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Teaching Mathematics Using Geometry: A Problem Solving Perspective

by

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ABSTRACT

This thesis investigated the nature of and compiled non-traditional mathematical activities involving geometric symbolism and concepts and described the teaching context in which these activities should be used to teach mathematics meaningfully. The main focus was on collecting and developing such activities that:

(i) covered topics from all strands of the Alberta Program of Studies in Mathematics for Grade 7;

(ii) developed mathematical thinking through exploration, discovery and problem solving;

(iii) was consistent with a constructivist perspective of learning and teaching.

The activities were from a variety of sources; self-made, journals, and books. They were adapted to fit the curriculum of studies, the constructivist perspective of teaching and learning, and the author's problem solving method.

The results involve a set of 66 activities following a similar format that presents the geometric ideas used in the activity, materials needed by the learners, a story line that could be used as an opener, key question(s) to be researched, a brief start-up for the learners, a description of what could be included in the published report, and some comments to teachers showing answers, connections to other mathematical concepts, and possible source. The results also provide a pedagogical approach to use the activities in the classroom.

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This thesis is dedicated to
my father and mother,

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Their sacrifice for me, providing a secure and loving environment where I could develop to my fullest potential, not afraid to risk, to love, to enjoy life, and to teach others the same.

They built into me a personal faith in God and by their example and attitude a set of virtues to live by.

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CHAPTER 1
INTRODUCTION

I. PURPOSE OF THIS THESIS

In this thesis my goal is to investigate the nature of and compile non-traditional mathematical activities involving geometric symbolism and concepts and to describe the teaching context in which these activities should be used to teach mathematics meaningfully. [Non-traditional activities are those that focus on developing mathematical thinking, in contrast to traditional activities which focus on drill and rote learning.]

My main focus will be on collecting and developing such activities that:

(i) cover topics from all strands of the Alberta Program of Studies in Mathematics for Grade 7 (Appendix A);

(ii) develop mathematical thinking through exploration, discovery and problem solving;

(iii) are consistent with a constructivist perspective of learning and teaching.

II. RATIONALE FOR STUDY

The rationale for this study involves several components. The most significant for me are as follows:

II.1 PERSONAL CONCERNS

My interest in this study grew out of my teaching of mathematics at the Junior High level for 22 years. During this period I "experimented" with different teaching approaches (described in a later chapter) as I struggled to move away from the traditional approach which was not producing the type of learning I wanted to see in students. I wanted to see students understanding mathematics and not simply memorizing mathematical facts. In general, I wanted them to develop mathematical thinking. This thesis will provide me with the opportunity to develop a meaningful collection of materials to help transform my teaching appropriately.

II.2 SHIFT IN FOCUS IN MATH EDUCATION

In the past few years, significant changes have been advocated to mathematics education internationally (NCTM,1989) and locally. In Alberta, significant revisions have been made and continue to be made to the mathematics curriculum to reflect these changes. These changes reflect a shift in focus from drill and practice learning to meaningful learning. The National Council for Teachers of Mathematics *Curriculum and Evaluation Standards,* describes this shift as follows: "The vision is that mathematical reasoning, problem solving, communication, and connections must be central (NCTM,1989) [in a mathematics classroom.]" This vision of the NCTM and its influence on the Alberta mathematics curriculum validates the need to shift to using materials and problems that will allow the students to construct their own meaning and understanding. This requires " a change in paradigm from school mathematics as a mind-independent reality recorded in textbooks to mathematics as a human activity that is carried out in the social context of the classroom, shifts the focus of mathematics teaching from a process of transferring information to students to interactive mathematical communication in a consensual domain of mathematical experience" (Steffe, 1990).

II. 3 INADEQUATE TEXTBOOKS

The above noted shift in mathematics education is not as yet fully integrated into mathematics textbooks. As the main form of resource materials available to teachers, mathematics textbooks need to be more non-traditional in their content and structure. Mathematics textbooks generally consist of pages and pages of similar type questions and exercises, and fragmented presentations that describe one way of getting the 'right answer' and that require memorization of a procedure, rather than understanding of the mathematical principle found in the concept. This became a major concern of mine after just a short period of teaching mathematics. I stopped using textbooks in my classroom for 10 years and only returned to using them because of budget constraints. The recommended mathematics textbooks in Alberta have not changed in their main focus they still emphasize skill development, finding the 'right answer', and standard procedures to getting that answer. I found that the enrichment activities, problems that developed mathematical thinking, and activities that used concrete materials were isolated examples, made little connection to the concepts being taught, and were not developed in a systematic way. Although some textbooks are currently being revised, it is not obvious that the changes will satisfy these concerns. This thesis will attempt to deal with these concerns, thus it will make a meaningful contribution to classroom teachers.

II.4 TEACHER'S NEEDS

Related to inadequate textbooks is the pressure on teachers to seek out alternative sources of mathematical activities to satisfy the shift in the mathematics curriculum. As Steffe suggests,

> Mathematics teachers generally resort to emphasizing the mechanics of doing mathematics out of desperation when attempting to teach an extant syllabus to 150 or so mathematics students a day (Steffe,1990).

As a mathematics teacher, I know that it can become very frustrating in trying to obtain adequate resources that can be used to develop mathematical thinking, when one has a limited time frame to seek out those kinds of materials and to cope with a curriculum that has too many mandated topics. This thesis will provide Grade 7 teachers with a sample of activities they could use in their teaching and will also be a basis for teacher development in terms of how to use these activities in a meaningful way.

II.5 NATURE OF GEOMETRY

The focus on activities involving geometric symbols and concepts is based on the fact that geometry lends itself to a variety of problem solving situations. Geometry provides a natural connection between mathematics concepts, between mathematics and the real world, and between mathematics and visual learning. These connections provide unique opportunities to make mathematics, the learning of it and the teaching of it meaningful.

III. Taking Action

This thesis is my effort to make a positive contribution to the teaching of mathematics. I have decided to take action to help bring about some needed changes in mathematics education. To begin my journey to this outcome, I consider it necessary to examine the personal experience, self and beliefs, that I am bringing to this situation that will unconsciously and consciously influence this outcome.

CHAPTER 2
MY JOURNEY TO THIS THESIS

In this chapter, I will take you through a journey of personal experiences involving my teaching of mathematics, my beliefs about teaching mathematics, and the circumstances that shaped and reshaped my beliefs and approaches to teaching mathematics. This journey should help one to understand the background of the research problem being investigated in this thesis and the perspective in which the study will unfold. [In appendix B, I have included a more general discussion of my history and beliefs that I have reflected on during my graduate program and that I think was a necessary part of my journey to this thesis, although, in a context that extends beyond it.] I will organize the chapter chronologically, based on the turning points in my teaching approaches. I will label these the traditional period, the affective-traditional period, the individualized learning period, and the problem solving period.

I. Traditional Period

My belief about teaching mathematics during this period was shaped by my experience as a student, particularly in High School. I was good at mathematics and had no problem with the teaching approaches I encountered. Consequently, I never questioned these approaches. In fact, I was most influenced by my Grade 12 mathematics teacher, who selected me to write several mathematics contests that year. He also was well organized and had all his "math in a binder", along with a set of jokes that he told during his math lessons. He knew how to joke but he also knew how to take a joke. I learned lots from him that year. My aim in teaching was to have all my materials in one binder and to tell lots of stories and jokes. I thought that if I could accomplish those two goals in teaching I was set for life in my teaching career.

At the point of beginning my teaching career, my goal was to get the students to learn math skills and concepts prescribed by the curriculum. I believed that students would learn by being told what to do and being made to do it. I would teach a concept by illustrating it with 2 or 3 examples on the board. I would then have the students work on practice pages of routine exercises (e.g. Table 1.1) then, I would take up the answers with the class.

Whole Numbers

I. Addition: Put in sign.

1.	34	57	317	1241	15434
	5	412	5142	17132	27112
2.	43	516	1618	30121	61013
	12	127	2206	12314	72342
	14	242	154	2121	53624

II. Subtraction: Put in sign.

3.	697	548	897	4309	6859
	234	126	586	108	4527
4.	932	473	914	873	613
	675	296	878	596	476

III. Multiplication:

5.	51	342	489	743	629
	X 6	X 9	X 6	X 8	X 9
6.	57	312	678	4273	256
	X 25	X 49	X 56	X 19	X 80

IV. Division: Recopy question and do the work in your scribbler.

7.	28910 ÷ 59 =	9012 ÷ 45 =	37352 ÷92 =
8.	20944 ÷68 =	21007 ÷ 35 =	17583 ÷ 57 =

TABLE 1.1

This cycle would be repeated with each new concept. This approach was efficient for satisfying my goals of covering the required content and surviving. This was my first teaching assignment, as an elementary teacher, and the demands on my time were too overwhelming to try to be creative in my teaching. So I stuck with this approach for about 3 years after which I started teaching at a Junior High School and a new period evolved.

II. Affective-Traditional Period

This period began on October 16, 1971 when I got a teaching job at a Junior High School, teaching mathematics. The teacher I was replacing was a Department Head of Mathematics and so I became acting Department Head for the year. The principal told me that if I did a good job that position was mine for as long as I stayed at that school. Wow, I had nine preparation periods a week that year. It was the new lease on life that I had been looking for. I now had only one subject to prepare for and all the resources available to me. I spent hours developing my 'mathematics teaching binders' in each grade. I found a lot of interesting activities and materials in this school. The teacher before me had developed some interesting games, hands-on materials and activities that opened my eyes to what could be done with mathematics. I tried a few of her activities and immediately liked them. I loved organizing and searching for new and different ideas and because I now had the time I used it wisely. I also loved the students. They were more mature and independent as learners and people in comparison to the elementary students in my first 3 years of teaching. They loved my jokes and I loved their sense of humor. I found that all of my strengths were being utilized, independence, organizing, developing a variety of methods and activities, coaching at a higher skill level and I could act more like myself.

I loved trying new activities, games, and ideas with my students. If they worked, great, do it again next year, if they failed then chalk it up to experience. I evaluated the success of these activities by the reaction of the students. Excitement, fun, enjoyment, understanding by the majority, and a positive feeling during the class determined a successful activity. Unenthusiasm, boredom, and comments by the majority that they didn't understand triggered an unsuccessful experience. Whenever I found an activity, game, discovery lesson or project that seemed interesting, I tried it. In dealing with the rest of the curriculum, I reverted to my drill and practice sheets. I did try to find puzzle and funsheets to enhance the practice times.

As I reflect on this period in my teaching career I realize that although some of my teaching looked different on the surface, it was actually similar to the traditional stage below the surface. My beliefs about teaching mathematics had not really changed because my goal was still drill and practice, but in a fun way, wherever possible or convenient. This is why I refer to this stage as the affective-traditional period because I was now trying to get the students to enjoy mathematics by disguising the drill and practice. So on reflecting on this period, I realize that the activities, games, and worksheets that I selected were simply to motivate the students to enjoy and 'play' with mathematical ideas. I was having fun and I wanted the students to have fun. There was no overriding structure to the learning activities and games. My plan was to do basic drill and practice during this period but with the added twist of motivational materials.

Some of the motivational games that I used during this period were Krypto (mental computation), Prime Time, Chess and Checkers, Card Games (Cribbage, Black Jack, Bridge), Warri and 4-Level Tic-Tac-Toe (strategy games). The game of Krypto was my passion during those years and I challenged any and every student or group of students to beat me in a game to 300. I offered to buy a meal at their favourite hamburger stand if they could win. I only lost once to a group of three girls on a day when I had a excruciating headache. I still find that this is one of the best games for the students to practice their basic computational facts and the challenge is still given with each new group of students.

In spite of the underlying focus on computational skill, there were the odd discovery activities that I tried. Two of these activities are:

1) Finding the area formula of a circle (Table 1.2). The students were asked to cut a circle into many wedge shapes, re-arrange these wedge shapes into a parallelogram and then using the formula of a parallelogram discover the area of a circle. [This activity can now be found in Junior High textbooks.]

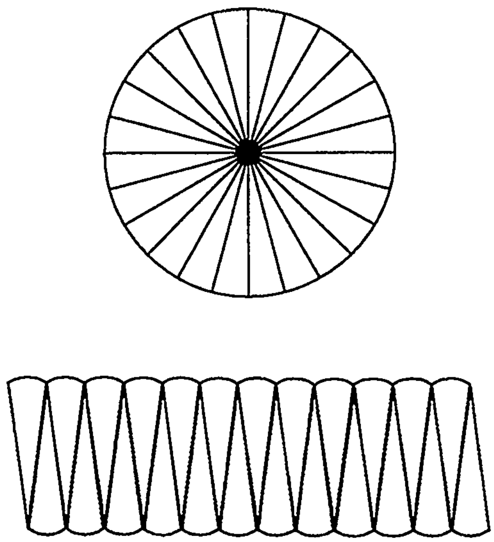


Table 1.2

2) What is the value of zero and negative exponents (Table 1.3)? Using the pattern method:

$2^4 = 16$	$(-3)^4 = 81$	$(1/4)^4 = 1/256$
$2^3 = 8$	$(-3)^3 = -27$	$(1/4)^3 = 1/64$
$2^2 = 4$	$(-3)^2 = 9$	$(1/4)^2 = 1/16$
$2^1 = 2$	$(-3)^1 = -3$	$(1/4)^1 = 1/4$
$2^0 = 1$	$(-3)^0 = 1$	$(1/4)^0 = 1$
$2^{-1} = 1/2$	$(-3)^{-1} = -1/3$	$(1/4)^{-1} = 4$
$2^{-2} = 1/4$	$(-3)^{-2} = 1/9$	$(1/4)^{-2} = 16$

TABLE1.3

The students were to discover from the patterns above that any number to the zero exponent would always result in +1 and, in finding the value of any number taken to a negative exponent the students should discover the rule that they need to do the reciprocal of the base, change the negative exponent to a positive exponent and then find the value of that power.

This affective-traditional period went on for about 12 years with a growing sense that there was something missing. That sense was that the level of mathematical understanding had not increased as I had expected. The main factors that could have contributed to this were;

(i) An overall lack of structure for using games, hands-on materials and discovery activities to help foster understanding.

(ii) A lack of planning on my part to connect the activities and the mathematical concepts being taught. The majority of students could not see the connections because I was using the activities on an experimental basis. This growing uneasiness led me to the individualized learning period.

III. Individual Learning Period

My focus about teaching/learning shifted from 'my way' to 'their way'. I had noticed in the discovery lessons I tried, that about 80% of the students couldn't come up with the mathematical discovery, and during the 'hands-on' activities the majority had difficulty connecting the concrete to the abstract. During the games days, the students played the games with enthusiasm but the connections between mathematics and the games eluded most. I also noticed when instructions on how to find the answer were given only verbally, that there was always a group of students that would ask me to 'show them' how to do it.

These observations left me open to any alternative approach to teaching mathematics that seemed more effective in helping students learn mathematics more meaningfully. Thus when I learned of a program that focused on individual learning styles in teaching, I took the opportunity to become familiar with it and to try it in my teaching.

In 1983, my wife, also a teacher, and I went to New York and worked with Rita and Ken Dunn, specialists in this individual learning approach. We had two weeks of intensive training on how to implement learning styles in a practical way in the classroom. In the fall of 1983, I radically changed my classroom (desks lined up in rows were shifted to grouping to accommodate for the sociological and environmental factors), my teaching practices (students were given the choice of learning in their best perceptual modality and the choice of activity to best accommodate their learning style), the types of learning activities (hands-on, drill and practice, discovery lessons, cassette and video tapes, visual materials, and a combination of ways of learning), and the way I related to the students. I tried to totally individualize the learning of mathematics from the student's individual learning style. The students chose the type of activity according to their style. I helped facilitate their learning through guiding, questioning, suggesting, discussing and then listening to what they had learned or looking at what they had created. It was hectic, chaotic, frustrating, enjoyable, exhilarating and tiring. The most challenging thing for me was the organization of activities, marking, keeping things running smoothly and trying to cover the curriculum mandated by the Department of Education.

In dealing with the curriculum during this phase, I designed mathematical resources that fit certain learning style characteristics, following the guidelines and sample activities by the Dunn's (1978) in their book, *Teaching Students Through Their Individual Learning Styles: a practical approach* . These activities are categorized in the following ways:

1. Contract Activity Packages

These are a set of activity alternatives and reporting alternatives that allow the students choice in what type of activity they wish to use for their learning. There should be at least one activity for each of the perceptual strengths. The Contract Activity Packages work best for students who are persistent, responsible, motivated, self-structured, able to work alone, visual, auditory, tactile, and kinesthetic.

2. Multisensory Instructional Packages.

The kits contain activities that include all perceptual strengths. Cassette tape, tactile instrument (electroboard, wheel of knowledge, task puzzles, etc.), worksheets or some type of skill activity, and games. Instructions are given on a cassette tape and the students follow the sequence given on the tape. The multisensory packages are best suited for the students who are unmotivated, not persistent, not responsible, need structure, low perceptual strengths or tactile, kinesthetic.

3. Programmed Learning Sequences.

These are sequences that give the students information, then ask questions about the information given, and then give the answers to the questions immediately. The micro-computers can do this type of work much more efficiently these days. These programmed learning sequences appeal to the students that are motivated, responsible, persistent, need structure, work alone, visual, auditory, and tactile.

Learning style helped me see that students need a lot of choice and so I developed all of the above categories of materials. The activity alternative/reporting alternative sheet (Table 1.4) is an example of what I used and developed during this learning period. I developed an activity alternative/reporting alternative for each concept found in the curriculum.

WHOLE NUMBERS-ROUNDING		
	Activity Alternative	Reporting Alternative
A	1. Create a tape which explains how to round whole numbers and decimal numbers.	1. Have two or three students play the tape.
A	2. Record news items from the radio and T.V. that give examples of rounding and approximation.	2. Have two or three students listen to your tape and also give the teacher your tape.
VT	3. Create a computer program that would demonstrate a knowledge of rounding in both whole and decimal numbers.	3. Have two or three students run your program.
VT	4. Create an artistic picture of situations where approximation is permissible to use and exact is necessary to use.	4. Share picture with two or three students and with math teacher.
VT	5. Create an electroboard which illustrates whole number rounding and then another one showing decimal rounding.	5. Have two or three students work on your electroboard.
VTK	6. Science uses rounding in many situations. Can you find as many examples as possible that would demonstrate rounding and approximation in the field of Science. Display in some creative form.	6. Display your results in the math classroom.
TK	7. Using a wooden cube or dice create a game showing rounding of whole and decimal numbers.	7. Have two or three students play your game.
K	8. Make a display (bulletin board or poster) of examples of rounding found in everyday life.	8. Display in your math classroom.
[A= Auditory; VT= Visual/Tactile; TK= Tactile/Kinesthetic; etc]		

Table 1.4

Several factors moved me away from the individual program approach to teaching. The most significant were:

1. I was exhausted trying to keep records of 160 individual programs.
2. The expertise that I had developed in the field of mathematics was often not utilized.
3. Mediocre work was being done or the 'easy way out' attitude was taken by many students because of time constraints and my limited time and involvement in each learner's individual work.
4. The students were not necessarily learning mathematics in a more meaningful way. There was still a focus on drill and practice underlying the individualized program. So the learning outcome of developing mathematical thinking was still not surfacing in the way I wanted to see it happen.

I learned many positive things about learning using this approach. The most important is that each individual will solve the problem or activity in their own unique way. This observation and my own experience at solving problems moved me into the next phase of my teaching, that of problem solving.

IV. PROBLEM SOLVING PERIOD

My focus for the next 5 years to the present has been problem solving. I shifted to a problem solving focus after I was exposed to a process writing program that seemed to be a way of approaching the teaching of mathematics in a more open-ended fashion than I had been doing. The shift to a problem solving focus also turned out to be one that I felt most comfortable with, perhaps because it seemed to be reflected in my personal learning approach, and this eventually allowed me to adapt to it in a more natural way. As a child growing up in a small town in Saskatchewan, I had to set farm machinery together in my father's business and generally would use a 'trial and error' approach. This now seems to be related to how I perceived learning of mathematics and now wanted to teach it. I was now beginning to find a teaching approach that more reflected my underlying personality and so I started to dedicate myself to develop it from this personal perspective. My first venture into achieving this was to conduct an action research on the use of writing in the mathematics class. At this point, having started a graduate program in mathematics education and using the ideas taught about action research, I did an independent study for a graduate course.

The investigation examined the relationship of problem solving and the writing strategy that is used by the Language Arts teachers in the Calgary Public School System. This writing format is done in five stages. These are: 1) Pre-Writing, 2) Writing, 3) Conferencing, 4) Revision, 5) Publishing. I adapted these five categories to teach mathematics in a problem solving context and adjusted them as deemed necessary to reflect the mathematics situation. The outcome of this adjustment is the following:

<u>Process Writing Format Adjusted for Mathematics</u>

1. Pre-writing stage.
    a) Understanding the problem.
        - words/ meaning
        - key words/ symbols
        - draw diagram
        - check assumptions
        - interpret pictures, graphs, charts or tables
    b) Specialize.
    c) Find examples.
    d) Organize examples.
    e) Make tables and charts.
    f) List patterns..
    g) Strategies.
        - start from simple to complex
        - guess and check
        - what type of operations
        - use manipulatives
        - works backwards
        - algebraic equations
        - logic or reasoning
        - recognize limits and eliminate possibilities

2. Writing stage.
    a) Guess or conjectures.
    b) Pattern finding.
    c) Find the rule or formula.

3. Conferencing stage (groups of 2 or 3).
    a) Check examples, specialization, charts and patterns.
    b) Look at each others' ideas, rules and conjectures.
    c) Come up with new ideas.

4. Revision stage.

a) Clarify ideas.
b) Evaluate ideas.
c) Rewrite conjectures, patterns and rules.
d) Re-organize specialization.

5. Publication stage.
   a) Place ideas, conjectures and answers in a finished form.
   b) Write a conclusion.
   c) Create new problems.

The application of this process revolved around non-routine mathematical problems and activities that led the students to discover mathematical patterns and concepts and a better understanding of mathematics. When I concluded this study I realized that this approach used a format that did not necessarily work for all the type of mathematical problems encountered in the curriculum and was similar to other problem solving methods, the only difference being the conferencing stage. I decided to change my categories to reflect a more open format that allowed for greater flexibility in solving most mathematical problems. I eventually ended up with 3 categories: Exploration, Confrontation, and Publication. I found that the stage of confrontation (conferencing) became a very important step in the process of solving a problem and this step is ignored by most other methods. I will elaborate on these categories later because they will be used as a basis of framing the problem solving approach to use the activities compiled in the thesis. But, for now, the overall process consists of: 1) Identifying the problem (this could be their own or one that I have given to them), 2) Understanding the question involved (its limits and assumptions) and stating this in their own terms either orally or in writing, 3) Reviewing all the information already known, 4) Exploring and gathering examples and specialization, 5) organizing of the data into tables or charts, 6) Reflecting on the data (analyzing, interpreting, conjecturing, finding patterns and rules, revising and collaborating with other students or the teacher), 7) Publication of their findings. I will use an example from my teaching to illustrate how I teach in this problem solving context.
THE PROBLEM: At a Junior High School there are 1000 students and 1000 lockers. The lockers are numbered in order from 1 to 1000. A student entered the building and opened every locker. A second student closed every even numbered locker. A third student changed every third locker, closing those that were open and opening those that were closed. A fourth student changed every fourth locker, and so on. This continued until all

1000 students passed through the locker room. What was the position of locker # 1000? Which lockers are open?

Teaching process: Students were placed into groups of 2, 3, or 4. They choose their own grouping. Each group was then given the locker problem. They were asked to read it over. I then asked if they understood the problem. Several hands went up. Two students already offered the answer. "The thousandth locker" and " Every odd numbered Locker." I replied, "I am not telling you if you are right or wrong, I want you to check out your answer to see if it is correct." I then asked if they all understood the problem and if anyone had any questions. Two groups wanted me to explain how the lockers were opened by the students. After that was done they went to work.

The majority of the groups had lots of discussion. Some of the ideas that were tried were: 1. Randomly come up with some numbers and then they would check to see if that locker was open or closed. 2. Simplify the problem to a smaller number of lockers. 3. Try ideas; every even numbered locker, every odd numbered locker, multiples of five, etc. This took a whole period, so I asked the students to think about the problem over night and come back the next day with their ideas.

The next day, I decided to work through the problem with the whole class. I first went over the problem again and then asked the groups to give me ideas on how to solve the problem. The first suggestion was to simplify the problem. So we systematically worked through the first twenty lockers, listing each locker that was open. When we completed the twenty lockers, I then posed the question, " Can you tell me the next number in this sequence?" Several students gave me the answer '25'. I asked them how they got that and they replied, " Between 1 and 4 there is a difference of 3, and between 4 and 9 there is a difference of 5, and between 9 and 16 there is a difference of 7, therefore, add 9 to 16 and you get 25, etc. The whole class could see this pattern and so I said, " Can you tell me if 850 will be in this sequence?" The learners then replied that they couldn't tell for sure, but if they followed the pattern of odd differences they then would be able to figure it out. I pointed out to the students that this was an elementary way of coming up with the next number in the sequence. An example that clarified what I meant by an elementary pattern was by asking the learners whether 103 follows 102? They all said 'yes' to that and I asked how many of them counted to 103 in order to discover whether 103 followed 102? They all replied 'no' to that and proceeded to tell me that they simply added 1 to 102 and got 103. They could see the difference so I then returned to the locker problem and asked them if they could find the pattern without using the adding method? There was a long silence while the students thought about this. After about three minutes (to the students this seemed like an eternity), a student raised his hand and told the class that they were square numbers. I then asked if locker # 1000 was open or closed and the reply came back, "only if locker 1000 is a square number?" We then used our calculators and the square root key and found out that it was not a square number.

Since I worked through the problem with the learners to show them how to gather and record information, look at the patterns formed and see the more powerful way of finding the next number in the pattern or whether a number actually fit in the pattern, and I did the revision or confrontation stage with them while we were working, they were told to write up the solution to the problem in a published form using the best information, conjecture, pattern, rules for finding any number in the pattern, and conclusions. I also asked them to describe how the problem was solved and to mention any area of interest that they would like to pursue or any area of concern about the problem as it was solved. When they handed in the published form, I read through them and the next day I selected the best write-ups and had the students read them. We then had a discussion about why these were the best and the types of information that should be included in their write-up. Along with the 'who discovered it' section on a bulletin board, I have a section that is reserved for excellent write-ups so that the learners may refer to these write-ups for guidance when they are asked to do this for another activity.

Some of the factors that have shaped my problem solving approach evolved from my graduate studies. These can be categorized in the context of education in general and problem solving in particular. In the former category, for example, I learned from Bruner (1966) that:

> a theory of instruction seeks to take account of the fact that a curriculum reflects not only the nature of knowledge itself (the specific capabilities) but also the nature of the knower and of the knowledge getting process. . . . To instruct someone in these disciplines is not a matter of getting him to commit results to mind. Rather it is to teach him to participate in the process that makes this possible- the establishment of knowledge. We teach a subject not to produce little living libraries on that subject but rather to get a student to think mathematically for himself, to conserve matters as a historian does, to take part in the process of knowledge getting. Knowing is a process, not a product. (1966, p.59-68)

I now believe that curriculum and instruction go 'hand in hand'. William Schubert summarizes this position as follows:

> One of the most recent positions to emerge on the curriculum horizon is to emphasis the verb form of curriculum, namely, currere. Instead of taking its interpretation from the race course etymology of curriculum, currere refers to the running of the race and emphsizes the individual's own capacity to reconceptualize his or her autobiography. The individual seeks meaning amid the swirl of present events, moves historically into his or her own past to recover and reconstitute origins and imagines and creates possible directions for his or her own future. Based on the sharing of autobiographical accounts with others who strive for similar understanding, the curriculum be comes a reconceiving of one's perspective on life. It also becomes a social process whereby individuals come to greater understanding of themselves, others, and the world through mutual reconceptualization. The curriculum is the interpretation of lived experiences. (1986, p. 33)

Finally, I learned of the constructivist perspective of how knowledge is developed. This concept of constructivism that I belief in is that knowledge does not exist outside of a person (Confrey, 1990; Piaget & Inhelder, 1969; von Glaserfeld, 1987a, b). True knowledge can only exist when it is constructed within the mind of a cognizing being. Understanding of any event, situation, or problem occurs only when relationships are made to existing understanding in a learner's mind (Cobb & Steffe, 1983). A person receives information through his/her senses (hearing, seeing, touching, tasting, smelling, manipulating, and observing) and this is at a perceptual level. Skemp (1978) describes this learning as instrumental understanding. The smallest of insects have an ability to perceive through their senses, but man is the only being that has an ability to conceive or create concepts. Skemp (1978) calls this kind of conception, relational understanding. This making sense of information and experience is the construction of knowledge. David Merrill (1991) gives me a good summary of this constructivist framework:

- knowledge is constructed from experience;
- learning results from a personal interpretation of knowledge;
- learning is an active process in which meaning is developed on the basis of experience;
- learning is collaborative with meaning negotiated from multiple perspectives;
- learning should occur (or be 'situated') in realistic settings; and

- testing should be integrated into the task, not a separate activity.

The influences from problem solving literature include Mason (1987), Polya (1963), Silver (1985), Meyers (1986), Ford (1990). This shift to problem solving reflects my current belief in a constructivist's perspective of learning (Cobb, 1988; Confrey, 1985, 1990; Davis, Maher, & Noddings, 1990; Duckworth, 1987; Fosnot, 1989; Lampert, 1988a; Simon, 1986). This perspective will be elaborated on in a later chapter, but for now, it proposes that knowledge is actively constructed by the learner and not passively received. As a result of accepting this view of learning, I have come to believe that students need to be taught how to learn. If we want them to construct their knowledge, we have to demonstrate to the students a proper way to do this construction.

Problem-solving does not just happen in the mathematics classroom, it is a process that must be learnt. Students need to be guided through the process with care and patience. They need lots of encouragement during the beginning stages and as they see that the process works they will gain confidence and courage to try the problems on their own. The process needs to be taught and demonstrated at each stage.

> Studies of students' difficulties in learning to write reveal that simply asking students to write, or giving them opportunities to write, does not produce better writers. Researchers have found that guiding students through a writing process is the key to producing better writers and better thinkers. The task of developing problem solving ability is similar because problem solving, like writing, is a process. Simply asking students to solve problems, or presenting opportunities for students to solve problems, does not produce better problem solvers. Guiding students through the processes of problem solving is essential if they are to become better problem solvers (Graves, 1983; Calkins, 1986 as cited in Ford,1990).

Recently, I have been trying to do this in my teaching of problem solving. At the beginning of the year I usually take one or two problems or activities and with the help of the students move through each of the steps of my problem solving process process. "Real teaching" begins with a clear presentation of the process (Schoenfeld 1987a). Throughout this process I have found that I am like a coach, helping them to analyze but not giving them the answers. Whenever they are stuck and ask me for the answer, I find that I am turning the questions back on them (Langer 1987) or taking them back to the point where there was understanding and right thinking and having them do their own thinking at that point. I want them to do their own reflection and find their own meaning.

This process of research, reflection and constructing their own knowledge is similar to Duckworth's (1987) action research, Schon's (1983) reflection and Connelly & Clandinin's (1988) autobiographical writing and storying. I want the students to do action research that moves from one mathematical problem to the next. I want them to gather information and evidence to support their conjectures and then finally construct their own knowledge. I see reflecting on the evidence through storying and collaborating together with others as important elements in constructing meaning. This is an area of teacher practice that needs to be expanded and revised. This means that communication through talking and writing (Skemp,1982) will become common occurrences in my classroom. The students' use of language and symbols as they relate to mathematics will be sharpened as they learn to communicate more effectively. I want to use whole class and small group discussions to help clarify, refine, correct and generate new ideas. I also want the students to be able to express their ideas orally, in writing, and with physical materials, pictures, and diagrams. I also want them to realize that representing, discussing, listening, writing, and reading are all vital aspects of the use and study of mathematics. The teacher's role is crucial in the timing of the discussions and in allowing for the students to exchange their ideas with one another.

In this process I have to become a very integral part of the communication, in that, I have to listen critically (Malave, Howlett & Collins, 1993) to the 'meanings' that students are creating (Steffe, 1988). I must be aware that "the student can generate a structural or conceptually ordered representation of the relations among the parts of the information to be learned and between this information or these ideas and one's knowledge base and experiences" (Wittrock, 1986, p. 308). These structures or schemata as they are frequently called can be thought of as one's beliefs, understandings, and explanations or the basis of a person's world view and actions. As Underhill & Jaworski (1991) put it "knowing is believing is simply another way of restating the constructivist point of view that reality is what we believe it is. When we say we know, it is a reflection that we believe that our beliefs are correct."

In this perspective I also have to be aware that often the mental constructions of the students will not be in accord with those of the mathematics community or those found in textbooks. There may be misconceptions, alternative conceptions and sometimes intuitive conceptions. This is where the fun begins in teaching. I will have to fully understand the mathematical concepts involved in the experience or activity, so that I can diagnose the type of conceptualization going on in the student's head. Cobb and Steffe (1983) point out that we must attempt "to 'see' both their own and the children's actions from the children's point of view". If it is a misconception then an alternative experience will need to be given to the student. If it is an alternative conception then it can be shared with others as an interesting way of looking at the concept or collaboration with other students and teacher to bring it more in line with standard mathematical thought. If it is an intuitive conception then verification and proof should be sought by the student through some experience or activity. As Underhill (1991) points out, " it is easier to develop a belief when one does not exist than to alter one once formed."

Finally, questioning becomes a very important aspect of this whole communication process. Some areas of questioning that I reflected upon and used throughout the process are:

- questions that point a direction for the students to proceed,
- questions that help a student recognize information that is readily available,
- questions that direct the student to relevant aspects of the problem,
- questions that ask value judgments,
- questions that are open-ended in nature, to encourage the students to generate new ideas, possibilities, and new areas of study (Wertheimer 1959).

This whole social process is a joint effort constructed by the teacher's and student's interpretation of each other's actions and intentions in the light of their own agendas (Mehan 1978).

Although I am emphasizing a problem solving approach, and am convinced that it is the best way to teach mathematics, my teaching today is not dominated by it. There are a few reasons for this.

(1) I still believe that attention should be given to traditional skills and practice. Core knowledge proponents believe, and I agree with them, that "it is important to begin building foundations of knowledge in the early grades because that is when children are most receptive, and because academic deficiencies in the first six grades can permanently impair the quality of later schooling." (Hirsch 1993). Hirsch also believes that, "a coherent approach to specific content enhances students' critical thinking and higher-order think-

ing skills." They use the constructivist psychology of the learning process and believe that having the prior knowledge helps students ask better questions which leads to understanding the new knowledge. They also believe that just as learning is cumulative, so are learning deficiencies. The modern classroom with 25 to 30 unique students in each prevents complete individualized instruction, so therefore, to help everyone receive the same level of education, core knowledge curriculum should be implemented in each school.

I have always felt that students that had a broad general knowledge in all subjects and had memorized the basic facts in mathematics, seemed to make better connections than other students who didn't have these skills. The argument that always stopped me from fully implementing this point of view- memorize as much knowledge as possible- was that, " knowledge changes so rapidly in our fast-changing world that we need not get bogged down with 'mere information', and it is more important to learn 'accessing' skills." I realize now, after some reflection and looking at the historical development of geometry, that the fundamentals of science, mathematics, history and geography change very slowly. A wide range of this stable, fundamental knowledge is the key to rapid adaptation and the learning of new skills.

(2) I am still influenced by the individual learning style approach and continue to use activities like the example given in table 1.4, i.e., I still try to integrate choice of activities by the students using their own learning styles.

(3) I have to teach in a context that is not changing fast enough to facilitate a problem solving teaching approach. Some of the sources of conflict in this context are:

1. Standardized testing that measures computational skills, not higher-order thinking.

2. Not enough time for students to construct their own meaning and understanding.

3. Parents that want their children to learn the same procedures, formulas and skills the way they learned them (even though most of them hated it). 4. Students, who when given the opportunity to construct meaning, in a more relaxed and uncontrolled environment, seek to disrupt the learning for themselves as well as for others.

5. Administration that requires classrooms where students are silent and working at seatwork instead of providing support in time, feedback, recognition and the means to do the teaching properly.

6. Students that say, " Just tell me how to do it or what formula should I use to get the right answer?" Many students do not view themselves as being participants in producing

mathematical results in the mathematics classroom and soon rely on the mathematics teacher, the textbook, or other sources of information, as the authority of the mathematics that they are supposed to learn. They knew that the teacher would eventually give them the basic principles or formulas, so why bother to actively search it out for themself.

Many days I have given in to these pressures and reverted to safe traditional practices with little satisfaction to myself and the students that I seek to teach.

In this chapter I did not make any reference to geometry in my teaching. This is because before this thesis, I had not consciously thought of the role of geometry in teaching in a problem solving context, although I always used it. It was in conceptualizing this thesis that I became overtly aware of its importance and I considered it a natural focus for what I wanted to do. Consequently, I decided to conduct a literature review on geometry to become more knowledgeable about its history and the role it can play in teaching mathematics. A short account of this is presented in the next chapter.

CHAPTER 3
GEOMETRY AS MATHEMATICS AND A TOOL FOR LEARNING MATHEMATICS

Based on my teaching experience and my understanding of the nature of mathematics, I believe that geometry is the great untapped source of ideas, processes and attitudes and can be challenging, exciting, rewarding, thought-provoking, and motivational in the teaching of all the mathematical concepts taught in our school systems across our nation. In this chapter I will discuss the nature of geometry that makes it suitable as a basis of teaching other concepts in mathematics. I will begin with a historical overview of geometry, then focus on some specific ways in which geometric ideas can be viewed as powerful tools in teaching mathematics.

I. WHAT IS GEOMETRY?

Geometry is being used in this thesis to refer to any spatial ideas or concepts embodied in natural and human-made objects or events. The ideas and concepts found in the study of space experiences come directly from Euclidean geometry. The main ideas being used will be point, line, rays, angles, and various shapes (plane and 3-dimensional). The geometric ideas found in the activities will be encountered in a natural or representational manner as experienced in everyday life. These geometric ideas will be used as a bridge to understanding the rest of mathematics. The geometric relationships will be used to link the visual, experiential, and everyday objects and events (natural and human-made) to the abstract numerical mathematics taught today in our educational system.

II. A HISTORICAL VIEW OF GEOMETRY

An important area that the teaching and learning of geometry fills, is that of our cultural or historical heritage. Many of the developments in our culture are directly or indirectly related to historical discoveries found in geometry.

In its earliest form, geometry, as developed by the ancient Babylonians and Egyptians, was concerned with the measurement of physical objects, particularly measuring objects for their length, area and volume. During this time mathematics was characterized by an empirical approach and generally asked the question "how", rather than developing a theoretical or systematic mathematics (O'Daffer & Clemens 1977, 9). Also early inhabitants of the earth in observing the world around them began to abstract the geometric ideas and draw pictures to represent them. Later they named these shapes, defined them and used them to communicate the relationships between the various abstract ideas.

Many great contributors to mathematics lived in the sixth century B.C. in Greece. During this age of rationalism the burning question was "why". The first great mathematician during this time was Thales of Miletus (640-546 B.C.) who formulated the abstract idea of physical space. He also used proof by deduction and collected many geometric facts and proofs (Smith 1923, 64).

The next great mathematician was Pythagoras (about 548-495 B.C.), a pupil of Thales, who organized a society of mathematicians called the Order of the Pythagoreans. Their scientific-philosophical-religious speculations led to many discoveries in geometry. The most famous were the Pythagorean Theorem, triangular numbers, space filling figures and regular solids (Williams & Mazzagatti 1986, 377-78; Calinger 1982, 50-51).

The next great mathematician during this time was Plato (431-351 B.C.). He believed that geometry became a branch of mathematics only when it was transformed into the study of mathematical space, a conceptual structure obtained through the idealization of physical space. This transformation took place with the development of the philosophy of Plato. Plato believed that their were three different types of being. The first is the idea, which is immutable, uncreated, and indestructible. The second is the object of sense perception, a mere imitation of the pattern. It is created, always in motion, and perishable. The third is space, the receptacle of all generation. Plato also developed deductive reasoning (Calinger 1982, 63-74) which was developed to a high level by Euclid (about 300 B.C.) in his book, *The Elements* (consisting of thirteen books and was a standard textbook for 2000 years (Calinger 1982, 105-22; Eves 1953, 111-122). This deductive system begins with certain undefined terms, and certain axioms about them and then on this foundation, defines all other terms used, and proves all other propositions that are asserted. Euclid is considered the greatest compiler of mathematics and has influenced the course of mathematics for over 2000 years.

Two other mathematicians during this time that developed some areas in geometry were: Archimedes (287 B.C.-212 B.C.) who examined the area of figures bounded by curved lines using essentially the theory of limits and also discovered ways of computing PI (Calinger 1982, 123-41) and Apollonius (246 B.C.-174 B.C.) who specialized in conic sections-ellipse, parabola, hyperbola, and circle (Boyer 1968, 157-74).

The field of geometry took a dramatic turn when Rene Descartes (1596-1650) and Pierre Fermat (1601-1665) developed a way of expressing all geometric relationships as relationships using numbers, thus making it possible to solve geometry problems by the techniques of arithmetic and algebra. This field of mathematics is known as analytic geometry. They also laid the foundation for the discovery of calculus (Neuman 1956, 129-33).

New perspectives and various branches of geometry continued during this time. Pascal (1623-1662), studied the cycloid and worked on projective geometry (Calinger 1982, 307-13). Newton (1642-1727) and Leibniz (1646-1716) independently invented calculus, which is the calculation of certain areas and volumes under curved surfaces (Goldberg 1987, 711-14; Boyer 1968, 437). Leonhard Euler (1707-1783) discovered a formula that relates to each other the number of vertices, the number of faces, and the number of edges of a simple polyhedron. Monge (1746-1818) united the study of calculus and geometry into differential geometry (Struik 1967, 145-47)

Problems with Euclidean geometry began surfacing with mathematicians like Gauss (1777-1855); Bolyai (1802-1860); and Lobachevsky (1793-1856) who independently discovered non-Euclidean geometry (Boyer 1968, 544-88). Riemann (1826-1866) worked on the elliptic non-Euclidean geometry that became the basis for Einstein's general theory of relativity (Calinger 1982, 524-25).

Other branches and programs of mathematics relating to geometry that began in this century were developed by Klein (1849-1925) who described the Erlanger program which is a way of characterizing various geometries using those properties of figures that remain invariant under a particular group of transformations (Boyer 1968, 592) and Cantor (1845-1918) who laid the foundation for topology (Calinger 1982, 591).Hausdorff (1868-1942) published a book that describes topology as a separate area of mathematics.

This area of mathematics is being worked on to the present time (Boyer 1968, 666-69).

III. GEOMETRY IN SCHOOL MATHEMATICS

P. M. van Hiele, studying students' mental development in geometry, identified five levels of thinking that a student passes through, assisted by appropriate instruction (Wirszup, 1976), they are:

Level O: (recognition). The student identifies, names, compares, and operates on geometric figures (e.g., triangles, angles, intersecting or parallel lines) according to their appearance.

Level 1: (analysis). The student analyzes figures in terms of their components and relationship among components and discovers properties/rules of a class of shapes empirically (e.g., by folding measuring, using a grid or diagram) and thinks of a shape in terms of its properties.

Level 2: (logical ordering). The student logically interrelates previously discovered properties/rules by giving or following informal arguments, but does not appreciate the role of deduction.

Level 3: (deduction). The students proves theorems deductively, establishes interrela tionships among networks of theorems, and understands the role of postulates, axoms, theorems, and proof.

Level 4: (rigor). The student establishes theorems in different postulational systems and analyzes/compares these systems (e.g., Euclidean and non-Euclidean geomtries).

This description, although by no means complete, provides a helpful framework for discussing the geometry curriculum as usually taught. The first three levels are usually taught in the elementary and junior high schools. The majority of activities are taught informally and usually involving manipulatives or concrete materials. During the construction skills, students will use a variety of tool such as, rulers, compasses, Miras, and protractors. The geometry program usually progresses from facts and skills of construction to generalizations and theorems. The higher level skills of deduction and proof are not taught in most high schools today. When geometry is taught in this fashion it is viewed as another unit in "school mathematics" that has very little connection to the real world. Too often mathematics has been viewed by teachers, students, and the general public as simply the study of computational skills, definition of terminology, formulas to find the correct answer, data management problems, and a few geometric plane and 3-D figures to create, and on which to do measurement calculations . The problem with this view is that it makes mathematics into a skill rather than practical, real-life knowledge. The relevance to a "lived experience" (Schubert, 1986) is lost in the sea of individual concepts and formulated computational skills.

IV. Importance of Teaching Geometry

It is important to teach geometry for several reasons. Some of these are:

1. Appreciation of Nature. We regularly see geometric forms and shapes in both natural and manufactured objects. Students should learn and appreciate both the aesthetic and practical reasons for these forms. The aesthetics can be learned by going on nature walks, visiting museums, and studying pictures of objects. The world that we live in is immersed with geometric form and structure, such as, seashells, sunflowers, honeycombs, snowflakes, stars, spiderwebs, etc. We as humans are able to create and admire beautiful objects. The ideas found in geometry are used to create paintings, sculptures, and other art objects. Leisure time of setting together of a puzzle provides satisfaction and pleasure when completed.
2. Geometry is also an important aid in communication. Many terms found in geometry are used in our speaking and writing, such as; line, point, parallel, perpendicular, square, circle, etc., and these terms help us to communicate to others location, size, and shape of places and objects.
3. Geometry provides a means for developing thinking, problem-solving skills and solving real-life problems. Geometry lends itself to effective development of thinking and learning in humans. Piaget's developmental psychology (Flavell,1963), and Bruner's symbolic stages (Bruner 1966) support the idea that human's action on objects is a perquisite to effective learning. Geometry ideas progress from the simplest forms of understanding to the complex encounters with shapes and ideas, thus providing for continuity and development of thinking and learning. Claude Janvier (1990) in his paper on *Contextualization and Mathematics for All* says that "efficient problem solvers rely on

the context of the problem and the fundamental relationships implicit in the problem to drive their reasoning rather than on transforming the relationships to equations .and then manipulating the equations."Janvier (1990) goes on to say that "there is a need to introduce contexts in the classrooms" and not just to applying mathematics.

Many of the excellent problems that develop mathematical thinking use geometric ideas as their basis. From history one will notice that the majority of mathematical problems that were solved came from the area of geometry, and that many of the present problems that students, people in the work force and at home seek to solve are usually geometric in nature. Using this context of geometric ideas allows for a lived experience, that will pose 'big questions' (Malave, Howlett & Collins, 1993) so that all children are engaged. The curriculum then opens to children's unique experiences, engages their imagination, and develops 'real knowledge' so as not to lose the children's connections with their own memories and histories. The many opportunities provided for students to compare, measure, guess and check, generalize, abstract, create, and imagine and think logically (Brumby,1982) abound in the field of geometry. Teaching geometry fits in with the research on the brain's two hemispheres (Wheatley et al. 1978), the left hemisphere involving logical, rule oriented, symbolic, and verbal types of thinking, and the right hemisphere involving spatial, intuitive, nonverbal, gestalt-type thinking. Using both hemispheres of the brain consistently, while developing mathematical thought, will help develop a balanced thinking person when practical problems arise in everyday life. Suydam (1985) implies that the teaching of geometry develops logical thinking abilities, develops spatial intuition about the real world, imparts the knowledge needed to study more mathematics, and teaches the reading and interpretation of mathematical arguments. Practical opportunities abound that use geometry in measurement situations around our home and work environments. We use linear, area and volume measurement when we hang pictures, paint and wallpaper walls, lay carpet and linoleum, plant a garden, buy loam for our flower gardens, buy cement for pouring a driveway, etc. We also use geometry skills when reading road maps. Grids and coordinate systems are use to locate places on city and country maps.

> Genuine learning involves interaction with the environment in such a way that what we experience becomes integrated into our system of meanings. Integration is some thing that we do ourselves: it is not done for us by others (Hopkins,1937), (Beane,1991).

It is interesting to note that many of the situations conducive to constructivist learning are geometric in nature and I will be describing these in a later chapter. These geometric activities are usually effective in arousing a student's curiosity and motivating the student to get involved in the learning. Many of the activities found in geometry are conducive to a 'hands on' or 'active' mode and therefore many of the reluctant learners can be drawn in to participate in an active, interesting way.

Another process skill that geometry develops in students is various ways of thinking. Geometric ideas allow for the following ways of thinking to be used. They are: comparing, contrasting, abstracting, inductive, deductive, intuitive, and creative. Two that are familiar to most teachers and students are the inductive thinking (formulating and testing conjectures), and deductive thinking (process of verification or proof of a generalization). Geometry is usually associated with the second or 'proof-oriented' thinking and therefore the inductive way becomes secondary in their minds. Geometry provides a fertile ground for intuitive and divergent thinking skills also. Asking oneself questions about a problem and creating a model or pictorial representation of an abstract situation is often helpful in solving the problem.

4. Activities in geometry are fun, add variety, and interest to any mathematics program. They help students acquire significant mathematical concepts and skills in both information processing, procedural and computational algorithms and problem solving. Geometry can be used to unify the entire mathematics program, in that, it is rich in visualization materials for the teaching of arithmetical, algebraic, statistical, and proportional concepts. Wheatley (1990) said, "Constructing new relationships and solving nonroutine problems are situations in which imagery is particularly valuable." Some example that I will be developing later are:

a) Arithmetical

The number line is useful in illustrating various number concepts and operations in integers, whole, natural, and rational numbers. Rectangular arrays are useful in developing concepts of multiplication, division, prime, composite, exponential, and odd and even numbers. Geometric shapes and regions are useful in developing meaning

(Fischbein,1987), equivalence, ordering and computing of fractions and decimal numbers. Linear, area and volume concepts are useful in measurement and computational skills. Student's "spatial capabilities frequently exceed their numerical skills, and tapping these strengths can foster an interest in mathematics and improve number understandings and skills."

(NCTM 1989, 48).

b) Algebraic

Coordinate geometry gives a visual picture of many abstract algebraic concepts and relationships.

c) Statistical

Data is depicted through bar, line or circle graphs that helps us to better understand and interpret the world of statistics and complex information.

d) Proportional

Scale drawings and rate pairs connect back and give visual pictures of rational numbers.

5. Geometry enhances student's spatial abilities (Young 1982; Battista, Wheatley, and Talsma 1982; Ben-Chaim, Lappan, and Houang 1989; Del Grande 1986). Two primary abilities that seem especially important in learning mathematics are spatial visualization and formal reasoning ability (Battista, Wheatley, & Talsma, 1982; Carpenter, 1980; Days, Wheatley, & Kulm, 1979; Gardner 1983, p. 8). We live in a visual world, surrounded by visual information but our understanding of the world is cognitive. As we receive this information mainly through our visual sense, we then begin processing it and later verbalization (written and talk) and categorization of the information into symbolic form begins to take place. "[Children's] spatial capabilities frequently exceed their numerical skills, and tapping these strengths can foster an interest in mathematics and improve number understandings and skills" (NCTM 1989, 48). Polya (1957) suggests asking the question, "Will a picture help?" Geometry provides the opportunity to represent the problem through a model or pictorial representation of many abstract situations that otherwise would be difficult to understand and to solve. Then language becomes important to help us define different aspects of the visual information.

> Language not only transmits, it creates or constitutes knowledge or "reality." Part of that reality is the stance that the language implies toward knowledge and reflection,

> and the generalized set of stances one negotiates creates in time a sense of one's self. Reflection and "distancing" are crucial aspects of achieving a sense of the range of possible stances-a metacognition step of huge import. The language of education is the language of culture creating, not of knowledge consuming or knowledge acquisition alone. (Bruner, 1986).

The third goal for students as found in the NCTM **Standards** document states: The students will learn to communicate mathematically.

> To express and expand their understanding of mathematical ideas, students need to learn the symbols and terms of mathematics. This goal is best accomplished in the context of problem solving that involves students in reading, writing, and talking in the language of mathematics. As students strive to communicate their ideas, they will learn to clarify, refine, and consolidate their thinking.

Polya (1963) so aptly summarized it for me:

> For efficient learning, an exploratory phase should precede the phase of verbalization and concept formation and, eventually, the material learnt should be merged in, and contribute to, the integral mental attitude of the learner.

There is also the aspect of the language of a visual expression, such as art and other representations of 3-dimensional space depicted in a 2-dimensional surface. This is not intuitive knowledge but something that requires learning and teaching.

Young advocates activities with geometric figures or tangrams in which students view objects from different perspectives, interpret drawings, generate new figures, and learn about flips, slides, and rotations to improve their spatial relations. Spatial ability has been identified by Gardner (1984) as one of the seven intelligences of the human mind. Many occupations require good spatial perception and as Moses (1980) showed, good problem solvers used spatial sense more than poor problem solvers. Seamstresses and clothes designers use the skills of laying out pattern pieces and then rotating and flipping them in order to make the best possible economical use of the materials being sewn. This type of skill can be taught in our schools with the use of tangrams (Dunkels 1990; Jamski 1989; Silverman 1990), jig-saw puzzles and other activities that allow for the filling of spaces. Many of us will face the task of putting together a piece of furniture, a bicycle, or a piece of play equipment for our children and all of these require spatial perception. We will all need to read a two-dimensional representation of a three-dimensional object sometime in our life. Floor plans for building a house or some other building are some examples of this type of activity. Manufacturing firms rely heavily on the use of blueprints. As we move away from procedural mathematics to a focus on connections and relationships, then spatial relations and spatial sense will take on increased importance (Skemp, 1978).

6. Geometry is a prerequisite for study in many other higher level courses and occupations. A course of study in higher level mathematics, that depends on the concepts of geometry to motivate and provide visualization of the situation, is calculus (Turner,1982). Geometry is also a prerequisite in other fields of study, such as, visual arts (artists such as Mondrian, Klee, Kandinsky and Miro used geometric form almost exclusively in their paintings), astronomy, physics, chemistry, biology, geology (structures of crystals displays symmetry in 3-dimensional figures), geography (maps and topology and the four-colour theorem), and mechanical engineering (where many scale drawings are needed). The skills developed in geometry also help in fields of study like, engineering, architecture and design, and the construction industry. Many jobs require mathematical computational skills which in today's society are being performed more accurately by technology and therefore the mathematical skills taught in geometry are useful in more practical ways and seen as necessary to keep options open for choosing any occupation. Some other occupations that require a good working knowledge of geometry are: Real estate agents (computational of the square footage of buildings), Insurance agents (insurance rates are calculated on the square footage of the buildings), Retail and wholesale firms (shelf and storage space for their goods), Pharmaceutical businesses (dispensing and counting of drugs, metric system knowledge).

V. Geometry as a Tool for Learning Mathematics

It is my belief that one can take the structure from the original construction of geometry and use the ideas found in that initial system to make mathematics easier to understand, to develop mathematical thinking, and to show the practicality of mathematics in real-life situations. Giving the students a sense of history and the development of geometry and mathematics, could help to remove the myth that the whole system of mathematics was dropped down ready-made from the skies, to be used and understood by only a select few.

Why have there been very few attempts at integrating all of mathematical thought around the one unifying theme of geometry? It seems so logical and sensible. As Veblen and Whitehead (1932) wrote " Any objective definition of geometry would probably include the whole of mathematics." Geometry is all around us and so enmeshed with all of mathematics and science that it is hard to isolate it as a simple school subject or as a single unit of study. As James Ulrich (1973) argues,

> In a backhanded way, geometry itself is at fault because it is so all-embracing, so ubiquitous, so important, and so capable of being interpreted in many different ways. Geometry can be taught intuitively or rigorously or by methods that lie somewhere between these extremes; to the learner, the study can be dull or exciting or some where in between. The mathematician whose speciality is algebra sees geometry as a vehicle for algebraic expression; if the mathematician's speciality is vectors, he believes that vectors should be used to interpret geometry. The theory-oriented mathematician views geometry as an excellent example of the group. The teacher who believes mathematics should be taught as it was developed historically by man will prefer Euclidean geometry; those who believe that mathematics education should exploit the most recent and modern advances in mathematics will favor an approach that features coordinates, vectors, or transformations.

These many points of view relating to content and method, the nature of geometry, the way it should be taught, the many ways that it can be interpreted and also its all-embracing nature, the fact that it is so ubiquitous, and its importance throughout history, give me reason to believe in integration. What is the missing ingredient for integration, and why did past attempts at integration fail? I would like to point out three programs that used the integrated approach to teaching mathematics and then discuss why I feel these attempts were not implemented by the majority of mathematics teachers.

V.1 ATTEMPTS AT INTEGRATION

The first, well thought-out program, was furnished by Felix Klein, the celebrated mathematician from Gottingen, Germany. In 1908 Klein gave a series of lectures and later published these lectures in three volumes, one called, *Elementary Mathematics from* *an Advanced Standpoint*, and two volumes on *Arithmetic, Algebra, Analysis and Geometry*. Klein used the phrase in his first book "fusion of arithmetic and geometry" indicating the need for viewing all mathematics as a whole. He was against separating mathematics into various disciplines, such as algebra, calculus, geometry, but was for seeing them as a "living interaction of the different branches of the science which have a common interest .... Each single branch should feel itself, in principle, as representing mathematics as a whole." (Klein, 1939). Klein pointed to the explosion of mathematics that was created when analytic geometry and algebra were shown to be related and he felt that that type of creation could go on in the minds of the students if all mathematics learning was treated as an "integrated, living organism" (Klein, 1939).

Another attempt that was made at integrating all of mathematics was done by John A. Swenson (1880-1944). He published five books that used many creative ways of teaching mathematics in an integrated manner. The integration appears to be only within the various branches of mathematics, ie. coordinate geometry with deductive proofs.

Another extensive integrated approach was written by a group of teachers over a period of several summers called the Secondary School Mathematics Curriculum Improvement Study (SSMCIS) (1968). The project was sponsored by Teachers College, Columbia University and directed by Howard F. Fehr and the series was called *Unified Modern Mathematics*. An important role was given to transformations, the coordinate plane and many different approaches were used throughout the level's published.

The remainder of the literature that I have reviewed has usually been of the nature where a single teacher has seen a relationship between a geometric idea and some other mathematical concept from a different unit of mathematics. An example of this type of relationship is the connection of geometric shapes divided into equal parts and then some of these parts being shaded to illustrate the concept of fractional parts.

V.2 Integrative versus Integration

The problem that I see with the above approaches to integration and why these integration approaches did not catch the interest of most mathematics teachers is the difference between integration and integrative curriculum. "Paul Dressel defines integrative curriculum as consisting of two parts. The first part provides learners with a unified view of commonly held knowledge. The second motivates and develops the learners' power to perceive and create new relationships for themselves" (Dressel, 1958, in Harter, & Gehrke, 1989). In the above approaches I see the first part accomplished, of providing the learners with a unified view of commonly held knowledge, but the second part of allowing the students to perceive and create new relationships is not apparent. The curriculum and the approach used is from the teacher's perspective and not from the perspective of the student. Herein lies the difference of what I am trying to promote in my thesis by providing examples of a perspective that can be used to teach all of mathematics using the unifying ideas in geometry with all of the other concepts in mathematics. The perspective I believe in is not merely from the teacher and curriculum but more important, from the perspective of the students, who they are, how they think and learn, how they have structured their world to this point in time, and what beliefs they presently hold as they are learning the ideas in mathematics. Thus my perspective is to use the integrative idea and present a unified view of the commonly held mathematical knowledge along with allowing the learners the power to perceive, make sense of and create new relationships from the activities and problems presented to them or they themselves have seen as problematic.

How do I see this integrative approach being worked out in my classroom? Figure 3.1 illustrates my conception using geometric shapes. I have used three concentric circles and within each circle a triangle. The triangle has always represented to me the idea of strength. In my diagram this idea holds true and illustrates where the strength is needed in education. The core of the illustration needs to be the strongest and therefore at the center I have placed the triangle which represents the learner. The other two triangles also illustrate where strength is needed. The second largest triangle is filled by the most significant people in the lives of the learner. They are the parents, teachers and other community resource people who help bring identification, belonging, and approval to the learner. The third triangle is where the on-going education of the learner is found. This area needs to be strong because as a person is exposed to learning experiences these experiences need to be of the kind that will impact the learner in positive, rather than negative ways. The context in which the learner grows and develops needs to be strong in order that the learner matures and begins serving others, as well as developing character and integrity for life situations that the learner may be exposed to that are not always easy to handle. Surrounding each of the triangles is a circle which has always represented to me the greatest amount of area for the least amount of boundary. The circle has also given me the idea of completeness and a continuous form. The three circular areas represent the areas of growth in relationships, service and character. The area contained in the circle allows for the learner freedom of movement, to a point, and a sense of security and protection. Growth in each of these areas allows for expansion in all directions as the learner chooses and as the learning experiences dictate. The concerns also expand from self to others and then to a global emphasis.

The methodology of constructivism that I will described in chapter four of this thesis, serves as an umbrella to developing a well-rounded learner where they can grow to their fullest potential, no matter what their perceived capabilities, and where all aspects of their personality-physical, mental, spiritual, and emotional-- can be realized. The activities and problems will all be chosen keeping in mind each individual's unique way of constructing their knowledge. These activities will be described in a following chapter of this thesis.

The discussion of geometry in this chapter extends beyond what will be accomplished in this thesis in terms of the activities that will be compiled. To keep the thesis manageable, the focus on geometry will be at times on its symbolism and at others on its concepts in integrating the teaching of Grade 7 mathematics. It will also be cast in situations that make use of its characteristics that enhance problem solving, thinking and reasoning, visualization and communication.

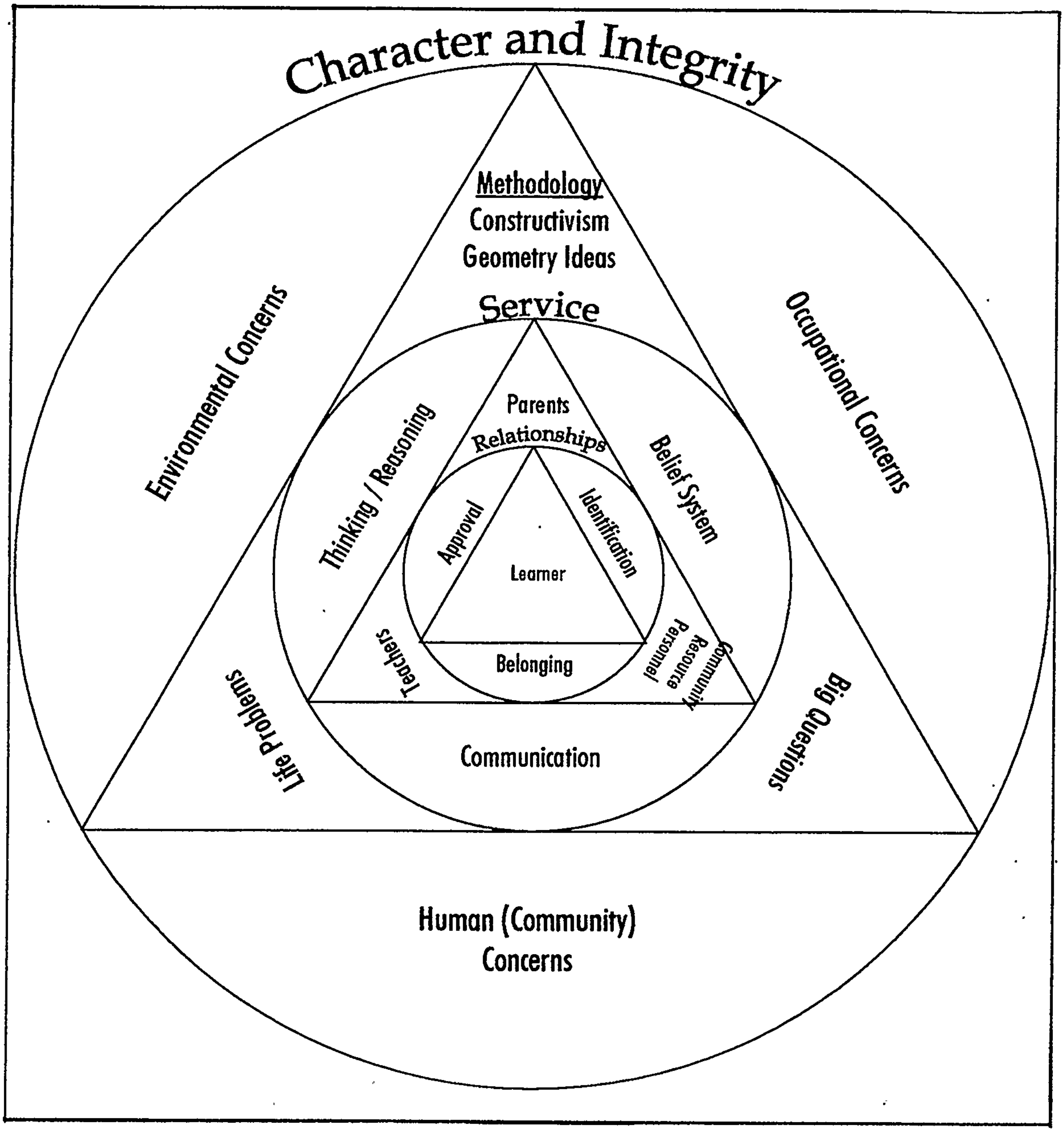

Character and Integrity
Methodology
Constructivism
Geometry Ideas
Service
Environmental Concerns
Occupational Concerns
Parents
Relationships
Thinking / Reasoning
Belief System
Approval
Identification
Learner
Teachers
Belonging
Community
Resource
Personnel
Life Problems
Big Questions
Communication
Human (Community)
Concerns

Figure 3.1

CHAPTER 4
PEDAGOGICAL PERSPECTIVE OF STUDY

In this chapter I will discuss some of the characteristics of the pedagogical perspective in which the study will evolve. These characteristics are those I consider to be best suited to foster the development of mathematical thinking and to form the basis of making the activities compiled in this thesis "non-traditional". The activities by themselves can be meaningless, it is in the way in which they are brought to life in the classroom that will make them traditional or non-traditional. This is why it is important for me to stress the pedagogical perspective that is most likely to allow the activities to satisfy the problem solving objective of the thesis.

The pedagogical perspective promoted here is one that is consistent with the National Council of Teachers of Mathematics Council Standards (1989) and reflects the current trend in mathematics education research of focusing on a constructivist perspective of teaching and learning mathematics. These ideas have been alluded to several times in preceding chapters. However, in this chapter, I will outline them as a way of establishing the formal framework in which the activities should be understood. The chapter will fall under the following categories: constructivist perspective of teaching, learning process, problem solving process, managerial organization of classroom and assessment.

I. CONSTRUCTIVIST PERSPECTIVE OF TEACHING

Constructivism is somewhat difficult to nail down because the label covers a wide spectrum of beliefs about cognition. In general, old style constructivists follow Piaget in emphasizing individual thinking and creation of meaning. This is the notion of constructivism familiar to most mathematics educators. Today constructivism includes social cognition theorists such as Bruner, Vygotsky and Ernest and radical constructivists such as von Glasersfeld and Polanyi.

The constructivist view of the world can inform teacher practice in very powerful ways. Some of these are as follows: Firstly, this shift in perspective from content or knowledge to learner, changes a teacher's methodology and practice. The teacher's most important task is now to understand the students' abilities, perceptions, understandings, meanings, motivation, and ways of looking at their experiences. We need to understand the thinking of the individual students. This requires careful listening and questioning skills. It also suggests sophisticated diagnostic and prescriptive tools to see the alternative ways of thinking. This also implies that the teacher understand the concepts being taught in an in-depth manner.

Secondly, the teacher also needs to understand the human character and its make-up so that a proper caring, learning environment can be cultivated. The environment must provide for psychological, emotional, intellectual and social safety for the individual student.

Thirdly, the practices should fit the diagnosis that takes place. This could allow for such practices as were found in the days of 'transmission' teaching (rote learning, drill and practice, memorization, telling and demonstrating). Some of the new (in a sense revived) practices of research, inquiry, integrated learning, community involvement, and discovery learning (Treffers 1987) could also be used when the experience demands it.

Fourthly, the area of teacher practice that needs to be expanded and revised is the area of students working together in consultation and collaboration. This means that communication through talking and writing will become common occurrences in every mathematics classroom. The teacher becomes an integral part of this communication process, in that, the teacher will have to listen critically to the 'meanings' that students are creating and also question, in the sense of pointing a direction for the students to proceed.

The possibilities of teacher practices from a constructivist perspective are limited only by the imagination and ingenuity of the teachers. However, in the context of this thesis, those stated here are important to create the appropriate circumstances to use the activities being compiled.

The constructivist perspective of teaching requires a focus on student needs. This need includes providing students with suitable activities to allow them to construct their knowledge. As Eleanor Duckworth (1964) in her book *Piaget Rediscovered* writes

> Good pedagogy must involve presenting the child with situations in which he himself experiments in the broadest sense of the term--trying things out to see what happens, manipulating things, manipulating symbols, posing questions and seeking his own answers, reconciling what he finds at one time with what he finds at another, comparing his findings with other children.

The following is a list of some of the students' characteristics that a teacher should consider in order to provide suitable activities and problems.

1) the students' perceptions, purposes, and ways of working things out.

2) the students' physical and cultural environments and how they interact with the various environments given to them. In the selection of the tasks I will need to understand what the students already know, and what will give them access to new concepts (Cohen et al., 1990; Shulman, 1986a, 1986b).

3) how each student thinks and therefore what types of activities and learning experiences will help them to best construct their knowledge.

These characteristics will be considered in selecting the activities for this thesis.

II. LEARNING PROCESS

Like many prominent educators, I believe that learning should be active (Davis & Hersh, 1980; Ernest, 1991; Lakatos, 1976; Lampert, 1988a; Scott-Hodgetts & Lerman, 1990; Tymoczko, 1986; Polya, 1981) and not passive. I have always loved to learn by doing and experimenting with materials and ideas. When I get actively involved in exploring and analyzing, I find that I then construct my own knowledge in a more meaningful way. I want my students to get actively involved in their learning and developing their own mathematical knowledge (Schifter & Fosnot 1993). Consequently, the process for learning that will be adopted when engaging students with the activities being compiled will be based on the following:

II.A. REFLECTION

The learning process often starts at a multi-sensory stage where data and information will be gathered through the five senses [hearing, seeing (reading, observing), touching (manipulating), tasting, smelling]. However this is only the beginning stage of learning and yet this is often where most education stops. Often the next step happens concurrently with the data gathering but it is under-emphasized, if emphasized at all. But here is where reflection should be emphasized.

Reflection is making sense of the information and data that was gathered. Reflection needs to be taught or at least time given for the students to relate the new information to what is already known, connecting the known to the unknown. In mathematics this is the stage where a student begins looking for patterns and forming conjectures. Piaget has demonstrated that students construct their own meanings and that their intellectual development is very gradual, therefore, the teacher's role is to develop an environment that provides for time. Time to reflect on ideas, try out new ideas (Mumme & Sheperd 1990, 20), and compare differing viewpoints.

II.B. Collaboration

Collaboration may occur at the same time as the the multi-sensory and reflective processes depending on the problem to be solved or the activity engaged in. Many researchers have done studies in collaboration and have said that in order to construct knowledge in an enriched fashion, cooperation and collaboration are necessary ingredients in this process (Yackel, Cobb, Wheatley, & Merkel, 1990; Johnson, Johnson, Holubec, & Roy, 1984; Davidson, 1990). This is where another person (teacher, peer) or group discuss the problem or activity, looking at the data gathered and looking for possible solutions and alternatives. They explain, clarify, elaborate, question, evaluate, justify, extend, and argue (Schifter & Fosnot 1993). The key to making sense is in this negotiation process. Vygotsky emphasized the need for social interaction and expert guidance within the *zone of proximal development* (Vygotsky, 1978). He also claimed that cognition is the internalization of social interaction (Vygotsky,1978; Schoenfeld, 1987b). Individual cognition is important but it should always be done in a social and cultural setting, and Bishop (1985) argues that successful communication requires the negotiation of meanings.

Skemp (1982) suggests that particularly in the early years we should "stay longer with spoken language. The connection between thought and spoken words are initially much stronger than those between thoughts and written words or symbols." Bruffee (1983) stresses that speech is important to writing and cooperative learning and conferring with others is essential to completing a written task. Vygotsky also highlighted dialogue (two way, interactive exchange between two speakers) in terms of human conversation, but today we are also seeing how technology is becoming a vital tool for interaction with students. Technology has helped to model and to simulate many mental and natural processes and these tools (computers, calculators, etc.) can provide both a context (Wheatley & Wheatley 1982) and support for meaningful problem solving activities (Cobb et. al. 1988). Skemp (1982) also suggested that the students use informal notation and symbols to express their meaning and then later the teacher should move them into the more formal language of mathematics. The concepts are more important at this initial stage of meaning-making than the formal notation. Skemp said that as the students see for themselves how lengthy, unclear, and the differences between individuals' ways of expressing the same concepts, that gradually there will be a move on the students part to express mathematical concepts in more powerful, convenient and socially accepted mathematical language.

Collaboration is often accompanied by sharing and, depending on the teacher and/or problem being solved, consensus. As the groups share their findings and reasoning with the rest of the class, new ideas and meaning can result for the other learners. It is important during this sharing time that the teacher provides psychological safety for each of the students so that students will be free to share their ideas without risk of harassment or embarrassment (Noddings 1984, 1989; Silver, 1985). S/he must communicate that it is human to make mistakes, that they are a natural and valuable part of the learning process because of the feedback they provide.

With respect to consensus, groups are expected to come to some agreement as to why and how the problem was solved. This could provides "equilibrium" (Piaget & Inhelder,1969) and "accommodation" to existing schemes, thus enriching and expanding existing constructs.

Some of the other useful outcomes of group work are:

1. Stronger groups and students helping the weaker groups and students.
2. Common or different strategies coming to light.
3. Students appreciating one another's opinion and class congeniality being formed.
4. Good examples of write-ups and answers being shared.
5. Students becoming teachers (Finkel & Monk 1983); having to explain and justify their reasons in clear and concise terms, resolving contradictions, verifying facts, and sometimes adjusting attitudes. This type of critical-thinking skill is best developed in an atmosphere of dialogue, debate, and problem solving, not merely by listening to a teacher that lectures (Meyers 1986).

6. Students formulating new questions which stimulate further investigation.
7. Students' perceptions being sharpened.

III. PROBLEM SOLVING PROCESS

In this section, I will elaborate on the problem solving approach I have used and will continue to use with the type of activities collected in this thesis. As noted in chapter 2, this approach follows three stages: exploration, confrontation, and publication. I will briefly describe each of these as a basis for providing an example of a specific context for using the activities. It should be noted that these stages may not always be rigidly adhered to, since each activity or problem usually will dictate a process to follow and/or individual learners may have their own method of development. The stages are as follows:

III.1. STAGE 1: EXPLORATION

The exploration stage has at least four phases.

III.1.A. UNDERSTANDING PHASE

In this phase the teacher introduces the activity with a story or along with the learners reads over the activity and then a brief discussion occurs around the following areas. The first is a discussion about what the activity is asking. Secondly, meanings of words, diagrams, and any information given is talked about until satisfactory understanding of the information is expressed by each of the learners present. Thirdly, the limits imposed by the activity and any that the teacher may see necessary- in order for the activity to be worked on- are discussed at this time. Fourthly, the assumptions that are made in the activity are addressed so that each learner is working with basically the same problem. This phase of the activity is usually done at the beginning but I have found that often throughout the activity discussions occur that relate to understanding, limits and assumptions. The teacher will need to discuss these as s/he sees fit, either with the whole class, groups or individuals.

III.1.B. INDUCTIVE PHASE

In this phase the learners will be collecting evidence or data in the form of examples, specialization, making diagrams, pictures, tables and charts. They will also be noticing and listing patterns, forming guesses and conjectures and then drawing conclusions from this information. In mathematics this is referred to as reasoning inductively and is similar to what scientists do as they experiment with nature and come up with some general laws of nature. This inductive phase is very important in developing mathematical ideas but it has a basic weakness, in that there may be more evidence and information that was not collected and that information may show that the conclusion drawn from the limited information was incorrect. Some examples of illusions and paradoxes that fool the senses have been seen by most learners. The teacher is cautioned during this phase to watch for over-generalization by the learners.

III.1.C. DEDUCTIVE PHASE.

During this phase the student is formulating rules and formulas that work for a general case. Deductive reasoning is a method of drawing conclusions by combining in a logical way other facts that we accept as true. The Greeks, particularly Euclid in 300 B.C., used this method to prove statements about geometric figures. Basically the learners at this phase- after finding the answer- will try to understand why it works. This usually involves the mathematical context that has produced these patterns.

III.1.D. CREATIVE PHASE

This phase is at work during the inductive and deductive phases but the reason why I have included it as a separate phase is that the learners may have seen some area of interest that was out of the limits or assumptions discussed at the beginning of the activity or problem and now the learner would like to pursue that 'rabbit trail'. Some very creative mathematics and unique discoveries have been made by mathematicians as they have followed these interests. This phase is where mathematical research begins.

III.2. STAGE 2: CONFRONTATION

This is the stage were learners move into groups or change groups and in these new-groups the learners check diagrams, examples, specialization, patterns, and rules for accuracy. They revise their work after careful discussion has taken place. This is also the stage when new ideas and new information are generated. This stage is similar to what the Language Arts teachers use when learners peer edit each others work or when teachers have individual interviews or conferences with learners about the writing that the learner has produced to this point in time.

III.3. STAGE 3: PUBLICATION

At this stage the learners will publish the best diagrams, examples, charts, patterns, conjectures, and rules/formulas in an organized form. They will show their conclusions and provide a brief description of how they solved the problem. This is the stage when reflection on the problem and solutions is encouraged and learners are asked to create new or similar problems and to research any area of mathematics that is connected or a tangent to the activity.

The published work could be handed in to the teacher for evaluation and assessment or it could be presented orally to the class. Since the learners are constructing and discovering their own mathematics, the first group or individual to discover the pattern, generalization or theorem is given credit by naming the constructed pattern, generalization or theorem after their own name or the group name that they have chosen. Instead of the Pythagorean theorem I would display " the Joe Smith theorem", and from that day forward we would refer to the Pythagorean theorem as the "Joe Smith" theorem. I would also tie their name to the historical mathematician that first discovered that particular theorem so that historical connections are made and appreciated.

IV. MANAGERIAL ORGANIZATION OF CLASSROOM

In this section, I will outline some of the factors a teacher needs to be aware of in terms of classroom organization and management of students and materials when using the type of activities being compiled. These are:

1. The classroom furniture needs to be easily changed to fit the size of the groups, or the type of activity being done. The teacher needs to be flexible in the set up of his/her classroom.

2. Materials. Materials or manipulatives are critical to the development of the student's thinking, and therefore proper management strategies need to be in place. Some areas of importance are:

a) Acquiring sufficient and good quality materials. Sharing intriguing materials that are in short supply can lead to conflicts among students. Materials that need constant fixing and replacing take up valuable teacher time.

b) Distributing the Materials: I have found that if I distribute the materials myself that sometimes the group that received them first are already finished and are waiting for the rest of the class to catch up. This can cause discipline problems. I now ask one person from each group to come and pick up the materials. Also, I have found that distributing intriguing materials prior to giving directions can sabotage my efforts with large groups, therefore, timing and locating of discussions and directions need careful thought and planning for each activity.

c) Collecting Materials: Giving time at the end of each class to collect materials must be structured and therefore, making one person in the group responsible for returning materials usually ensures that most material are returned.

d) Selecting Materials: As a facilitator of students' interaction with materials, I have the task of understanding, organizing, adapting, and creating materials that will fit each activity. The materials need to be rich enough for the simplest tasks but also lend themselves to making the students think deeply. This is a large assignment and requires a great deal of thought.

e) Exploring Materials: Whenever any new materials are introduced to the students a period of play time should be given to allow the students to explore and become familiar with the materials. This tends to cut down on teacher frustration.

f) Student Organization: One of the things that I have observed over my teaching career is the lack of organizational skills of the students. I assumed that if I was organized and that the students recognized this about me, that somehow this skill would be picked up by them. I now realize that it never happens for the majority of students and so an important component of my teaching in the last few years has been a way of helping students to organize their notebooks. This has taken on the form of a daily record sheet (appendix C) where the students record what has happened in the class each day. At the beginning of the year I have to remind them every day to make the entry but after a few weeks of this type of daily reminder it usually becomes an automatic routine for most of the students. In problem solving I show them how I organize my examples, ideas and whatever is involved in the exploration part of the problem or activity. This modeling has helped the students organize their work and now more students are seeing the patterns involved in the problems. I have also noticed that the ideas and conjectures flow more quickly when the data discovered by the students is in an organized form.

V. ASSESSMENT

Skemp (1980) has indicated that emotions may signal danger and an attack upon oneself. The learning experiences that are presented to us lead one to believe that demands are being made. When presented with a new learning experience, the demand is that a response be made, such as imitating it, understanding it, or working something out. The threat lies at the end of that process, because failure to satisfy the teacher's wishes or the learners own expectation can often have a negative emotional outcome! I believe that one of the ways to measure the success of a learning experience is to check for "emotional acceptability" by the learner and/or the teacher. Often math anxiety is described as both an emotional and a cognitive dread of mathematics (Hodges, 1983; Reyes, 1980; Seguin, 1984; Tobias, 1976, 1978).

Assessment cannot follow traditional methods of grading a final answer in the pedagogical perspective being promoted here. A more holistic approach to assessment is required. Such approaches are described in detail in the literature, for example, NCTM (1990) and O. Chapman (1992) and will not be discussed here. But to complete this chapter on pedagogy, I will make some general points about the nature of the assessment that is relevant when activities like those compiled in this thesis are used.

The areas that should be evaluated during the learning process are:

1. Skill development.

This could be done as the activity is in progress or through some diagnostic test.

2. Social development.

This would be done mainly through observation and feedback during the learning process.

3. Process and mathematical thinking development.

This would be done mainly through the following areas:

a) Questioning the students' thought process while in group setting or class setting.

b) Individual interview: Again, this should be on-going and happening during the active part of the activity.

c) Group or individual report: A student report is a written or audio-recorded retrospective report on a problem solving experience. The student is asked to think back over the problem solving experience and describe how s/he found the solution. The studentsares asked to tell or write about their thinking as they went through the problem. The writing component confirms that students who write down their thinking do retain the concepts better than those who do not write them down (Robertson & Miller 1988). This would involve a variety of areas depending on the activity or problem. Some things that would be looked at in the report are:

1. Showing an understanding of the problem.
2. Formulating and explaining a strategy.
3. Analysis.
4. Making clear explanations.
5. Systematic attack of the problem.
6. Formulating a general rule verbally or algebraically.
7. Looking at alternate solutions.
8. Seeing the pattern.
9. Correct solution.

In conclusion, assessment should be an on-going activity that determines whether the students have grasped what was needed to be learned and to point a direction for the next learning activity.

CHAPTER 5

METHODOLOGY IN COMPILING ACTIVITIES

I. INTRODUCTION

Given the objectives of the study, the research process involves a literature review of mathematics activities to arrive at a collection of mathematics activities involving geometry to teach Grade 7 mathematics. The study focuses on the Grade 7 curriculum for several reasons. The first reason comes from my own personal teaching career which involved teaching at the junior high level for 22 years. I would like to progressively build in the lives of the students the geometry concepts throughout their junior high experience. Secondly, the basic geometry ideas are introduced and reviewed in the Grade 7 curriculum. This seems to be a natural starting point for developing a mathematics curriculum based on geometry ideas. Thirdly, many of the good geometry activities that I had collected and used in previous years were used to teach other mathematical concepts found at the Grade 7 level. Fourthly, the nature of the Grade 7 students and the fact that Junior High seems to be a new beginning, a new school, and many new teachers. They usually have to adjust to new ways of learning, adapting, and trying new methods.

The Grade 7 mathematics curriculum consists of 9 strands: Problem Solving, Whole Numbers, Decimals, Fractions, Integers, Ratio and Proportion, Measurement and Geometry, Data Management, and Algebra. A summary of the curriculum is attached in Appendix A. Activities for all strands will be sought. The availability of these activities will determine the relative emphasis on each strand. The goal is to obtain as many activities as possible for each strand within the time constraints to complete this thesis. However, problem solving will be a part of all the activites because they will be framed in a problem solving context as described in earlier chapters.

II. SOURCES OF ACTIVITIES

The areas for locating and collecting the materials to be used in teaching the Grade 7 curriculum have come from the following sources:

II.1. JOURNALS AND ERIC DOCUMENTS

Using the ERIC retrieval system, I used the descriptor "Geometry" and asked that all references found in the ERIC system from its inception to the present be looked at. There was a total of 749 records with the Eric subject headings of Geometry. I then made a list of all the primary journals (those listed over and over) and all of the secondary journals (those only listed once or twice). The primary journals are: *!) School Science and Mathematics, 2) Arithmetic Teacher, 3) Mathematics Teacher, 4) Mathematics in Schools, 5) Phi Delta Kappa.* The secondary journals are: *1) Psychometrika, 2) Learning and Instruction, 3) Ohio Reading Teacher, 4) Journal for Research in Mathematics Education, 5) Journal of Experimental Child Psychology, 6) Science, 7) Educational Studies in Mathematics, 8) American Mathematical Monthly,9) Applied Psychological Measurement.* As I began going through each of the items found in the primary journals I noticed that often an article found in the same journal and having a geometry idea, was not listed in the Eric draw, so I decided to systematically go through each of the primary journals. I started with the oldest copy to the present copy of that journal and paged through from cover to cover scanning each of the articles and activities. Often articles that seemêd unrelated to my topic were useful to me in that they provided information about methodology, theory, assessment, and practical hints about the whole area of teaching and learning. The articles from secondary journals and Eric documents were looked at as suggested by the ERIC draw.

I also used journal materials that were distributed during my Graduate classes at the University of Calgary.

II.2. BOOKS

Three areas became my main source of books for this thesis. The first was the

Calgary Board of Education Professional Library. I went through the entire collection of mathematics books and texts found in the QA section of the library. If the title of the book indicated that there were some geometric ideas to be found in that book, I read the table of contents and then scanned any articles or activities found in that book. This again proved to be a valuable source for ideas in methodology, theory and activities.

The second source of books was the University of Calgary Professional Library. I found a group of older mathematics books that were being discarded and again found some valuable activities in these books. I also scanned the mathematics section and only looked at those books that were different from the ones found in the Calgary Board of Education collection. I went through the same process of collection as before.

A third source of books were those that I had collected over the 22 years of teaching mathematics and my faculty advisor's collection. I went through each of these books with the same system.

II.3. SELF-MADE ACTIVITIES AND SELF-ADAPTED ACTIVITIES FROM UNKNOWN SOURCES

Most of these self-made or adapted ideas were created through reading a teaching idea found in a textbook, curriculum document, a problem, or from students' difficulty in understanding a concept when taught in a traditional way (giving examples or procedures to follow and then students working on similar type questions). If the idea came from a textbook or curriculum document then I would try to put some flesh on the idea and make the idea into something visual or turn the idea into an investigation. I also realized that the majority of students' lack of understanding of abstract concepts is a problem of visualization and so I would try to make some type of activity that would help them to visualize the answer. An example of this type of problem arose when learners in Grade nine were trying to understand the value of multiplying the difference of perfect squares. They see very little value or a practical application to this type of question. I usually show them how two numbers can be multiplied using the difference of perfect squares concept. If you multiply 68 X 72 mentally, you can do it by squaring 70 and subtracting the square of the difference 2. $70^2 - 2^2 = 4900 - 4 = 4896$. In Algebra this translates as $(A - B)(A + B) = A^2 - B^2 = (70-2)(70+2)$. The students can check to see that the answer is correct by using a calculator, but they really can't see it or understand how this works so I usually show them an example using smaller numbers, like 7 X 9. Square 8 and minus the difference of 1 squared and you get 63. I then draw them a visual picture using squares and rectangles. Table 5.1 below.

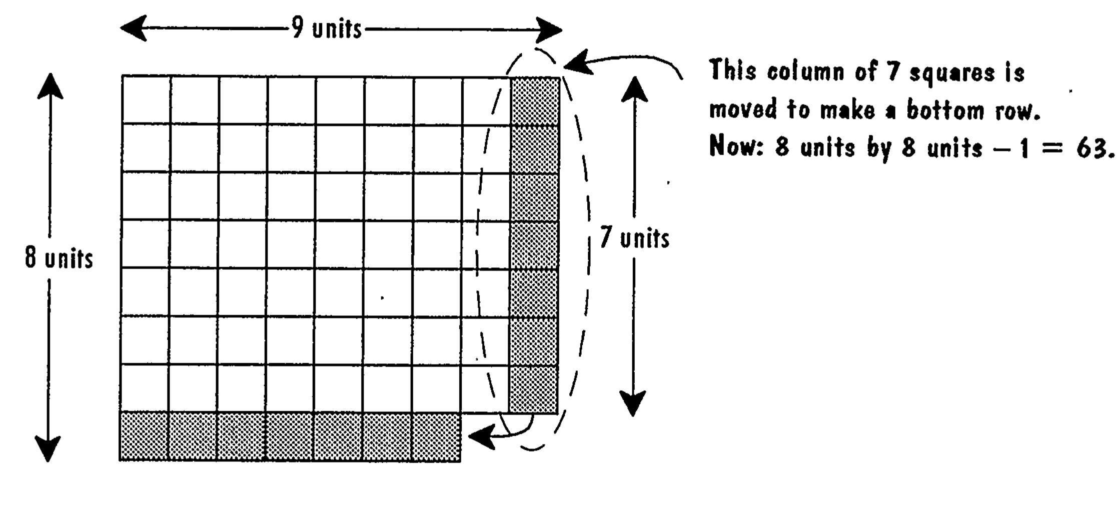


Table 5.1

This visual idea usually take on the form of some geometric idea. I will use this idea at the Grade 7 level to get them used to visualizing algebraic ideas in a geometric way and also when teaching mental multiplication skills.

I have observed that most good teaching ideas that give students a proper understanding are usually associated with something visual or concrete in nature and therefore when a problem of understanding arises in my students it is usually connected to a lack of visualization. Some of the ideas were developed through mathematics staff consultation, again, because problems had arisen when students couldn't understand a concept.

Once the activity was developed I would try it in my classroom and then evaluate the results to see if understanding had occurred. Evaluation was on-going throughout the activity, in that, if there were similar types of questions asked during the activity, I knew that I had to add, change or delete something at those points in the activity where the questions were being asked. I would also ask other teachers to try the activity with their learners and see what they thought. We would then compare notes and again change the activity or eliminate it completely. The students were usually asked whether they thought that they understood the concept better now that it was taught through this visual sense.

The process that I went through in gathering these activities for this thesis was one that began two years before I started writing the thesis. I had organized all of my materials into strands (Whole numbers, Number Theory, Decimals and Fractions, Integers, Exponents, Ratio and Proportion, Algebra, Graphing, Data Management and Geometry) and also under various concepts within each strand and so I simply went through all of my binders and removed each activity, problem, and game that had some geometry idea at its core and began making a pile for each of the above strands. Then I took all of the other articles that I had collected from the journals, books, and university classes and sorted these into the strand piles or into another pile called theory. This was my initial organization.

III. SELECTION OF ACTIVITIES

III.1. SORTING THE ACTIVITIES

I started with the activities sorted into strands. For each of the strand stacks I began sorting these into separate piles for each concept, a pile of activities that covered more than one concept, another pile for games, one for stories, and another for problems. To determine which pile to place the activity, I skim read each of the articles.

Then I went through each of the piles and divided them into three piles; Grade 7, Grade 8, and Grade 9. This required a lot of time since each article had to be read carefully to see its intent, its outcome, and its level of difficulty. It was at this stage of organizing that I began noticing general patterns from the materials I had collected. Some of the patterns emerging were:

1) I noticed that some activities only covered a specific concept.
2) Some activities were good starter activities for a unit study.
3) Some activities covered a range of concepts either within a unit or over several units.
4) Some ideas allowed the learners to explore the mathematical ideas while others could best be used as a demonstration by the teacher.

5) Some problems opened up mathematical ideas, while others simply required a single answer.
6) Some ideas were fundamental to teaching all the concepts found in that unit.
7) Some games were good for reinforcing the mathematical concepts taught, while others were simply games to develop strategies in winning.
8) The ideas I had collected were not skill development activities, problems or games. They were ideas that allowed for exploration, meaning seeking, were motivational in nature, and would develop mathematical thinking.

These patterns helped me to determine what ideas I should include in my final draft but they also became a source of frustration since I would have to eliminate some excellent ideas in teaching and learning.

The unit that took the longest to organize was that of measurement and geometry. I began by placing the articles into two piles. A pile that could be used to teach other concepts found in the mathematics curriculum and a pile that only covered concepts specific to the geometry and measurement units. I then took the pile of ideas that could be used to teach other mathematical concepts and put them into the grade 7 ,8, 9 unit piles. Then the specific geometry and measurement concepts were sorted into three piles of grade 7, 8, 9. These were placed into file folders for future reference when I began teaching those specific concepts next year and therefore were eliminated from my thesis.

III.2. CRITERIA AND SELECTION OF ACTIVITIES

Once I got to this stage of organization, I realized as I began trying to eliminate or make the selection of the materials that I would include in my final draft for this thesis that I needed to look at what my basic philosophy of teaching was, what things I valued in teaching and learning, and what I saw as important. The over-riding principle that I used in the selection of these activities was- will these activities form a network that make connections with others areas in mathematics and will they use the principle of integrative curriculum described in a previous chapter of this thesis.

The criteria that I used at this stage in selecting of the materials was as follows:
1) It must use an idea found in geometry.
2) The student should be able to construct the knowledge rather than be told by examples

or demonstration.

3) The activity should allow for inductive, creative, divergent, and deductive thinking.

4) Theactivity should connect to real-life.

5) The activity should connect to other areas of mathematics, other subjects, and other fields of study and occupations found in our society.

6) The activity should promote cooperation of learning, discussion, writing and a greater emphasis on process than on product.

7) The activity should develop meaning and understanding and allow for alternative ways of knowing.

8) The activity should be interesting and instill motivation.

9) The activity should develop an idea or concept that is to be taught in the Grade 7 Alberta curriculum.

Using this set of criteria I went through all the materials that I had selected as Grade 7 material and made two piles for each unit of study, one that fit the above criteria and one that missed the criteria. I then went through the pile that fit the above criteria and tried to make some sense of what I had selected. I found that the activities fell into three categories. 1. Exploratory activities. 2. Problems that develop mathematical thinking and allow the learner to explore mathematical concepts. 3. Games that reinforce the mathematical skills developed in the previous activities, develop mathematical thinking, and develop strategies in winning or losing.

There were many activities that fit under these three categories and also fit the criteria that I had selected and so I had to do some more eliminating and concentrating and this meant refining my criteria to become more specific.

The final criteria that are satisfying to myself and that helped me select the final set of activities and problems, are as follows:

1. The activity should begin with some geometric idea and expand to other fundamental areas of mathematics.

2. The activity should have emotional appeal, that is, look interesting, call to the learner to find a solution, touch some emotional cord in the learner.

3. The learner will have to use the approach of gathering and generating data and examples in order for the activity to be solved. The activity should allow for inductive, creative, divergent, and deductive thinking. The materials should promote cooperation of

learning, discussion, writing and a greater emphasis on process than on product.
4. The activity's solution should not be apparent after a simple reading of the problem but should call for immediate conjectures or guesses.
5. The activity should have several levels of solutions and/or interest. Level one: A simple answer to a specific question. Level two: Seeing a pattern and understanding the pattern at an elementary level. Level three: Understanding the pattern in a more powerful mathematical form, such as a general formula or arithmetic algorithm. Level four: Move the learner to look at similar activities or other connected fields of mathematical application. The solution should be satisfying emotionally, and bring a sense of fulfillment to the learner. Math anxiety should be limited and if possible eliminated as a result of finding a solution.
6. The activities found within each unit of mathematical study should declare the 'ground' upon which the concepts will be built. The ground-work or foundation must be properly laid so that when an idea is developed then that idea should be carried through to the end of the unit of study.

III. 3. ORGANIZATION OF ACTIVITIES

After deciding on the final set of activities within each unit of study, I then had to decide on the way to organize these units. Because geometry has changed in its shape and content over the many years of existence, I decided to structure a learning program that follows the historical development of geometry from its beginning and including the Euclidean era. The formal "proof-oriented" geometry of Euclid is not covered but deductive thinking is stressed whenever the activity allows for this kind of thinking. I also use simple activities that are found in the field of analytic geometry. This connection between geometry and algebra will be stressed through visual presentations. Three other areas beyond Euclidean geometry that have a few activities presented in basic, simple ways are: Projective geometry, Topology, and Erlanger Programm (transformations and translations).

The first major area in historical development of geometry that I would cover is what Eves (1983) calls "Subconscious Geometry." This geometry is what is observed by each one of us on a daily basis. The examples of geometry are found in the physical environment around us. The second major area is what Eves (1983) calls "Scientific Geometry." In this stage of development the observed properties and connections between the various shapes, sizes, and spatial relations are categorized, classified, defined, compared and contrasted. Problems that arise from these shapes are usually solved by rules or some general procedure. Geometry, in this scientific view, uses the inductive method of thinking, where information is gathered and organized, patterns are observed and generalizations are formed. It is in these two major areas that the materials for this thesis are organized and presented.

The organization and sequence of these activities, problems, games or skill development are as follows:

~ Basic geometry ideas [points, lines, rays, space].
~ Geometric or Figurate numbers [geometry shapes].
~ Number theory/Number bases/Patterns in mathematics.
~ Whole number operations.
~ Exponent numbers.
~ Integer numbers [number line, topology].
~ Decimals, Fractions, and Rationals.
~ Ratio, Proportion, and Percent [golden ratio, scaling].
~ Algebra, and Coordinate geometry.
~ Graphing, Probability, and Data Management.
~ More geometry ideas [symmetry, transformations, and tessellations].
~ Geometry and Art
~ Strategy games.

Each activity is presented in a format that is intended to be teacher friendly. This format consists of identifying for each problem the geometry ideas and procedures, and, where applicable, special materials required, a story line, suggestions for posing questions, student report and comments on making connections, for example. To also provide a sense of the complete set of activities, a summary page is provided, identifying the curriculum topic(s) each activity best fits and the geometry idea(s) used. A total of 66 activities have been included.

CHAPTER 6
THE ACTIVITIES

The outcome in this thesis is the set of activities that was compiled to fit the objectives of the study. A summary of these activities is provided followed by a description of each.

I. SUMMARY OF ACTIVITIES

Activity	Grade 7 Curriculum Topic	Geometry Used
1. Peter Path Problem.	patterns, addition of whole numbers, exponent numbers.	Topology, lines, rays.
2. Peter Moves to Diagonal City.	number patterns, addition of whole numbers, Golden Ratio	Topology, lines, rays.
3. Draw a Line Add a Dot Game.	problem solving, strategy.	lines, points.
4. Triangular Numbers:		
a) Collinear Points and Line Segments.	number patterns, addition of whole numbers	naming segments, shapes, points, line.
b) Triangular Dots.	addition of whole numbers.	shapes, points.
c) Triangular Shapes.	multiples, square numbers, addition of whole numbers, number patterns.	shapes.
d) Doubling the Triangular Shape.	multiplication of whole numbers,exponent numbers.	shapes, lines, points.
5. Square, Rectangular and Oblong Numbers.	number patterns, exponent numbers, addition and multiplication of whole numbers.	shapes.
6. Cutting a Pizza.	number patterns, addition of whole numbers.	lines, circles.
7.Twenty-Four Square Tiles.	square numbers, factors, primes,composite, multiples,	shapes.

	even/odd numbers.	
8.Numbered Points on a Circle.	factors, prime and composite numbers.	points, shapes.
9. Coded Number Line.	factors, prime and composite numbers, LCM, GCF, exponent numbers, square root.	shapes, lines, points.
10. Arranging Problems. a) Spelling 1-10. b) Josephus Circle. c) Turks and Christians.	problem solving, arranging, multiples, division even/odd.	space, shapes.
11. Nim	multiples, even/odd.	space.
12. Patterns in the Addition Table.	number theory ideas	line, diagonal, shapes, parallel, perpendicular.
13. Hundred's Chart.	factors, primes, composite, multiples.	shapes.
14. Magic Squares.	patterns, addition, subtraction, combinations.	shapes.
15. Magic Triangles.	patterns, addition, subtraction.	triangular shapes.
16. Adding Counting Numbers.	addition, multiplication.	shapes.
17. Pick's Theorem.	patterns, addition, multiplication.	area.
18. Hundred Squares.	patterns, addition.	lines.
19. Area Models.	multiplication and division of whole, decimals, fractions, and percent numbers.	area.
20. Gelosia or Lattice Multiplication.	multiplication	squares, diagonals.
21. A Vedic Method.	multiplication.	lines.

22. Multiplication	addition, multiplication, division, palindromic numbers, square numbers, multiples.	line, row, column, parallel, diagonal.
23. Origami.	construction.	geometric ideas.
24. Paper Airplanes. (4 lessons)	directions, measurement, area, perimeter, angles.	classification of shapes, area, angles, perimeter.
25. Art Contest.	all geometric concepts.	geometric ideas.
26. Tangrams.	construction ideas.	construction.
27. Paper Folding and Hole Punching. a) Values for Powers. b) Power Patterns. c) Constructing the Pattern.	number patterns, exponents.	shapes.
28. Pentominoes.	problem solving, strategy.	shapes.
29. Numbering Pages.	spatial relations.	shapes.
30. Integer Bulletin Board.	everyday examples of integers	lines, points, measurement skills
31. Integer Number Line.	number line ideas.	lines, points, measurement skills.
32. Simon Says.	addition, subtraction.	points.
33. Tug-o-war.	addition, subtraction.	points.
34. Electrical Charges.	addition, and subtraction, multiplication, division.	circles.
35. Postman Stories.	addition, subtraction, multiplication, division.	shapes.
36. Rectangular Models.	equivalence, ordering.	rectangular shapes.
37. Circle Models.	equivalence, ordering, addition, subtraction, multiplication, division.	circles, semi-circles, pie shapes.

38. Folding One Circle.	fractions, volume, shapes, surface area.	shapes, volume, surface area.
39. Folding Strips.	equivalence, ordering, addition, subtraction, division.	lines, points.
40. Folding and Shading.	multiplication.	shapes.
41. Using Arrays.	addition, subtraction, multiplication, division.	points, shapes.
42. Decimal Ruler.	decimal meaning.	lines, points.
43. Fibonacci Sequence and the Golden Ratio.	rate pairs, ratio, fractions.	shapes.
44. Pleasing Rectangles.	ratio.	shapes.
45. Constructing a Golden Rectangle.	rate pairs, ratios,	shapes, constructing skills, measurement skills.
46. The Pentagram.	ratio.	pentagon shape, constructing skills.
47. The Spear Thrower.	ratio.	measurement skills.
48. A Smile Contest.	ratios.	measurement skills
49. Map of Calgary.	scale, ratios.	scaling.
50. Environmental Problem Solving.	ratio, perimeter, area, estimation.	shapes, symmetry, similar triangles, area.
51. Scale Drawings.	ratio, scaling.	scaling.
52. Guess My Rule or "I Say; You Say".	tables, ordered pairs, algebra.	shapes.
53. Think of a Number.	solving equations.	shapes.
54. Coordinate Drawings.	ordered pairs, Cartesian plane.	shapes.

55. Mazes.	problem solving.	rays, points, lines, shapes.
56. Tread Design on Sneakers.	graphs, classification of shapes.	shapes.
57. Dice Graphing.	ordered pairs, graphing.	lines, points, coordinate plane.
58. Battleship.	ordered pairs, graphing.	lines, points, coordinate plane.
59. Hidden Treasure.	ordered pairs, graphing.	points, coordinate plane.
60. Cartesian Race.	ordered pairs, graphing.	shapes.
61. Pascal's Triangle.	patterns, combinations.	shapes.
62. Points on a Circle.	patterns, combinations.	lines, points, circles.
63. Symmetry: a) Penny Problem. b) Trademarks, Flags and Hub Caps. c) A Wardrobe.	symmetry.	symmetry.
64. Transformational Geometry: a) Amusement Park. b) The Route. c) Building a Parking Lot.	slides, flips, turns.	slides, flips, turns.
65. Tessellations: a) Tiling a Floor. b) Semiregular Tessel-lations. c) Capital Letters/ Block Numbers. d) Pentagons.	shapes, angles.	shapes, angles.

e) Unique Shapes that Tessellate.		
66. Tic-Tac-Toe.	strategy, problem solving.	shapes

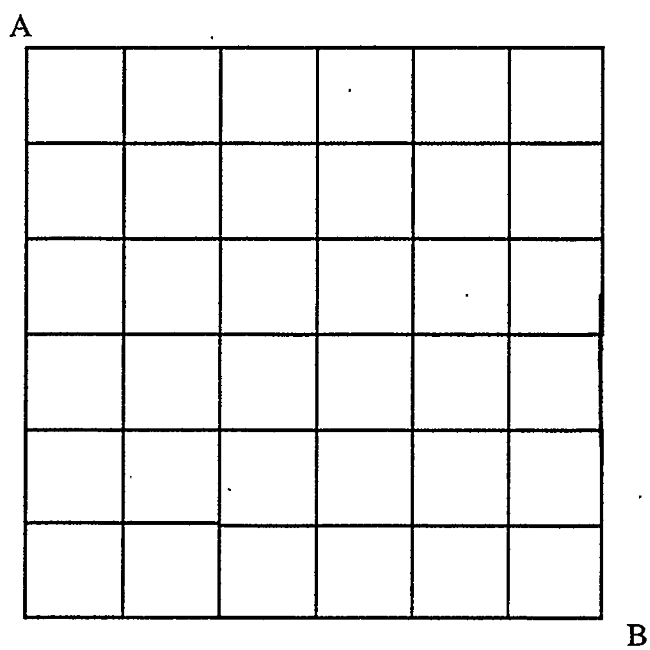
II. ACTIVITIES

#1
PETER'S PATH PROBLEM

Geometry Ideas: Topology, lines, rays.

Materials: 6X6 square grid.

Story line:
Peter's family moved to a city that was perfectly flat, like Regina, and the city blocks were all perfect squares, that is , the same length and width. He lived six blocks to the west and six blocks to the north of the school he was to attend in fall. He decided to ride his bike to school every day of the year and he wondered if he could take a new route to school each day. A is where Peter lives, and B is where the school is located.



Key Questions:
Do you think Peter will be able to take a new route to school everyday? (Assume there are 180 days in a school year and Peter can only go right and down). How many different routes could he take to get from his home to school?

Procedure and Start-up:

Give each student a square grid paper and have them begin by finding the different routes on a 1X1grid, a 2X2 grid, a 3X3 grid, etc, and then by seeing the pattern of different routes on these smaller grids come up with the results on a 6X6 grid.

Student Report:

The learners are then asked to publish their results showing grids, patterns, and conclusions.

COMMENTS: Difficulty level: High

#2

PETER MOVES TO DIAGONAL CITY

Geometry ideas: Topology, lines, intersections, rays.

Materials: A grid like Diagram #2.1.

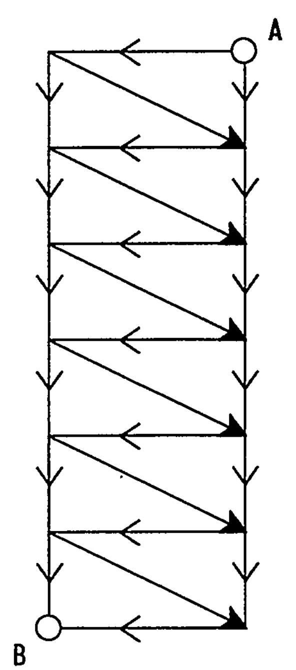


Diagram # 2.1

Story line:

In this city the blocks are in the shape of a rectangle and there is also a street that runs diagonal from the northwest corner to the southeast corner. Peter now lives one block east and six blocks north of the school he will be attending. He decided to see if he could ride his bike a different route to school every day of this new school year The diagonal streets were all one-way and Peter never wanted to ride into the sun so he could only go on the east-west streets from east to west. Diagram #2.1 shows the only way that Peter could travel.

Key Questions:

Could Peter take a different route to school each day during this new school year? (Assume there are 180 days in a school year and Peter can only follow the arrows in Diagram #2.1).

How many different routes can Peter take to get from his home to school?

What number pattern do you notice?

Procedure and Start-up:

Give each learner a grid as in Diagram #2.1.

Using the ideas learned in activity 1, the learners should be able to find the number of routes from home to school.

Student Report:

The learners are asked to write up their results showing diagrams, patterns, and conclusions.

COMMENTS: Difficulty level: High

#3

DRAW A LINE, ADD A DOT GAME

Geometry ideas: lines and points.

Materials: paper.

Story line: not applicable.

Key Questions:

What is the greatest number of moves possible in a 3-dot game? 4-dot game? 5-dot game?

What is the general rule for any amount of dots at the beginning of the game? Why?

Procedure and Start-up:

This game is played by two people. The game starts by placing 3 dots on a sheet of paper. The first player to play draws a line (not necessarily straight) from one dot to another and somewhere on the line that s/he drew, places a dot. The second player now chooses two dots and draws a line between them and then places a dot on the line that s/he drew. The players alternate drawing lines until one player cannot draw a line and therefore the last player to draw a line wins. There are only two rules you must follow as play continues in this same way.

1. You may not cross a line already drawn.
2. Any single dot may have no more than three lines connected to it.

Diagram 1 shows a completed 3-dot game.

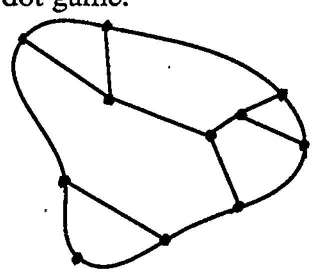


Diagram #3.1

Student Report:

Write up your results showing diagrams, answers, and conclusions.

Comments:

Variation:

Begin with four (or more) dots.

Difficulty level: Average

#4
TRIANGULAR NUMBERS

Geometry ideas: points, lines, triangular shapes, geometric symbolism, naming segments.

Materials: Triangular dot paper, triangular grid paper.

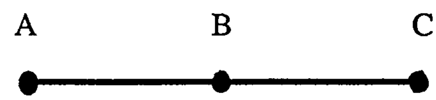
Story line:
The representing of numbers in a physical or geometric form was investigated by the Pythagoreans. The simplest polygonal numbers are the triangular and square numbers. The first figurate numbers that the Greeks investigated were the triangular numbers. This series of numbers will come up time and again in various mathematical problems and activities. The learners have seen examples of these triangular shapes from a young age and can learn some interesting mathematical facts from what they have observed. For instance, some stools and patio tables have only three legs, and bowling pins form a triangular shape. In the example of the bowling pins the total number of pins in the first row is 1, in the second row 2, third row 3, fourth row 4, If you add the first and second row you get 3, then add to that the third row you get 6, then add the fourth row to that and you get 10. If you continue this pattern by adding additional rows of pins you would then have the series of triangular numbers; 1, 3, 6, 10, 15, 21, etc. The number patterns formed by geometric shapes will be repeated in many activities and problems found in later mathematical strands. Each figurate number begins with a single point so that the first figurate number of any geometric shape is one (1). Several activities or investigations that can be done to illustrate this series of numbers are:

A) COLLINEAR POINTS (POINTS IN A ROW) AND LINE SEGMENTS.

Key Question:
How many line segments are formed with 10 collinear points?

Procedure and Start-up:
The learners should investigate this question by starting out with 2 points. They will get 1

segment. Then put three points in a row and they should get 3 segments.



Student Report:
Write up the results using diagrams, patterns, answers, algebraic generalization and conclusions.

Comments:
For ten collinear points the number of segments can be found by adding 1+2+3+4+5+6+7+8+9= 45 or $n(n-1)/2$, where n stands for the number of points in a row.
This simple activity connects nicely to number line concepts and all of its uses in fractions, decimals, and integer numbers and also to addition of numbers, particularly the method that Gauss used to add up the first 100 whole numbers, using the formula $n(n+1)/2$, where n stands for the last number to be added e.g. $100(100+1)/2= 5050$.
Difficulty level: Average

B) TRIANGULAR DOTS.

Key Questions:
How many dots would it require to form a triangular shape where the bottom row contained 10 dots?
What sequence of numbers do these triangles create?

Procedure and Start-up:
Start with the following diagram of dots to find the answers.

Student Report:

Write up the results giving diagrams, answers, and conclusions.

Comments:

The sequence of numbers discovered are the triangular numbers 1, 3, 6, 10, 15, 21, etc. and the answer is 55 dots.

Difficulty level: Average

C) TRIANGULAR SHAPES.

Key Questions:

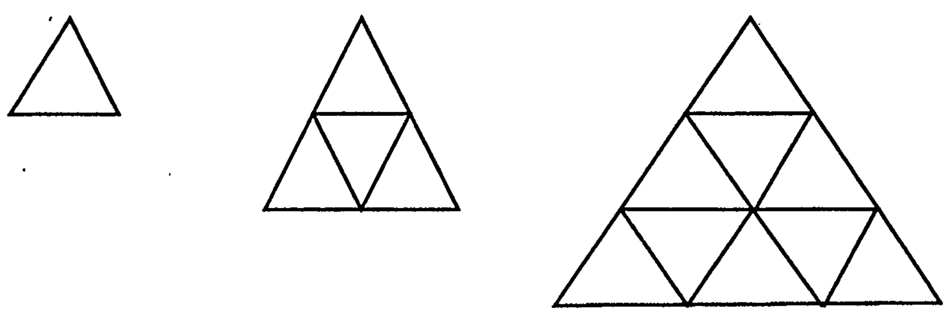
1) How many points are found on only the outside edge of each triangle? What sequence of numbers have you discovered?

2) What is the total number of the same size of triangles found in each of the larger triangles? What sequenceof numbers have you discovered?

3) What is the total of all the different sizes of triangles found in each of the larger triangles? What sequence of numbers have you discovered?

Procedure and Start-up:

Ask the learners to draw all line segments joining the points, as illustrated in the diagram below.



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tinue this pattern of triangles and then have the learners answer the key questions.

Student Report:

Write up the results using the diagrams, patterns, answers, and any conclusions.

Comments:

1. The numbers are 3, 6, 9, 12, 15, etc. These are the multiples of three.
2. The numbers are 1, 4, 9, 16, 25, etc. These are square numbers.
3. They are the numbers 1, 5, 13, 26, 45, etc.

Difficulty level: Average

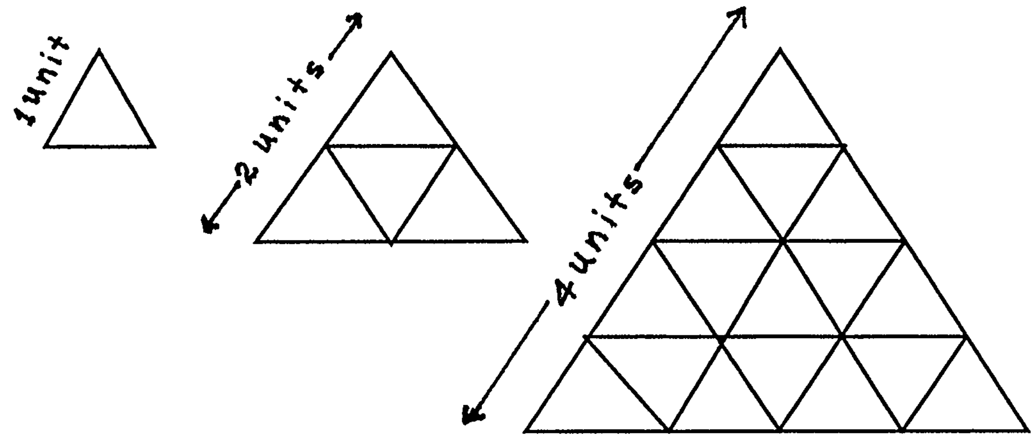
D) DOUBLING THE TRIANGULAR SHAPE.

Key Questions:

1. What sequence of numbers would result from doubling the outside edges of a triangular shape?
2. Describe this pattern using exponent numbers.

Procedure and Start-up:

Ask the learners to double the length of the side of the original triangle and then draw the smaller triangle within the larger triangles, as illustrated in the diagram below. Then ask the learners to answer the key questions.



Student Report:
Write up the results of this investigation using diagrams, patterns, answers, and conclusions.

Comments:
The sequence of numbers are; 1, 4, 16, 64, etc. They are 4^n where n is a whole number. This set of activities connects nicely to activities involving square, rectangular and oblong numbers.
Difficulty level: High

#5

SQUARE , RECTANGULAR, AND OBLONG NUMBERS.

Geometry ideas: Square, rectangular, and oblong shapes.

Materials: 8x8 checkerboard or square grid paper.

Story line: not applicable.

Key Questions:
1. How many squares are found on an 8x8 checkerboard?
2. How many rectangles are found on an 8X8 checkerboard?

Procedure and Start-up:

The learners are first asked to start with a 1X1 square and find the number of squares. Then they are told to use a 2X2 square grid and again find the number of squares in that shape. They continue this method until they can see the pattern and find the answer to an 8X8 square grid or they find the answer to each size of square grids until they arrive at an 8X8 square shape.

Then the learners are asked to find all of the different sized rectangles found in an 8X8 square grid. They should follow the same procedure as with finding the total number of squares in an 8X8 checkerboard. The learners are reminded that squares are classified as rectangles and therefore they need to count every square from the previous investigation.

Student Report:

Write up the results giving diagrams, patterns, answers, algebraic formulas, and conclusions.

Comments:

The learners should discover that there are 64-1x1 squares; 49-2x2 squares; 36-3x3 squares; 25-4x4 squares; 16-5x5 squares; 9-6x6 squares; 4-7x7 squares; and 1-8x8 square. When you add them up 1 + 4 + 9 + 16 + 25 + 36 + 49 + 64 = 204. The general rule for the total number of squares is $n^2 + (n-1)^2 + (n-2)^2 + (n-3)^2 + \ldots + (n-n)^2$.

There are 204 squares from the previous activity; 112-1x2 rectangles; 96-1x3 rectangles; 80-1x4 rectangles; 64-1x5 rectangles; 48-1x6 rectangles; 32-1x7 rectangles;16-1x8 rectangles; 84-2x3 rectangles; 70-2x4 rectangles; 56-2x5 rectangles; 42-2x6 rectangles; 28-2x7 rectangles; 14-2x8 rectangles; 60-3x4 rectangles; 48-3x5 rectangles; 36-3x6 rectangles; 24-3x7 rectangles;12-3x8 rectangles; 40-4x5 rectangles; 30-4x6 rectangles; 20-4x7 rectangles;10-4x8 rectangles; 24-5x6 rectangles; 16-5x7 rectangles; 8-5x8 rectangles; 12-6x7 rectangles;6-6x8 rectangles; 4-7x8 rectangles = 1296 rectangles.

The pattern can be simplified by showing the numbers in a vertical table:

Dimensions of the square	Total number of rectangles
1x1	1 =1x1
2x2	9 =3x3
3x3	36= 6x6
4x4	100 =10x10
5x5	225 =15x15
6x6	441 =21x21
7x7	784 =28x28
8x8	1296 =36x36

The numbers 1, 3, 6, 10, 15, 21, 28, 36, should be recognized by the learners as the triangular number series.

This activity also connects to the multiplication table. If you use the gnomon shape, the number of rectangles-including squares- is found by adding all of the numbers found in the gnomon shapes up to and including the 8x8 gnomon.

This grouping of activities ties in with prime and composite numbers, odd and even numbers, factors, and multiples.

Difficulty level: High

#6

CUTTING A PIZZA

Geometry ideas: lines and circle shape.

Materials: Compass, ruler, paper,

Story line:

There are 100 guests coming to a pizza party. A pizza parlour has advertised that they can make a pizza 200 cm in diameter that can feed 100 guests. You have asked the pizza parlour to cut the pizza in unequal pieces because the appetite of each person is different and they all know what size of piece will fit their appetite. The pizza parlour manager replies that he can do it with only 14 straight cuts. Prove that the manager is right or wrong.

Key Question:

Can the manager of the Pizza Parlour make enough pieces to feed 100 people of various appetites?

Procedure and Start-up:

Have the learners start by cutting a circle with only one cut or line. Then 2 cuts, 3 cuts, etc. The results should be recorded in a table and a pattern of cuts and pieces should be discovered. The learners will soon discover that it is almost impossible to make 14 cuts and count the number of pieces in a circle. They will be forced to seek a pattern from the table recorded earlier.

Student Report:

The results should be written up giving diagrams, table of results, patterns (both numerical and algebraic), answers and conclusions.

Comments:

The manager can do this with only 14 straight cuts. The table below gives the information:

Number of cuts	Number of pieces
1	2
2	4
3	7
4	11
5	16
6	22
7	29
8	37
9	46
10	56
11	67
12	79
13	92
14	106

The general formula for this activity is: $n(n+1)/2 + 1$.
This problem with the pizza connects well with placing different amounts of points on the circumference of the circle and then connecting the points with straight lines in as many ways as possible. Naming these various points and then the resulting figures leads to permutations and combinations. Later, I will describe this activity in greater detail in this paper.
Difficulty level: High

#7
TWENTY-FOUR SQUARE TILES

Geometry ideas: shapes.

Materials: 24 squares, results sheet.

Story line: not applicable.

Key Questions:
1. Circle all the numbers in column 1 that have only 1 rectangle or square in column 3 and that have different lengths for the sides. Ask the learners if they know the name given to these numbers ?
2. Put a diamond shape around all the numbers in column 1 that have two or more rectangles or squares in column 3. Ask the learners if they know the name given to these numbers ?
3. Put a square around all the numbers in column 1 that have formed a perfect square. Check dimensions in column 2 that have the same length and width. Ask the learners if they know the name given to these numbers?
4. Ask the learners to look at all the lengths (column 4) for the number 12. What do all these lengths do to the number 12 ? Do you know the name for all the numbers that divide evenly into another number ?
DEFINITIONS: Prime numbers have how many factors?____ Composite numbers have how many factors?______ Square numbers always have an ______ number of factors?

Procedure and Start-up:

1. Students are given 24 square tiles. They are asked to use 1(one) square tile and make as many squares or rectangles using that tile. They must write the dimension of the square(s) or rectangle(s) formed. Then they are given 2(two) square tiles and asked to do the same thing as with one square tile. Then they are given 3(three) square tiles and asked to do the same thing again. This continues until all 24 square tiles have been used. The results are recorded on a sheet. Once the chart is completed the teacher checks to see that all the information is correct.

Student Report:

The results, notes, definitions, patterns and names of patterns, and conclusions are written up and handed in for evaluation.

Comments:

This activity connects well with the locker problem.

Difficulty level: Easy

RESULTS

Column 1 Number of square tiles	Column 2 Dimensions of square(s) or rectangle(s)	Column 3 Number of different square(s) or rectangle(s) found	Column 4 Give all the different length of the sides of the squares and the rectangles.
1			
2			
3			
4			
5			
6			
7			
8			
9			
10			
11			
12			
13			
14			
15			
16			
17			
18			
19			
20			
21			
22			
23			
24			

#8
NUMBER POINTS ON A CIRCLE

Geometry ideas: points, lines, naming of shapes, circles.

Materials: compass, rulers, paper.

Story line: not applicable

Key Questions:

1. How many regular shapes of the same size can you find when you construct a circle with 6 equal points on the circumference? 12 equal points on the circumference? 24 equal points on the circumference? 36 equal points on the circumference?
2. What are the names of each of these regular polygons?
3. What relationship is there between the number of regular polygons and the number of sides found in each polygon, to the number of points found on the circumference of the circle?

Procedure and Start-up:

The learners are asked to construct a circle with 6 equal points found on the circumference. Hint: Use the central angle idea or the 6 arcs the same length as the radius of the circle.

Ask learners to join every point, every other point and then answer the key questions.

They then construct the 12, 24, and 36 point circles and follow the same procedure as with the 6 point circle.

Ask learners to experiment with other circles with different numbers of equal points on the circumference.

Student Report:

Write up the results showing diagrams, tables, patterns and pattern names, and conclusions.

Comments:

I have adapted this activity from Claire M. Newman and Susan B. Turkel (1989)

Integrating Arithmetic and Geometry with Numbered Points on a Circle, published in the January Arithmetic Teacher. They connect factors of numbers with numbered points on a circle. They inscribe regular geometric shapes with 12, 24, and 36 points on the circumference of circles. They then show how the learners can join every point, every second point, every third point, and every fourth point in the twelve point circle, that produce all of the factors of twelve along with the names of each of the regular polygons. They also produce all of the factors of 24 and 36 by using this same method. This activity could be extended to find all of the factors for any number of points on the circumference of a circle. If only one regular polygon is produced you have discovered prime numbers e.g. a seven point heptagon produces only one seven sided regular polygon. If more than one polygon can be produced at different positions then you would have demonstrated that that number is a composite number.
Difficulty level: High

#9
CODED NUMBER LINE

Geometry ideas: points, lines and shapes.

Materials: 10X10 square grid.

Story line: not applicable.

Key Questions:
1. Which number is a factor of every other number?
2. Which numbers have exactly two factors?
3. Which numbers have an odd number of factors?
4. How could you express all the other numbers(composite numbers) as a product of prime numbers?
5. What geometric shapes would best suit the various numbers from 1 to 100?

Procedure and Start-up:
Place numbers 1 to 100 in each of the 100 squares.
Then, since every number has one as a factor a black dot is placed over every number.

Next, each of the twenty-five prime numbers is assigned a different coloured geometric symbol such as 2-a red dot, 3-a green dot, 5-a blue dot, 7-an orange square etc. The composite numbers are then given appropriate symbols for their prime factorization. For example 6 has a red and green dot over it, 12 has 2 red dots and a green dot. Once the number line has been coded then four different mathematical concepts can be taught by using the coded number line.

Student Report:
Write up the results of your findings and reasons for choosing the geometric shapes for certain numbers.

Comments:
This activity is adapted from Tom Graviss and Joanne Greaver's (1992) article Extending the Number Line to Make Connections with Number Theory. Some number theory concepts that can be taught and reinforced with the coded number line are:

1. Greatest Common Factor (GCF).

This is done by using the prime factorization of each of the numbers and circling the common factors and then multiplying them together.

e.g. 12- 2x2x3
18-2x3x3
Common factors-2x3 = 6

2. Least Common Multiple (LCM).

This is done in a similar fashion as the GCF, the common factors are first paired and then all the different factors along with the paired factors are multiplied together.

e.g. 10- 2x5
15-3x5 LCM-2x3x5 =30

3. Square Roots.

The square root of a perfect square is found by again pairing common factors and then multiplying these common factors together.

e.g. 36- 2x2x3x3
Common factors- 2x3 = 6.

4. Exponent Numbers.

The prime factorization again is used and the number of the same symbols used determines the exponent number.

e.g. 36- 2x2x3x3 = $2^2 \times 3^2$

54- 2x3x3x3 = $2^1 \times 3^3$

The exponent 1 is often forgotten when students get to the polynomial section when variables are used instead of numbers. Hopefully, this visual, tactile reminder will help the students remember.

Difficulty level: Average

#10
ARRANGING PROBLEMS

Geometry ideas: Space, shapes, points, dividing a line.

Materials: Cards numbered from 1 to 10, paper.

A) SPELLING 1-10.

Story line:

The problem that is posed to the learners is they must arrange the deck of 10 numbers so that as they spell out the number-starting at one- the next card that appears will be that number. I demonstrate how this works: As I deal through the deck I spell the letters of the first number,

Spell	O-N-E	next card that appears is a 1 (set aside this card)
	T-W-O	next card is a 2 (set aside)
	T-H-R-E-E	next card is a 3 (set aside)
	F-O-U-R	next card is a 4 (set aside)

This continues until the only card left is a 10 set it aside.

Key Questions:

How can you arrange the deck of ten cards so that as you spell through the numbers from

1 to 10, that numbered card will appear next in the deck?

Student Report:

Write up the results showing your method of arranging the ten numbered cards.

Comments:

This activity can be done in any language French, Spanish, etc.

The order of the deck is: 4-9-10-1-3-6-8-2-5-7 (for English).

Difficulty level: Average

B) JOSEPHUS CIRCLE.

Story line:

When the Romans were attacking the Jews during the war about 67-70 AD, Josephus described in his book De Bello Judaico how he and forty other Jews hid in a cave. Not wanting to be taken captive they decided to kill themselves. Josephus suggested that they form a circle and be counted one by one, and every third person be killed until there was only one person left and that person would commit suicide. Josephus arranged the counting so that he and a companion came out last and they decided to be taken prisoner.

Key Question:

In what positions did Josephus place himself and his companion?

Student Report:

Write up your results using diagrams, and method used to solve this problem.

Comments:

The answer is the 16th and 31st positions.

This problem could be followed by another posed during Medieval times. It is: "There

are fifteen Christians and fifteen Turks and all the Christians are to be saved. Where should the Christians be placed in order that all are saved?"
Difficulty level: Average

#11
GAME OF NIM.

Geometry ideas: space, rows.

Materials: paper.

Story line:
The game of Nim is played by two people. The simplest game requires 12 markers placed in three rows- 3 markers in row 1, 4 markers in row 2, and 5 markers in row 3.
Each player in turn takes any number of markers from exactly one row. At least one marker must be taken each turn. The winner is the player who removes the last marker or group of markers.

Key Question:
What strategy should be used in order to win no matter who begins the game?

Procedure and Start-up:
Begin by playing several games with the whole group of learners.
Then have the learners form pairs and try out their strategies.

Student Report:
Write up your results explaining your strategy for winning.

Comments:
Variations: The amount of markers per row can be increased.
More rows of markers may be added.
The loser is the last person to remove a marker.
This strategy game develops concepts of odd and even numbers.
Difficulty level: Average

#12
PATTERNS IN THE ADDITION TABLE.

Geometry ideas: line, diagonal, shapes, parallel, perpendicular.

Materials: 20X20 square grid.

Story line: not applicable.

Key Question:
Using any geometry ideas, what numerical patterns can you find in the addition table?

Procedure and Start-up:
The learners are asked to construct an addition table, using the numbers from 1-20. They are then asked to find as many patterns as possible but these patterns must be connected to some geometric idea such as, line, diagonal, square, triangle, other shapes, parallel, perpendicular, etc.

Student Report:
Write up all the patterns that you have found and show how they were connected to some geometric idea.

Comments:
A connection to a 10x10 multiplication table could be done at this point with similar instructions given to the learners.
Difficulty level: Average

#13
HUNDREDS CHART

Geometry ideas: shapes.

Materials: Hundred's chart.

Story line:

This activity is similar to the addition and multiplication tables, but it has the added feature of larger numbers. The patterns that can be found also relate to many of the concepts found in number theory.

Key Questions:

Using any geometry ideas, what numerical patterns can you find in the hundred's chart?

Procedure and Start-up:

Hand out the hundred's chart and ask the learners to find as many patterns as possible but these patterns must be connected to some geometric idea such as, line, diagonal, square, triangle, other shapes, parallel, perpendicular, etc.

Student Report:

Write up all the patterns that you have found and show how they were connected to some geometric idea.

Comments:

Bennett and Nelson (1988) used a hundreds chart arranged in a spiral pattern. This hundreds chart creates new possibilities for discovering new patterns and reinforcing some previous number theory concepts such as primes and composites.

Difficulty level: Average

#14
MAGIC SQUARES.

Geometry ideas: square and other shapes.

Materials: Picture of *Melencolia* by Albrecht Durer, Albrecht Durer magic square.

Story line:

The most interesting and motivational magic square appears in the famous engraving *Melencolia* by Albrecht Durer.

Key Questions:

What patterns do you see in this magic square?

Why is it called the most famous magic square of all magic squares?

Procedure and Start-up:

The learners are given this magic square and then asked to search for as many ways that 34 is found in the magic square by adding four numbers together.

Student Report:

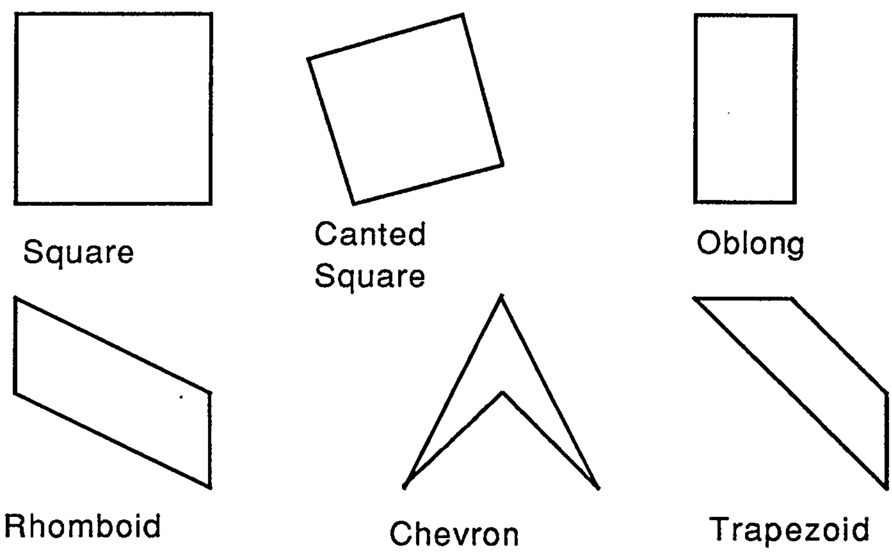
Write up all your findings, including all the ways that 34 is found in the magic square.

Comments:

There should be 86 combinations.

Herbert Wills III (1989) gives some interesting activities to try with your learners. They are: 1. Use the first nine digits (1, 2, 3, 4, 5, 6, 7, 8, 9) and create a magic square that adds to fifteen.

2. Using various quadrilaterals (small square, canted square, oblong rectangle, rhomboid, chevron, trapezoid) or other geometric shapes discover patterns of 34 found in the magic square.



A further investigation is to have the students develop a general method for creating magic squares of various dimensions and using different numbers. Is there a way of creating a magic square that is 5x5, the sums add up to 65 and uses the ideas found in the famous 4x4 magic square of Albrecht Durer?
Difficulty level: Average

Durer's *Melencolia*.

Courtesy of Kennedy Galleries, New York

The Dürer Square **HANDOUT**

16	3	2	13	16	3	2	13	16	3	2	13
5	10	11	8	5	10	11	8	5	10	11	8
9	6	7	12	9	6	7	12	9	6	7	12
4	15	14	1	4	15	14	1	4	15	14	1

16	3	2	13	16	3	2	13	16	3	2	13
5	10	11	8	5	10	11	8	5	10	11	8
9	6	7	12	9	6	7	12	9	6	7	12
4	15	14	1	4	15	14	1	4	15	14	1

16	3	2	13	16	3	2	13	16	3	2	13
5	10	11	8	5	10	11	8	5	10	11	8
9	6	7	12	9	6	7	12	9	6	7	12
4	15	14	1	4	15	14	1	4	15	14	1

16	3	2	13	16	3	2	13	16	3	2	13
5	10	11	8	5	10	11	8	5	10	11	8
9	6	7	12	9	6	7	12	9	6	7	12
4	15	14	1	4	15	14	1	4	15	14	1

#15
MAGIC TRIANGLES

Geometry ideas: triangular shapes.

Materials: paper.

Story line: not applicable.

Key Questions:

1. Using the numbers from 1 to 6, how many situations can you find where the sums are the same? differ by 1? differ by 2? differ by 3? and differ by 4?
2. The learners are asked to make some general rules about how different combinations are found, how many possible differences between consecutive sums, and what sequence of numbers are found in determining the number of points on a side of a triangle?

Procedure and Start-up:

This activity has the learners place the digits 1, 2, 3 in a 3-pointed triangle and then sum the sides. The learners will notice that no matter where the digits are placed the sum on the sides will always have a difference of 1.

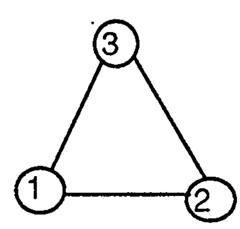


Diagram #15.1

The learners are next asked to use the digits 1, 2, 3, 4, 5, 6 and to place these numbers in the 6 circles that form the triangle. An example of differing by one is given in Diagram #15.2.

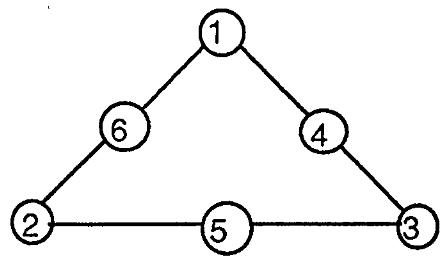


Diagram #15. 2

This activity is now increased to nine digits 1, 2, 3, 4, 5, 6, 7, 8, 9 and the same investigation takes place except that now there are more possibilities or combinations.

Student Report:
Write up the results using diagrams, tables, patterns, and conclusions.

Comments:
This investigation could be extended depending on the group of learners and their motivational level.
I have adapted this activity from John Firkins (1983) article on Manipulating Numbers.
Difficulty level: High

#16
ADDING COUNTING NUMBERS

Geometry ideas: shape, translations.

Materials: Paper.

Story line:
This problem was solved by Gauss while in elementary school. The problem is to add up the first 100 whole numbers. A geometric way of visualizing this problem is by using a staircase and then rotating the staircase so that they fit together (Diagram #16.1).

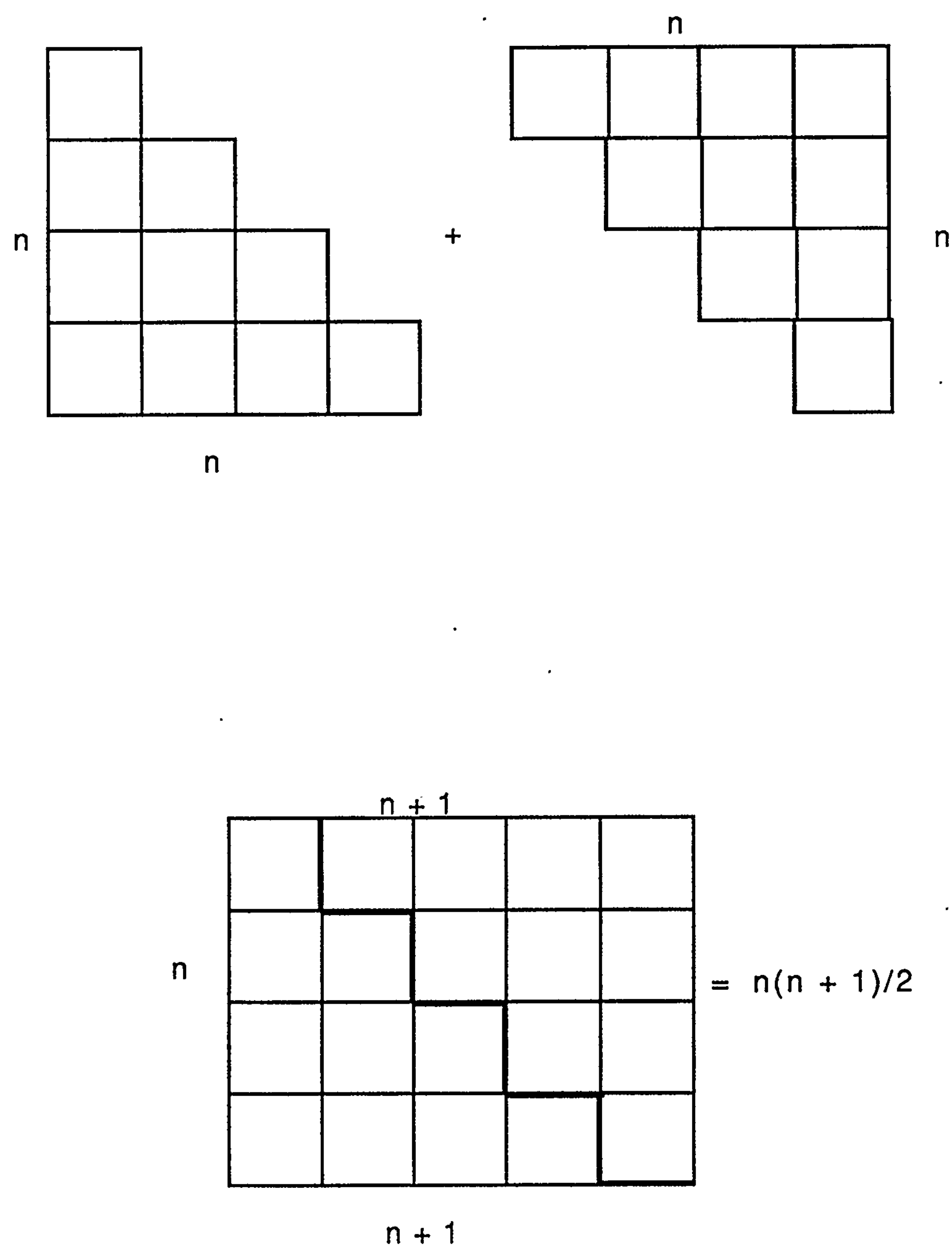


Diagram #16.1.

Key Questions:

1. Add the first 100 counting numbers?
2. Can you find an easier way to add up the 100 numbers instead of writing them out and

then adding them?
3. Can you find a visual way using geometry ideas that can help you solve this problem?
4. Can you find an algebraic formula to solve this problem?

Procedure and Start-up:
Have the learners begin by adding up the first 10 numbers, then see if they can see a pattern. Work through larger numbers until a pattern emerges.

Student Report:
Write up the results showing diagrams, patterns discovered, answers, and conclusions.

Comments:
Variations: 1. Find the sum of the numbers from 50 to 100?
2. Find the sum of the odd numbers from 1 to 50?
3. Find the sum of the days of any week on a calendar?
Difficulty level: Average

#17
PICK'S THEOREM.

Geometry ideas: area.

Materials: geoboards, dot paper.

Story line:
This investigation involves finding the area of an irregular geometric shape by using dot paper or a geoboard. The learners are asked to find the area of any shape by counting the number of dots or points on the outside edge of the shape and also the number of dots or points found within the shape. They are to come up with a general formula and if possible an informal proof of why this works.

Key Question:
1. Find the area of an irregular geometric shape.

Procedure and Start-up:

Using elastics and pegs on a geoboard or drawing on dot paper make irregular geometric shapes. Find the area of these shapes. You may want to start with regular shapes and find the pattern with these and then make small changes to the regular shape and try to find the area.

Student Report:
Write up the results showing examples, patterns, method used to solve problem, a formula, and conclusions.

Comments:
The general formula is 1/2 s + (i) - 1, where s represents the number of points on the sides and (i) represents the number of points in the interior of the polygon.
Difficulty level: High

#18
HUNDRED SQUARES

Geometry ideas: lines.

Materials: paper.

Story line: not applicable.

Key Questions:
How few lines are required on a page in order to have drawn exactly 100 squares?
Could you find a general formula for any number of squares drawn with the least number of lines?

Procedure and Start-up:
Have the learners experiment with finding the least number of lines required to draw 100 squares.
Then when they have found the answer have the learners start with fewer squares, such as 1, 4, 10, and 16. Then they should come up with a pattern and a generalization or formu-

la.

Student Report:
Write up the results giving answers, diagrams, patterns, formulas, and conclusions.

COMMENTS:
Difficulty level: Easy

#19
AREA MODELS

Geometry ideas: area of shapes.

Materials: grid paper, adding machine tape.

Story line:
The use of area models has been shown by James E. Schultz (1991) to span over mathematical concepts in whole number, fraction, and decimal arithmetic, as well as, probability, and algebra. In whole numbers you can use simple calculations for finding the area of quadrilaterals. Using grid paper marked off in hundreds, many decimal calculations can be made. Multiplication and division of fractions have been demonstrated using rectangular shapes. Percents equivalent to simple fractions can be demonstrated using percent bars in the shape of rectangles. Algebra tiles and their equivalent area models help make multiplication of binomials visual and concrete for junior high learners. Once area models have been introduced at the simplest level-area of quadrilaterals, using whole numbers-then as each new topic involving operations in decimals, fractions, percents, and algebra are taught to the learners, these topics should be introduced through area models to help bring consistency to student's learning. Since many activities abound in each of these areas and are available to most teachers, I will not explain any in this paper but simply recommend their usage.

#20

GELOSIA OR LATTICE MULTIPLICATION

Geometry ideas: Squares, diagonals.

Materials: Square or rectangular grid paper.

Story line: not applicable.

Key Questions:

How is this method different, the same, as the algorithm you have been taught in elementary school?

What advantages and disadvantages does this method have over the traditional algorithm?

Using this method of multiplication, find a method for multiplying decimal numbers.

Procedure and Start-up:

Demonstrate this method to the class of learners. Then have them practice this method on some multiplication questions.

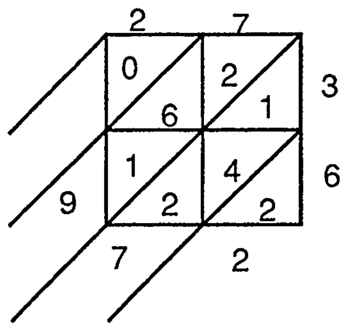
Ask the learners to answer the key questions.

Student Report:

Write up the results giving answers, examples, method for multiplying decimal numbers, and conclusions.

Comments:

This method of multiplication uses geometric ideas to help learners deal with the numbers involved in an organized manner. This method can be used for whole numbers as well as decimal number operations. Memorized multiplication facts are needed up to and including the nine times table. Diagram # 20.1 gives an example of how this method works.



27 x 36 = 972

Diagram #20. 1.

Difficulty level: Average

#21
A VEDIC METHOD

Geometry ideas: diagonals, parallel lines, columns, rows.

Materials: Paper.

Story line:
This method helps learners hold information in their heads and therefore helps them develop mental computational skills.

Key Questions:
How is this method different, the same, as the algorithm you have been taught in elementary school?
What advantages and disadvantages does this method have over the traditional algo-

rithm?

Demonstrate a way to multiply numbers from left to right instead of right to left.

Procedure and Start-up:

Demonstrate this method to the class of learners. Then have them practice this method on some multiplication questions.

Ask the learners to answer the key questions.

Student Report:

Write up the results giving answers, examples, method for multiplying right to left, and conclusions.

Comments:

This method will also be useful when teaching the multiplication of two binomials in algebra. Diagram #21.2 gives an example of this method.

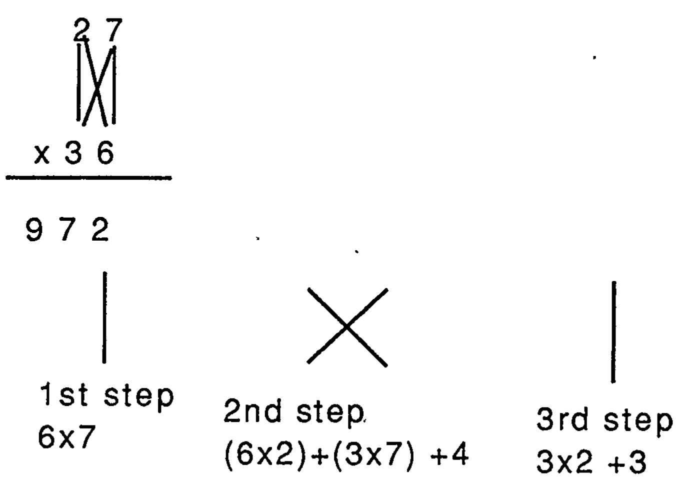


Diagram #21.2

Difficulty level: Average

#22
MULTIPLICATION TABLE

Geometry ideas: line, diagonal, shapes, parallel, perpendicular.

Materials: 20X20 square grid.

Story line: not applicable.

Key Question:
Using any geometry ideas, what numerical patterns can you find in the multiplication table?

Procedure and Start-up:
The learners are asked to construct a multiplication table, using the numbers from 1-10. They are then asked to find as many patterns as possible but these patterns must be connected to some geometric idea such as, line, diagonal, square, triangle, other shapes, parallel, perpendicular, etc.

Student Report:
Write up all the patterns that you have found and show how they were connected to some geometric idea.

Comments:
Some patterns are:
1. The diagonal shows square numbers;1, 4, 9, 16, etc. and also divides the table symmetrically. The diagonal going the opposite way indicates palindromic numbers; 58985, 4664.
2. All numbers in a row or column have a constant difference and create a sequence of numbers. e.g. 4, 8, 12, 16, etc.
3. When the numbers in a gnomon shape are added the resulting sum is a cubic number; $3+6+9+6+3 = 27 = 3^3$.

4. The numbers closest and parallel to the main diagonal (2, 6, 12, 20, 30, 42, 56) are related to the triangular numbers (1, 3, 6, 10, 15, etc.).
Difficulty level: Average

#23
ORIGAMI

Geometry ideas: terminology of geometry.

Materials: origami paper, paper, origami books and ideas.

Story line:
Origami is an oriental art-form of paper folding. It can be used successfully to teach many of the basic geometric skills. When used to create an art-form it acts as a geometric algorithm. The geometric concepts and skills that can be taught through origami are; types of polygons, angles, measurement (area and perimeter), symmetry, and congruency. Many geometric terms are used informally as the paper folding takes place. Some of these terms are: length, width, dimensions, depth, plane, diagonal, degree, bisect, perpendicular, parallel.

Key Question:
How can the learning of origami help me in real life?

Procedure and Start-up:
Before beginning a project, the learners should understand some standard origami symbols, some guidelines for folding, and origami books should be made available for reference.
Then begin with some simple folding activities following the instructions found in an origami book.
1. Create a square, pentagon, hexagon, and octagon shapes.
2. Make a five pointed star.
3. Make a fish.

Next have the learners create their own origami art-form and have the learners write up instructions for other learners to follow.

Student Report:
Have the learners hand in the art-form they created along with the instructions on how to do it.

Comments:
Paper folding activities lead to connections in exponent concepts in mathematics, design occupations, scale drawings, and leisure time activities.
Difficulty level: Average

#24
PAPER AIRPLANES

Geometry ideas: classification of shapes, area, perimeter, angles, measurement skills.

Materials: paper, paper airplane design, measuring tools(rulers, protractors).

Story line: not applicable.

Key Questions: found throughout the lessons.

Procedure and Start-up:
I have designed and used a series of lesson that incorporate paper folding, choice, art and design.

SERIES OF LESSONS- GRADE 7
Lesson 1: Making a paper airplane
Lesson 2: Working with a Designer Plane
Lesson 3: Construct your own plane.
Lesson 4: Competition Day

LESSON 1:

PROBLEM 1: Make a paper airplane in 5 minutes. You may use any design and if you have time you may make more than one. (You may work in pairs)

Supplies: Foolscap paper, glue, scissors, rulers, protractor.

Try flying your plane.

PROBLEM 2: Write instructions on how you made your paper plane.

PROBLEM 3: Give your instructions to another group of learners. When you have received the instructions from someone else then you are to follow their instructions exactly as they have written them, and make the paper airplane.

PROBLEM 4: Try flying this plane while the other learners watch.

KEY QUESTIONS:

1. How does the original plane compare to the copy?
2. How many found that the instructions where sufficient?
3. Were the instructions in a nice sequence?
4. How many groups included a diagram or drawing in the instructions?
5. How many planes flew the second time?
6. What are your feelings on reading other people's instruction?
7. What are some of the important things to include in the writing of instructions? (Make a list of these important items on the chalkboard and have students take notes.)

PROBLEM 5: Have learners re-write their original instructions and have them try it with a different learner. The learners should evaluate each other's instructions.

EXTENSION AND REINFORCEMENT: Use the "PAPER FOLDING" exercise found in the book Problems with Patterns and Numbers, Joint Matriculation Board Shell Centre for Mathematical Education.

I would include this activity if I found that the learners needed more practice in setting up instructions on how to fold and illustrate their instructions. I would also use it for those students that wanted to investigate the patterns involved in paper folding . I would allow the students to go as far as they wanted to in this activity.

LESSON 2: DESIGNED PLANE

1. Working in pairs, take the designed plane and following the instructions construct the plane.
2. Try flying this plane. Note the special features of how this plane flies.
3. Unfold this plane and then analyze it by filling in the following categories.

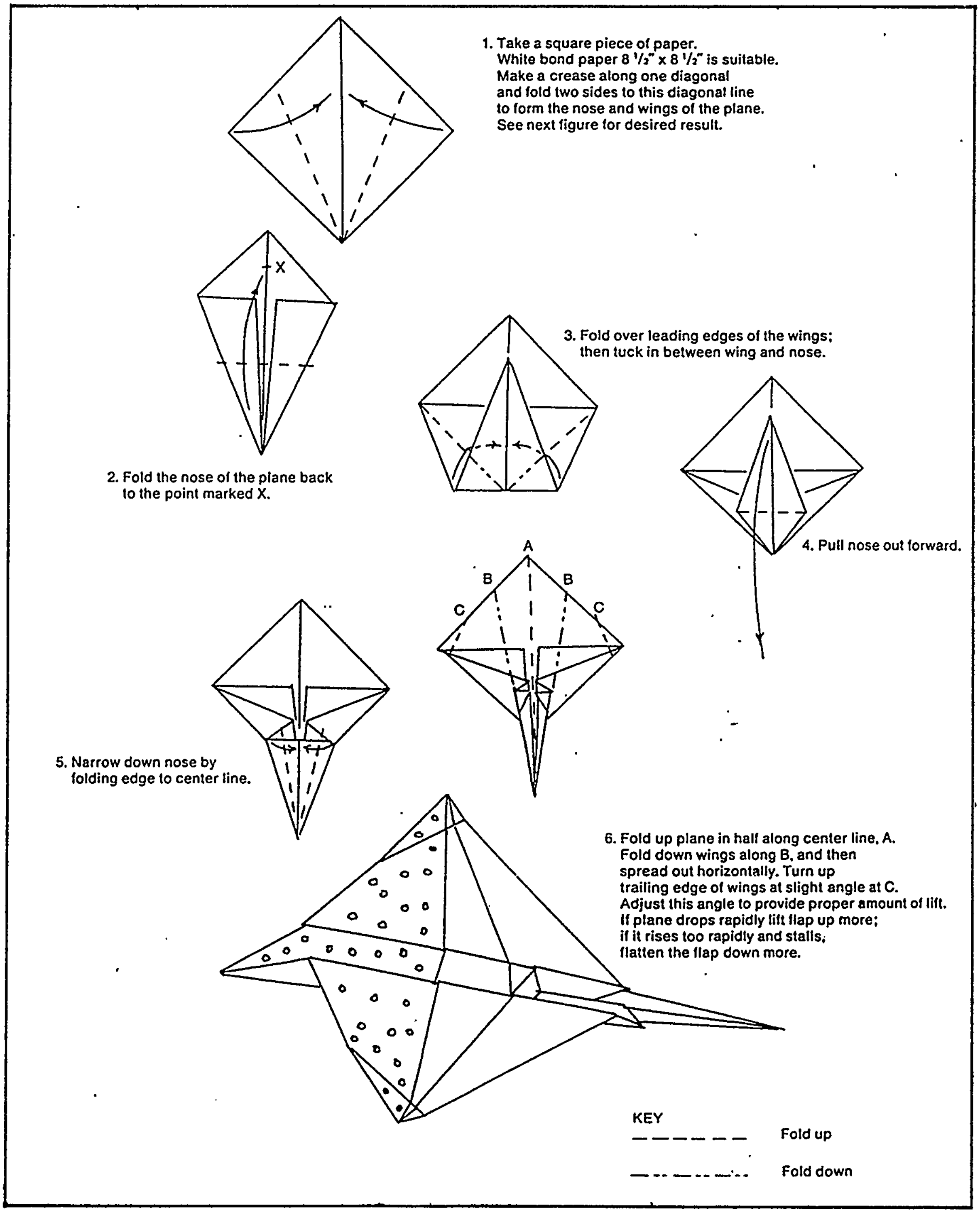
CLASSIFY AND IDENTIFY ALL THE DIFFERENT SHAPES AND ANGLES (record results)

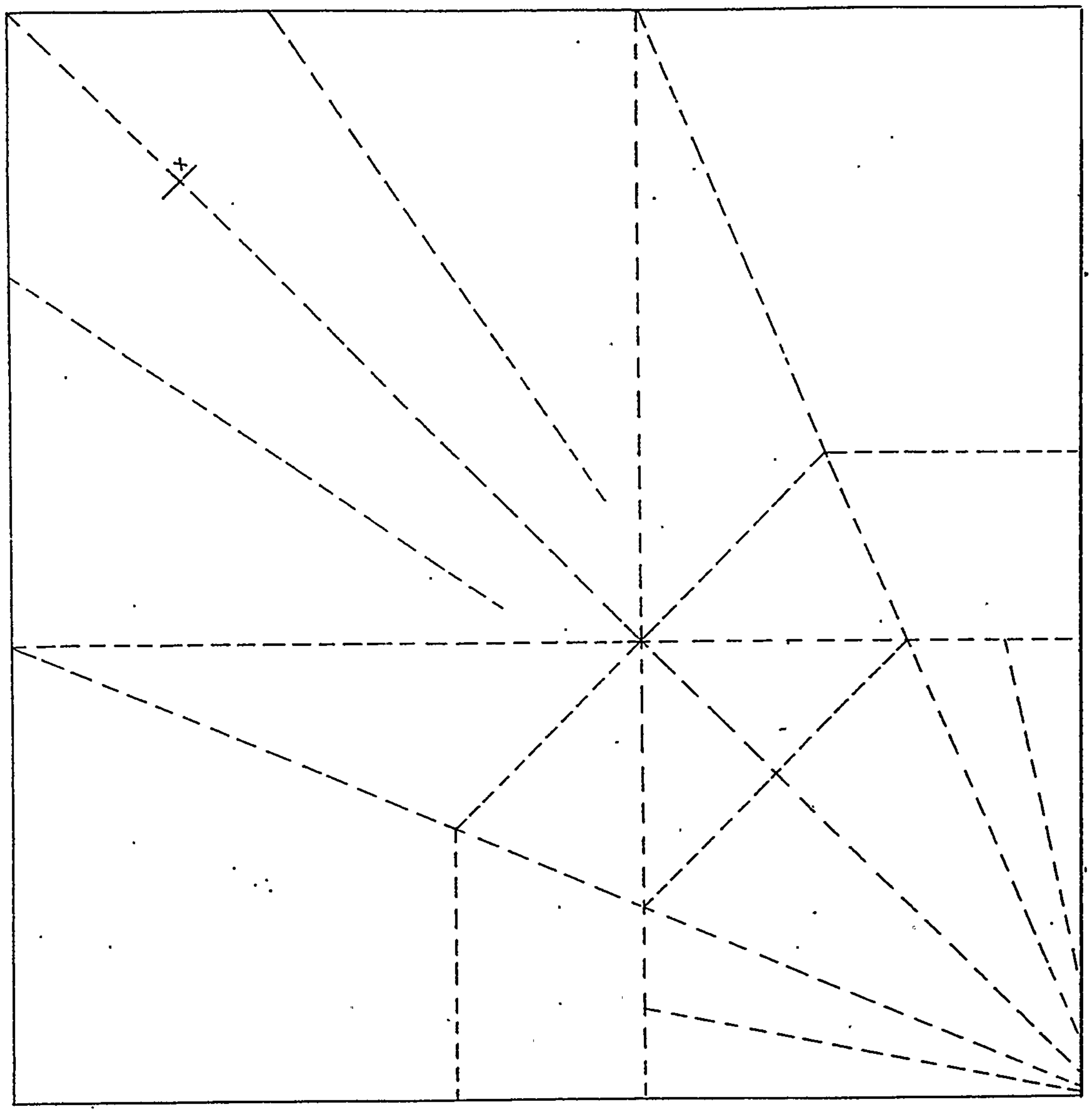
MEASURE ALL OF THE ANGLES (record results)

MEASURE ALL OF THE LINE SEGMENTS ON THIS SHEET (record results)

CALCULATE EACH AREA AND PERIMETER ON THIS SHEET (record results)

VARIATIONS: 1. Pictures only then construct.
2. Plans only then construct.
3. Plans and template then construct.


1. Take a square piece of paper.
White bond paper 8 1/2" x 8 1/2" is suitable.
Make a crease along one diagonal
and fold two sides to this diagonal line
to form the nose and wings of the plane.
See next figure for desired result.
X
2. Fold the nose of the plane back
to the point marked X.
3. Fold over leading edges of the wings;
then tuck in between wing and nose.
4. Pull nose out forward.
A
B
B
C
C
5. Narrow down nose by
folding edge to center line.
6. Fold up plane in half along center line, A.
Fold down wings along B, and then
spread out horizontally. Turn up
trailing edge of wings at slight angle at C.
Adjust this angle to provide proper amount of lift.
If plane drops rapidly lift flap up more;
if it rises too rapidly and stalls,
flatten the flap down more.
KEY
Fold up
Fold down



LESSON 3: CONSTRUCT YOUR OWN PAPER AIRPLANE

The students will work in pairs and construct their own model paper airplane. A contest will be held when the planes are completed. The paper airplanes will be judged in four categories;

-duration aloft
-distance flown
-aerobatics
-Origami

LESSON 4: PAPER AIRPLANE COMPETITION DAY

The school gymnasium will be booked for an afternoon and the parents and school board personnel will be invited to the competition. Letters will be written by the students inviting these V. I. P.'s to this afternoon of fun. If this competition cannot be held in the gymnasium then the teacher should arrange it in the classroom. Trophies should be bought or made to be awarded to the best plane in each category. When the competition is finished then all the planes should be displayed in the classroom or the school.

SOME ALTERNATIVE LESSONS

The following alternative lessons should be used only if time or personnel permit it.

1. Guest Speaker. a) Pilot
b) Paper Plane Enthusiast
c) Airport Personnel
2. Re-design a balsa airplane

Difficulty level: Average

#25

ART CONTEST

Geometry ideas: All geometric concepts.

Materials: paper, wood, metal, string, wire, etc.

Story line: not applicable.

Key Questions: not applicable.

Procedure and Start-up:

This art contest can run concurrently with this paper airplane project since the work is not done during school hours but at home. The art categories all help to reinforce the objectives that we are covering during these lessons on paper airplanes.

Contest Rules.

1. Any learner may enter.
2. All projects must be home constructed (outside of school time).
3. Emphasis is on mathematical accuracy and artistic impression.
4. Materials are not supplied by the school.
5. Any medium may be used (paper, wood, metal, string, wire, etc.).
6. Contestants must enter in one of the following categories:
    a. Curved Stitchery - 2 dimensional
    b. Curved Stitchery - 3 dimensional
    c. Geometric Constructions (compass and straight edge only)
    d. Games and Puzzles (Geometry related)
    e. Mobiles
    f. Golden Rectangle or Golden Ratio Designs
    g. Architectural Designs
    h. Sculpture
    i. Computer Graphics (own design only)
    j. Perspective Drawings
    k. Miscellaneous (combinations of the above)
    l. Tessellations
7. Prizes are awarded. a) These consist of ribbons.
    b) 1st prize in each category $10.00
    c) 2nd prize in each category $ 5.00

Student Report:

The learners will hand in their art projects, and these will be judged by interested personnel in the school.

COMMENTS:
Difficulty level: Average

#26
TANGRAMS

Geometry ideas: construction concepts.

Materials: paper.

Story line: not applicable.

Key Questions:
How could you construct the 7 tangram pieces by only folding and cutting on folded lines?

Procedure and Start-up:
The learners are given a sheet with the seven tangram pieces intact. Diagram #26.1

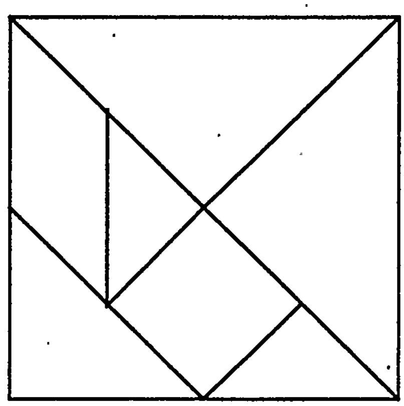


Diagram #26.1

The learners are then given a 8.5 x 11 sheet of paper and asked to construct the seven tangram pieces by only folding and cutting on folded lines. Once they have completed their tangram construction, they are then asked to make various tangram pictures of real life objects or animals.

Student Report:
Write up the instructions of how you folded the paper to form the 7 tangram shapes.
Give examples and drawings of some of your best pictures of real life objects and animals.
COMMENTS:
Difficulty level: High

#27

PAPER FOLDING AND PUNCHING HOLES

Geometry ideas: shapes.

Materials: paper, single-hole punch.

Story line: not applicable.

Key Questions: found within each investigation.

A) VALUES FOR POWERS

Procedure and Start-up:
The learners are given a piece of paper. They are asked to fold the paper once and to punch the folded paper in the top left corner. They open the paper and count the number of holes. The results of how many folds and holes is recorded on a separate sheet. They fold the paper a second time and again punch a hole in the top left corner exactly in the same spot as they did the first time. They again open the paper and record the number of folds and holes. This procedure is repeated until they cannot fold the paper again. The results are recorded each time.

Student Report:

When the investigation is complete the learners are asked to write up their conjectures, patterns, express the pattern in exponent form and draw conclusions.

Comments:

Diagram #27.1 shows the results.

# of folds	# of holes	Powers of 2
1	2	2^1
2	4=2x2	2^2
3	8=2x2x2	2^3
4	16 2x2x2x2	2^4

Diagram #27.1

COMMENTS:

Difficulty level: Average

B) POWER PATTERNS

Procedure and Start-up:

The learners are given a piece of paper. They are asked to punch a hole in it. Record the results of folds and holes. Then they are to fold the paper once and punch another hole. They unfold the paper and record the number of folds and holes. Again they are told to fold the paper a third time and punch another hole. They again unfold the paper and record the number of folds and holes. This procedures is repeated until they cannot fold the paper anymore.

Student Report:

Write up the results showing table, patterns, exponent forms, and conclusions.

Comments:

The results are recorded in Diagram #27.2.

# of folds	# of holes	Power Form
0	1	2^1-1
1	3	2^2-1
2	7	2^3-1
3	15	2^4-1

Diagram #27.2

The results of this activity are identical to the "Tower of Hanoi" activity.

COMMENTS:

Difficulty level: High

C) CONSTRUCTING THE PATTERN

Procedure and Start-up:

The learners are given this chart and asked to make the pattern with their paper and then add 2 more sections to the chart.

# of folds	# 0f holes	Pattern
0	1	1x1
1	5	1x1+2x2
2	17	1x1+2x2 + 4x3

Student Report:
Write up the results to your findings, showing the method you used to create the pattern in the chart and any conclusions drawn.

COMMENTS:
Difficulty level: High

#28
PENTOMINOES

Geometry ideas: shapes.

Materials: 5 squares, 8X8 square grid.

Story line: not applicable.

Key Question:
How many different ways can five squares be arranged in a single shape so that if two

squares touch, they border along a full side?

Procedure and Start-up:

The learners are given 5 equivalent squares. They are asked to create all possible different shapes with the five squares. They are to trace these shapes on a piece of paper and cut them out. Then two learners are given a checkerboard or a 64 square grid and asked to play the game of cover-up. The rules are as follows:

1. Place the twelve pentominoes in a pile.
2. Each player draws in turn from the pile until all are gone.
3. Each player in turn plays one of his/her pentominoes anywhere on the playing board.
4. Play continues until a player cannot place any more pentominoes on the board.
5. The winner is the player with the least number of pentominoes left.

Student Report:

Write up the results showing all the pentominoes found, a strategy for winning the game, and any conclusions drawn.

Comments:

The learners should discover 12 different shapes. The term *pentominoe,* for a 5-omino, is a registered trademark of Solomon W. Golomb (No. 1008964, U. S. Patent Office, April 15, 1975).

Difficulty level: Average

#29

NUMBERING PAGES

Geometry ideas: shapes.

Materials: paper.

Story line: not applicable.

Key Question:

How would you number the pages of a small booklet before any folds are made?

Procedure and Start-up:
The learners are asked to describe and illustrate how to number the pages of a booklet before a fold is made in forming a small booklet.

Student Report:
Write up your method of numbering and folding.

COMMENTS:
Difficulty level: Easy

#30
INTEGER BULLETIN BOARD

Geometry ideas: line, division of a line, points, measurement skills.

Materials: Bulletin board.

Story line: not applicable.

Key Questions:
How does this example or picture illustrate an integer?
What words used in this example show direction? What direction?

Procedure and Start-up:
Ask the learners to search through books, magazines, newspapers and everyday things around their house that illustrate the set of integer numbers. These should be brought to school over the next couple of days.
A discussion should ensure with each example brought in using the key questions and any other questions that may arise from the examples.
The examples should be displayed on a bulletin board that is prepared with a title (Everyday Examples of Integers and Rationals) and possible categories (Stock Market, Temperature, Money Markets, etc.).

Student Report:
Write up a page report giving examples of integers and how integers are useful in our everyday lives.

Comments:
The ideas found in this section are adapted from Joe McAuley (1990) article "Please Sir i Didn't Do nothing".

An appropriate way to start any unit of mathematical study is to declare the 'ground' upon which the concepts will be built. I have found that often misconceptions can be reduced if the ground work or foundation is properly laid. Also when an idea is developed then this idea should be carried through to the end of the unit of study.

The Foundation
In the Integer unit, Joe McAuley points out that "an appropriate place to start with any outline of this topic ..... is to make the distinction between "Binary operations" and "Unary operators." By Binary operations he means that the operations of addition, subtraction, multiplication, and division on the set of Integers operate on pairs of elements of the set Z (Integers) e.g. Z X Z = Z where Z is an element of the Integer set. "The positive and negative operators on the other hand operate on a single element (hence unary). They may be thought of as instructions to move to a specific position to the right or left, respectively, of zero on an Integer number line." McAuley defines the set of Integers in terms of the set of Naturàl numbers ($x \xrightarrow{+} {}^{+}x$), thus moving from the known to the unknown. This development and distinction has been appropriately called "Directed Numbers" in some textbooks.

#31
LARGE INTEGER NUMBER LINE

Geometry ideas: points, lines, measurement skills.

Materials: Construction paper.

Story line:

McAuley suggests that "an Integer number line, preferably large scale and on a permanent display is an invaluable visual aid. It is a concrete focal point to which reference may be made at any time during the general exposition and whenever individual students experience difficulty."

Key Questions:

How should the construction paper be divided so that all sections of the number lines will match?

What operations can be done using the number lines?

Procedure and Start-up:

The learners should be divided into 3 groups. One group will make the natural number line from 1 to 50.

The second group will create an integer number line from -50 to 0.

The third group will create an integer number line from +1 to +50.

Each large construction paper should be divided into 10 equal parts and labeled appropriately. The three groups will need to coordinate their efforts so that the final product will match together.

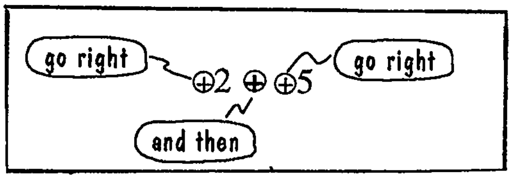
Student Report:

Write up your method of creating your part of the number line. Give examples of the type of integer operations that can be performed using the number lines.

Comments:

The number lines can be used to do addition, subtraction and multiplication. The use of "direction" should be emphasized in each operation.

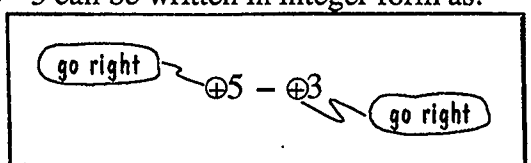
I would start the exploration of the topic of addition with a problem to which the students already know the answer, and then using the isomorphism(Pimm, 1981) between the set of Natural numbers under the operation "+" and the set of positive Integers under the operation "+". For example $2 + 5 = 7$ may be written as: $^{+}2 + {}^{+}5 = {}^{+}7$. At this stage we can distinguish between the "positive" sign as an instruction to move to a specific position to the right of zero, and the "addition" sign as a conjunction between the two instructions of $^{+}2$ and $^{+}5$.



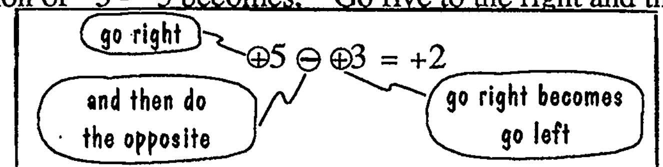
Spending time to translate the problems into English is necessary and recommended. Have students work through as many examples as necessary to understand that this works in every case.

The Minus Operation

I would again start with a familiar subtraction problem to which many students can arrive at an understanding of the operation performed in subtraction with little assistance. For example, 5 - 3 = 2, but 5 - 3 can be written in integer form as:



Knowing that the result of the operation is $^{+}2$ how can instruction of "5 to the right" and "three to the right" give an answer of only 2 to the right of zero? The answer to this question lies in the rule of the minus operation which may then readily be interpreted as the conjunctive: "**And Then** do exactly the opposite to the next instruction". Therefore the translation of $^{+}5$ - $^{+}3$ becomes: "Go five to the right and then go 3 to the left." So



Some further example with less obvious results can be tried at this point. What is preserved is the idea of 'direction' when looking at the minus operation.

Multiplication of Integers

Positive x Positive

Few problems will arise here since we can make use of the isomorphism between the set of integers and the set of natural numbers. Simply;

$$^{+}2 \times {}^{+}3$$

may be treated as;

$$2 X 3.$$

Positive X Negative

Problems such as ${}^{+}3$ X ${}^{-}4$ can be thought of as;

$$3 X {}^{-}4$$

The translation of this into English is:

"3 lots or groups of 4 to the left"

The result of this instruction is 12 to the left of zero i.e.: ${}^{-}12$. Hence;

$${}^{+}3 X {}^{-}4 = {}^{-}12$$

${}^{-}4$ X ${}^{+}3$ may be dealt with as above using the commutative property of multiplication or literally may be translated to: "go 4 to the left 3 times" again giving the answer ${}^{-}12$.

Negative X Negative

Now the thorny problem; "Why should the multiplication of two negative numbers produce a positive result?" What I am going to do here is not a proof but an argument that appeals to mathematical rather than primitive principles such as pattern or common sense found in grammar. e.g.

$${}^{-}4 X {}^{-}5$$

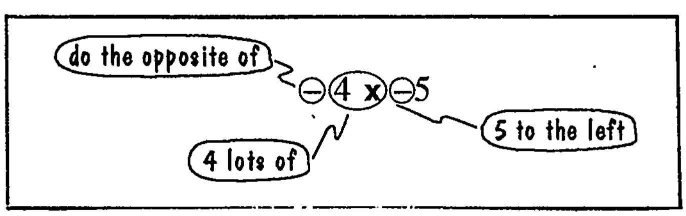
I would first argue for the equivalence of ${}^{-}4$ and - ${}^{+}4$ by using the number line and showing that you arrive at exactly the same spot on the number line, namely 4 to the left of zero. Then we know that ${}^{-}4$ X ${}^{-}5$ is equivalent to:

$$- {}^{+}4 X {}^{-}5$$

which is equivalent to:

$$-4 X {}^{-}5$$

i.e. ${}^{-}4$ X ${}^{-}5$ = - 4 X ${}^{-}5$; translated into English:



and this of course is the same as; " 4 lots or groups of 5 to the right."

Hence ${}^{-}4$ X ${}^{-}5$ = ${}^{+}20$. I have found that being consistent with the positive and negative signs as "instructions for direction" and that the minus sign is translated as "do the opposite of" helps the students to understand the operations of integer numbers.

#32
SIMON SAYS

Geometry ideas: points.

Materials: 7 chairs, two cards with a plus sign on one and a minus sign on the other.

Story line: not applicable.

Key Questions:

If you are the counter what do you do when the plus operator gives you a command?
If you are the counter and the minus operator gives you a command, what should you do?

Procedure and Start-up:

All that is required for this activity is row of seven chairs, three students, two cards with a plus sign on one and a minus sign on the other. The chairs are set up in a row with the middle chair being different than the other chairs so as to distinguish it as the zero chair. One student holds the plus sign, one the minus sign, and the third student is the 'counter' and sits on the middle chair or the zero chair to begin the activity. To avoid any confusion between left and right it is best if Plus, Minus , and Counter all face the same direction. Plus and Minus then issue instructions to the Counter, such as: "right 2" "left 1" etc. After a few such practices the instructions may then be changed to the form "Positive 2" "negative 3" etc. The Counter must follow the instructions of the Plus operator and do exactly the opposite of the Minus operator.

Student Report:

Write up a report giving your impressions of the game and whether or not you learned something.

Comments:

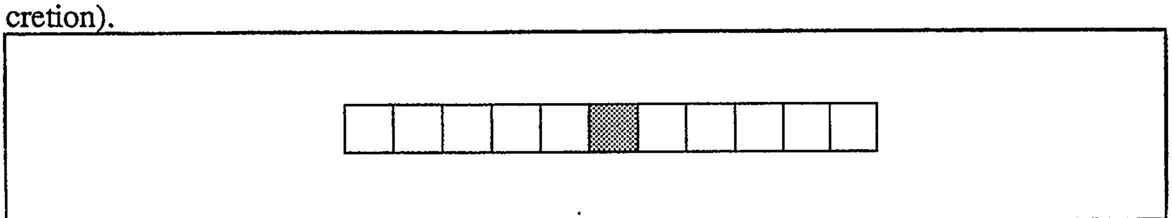
For students having conceptual difficulties with the topic, a spell in the "hot seat" as counter usually clears up any problems.

#33

TUG-O WAR

Geometry ideas: squares, cubes.

Materials: Two wooden cubes, a strip of card and a counter.
On one cube write the integers 0, $^{+}1$, and $^{+}2$, with those of equal magnitude on opposite faces of the cube. On the other cube mark 3 plus signs and 3 minus signs. The card can be divided into a row of 11 squares as shown below(this could be extended at your discretion).



Story line: not applicable.

Procedure and Start-up:
This is a game for 2 players seated side by side. The counter is placed on the coloured square in the center of the strip of card and the pair of dice is thrown in turn by each player. At the end of each turn the counter is moved according to the outcome of the dice. A player is declared the winner when the counter is legally moved onto the square at his/her end of the board.

Student Report:
Write up a report giving your impressions of this game.

COMMENTS:
Difficulty level: Average

#34
ELECTRIC CHARGES

Geometry ideas: shapes.

Materials: blue and red chips (+ and - charges), beakers or containers.

Story line: not applicable.

Key Questions:
How would you explain or make meaning of the four operations in the set of integers using electrical charges?
How would you demonstrate the four operations in the set of integers using electric charges?

Procedure and Start-up:
The learners are asked to construct their own way of explaining or making meaning of the four operations of the set of Integers using + and - charges. They are given blue and red chips and containers to hold the chips. The construction should be consistent with all the principles of logic and reason found in mathematics.

Student Report:
Write up your explanation showing examples of the four operations and have the other learners try out your construction and method for adding, subtracting, multiplying and dividing integers.

COMMENTS:
Difficulty level: High

#35
POSTMAN STORIES

Geometry ideas: shapes.

Materials: not applicable.

Story line:
Addition: A postman comes to your house with a letter containing a cheque for $5.00. Ask learners whether they would be better off than before? Use the terms positive for better and negative for worse off. How much better? The next day when the postman arrived he again gave you a letter containing a cheque for $4.00. Ask learners if they would be better or worse off? By how much better than two days ago? This is only an example of the type of story that you could use to introduce the four operations of integers.

Key Questions: These are found within the body of the stories.

Procedure and Start-up:
Start by giving an example of a story as in the story line above. Then I would ask the class to work in groups and develop the rest of the Postman stories for the remaining addition operations and stories using the other three operations (subtraction, multiplication and division).
When their stories are complete then ask the groups to share their finding with each other and see if all their stories are clear and work.

Student Report:
Write up all your stories and examples of how each story can be illustrated by the abstract integer numbers.

COMMENTS:
Difficulty level: Average

#36
RECTANGULAR MODELS

Geometry ideas: Rectangular shapes, construction skills.

Materials: 5 different coloured construction paper.

Story line: not applicable.

Key Question:
Using the models have the learners demonstrate how they would illustrate the concepts of equivalence, how many, more than, less than, and other ways of folding and cutting these 5 coloured shapes?

Procedure and Start-up:
The learners are given 5 rectangular pieces of construction paper. They are all the same size but different colours. The gold should be kept as a unit. The red is to be folded in half length-wise, and then cut on the fold. The blue should be folded into thirds and then cut at the folds. The tan should be folded in half twice and then cut along the folds. The black should be folded in half and then into thirds and then cut on the fold lines. Diagram #36.1 shows a picture of these five pieces before being cut.

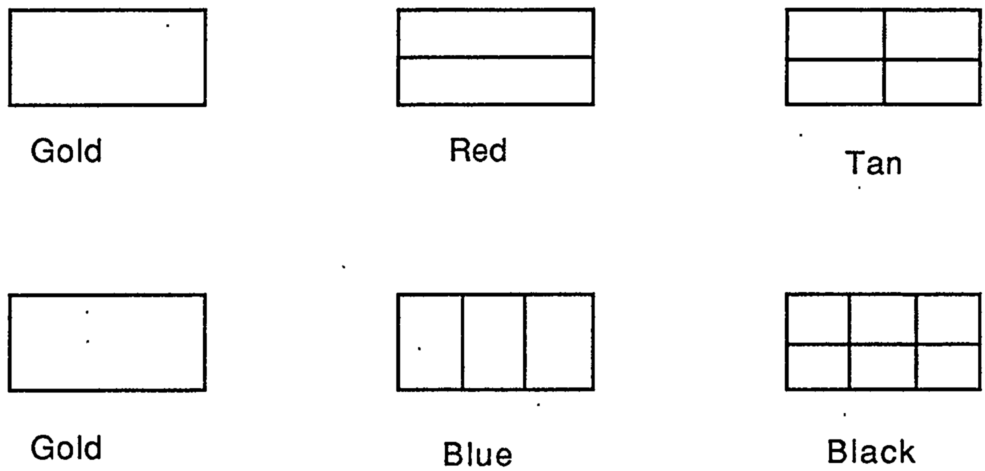


Diagram #36.1

Student Report:

Write up your examples of how you would use the rectangular models to illustrate the concepts of equivalence, how many, more than, less than and other ways of folding and cutting the 5 different models.

Comments:

One of the most difficult sections in mathematics for most learners is fractions. Some of the reasons for this difficulty are the various interpretations associated with the meaning of fractions. Kennedy and Tipps (1988) identify five interpretations of one-half:

1. A unit subdivided into equal-sized parts. This part-whole interpretation gives rise to the familiar "half of a pie" model. Generally the unit is a region.

2. A set subdivided into equal-sized parts. In this part-whole interpretation the unit is a set: "half of the children are girls."

3. A ratio. This use describes the relationship between two quantities: "I have half as much lemonade as you do."

4. An indicated division. This more abstract notion is the quotient of two integers: one-half is the result when one is divided by two.

5. An expression of rational numbers. One-half is the point on a number line that is midway between zero and one.

I have also seen misconceptions formed by the learners because of various ways of representing fractions. The spoken language "one-half" has been taken as one and one-half by many listeners. Using different symbols to represent the same fraction also causes confusion for some learners. In most other areas in mathematics the representation for most ideas remains constant throughout the teaching of those concepts.

There are many good activities to help teach the fraction concepts and so to lessen the confusion found in representation and interpretation, I have chosen to use only one approach throughout this unit. I want to describe only those activities that use the idea of folding and cutting. I believe that many problems with fractions occur in the classroom because the abstract numbers that represent the models are introduced before the basic concepts have been fully developed. The models are usually taken away by the teacher before the learners are ready to give them up. I feel that the learner will give up any concrete models when they feel comfortable with only using the symbols or numbers.

Difficulty level: Average

#37
CIRCLE MODELS

Geometry ideas: shapes (circle, semicircle, quarter circles, pie shapes).

Materials: 7 different coloured construction paper or circles.

Story line: not applicable.

Key Questions:

How could you fold and cut the circle shapes to create equal fractional parts of halves, thirds, quarters, sixths, eighths, and twelfths?

Illustrate the concepts of equivalence, addition, subtraction, multiplication, and division using these fractional models.

Procedure and Start-up:

The learners are given 7 different coloured circles. They are then asked to fold and cut the circles into the following equal pieces: whole, halves, thirds, quarters, sixths, eighths, and twelfths. The thirds, sixths, and twelfths could pose a problem for some learners because they are not allowed to use a measuring device.

Student Report:
Write up your method of folding and cutting to get the various fractional models.
Give an example for each of the operations (equivalence, addition, subtraction, multiplication, and division).

Comments:
With this set of models the fractional concepts that can be developed are; equivalence, addition, subtraction, multiplication and division. The symbolism should be used along with pictures after each learner is comfortable with the models.
Difficulty level: Average

#38
FOLDING ONE CIRCLE

Geometry ideas: shapes, volume, surface area.

Materials: circle shape, activity questions.

Story line: not applicable.

Key Questions: found throughout the activity.

Procedure and Start-up:
This activity begins with a circle. The circle should be at least 15 cm in diameter. The learners are given a set of instructions to follow and questions to answer as they fold the circle. The instructions and questions are as follows:

1. Mark the center of the circle. Fold the circle in half. What is the name of the fold line? Fold in half again. Unfold both folds. Mark the true center. What are the smaller fold lines called? What angle has been formed? How many angles were created by the two folds? How many degrees are there in this angle? How many degrees in a circle?

2. Mark a point on the circumference of the circle. Fold that point to the center of the circle. What is this new fold line called?

3. Using one endpoint of the chord fold outside edge to the center. Then repeat with the last arc and fold it to the center. Diagram #38.1. What shape have you formed? Be specific. This triangular shape will be given the area of one (1), or think of this as a unit.

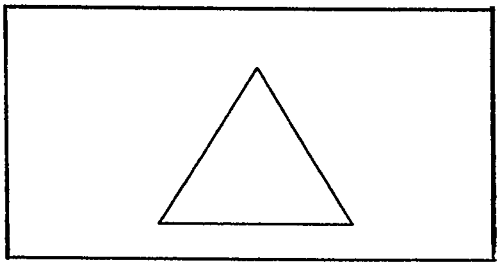


Diagram #38.1

4. Find the midpoint of one of the sides of the triangle. Fold the opposite vertex to that midpoint. What shape have you formed? Be specific. What is the area of this shape, if the area of the original triangle is one unit? How many equal triangles have you formed with this shape?Diagram #38.2. Think back to triangular numbers. Do you see a pattern forming?

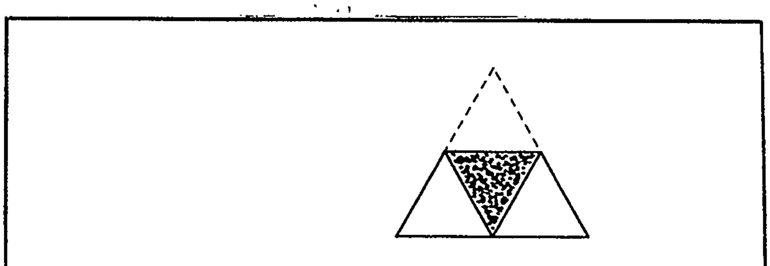


Diagram # 38.2.

5. Fold one of the smaller triangle over the top of the middle triangle. What shape have you formed? What is its area? Diagram #38.3.

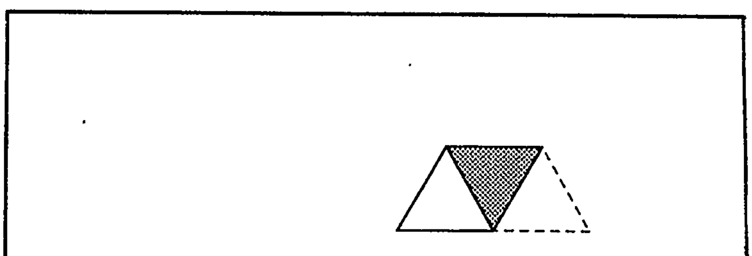


Diagram # 38.3

6. Fold the remaining triangle over the top of the other two. What shape have you created? What is its area?

7. Place the three folded over triangle in the palm of your hand and lift up three triangles to form a 3-dimensional shape.Diagram #38.4What is this shape called? What is the surface area of the shape?

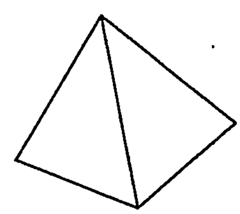


Diagram # 38.4

8. Open up the shape to its original triangle. Then fold each of the vertices to the center of the circle. What shape have you formed? What is the area of this new shape? Diagram #38.5.

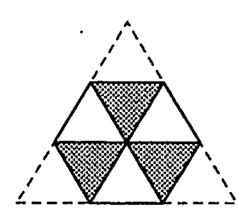


Diagram # 38.5

9. Turn the hexagon over and with a crayon, pen or pencil shade the hexagon. Turn the hexagon over again and gently push toward the center so that the hexagon folds out to form a truncated tetrahedron. What is the surface area of this truncated figure? Diagram #38.6.

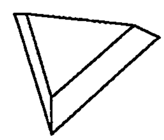


Diagram # 38.6

10. Using only the fold lines already determined, create different polygonal figures and determine there area? Using only the existing fold lines, can you construct figures with the following areas? 1/4, 1/2, 19/36, 23/36, 2/3, 3/4, 7/9, 8/9. Trace around each geometric figure that corresponds to one of the fractions listed. Find as many other fractions as possible.

Student Report:

The learners are asked to hand in the answers to the above questions along with all the equivalent shapes and fractions that they represent.

COMMENTS:

The diagrams throughout this activity are not drawn to scale or accurate in angle or linear measurement. They are merely a pictorial representation of what the shape should look like when folded.

Difficulty level: Average

#39

FOLDING STRIPS

Geometry ideas: points, lines.

Materials: adding machine tape.

Story line: not applicable.

Key Questions:
What methods did you use to fold the strips so that the parts created were equal?
Can you give an example of how to use these folded strips with the creases and labels to create a number line greater than one(1)?
Can you give an example of how to use these folded strips and the number line created to do the operations of equivalence, addition, subtraction, and division of fractions?

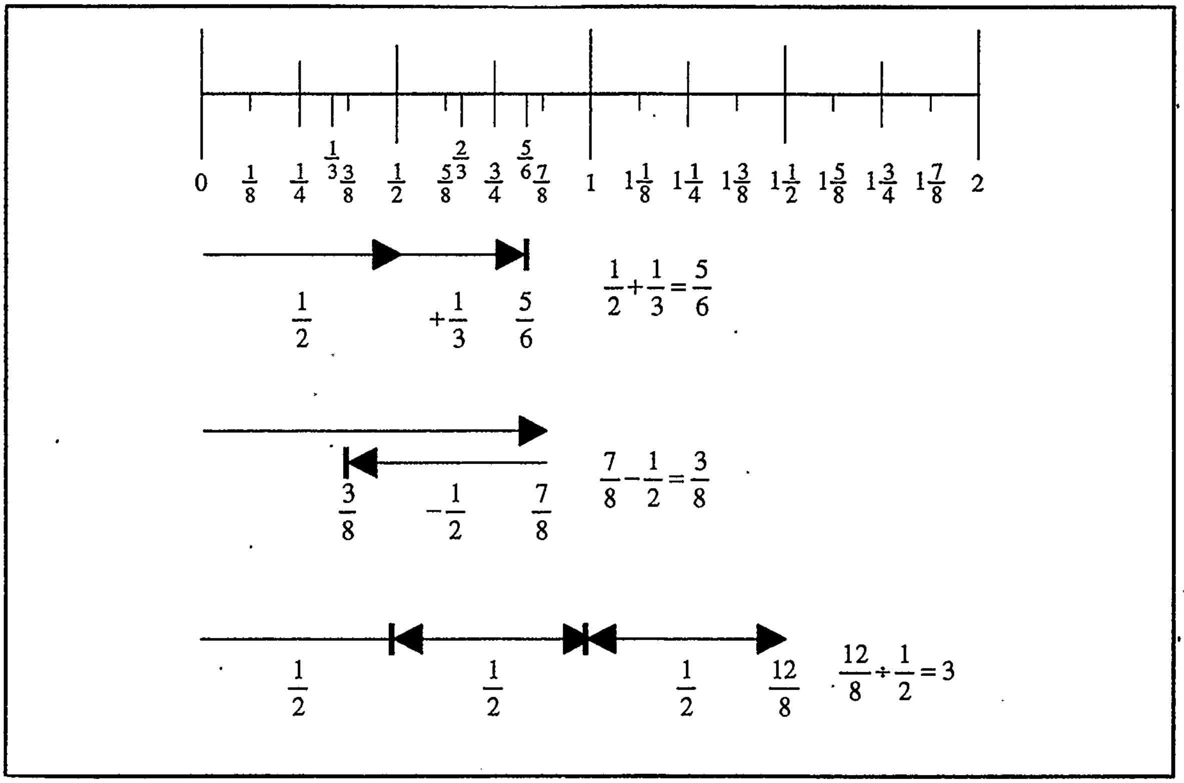
Procedure and Start-up:
The learners are given a group of equivalent paper strips.
They are asked to fold the strips into various parts (2, 3, 4, 5, 6, 8, 10, 12, 16, 24). When the folds are made the number of parts should be labeled and the fold or crease lines should be penciled in and marked by the appropriate fraction number.
Then a number line should be created on a larger strip of adding machine tape using the accumulated information from each of the separate strips and this number line should be extended beyond one unit.

Student Report:
Write up the results of this activity by showing the number line created and examples of how to use these folded strips and the accumulated number line to understand the various concepts in fractions.

Comments:
Difficulty level: Average

The following fractional concepts can be observed by using these folded strips; ordering, greater and less than concepts, equivalence.

When the extended number line is drawn then improper fractions and mixed numerals can be introduced. The use of the separate fraction strips and the accumulated number line created from those fraction strips will allow the learners to do simple addition and subtraction, and division questions.. e.g. 1/2 + 1/3 = 5/6



#40

Folding and Shading

Geometry ideas: shapes.

Materials: paper.

Story line:
The equivalence between (X) and (of) should be established before this paper folding activity begins. I would suggest that going back to examples from the set of whole numbers will establish this point.
e.g. 2 **X** 3 = 6 is the same as saying 2 groups **of** 3 people = 6 people.

Key Questions:
Explain how this folding and shading activity illustrates multiplication of fractions?
A set of multiplication questions that use proper fractions should be available to the learners to practice this folding and shading activity.

Procedure and Start-up:
The learners are given pieces of regular (8.5 X 11) paper. They will be asked to fold and shade in this activity.

Example: 1/3 of 1/2

Step 1: Fold the paper in half.

Step 2: Fold one-half paper into three equal parts.

Step 3: Shade that part (which is 1/3)

Step 4: Unfold the paper and the shaded part shows the answer, which is 1/6.

Give the learners a set of multiplication questions that use only proper fractions. Have the learners practice this folding and shading activity until they are proficient at it. Once the learners are proficient with this folding activity then the symbols (numbers and operation) should be introduced along with the algorithm.

Student Report:
Explain how this activity works.

COMMENTS:

Difficulty level: Average

#41
USING ARRAYS

Geometry ideas: shapes, points, lattice array.

Materials: dot paper.

Story line: not applicable.

Key Questions:

How do you determine the size of the array?

What operations of fractions can this activity perform?

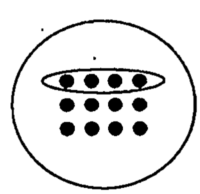
Procedure and Start-up:

Give the learners several sheets of dot paper.

Have them illustrate each of the five operations (equivalence, addition, subtraction, multiplication, and division) using the dot paper.

Give them an example of equivalence and multiplication of fractions, such as:

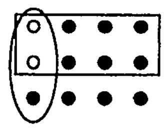
1. Equivalent fractions. 1/3 of 12 dots equal 4 dots.



$$\therefore \frac{1}{3} = \frac{4}{12}$$

2. Multiplication.

1/4 X 2/3 = 2/12.



$$\frac{1}{4}\times\frac{2}{3}=\frac{2}{12}$$

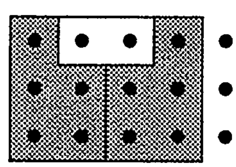
The array of 12 is chosen from the denominators of 3 and 4. Then the fraction 2/3 is found in the rectangle. The fraction 1/4 is found in the circle. The answer is the number of dots found in the circle and rectangle. There are 2 dots out of 12 dots therefore the answer is 2/12.

Then have the learners illustrate the following examples of the other operations.

Examples:

Division.

Divide 4/5 by 1/3



Rectangle $=\frac{4}{5}$

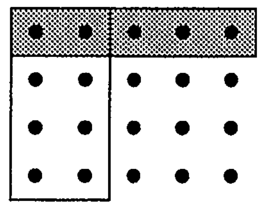
Shaded $=\frac{1}{3}$

Two shaded regions contain 1/3 (5 dots) and the 2 dots are 2/5.

$$\therefore \frac{4}{5}\div\frac{1}{3}=2\frac{2}{5}$$

Addition.

Add 2/5 plus 1/4.



Rectangle $= \frac{2}{5}$

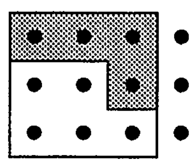
Shaded $= \frac{1}{4}$

Count all the dots in the shaded region (5 dots) and in the rectangle (8 dots).
Total = 13 dots

$$\therefore \frac{2}{5} + \frac{1}{4} = \frac{13}{20}$$

Subtraction.

Subtract or "take away" 3/4 - 1/3.



Rectangle $= \frac{3}{4}$

Shaded $= \frac{1}{3}$

$$\begin{array}{l} \frac{3}{4} = 9 \text{ dots} \\ -\frac{1}{3} = 4 \text{ dots} \\ \hline \frac{5}{12} = 5 \text{ dots} \end{array}$$

$$\therefore \frac{3}{4} - \frac{1}{3} = \frac{5}{12}$$

Student Report:

Write up your results showing how to do the 5 operations of fractions using this array method.

Comments:

I have adapted this activity from an article written by John G. Van Beynen and Robert L. McGinty (1979), Fractions Revisited. The arrays selected for each operation are established by the denominators of the fractions involved. Beynen and McGinty establish at

the beginning of this activity that " for the following work we will be using *arrays as units*, where the denominators of the fractions will determine the size of the array."
Difficulty level: High

#42
DECIMAL RULER

Geometry ideas: points, lines.

Materials: strips of construction paper about 35 cm long.

Story line: not applicable.

Key Question:
What is the best method for dividing up the unit length into 10 equal parts?

Procedure and Start-up:
The learners are asked to design a decimal ruler using the length of a given segment equal to 1/10 or 0.1. The learners are given a strip of heavy construction paper on which to create their ruler. The unit segment they are to use is: ______________
They are then asked to make 10 equal division points between each unit length.
Then this ruler is used to measure various objects around the classroom.

Student Report:
Explain your method of dividing the unit length into 10 equal segments. Show your method.

COMMENTS:
Difficulty level: Average

#43

Fibonacci Sequence and the Golden Ratio

Geometry ideas: rectangular shapes.

Materials: calculator and the Fibonacci sequence.

Story line:

A good introduction to ratios is to look at the film, "Donald in Mathmagic Land." This film provides many interesting visual examples of the most famous ratio, the Golden Ratio.

Key Question:

What is the Golden Ratio? Express in a common fraction form and as a decimal equivalent.

Procedure and Start-up:

Along with this introduction of "Donald in Mathmagic Land", I would use the Fibonacci sequence, 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233, etc. The learners are asked to calculate the number when they divide the consecutive numbers in the sequence, e.g. 8/5 or 21/13. They are asked to make a table of values and to notice the pattern.

Student Report:

Write up your findings and conclusions.

Comments:

Difficulty level: Average

#44

Pleasing Rectangles

Geometry ideas: rectangular shapes.

Materials: a set of rectangle.

Story line: not applicable.

Key Questions:

Which 2 rectangles are the most pleasing to your eye?

Why did you choose those 2 rectangles? Give reasons.

Procedure and Start-up:

The learners are given a sheet of different sized rectangles Diagram #44.1 and ask to choose 1 or 2 rectangles from the group of rectangles that are the most pleasing to their eye.

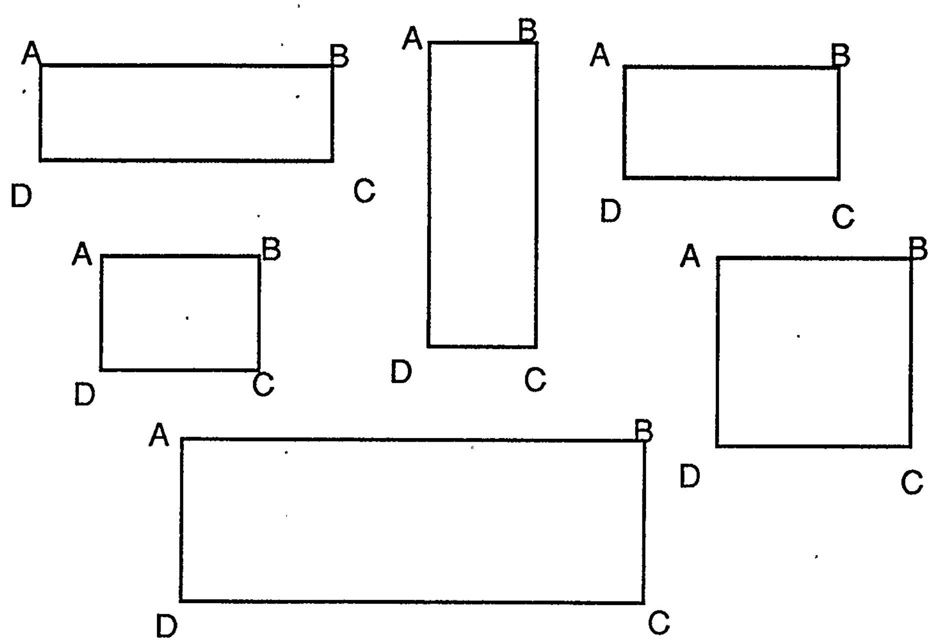


Diagram #44.1

Then they are asked to make a tally and graph the choices of their class.
They are also asked to divide the length by the width for each of their choices.. What interesting number did you find?

Student Report:
Write up the results of you finding giving reasons for your choice and how close were your choices to the golden ratio when you divided the length by the width? Include your tally and graph in the report.

Comments:
The majority of the learners should choose 2 or 5 which best fit the golden rectangle ratio.
Difficulty level: Easy

#45
CONSTRUCTING A GOLDEN RECTANGLE

Geometry ideas: Measurement and construction skills, rectangular shapes.

Materials: paper, compass, ruler.

Story line: not applicable.

Key Questions:
What doės a golden rectangle look like?
What is the Golden Mean or Ratio ?

Procedure and Start-up:
The learners are given a set of instructions to follow.

1. Draw a line segment 233 mm, label it AB.
2. At point A construct a right angle and a line segment 144 mm in length. Label the

segment draw AD. Complete the rectangle by drawing the other segment and label the vertex C.

3. On AB, locate a point E so that AE = AD.
4. On DC, locate a point F so that DF = AD.
5. In the smaller rectangle locate a point G on BC so that CG = FC.
6. On the line EF locate a point H so that FH = FC.
7. Continue this pattern in the smaller rectangle so that a square is formed equal to the length of the width of that rectangle.

Student Report:

Hand in your results of the construction and findings.

Comments:

I have chosen the two lengths to match two consecutive numbers in the Fibonacci sequence, 233 mm and 144 mm. These 2 numbers when divided will give theapproximate Golden Ratio or Mean.

Difficulty level: Average

#46
THE PENTAGRAM

Geometry ideas: construction skills, pentagram.

Materials: paper, compass, ruler.

Story line: This was the symbol of the Pythagorean Society.

Key Questions:

What ratio do you notice when measuring the lengths of the various lines?

How many lines can you find that illustrate this ratio? Illustrate these.

Procedure and Start-up:

The learners are asked to construct the Pythagorean Society's symbol, the Pentagram.

From that pentagram find all the segments that illustrate the Golden Ratio.

Student Report:
Illustrate your construction of the Pentagram and all the line segments that illustrate the golden ratio.

COMMENTS:
Difficulty level: High

#47

THE SPEAR THROWER

Geometry ideas: measurement skills.

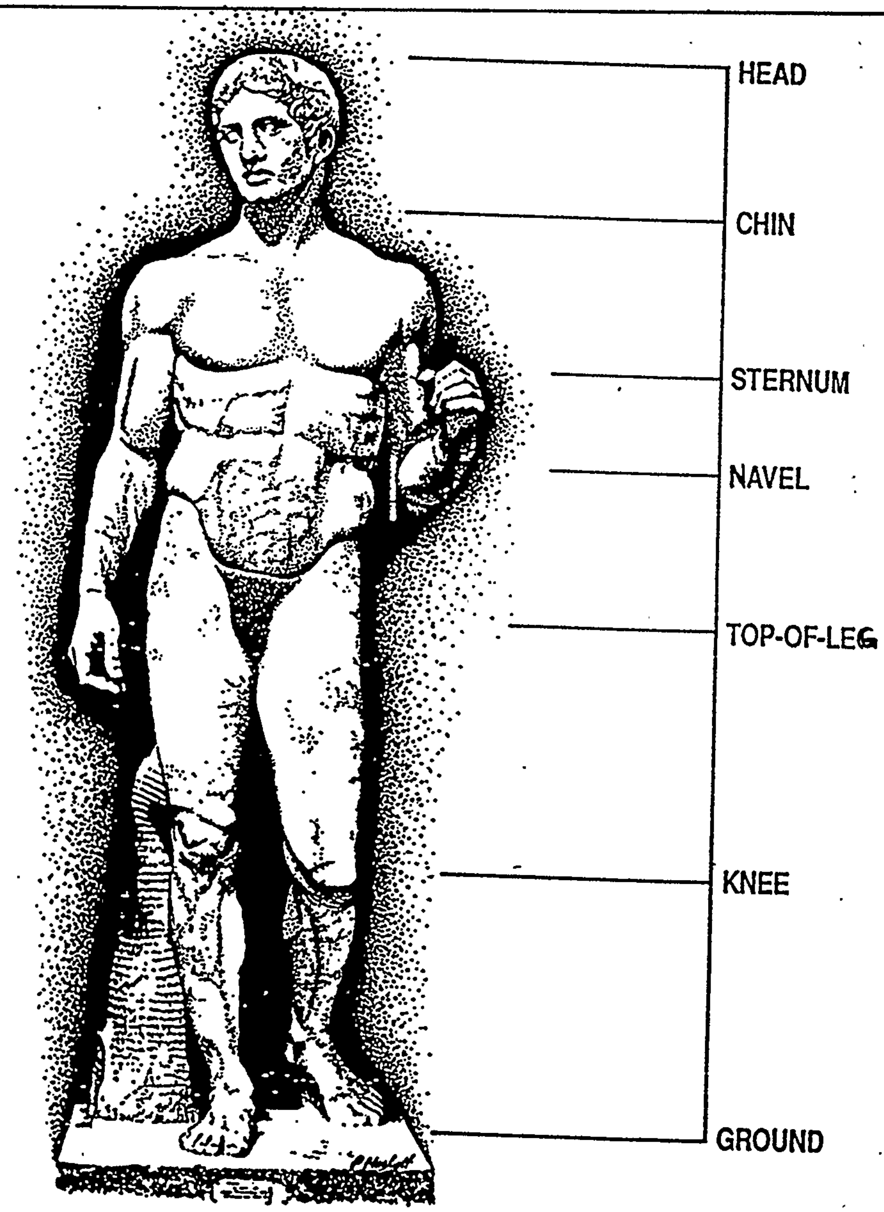
Materials: Picture of "The Spear Thrower" by Polyleitos of Argos, measurement tools.

Story line:
Another activity that illustrates the Greeks passion for this ratio, is found in their art. " The Spear Thrower" by Polyleitos of Argos, a 7' bronze statue sculpted in Greece in 440 BC. has many lengths that have the Golden Ratio property.

Key Question:
Measure the body and find the many Golden Ratios in the "Classic Greek" body.

Procedure and Start-up:
A picture of this statue should be given to the learners and they are asked to measure the various body parts (head, neck, upper body, legs, arms, etc.) and to compare various measures of body parts that illustrate the Golden Ratio.

Student Report:
Write up your results showing as many comparisons that illustrate the Golden Ratio.



"The Spear Thrower," by Polyleitos of Argos,
a 7' bronze statue sculpted in Greece in 440 BC.

#48
A SMILE CONTEST

Geometry ideas: measurement skills.

Materials: tools for measuring.

Story line:
Many of the best smiles are attributed to the face, eyes, lips, and teeth having the golden proportion. Edwin I. Levin (1978) in an article on dental esthetics, shows how many of the qualities of a good smile are quantifiable. In this contest the learners are asked to choose a male and female who they feel exhibit the best smiles. Once this is done various measurements are taken of the face and teeth and the golden mean is calculated. The learners are asked to do similar measurements on each other's faces and see if the golden mean is found. The learner should choose partners who they feel comfortable with. This is an optional activity but can prove to "put a smile on their faces."

Key Questions:
Calculate the ratios after various facial measurements are taken?
How do they compare to the Golden Ratio?

Procedure and Start-up:
Ask the learners to choose a male and female in the class that they feel exhibit the best smile.
Have several learners who feel comfortable with the two subjects chosen do the following measurements as suggested by Levin (1978):
1. Measure the width of the central incisor and the lateral incisor.
2. The distance from the nose to the incisal edge of the central incisor compared to the distance from that edge to the chin and that to the total distance.
3. The interdental papilla tip divides the length of the clinical crown of the central incisor. Measure these two distances.
4. Measure the length of the smile and the width of the opening of the lips.

5. Measure the width of the eye and the width between the eyes.
The ratio should be calculated and each learner that is participating should see how close they come to the golden ratio.
The other learners should choose partners with whom they feel comfortable making measurements and they should follow the same procedure listed above.
If some learners feel uncomfortable with this activity then they should search through some magazines until they find a male and female picture whom they feel have the best smile and then follow the above procedure.

Student Report:
Write up your or your group results of the findings and hand in report to the teacher.

COMMENTS:
Difficulty level: High

#49
MAP OF CALGARY

Geometry ideas: scale drawings.

Materials: map of your town or city, measurement tools.

Story line: not applicable.

Key Questions:
Make up a set of questions using the map of Calgary and the given scale or ratio found on that map.

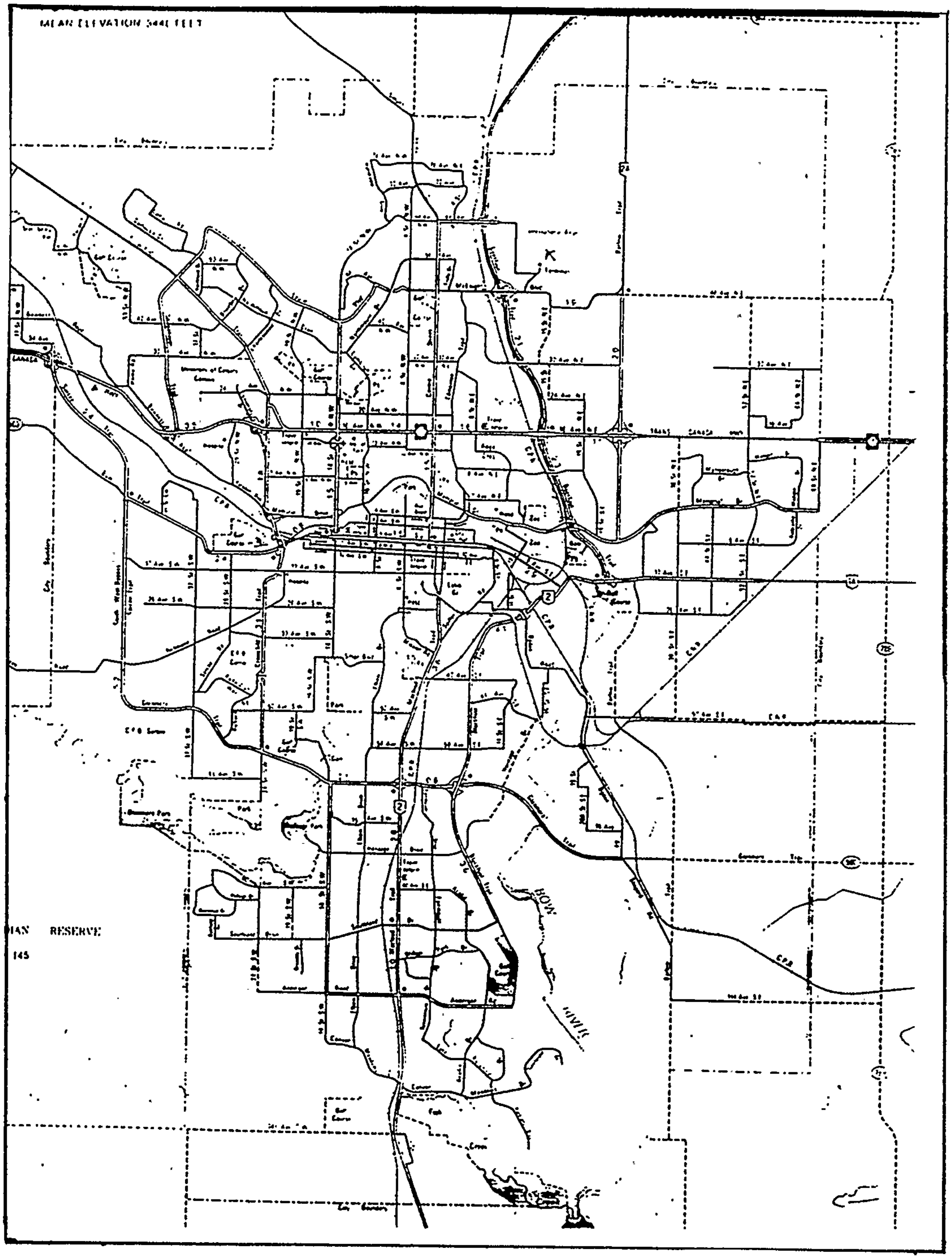
Procedure and Start-up:
Each learner is given a map of Calgary with a scale or ratio.
The learners, using string and the scale given on the map of Calgary, are asked to develop questions and the answers using familiar places in the city.
Once these questions are made, the groups exchange questions and find the answers.

The answers are then compared for accuracy.

Student Report:

The learners are asked to hand in their sets of questions.

They are also asked to hand in the answers that they got after exchanging questions with another group of learners.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#50
Environmental Problem Solving

Geometry ideas: estimation, shapes, symmetry, similar triangles, perimeter, area, measurement skills.

Materials: string, meter sticks, compasses.

Story line:
Every year the Grade Seven learners from our school go to Fish Creek Park in Calgary (these activities could be done in most parks). The following problems are given to the students once they arrive at the park. The materials that the students might request should not be shown so that the originality of the students in solving the problems is not interfered with.

Key Questions:

1. Estimate the height of a tall tree chosen by your teacher and tell, using diagrams, how you did it.

2. How deep is the Fish Creek off the main bridge? Draw a picture or a diagram to show how you arrived at the answer.

3. What is the approximate perimeter of a poplar leaf or any large leaf? Explain how you arrived at the answer.

4. What is the approximate speed of the Fish Creek? Explain how you arrived at your answer.

5. About how many blades of grass are there in a square metre? Estimate how many blades of grass are found in the Park. Explain and show your answer and calculations.

6 Find as many examples of symmetry as you can. Display your findings when you get back to school in the form of a tessellation.

Procedure and Start-up:

The field trip needs to be arranged.

Give out the questions or problems to be solved at the park location.

Have learners work in small groups.

The teacher should only give out materials after a request is made by a group of learners.

The teacher should circulate among groups to answer any questions that may arise.

Student Report:

The solution and how the problem was solved should be handed in to the teacher.

COMMENTS:

Difficulty level: Average

#51

SCALE DRAWINGS

Geometry ideas: scaling.

Materials: baby picture, or other pleasing picture, construction paper, measurement tools.

Story line: not applicable.

Key Question:

What scale was use to increase or decrease your photo?

Procedure and Start-up:

The learners are asked to choose and bring a small baby picture of themself to class. They are then asked to use scale drawing and grids to create a portrait of themself that is 2, 3, or 4 times bigger. The small and large pictures are displayed on a poster and in the class-room. A guessing contest could be done between two Grade 7 classes if you teach both. If the learners are reluctant to do a portrait of themself, then any picture that the learner finds interesting could be used.

Student Report:
The scale used to increase the photo should be displayed on the construction paper and then the scale drawings should be handed in.

COMMENTS:
Difficulty level: Average

#52
GUESS MY RULE OR "I SAY; YOU SAY."

Geometry ideas: shapes.

Materials: paper.

Story line: not applicable.

Key Question:
What arithmetic rule is being used to come up with the second number?

Procedure and Start-up:
The teacher and class should play a couple of rounds of this game. The teacher asks the learners to give him any number. The teacher responds with a number. Then asks, "What is my rule?" If no one responds with an answer then a second number is asked for from the learners. The teacher again responds with a number that illustrate his/her rule. The learners are again asked, "What is my rule?"
An example: Class says 2. Teacher responds 4. Class says 6. Teacher responds 12. Rule is multiplying by 2 or double learners' number.
Now learners are asked to pair off and try the game.
The game ends when the person giving the number has figured out the rule of the other

person. Reasons must be given explaining the rule. Several rounds should be played until both partners are efficient rule breakers.

Comments:

This activity is played like a game but it leads to mathematical connections in algebra, tables for ordered pairs and finding relational equations. The patterns, and ways of figuring out those patterns by the learners, will often show originality, creativity, and some interesting logic. The rules usually can be expressed by algebraic symbols but for this activity that is a secondary objective. If the learners keep a record-in a chart form- of the numbers and responses given, then the ordered pairs can be graphed on the Cartesian Plane and the relationship can be seen visually.

COMMENTS:

Difficulty level: Average

#53
THINK OF A NUMBER?

Geometry ideas: shapes.

Materials: envelopes, poker chips or discs.

Comment:

This activity begins by using mental arithmetic skills and then progresses to a visual representation using geometric shapes and then finally to the algebraic symbols, variables and constants. The distributive property is used and gathering similar terms is introduced and used frequently in this activity.

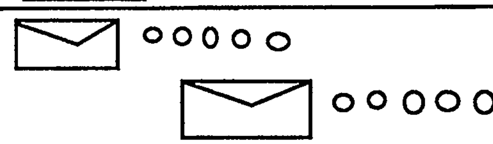
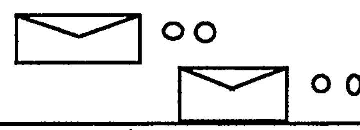
Key Question:

What is the number?

Procedure and Start-up:

The learners are given envelopes, and poker chips or discs. The number thought of is written on a piece of paper and placed in the envelope. The learners use these visual ob-

jects to show what is happening in the mind.
An example:

Think of a number	
Add 5	
Double it	
Subtract 6	
Find 1/2 of the result	
Subtract the original number	

The answer will always be 2, which is illustrated by the pictures. The learners work with each other developing verbal sequences of mental arithmetic and then the other student illustrates these with models and numbers.

Secondly, the learners are guided into the algebraic representation and properties used. The algebraic and numerical representation of the above sequence is:

Algebraic	Numerical
x	2
x + 5	2+5=7
2 (x + 5)= 2x + 10 (Distributive Property)	2(7)=14
(2x + 10) - 6 = 2x + 10 - 6 = 2x + 4 (Gathering like terms)	14-6=8
1/2 (2x + 4) = x + 2 (Distributive Property)	1/2(8)=4
(x + 2) -x = x -x + 2 = 2 (Gathering like terms)	4-2=2

Thirdly, when an algebraic equation is given to the learners to solve, they will be asked to first verbalize it in words, then illustrate it with models (envelopes and discs), gather like terms and use properties, and then check for numerical plausibility by using only numbers. I will only illustrate one example in this thesis but when teaching this unit I would start with the simplest equations and work through to the more complicated equations. Finally when I feel, or the learners feel, they can handle the equations without models, then I would show them only the algebraic method.

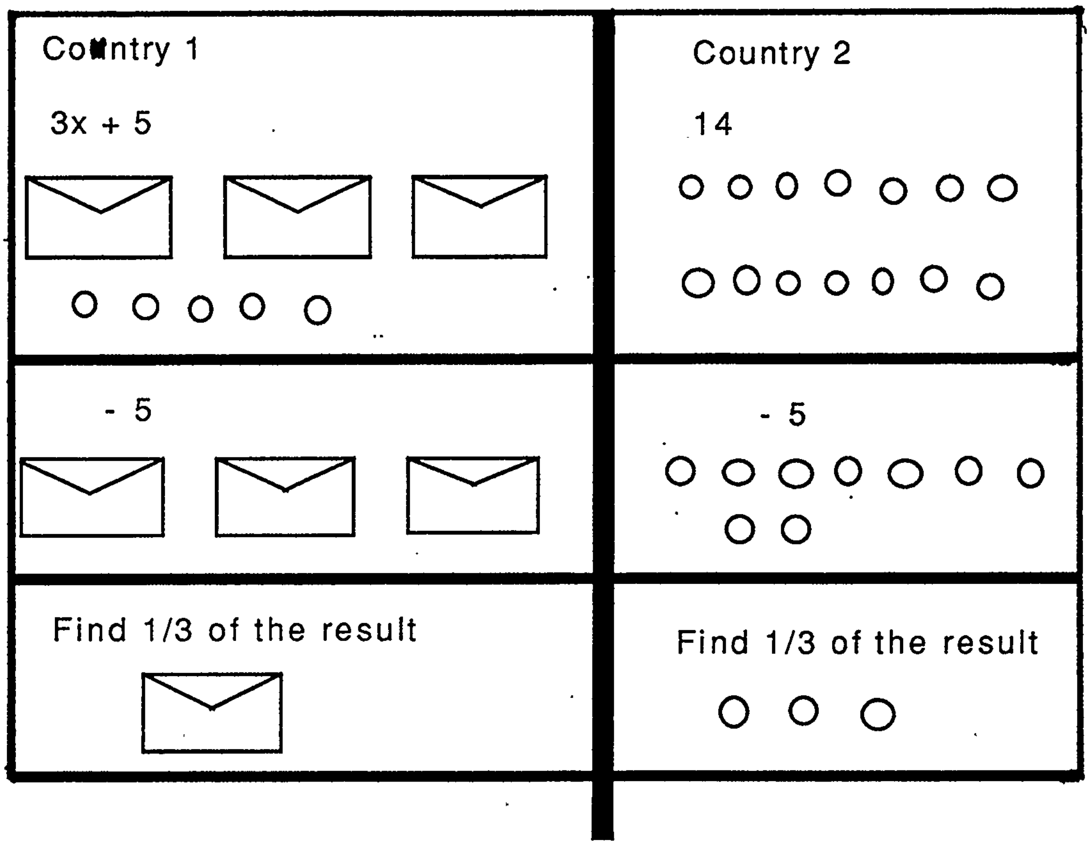
An example: $3x + 5 = 14$.

The first idea that needs to be developed is the idea of equal (=). The learners need to understand that what is on the left-hand side of the equal sign is exactly the same as what is on the right-hand side even though the symbols look different. To help the learners during the modeling stage, I put a piece of masking tape on each learner's desk splitting it in half. The learners are to think of the two halves as two islands with water between or two countries with a border between them and that if something happens in one country then the identical thing must happen in the other country.

Verbal stage:

x	think of a number
3x	multiply it by 3
3x + 5	Add 5
3x + 5 =	is the same as or equals
3x + 5 = 14	fourteen or what is your answer?

Visual or model stage: $3x + 5 = 14$



Numerical stage:

3	$3x + 5 = 14$
3x3=9	$3\ (3) + 5 = 14$
9 +5 =14	$9 + 5 = 14$
14=14	$14 = 14$

Algebraic and Property stage:

Before the formal stage it is good to go back to the verbal stage and then reverse the steps.

Think of a number	x
Multiply by 3	3x
Add 5	3x + 5

The number in the person's head should be 14.

Reversal of steps:

What is the number in your head:	14
Subtract 5:	14-5=9
Divide by 3 or take 1/3 of the number:	9/3 or 1/3 (9) = 3

The number you thought of is 3.

Formal way: (Do steps in reverse order)

$3x + 5 = 14$

$\quad - 5 = -5$ (Identity Element)

$3x \quad = 9$

$1/3\ (3x) = 1/3\ (9)$ (Multiplicative Inverse)

$x = 3$

The learners are given various equations and asked to verbalize and visualize each equation to arrive at the answer.

Student Report:

The learners are asked to hand in the results of their visualization and algebraic representations.

Comments:

This activity builds on the previous activity "Think of a Number." The learners now are working the problem in reverse or the inverse. I have found that a tremendous amount of mathematical thinking takes place and concepts are solidified when the inverse operation is taught and worked on by the learners. Some examples of inverse concepts are subtraction, and division. When these are understood then the concepts of addition and multiplication are usually fully understood. Another inverse that requires much more understanding and mathematical thinking is that of factoring polynomials. The multiplication of polynomial is usually done in a rote fashion but factoring requires a depth of thinking and understanding and cannot be done in a rote fashion. I feel that good teaching needs to be done always, using visuals and concrete materials, but especially when the inverse operation is being taught. The concept of solving equations requires a careful flow of steps so that the abstract ideas developed at the end give meaning and understanding to the learners.

COMMENTS:
Difficulty level: Average

#54
COORDINATE DRAWINGS

Geometry ideas: shapes, points, lines.

Materials: Cartesian coordinate plane, geometry tools.

Story line: not applicable.

Key Question:
What order do the numbers represent?

Procedure and Start-up:
The learners are asked to create a geometry shape or a picture of some object on the coordinate plane and then generate the points (ordered pairs) from these pictures. The ordered pairs are exchanged with other learners and the pictures are created.

Student Report:

The learners are asked to hand in their drawings and generated ordered pairs.

Comments:

There are several types of graphing skills that need to be developed with the Grade Seven learners. The connections from Algebra and its relational qualities can be graphed on a Cartesian coordinate plane.

COMMENTS:

Difficulty level: Average

#55
MAZES

Geometry ideas: lines, points, rays, and shapes.

Materials: maze puzzles.

Story line: not applicable.

Key Question:

What is a basic rule to follow in order to move successfully through a maze?

Procedure and Start-up:

Have the learners look at and work through a variety of mazes and then have the learners create an intricate maze using various geometric ideas and shapes.

Student Report:

Have the learners hand in their creative maze and a solution to the key question.

COMMENTS:

Difficulty level: High

#56

TREAD DESIGN ON SNEAKERS

Geometry ideas: shapes, lines.

Materials: learner's sneakers, pencils, crayons, Jiffy markers.

Story line: not applicable.

Key Question:
Classify and list the various geometric ideas found on the tread of a sneaker.

Procedure and Start-up:
The learners are asked to make crayon, pencil or Jiffy pen rubbings of the various types of treads found on their classmates runners. The geometric designs are then classified, organized and graphed using an appropriate graph for their information gleaned from the rubbings.

Student Report:
Write up the results showing the types of geometry used, the graphs of ideas, and conclusions.

COMMENTS:
Difficulty level: Average

#57

DICE GRAPHING

Geometry ideas: points, lines, coordinate plane.

Materials: Cartesian coordinate plane, dice.

Story line: not applicable.

Key Questions:
What is the probability of rolling the dice only 36 times and marking off all the points?
How many rolls of the dice did it require to finish a game?
What is the difference between the experiential and theoretical probabilities of the game you played?

Procedure and Start-up:
This game is played by 2 learners. A green and red dice are used as well as a coordinate plane(first quadrant only) and up to points (6,6). Red being the x-axis and Green being the y-axis. The first player rolls both dice and marks off the corresponding point on his coordinate plane. The second player rolls the dice and marks the corresponding point on his coordinate plane. The first player to successfully mark off all points is the winner. The players are asked to keep a tally of how many times they rolled the pair of dice.

Student Report:
Answer the key questions and then for each game played make a table of values and draw a graph illustrating the differences between the theoretical and experiential probability.

COMMENTS:
Difficulty level: Easy

#58
BATTLESHIP

Geometry ideas: points, lines, coordinate plane.

Materials: coordinate plane paper.

Story line: not applicable.

Key Questions:
What is the best strategy for winning this game?

Procedure and Start-up:

This game is also played on the coordinate plane except all quadrants are used. Each player hides his/her ships on points corresponding to ordered pairs. The players take turns giving out ordered pairs and trying to sink the other pla'yers ships. If a player makes a strike, s/he guesses again. The winner is the player who successfully sinks all of the other' players ships.

Student Report:
Write up your best strategy for winning at the game of Battleships.

COMMENTS:
Difficulty level: Average

#59

HIDDEN TREASURES

Geometry ideas: points lines, coordinate plane.

Materials: coordinate plane paper.

Story line: not applicable.

Key Question:
What is the best strategy for winning at the game of Hidden Treasures?

Procedure and Start-up:
This game is also played on the coordinate plane (1st quadrant) by 2 players. The plane goes to (10, 10). One player hides the treasure at a certain ordered pair. The other player guesses points until they have located the treasure. After each guess the player who hid the treasure gives one direction, such as, "go north" "go south" "go north-east" etc. The object is to find the treasure in the least amount of guesses. When the player has found the treasure then the players switch roles. The winner is the person with the least number of guesses.

Student Report:
The players are asked to describe their strategy for finding the treasure in the least amount of moves.

COMMENTS:
Difficulty level: Average

#60
CARTESIAN RACE

Geometry ideas: shapes, lines, points.

Materials: coordinate plane.

Story line: not applicable.

Key Question:
What is the best strategy for winning at the game of Cartesian Race?

Procedure and Start-up:
This is also a strategy game played on the first quadrant of the coordinate plane. The play begins at the bottom left corner where a player puts an x in the square. The next player can put an x into a square; directly above or directly to the right of, or diagonally above and to the right of the last mark made by his opponent. The players alternate moves until one player reaches the square at the top right hand corner.

Student Report:
Write up your winning strategy in a paragraph.

COMMENTS:
Difficulty level: Average

#60
PASCAL'S TRIANGLE

Geometry ideas: shapes.

Materials: penny, nickel, dime, and quarter.

Story line: not applicable.

Key Question:
What is an easy way of finding the results for any number of different coins?

Procedure and Start-up:
Each learner is given 4 different coins (penny, nickel, dime, quarter). The learners are asked to generate all the different combinations of heads and tails using these four coins. Using 3 different coins; Using 2 different coins ; Using 1 coin; Using 0 coins. The learners are asked to organize the data so that they could determine the number of different combinations using 5 different coins, using only the patterns generated from the previous experiment.

Student Report:
Once the triangle has been completed ask the learners to find interesting patterns that have already been discovered from previous investigations. (Triangular numbers, Pyramidal numbers, Fibonacci sequence. etc.)
Write up results showing the table of values, the organized data, the different patterns discovered, an algebraic formula, and any conclusions.

COMMENTS:
Difficulty level: High

#62

POINTS ON A CIRCLE

Geometry ideas: points, lines, circle shapes.

Materials: compass, ruler, paper.

Story line: not applicable.

Key Question:

What is the number of different ways to draw and label the four points on the circumference of a circle?

Procedure and Start-up:

1) The learners are asked to draw a circle.

2) They are to place 4 points on the circumference of the circle.

3) They are to start at any point on th circumference of the circle and trace the figure completely, then return to the original point.

4) Then they are to name the points A, B, C, D, and then name the shape drawn in as many different ways as possible.

5) Then steps 1 to 4 should be repeated for a different way of tracing the figure starting at a different point. When this is completed re-do steps 1 to 4 again starting at a different point than the first two times.

Student Report:

Write up all of your findings including your diagrams, list of different ways to name the figures drawn, and a simple way to find the number of different ways to draw and label four points on the circumference of a circle.

Comments:

The teacher may have to start out this activity showing how to draw and label and then let the learners complete the investigation.

The investigation should lead to 24 different possibilities or permutations. (4x3x2x1=4!=24)

Difficulty level: High

#63
SYMMETRY

Geometry ideas: symmetry.

Materials: pennies, newspapers, magazines, yellow pages of a telephone book, pictures of flags of various countries of the world, paper.

A) PENNY PROBLEM.

Story line:
CHALLENGE: Sourdough Sam and Pecos Pete were sitting around the table in the mess hall. " Sam, dig out your silver dollars," ordered Pete. " You and I are going to have a little contest. We'll take turns placing silver dollars around the edge of this table. If you're able to put the last dollar on the edge you get to keep all of the money. Otherwise I win. Just to give you a fighting chance I'll let you start first."

Key Questions:
If you were Sourdough Sam would you accept the challenge?
How would you play to win?

Procedure and Start-up:
To investigate this challenge, the learners are given a bag of pennies and asked to play this game or the challenge given in the story of Sourdough Sam.

Student Report:
Write up your strategy for winning.
Challenge 2:
This game is played in pairs. Suppose that each of you has a bag of pennies. You decide to take turns placing a penny on the desk. Each penny must lie completely on the desk, none can overlap, and once a penny is positioned it cannot be removed. The last person able to place a penny on the desk wins both bags of pennies.

Key Questions:

How would you play to win?

Do you think it is possible to have a winning strategy regardless of the size of the desk? Shape?

If so, do you want to have the first turn or the second? Where would you place the first penny?

How would you make each successive play to ensure winning?

Student Report:

Answer the key questions.

COMMENTS:

Difficulty level: High

B) TRADEMARKS, FLAGS, AND HUB CAPS.

Story line: not applicable.

Key Question:

What geometric concept is used in each of these activities?

Procedure and Start-up:

Choose one of the following: Trademarks, Flags, or Hub Caps.

1.Trademarks.

Find as many trademarks in the newspapers, magazines, and the yellow pages.

a. Make a display of all of the trademarks that you have found and draw in all of the lines of symmetry on the trademarks.

b. Create your own trademark for a company that you are planning on starting.

2. Flags.

Find as many flags as you can or draw them.

a. Make a display of all of the flags that you have found or drawn and draw in as many lines of symmetry as you can.

b. Create your own flag for a country that belonged exclusively to yourself.

3. Hub Caps.

Find examples of hub caps in magazines.

a. Make a poster of these hub caps and draw in lines of symmetry.

b. Create your own hub cap design.

Student Report:

Display your posters on a bulletin board in the classroom.

COMMENTS;

Difficulty level: Average

C) A WARDROBE.

Story line: not applicable.

Key Question:

What geometric concepts did you use in creating your wardrobe?

Procedure and Start-up:

Using geometric shapes, folding and cutting, and ideas of symmetry, create or design a wardrobe for yourself. The wardrobe should include at least 2 outfits for different seasons of the year.

Student Report:

The results of your wardrobe should be made into a poster and displayed in the classroom.

COMMENTS:

Difficulty level: High

#64
TRANSFORMATIONAL GEOMETRY

Geometry ideas: slides, flips, and turns.

Materials: diagrams of problems, 36-4cm X 4cm construction paper, large poster paper.

A) AMUSEMENT PARK.

Story line:
Plans for a new amusement park include a refreshment stand that will be located on the main street, MS. The main street is located near the roller coaster (R) and the bumper cars (B).

Key Question:
Where should the refreshment stand be built so that the distance from the roller coaster to the refreshment stand and then to the bumper cars will be as short as possible?

Procedure and Start-up:
The learners are given a drawing of the problem. They are to solve the problem showing the answer. A systematic method and proper drawings should be made.

Student Report:

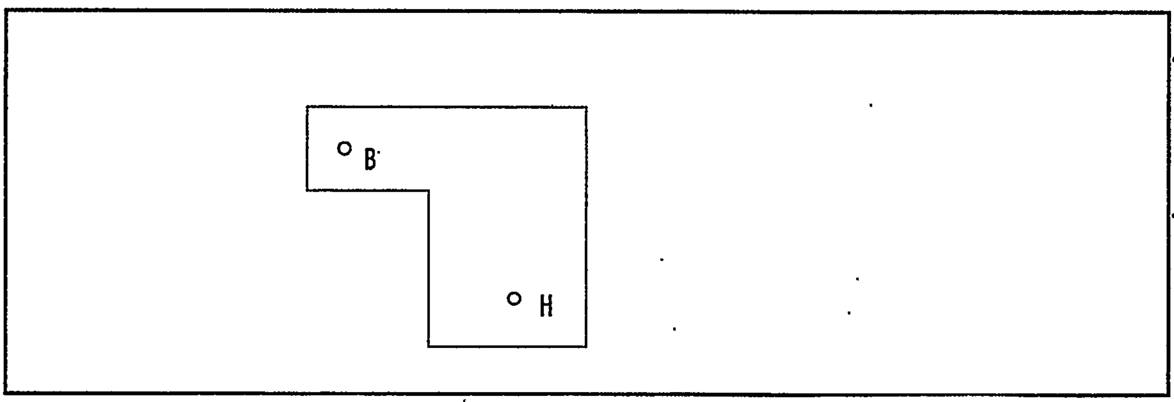
Write up the results showing your systematic method. Show diagrams

Comments:

The learners should use some type of transformational geometry concept in drawing the diagram.

Variation:

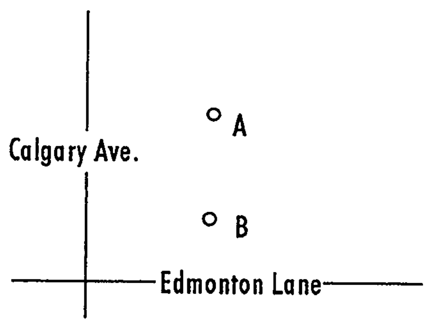
1.The amusement park has a miniature golf course. The green around each hole is bounded by a wooden frame. You are playing a game of miniature golf and are at hole #4. Point B represents the location of the ball. Point H represents the position of the hole. Can you make a hole in one? Show the path of the ball.



B) THE ROUTE.

Story line:

A policeman at station A wishes to leave the station, check traffic on Calgary Avenue, and then drive to Edmonton Lane to check the traffic there. After he has checked the traffic on these two streets, he must go to the county seat located at B.



Key Question:
What is the shortest route he can take?

Procedure and Start-up:
Give the learners a drawing of the problem.
Learners are asked to use drawings to illustrate their answer.

Student Report:
Write up your results showing diagrams and systematic method used to solve the problem.

COMMENTS;
Difficulty level: Average

C) BUILDING A PARKING LOT.

Story line: not applicable.

Key Questions:
What is the rectangular parking lot that is the cheapest to fence?
What is the most expensive?
What is the cheapest to pave?

What is the most expensive to pave?

Procedure and Start-up:
Students are given 36 square pieces of construction paper, 4 cm x 4 cm in length. Each square piece of paper is enough area to park one car or room to drive in or back out of a parking stall. Your task is to build as many different rectangular parking lots as possible. Display each on a bigger piece of paper. Record all lengths, widths, perimeters, and areas. Show all of your work.
Name all of the different sized rectangles, if you had 25-4 cm X 4 cm squares, then 64- 4 cm X 4 cm squares. Do not draw all of the rectangles but name them from the information learned from doing the 36 squares.

Student Report:
Write up the results showing all diagrams, answers to questions, and conclusions.

COMMENTS:
Difficulty level: High

#65
TESSELLATIONS

Geometry ideas: Shapes, angles.

Materials: construction tools, paper.

Story line:
The word *tessellation* comes from a Latin word meaning “to pave with tiles” or “ to make a mosaic of.”

A) TILING A FLOOR.

Key Questions:
Can I tile a floor or a bathroom wall with any other shape than a square? If so, what other shapes would work?

Why does this shape work or not work?

Procedure and Start-up:

Some basic rules must be mentioned at this time. They are:

1. The tiles should not overlap and,
2. No space should be left between tiles.
3. The edges of the paper are not important since the shapes can be cut to fit into the larger area but what is important is that in the middle of the area to be covered there is no overlapping or space left.

The learners begin with triangles (all types).

Then they try all types of quadrilaterals.

Then they should try regular shapes that have sides greater than 4.

The learners should show diagrams for each shape that works.

Student Report:

Write up the results showing illustrations of each type of polygon that works, reasons why they work and conclusions.

COMMENTS:

Difficulty level: Average

B) SEMIREGULAR TESSELLATIONS.

Key Questions:

What combinations of geometric shapes will create a tiling pattern?

Can you develop a method for naming your tessellation? Explain your method.

Procedure and Start-up:

The learners are asked to find combinations of geometric shapes that will create a tiling pattern that fit the rules in activity A. Draw as many diagrams of semiregular tessellations that work.

Student Report:
Write up your results showing diagrams of tessellations, method of naming your tessellations, why these combinations work and conclusions.

COMMENTS:
Difficulty level: High

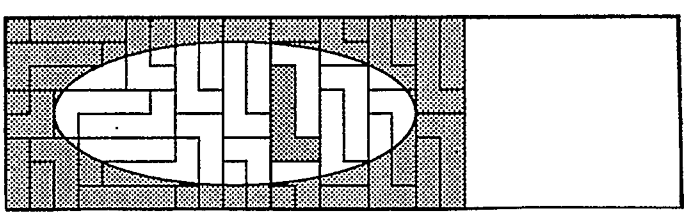
C) CAPITAL LETTERS/BLOCK NUMBERS.

Key Question:
Using only capital letter or block numbers can you find which of these would work to tile a room or a wall of a bathroom?

Procedure and Start-up:
Using only capital letters and block numbers, draw diagrams of each capital letter and block number that will tile a floor or wall.
Use the same rules found in activity A.

Student Report:
Write up your results showing diagrams of tessellations, why these letters and numbers work and conclusions.

COMMENTS:
Each part of the block letter must have the same thickness orwidth so that the arms of the letter will exactly fit into the spaces of the other letter. See diagram below.



Difficulty level: High

D) PENTAGONS.

Key Questions:
Which pentagonal figures tessellate?
Why do these pentagonal shapes tessellate and others will not? Explain your reasoning.

Procedure and Start-up:
Design as many pentagon shapes that will tessellate. Illustrate your findings. Use the same rules as found in activity A.

Student Report:
Write up the results showing illustrations of each type of pentagon that tessellates, reasons why they work and conclusions.

COMMENTS:
Difficulty level: High

E) UNIQUE SHAPES THAT TESSELLATE.

Key Questions:
How could you cut and paste a triangular shape or a quadrilateral shape, preserving the basic area of that shape and create a shape that will tessellate?
What method(s) did you use in cutting and pasting?

Procedure and Start-up:
Using a triangle or a quadrilateral that tessellates, by cutting and pasting create your own unique shape that will tessellate.
Use shading and/or creative design to make a unique wallpaper design. Display your wallpaper on poster paper.

Student Report:
Show your method(s) of cutting and pasting, using diagrams and illustrations, and display your finished poster or wallpaper design.

Comments:

Geometry and Art are connected in many ways. Previously, I have mentioned some areas where geometry ideas connect nicely with art ideas. The art contest, tessellation art (wallpaper design, floor coverings), scale drawings using grids and the Greeks' use of the Golden Ratio in art works are all ideas that can be used in the classroom while teaching geometry. Some projects or optional ideas that usually are of interest to some learners and that I want to mention briefly are:

a. String designs.

b.Perspective Drawings.

c. Illusionary Art.

These art ideas are mentioned in case some learner see connections from some activity and would like to pursue that interest. I have chosen not to describe these in detail since many good books have been written about each of these areas and the learners are free to do their own research if they so choose.

Difficulty level: High

#66
TIC-TAC-TOE

Geometry ideas: shapes, points, lines, spatial relations.

Materials: Tic-Tac-Toe game boards, Unique design Tic-Tac-Toe game boards, 4 X 4 grids side by side.

Story line:

I have listed this as a separate topic because there are some games that learners have found intriguing from birth to death. The one set of games that has appeal to all ages is that of Tic-Tac-Toe. I feel that the very young can play with a great deal of understanding as well as the adult. The majority of learners have never analyzed the winning combinations of the simple game of Tic-Tac-Toe, and the reason for its continued appeal throughout adult life.

Key Questions:

What is a winning strategy in the game of Tic-Tac-Toe?
Is there a way to get "the cat" to win each time?

Procedure and Start-up:

The learners are asked to play a few rounds of Tic-Tac-Toe and then analyze the games that were won or tied.

Student Report:

Write up your analysis of a winning strategy, illustrate with examples of winning combinations and give some conclusions.

Comments:

Using the above format use the following variations for the game of Tic-Tac-Toe.
Variation 1: The player who gets three in a row is the loser.

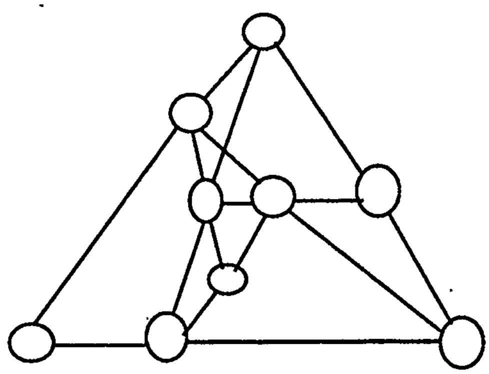
Variation 2: The players each have only three discs of a different colour. If neither player has three in a row then they alternate by moving their token horizontally or vertically, not diagonally, into an empty space until one person has three in a row.

Variation 3: The players alternate moves by placing only x's in the spaces. On his or her turn, a player may fill one, two, or three square with an x. If two or three square are marked they must be in the same row or column but not diagonally. They DO NOT have to be adjacent to each other as long as they are in the same row. The player to play the last remaining square or two or three squares is the winner.
Variation 4: Place discs on all squares of a Tic-Tac-Toe game board. Players, in turn, remove as many discs from a row or column as they wish, providing the counters are adjacent to each other before they are removed. The player who takes the last disc loses.

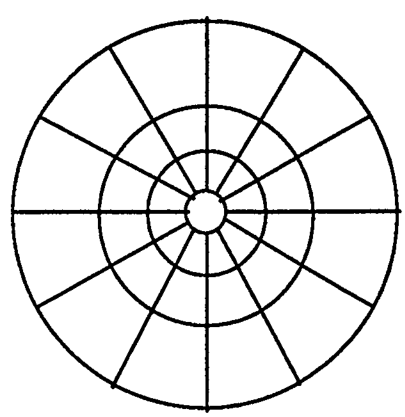
Variation 5: The game board is now changed to 4x4 and you need to get four in a row. The regular game and the four variations can be played and analyzed as before.

Variation 6: Change the game board to a 5x5 square grid and again whole new strategies for each of the games described above are needed.

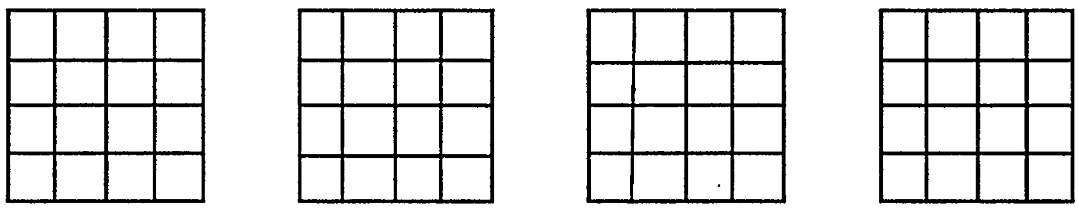
Variation 7: Change the game board to some other shape but still using a 3x3 cell idea. Here is a suggestion:



Or a 4x4 circular board. Different strategies are needed but the various games can be played on these two boards.



Variation 8: This game uses a 3-dimensional board with four tiers. It can be bought commercially. It is an excellent game for developing spatial relations. A four tier game on a 2-dimensional plane was suggested by H.M. Snook (1981) in an article called 3-D in 2-D is One Demanding Game. The game is played on four 4x4 grids placed side by side.



These four squares can be duplicated to form many game boards. The strategy for this game is the same as with the commercial game but the difficulty in this game is the mental visualization of the four in a row. This moves the game of Tic-Tac-Toe to a new dimension (pardon the pun).

CHAPTER 7
CONCLUSIONS AND IMPLICATION FOR FUTURE RESEARCH

I. CONCLUSIONS

On reflecting on the collection of activities, I observe many of them do not involve geometry on a conceptual level. Instead, they involve geometry on a symbolic level. The use of the geometry idea in the activity in a symbolic way means that the learners do not need to know the concept associated with the idea in its fullest definition but can understand the idea through association with the everyday things or general meaning found around them, i.e., any other symbol could be used instead of a geometric one. To use the geometry idea in a conceptual sense means that the fullest definition of the idea is needed in order to grasp the concepts being taught. About two-thirds of the activities reflect the symbolic use rather than the conceptual use of geometry. This was not intentional, but since a big portion of the Grade 7 curriculum focuses on arithmetic operations, it seems that they were more compatible with the symbolic rather than the conceptual side of the geometry ideas.

This bias also occurred in the distribution of the activities across the strands in the curriculum where, as can be seen on the summary of activities, number concepts were most represented.

This outcome can be considered a limitation of the study in that a better distribution of the activities across strands and greater emphasis on using geometry conceptually to teach the other strands would have been a better reflection of the goals of the study. This leaves a door open for future work in this area to fill the gaps.

As I reflect on this thesis, I realize that the development has been based on several assumptions. They are:

1. Students learn mathematics best when they are allowed to construct meaning and understanding of the mathematical ideas and to be creative in the constructing and in the usage of these ideas. This changes the focus of learning and teaching from the teacher as the center of communication and authority to a facilitator and organizer of a flow of activities that generate ideas, communication, authority and learning among the learners, of which the teacher becomes one. The success of this type of learning depends upon stu-

dents and teacher listening to each other, questioning each other, respecting each other, and growing together cooperatively. The learners, who they are and how they learn and gain meaning and insight are more important than the 'what' that is being learned. The learning environment becomes an integrative process where the whole-learners (their nature and nurture), content (the nature of mathematics), methodology (the vehicle), assessment (where do we go from here)- become more important than any one part of the process.

2. The students learn effectively when the mathematics is connected to their everyday lives and to the cultural and societal structures that they see around them. The teacher plays a crucial role in making these connections happen in the classroom. S/he can establish a network of mathematical investigations that builds, extends, deepens and unifies the mathematical relations and experiences of each student.

3. The students will benefit from an in-depth exposure to a few geometric ideas and concepts that make connections to other areas of mathematics and knowledge. Geometry is more than just a unit of study in school- it is a vehicle for understanding the world around and in us, it is visible and can be manipulated, it can be used to help us discover relationships, derive formulas, rules and generate data, it truly is ubiquitous.

II. IMPLICATIONS FOR FURTHER RESEARCH

I believe that I have only scratched the surface of the many connections and relationships that can and should be made between the ideas in geometry and the rest of the ideas in mathematics and living. The door for more research and development of activities and problems is wide open. I noticed as I gathered the many activities for the Grade 7 curriculum that many excellent geometric activities fit better into the Grade 8, 9, or10 curriculum, especially at the conceptual level.

Future research needs to consider how visual thinking affects the auditory learners and higher levels of mathematical thinking. The area of gender differences in geometry thinking, especially spatial reasoning, needs further investigation. I believe that it is not as important to look at the gender difference as it is to look at the different ways that humans think. How will teaching- using geometry ideas alone- affect the self-concept, success or achievement in mathematics, and interest of the students? How will empowering students as to what and how they learn and giving them more autonomy in the interpretation of mathematical thought and meaning affect the learning environment? This could become an area of great concern over rights and responsibilities of students, the marginal student, and the role of the teacher and society in general.

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APPENDIX A

ALBERTA JUNIOR HIGH MATHEMATICS PROGRAM-GRADE 7

> Applies and practices problem solving skills in new situations.

> Uses mental computation, paper-and-pencil algorithms, estimation and calculators to perform computations.

Whole Numbers:

> Maintains previously developed skills with whole numbers (place value, standard and expanded forms, adding, subtracting, multiplying and dividing whole numbers).

> Understands properties of number operations and uses properties and relationships to perform mental computations (e.g. associative, commutative, distributive).

> Understands that division by zero is undefined.

> Writes the value of a power (whole number base and exponent).

> Applies the rules for the order of operations to evaluate expressions.

> Recognizes prime and composite numbers (limit: primes to 50).

> Lists the factors for whole numbers up to 200.

> Expresses a number as a product of its prime factors.

> Uses a calculator or microcomputer to generate multiples of a given number.

> Determines whether a number is divisible by 2, 3, 5, 6, 9 or 10.

Decimals:

> Maintains previously developed skills with decimal numbers (place value, expanded and standard forms, adding, subtracting, multiplying and dividing decimal numbers).

> Compares and orders decimal numbers.

> Rounds decimal numbers.

Fractions:

> Maintains previously developed skills with fractions (concept of a fraction, need for fractional numbers, equivalent fractions, basic fractions) at a concrete level.

> Identifies mixed numbers and improper fractions and converts from one to the other.

> Orders fractional numbers.

> Uses concrete manipulatives to demonstrate the addition and subtraction of fractions with and without common denominators.

> Writes number sentences to describe the addition and subtraction of fractions.

> Uses concrete manipulatives to demonstrate the multiplication and division of proper fractions.

> Writes number sentences to describe the multiplication and division of fractions.

Integers:

> Maintains previously developed skills with integers (concept of integers, need for integers, ordering of integers).

> Uses concrete manipulative to demonstrate the addition of integers.

> Writes number sentences to describe addition of integers.

Ratio and Proportion:

> Applies and practices problem-solving skills in new situations.

> Maintains previously developed skills (identifies ratios as ordered pairs of numbers related to concrete situations; uses whole number constants to generate equivalent ratios).

> Uses concrete manipulatives to construct ratios in the following forms: a : b, a to b, and a/b.

> Verifies the equivalence of two ratios using common multiples or factors.

> Finds a missing element of a proportion using a common multiple of the elements.

> Identifies percent as a ratio.

> Expresses ratios as percents and decimals and vice versa (limit: ratios in the form a/b, where b = 2, 4, 5, 10, 20, 25, 50).

> Finds the percent of a number.

> Expresses one number as a percent of another number.

Measurement and Geometry:

> Applies and practices problem-solving skills in new situations.

> Maintains previously developed skills (concepts of linear, perimeter, area, volume, capacity and mass measures in concrete and pictorial forms; determines perimeter and area of right triangles and rectangles, and volumes of rectangular solids, without formulas; uses protractor to determine the measure of an angle; transformational geometry).

> Expresses equivalent measures in SI units (linear).
> Understands and uses the terms similar and congruent with respect to geometric figures.
> Understands and uses the term symmetry with respect to geometric shapes (line and turn symmetry).
> Constructs geometric designs using tools such as a computer, compass, straight-edge, ruler or Mira.

Data Management:
> Applies and practices problem-solving skills in new situations.
> Demonstrates a knowledge and understanding of the use and purposes of statistics as it affects daily living.
> Collects and records data (tally sheets and frequency tables).
> Understands and uses the term average (mean) as related to practical situations (e.g. test marks).
> Maintains previously developed skills (Interprets data from pictographs, bar graphs, line graphs and circle graphs).
> Understands when and how to represent data in the form of pictographs, bar graphs, line graphs and circle graphs.

Algebra:
> Applies and practices problem-solving skills in new situations.
> Understands and uses the term variable and uses variables to describe a concrete situation (e.g., number of jelly beans in a jar).
> Uses variables to write mathematical expressions to represent practical situations (e.g., age of the students in the class in three years will be $X + 3$ years).
> Evaluates expressions for given values of the variable (limit: whole numbers, decimals).
> Uses variables to write mathematical sentences to represent practical situations (e.g., people in a classroom = boys + girls + teachers or $P = b + g + t$).
> Uses concrete manipulatives to demonstrate the concept of "equals" (i.e. equality).
> Uses estimation, and guess and test procedures to solve equations of the form: $X + a = b$, $aX = b$, $aX + b = c$, and $X/a = b/c$.

> Verifies solutions to equations by substitution.
> Given ordered pairs, plot points on a coordinate plane.

Appendix B

MY PRESENT-STORY

As I ponder where I am today, my experiences in life and teaching, the intense learning and reading that I have done over the past 5 years, and the many changes that our society (particularly in education) is going through, I hear many "voices" from all segments of society telling me what to do and how to do it. It seems that everyone has an opinion, some strong, some weak, some passionate and some very well organized. Some of the voices that are impacting or will be impacting our educational system are: business, the environment, parents, students, government, Institutions of higher learning, friends, religious groups, the media, other professions, family, pressure groups, and internal organizations within our educational system such as, Mathematics organizations, Departments of Education, School Boards, education personnel, etc.

I would like to select those "voices" that are important to me and show, through research literature and my own experience, how these form my current belief system and why and how these beliefs lead me to teach mathematics through geometry.

An overall guiding principle that has guided my life is: You can never give another person that which you have found, but you can make him homesick or thirsty for that which you have. Experience is a gateway, not an end. Therefore, as I teach I want the students to understand the "whys" and "what ifs" of life and mathematics, the meaning and understanding of life and mathematics, the nature, role, fascination, and beauty of life and mathematics, the thrill of discovery and creation of life and mathematics and a realization that others around them are in this same process of construction.

One of the strongest "voices" that summarizes many of the other concerns and opinions about teaching mathematics today comes from the National Council of Teachers of Mathematics. In the document called the *Curriculum and Evaluation Standards for School Mathematics* (1989) the over-all focus is summarized on the last page of the *Standards* and it is:

"The National Council of Teachers of Mathematics has created a vision of-

> mathematical power for all in a technological society;

> mathematics as something one does- solve problems, communicates, reasons;

> a curriculum for all that includes a broad range of content, a variety of contexts,

and deliberate connections;

> the learning of mathematics as an active, constructive process;

> instruction based on real problems;

> evaluation as a means of improving instruction, learning, and programs."

As I read this vision I see all of these themes (curriculum, learners, teachers, instruction, learning, teaching, evaluation, communication, reasoning, problems) as being interconnected. During this past year while being on sabbatical, I have reflected about life, learning and teaching. From my reflection and personal experience the following aspects of the teaching/learning process are deeply significant. They are: One, what is the nature of a human being. Two, what it is that is valuable for that being to know. Three, what educational pedagogy would best accommodate the needs and understandings of human nature and their attendant value system. These three aspects are reflexively related in that the type of pedagogy that I choose will drive the curriculum or what is valuable to teach and that in turn will bring out various aspects of character and identification in a person. My journey has brought me to the following thoughts about what is important in each of these three categories.

NATURE OF THE HUMAN BEING (WHO I AM).

I. STAGES IN HUMAN DEVELOPMENT.

I believe that there are three stages that each of us as human beings should develop and move through. They are: developing relationships, developing a sphere of service, and developing character. The three stages overlap and growth and development in relationships, service and character should happen throughout life.

A. DEVELOPING RELATIONSHIPS.

1. DEVELOPING A PROPER SELF CONCEPT.

When I reflect on my perspective of teaching and what it means to teach, I realize my teacher knowledge is embodied in who I am, what and how I think and what experiences I have had in life. I now realize that my actions and intentions in teaching have been shaped by this personal knowledge. Every new experience and information that comes my way is interpreted through my autonomous, independent nature. I reflect on previous experiences and the knowledge that I have 'constructed' from those experiences and then

I will either fit this new knowledge into my scheme of things or discard it as not relevant for me at this time in my life. As I reflected on my teaching I realized that I have been learning and constructing my knowledge of teaching with each new experience in life. There have been some significant leaps in my teaching career i.e. moving from the elementary school level to the junior high school level, becoming aware of learning styles of students, an emphasis on problem solving and teacher thinking. Each significant leap in my teacher thinking has always been accompanied with lots of theoretical knowledge. This theoretical knowledge has had to work out experientially for me in my classroom in order for me to adopt it into my schema of teaching. I feel that one of the most important things in teaching is the I-I relationship that Martin Buber (1969) talks about. The dialogue or the dialectic conversations that go on in my head will display themselves in the way I behave in each situation as a teacher. The whole area of trusting myself to make good decisions no matter what the dilemma whether in my own personal life or in the lives of the students I teach is extremely important. There are three aspects to developing a proper, balanced I-I relationship. They are:

A. IDENTIFICATION.

I have thought a lot about how a person develops and feels value and worth. I pondered why certain students appear to have a poor self concept and lack confidence in their ability. The incident of building the coal shed for my parents sheds some light on why I have a healthy self concept.

Even though I have 6 brothers and 3 sisters, I always felt acceptance, love and forgiveness. Whenever I asked my mother if I could help bake, cook or just watch she always gave me the time and explained exactly what she was doing. I remember my Dad asking me to build a shed to store the coal for the winter. To this day I still wonder why he chose me instead of some of my older brothers. My sense is that my Dad knew instinctively that that job would make me feel really good. He seemed to sense this with all of his children. He never kept a record of who did what and he always seemed to choose the right person for the right job. To this day I am not afraid to tackle any job given to me. This story about building the shed is always the first to come to my mind when I think of the most significant accomplishment in my life. When I analyzed 'why' I realized that my Dad gave me free reign to do what I wanted in constructing the shed. He believed in me and when I had finished he praised me for the excellent job. It wasn't a perfect shed but my Dad understood the flaws and listened to my explanation of why it turned out the way it did. This incident in my life has molded the way I approach life and particular my teaching career- autonomy.

I believe that human beings are not self-determined. We need others to 'name us' or identify who we are. I believe that the best people to give us an identity are those closest to us, parents, relatives, teachers, and friends. This process of identification will go on throughout our lifetime and certain aspects of our character will be identified at different times in our lives. This is part of a continuous process of growing and learning.

B. BELONGING.

The second aspect that is as important as identification is that of belonging. Identity is connected to belonging. The initiative must always come from others. The parents should be the first to provide this place of belonging in the family. It is not good enough to just say it-you belong- but there is need for action, something that shows it, a drawing in. As a teacher this type of relationship begins from the first encounter with each student. Trust and respect and a sense of belonging are formed in a relationship. I will try to form a relationship with my students from the first moment they walk into my room. I do this with a greeting, a look, knowing and remembering their names, making some connection to previous siblings that I have taught, humour and an excitement that they are in my classroom to learn the best subject in school. What I teach and how I teach always take second place to who I am and who I teach. Because I like myself and like what I do I now am free to find out who they are and what they like. I want a mutuality (Buber, 1969) to be established between the two of us. This forming of a relationship built on trust and mutuality will allow me to instruct, demonstrate, lecture, guide, direct or discipline as I see necessary because I respect them as learners and they respect and trust me to provide them with the best learning for their individual selves. This relationship building is similar to what Carl Rogers (1961) talks about, of realness in the teacher and prizing, acceptance, trust and understanding of the students. This proper relationship with myself will free me as a teacher to have a proper I-Thou relationship Buber (1969) with my students or with any learning situation I may encounter. I found that if I feel secure and successful at learning I will take risks in learning and therefore in teaching. Many researchers have found that students' self concept is related to their academic achievement (Burns 1979; Purkey, 1978; Kremer & Walberg, 1981; Raven & Adrian, 1978; Starr, 1975) and Hamachek (1972) found that students with a high self-concept were more confident, could handle risks and failures better and could defend their discoveries better. Also, this security that I have will be picked up unconsciously by my students and therefore they will be willing to take risks and if they do not succeed they will not feel judgment from me as a teacher. If I am secure in my I-I relationship then as a teacher I will be able to set up a level of trust with my students. Teacher influence on self-concept (Burns, 1979, Covington & Beery, 1976; Purkey, 1978; Simpson, 1978) and teacher expectations, interactions and feelings about students all contributed to student's feelings of worth and performance in their school achievement. My trust and understanding of the students does not go as far as what Carl Rogers wants. I can understand the students but I can not totally "understand the student's reactions from the INSIDE", I would need to have lived their every experience and been born exactly like them. The third aspect of building a proper self concept is that of approval.

C. APPROVAL.

Approval must always be based on identity and belonging. My parents approval of me always came in small doses but it was always timely and based on who I was. In our society today I see a chasing after this approval without proper identity and belonging and it has produced a group of people called 'celebrities', or 'I am special', and a value for narcissism. Seeking approval without proper identity and belonging will produce a cycle of approval seeking that is never satisfying or never enough. The freedom to just 'be' is lost and we become people bound to having approval. As a teacher I can help stop this cycle by giving the students a proper identity, a feeling of belonging, and deserved approval. What are some of the characteristics that a teacher should possess or be developing in order to pass on this blessing? Some characteristics are:

1) Empathy and love.

Empathy is a 'feeling with.' It is an ability to experience, in a sense, what the other person is experiencing. The empathy sought here is to cause me to do something helpful, by lending support in an emotional, understanding and acceptable way. Love is of the acceptance kind. I do not believe that we can like or love every student that we teach, but I do believe that it is our responsibility to accept and try to understand each student as a human being with thoughts, opinions, emotions, attitudes, and personalities that may be different from ours. This intimacy is something higher than merely sensory experience for it builds self-worth, gives a sense of liveliness, is elevating, and models giving and receiving in appropriate balance.

2) Genuineness.

If I am genuine I am, for the most part, what I seem to be to those who know me best.

3) Problem-solving attitude.

Having a positive problem-solving attitude means that you view problems as opportunities or as a challenge to survive, rather than reacting, taking offense, or assuming the worst. Your attitude to the problem is a calm and rational approach. You do not react immediately but take time to reflect and get as much information as possible by talking to all the parties involved or researching every possible avenue. Teachers need to become adequate problem solvers themselves in order to deliver effective instruction in problem solving (Krulik & Rudnick 1982).

4) Listening

No one fully understands the motivations, feelings, and understandings of others. But to get as close to understanding as possible, each person must teach himself to listen to whatever is available. We must become sensitive and respectful of the children in our charge. This will require a lot of time and effort to uncover the facts, feelings, assumptions, concerns and arguments of all concerned. We will have to listen critically, so that we can reflect on and analyze the information given, see the points of strength and weakness, uncover the hidden meanings and give a proper plan of action for each individual's learning.

5) Questioning.

One of the best tools for understanding individuals and also for solving problems-interpersonal to international- is asking questions. These are not judgmental questions whose answers are implied or inferred in the question itself but questions designed for learning, to elicit information and gain an insight into the students we teach. They help us to draw an arrow or direction for the students' progress in learning. In this sense they are questions that protect the individual's ego.

In conclusion, the most important ingredient in the learning/teaching process is not the type of approach we use, for often the problem needed to be solved or the knowledge needed to be learnt dictates the approach, but the interaction between the humans involved in the learning. The responsibility for the type of interaction that happens in education is mainly on the teachers shoulders. The students who are moving toward maturity, do have responsibilities in the process, but the teacher should have developed maturity and therefore a greater understanding of life situations. Who you are, your **being**, should be your motive for doing. The teaching/learning process is so wrapped up in our humanism that the approaches used take second place to the human character being developed.

2. DEVELOPING A PROTECTED, SAFE ENVIRONMENT.

The teacher also needs to understand the human character and its make-up so that a proper caring, learning environment can be cultivated. The environment must provide for psychological, emotional, intellectual and social safety for the individual student. Teachers are being asked, and are taking the responsibility of being the primary care-givers to many of today's children. The children need to know that they are in a safe environment, where communication is open, honest, and ideas, imagination and creativity are listened to and not criticized, play is allowed and encouraged, and acceptance and respect for one another and a feeling of understanding and growth are encouraged.

3. DEVELOPING A PRINCIPLE OF GUIDANCE.

Rogers (1961) says that we should trust "the capacity of the human individual for developing his own potentiality". I disagree. I needed someone to help me make good decisions and to teach me as a youngster. My parents and teachers saw my potential and therefore put time in on me to develop those potentials to the fullest. I have seen many students with loads of capability who never developed their potential because no one put quality time into them. Students need to be guided throughout their learning until they can activate themselves. This need for guidance and its fulfillment in us helps us to develop competence. This guidance should always be done through principles rather than rules because principles provide true guidance rather than thoughtless, rote procedures and regulations. Principles also give needed flexibility when rules don't apply.

Several principles that are important to me are: one, authority. I believe in authority, not as a set of rules set up by me for the students to follow but authority built on a relationship of trust and respect. I must show to the students that we are fellow travelers in this process of learning. True authority built on a relationship responds in obedience, trust, humility, belief in yourself and honor of one another. Rules on the other hand set up a tension, fear, pride, honoring self, competition and a wanting to gain status or power. I always wondered how I had learned to submit to authority. I realized as I was reflecting on this that it was not something my parents did but something that they did not do that taught me this principle. My parents never talked against authority figures whenever their children were present. They told me as an adult that they didn't always agree with those in authority and that they would go to them and voice their disagreement in private. I realized that I had learned this principle through modeling. I cannot truly teach another unless I have established a genuine relationship with my students. This relationship takes time and a commitment on both parties concerned but I believe it must first come from me as the teacher. This type of authority seems to fit in nicely with my view of democracy.

Two, "positioning." I also believe that in this relationship there is a 'positioning' (Buber, 1969) of teacher and student. They understand that I am an 'expert' - not someone who knows it all- but someone who has probably experienced and learned more about the subject at this point in time but who is still learning and growing.

4) DEVELOPING A PROPER ATMOSPHERE OF AUTONOMY.

There needs to be a balanced atmosphere of dependence/ independence. A push for too much independence can cause a learner to not develop proper balanced relationships with others and too much dependence on others can cause a learner to not take risks and avoid making decisions by themselves. Developing a balanced identity in dependence/independence is essential in the self-esteem and well-being of a personality.

B. DEVELOPING A SPHERE OF SERVICE.

A second stage in the development of who I am is that of service. Evidence that the student is changing from self-orientation to maturity is found in the individual's conduct.

The concern for others becomes stronger and trying to please and tolerate other points of view are seen on a consistent basis. There needs to be a balance of competition/cooperation. A spirit of servanthood(not servitude) needs to be the attitude with all participants in the learning process. To help out and cooperate with one another develops a connectedness, a sharing, a feeling of worth, a feeling that we are in this together, instead of "I am the best", "I don't need you", which will only lead to judging whether I am superior or inferior to the other person. Good competition can lead to a striving on the part of the individual to do their best and therefore competition that does not lead to judging is needed. If there is one thing that is needed in our society in general and in education specifically, is that we as humans need each other and are connected to each other. We need to place the good of others over our own personal benefit because as the Scriptures declare, " It is more blessed to give than to receive" and "the good we have done will be returned to us a hundred-fold."

C. DEVELOPING CHARACTER.

Character development is the final stage of knowing and understanding who you are. At this stage a person has developed to a level of idealistic motivation. "All education worthy of the name is education of 'character', writes Buber, and education of character takes place through the encounter with the image of man that the teacher brings before the pupil in the material he presents and in the way he stands behind this material." It is at this stage where you do not blame others for your problems, the responsibility rests squarely on your own shoulders.

II. SOME DIAGNOSTIC TOOLS TO HELP DETERMINE WHO I AM.

I have selected the following four learning styles models: 1. The Dunn and Dunn, Elements Of Learning Style. 2. David Kolb's, Experiential Learning Theory. 3. Anthony Gregorc, An Adult's Guide To Style. 4. Bernice McCarthy, The 4MAT System Model. These learning styles models can help students to reflect on who they are and also give the teacher insight into the student's uniqueness.

DESCRIPTION OF LEARNING STYLES MODELS

1.The Dunn and Dunn, Elements Of Learning Style.

The Dunn and Dunn model has a total of 21 elements that are broken down into 5 catagories. They are: A. Enviromental--noise, light, temperature, and design.B. Emotional--motivation, persistence, responsibility, and structure. C. Sociological--colleagues, self, pair, team, authority, and varied. D. Physical --perceptual (visual, auditory, tactile, kinesthetic), intake, time, and mobility. E. Psychological--analytic, global, field dependent, field independent, reflective, and impulsive.

A Learning Styles Inventory (LSI) is given to the students. This inventory contains 100 questions, which the students answer. It is then processed by Price Systems in Kansas and an Individual Profile is sent to each student This profile shows the elements that make up the student's learning style preferences.

2.David Kolb's, Experiential Learning Theory

Kolb suggested that in early life, people are primarily engaged in acquiring information and the basic skills needed for effective functioning. By adolescence, people come to deal with situations in characteristic ways and develop certain preferences in terms of how they grasp experience and transform it. To determine people's learning preferences, or styles, Kolb developed a Learning Style Inventory (1985), which identifies people's styles.

The first group of people are "divergers," who prefer to grasp experience by concrete experience and transform it by reflective observation. The second type are "assimilators," who prefer to grasp experiences by abstract conceptualization and transform it by reflective observation. The third type are the "convergers," who prefer to grasp experience by abstract conceptualization and transform it by active experimentation. The fourth type are "accommodators," who prefer to grasp experience by concrete experience and transform it by active experimentation.

3.Anthony Gregorc, An Adult's Guide To Style.

The *Guide* uses the *Gregorc Style Delineator* to identify two crucial sets of qualities: where a person's sense of reality is anchored, and how that person orders information. Each quality is a duality. The anchoring point, the internal view of reality, is described in terms of Concrete and Abstract qualities. Information ordering, how a person organizes data, is portrayed as Sequential and Random qualities. The *Style Delineator* identifies combinations of these dualities. Each combination represents a different style type: Concrete Sequential, Abstract Sequential, Abstract Random, Concrete Random. And each has its own very special set of characteristics. We all possess some of the features of each style, yet we all, to a more or less intense degree, have a natural "dominant" point.

4. Bernice McCarthy, 4mat System Model.

The 4Mat System is based on learning style research, especially David Kolb's Experiential Learning Model and research on brain dominance. The resulting quadrant system formed the conceptual rationale for four different learning styles. Type One Learners: They usually ask the question Why?, and their primary concern is for personal meaning. Type Two Learners: They usually ask the question What?, and their primary concern is to get information. Type Three Learners: They usually ask the question How?, and their primary concern is to try things for themselves. Type Four Learners: They usually ask the question If?, and their primary concern is to adapt learning to their own life situations, to make more of what they learn.

APPENDIX C

TABLE OF CONTENTS

UNIT NAME: ______________________________

Date	Page(s) Name	Page number(s)

NOTEBOOK CHECK

NAME:____________________ **CLASS:**__________

CORRECTED BY:______________________________

PRESENTATION/GENERAL APPEARANCE	5 4 3 2 1
-notes neatly done/legible -titles or headings on pages	
ORGANIZATION	10 8 6 4 2 1
-table of contents complete -all notes in the correct place -no notes/pages missing	
COMPLETENESS	10 8 6 4 2 1
all notes and homework assignments completed -corrections done	
TOTAL	________ 25