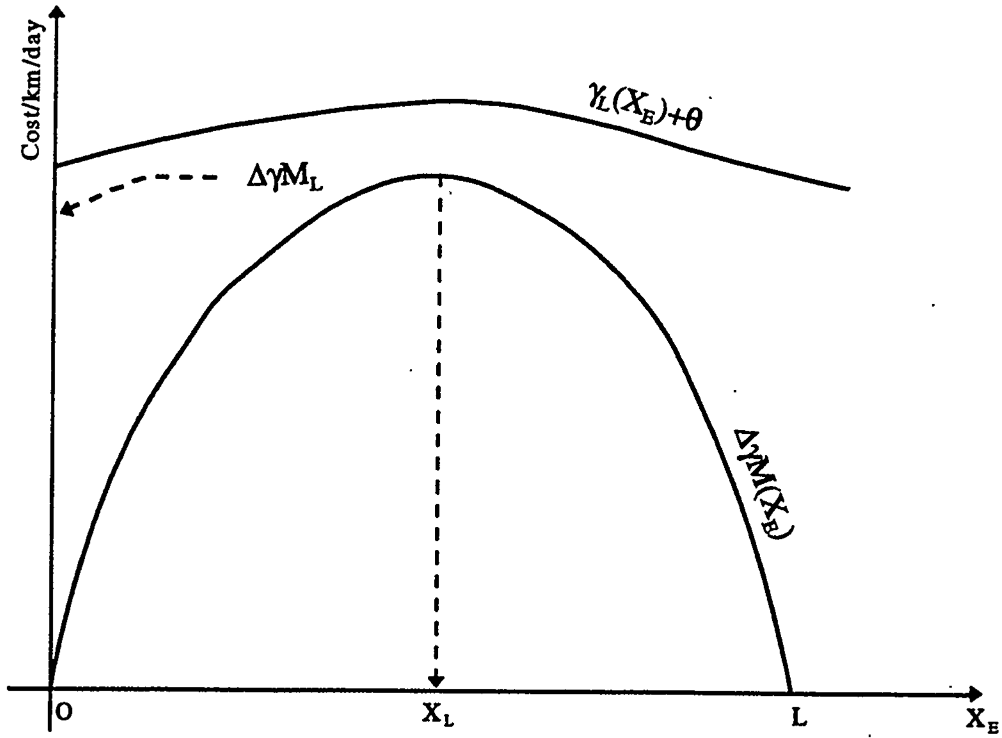

Figure 3.10.1 Optimal Line Length (Non-Uniform Line Cost)

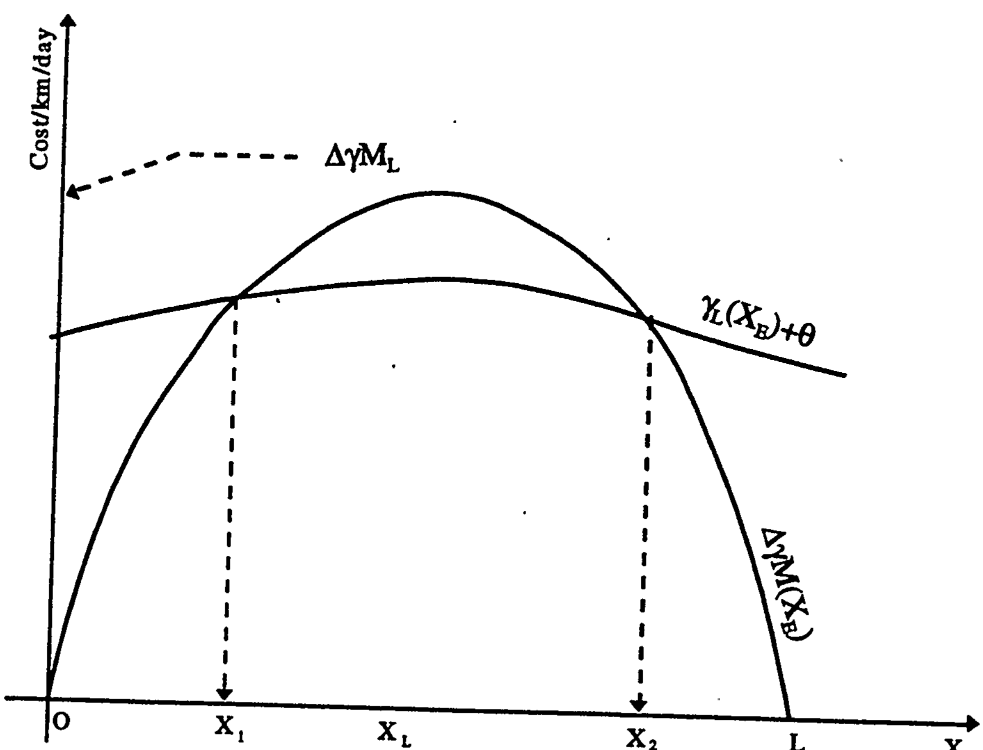

Figure 3.10.2 Optimal Line Length (Non-Uniform Line Length)

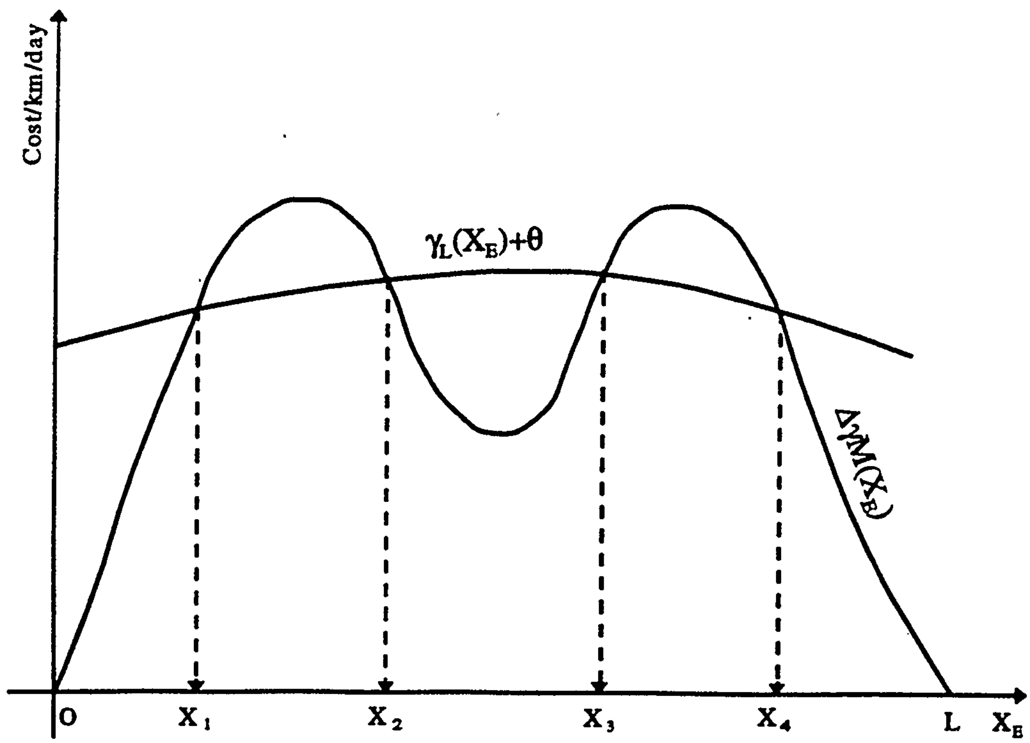

Figure 3.10.3 Optimal Line Length (Non-Uniform Line Cost)

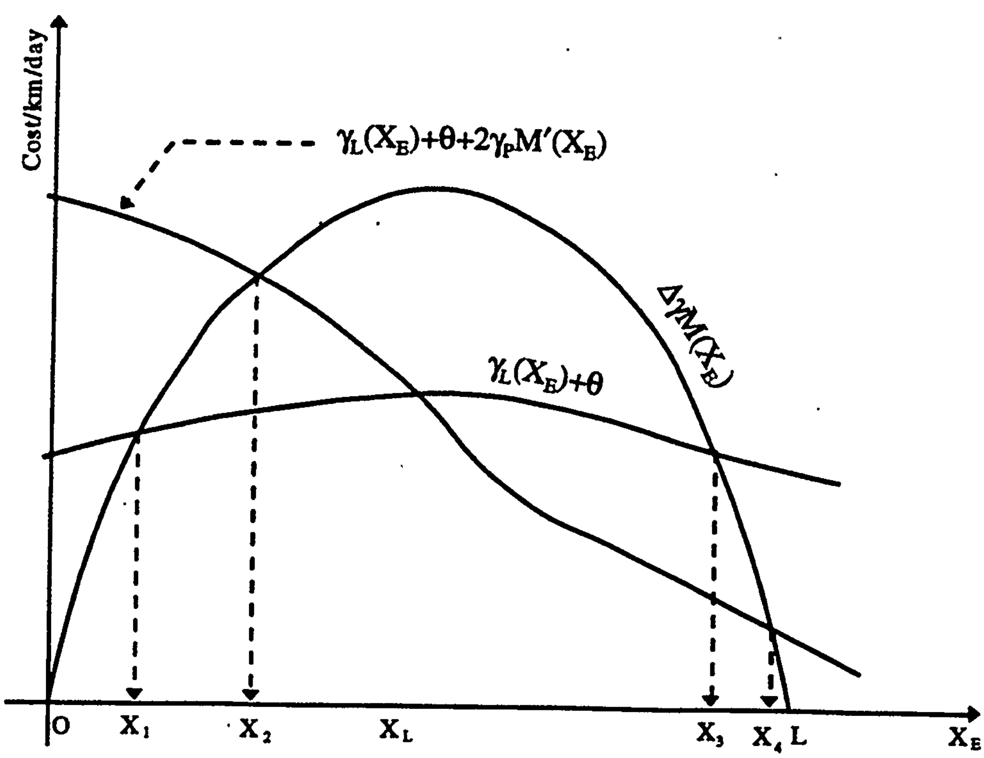

Figure 3.10.4 Optimal Line Length (Non-Uniform Line Cost)

then:

$$M_L < \frac{\gamma_L(X_E) + \theta}{\Delta\gamma} \qquad (3.30.2)$$

3.16.2 CASE 2: $\gamma_L(X_E) + \theta < \Delta\gamma M_L$

It is possible to obtain two solutions of X_E under this condition (Figure 3.10.2). The total cost is locally minimized at X_2 since $Z''(X_2) \succ 0$ at X_2. Hence the optimal X_E=X_2. However, the total cost is maximized locally at X_1 since $Z''(X_1)<0$.

3.16.3 SPECIAL CASE

The possibility of obtaining four solutions of X_E is graphically illustrated in Figure 3.10.3. In this case, the total cost is minimized locally at X_2 or X_4. The total cost at X_2 and X_4, i.e. $Z(X_2)$ and $Z(X_4)$, are enumerated and compared. The length that gives the overall minimum cost is the global optimal.

3.16.4 CASE 4: GENERAL CASE

The effect of passenger transfer penalty cost (Equation 3.9) on the optimal X_E is graphically explained using Figure 3.8.4. By considering passenger transfer penalty cost, Equations 3.25 and 3.26 respectively become:

$$Z'(X_E) = -\Delta\gamma M(X_E) + \gamma_L(X_E) + \theta + 2\gamma_P M'(X_E) \qquad (3.31.1)$$

and

$$Z''(X_E) = -\Delta\gamma M'(X_E) + \gamma_L'(X_E) + 2\gamma_P M''(X_E) \qquad (3.31.2)$$

It is observed from Equation 3.31.2 that $Z''(X_E)>0$ if $M'(X_E)<0$, $\gamma_L'(X_E)>0$ and $M''(X_E)>0$. However, $Z''(X_E)<0$ provided $M'(X_E)>0$, $\gamma_L'(X_E)<0$ and $M''(X_E)<0$. Two solutions of X_E (i.e. X_2 and X_4) are obtained under this condition. The total cost is minimized locally at X_4 (Figure 3.10.4). From Figure 3.10.4, it is observed that if transfer penalty cost is neglected, the optimal X_E is X_3, which is less than X_4. Thus, the effect of the transfer penalty cost is to increase the optimal X_E.

3.17 MODEL APPLICATION

An attempt to determine the optimal location of the starting and ending points of a crosstown LRT line using graphically technique presented in Section 3.12 is discussed below. The Northwest-South LRT line in Calgary is used as a case study (Figure 3.11). A crosstown transportation corridor originating from Crowfoot bus-stop and terminating at Shawnessy bus stop is assumed to exist. A 1991-92 daily passenger ridership data is shown in Table 3.1.

Figures 3.12 and 3.13 respectively depict the daily number of passenger boarding [b(x)] and alighting [a(x)] at point x on the haul line and the daily cumulative of the number of boarding [B(x)] and alighting [A(x)] passengers at point x. The difference between the daily cumulative of the number of boarding and alighting passengers [M(x)] is shown in Figure 3.14. Table 2.2 shows the values of the unit cost and design parameters.

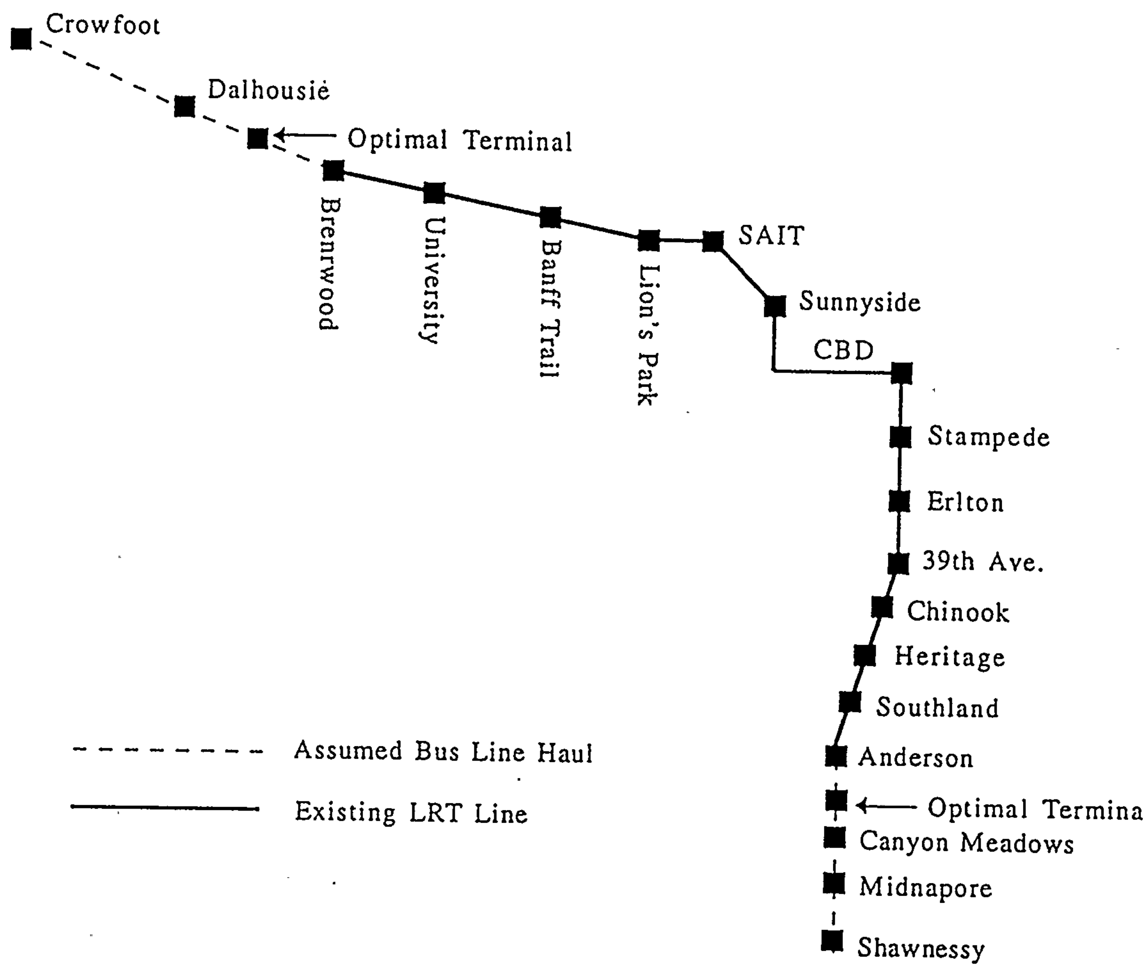

Figure 3.11 Northwest-South Transit Line Haul in Calgary, Alberta

Table 3.1 **1991-92 Daily Passenger Demand in Calgary, Alberta**

Sta. No.	Sta. Name	Dist. (x)	b(x)	a(x)	B(x)	A(x)	M(x)
1	Crowfoot	0.00	0	0	0	0	0
2	Dalhousie	4.74	3460	100	3360	100	3260
3	Brentwood	8.45	2629	200	5989	300	5689
4	University	9.48	4370	1030	10359	1330	9029
5	Banff Trail	10.84	3400	100	13759	1430	12329
6	Lion's Park	11.64	1550	350	15309	1780	13529
7	SAIT	12.73	1970	470	17279	2250	15029
8	Sunnyside	13.70	1675	680	18954	2930	16024
9	CBD	16.75	8314	6646	27268	9576	17692
10	Stampede	18.14	830	1070	28098	10646	17452
11	Erlton	18.87	72	562	28170	11208	16962
12	39th Avenue	20.57	250	1070	28420	12278	16142
13	Chinook	22.90	1556	3340	29976	15618	14358
14	Heritage	25.10	910	2750	30886	18368	12518
15	Southland	26.78	1190	3826	32076	22194	9882
16	Anderson	27.79	300	5049	32376	27243	5133
17	Canyon Meadows	30.47	200	1940	32576	29183	3393
18	Midnapore	31.50	100	3493	32676	32676	2000
19	Shawnessy	33.35	0	0	32676	32676	0

1991-92 DATA ON PASSENGER TRAVEL DEMAND
CROWFOOT-SHAWNESSY TRANSIT LINE HAUL IN CALGARY, ALBERTA

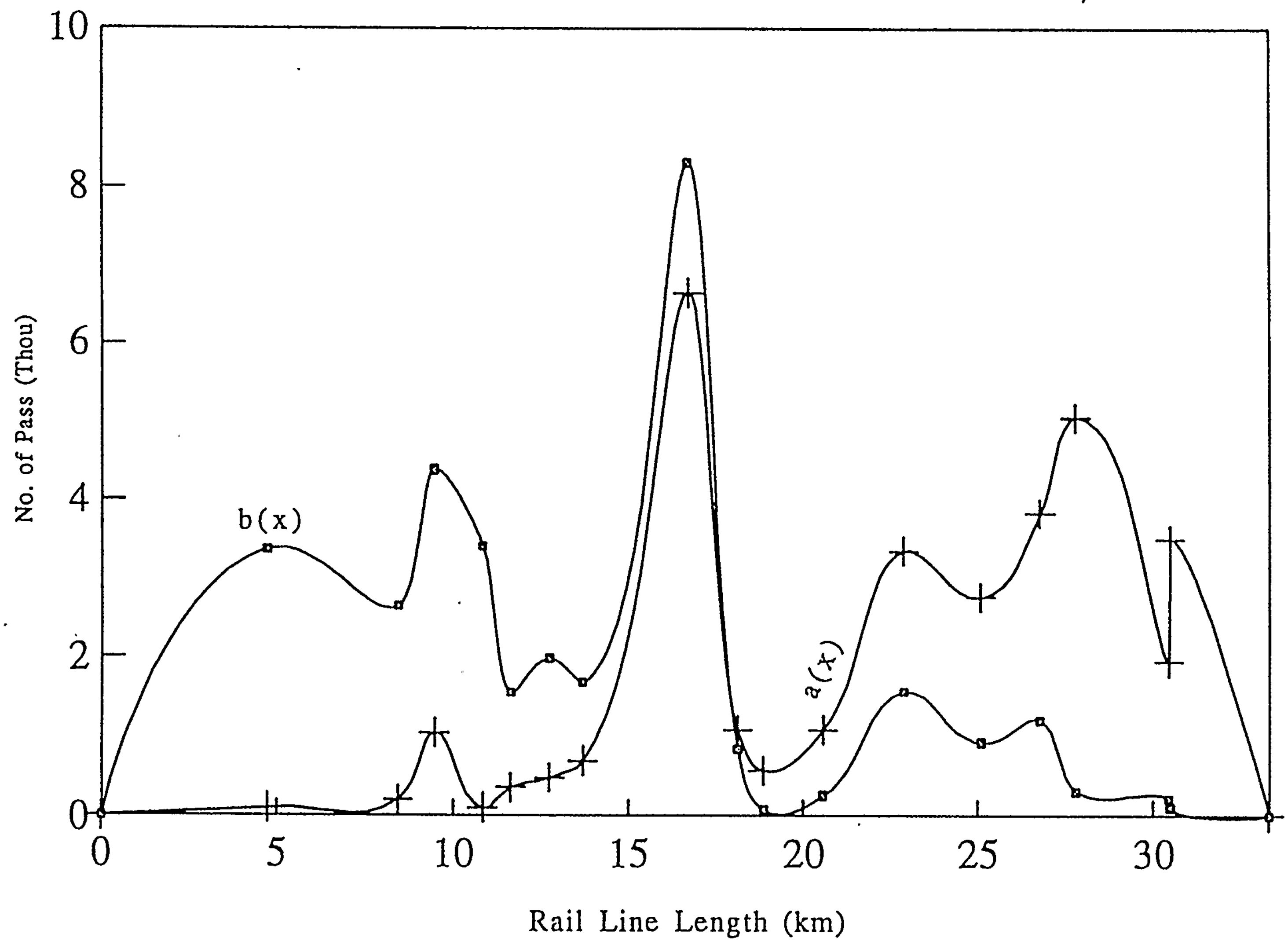

Figure 3.12 Daily Number of Boarding and Alighting Passengers

1991-92 DATA ON PASSENGER TRAVEL DEMAND
CROWFOOT-SHAWNESSY TRANSIT LINE HAUL IN CALGARY, ALBERTA

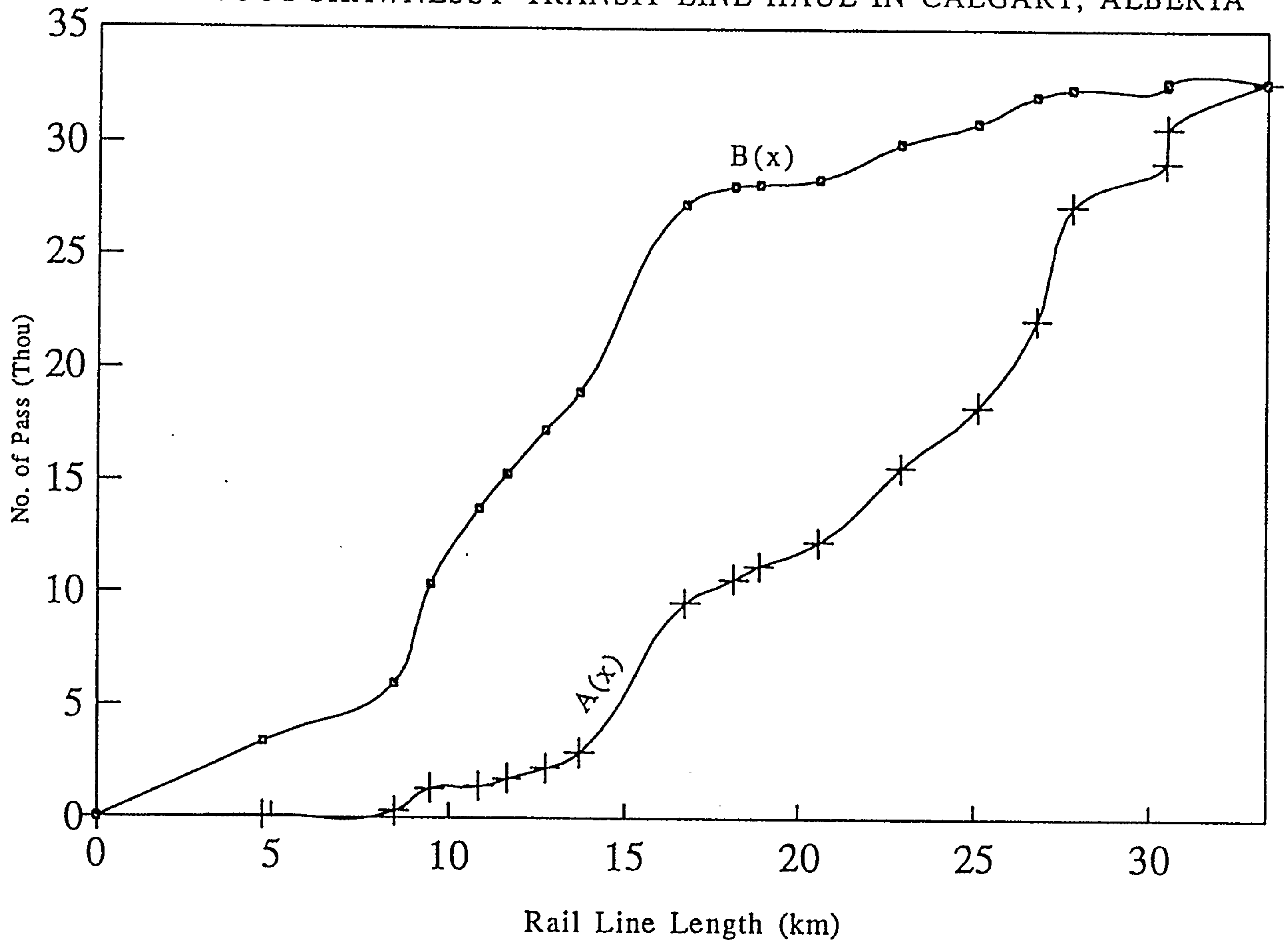

Figure 3.13 Cumulative of Daily Number of Boarding and Alighting Passengers

1991-92 DATA ON PASSENGER TRAVEL DEMAND
CROWFOOT-SHAWNESSY TRANSIT LINE HAUL IN CALGARY, ALBERTA

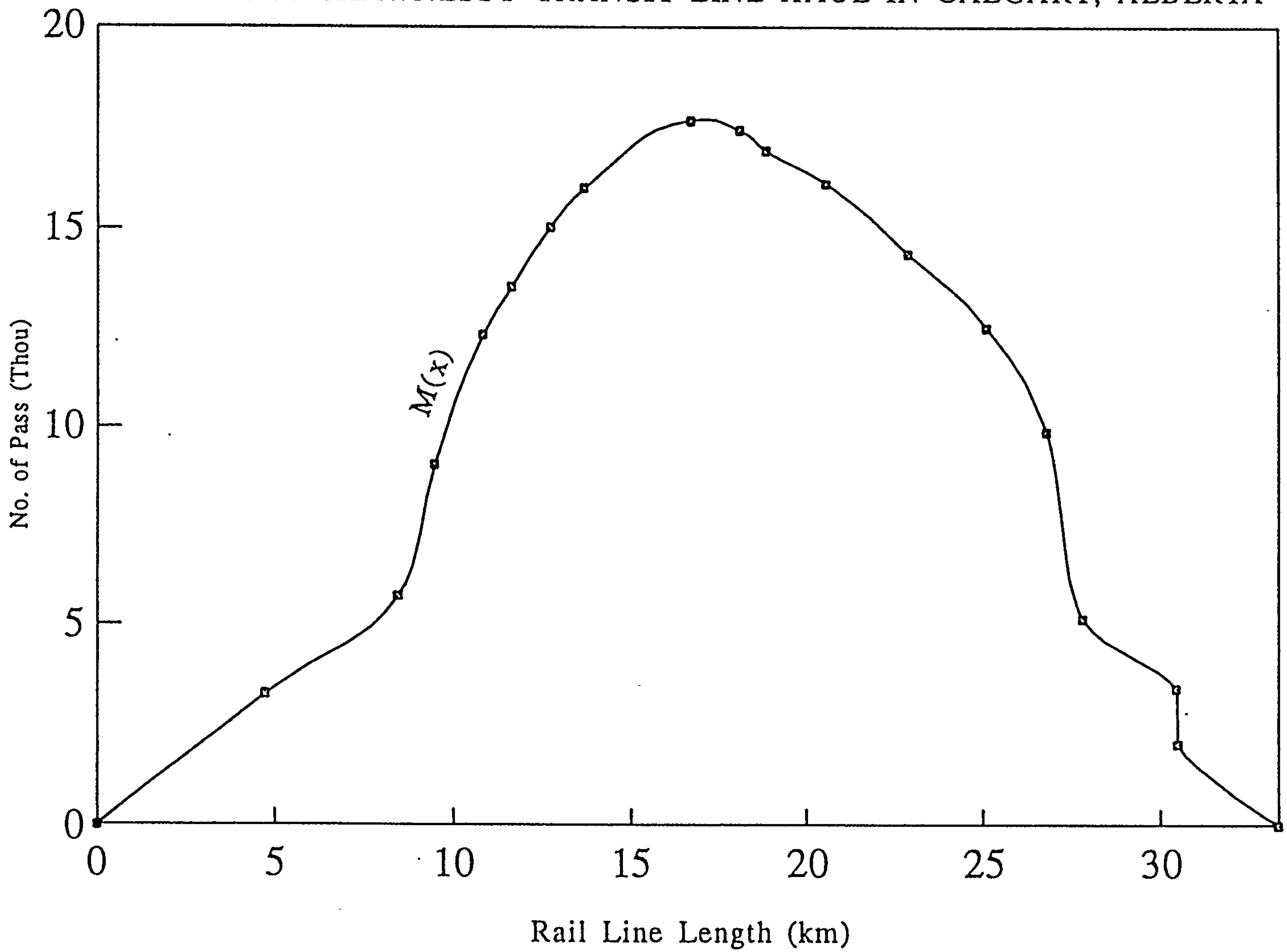

Figure 3.14 Daily Through Passenger Load

Table 3.2 Determination of Optimal Starting Point of a Rail Line

Sta. No.	$M(X_S)$ (pass/day)	$M'(X_S)$ (pass/km/day)	$\Delta\gamma M(X_S)$ + (S/km/day)	$\gamma_L - 2\gamma_P M'(X_S) + \theta$ * (S/km/day)
2	3360	673	2285	3519
3	5689	1429	3868	2975
4	9029	2000	6139	2564
5	12329	1818	8384	2695
6	13529	1427	9199	2977
7	15029	1333	10219	3044
8	16024	777	10896	3444
9	17692	0	12030	4004
10	17452	-500	11867	4364
11	16962	-554	11534	4403
12	16142	-647	10976	4469
13	14358	-798	9763	4578
14	12518	-1111	8512	4804
15	9882	-1517	6719	5096
16	5133	-1875	3490	5354
17	3393	-2031	2307	5466
18	2000	-2356	1360	5700
19	0	-2835	0	6045

+ LHS of Equation 3.13
* RHS of Equation 3.13

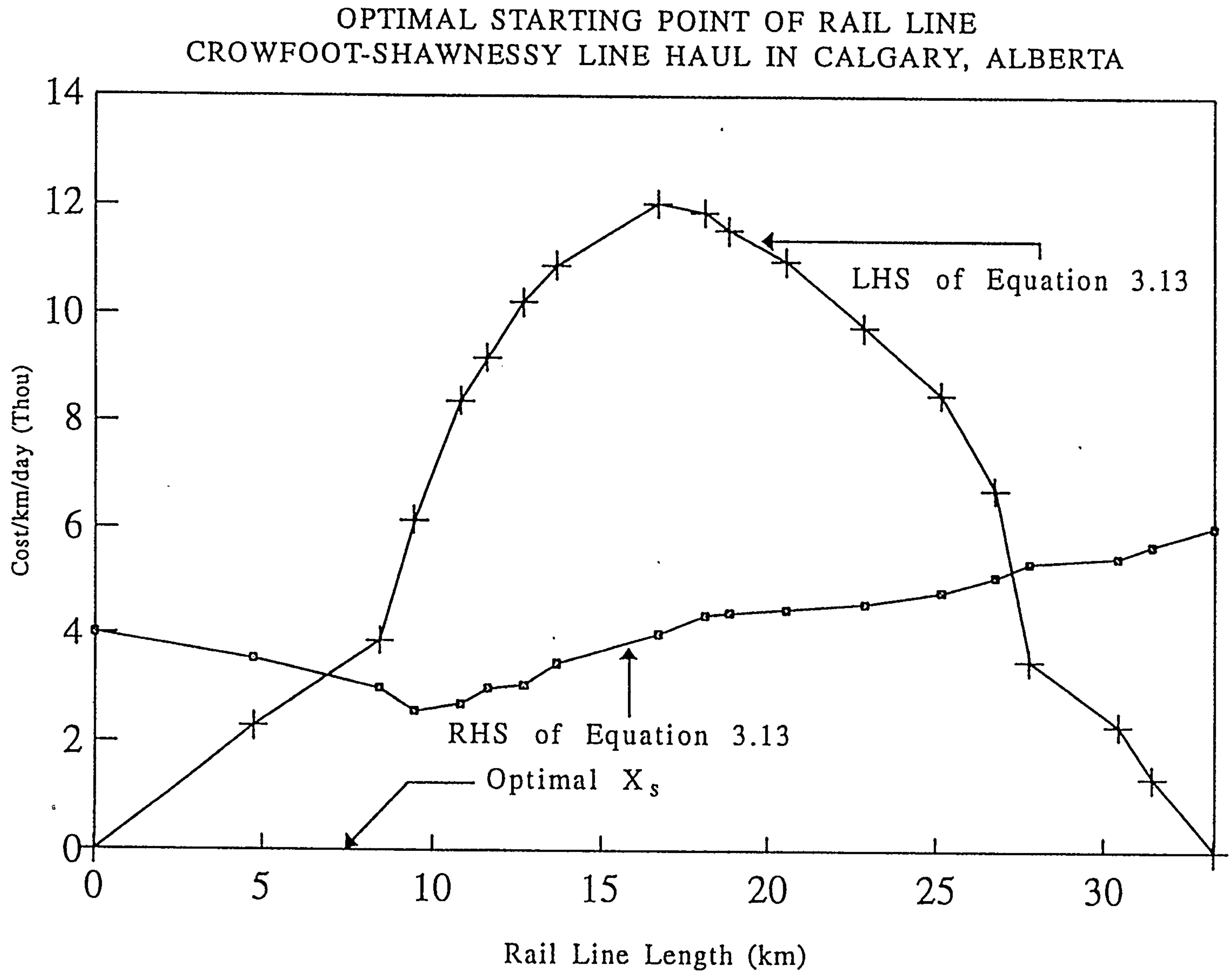

Figure 3.15 Optimal Location of Starting Point of a Cross-town Rail Line

Table 3.3 **Summary of Sensitivity Test Results on X_S**

Unit Cost Parameters: Symbol	Original Optimal X_S (km)	At -25% Sensitivity Test: Optimal X_S (km)	% Change in length: Inc.	% Change in length: Dec.	At +25% Sensitivity Test: Optimal (km)	Change in length: Inc.	Change in length: Dec.	Sensitivity Rating of Unit Cost Parameters on Optimal X_S
γ_B	6.85	8.48	23.79		5.87		14.31	Sensitive
γ_R	6.85	6.30		8.03	7.39	7.88		Sensitive
λ_B	6.85	7.83	14.31		5.11		25.40	Highly Sensitive
λ_R	6.85	6.83		0.20	6.87		0.30	Insensitive
γ_P	6.85	7.17	4.67		6.52		4.82	Sensitive
λ_F	6.85	6.68		2.41	6.93	1.26		Insensitive
γ_L	6.85	5.53		20.73	7.17	20.58		Very Sensitive
$M(X_S)$	6.85	7.53	9.92		8.26		10.96	Sensitive

Table 3.4 **Summary of Sensitivity Test Results on Daily Transportation Cost**

Unit Cost Parameters	Original Optimal Cost ($)	At -25% Sensitivity Test			At +25% Sensitivity Test			Sensitivity Rating of Unit Cost Parameters on Optimal Cost
Symbol		Optimal Cost ($)	% Change in Cost		Optimal Cost ($)	% Change in Cost		
			Inc.	Dec.		Inc.	Dec.	
γ_B	233,221	206951.00		11.26	206,951		11.26	Sensitive
γ_R	233,221	229,593.00		1.55	291,923		25.17	Sensitive
λ_B	233,221	245,995.00	5.48		222,297		0.40	Sensitive
λ_R	233,221	232769		0.19	232,769		4.68	Insensitive
γ_P	233,221	221705		4.94	221,007		5.24	Sensitive
λ_F	233,221	232436		0.33	232,436		0.34	Insensitive
γ_L	233,221	231007		0.95	239,952	2.88		Sensitive
$M(X_S)$	233,221	198776		14.76	274,338	17.63		Sensitive

Table 3.5 **Determination of Optimal Ending Point of a Rail Line**

Sta. No.	$M(X_E)$ (pass/day)	$M'(X_E)$ (pass/km/day)	$\Delta\gamma M(X_E)$ + ($/km/day)	$\gamma_L+2\gamma_P M'(X_E)+\theta$ * ($/km/day)
2	3360	673	2285	4488
3	5689	1429	3868	5032
4	9029	2000	6139	5444
5	12329	1818	8384	5313
6	13529	1429	9199	5031
7	5029	1333	10219	4963
8	16024	777	10896	4563
9	17692	0	12030	4004
12	17452	-500	11867	3644
11	16962	-554	11534	3605
12	16142	-647	10976	3538
13	14358	-798	9763	3429
14	12518	-1111	8512	3204
15	9882	-1517	6719	2911
16	5133	-1875	3490	2654
17	3393	-2031	2307	2541
18	2000	-2355	1360	2307
19	0	-2835	0	1963

+ LHS of Equation 3.16

* RHS of Equation 3.16

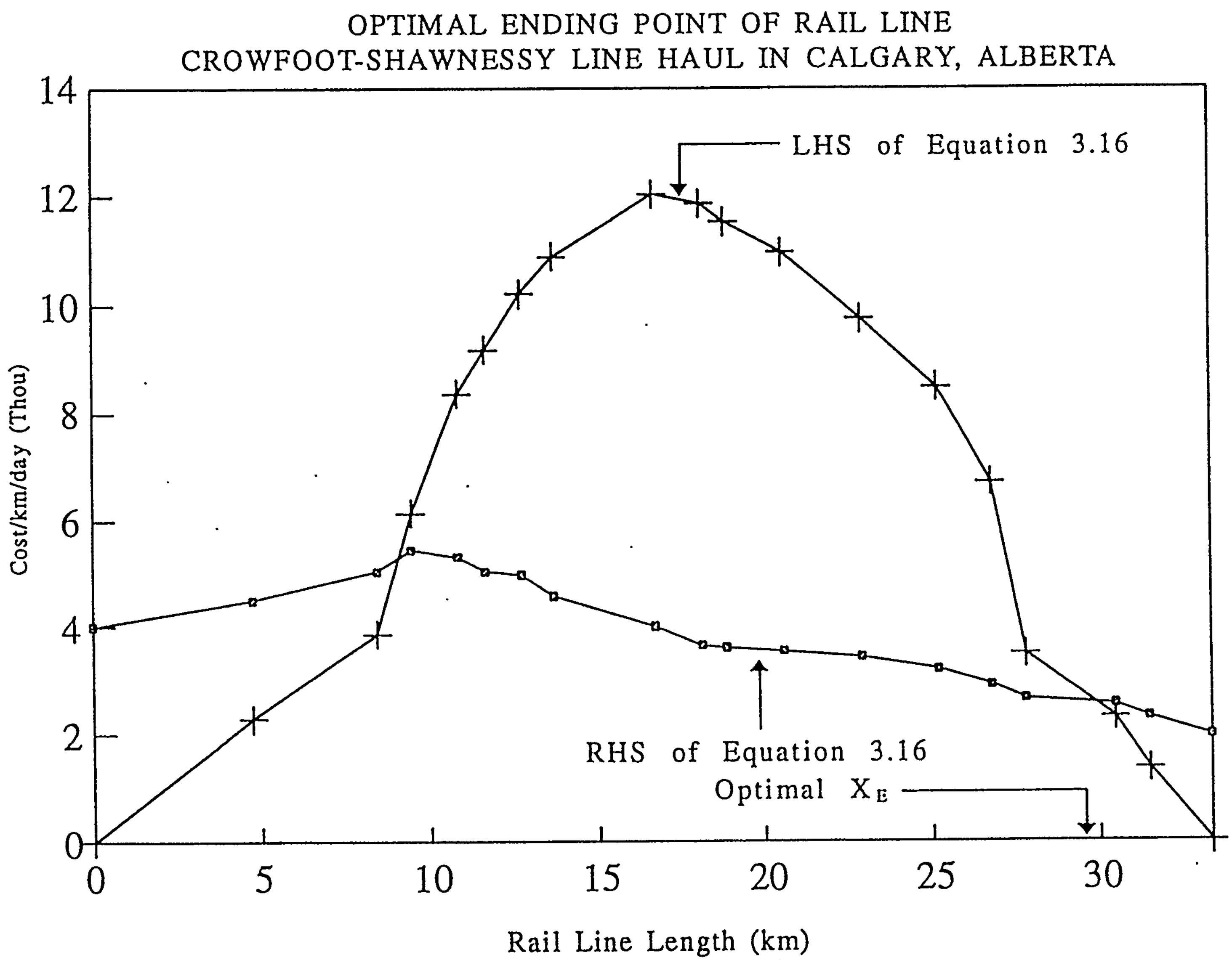

Figure 3.16 Optimal Location of Ending Point of a Cross-town Rail Line

Table 3.6 **Summary of Sensitivity Test Results on Optimal X_E**

Unit Cost Parameters	Original Optimal Length	At -25% Sensitivity Test			At +25% Sensitivity Test			Sensitivity Rating of Unit Cost Parameters on Optimal X_E
Symbols		Optimal Length (km)	% Change in Length (km)		Optimal Length (km)	% Change in Length (km)		
	(km)		Inc.	Dec.		Inc.	Dec.	
γ_B	29.67	28.04		5.49	30.43	2.56		Sensitive
γ_R	29.67	30.43	2.56		29.24		1.44	Sensitive
λ_B	29.67	28.69		3.30	30.43	2.56		Sensitive
λ_R	29.67	30.00	1.11		29.56		0.37	Insensitive
λ_F	29.67	30.43	2.56		30.43	2.56		Insensitive
γ_P	29.67	29.02		2.19	30.43	2.56		Sensitive
γ_L	29.67	31.53	6.36		27.61		6.90	Sensitive
$M(X_E)$	29.67	25.62		13.65	32.89	10.85		Sensitive

3.17.1 OPTIMAL X_S

The computation of the left hand side (LHS) and right hand side (RHS) of Equation 3.13 using the demand data shown in Table 3.1 and the values of the unit cost parameters given in Tables 2.2 is shown in Table 3.2. The calculated values (Columns 5 and 6, Table 3.2) are plotted against the rail line length X_S (Figure 3.15). The optimal X_S is found to be 6.85km. Hence the starting point of the rail line should be located at 6.85km measured from Crowfoot Station. A computer program (Appendix II) developed using Equation 3.14 gives the value of the optimal X_S as 6.85km. This brings the optimal rail terminal close to Northland Mall. Hence, the existing Brentwood Station is found to be located at a distance which is approximately 1.60km less the optimal terminal.

Sensitivity analysis is conducted to investigate the parameters that are sensitive to the optimal location of the starting point of the rail line. As usual, the test is conducted at ± 25% of the central values of the parameters given in Table 2.2. The grand summary of the sensitivity test results is shown in Table 3.3. It is found that the optimal X_S is sensitive to the parameters γ_B, γ_R, γ_L, γ_P, λ_B and $M(X_S)$. However, the optimal X_S is insensitive to the parameters λ_F and λ_R.

Furthermore, the sensitivity of the unit cost parameters on the optimal daily total cost is tested under similar conditions discussed above. Table 3.4 gives the overall summary of the test. It is also found that the optimal daily total cost of locating the rail terminal at X_S=6.85km is sensitive to the parameters γ_B, γ_R, γ_L, γ_P, λ_B and $M(X_R)$. The daily total cost is however, insensitive to λ_F and λ_R.

3.17.2 OPTIMAL X_E

Columns 6 and 7 of Table 3.5 respectively present the evaluation of the left hand side (LHS) and right hand side (LHS) of Equation 3.16 using the M(x) data given in column 6 of Tables 3.1 as well as the values of the parameters given in Tables 2.3. These computed values are plotted against the rail line length X_E (Figure 3.16). The optimal X_E is 29.67km. A computer program (Appendix II) developed using Equation 3.16 gives the value of the optimal X_E as 29.67km. This brings the optimal ending point of the rail line near Canyon Meadows. In particular, the optimal X_E is approximately 1.88km beyond the existing terminal at Anderson Station, which is located at chainage 27.79km. The difference between X_S and X_E is the required optimal rail line length X_R, which is found to be approximately 22.82km. The optimal line length is found to be 3.48km longer than the existing LRT line of length 19.34km.

The sensitiveness of the unit cost parameters on X_E is explored as well. Table 3.6 presents the overall summary of the sensitivity test. The optimal X_E is determined to be sensitive to the parameters γ_B, γ_R, γ_L, γ_P, λ_B and $M(X_R)$. However, the optimal X_E is insensitive to λ_F and λ_R.

In concluding this section, it is imperative to remark that an optimal X_S of value 8.446km (i.e. at Brentwood Station) is however obtained if values of γ_B=0.20, γ_R=0.19, λ_B=0.15, λ_R=0.10, λ_F=5.50, γ_P=0.19 and γ_L=3950 are independently used. It is found that low values of optimal X_S are obtained if unreasonably high values of γ_B, λ_B and γ_P are used individually. Contrarily, high values of X_S are obtained if considerably high values of γ_R, λ_R, λ_F and γ_L are separately used. Also, an optimal X_E of value 27.79km (i.e. at

Anderson Station) is obtained if values of γ_B=0.20, γ_R=0.20, λ_B=0.15, λ_R=0.10, λ_F=5.50, γ_P=0.19 and γ_L=3950 are separately used. It is found that no optimal X_E is obtained if considerably high values of γ_R, λ_R, λ_F and γ_L are individually considered. Besides, high positive values of X_E are obtained if very high values of γ_B, λ_B and γ_p are individually used in the analysis.

The analysis indicates that the optimal X_S decreases with increasing values of γ_B, λ_B and γ_P, and vice versa. High values of these parameters will cause an increase in demand [i.e $M(X_S)$] for rail systems. This requires optimal X_S to be shorter. The converse is true. However, the optimal X_S is found to increase with increasing values of γ_R, λ_R, λ_F and γ_L and vice versa. More particularly, high values of these parameters will result in a decrease in rail demand, causing optimal X_S to be longer. The converse is also true.

It is also observed that the optimal X_E increases with increasing values of γ_B, γ_R and γ_P and vice versa. High values of these parameters will cause an increase in demand [i.e $M(X_E)$] for rail systems. Hence a longer optimal X_E is required. The optimal X_E is, however, found to decrease with increasing values of γ_R, λ_R, λ_F and γ_L, and vice versa. Thus high values of these parameters will cause rail demand to decrease, and a corresponding decrease in optimal X_E. The opposite is also true. From the foregoing observation, it is concluded that high values γ_B, λ_B and γ_P will results in high demand for rail systems, and consequently the provision of a longer cross-town rail line, and vice versa. However, high values of the parameters γ_R, λ_R, λ_F and γ_L will cause low rail demand, resulting in the provision of a shorter rail line length. The reverse is also true.

3.18 MODEL EXTENSION

The model presented above can be extended to investigate the optimal termini of a rail line length. A slightly different transit network is considered in this case. A rectangular local road network consisting of two distinct sets of parallel curvilinear roads(x and y) is considered. An existing cross-town haul bus line CS, which is assumed to be parallel to the x-roads (Figure 3.17), is to be replaced by a haul rail line T_ST_E (Figure 3.18). The rail line T_CT_R, to be located closely parallel to the existing bus line haul, will emanate from a point T_S located in the suburban region to point T_E located in another suburban region but not necessary to the ends of the suburban regions at T_O and T_D. The rail line will pass through the point T_C located at the CBD. However, feeder bus service will be provided to the rail line from all areas, including those beyond the rail termini.

All assumptions regarding the operations of trains and feeder buses as well as nature of passenger accessibility to the transit systems discussed in Section 3.2 will apply in this case. It is further assumed that feeder buses will use the haul bus lines T_OT_A and T_DT_C to access the rail termini T_S and T_E respectively. Besides, all assumptions considered in the formulation of user time cost, rail and bus operating costs, rail line cost and rail fleet cost will be considered in this case. Hence, the expressions for user time cost, rail and bus operating costs, rail line cost and rail fleet cost given by Equations 3.4, 3.5, 3.6 and 3.9 respectively, will apply in this case.

However, the passenger transfer penalty cost (Equation 3.10) will not apply in this case. The reason is due to the different operating strategies of feeder buses assumed in

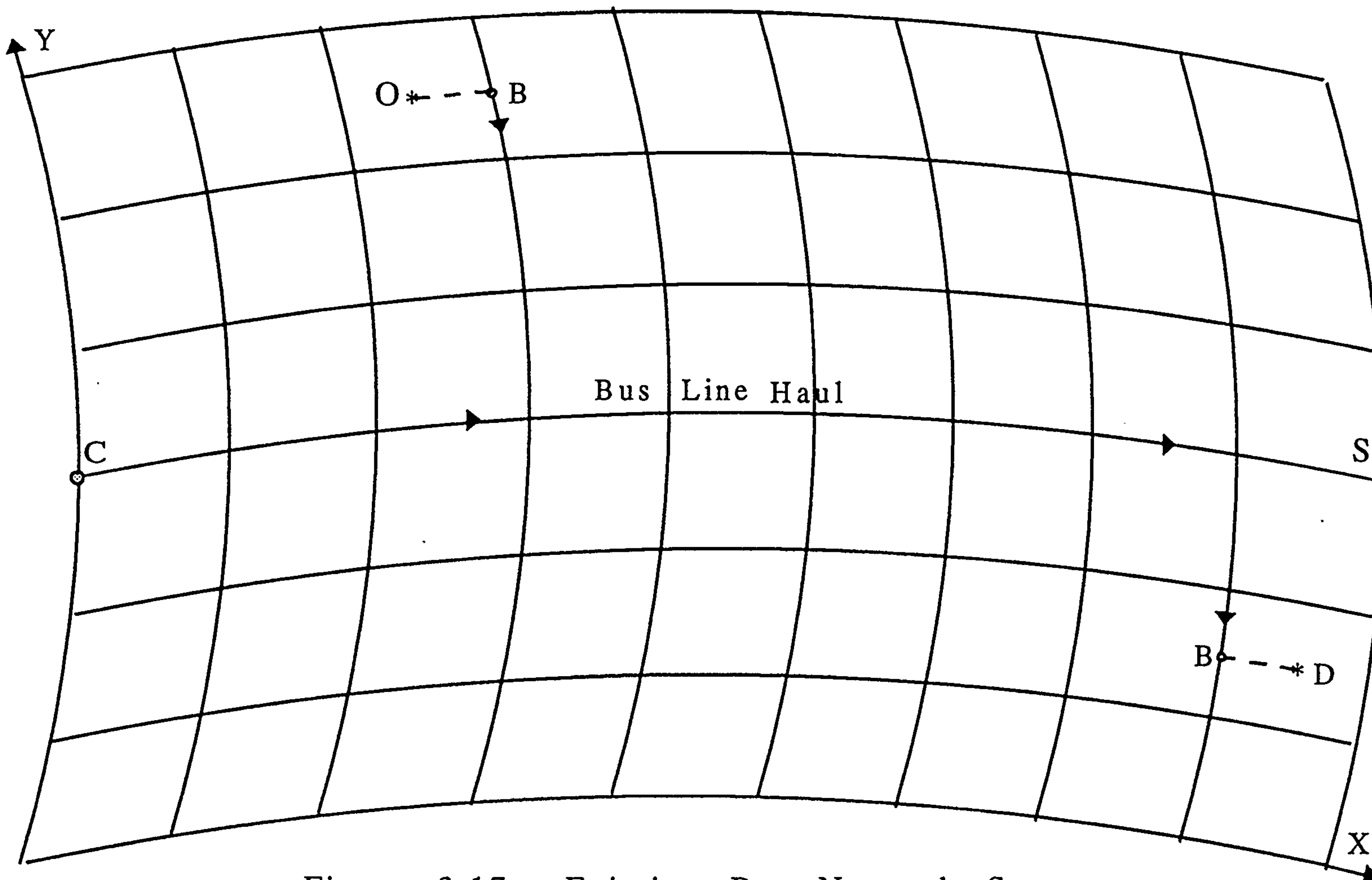

Figure 3.17 Existing Bus Network System

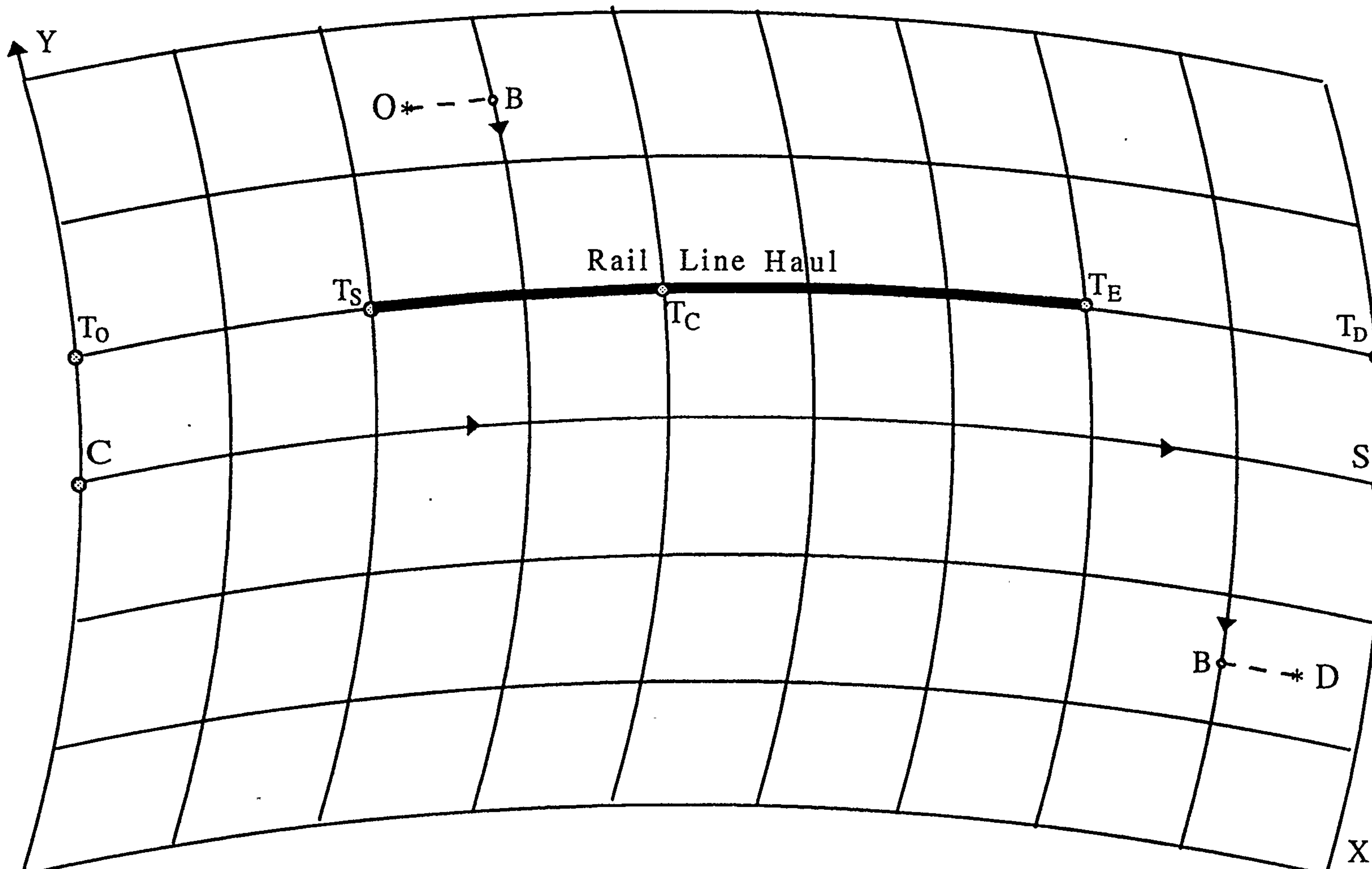

Figure 3.18 Proposed Bus and Rail Network System

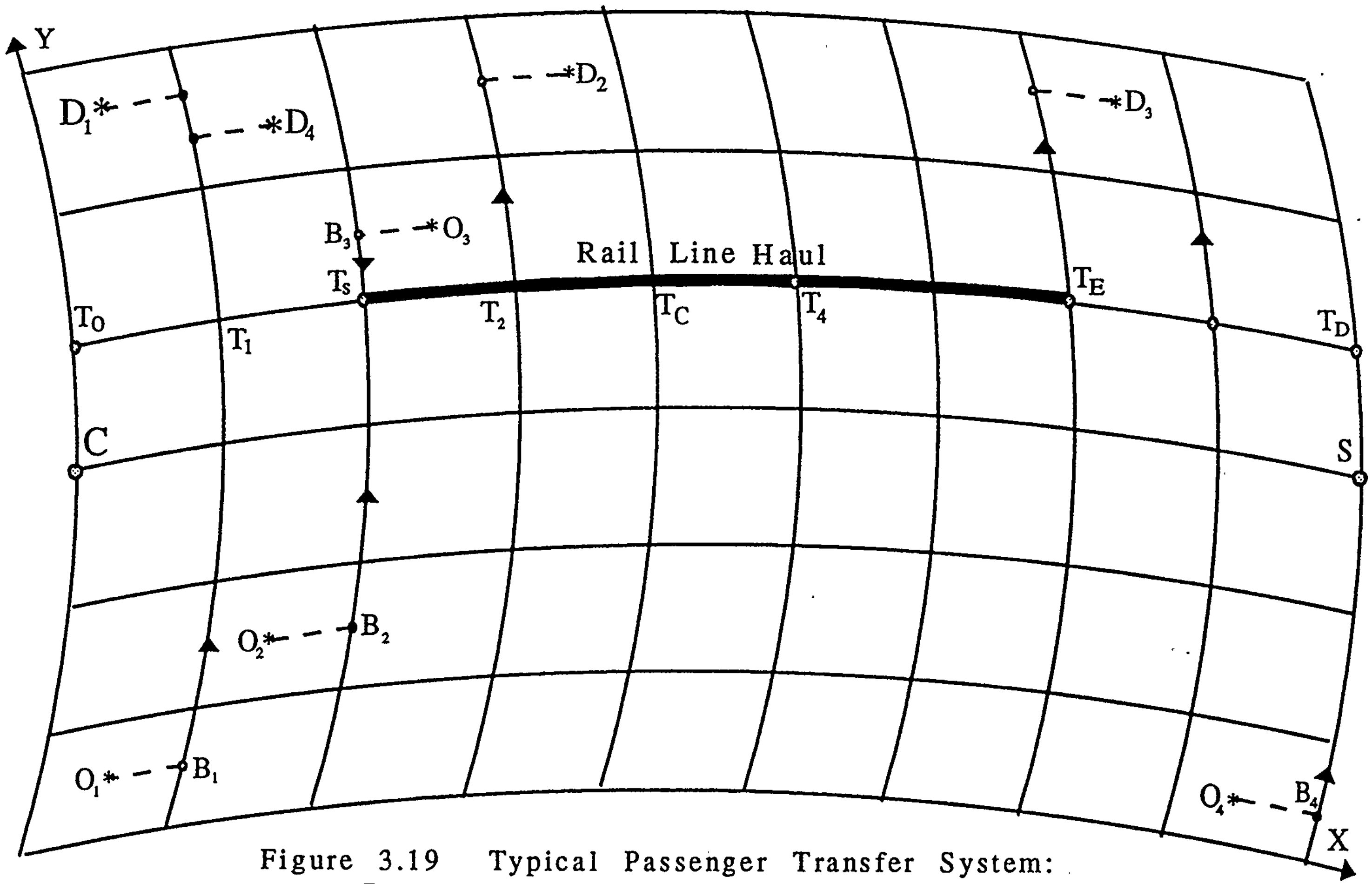

Figure 3.19 Typical Passenger Transfer System: Crosstown Transportation Corridor

the formulating the proposed model. An assessment of the model indicates that the introduction of rail line will generate an additional transfer for all passengers boarding the train. The total number of passengers boarding the train is therefore considered in formulating the related transfer penalty cost. An analysis to determine the passenger transfer penalty cost under this scenario is presented below.

It is observed from Figure 3.19 that a passenger travelling from O_1 to D_1 will experience one type of transfer from bus to bus at T_1 or no transfer depending on the configuration of the bus routes. A passenger travelling from O_2 to D_2 will experience two types of transfer. These are transfer from bus to train at T_S and train to bus at T_2. Also, a passenger travelling from O_3 to D_3 will be subjected to two types of transfers. These consist of transfer from bus to train at T_S and then transfer from train to bus at T_E. Moreover, a passenger travelling from O_4 to D_4 will experience two types of transfers; transfer from bus to train at T_E followed by transfer from train to bus at T_S. Relating to the above, it is found that with bus network only (Figure 3.17) at most one transfer is required. An introduction of a rail line (Figure 3.18) will result in an additional transfer is required for all passengers boarding the train. The relevant transfer penalty cost is expressed as:

$$2\gamma_P\int_{X_S}^{X_E} B(x)\,dx \tag{3.32}$$

3.19 OPTIMIZATION

The overall transportation cost is obtained by summing Equations 3.4, 3.5, 3.6, 3.10 and 3.32:

$$Z(x) = 2\gamma_B\int_0^{X_s} M(x)\,dx + 2\gamma_R\int_{X_s}^{X_E} M(x)\,dx + 2\gamma_B\int_{X_E}^{L} M(x)\,dx$$
$$+ 2\lambda_B\int_0^{X_s} M(x)\,dx + 2\lambda_R\int_{X_s}^{X_E} M(x)\,dx + 2\lambda_B\int_{X_E}^{L} M(x)\,dx \qquad (3.33)$$
$$+ \int_{X_s}^{X_E}\gamma_L(x)\,dx + 2\gamma_P\int_{X_s}^{X_E} B(x)\,dx$$
$$+ [2(X_E-X_S)\Lambda_R+\tau_R]\lambda_F M^*$$

Keeping X_E constant and differentiating Equation 3.33 with respect to X_S gives:

$$\frac{\partial Z}{\partial X_S} = 2\gamma_B M(X_S) - 2\gamma_R M(X_S) + 2\lambda_B M(X_S) - 2\gamma_R M(X_S) \qquad (3.34)$$
$$- \gamma_L(X_S) - 2\gamma_P B(X_S) - 2\Lambda_R\lambda_F M^*$$

Setting Equation 3.34 to zero gives:

$$2[(\gamma_B-\gamma_R)+(\lambda_B-\lambda_R)]M(X_S) = \gamma_L(X_S) + 2\gamma_P B(X_S) + 2\Lambda_R\lambda_F M^* \quad (3.35)$$

The minimum transportation cost is obtained by taking the second derivative of Equation 3.33 with respect X_S. This gives:

$$\frac{\partial^2 Z}{\partial X_S^2} = 2[(\gamma_B-\gamma_R)+(\lambda_B-\lambda_R)]M'(X_S) - 2\gamma_P B(X_S) - \gamma_L'(X_S) > 0 \quad (3.36)$$

Keeping X_S constant and differentiating Equation 3.33 with respect to X_E gives:

$$\frac{\partial Z}{\partial X_E} = 2\gamma_R M(X_E) - 2\gamma_B M(X_E) + 2\lambda_R M(X_E) - 2\lambda_B M(X_E) \qquad (3.37)$$
$$+ \gamma_L(X_E) + 2\gamma_P B(X_E) + 2\Lambda_R\lambda_F M^*$$

Hence setting Equation 3.37 to zero gives:

$$2[(\gamma_B-\gamma_R)+(\lambda_B-\lambda_R)]M(X_E) = \gamma_L(X_E) + 2\gamma_P B(X_E) + 2\Lambda_R\lambda_F M^* \quad (3.38)$$

For minimum transportation cost, the second derivative of Equation 3.26 with respect to X_E should be positive. Thus:

$$\frac{\partial^2 Z}{\partial X_E^2} = 2\left[(\gamma_R-\gamma_B)+(\lambda_R-\lambda_B)\right]M'(X_E)+\gamma_L{}'(X_E)+2\gamma_P B'(X_E) > 0 \qquad (3.39)$$

3.20 MODEL APPLICATION

The validity and applicability of the proposed model is investigated using the existing Northwest-South LRT line in Calgary, Alberta. The transit ridership data and the values of the unit cost parameters given respectively in Tables 3.1 and 2.7 are used in the test. The computation of the LHS and RHS of Equation 3.35 are shown respectively at Columns 6 and 7 of Table 3.7. By plotting these values against the rail line length (Figure 3.20), an optimal X_S value of 0.00km is obtained. Moreover, a computer program (Appendix III) developed using Equation 3.35 gives the optimal X_S as 0.00km. Furthermore, the computation of the LHS and RHS of Equation 3.38 is shown in Table 3.8. The values are plotted against X_E (Figure 3.21). A zero optimal value of X_E is obtained. A computer program (Appendix) developed using Equation 3.38 gives the optimal X_E as zero as well.

The model extension revealed that no optimal X_S and X_E exist using Calgary as a case study. However, it is possible to obtain positive values of X_S and X_E under some conditions. For instance, using γ_B=0.81, γ_R=0.69 and γ_P=0.01 separately, optimal X_S of values 4.78km, 4.78km and 1.52km are respectively obtained. Furthermore, by increasing the values of M(x) as given in Column 8 of Table 3.1 by 1000%, an optimal X_S of value 0.65km is obtained. Also, using γ_B=2.43, γ_R=2.30 and γ_P=0.01 independently, optimal X_E

Table 3.7 Determination of Optimal Starting Point of a Rail Line

Sta. No.	$M(X_S)$ (pass/day)	$M'(X_S)$ (pass/km/day)	$\Delta\gamma M(X_S)$ + ($/km/day)	γ_L-$2\gamma_P B(X_S)+\theta$ * ($/km/day)
2	3360	673	2284	5616
3	5689	1429	3868	6878
4	9029	2000	6139	8976
5	12329	1818	8384	10608
6	13529	1427	9199	11352
7	15029	1333	10219	12298
8	16024	777	108896	13102
9	17692	0	12030	17093
10	17452	-500	11867	17491
11	16962	-554	11534	17526
12	16142	-647	10976	17646
13	14358	-798	9763	18393
14	12518	-1111	8512	18829
15	9882	-1517	6719	19401
16	5133	-1875	3490	19545
17	3393	-2031	2307	19641
18	2000	-2356	1360	19689
19	0	-2835	0	19689

+ LHS of Equation 3.29

* RHS of Equation 3.29

OPTIMAL STARTING POINT OF RAIL LINE
CROWFOOT-SHAWNESSY LINE HAUL IN CALGARY, ALBERTA

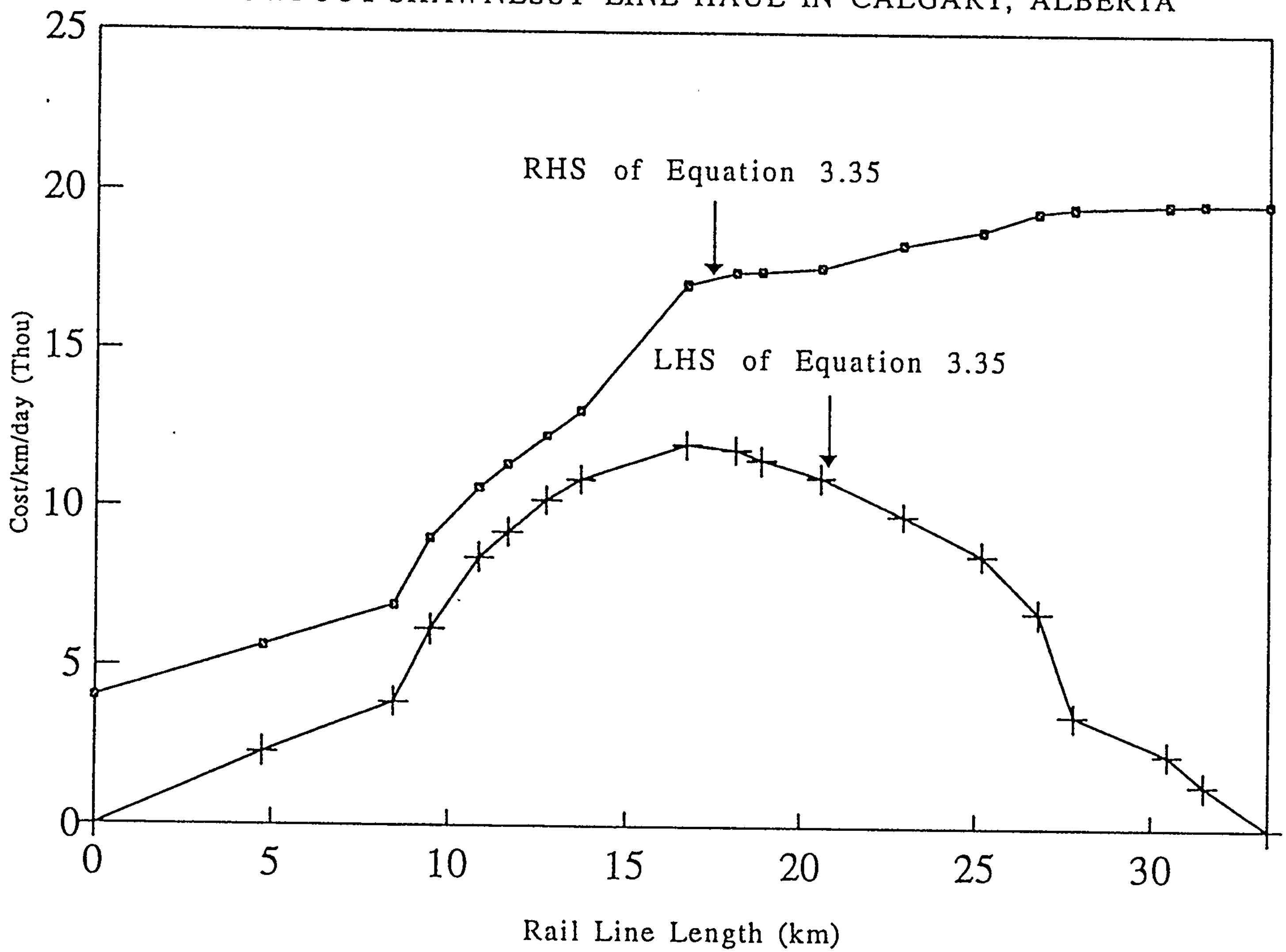

Figure 3.20 Optimal Location of Starting Point of a Cross-town Rail Line

Table 3.8 Determination of Optimal Ending Point of a Rail Line

Sta. No.	$M(X_E)$ (pass/day)	$M'(X_E)$ (pass/km/day)	$\Delta\gamma M(X_E)$ + ($/km/day)	$\gamma_L+2\gamma_P B(X_E)+\theta$ * ($/km/day)
2	3360	673	1478	7030
3	5689	1429	2503	9554
4	9029	2000	6139	13948
5	12329	1818	8383	17212
6	13529	1429	9199	18700
7	15029	1333	10219	20591
8	16024	777	10896	22199
9	17692	0	12030	30181
12	17452	-500	11867	30978
11	16962	-554	11534	31047
12	16142	-647	10296	31287
13	14358	-798	9763	32781
14	12518	-1111	8512	33654
15	9882	-1517	6719	34797
16	5133	-1875	3490	35085
17	3393	-2031	2307	35277
18	2000	-2355	1360	35373
19	0	-2835	0	35373

+ LHS of Equation 3.33

* RHS of Equation 3.33

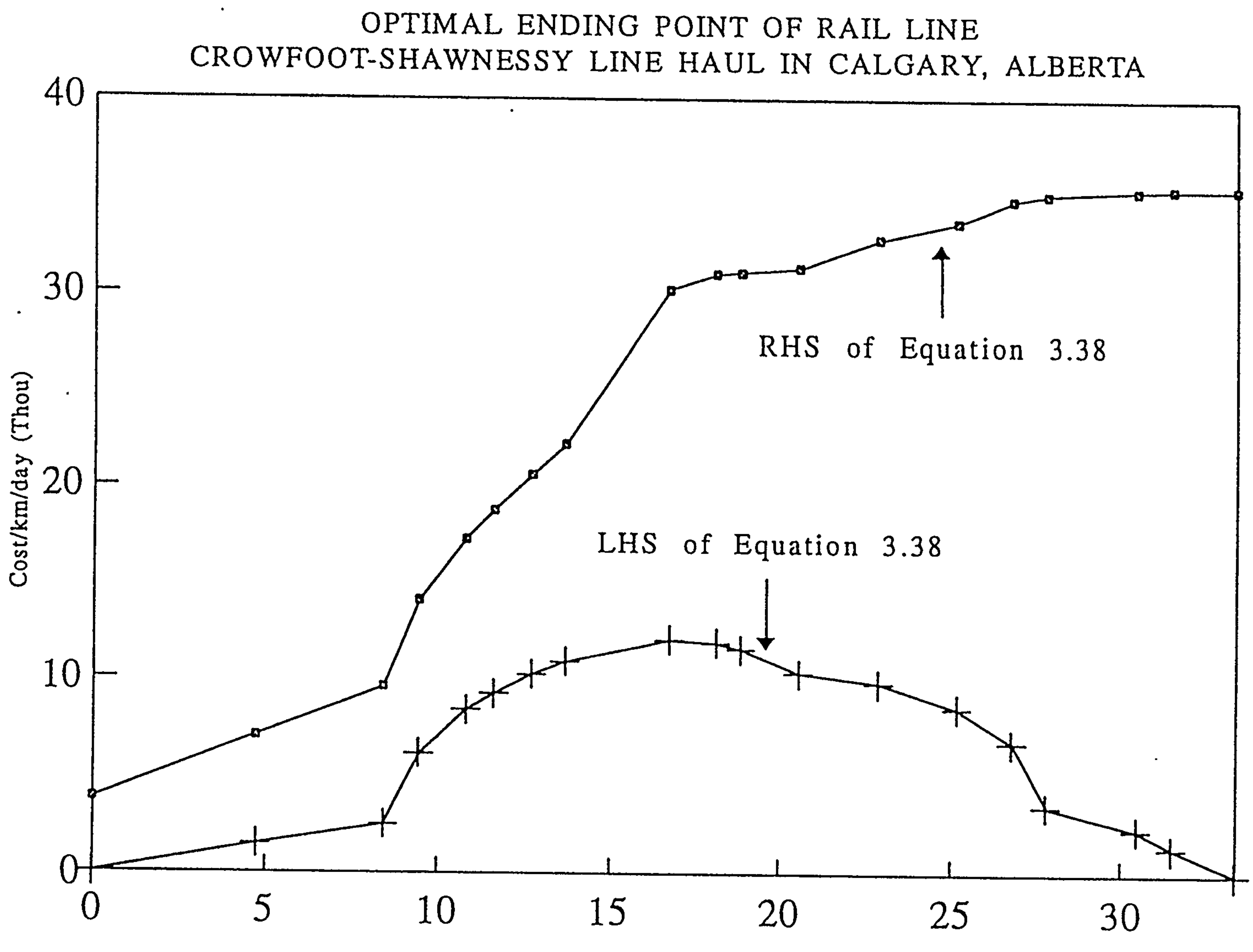

Figure 3.21 Optimal Location of Ending Point of a Cross-town Rail Line

of values 27.89km, 27.78km and 28.00km are obtained. respectively. Besides, by increasing the values of M(x) by 1000%, optimal X_E of value 27.78km is obtained.

CHAPTER FOUR

LOCATION OF RING RAIL LINE

4.1 INTRODUCTION

Many North American cities have inherited a rail network which is basically radial. The radial rail network provided a socio-economic benefit to individual passengers and the society as a whole (Yeates et al,1980). However, factors such as suburbanization and decentralization of socio-economic activities have resulted in the reduction of passenger travel from suburban regions towards the CBD along radial rail lines in recent years (Vuchic et al, 1968). Contrarily, passenger trips not oriented towards the CBD but rather connecting suburban regions in some cities are reported to increase at high rate (Potter, 1992). For such cities, it is necessary to consider providing some ring rail lines so as to accommodate and improve the Suburban-Suburban travel along the ring rail line.

However, the effective operation of a rail transit system on a ring rail line is chiefly dependent on the size of the entire urban area, degree of dispersion of residences and most importantly, the density of passenger travel demand. Typically, ring rail facilities are attractive in large urban areas with a high concentration of passenger transit demand spread over a large area. However, such high capital intensive facilities are less attractive where passenger demand is scattered over a large area at low densities (Clarens et al, 1975).

Some transportation planners (Abraham et al, 1993) suggested the provision of several ring roads on which transit buses are operated as an effective measure of improving passenger trips linking suburban regions in a large metropolitan area. They argued that a relatively low initial capital investment is required to construct a ring road as compared to the provision of a ring rail line of the same radius. Although their arguments is sensible, one cannot rule out the possibility of occurrence of high vehicular congestion and associated traffic problems on the road network that will make the bus routes unattractive. However, exclusive ring bus lanes can be analyzed in a manner very similar to that for a ring rail line. Abene (1992) cited Ghana as one of the countries experiencing very serious transportation problems. Other countries like Nigeria, Nepal and India are currently facing atrocious traffic problems which cannot be solved even by the application of existing sophisticated Traffic Demand Management (TDM) tools.The introduction of an efficiently integrated bus and rail transit systems to be operated on a combined radial and ring rail network, can be ideal solution to the complex transportation problems prevailing in these countries.

Ironically, rail infrastructure projects require a high initial capital upfront, and public funds needed to construct the rail and other supporting facilities are very scarce. Accordingly, the construction of several ring rail lines might not be plausible in the interim. However, the provision of one or two well-placed ring rail lines will, to a very large extent, improve passenger travel especially along transportation corridors connecting suburban regions and generate tremendous benefits in the not too distant future. This line of reasoning leads us to the concept of developing an optimization model to analytically investigate the location of a single ring rail line based on some reasonable and realistic assumptions.

A circumferential transportation corridor is described as a route that accommodates most trips destined to places located in the Suburban Regions other than the CBD. The corridor serves multiple destinations and hence provides a high level of connectivity. It is, however, characterized by uncoordinated transfer. In some cases, transfer can be lengthy and inconvenient so that during the off-peak periods, the resistance to travel using a circumferential route may be high (Canadian Transit Handbook, 1980). The benefits associated with the provision of a circumferential route include savings in user riding time cost, savings in systems operating costs and reduction in environmental pollution (Potter, 1992).

Vuchic et al (1988) described a circumferential line as a line that serves non-centrally oriented trips, but rather trips in circular form. They highlighted the important role played by ring lines in an urban network. In addition to serving circular trips, they connect radial lines in the city, shortening trips among them, and thus distribute their trips to various points in the city. Due to their multiple purpose, ring lines often have rather even passenger loadings along their length and during different periods of day. This results in high utilization of capacity and makes operations economical. They also identified some operational problems of ring lines. The most serious is the absence of terminal times, which prevents recovery of delays and reduces their reliability. Besides, their speeds can be changed only in certain increments, due to the fixed ratio between headway and cycle time.

An analytical model designed to determine the optimal location of a ring rail line in a large densely populated metropolitan region is presented in this chapter. Precisely, the analysis involve the determination of optimal radius of a proposed ring rail line with the objective of minimizing user costs and rail line costs. More essentially, the analysis will consider passenger demand travel pattern at both peak and off-peak periods. The validity and applicability of the model is explored using Calgary, Alberta as a case study. Sensitivity analysis is also conducted to test the robustness of the proposed model.

4.2 DECISION CRITERIA ASSOCIATED WITH DETERMINATION OF OPTIMAL LOCATION OF A RING RAIL LINE

The concerns underlying the provision of a ring rail line in a large metropolitan area, which is characterized by high and uniform passenger demand, are socio-economic related. To this end, constructive decisions have to be considered and agreed upon by all decision-making bodies (including transportation planners, politicians and the general public as a whole) before the final approval for provision of a ring rail line is granted. Some important decision criteria which are of great interest so far as the provision of a ring rail line is concerned are discussed below. These decisions consist of the consideration of concentration and density of passenger travel demand in a given zone, rail line cost, user total travel time, total systems operating cost and more perhaps passenger transfer at major transit transfer points.

4.2.1 CONCENTRATION OF PASSENGER TRAVEL DEMAND

A discussion on the passenger travel demand as the most important determinant that will justify the provision of a rail line in an urban transportation corridor is given in Section 2.11. More essentially, concerning the provision of a ring rail line, the locations of high concentration and density of passenger travel demand at the suburban regions will govern the location of the ring rail line that will connect the regions. Consider a large

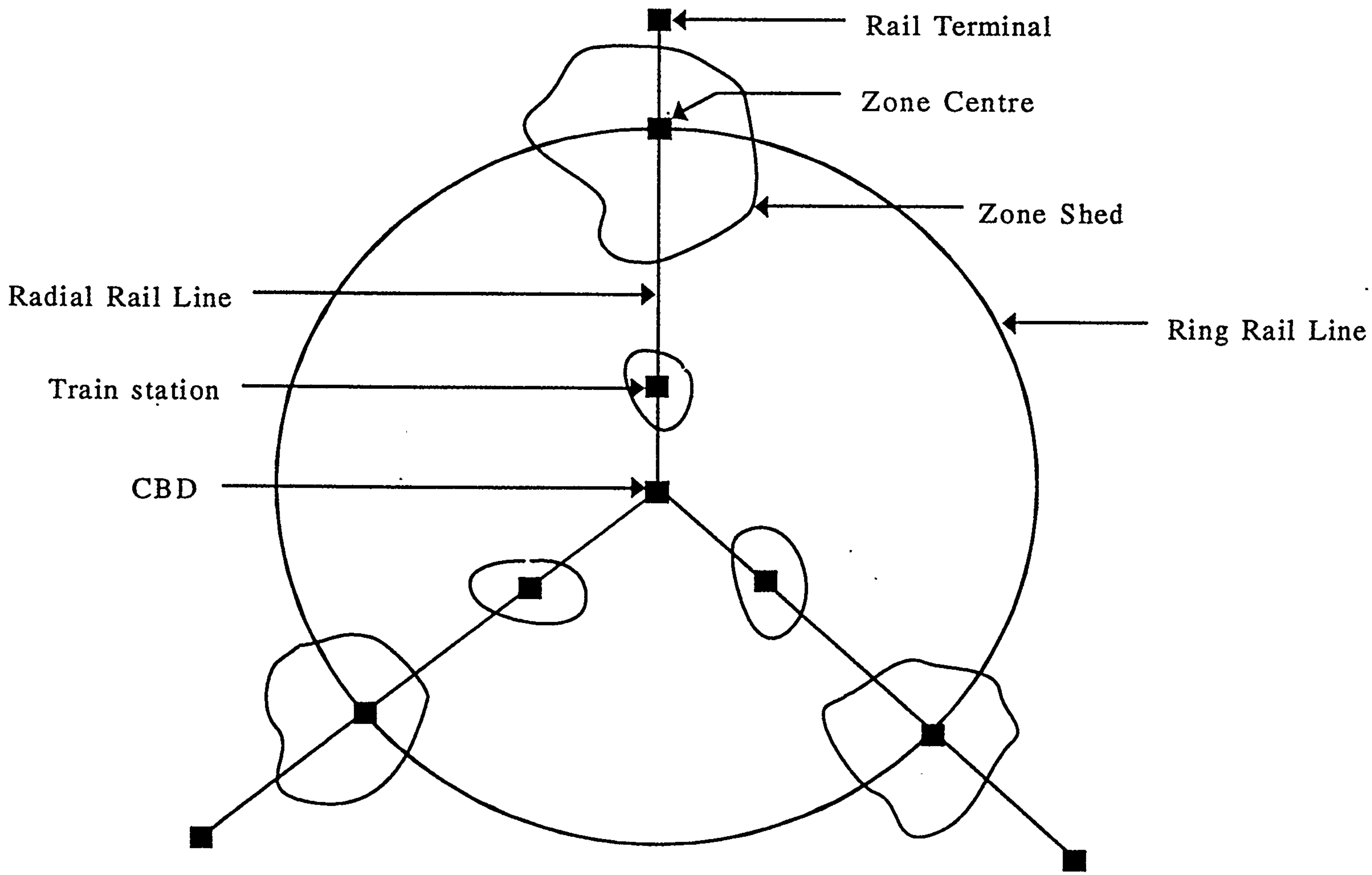

Figure 4.1 Optimal Location of Ring Rail Line Based on Passenger Demand Density

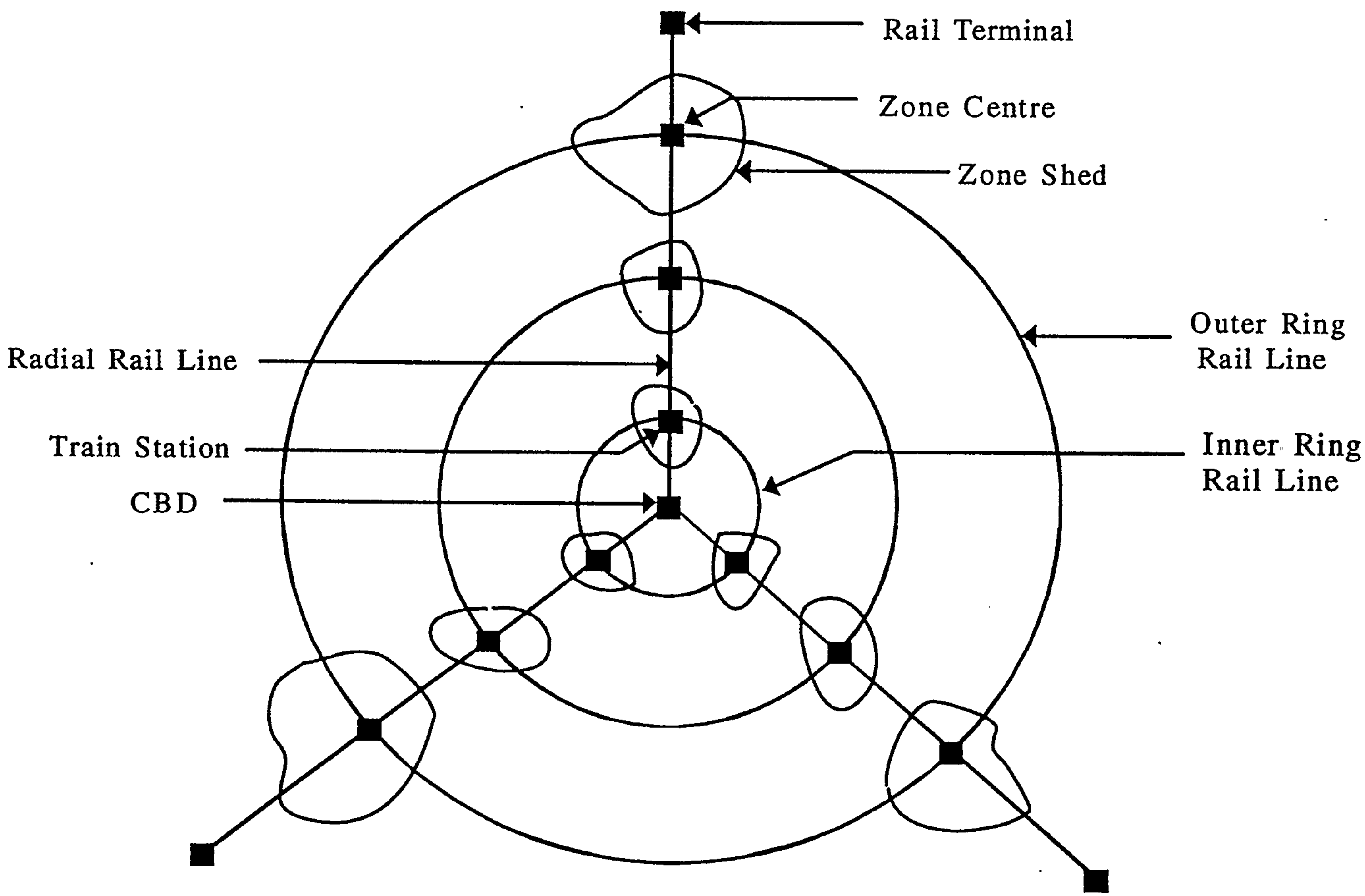

Figure 4.2 Optimal Location of Three Ring Rail Lines Based on Passenger Demand Density

metropolitan area with a relatively small CBD at which several radial rail lines are centred (Figure 4.1), and assume that passenger demand varies along the radial rail lines. In reality, there will be locations or zones on the radials where passenger demand is highly concentrated.

The generation of high concentration of passenger demand is mainly due to the development of high residential settlements and occurrences of high socio-economic activities within the suburban regions. These factors will lead to the growth and expansion of the suburban regions as well as the interaction of socio-economic activities between the suburban regions. In such a case, it is appropriate to locate a ring line to connect the regions of high concentration of passenger demand with the aim of improving passenger trips between the suburban regions. Moreover, depending on the locations of the passenger travel demand densities, it is appropriate to construct two or three ring rail lines (Figure 4.2) in order to meet the ultimate objective of providing a more efficient means of transportation services to the general public.

4.2.2 RAIL LINE COST

If the location of a circumferential rail line is based on the minimization of construction and maintenance of the ring rail line and stations costs, then obviously the concern will be to locate the rail line with a minimum radius R_{min} close to the neighbourhood of the CBD (ring 1, Figure 4.3). Ironically, whilst such a ring rail line is most likely to favour passenger trips in the vicinity of the CBD, a large portion of trips originating and destinating at an appreciable distance away from the CBD will not be satisfied. In such a case, it is most likely that the economic benefits will not be maximized even in a long period of time.

Conversely, the location of the ring rail line of maximum radius R_{max} at the outermost areas of the metropolitan area (ring 3) will require a high initial capital upfront for which funds required for construction might not be available sooner or later. Besides it might only favour passenger trips originating and destinating in the outermost regions.

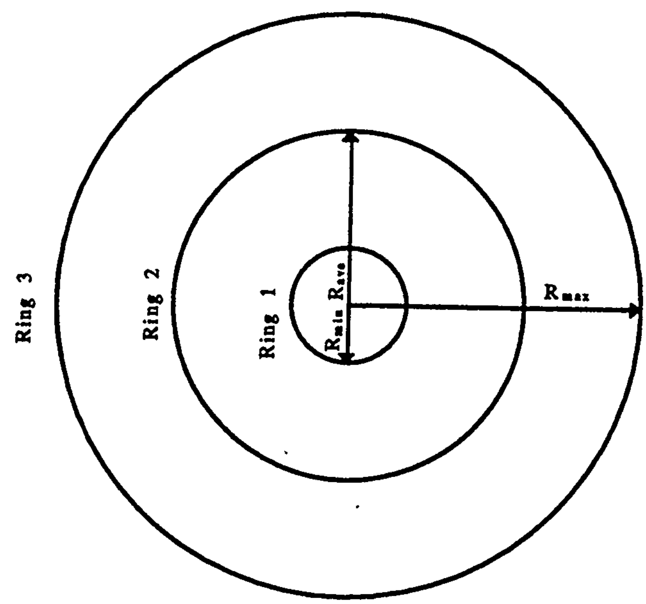

Figure 4.3 Optimal Location of Ring Rail Line

For that matter, it is appropriate to locate the rail line at average radius R_{ave} (ring 2) measured from T_O, where R_{ave} is approximately the average value of R_{min} and R_{max}.

4.2.3 SYSTEMS OPERATING COSTS

Operating cost is one of the important cost elements that should be given prominence when the planning for provision of a rail line is desired. In this context, systems operating cost include bus and rail fleet costs and bus and rail operating costs. Generally, transit operators will prefer to run transit systems haul lines which will minimize their total operating costs. Accordingly, the operations of trains on relatively short haul line (ring 1) will obviously meet this objective. The question then arise as to whether the operation of trains on ring 1 will generate the funds adequately enough to maintain the existing transit systems as well as to purchase new bus and rail vehicles, if and when necessary.

The crux of the matter is that operating trains on ring 1 will not generate enough passenger demand to meet the ever-increasing systems operating costs. For the same reasons, it is economically unjustifiable to provide ring 3. However, the provision of ring 2 is most likely to generate high passenger demand and associated benefits and funds needed to sustain the operations of the transit system and, more perhaps, to purchase new vehicles.

4.2.4 PASSENGER TRANSFER PENALTY COST

One major factor affecting passenger travel along radial lines connected at CBD (Figure 4.2) is passenger transfer from one mode to a similar or dissimilar mode (TRRL, 1990). This problem, to a very large extent, has tremendously caused a decline of passenger demand for transit systems (Doornenbal et al, 1985). Hence, an attempt to develop a ring rail line model basically to promote and enhance passenger trips connecting suburban areas is a step in the right direction.

The provision of a ring rail line will enhance passenger trips linking suburban regions by reducing the average total travel time of passengers. Some of the anticipated benefits as a result will include attraction of private automobile users to the usage of public transit systems, production of high passenger demand and generation of high revenue needed for the sustenance and improvement of the overall transit services in the entire metropolitan area.

4.3 THE MODEL

An idealized metropolitan region (Figure 4.4) with a relatively small CBD located around T_O, on which several existing radial railway lines are centred, is considered. The regional highway grid is assumed to be radiocentric and centred at T_O as well. However, the local roads are assumed to consist of a rectangular grid network. Both the radiocentric and rectangular roads are assumed to be shared by feeder buses and other class of vehicles. Rail service is provided between the CBD and stations in the residential zones. It is assumed that trains stops at every station to allow passengers to board and alight from the train.

In addition, feeder bus services are provided to the rail line from all residential zones. It is postulated that each residential zone is served exclusively by feeder buses. The feeder buses stops at bus stops located in the zones to allow for boarding and alighting of passenger from the buses. It is particularly assumed that passenger trips originating at any point in the residential zones are served by feeder buses to a selected station from which rail service is available. The points T_O is assumed to be a central terminal for operations of LRT systems on the radial haul rail lines. Besides, T_O is assumed to be a major transfer point.

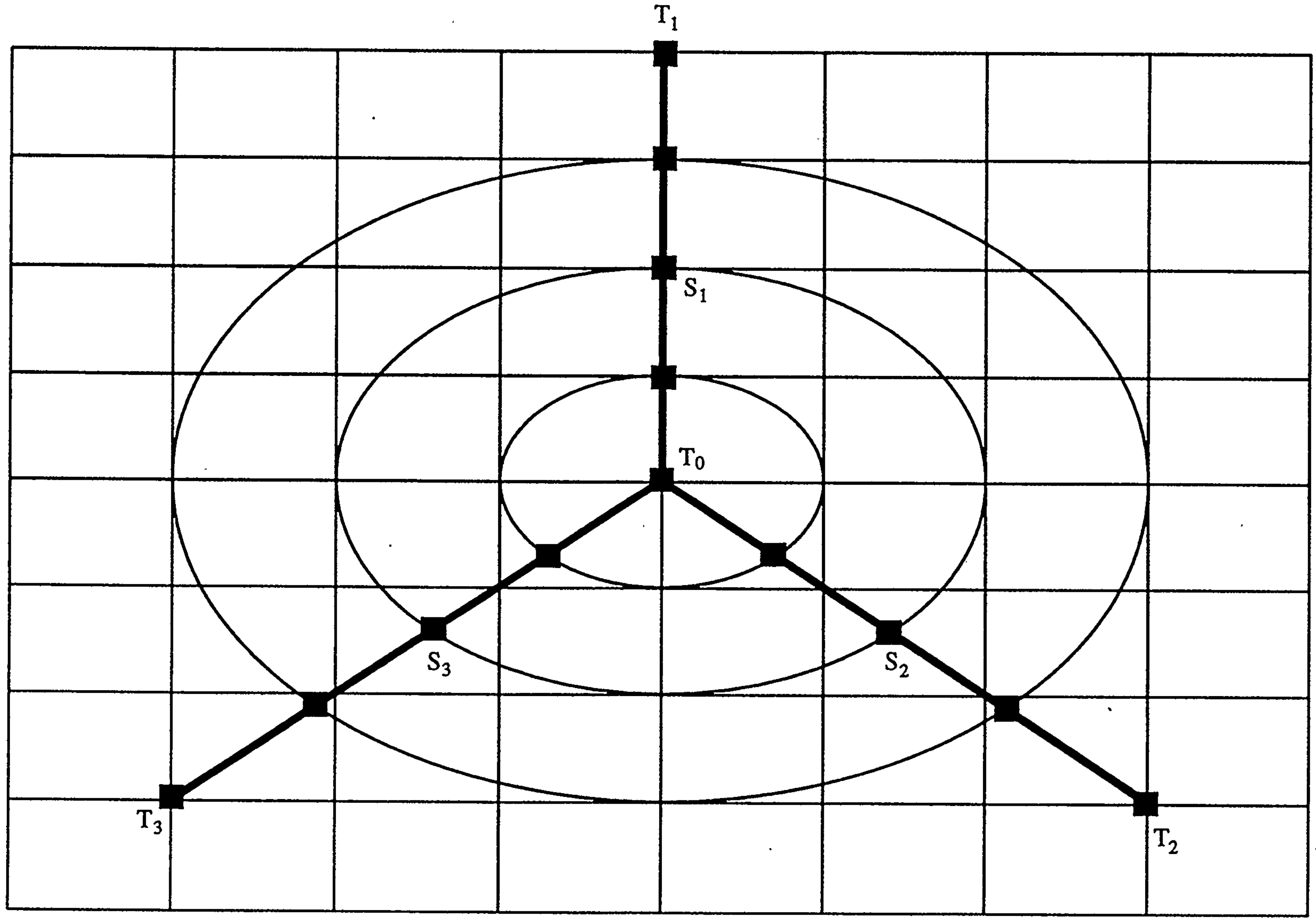

Figure 4.4 An Idealised Metropolitan Region

Trains operating on ring rail line are assumed to stop at stations on the haul line to allow for boarding and alighting of passengers at the stations. These trains are fed by feeder buses as well. In particular, the station S_1 is assumed to be the central terminus for operations of LRT systems on the ring rail line. It is assumed that feeder buses will travel at a uniform speed. Trains are assumed to travel at constant speed as well. Since passengers can board and alight the transit vehicles at every transit stops and stations, the maximum number of passengers on the vehicle at any space or time is assumed to be less than or equal to the vehicle capacity.

Also trains are subjected to the same operating conditions. Bus and rail fleet as well as the operating costs are not considered in the analysis since the transit systems are assumed to be existing. The cost of constructing the radial rail lines is not considered since the radial lines are existing. However, the cost of constructing the ring rail line is considered in the analysis. The rail fleet cost and the cost of dispatching trains on the ring rail line are considered.

Figure 4.5 shows a circular city $S_4S_5S_6$ of uniform radius b and centre at T_O. T_OT_1, T_OT_2 and T_OT_3 are radial rail lines, all connected at T_O. $S_1S_2S_3$ is a ring rail line of radius R and centre at T_O. O_1 is the origin point of a passenger located at the right side of the radial line T_OT_1 and outside the ring rail line. Passenger O_1 can access the ring rail line by two ways. These consist riding in a train from T_4 to S_1 or access the ring rail line at T_4. O_2 is the origin point of a passenger located at the left side of the line T_OT_1 and outside the ring rail line. Similarly, passenger O_2 has two ways of accessing the ring rail line. These are riding in train from T_4 to S_1 and accessing the train at T_5. O_3 is the origin

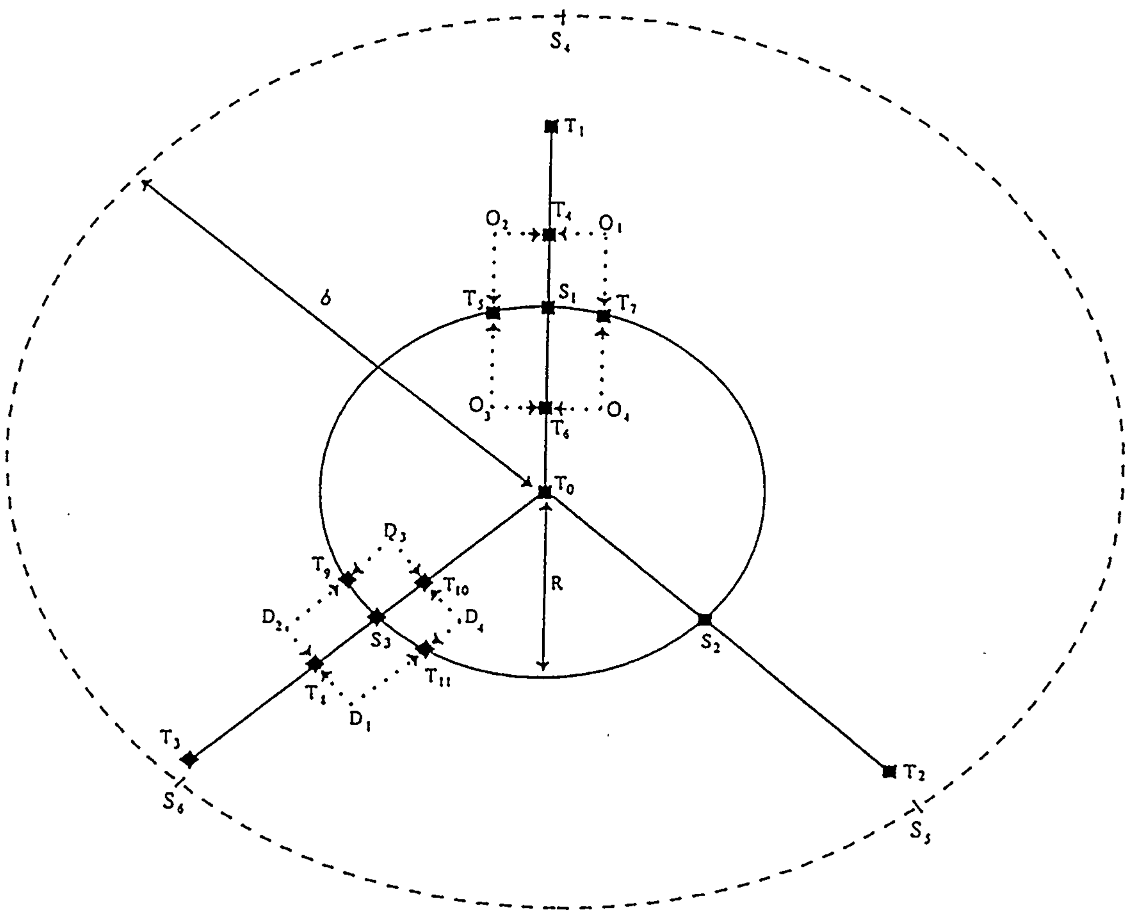

Figure 4.5 Typical Passenger Trip Assignment

point of passenger located at the left side of the line T_OT_1 and inside the ring rail line. O_3 can access the ring rail line by riding in train from T_6 to S_1, or accessing the ring rail line at T_5. O_4 is the origin point of a passenger located at the right side of line T_OT_1 and inside the ring. O_4 can access the ring either at T_7 or by riding in train from T_6 to S_1.

D_1 is the destination point of a passenger located at the right side of radial rail line T_OT_3 and outside the ring rail line. A passenger originating at O_1, O_2, O_3 or O_4 will get to D_1 by two ways. These consist of egressing the ring rail line haul either at T_{11} or riding in train from S_3 to T_8. D_2 is the destination point of a passenger located at the left side of the radial line T_OT_3 and outside the ring rail line. A passenger from O_1, O_2, O_3 or O_4 will get to D_2 by egressing the train at T_9 or by riding in train from S_3 to T_8. D_3, the destination point of a passenger located at the left side of the radial line T_OT_3 and inside the ring rail line, will be reached by a passenger from O_1, O_2, O_3 and O_4 by either egressing the train at T_9 or riding in train from S_3 to T_{10}. D_4 is the destination point of a passenger located at the right side of the radial line T_OT_3 and inside the ring rail line. A passenger from O_1, O_2, O_3 and O_4 will get to D_4 either by egressing the train at T_{11} or by riding in train from S_3 to T_{10}.

It is observed from the above passenger travel patterns that sixteen passenger trip types will be obtained. A model that accounts for all of the trip scenarios will be much too detailed, and more perhaps, result in more complexities. For mathematical simplicity, consideration will be given to passenger trips originating at both inside and outside the ring and destinating at both inside or outside the ring.

4.4 TRIP DEMAND DENSITY

Passenger many to many demand is considered in the analytical determination of the optimal location of a ring rail line within a large metropolitan area. Two types of transit trip demand densities will be considered in the analysis. These are uniform and non-uniform demand densities. With regard to uniform demand density, the origin and destination points of passengers are assumed to be uniformly spread over the entire metropolitan region. The passenger many to many demand is uniform in space and therefore independent of polar radius R and angle θ. The symbol M, with units expressed in passengers per square kilometre, will be used to highlight the fact that the daily passenger demand density as used in the model formulation is constant and independent of R and θ.

Concerning the non-uniform demand density, it is assumed that the origin and destination points of passengers are unevenly spread over the entire region. The demand is therefore variable in space, and is a function of location (R,θ). The notation $M(R,\theta)$ is used to represent variable trip demand density.

4.5 ACCESSIBILITY COSTS

Transportation Planners have frequently used the concept of accessibility, mainly in connection with trips generation models and within the context of evaluation of transportation systems. Accessibility has played a major role in spatial economic theories of cities. It has been considered as a key role in the determination of urban densities, land-use and effective operation of transit systems. Attention has been focused on the role of accessibility in urban interaction and trip demand models. Unfortunately, the term is rarely defined, let alone measured in quantifiable terms. Consequently, the conceptual nature of accessibility results in difficulties in evolving a truly satisfactory measure for it. This in turn complicates its use as an explanatory variable for transportation planning.

In his paper entitled "The Role of Accessibility in Basic Transportation Choice Behaviour", Burns et al (1976) discussed that accessibility measures reflect the level of service provided by transportation systems to various locations. More importantly, they discussed that accessibility to public transit systems by transit riders is a very important factor which affect patronage of public transit systems. This explains why it is given much prominence in several literature related to urban transit planning. In this analysis, it is particularly assumed that passengers will access the ring rail line using the existing rail lines.

4.6 UNIFORM DEMAND ANALYSIS

4.6.1 MINIMIZATION OF USER COSTS

Suppose a ring rail line of radius R and centre at T_O (Figure 4.5) is to be located within the metropolitan region. It is required to determine the optimal radius of a ring rail line with the objective of minimizing the total accessibility (user) costs considering a daily uniform demand density M. The user cost include walking (access and egress) cost, waiting time cost, riding time cost and transfer penalty cost. Consider a passenger whose origin point (r,θ) is located inside the ring rail line (Figure 4.6). The elemental area ΔA of the passenger origin point (r,θ) is $rdrd\theta$. By considering the locations of all passengers residing inside the ring rail line in the entire polar region, the daily cost of accessing the

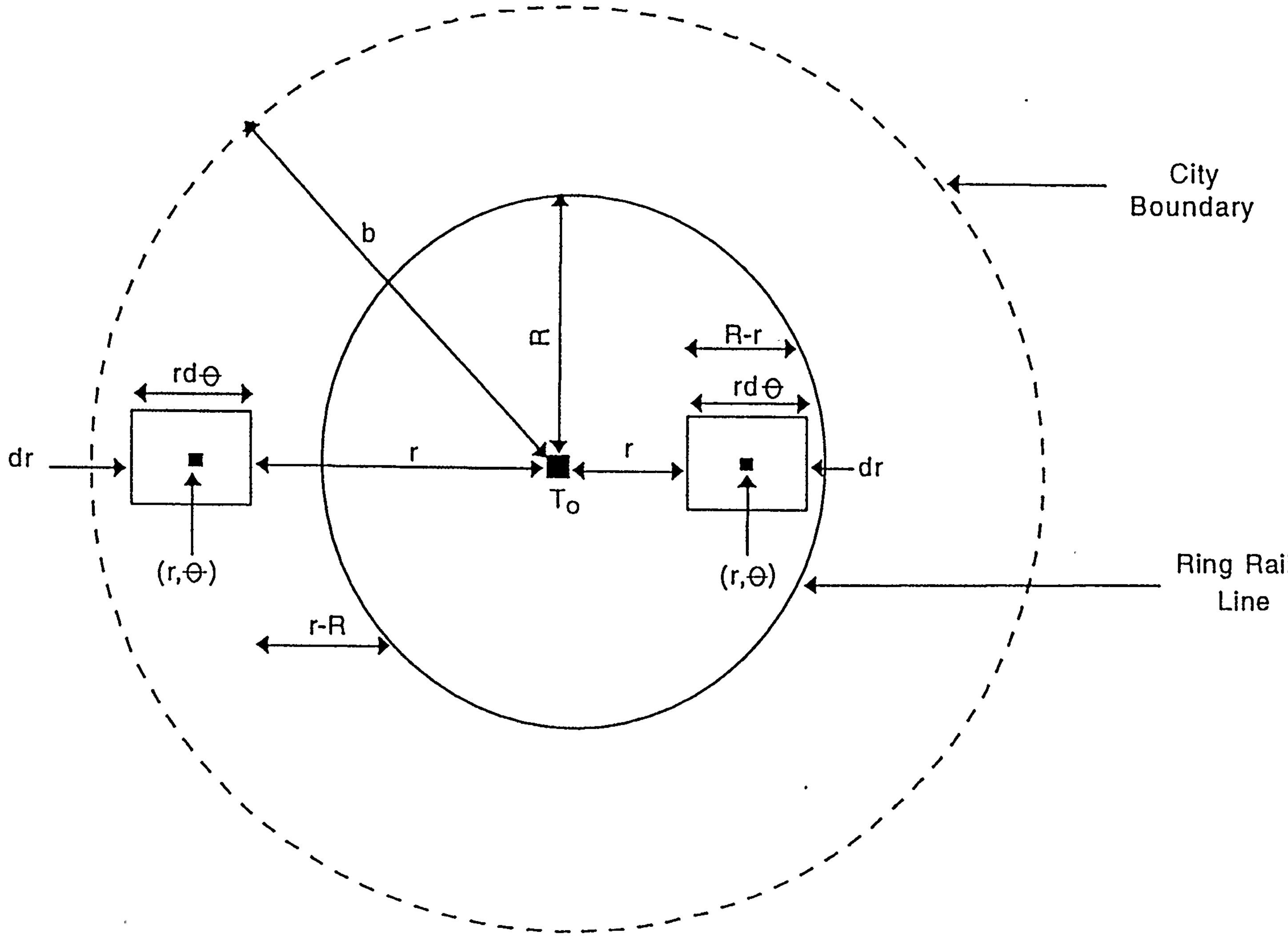

Figure 4.6 Elemental Area Located at Typical Origin Point of Passenger

ring rail line is:

$$\gamma_A M \int_0^{2\pi}\int_0^{R} (R-r)\, r\, dr\, d\theta \tag{4.1}$$

where γ_A is the cost of accessing the ring rail line per passenger per kilometre.

The daily cost of accessing the ring rail line by all passengers whose origin points are located outside the ring rail line is given by the expression:

$$\gamma_A M \int_0^{2\pi}\int_R^{b} (r-R)\, r\, dr\, d\theta \tag{4.2}$$

The daily total access cost is given by the sum of Equations 4.1 and 4.2, which reduces to:

$$2\pi\gamma_A M[\frac{R^3}{3} - \frac{Rb^2}{2}] \tag{4.3}$$

All passengers egressing the ring rail line haul will get to their destination points which are located at both inside and outside the ring rail line. The required total user access cost [Z(R)] is obtained by doubling Equation 4.3. Hence:

$$Z(R) = 4\pi\gamma_A M[\frac{R^3}{3} - \frac{Rb^2}{2}] \tag{4.4}$$

Differentiating Equation 4.4 with respect to R and setting the resulting expression to zero gives the required optimum radius (R_O) of the ring rail line that minimizes access cost as:

$$R_O = \frac{b}{\sqrt{2}} \tag{4.5}$$

It is concluded from Equation 4.5 that the optimal radius is greater than half of the city radius. Hence the optimal location of the ring rail line is found to be more closer to the city boundaries. For minimum total cost to be obtained, the second derivative of Equation 4.4 with respect to R should be positive. Thus:

$$Z''(R) = 8\pi R\gamma_A M > 0 \tag{4.6}$$

4.6.2 MINIMIZATION OF USER AND LINE COSTS

It is possible to obtain an expression for the optimum radius of a ring rail line with the objective of minimizing the sum of user access cost and rail line cost. All capital costs associated with the construction and maintenance of the a ring rail line are categorized as rail line cost. These include land acquisition cost, design cost, rail track acquisition cost and rail track construction cost. Others are station construction cost, parking lots construction cost, railcars garages construction cost, utility relocation cost. In this analysis, the cost of dispatching trains on the ring rail line is included in the rail line cost. If a uniform rail line cost per kilometre per day γ_L is assumed, then the rail line cost per day is:

$$2\pi R\gamma_L \tag{4.7}$$

In this case, the daily total cost [Z(R)], which is redefined as the sum of user costs and rail line costs, is given by the sum of Equations 4.4 and 4.7. Thus:

$$Z(R) = 4\pi\gamma_A M[\frac{R^3}{3} - \frac{Rb^2}{2}] + 2\pi R\gamma_L \tag{4.8}$$

By differentiating Equation 4.8 with respect to R and setting the resulting expression to zero, the optimum radius R_O of the ring rail line becomes:

$$R_o = \sqrt{\frac{b^2}{2} - \frac{\gamma_L}{2\gamma_A M}} \qquad (4.9)$$

Relating Equation 4.9 to Equation 4.5, it is observed that the addition of rail line cost in the total cost function tends to reduce the optimal radius of the ring rail line. The reduction in optimal radius tends to be pronounced if the rail fleet cost and cost of operating trains on the ring rail line are included in the rail line cost. Moreover, by including transfer cost due to transfer of pasengers on the ring rail line as well as user riding cost in the objective function, the optimal radius will further decrease.

In reality, trips are made in order to achieve some socio-economic gains and related benefits. Thus passenger trips on the ring rail line will generate some benefits. The resulting effect of the derived benefits is likely to be to allow the radius of the ring rail line to be longer. However, the benefits associated with passenger travel on the ring rail line is beyond the scope of this thesis. The minimum total cost is obtained by setting the second derivative of Equation 4.8 with respect to R to be positive. This gives:

$$Z''(R) = 8\pi M R \gamma_A > 0 \qquad (4.10)$$

4.7 MODEL APPLICATION

The applicability of the proposed model is explored using Calgary, Alberta, as a case study. The centre of the city is assumed to be located at the intersection of 7th Avenue and Centre Street. The average city radius (b) and current average daily transit trip density (M) are 14.63km and 200pass/km^2 (Calgary Transit). From Equation 4.5, the optimum radius of the ring rail line is 10.34km, which is greater than half of the city radius. The proposed ring rail line (ring 1) is observed to pass through such areas as

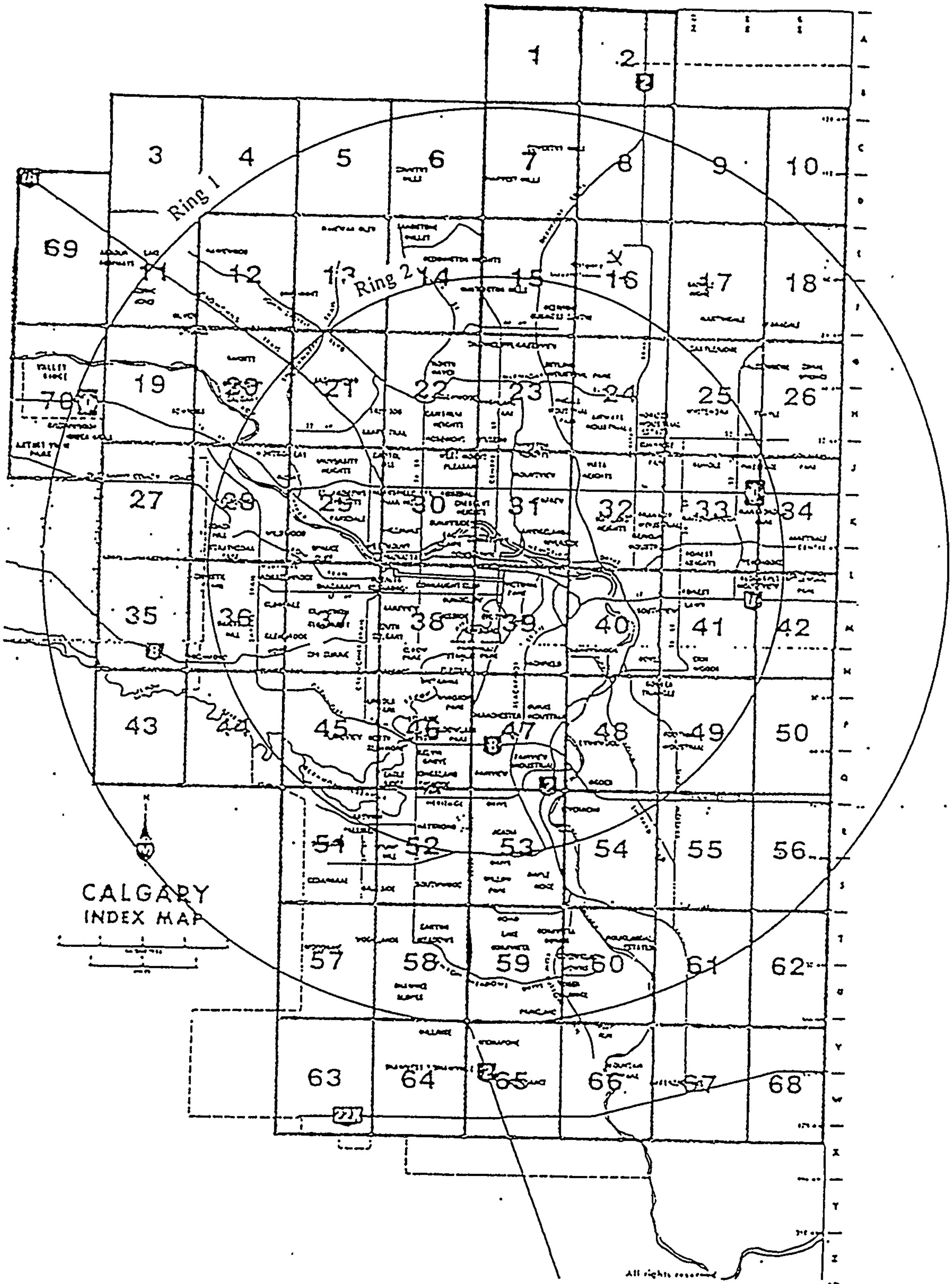

Figure 4.7 Optimal Location of Proposed Ring Rail Line in Calgary, Alberta

Table 4.1 **Summary of Sensitivity Test Results on Optimal Radius**

Unit Cost Parameters	Original Optimal Radius (km)	At -25% Sensitivity Test			At +25% Sensitivity Test			Sensitivity Rating of Unit Cost Parameters on Optimal Radius
Symbols		Optimal Radius (km)	% Change in Radius		Optimal Radius (km)	% Change in Radius		
			Inc.	Dec.		Inc.	Dec.	
b	6.82	0.00		100.00	10.33	51.47		Very Sensitive
γ_L	6.82	7.80	15.10		5.60		17.89	Sensitive
γ_A	6.82	5.13		24.78	7.65	12.17		Sensitive
M	6.82	5.13		24.78	7.65	12.17		Sensitive

Arbour Lake and Scenic Acres in the North-West, Saddle Ridge and Taradale in the North-East as well as Douglas Estates and Canyon Meadows in the Southern, part of Calgary.

The user access cost γ_A and rail line unit cost γ_L are respectively \$0.13 per passenger per kilometre and \$3148 per kilometre per day (Appendix 1). The substitution of the values of b, γ_L, γ_A and M into Equation 4.9 gives the optimum radius of the ring rail line as 6.82km. Hence the effect of line costs is to decrease the optimal radius. In this case, the proposed ring rail line (ring 2) is observed to run through areas including Brentwood and Beddington Heights in the North-West, Whitehorn and Penerooke in the North-East, Foothills Industrial and Acadia in the South-East, and Richmond and Patterson in the South-West, part of Calgary (Figure 4.7).

Sensitivity test is conducted to test the robustness of the model. The test is conducted at ±25% of the values of b, γ_L, γ_A and M. Table 4.1 depicts the overall summary of the sensitivity test results. It is found that the optimal radius is very sensitive to the parameters b, M, γ_A and γ_L.

It is worth indicating that the rail fleet cost, cost of operating trains on the ring rail line, user cost of riding in trains on the ring rail line and transfer cost due to transfer of passengers occuring at stations on the ring rail line are not considered in the analysis. The inclusion of these other costs in the objective function will cause the radius of the ring rail line to decrease. Generally, trips are made to achieve some socio-economic goals and related benefits. The inclusion of the derived benefits in the objective function will cause the radius of the ring rail line to increase.

4.8 VARIABLE DEMAND ANALYSIS

4.8.1 MINIMIZATION OF USER COSTS

An analysis to determine the optimal radius of a ring rail line with the objective of minimizing total user costs considering variable trip demand density M(R,θ) is explained as follows. M(R,θ) is define as the number of passengers per sqaure kilometre at θ. Suppose a ring rail line of varying radii R(θ) and centre at T_O is to be located within a city of radius b (Figure 4.8). Considering the locations of all passengers residing inside the ring rail line in the entire polar region (Figure 4.9), the related daily cost of accessing the ring rail line is:

$$\gamma_A \int_0^{2\pi} \int_0^{R(\theta)} [R(\theta)-r]M(r,\theta)\,rdrd\theta \qquad (4.11)$$

where γ_A is the cost of accessing the ring rail line per passenger per kilometre and M(R,θ) is daily non-uniform demand density. The daily cost of accessing the ring rail line by all passengers whose origin points are located outside the ring rail line is given by the expression:

$$\gamma_A \int_0^{2\pi} \int_{R(\theta)}^{b} [r-R(\theta)]M(r,\theta)\,rdrd\theta \qquad (4.12)$$

The daily total access cost is given by the sum of Equations 4.11 and 4.12. It is assumed that all passengers will egress from the ring rail line to reach their destination points which are located at both inside and outside the ring rail line. The required total user costs [Z(R,θ)] is given by twice the sum of Equations 4.11 and 4.12. The total user cost then becomes:

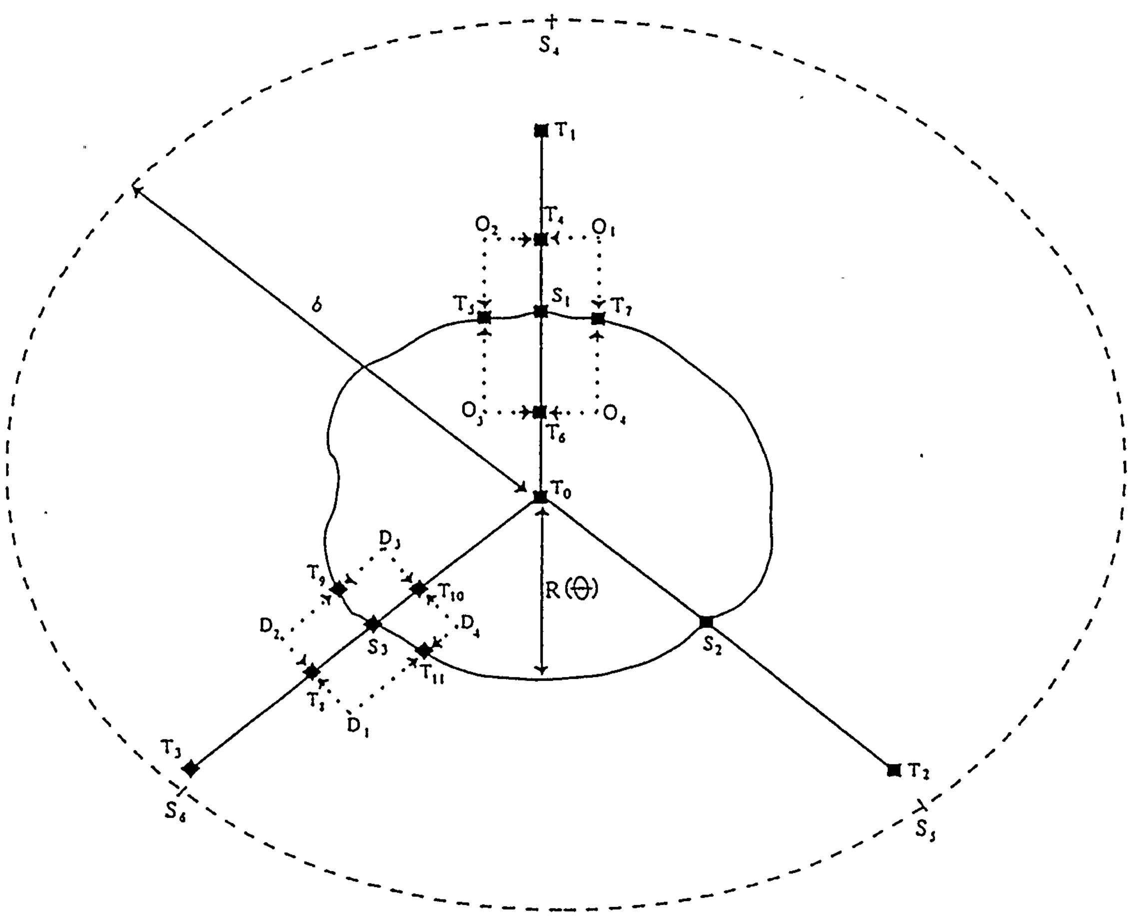

Figure 4.8 Typical Passenger Trip Assignment

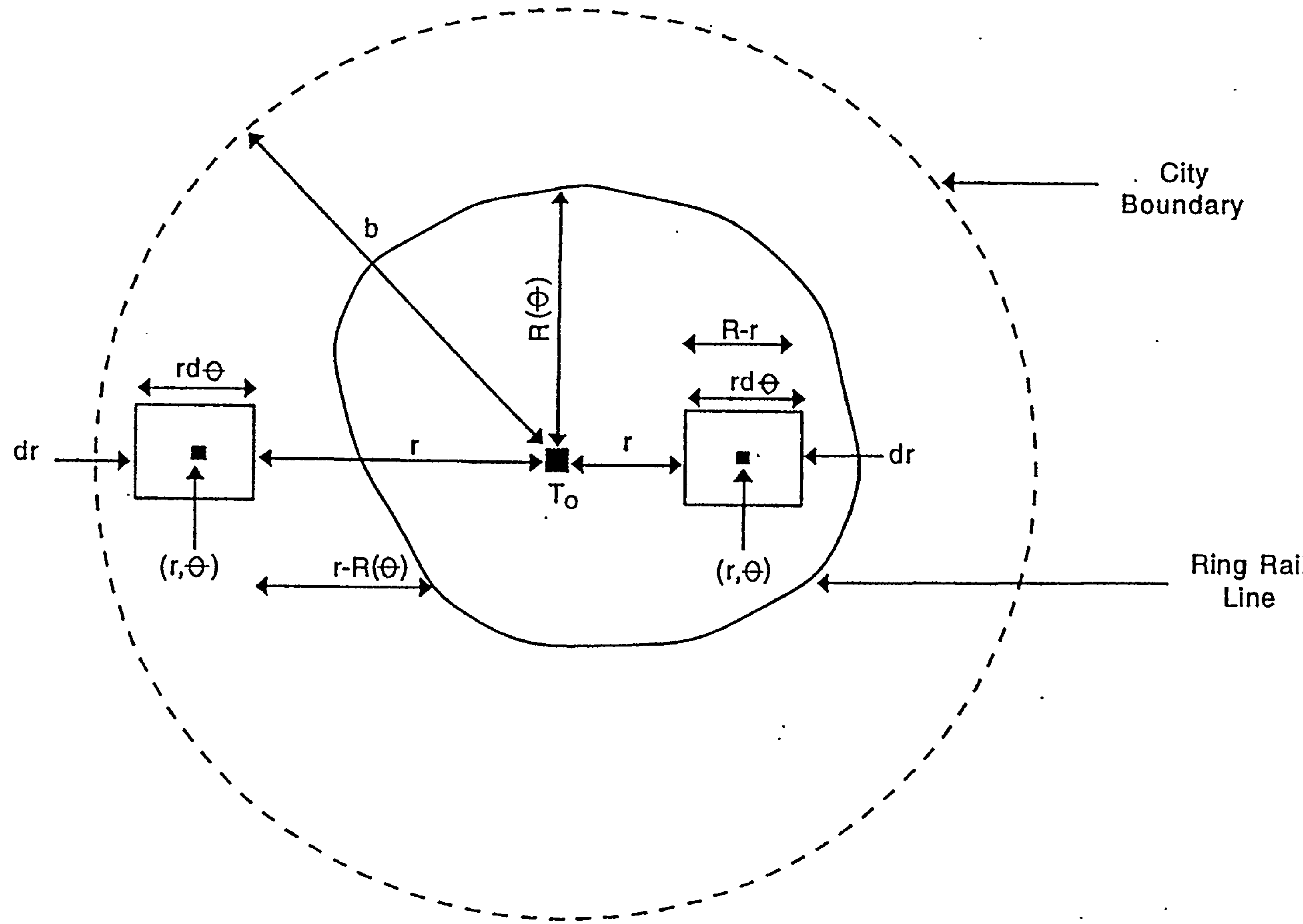

Figure 4.9 Elemental Area Located at Typical Origin Point of Passenger

$$Z(R,\theta) = 4\pi\gamma_A[\int_0^{R(\theta)} R(\theta) rM(r,\theta)dr - \int_0^{R(\theta)} r^2M(r,\theta)dr] + 4\pi\gamma_A[\int_{R(\theta)}^{b} r^2M(r,\theta)dr - \int_{R(\theta)}^{b} R(\theta)rM(r,\theta)dr] \tag{4.13}$$

The optimal radius which minimizes the total user access costs is obtained by taking the first derivative of Equation 4.13 with respect to R(θ) and setting the resulting expression to zero. This gives:

$$\int_0^{R(\theta)} rM(r,\theta)\,dr = \int_{R(\theta)}^{b} rM(r,\theta)\,dr \tag{4.14}$$

The left and right hand terms of Equation 4.14 are interpreted as the total demand at θ for all passengers residing at inside and outside of the ring rail line respectively. These terms can be obtained by numerical or graphical integration rM(r,θ) with respect to r. From Equation 4.14, it is deduced that the required optimal radius R(θ) is the value of R(θ) which makes the total number of passenger residing inside the ring rail line to be equal to the number of passenger residing outside the ring rail line. It is worth mentioning that the second and third terms on the right side of Equation 4.13 are respectively interpreted as the first moment of the passenger demand at inside and outside the ring rail line about T_O. Also, these terms can be obtained by numerical or graphical integration of $r^2M(r,\theta)$ with respect to r. For minimum total cost to be realised, the second derivative of Equation 4.13 with respect to R(θ) should be positive. Thus:

$$Z''(R,\theta) = 8\pi\gamma_A R(\theta)M(R,\theta) > 0 \tag{4.15}$$

4.8.2 MINIMIZATION OF USER AND LINE COSTS.

The possibility of a non-zero optimum radii of a ring rail line with the objective of minimizing the sum of user costs and rail line costs exists. If a non-uniform rail line cost per kilometre per day $\gamma_L(R,\theta)$ is assumed, then the rail line cost per day is:

$$2\pi R(\theta)\gamma_L(R,\theta) \tag{4.16}$$

The daily total cost [Z(R,θ)] is then given by the sum of Equations 4.13 and 4.16. Thus:

$$\begin{aligned} Z(R,\theta) &= 4\pi\gamma_A\left[\int_0^{R(\theta)} R(\theta)rM(r,\theta)\,dr - \int_0^{R(\theta)} r^2M(r,\theta)\,dr\right] \\ &+ 4\pi\gamma_A\left[\int_{R(\theta)}^{b} r^2M(r,\theta)\,dr - \int_{R(\theta)}^{b} R(\theta)rM(r,\theta)\,dr\right] \\ &+ 2\pi R(\theta)\gamma_L(R,\theta) \end{aligned} \tag{4.17}$$

The optimal radius which minimizes the total user and line costs is obtained by taking the first derivative of Equation 4.17 with respect to R(θ) and setting the resulting expression to zero. This simplifies to:

$$2\gamma_A\int_{R(\theta)}^{b} rM(r,\theta)\,dr - 2\gamma_A\int_0^{R(\theta)} rM(r,\theta)\,dr = \gamma_L(R,\theta) \tag{4.18}$$

From Equation 4.18, it is found that the optimal radius R(θ) is the required value of the ring rail line such that twice the difference between the daily user access costs per kilometre for passengers residing ouside and inside the ring rail line equals the daily rail line cost per kilometre. For minimum total cost to be realised, the second derivative of Equation 4.18 with respect to R(θ) should be positive. Thus:

$$Z''(R,\theta) = 8\pi\gamma_A R(\theta)M(R,\theta) + 2\pi\gamma_L'(R,\theta) > 0 \tag{4.19}$$

4.9 MODEL APPLICATION

For illustrative purposes, the entire city of Calgary is divided into four sectors Figure 4.10). Sector one, located at the North-West part of Calgary, is enclosed by Bow River, Downtown Corridor and Edmonton Trail. Sector two, which is located at the North-East part of Calgary, is surrouded by Memorial Drive and Edmonton Trail. Sector three is assumed to be located at the South-East part of Calgary. This sector is enclosed by Memorial Drive and Elbow River. Sector four, located at the South-East part of Calgary, is surrounded by the Elbow River and Bow River. Data on zonal population of the city (Appendix IV), obtained from Department of Transportation Planning in City of Calgary, is used in the analysis.

Based on the objective of minimizing only user access cost (Equation 4.14), the values of the optimal radius obtained for sectors one, two, three and four are 4.06km, 3.51km, 5.01km and 3.29km respectively. The proposed ring rail line (ring 3, Figure 4.10) is observed to run through such areas as Banff Trail and Highwood in the North-West, Vista Heights and Franklin Industrail in the North-East, Erin Woods and Fairview in the South-East, and Spruce Cliff and Elbow Park in the South-West part of Calgary.

Using Equation 4.18, it is possible to obtain optimal radius of the ring rail line considering the objective of minimizing both user access and line costs. The values of the optimal radius obtained for sector one, two, three and four are respectively 2.74km, 3.07km, 4.00km and 2.47km. The proposed ring rail line (ring 4, Figure 4.10) is observed to run through areas including Hillhurst and Rosemount in the North-West, Winston Heights and Mayland Heights in the North-East, Southview and Manchester in the South-

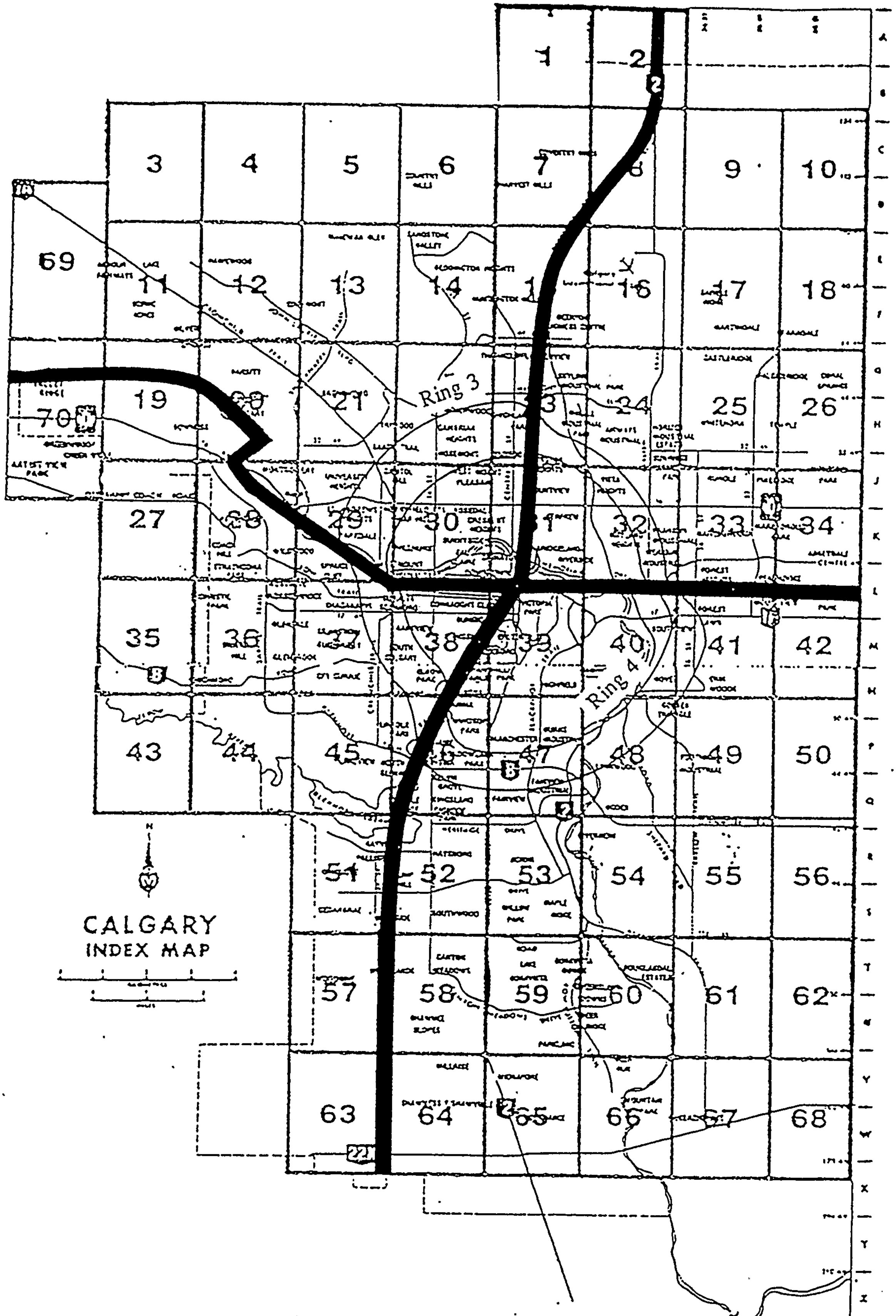

Figure 4.10 Optimal Location of Proposed Ring Rail Line in Calgary, Alberta

West, and Bankview and Sunalta in the South-East. Also, it is found that the optimal radius for each sector decreases if the line cost is included in the objective function. The values of optimal radii obtained in this case are observed to be unrealistically low. However, by including the benefits associated with the provision of the ring rail line in the total cost function (Equation 4.18), reasonable and realistic values of the optimal radii will be obtained.

Suffice it to say that the concept of provision of a ring rail line in Calgary has not received sufficient attention. Transit planners in Calgary emphasised that the benefits associated with the provision of the ring rail line in Calgary is mainly dependent on high passenger demand. Hence a high passenger demand is required to warrant the provision of the ring rail line. They remarked that the current transit demand in Calgary is insufficient to justify the provision of the ring rail line. The benefit of providing a ring rail line is not incorporated in the model. Furthermore, the rail fleet cost, cost of operating trains on the ring rail line, user cost of riding in trains on the ring rail line and transfer cost due to passenger transfer occurring on the ring rail line are not considered as well. This makes the proposed model for determining the optimal location of a circumferential rail line somewhat imperfect.

CHAPTER FIVE

CONCLUSIONS AND RECOMMENDATION

5.1 CONCLUSIONS

A set of analytical models for optimal rail line length, optimal rail termini as well as optimal location of a ring rail line are developed considering relevant cost parameters, which are user costs, systems operating costs, rail line and fleet costs and passenger transfer penalty cost. Many to many demand at both peak and off-peak periods is considered. The mathematical methods used are generally restricted to calculus with graphical and numerical illustrations.

Application of the models to Calgary's transit line demonstrated their usefulness in providing simple and ready answers. Considerable planning insights can be obtained with the use of the proposed models. Most importantly, the proposed models, in conjunction with the sensitivity analyses are expected to be of practical use for determining optimum value of rail line length and location of rail termini and location of a ring rail line. Moreover,the models provide useful information to planners on ways to provide an efficient and economic policies that will improve the current practices in rail planning.

The models for optimal rail line length and optimal location of rail termini are designed considering important factors which include passenger many to many demand at both peak and off-peak periods as well as passenger transfer penalty costs. These factors are overlooked in previous studies. This makes the proposed models more realistic and applicable to real life situations.

It should be mentioned that the models are applicable to LRT lines serving many to one or one to many trips. It is also applicable to various rail technologies such as Rapid Rail Transit (RRT) systems and Commuter Rail Transit (CRT) systems, and can be easily adapted to the operations of other public transit modes.

To date, no literature on optimal location of rail termini and optimal location of a ring rail line are documented. The determination of these important rail parameters are explored and presented in this report. The research therefore contributes a great deal towards effective rail planning.

5.2 RECOMMENDATIONS FOR FUTURE RESEARCH

The research investigated the effect of passenger transfer penalty cost on the desired parameters. However, the assumption that all passenger travelling beyond the rail termini will transfer from rail to bus creates room for criticism. This assumption is considered for the sake of mathematical simplicity. But simplicity curtails accuracy. In reality, not all passengers will transfer into buses. Some passengers will continue their trips to their destination points by using their private automobiles or walking. Therefore it is more desirable to investigate the possibility of using a modal split analysis to determine a more accurate number of transfer passengers and their related cost.

The model developed to determine the optimal location of a ring rail line, which considers only few parameters such as user access cost and line cost, is expected to be a starting point for further research. A more comprehensive research, encompassing many other factors including user riding cost, transfer cost, fleet cost and operating cost, is therefore required in order to develop a more realistic model to determine the optimal radius of a ring rail line.

The determination of realistic optimal radius is, to a very large extent, dependent on user access cost and user riding cost. These cost parameters can be properly formulated by an efficient trip assignment technique. In this analysis, the passenger trip assignment technique used to formulate the user access cost is really not constructive. It is therefore recommended that an effective trip assignment model to be developed in order to properly formulate the user access and riding costs. Although the analysis gave prominence to the optimal location of a single ring rail line, it is imperative to remark that the proposed model can be used to determine the optimal location of two or three ring rail lines in a large metropolitan area characterized by a high concentration of passenger demand for public transit systems. Moreover, the optimal spacing between the ring rail lines can be determined using the model.

It is imperative to remark that the research is focused on the supply side of transportation systems, with little prominence given to the demand side of the transportation systems. This research therefore provides simple economic tools to rail planners transportation researchers and analysts. The lack of guidance to provision of a rail line is no longer an obstacle when experience and judgement are used in conjunction with the information proposed by these models. It is demonstrated that the models meet their designed objectives fairly well. However, due to the deficiencies identified in the foregoing discussions, with particularly reference to the model developed to determine the optimal radius of a ring rail line, it is necessary that more research work to be undertaken to establish more confidence in the validity of the proposed models.

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APPENDIX 1

UNIT COST ESTIMATES

In this section, various unit cost parameters required for validating the presented models are estimated. Basic assumptions made for this purpose are described as well. Costs are updated to reflect their current value and tailored to meet our requirements. The formulas

$$F = P(1+i)^{n}$$

and

$$P = F(1+i)^{-n}$$

are used to estimate the future sum F and present sum P, where i is the inflation rate and n is the duration. Moreover, the formula

$$A = P[I(1+I)^{N}]/[(1+I)^{N}-1]$$

is used to estimate the yearly costs, where A represent the annual cost, P is the present cost, I is the interest rate (i.e. rate of return) on investment and N is the design life span of project.

1.0 TRAVEL TIME UNIT COSTS

The average cost per passenger per hour by walking, waiting, riding and transferring in Calgary are respectively \$11.70, \$3.40, \$5.40 and \$16.69 (Hunt et al, 1993). Given that the average walking speed of a passenger is 1.2m/s (Teply, 1984), the average cost of walking per passenger per kilometre (i.e.γ_A) is \$2.71. If the average speed of buses and trains are respectively 20.0km/h and 35.0km/h (Calgary Transit, 1993), then the average cost of travel by bus and train (i.e. γ_B and γ_R) are $0.27 and $0.13 respectively. The average cost of transferring per passenger per kilometre γ_P by bus and train are respectively $0.72 and $0.48.

2.0 RAIL LINE UNIT COST

The cost of constructing the south LRT line of length 10.9km in Calgary in 1981 is $136,200,000.00 (Calgary Transit, 1993). Based on 2% inflation rate, the present construction cost is $172,734,532.40. At 7% rate of return on investment (Calgary Transit, 1994), the annual line cost based on 50 years design period is $12,523,253.60. For 365 number of days in a year, the average discounted rail line cost per kilometre per day (i.e. γ_L) is $3148.00.

3.0 RAIL FLEET UNIT COST

The cost of a rail vehicle in Calgary in 1981 is $1,400,000.00 (Calgary Transit, 1993). At 2% inflation rate, the current (1993) rail vehicle cost is $1,775,538.51. At 7% interest rate and 20 years design life span of vehicle, the annual rail fleet cost per vehicle is $128,726.54. Based on 365 number of days in a year, 162 places (seating and standing spaces) per LRT vehicle and a place for each passenger, the average rail fleet cost per place per day (i.e. λ_F) is $2.84.

4.0 RAIL OPERATING UNIT COST

The total cost of operating rails on the North-West, North East and South rail lines in Calgary in 1991 is $9,955,791.00 (Calgary Transit, 1993). The current (1993) rail operating cost based on 2% inflation rate is $10,358,004.96. The daily total rail demand expressed in passenger-kilometre, on all the exiting lines in Calgary, in 1991 is 856,328

(Calgary Transit, 1993). At 2% growth rate of transit demand (Calgary Transit) the present total rail demand is 890,924 passenger-kilometre per day. The average rail operating cost per passenger per kilometre (i.e. λ_R) is therefore \$0.03.

5.0 BUS OPERATING UNIT COST

The cost of purchasing a bus in Calgary in 1975 is \$215,000.00 (Calgary Transit, 1993). For a total number of 526 buses purchased in 1975, the associated total bus fleet cost is \$113,090,000.00. At 2% inflation rate, the current (1993) bus fleet cost is \$161,520,368.10. Each bus has a salvage value of \$4000.00 at the end of 20 years. Hence the present total salvage value is \$1,415,992.00. Based on 7% interest rate at 50 years project life span, the effective annual bus fleet cost is \$11,607,567.27.

The total bus operating cost in Calgary in 1991 is \$63,012,520.00. Considering a 2% average inflation rate, the present (1993) bus operating cost in Calgary is \$65,558,225.80. In this analysis, the required operating cost is taken to be the sum of annual bus fleet and annual operating cost of buses. The required annual operating cost is therefore \$77,165,793.07. This simplifies to daily bus operating cost of \$211,413.13 based on 365 number of days in a year. The daily total bus demand in Calgary in 1991 is 919,186 passenger-kilometres (Calgary Transit, 1993). At 2% transit demand growth rate, the present (1993) daily total bus demand is 972,066 passenger-kilometre. Hence the average bus operating cost per passenger per kilometre (i.e. λ_B) is \$0.23.

APPENDIX II

```
100 ' MAIN PROGRAM
110 '
120 '
130 N=1 : DIM X(N), Y(N), HX(N), DF(N)
140 '
150 '
160 ' GET PARAMETERS
170 '
180 GMB=.27 : GMR=.13 : LMB=.23 : LMR=.03 : GML=3148 : GMP=0.72
190 ALFR=1.0 : CLMR=.0314 : LMFR=2.84 : MMR=4800
200 , XR0=8
210 '
220 ' GET XR(i) AND MXR(i)
230 '
240 READ NPT
250 DIM XR(NPT), MXR(NPT)
260 FOR I=1 TO NPT
270 READ XR(I), MXR(I)
280 NEXT I
290 DATA 10
300 DATA 0, 0, 2, 9700, 3.045, 10589, 4.015, 9329, 5.106, 8309
310 DATA 5.909, 7789, 7.273, 6029, 8.303, 3900, 12.011, 1800, 16.479, 0
320 '
330 '
340 ' CALCULATE CONSTANTS
350 '
360 '
370 CST1=2*(GMR-GMB+LMR-LMB)
380 CST2=GML+2*ALFR*CLMR*LMFR*MMR-2*ALFB*CLMB*LMFB*MMB
390 'INPUT"xr0=",XR0
400 X(1)=XR0
```

```
410 'GOSUB 750
420 'PRINT "x=";X(1),"y=";Y(1)
430 'GOTO 365
440 GOSUB 550
441 CLS
450 PRINT "********* INPUT DATA ***********" : PRINT
460 PRINT "gmb=";GMB, "gmr=";GMR, "lmb=";LMB, "lmr=";LMR
470 PRINT "gml=";GML, "gmp=";GMP
480 PRINT "alfr=";ALFR, "lmfr=";LMFR,
490 PRINT "INITIAL VALUE=";XR0 : PRINT
500 PRINT "********* OUTPUT DATA ***********" : PRINT
510 PRINT "XR=";X(1), "ERROR=";Y(1), "NO. OF ITERATIONS=";ITE
520 END
530 '
540 '
550 ' GRADIENT METHOD TO SOLVE A GROUP OF N NON-LINEAR EQUATIONS:
560 ' Y(I)=f(X(1),X(2),...,X(N))=0  for I=1, 2,..,N
570 ' E - GIVEN ERRORS;  H - STEP.
580 ' EQUATIONS ARE DEFINED STARTING AT LINE ****
590 '
600 '
610 E=.01 : H=.0001 : ITE=0
620 FOR I=1 TO N
630 IF X(I) <> 0 THEN HX(I)=H*X(I) ELSE HX(I)=H
640 NEXT I
650 GOSUB 890
660 F=F1
670 IF F<E THEN 860
680 ITE=ITE+1 : PRINT"No. of iteration=";ITE
690 FOR I=1 TO N
700 PRINT "x(";I;")=";X(I),"y(";I")=";Y(I)
710 NEXT I
```

```
720 SUM=0
730 FOR I=1 TO N
740 X(I)=X(I)+HX(I)
750 GOSUB 890
760 FH=F1
770 DF(I)=(FH-F)/HX(I)
780 SUM=SUM+DF(I)^2
790 X(I)=X(I)-HX(I)
800 NEXT I
810 RLMT=F/SUM
820 FOR I=1 TO N
830 X(I)=X(I)-RLMT*DF(I)
840 NEXT I
850 GOTO 620
860 RETURN
870 '
880 '
890 'USER DEFINED A GROUP OF N NON-LINEAR EQUATIONS
900 '
910 '
920 XRV=X(1) : GOSUB 990 : Y(1)=CST1*MXRV+CST2
930 XRV1=XRV : GOSUB 1140
940 Y(1)=Y(1)+2*GMP*DMXRV
950 F1=Y(1)^2
960 RETURN
970 '
980 '
990 ' SUBROUTINE FOR INTERPOLATION: USE ALL POINTS
1000 ' GIVEN A XRV, RETURN A MXRV
1010 '
1020 '
1030 MXRV=0
```

```
1040 FOR PT=1 TO NPT
1050 MULTI=1
1060 FOR MPT=1 TO NPT
1070 IF PT<>MPT THEN MULTI=MULTI*(XRV-XR(MPT))/(XR(PT)-XR(MPT))
1080 NEXT MPT
1090 MXRV=MXRV+MULTI*MXR(PT)
1100 NEXT PT
1110 RETURN
1120 '
1130 '
1140 ' SUBROUTINE FOR CALCULATING DERIVATIVES
1150 ' GIVEN A XRV1, RETURN A DMXRV
1160 '
1170 '
1180 DELTA=.05
1190 XRV=XRV1+DLTA
1200 GOSUB 990
1210 VALUE1=MXRV
1220 XRV=XRV1-DLTA
1230 GOSUB 990
1240 VALUE2=MXRV
1250 DMXRV=(VALUE1-VALUE2)/(2*DLTA)
1260 RETURN
```

APPENDIX III

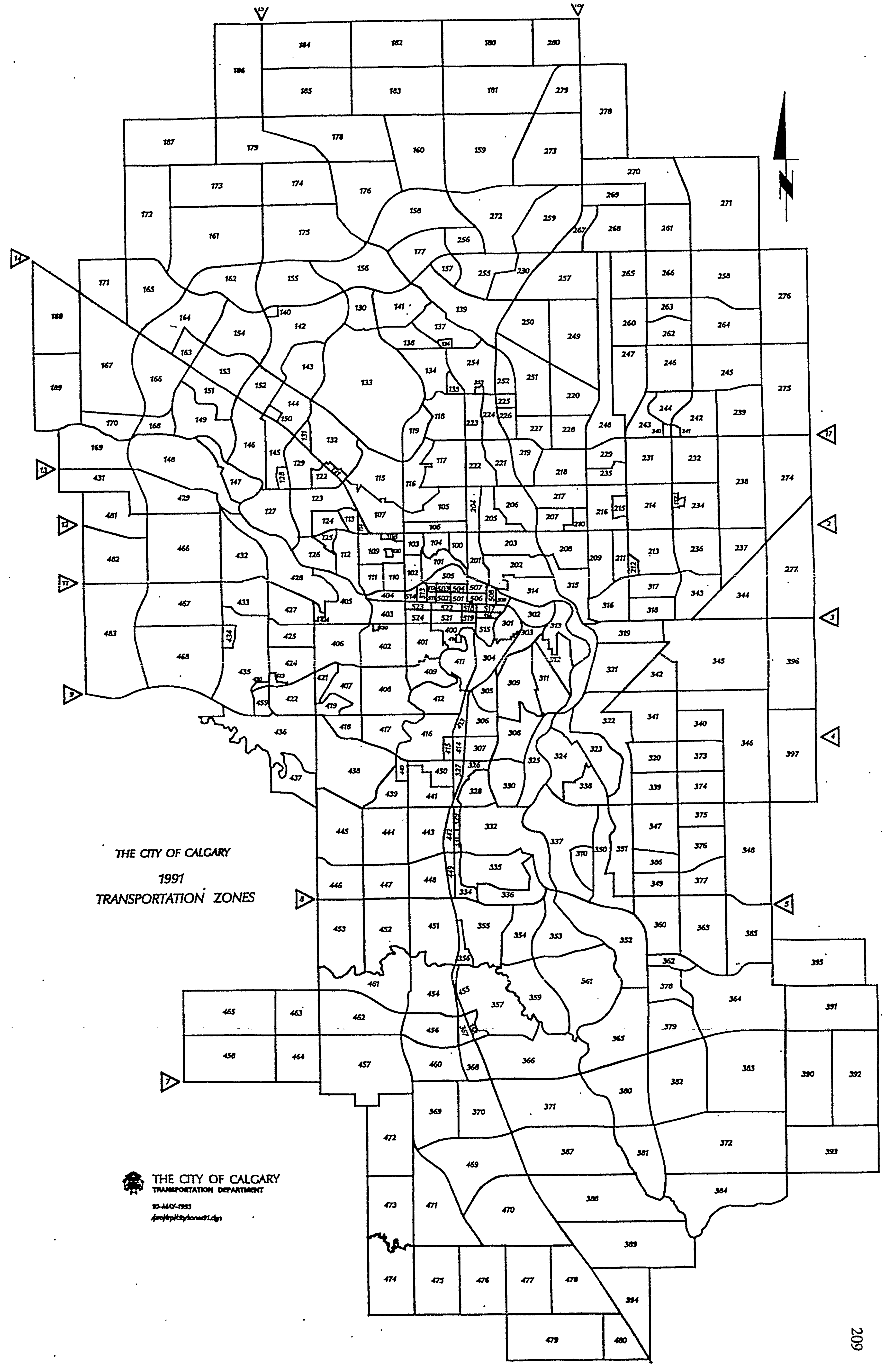
```
100 ' MAIN PROGRAM
110 '
120 '
130 N=1 : DIM X(N), Y(N), HX(N), DF(N)
140 '
150 '
160 ' GET PARAMETERS
170 '
180 GMB=.27 : GMR=.13 : LMB=.23 : LMR=.03 : GML=3148 : GMP=-.54
190 ALFR=1.0 : CLMR=.0314 : LMFR=2.84 : MMR=4800
200 XR0=1
210 '
220 ' GET XR(i) AND MXR(i)
230 '
240 READ NPT
250 DIM XR(NPT), MXR(NPT)
260 FOR I=1 TO NPT
270 READ XR(I), MXR(I)
280 NEXT I
282 READ NPTB
284 FOR I=1 TO NPTB
286 READ BXR(I) : NEXT I
290 DATA 10
300 DATA 0,0, 2.00, 10729, 3.045, 10589, 4.015, 9329, 5.106, 8309
310 DATA 5.909, 7789, 7.273, 6029, 8.303, 3900,12.011, 1800,16.749, 0
312 DATA 10
314 DATA 0,10729,11379,11919,12269,12409,13149,13349,13449,13449
320 '
330 '
340 ' CALCULATE CONSTANTS
350 '
360 '
370 CST1=2*(GMR-GMB+LMR-LMB)
380 CST2=GML+2*ALFR*CLMR*LMFR*MMR
```

```
390 'INPUT"xr0=",XR0
400 X(1)=XR0
410 'GOSUB 750
420 'PRINT "x=";X(1),"y=";Y(1)
430 'GOTO 365
440 GOSUB 550
441 CLS
450 PRINT "********* INPUT DATA ***********" : PRINT
460 PRINT "gmb=";GMB, "gmr=";GMR, "lmb=";LMB, "lmr=";LMR
470 PRINT "gml=";GML, "gmp=";GMP
480 PRINT "alfr=";ALFR, "lmfr=";LMFR,
490 PRINT "Initial XR value=";XR0 : PRINT
500 PRINT "********* OUTPUT DATA ***********" : PRINT
510 PRINT "XR=";X(1), "ERROR=";Y(1), "NO. OF ITERATIONS=";ITE
520 END
530 '
540 '
550 ' GRADIENT METHOD TO SOLVE A GROUP OF N NON-LINEAR EQUATIONS:
560 ' Y(I)=f(X(1),X(2),...,X(N))=0  FOR I=1, 2,..,N
570 ' E - GIVEN ERROR;  H - STEP.
580 ' EQUATIONS ARE DEFINED STARTING AT LINE ****
590 '
600 '
610 E=.01 : H=.0001 : ITE=0
620 FOR I=1 TO N
630 IF X(I) <> 0 THEN HX(I)=H*X(I) ELSE HX(I)=H
640 NEXT I
650 GOSUB 890
660 F=F1
670 IF F<E THEN 860
680 ITE=ITE+1 : PRINT"NO. OF ITERATION=";ITE
690 FOR I=1 TO N
700 PRINT "x(";I;")=";X(I),"y(";I")=";Y(I)
710 NEXT I
720 SUM=0
```

```
730 FOR I=1 TO N
740 X(I)=X(I)+HX(I)
750 GOSUB 890
760 FH=F1
770 DF(I)=(FH-F)/HX(I)
780 SUM=SUM+DF(I)^2
790 X(I)=X(I)-HX(I)
800 NEXT I
810 RLMT=F/SUM
820 FOR I=1 TO N
830 X(I)=X(I)-RLMT*DF(I)
840 NEXT I
850 GOTO 620
860 RETURN
870 '
880 '
890 'USER DEFINED A GROUP OF N NON-LINEAR EQUATIONS
900 '
910 '
920 XRV=X(1) : GOSUB 990 : Y(1)=CST1*MXRV+CST2
930 GOSUB 1121
940 Y(1)=Y(1)+2*GMP*BXRV
950 F1=Y(1)^2
960 RETURN
970 '
980 '
990 ' SUBROUTINE FOR INTERPOLATION: USE ALL POINTS
1000 ' GIVEN A XRV, RETURN A MXRV
1010 '
1020 '
1030 MXRV=0
1040 FOR PT=1 TO NPT
1050 MULTI=1
1060 FOR MPT=1 TO NPT
1070 IF PT<>MPT THEN MULTI=MULTI*(XRV-XR(MPT))/(XR(PT)-XR(MPT))
```

```
1080 NEXT MPT
1090 MXRV=MXRV+MULTI*MXR(PT)
1100 NEXT PT
1110 RETURN
1120 '
1121 ' SUBROUTINE FOR INTERPOLATION: USE ALL POINTS
1122 'GIVEN A XRV, RETURN A BXRV
1123 '
1124 BXRV=0
1125 FOR PT=1 TO NPTB
1126 MULTI=1
1127 FOR MPT=1 TO NPTB
1128 IF PT<>MPT THEN MULTI=MULTI*(XRV-XR(MPT))/(XR(PT)-XR(MPT))
1129 NEXT MPT
1130 BXRV=BXRV+MULTI*BXR(PT)
1131 NEXT PT
1132 RETURN
1133 '
1140 ' SUNROUTINE FOR CALCULATING DERIVATIVE
1150 ' GIVEN A XRV1, RETURN A DMXRV
1160 '
1170 '
1180 DLTA=.05
1190 XRV=XRV1+DLTA
1200 GOSUB 990
1210 VALUE1=MXRV
1220 XRV=XRV1-DLTA
1230 GOSUB 990
1240 VALUE2=MXRV
1250 DMXRV=(VALUE1-VALUE2)/(2*DLTA)
1260 RETURN
```

Appendix IV



THE CITY OF CALGARY
1991
TRANSPORTATION ZONES
THE CITY OF CALGARY
TRANSPORTATION DEPARTMENT
10-MAY-1993

APPENDIX IV

Zone	Pop	Jobs
2	0	652
3	0	154
4	0	70
5	0	0
6	0	1629
7	0	0
8	0	20
9	0	22
11	0	11
12	0	0
13	0	1041
14	0	679
15	0	43
16	0	2122
17	0	24
100	1443	818
101	3548	687
102	2213	1783
103	454	1824
104	1588	99
105	5239	736
106	2749	1158
107	5617	863
108	0	1317
109	3626	747
110	2226	1148
111	2179	456
112	2957	661
113	0	126
114	50	562
115	4709	658
116	4250	454
117	4207	524
118	6224	654
119	3418	171
120	537	749
121	0	1218
122	452	869
123	1882	5531
124	2846	395
125	499	5698
126	3005	574
127	4135	1072
128	0	2291
129	5161	646
130	3818	63
131	0	1322
132	7725	987
133	0	256
134	6759	372

135	753	456
136	0	760
137	6814	414
138	6186	74
139	0	0
141	5112	223
142	7468	282
143	4827	264
144	6374	742
145	4443	570
146	2846	138
147	3867	620
148	9704	1165
149	6047	196
151	4344	706
152	3725	120
153	8727	890
154	7965	442
155	271	31
156	52	0
157	0	0
158	47	10
159	18	0
161	565	283
162	27	0
163	0	1205
164	11	30
165	212	152
166	4279	113
167	49	102
168	76	78
169	26	26
201	4355	1029
202	5992	858
203	3626	559
204	2330	852
205	3592	1030
206	0	255
207	2435	107
208	6283	496
209	0	4531
210	2	643
211	0	2787
212	0	1369
213	9427	805
214	11665	591
215	0	1603
216	0	2781
217	0	6694
218	1	3570
219	0	3631
221	2	2735

222	2561	2612
223	4700	271
224	0	904
225	0	729
226	0	1241
227	0	1508
228	0	1003
229	0	553
231	12002	582
232	11996	660
233	0	694
234	10513	544
235	0	3617
236	9411	1067
237	6786	182
238	2175	122
239	5	9
241	0	224
242	10652	311
243	13	20
244	6462	322
245	1465	20
246	2777	59
247	103	80
248	1	0
249	0	3339
251	0	412
252	0	1459
253	0	373
254	8074	343
255	445	43
256	0	0
257	21	10
258	145	0
259	31	0
261	76	47
301	2228	348
302	1489	1116
303	0	472
304	0	3290
305	0	2750
306	0	2956
307	0	3008
308	0	3504
309	0	5219
311	25	2502
312	2	1212
313	1055	263
314	755	3765
315	2	3159
316	6541	1820
317	7005	698

318	4406	901
319	6538	871
321	11543	733
322	63	4437
323	0	1124
324	6173	346
325	0	369
326	0	1754
327	2	1707
328	3925	298
329	0	2146
330	0	2374
331	0	653
332	11226	1226
334	2	3788
335	6746	623
336	1655	201
337	4927	383
338	4622	823
339	5	5664
341	1	2156
342	4534	207
343	9720	513
344	2411	49
345	150	1425
346	43	122
347	20	1682
348	22	325
349	0	30
351	18	930
352	2520	116
353	6575	378
354	6000	318
355	7357	863
356	0	220
357	8762	1035
358	0	1025
359	4733	261
361	10580	1369
362	0	0
363	3	69
364	8	0
365	3855	151
366	7291	274
367	5	113
368	0	0
369	27	0
371	1	0
372	33	6
400	6163	3451
401	5330	1465
402	10697	1194

403	4077	1837
404	0	851
405	2050	476
406	7371	1224
407	479	2576
408	7182	891
409	2610	284
410	0	1367
411	2772	659
412	3132	666
413	274	1680
414	3	1165
415	0	3309
416	5353	905
417	1961	578
418	674	424
419	761	1010
420	0	1469
421	2346	204
422	6395	808
423	0	659
424	7344	469
425	2903	480
426	0	704
427	6635	1134
428	5981	766
429	23	11
431	16	0
432	4426	547
433	4937	241
434	0	10
435	3544	79
436	802	272
437	0	0
438	6535	793
439	315	143
440	210	2176
441	4121	803
442	578	1753
443	6457	516
444	5815	1415
445	7042	981
446	6920	437
447	7067	655
448	6094	591
449	0	3082
450	4019	2105
451	8724	864
452	6316	351
453	11170	561
454	4830	260
455	0	45

456	5511	204
457	345	84
458	63	0
501	27	19894
502	265	10136
503	692	9260
504	0	20672
505	1822	2794
506	1002	6829
507	861	6385
508	914	558
509	0	69
511	2821	5403
512	1152	3227
513	467	1160
514	0	290
515	0	1542
516	688	33
517	243	840
518	797	1897
519	908	1008
521	6288	3357
522	314	5016
523	505	3872
524	6375	1785