THE UNIVERSITY OF CALGARY

A comparison of two models for the development of expert schemata

by

Darcy N. Rollack

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The undersigned certify that they have read, and recommend to the Faculty of Graduate Studies for acceptance, a thesis entitled "A comparison of two models for the development of expert schemata" submitted by Darcy N. Rollack in partial fulfillment of the requirements for the degree of Master of Science.

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ABSTRACT

This thesis reviews the research which investigates the utility of schema theory in explaining expert problem-solving behavior, and describes a model for the development of expertise which relies on a combinatorial process of schema formation. In this model, new schemata are developed from existing problem-solving procedures or schemata by repeated operations of combination and generalization. Evidence for an alternate model of development, in which this combinatorial process is complemented by a process of schema formation through schema separation, is presented, and such a model is proposed. The results of an experiment designed to test these two models by comparing the actual ability of novice, advanced novice, and intermediate physics students to rate the perceived relevance of equations to a set of test problems, against their ability as predicted by these two models are reported. These results were inconclusive. This was apparently due to the need to apply very strict criteria for separating schematized and unschematized problems for analysis. Implications of this study for further research are considered.

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CHAPTER I

INTRODUCTION

One reason for investigating the nature of expertise is the possibility that, through such research, we may be able to develop more effective educational techniques for promoting and guiding the development of expertise. If, as this author does, one views expertise as a continuous variable, defined as the facility with which one solves problems or manages other tasks in a particular domain, then such a goal would seem inseparable from that of improving education as a whole. If, on the other hand, one defines expertise more stringently, as a state characterized by an extraordinary facility in problem-solving and a high degree of automaticity in routine tasks, then the importance of such a goal can be measured by the high value, economic and otherwise, which is placed on the expert in our society.

In pursuing such a goal, it is not enough to understand the result of the developmental process, one must also understand the process itself. Certainly, there is much about the development of expertise which might be inferred from an understanding of expert behavior, but prescribing educational techniques solely on the basis of such inferences would seem unnecessarily risky where direct investigation of the developmental process is possible. In addition, there may be aspects of expert behavior which can best -- if not only -- be understood through a study of their development.

Of course, the above could be said of the development of almost any human characteristic, cognitive or otherwise; it might even be considered as a statement of the obvious. Nonetheless, its importance in the investigation of expertise should not be undervalued. Expert behavior is, to a great degree, guided by automatized processes; critical aspects of expert cognition may, therefore, be largely invisible, and the process by which expertise is developed may include significant departures from that which would be predicted by a simple interpolation based on expert and novice behaviors. As a well-documented example of this, one might consider the "intermediate effect" which has been observed in the clinical diagnosis of medical conditions: a number of studies (see Boshuizen & Schmidt, 1990, for a review of the relevant literature) have reported that experts use less biomedical knowledge in the development of such diagnoses than either novices or intermediates; intermediates use the most. Boshuizen and Schmidt (1990), using a form of propositional analysis, were able to show that this finding is, in fact, quite misleading. Experts do apply more biomedical knowledge in developing their diagnoses than either of the other two groups. However, this knowledge has been reorganized and integrated with their clinical knowledge base to form highly automatized diagnostic procedures; its application in diagnosis is, therefore, not readily observable.

Of course, the obviousness of this example also makes it somewhat trivial. It is highly unlikely that anyone would actually prescribe educational techniques which were based on the belief that experts have or use less knowledge than novices. However, the cautionary note which this strikes cannot be ignored: educational prescriptions based on an incomplete understanding of the development of expertise may not merely fail to promote that development, but may actively hinder it. As a corollary to this, where there is some question about the theoretical basis for such a prescription, further investigation of the relevant area would seem to be strongly indicated.

One such case may be the recommendation, by Owen and Sweller (1989), that problem-solving should be de-emphasized, if not discontinued, as an instructional device in the teaching of mathematics, and replaced with the study of solved problems and the use of goal-free problems -- word problems in which a problem situation is presented for exploration, but no specific quantity is required as a solution. This recommendation is based on research by Sweller, Mawer, and Ward (1983) which has shown that students learn to solve problems more quickly when the two latter techniques are used for instruction, and on a theoretical model proposed by Cooper and Sweller (1987), in which expertise is developed by the combination of procedures for solving individual problems into successively more general structures. Given this model, and the demonstrated benefits to be obtained by the use of the recommended techniques, Owen and Sweller's recommendation is both reasonable and appropriate. However, Sweller et al.'s finding, reported in the same study, that this performance benefit does not extend to problems which are even moderately different from those used for instruction must raise some question about the effectiveness of these techniques in developing expertise and, hence, about the theoretical model by which they are supported.

This study will address this question by (1) examining the model developed by Cooper and Sweller (1987) and the research on which this is based, (2) proposing a modified model which might be more useful in explaining the development of expertise, and (3) testing these two models by comparing the actual performance of physics students against their performance as predicted by each of these models.

CHAPTER II

REVIEW OF THE RELATED LITERATURE

Introduction

Research in a variety of domains has identified a number of problem-solving behaviors which develop with expertise and which might explain, in some measure, the extraordinary facility with which experts solve problems in their domains. In particular, fairly robust effects of expertise on problem perception, problem representation, and solution strategy have been observed. These effects are consistent with a model of expertise in which expert problem-solving behaviours are guided and partially automatized by hierarchical knowledge structures, and thus, with the major predictions of schema theory as proposed by such researchers as Case (1985), Neves and Anderson (1981), Rumelhart and Norman (1976), and Schank and Abelson (1977).

In such a model, the development of expertise can be traced to the basic mechanism by which experience is organized and stored in long-term memory, and expert performance can be directly equated to the facility which individuals display in the solution of routine, "real-world" problems. However, it must be noted that an ascription to the general outlines of schema theory, here, is not intended to suggest a strict and complete conformity to the theoretical models proposed by such researchers as the above. The available research on the effects of expertise on problem-solving behavior is simply insufficiently fine-grained either to require or to support the details of development, operation, and internal structure provided by schema models.

The effects of expertise

Differences in problem perception

Perhaps the most robust effect of expertise on problem perception is the interaction between expertise and meaningfulness in reconstructive recall, first observed by Chase and Simon (1973) in the recall of chess positions by chess players of various levels of skill. When random piece arrangements were used, no significant difference was found in players' ability to reconstruct the arrangement after a five-second exposure. However, when positions drawn from actual games were used, recall increased dramatically with expertise. Similar results have been obtained using piece positions in the game of Go (Reitman, 1976), electronic circuit diagrams (Egan & Schwartz, 1979), and segments of musical notation (Halpern & Bower, 1982). Both Chi (1978) and Gruber, Gold, Opwis and Schneider (cited in Schneider, 1990) observed the same effects in child, as well as adult, chess players; that is, both found no significant effect of age on performance.

Shneiderman (1976) varied this methodology slightly by using two short Fortran programs in a recall task. One program was presented in its original, executable form; the other was presented with its individual lines randomly ordered. The general pattern of results resembled that obtained by Chase and Simon (1973): there was a strong interaction between expertise and program form, with experts benefiting far more from a meaningful arrangement of stimulus materials than novices. Similar results have been obtained using scrambled/unscrambled versions of ALGOL (McKeithen, Reitman, Rueter, & Hirtle, 1981) and BASIC (Barfield, 1986) computer programs, and suit-ordered/unordered bridge hands (Charness, 1979; Engle & Bukstel, 1978).

Thorndyke and Stasz (1980) did not find improved recall of geographic map elements by experienced map users. In a study which used two specially prepared maps -- a Town Map and a Countries Map -- the three subjects they identified as expert map users ranked first, third, and eighth of 8 subjects in the number of map elements correctly recalled. However, this failure would seem to be due (1) to a lack of sufficient meaning in the materials used, and (2) to the use of materials which experts would not typically encounter in their professional work and, consequently, with which they might not be sufficiently familiar to function as experts.

Ormrod, Ormrod, Wagner, and McCallin (1988) argue that the maps used by Thorndyke and Stasz (1980) did not provide the functional groups which, as will be seen, are used by experts to organize information for recall and on which their superior recall performance depends. Where functional groups were incorporated in the maps used in the experimental task, they contained a limited number of elements; an informal count by this author found a mean of 2.1 elements in the 10 groups presented in the Town Map used in this study. Some of the relationships presented were also extremely atypical (Ormrod et al., 1988).

To support the importance of meaningful relationships on memory for map elements, Ormrod et al. (1986, 1988) compared the recall performance of university faculty in geography, educational psychology, and sociology using two maps similar to those used by Thorndyke and Stasz (1980). One of these maps, the Town Map, was redesigned to incorporate typical relationships between map elements; the Countries Map was deliberately designed to be non-congruent with the relationships which would typically exist between map elements. A significant interaction was found between discipline, map, and number of map elements correctly recalled. The non-geographers showed no significant effect of map congruence on recall of map elements. The geographers recalled more elements than either of the other groups on the first, congruent, map; on the second, non-congruent, map, their performance declined to the level of the educational psychologists, the better of the two novice groups. Ormrod et al. also note that the geographers "tended to voice frustration with the illogic of the country map, something the other two groups did not do" (Ormrod et al., 1988, p. 431.). When students in the three disciplines were used, instead of faculty members, a similar but more extreme pattern of results was found. No significant difference in recall performance was found between sociology and educational psychology students or between maps for these two groups; no significant difference was found between the geography students and the other two groups on the non-congruent map, but geography students had significantly higher recall of map elements on the congruent map. The results for both of these experiments are consistent with those reported by Chase and Simon (1973) and the other researchers cited above.

In discussing the effect of materials on recall performance, Thorndyke and Stasz (1980) state that the observed differences in recall between subjects could not be due to differences in their familiarity with the material or task because "If familiarity with maps were the critical variable, then all experienced users should have performed well" (Thorndyke & Stasz, 1980, p. 156). Insofar as the variability of expert performance is concerned, this statement is certainly true: The lack or incongruence of functional relationships in the stimulus materials does not, of itself, explain the wide range of scores obtained by the experienced map users. This can, however, be explained by this study's inappropriate equating of task and domain expertise; expertise in the use of maps does not necessarily imply expertise with the type of maps used in this study. Gilhooly, Wood, Kinnear, and Green (1988) have noted that the maps used by Thorndyke and Stasz are planimetric maps, showing only the horizontal location of geographic features, while the professional experience of most cartographic experts would be with topographic contour maps, which are abstract representations of all three dimensions of a geographic area. Using high- and low-skill subjects selected on the basis of a test of contour-map reading and a biographical questionnaire, and both planimetric and topographic maps, Gilhooly et al. found no effect of group on performance in a multiple choice memory test when planimetric maps were used; the high-skill group performed significantly better on a similar test when topographic contour maps were used. Similar results were obtained when a reconstructive recall task was used. Skill in reading topographic contour maps clearly does not transfer to the reading of planimetric maps.

The three experts used by Thorndyke and Stasz (1980) came from very different professional backgrounds. One was a "retired Army officer who had field map-using experience and had taught map reckoning to recruits" (Thorndyke & Stasz, 1980, p. 140), the second was a retired Air Force pilot, and the third was "a scientist who regularly used graphics display systems for geographic data bases" (Thorndyke & Stasz, 1980, p. 140). It would, therefore, seem reasonable to conjecture that the varied ranks of these experts (first, sixth and eighth of 8 subjects, respectively) actually reflect differences in the amount of their professional experience with planimetric maps. It might also be conjectured that the undergraduate students who composed the novice group in this study had existing skills which could be applied to the task of memorizing planimetric maps and which, in the expert group, would have been largely supplanted by skills specific to the type of materials normally encountered in their professional experience.

Attempts to replicate the effect of expertise on problem perception using patient information in the field of medical diagnosis have produced mixed results. Norman, Jacoby, Feightner and Campbell (1979) had medical students, first-year medical residents, and physicians read and attempt to recall patient histories which were typical of actual diseases or which contained randomly selected information. Presentation time for the histories was not controlled and varied from 65 seconds to 210 seconds. It was found that the number of items recalled per unit of presentation time increased with expertise for the meaningful case histories; no significant effect of expertise was found when randomly-generated case histories were used. Norman, Brooks, and Allen (1989) observed similar results using meaningful or randomly generated data from laboratory analyses. Norman et al. (1989) also found that the interaction between expertise, meaningfulness, and recall performance was stronger when stimulus materials were presented in the order in which they would normally be encountered in a clinical setting than when the same materials were presented in a randomly determined order.

While the above results are consistent with those obtained in other domains, other studies using medical data have failed to replicate these findings. Muzzin, Norman, Jacoby, Feightner, Tugwell, and Guyatt (1982) modified the procedure used by Normal et al. (1979) by limiting the presentation time of the case histories to 120 seconds and by replacing the randomly generated case histories used by Norman et al. with atypical case histories which presented a realistic but uncommon pattern of patient information. It was found that neither expertise nor meaningfulness of the stimulus materials had a significant effect on the number of items recalled. Using pauses in recall as indication of chunk boundaries revealed that the experts did partition the information into larger chunks but produced fewer chunks than novices. Muzzin, Norman, Feightner, Tugwell, and

Guyatt (1983) replicated these findings and, in addition, found that, when presentation times were reduced to 45 seconds, the pattern of results was the reverse of that observed in other domains: Novices recalled more information than experts when typical case histories were used; no significant differences were observed for atypical case histories.

Given the extended presentation time used for the stimulus materials -- 45 seconds to 210 seconds -- it is possible that these failures to replicate, in the field of medical diagnosis, the effects of expertise on problem perception which have been observed in other domains are due to additional processing of the stimulus materials; that is, what is actually being observed may not be the effects of expertise on problem perception, but the effects of expertise on problem representation.

This possibility is supported by a reanalysis of the data obtained by Muzzin et al. (1982) by Patel, Groen, and Frederikson (1986), which isolated two additional factors: the relevance of the recall items and whether the items were recalled as presented in the case history or showed evidence of processing prior to recall. Significant interactions were found between expertise, case typicality, and processing and between expertise, case typicality and relevance. While the complexity of these interactions makes a more precise interpretation rather problematic, these results do suggest differences in the depth and manner of processing in the novice and expert groups. An additional experiment by these researchers, which simplified the design by the elimination of the atypical case histories, found that the items recalled by experts were more likely to be highly relevant to the final diagnosis than those recalled by novices, and were particularly likely to be highly relevant items which had received some degree of processing before recall. Coughlin and Patel (1986) report a similar interaction between relevance, expertise, and recall, and an even more pronounced interaction when items were classified as critical or noncritical to the diagnosis of the cases presented. The pattern of results observed in these two studies strongly supports the hypothesis originally advanced by Muzzin et al. (1982) that experts were more selective than novices in the items they recalled, excluding superficial items which were included in novice recall. As will be seen below, this is typical of the problem representations formed by these two groups

As might be suggested by the above, experts' increased recall of meaningful material seems to be a product of their ability to identify functional relationships between problem elements and to use these to chunk the problem space more effectively than novices, who are forced to rely on more superficial features to partition the same space. Egan and Schwartz (1979) analyzed the chunks formed by novices and experts in reconstructing schematic diagrams of electronic circuits and observed that experts could use functional features to chunk spatially separated components, while novices' chunks could be almost completely predicted on the basis of proximity alone.

Adelson (1981), working with computer programmers, also observed a shift in chunk organization with increasing expertise. In a recall task using scrambled lines from three short PPL programs, novices chunked according to the syntactic category of individual lines, while experts formed chunks on the basis of program membership. McKeithen et al. (1981) observed the same bases used for chunk formation in the recall of a list of reserved words from the ALGOL programming language.

Halpern and Bower (1982) had musicians and non-musicians partition short, written melodies. Non-musicians formed chunks by using a simple obvious feature: Melodies were partitioned into chunks formed of consecutive notes whose stems all pointed in one direction. Expert partitions were more varied, and a significant correlation was found between the ease with which a melody was recalled, and the consistency of its partitioning by the expert subjects -- presumably, a measure of the typicality of that melody's chunks.

Gilhooly et al. (1988) analyzed novice and expert protocols obtained during the memorization of maps and found that the major difference in expert and novice behavior was the experts' greater use of domain-specific relationships to organize materials for memorization. The significance of this difference is, however, open to question, as individual t-tests seem to have been used for the 15 memorization behaviours examined.

Chase and Simon (1973) did not find a significant difference in the types of chess relationships used by experts and novices in chunk formation. However, they did find that experts' chunks included more relationships than novices', and note that even highly significant differences could be concealed by the confounding of surface and functional features in the game of chess, where a piece's defensive or offensive capabilities -- its role in the deep structure of a position -- are determined, in large measure, by superficial features such as its color and its proximity to other pieces. An analysis of a Grandmaster's chunks (Simon & Chase, 1973) found that 75% could be classified as one of three functional configurations which occur with high frequency in actual chess positions.

Differences in problem representation

There is obviously a close relationship between the way a problem is perceived and the way it is represented internally. The details and organizational principles which are selected from the problem environment clearly limit the nature of the representation which can be formed; at the same time, preferred or habitual forms of representation can be expected to condition the kind of information which is abstracted from the problem space. Given this, it is not surprising that expert-novice differences in problem representation closely parallel those in problem perception.

In a number of experiments using physics problems, Chi, Feltovich, and Glaser (1981) found strong evidence that novices and experts maintain, at the representational level, the preferences for organizational principles which have been observed at the perceptual level. When asked to group problems from a standard physics textbook (<u>Fundamentals of Physics</u>, Halliday & Resnick, 1974) according to their similarity, novices formed categories on the basis of surface features such as key words or physical objects mentioned in the problem statement, while experts grouped the same problems by the general principles of physics involved in their solution -- their deep structure. These same categorization schemes were observed in a second experiment in which the problems to be sorted were constructed to deliberately cross deep and surface features. Intermediate and advanced novices included in this second experiment used a combination of surface features and deep structures in their categorization: categories formed on the basis of the problems' deep structures were subdivided according to the problems' surface features. In both experiments, the category labels provided by subjects support the researchers' identification of the categorization schemes used. Novices tended to label categories with words or objects taken from the problem statement, while experts tended to use laws of physics as category labels. Similar results have been obtained by Veldhuis (1990), and using mathematics problems, by Schoenfeld and Hermann (1982). Chi, Glaser, and Rees (1981) found a similar pattern of verbal characterization when experts and novices were asked to rate the difficulty of physics problems and to explain their ratings. Thirty percent of the reasons given by experts referred to abstract physics principles, while 28% were not physics-related. The reasons provided by novices referred far less often (9%) to abstract principles, and more often (40%) to problem characteristics unrelated to physics.

Chi, Feltovich, and Glaser (1981) also used the category labels generated by subjects in their sorting task to examine the knowledge structures invoked by problem groups by analyzing the protocols of subjects asked to elaborate on the information they possessed about problems of that type. These protocols show the same variation observed in the sorting task: novices' protocols focused on physical objects and their properties, while experts, even when prompted by surface feature labels, initially produced fairly well-organized groups of general physics principles which might be applicable to solution, and statements about the conditions under which these would be applied. Descriptions of objects and other surface features were produced only after the applicable general principles had been exhausted.

Dufresne (1989) and Hardiman, Dufresne, and Mestre (1989) investigated expert and novice categorization of physics problems in two experiments which used a similarity judgement task. Given a "model" problem, subjects were asked to decide if a "comparison" problem was solved the same way as the model problem (Dufresne, 1989; Hardiman et al., 1988) or which of two comparison problems was solved in the same way as the model problem (Hardiman et al., 1989). The comparison problems in both studies were constructed to incorporate different combinations of surface-feature and deep-structure similarity. The results obtained support the difference in problem representation discussed above: Experts tended to identify comparison problems as similar to model problems if they had the same deep structure, while novices tended to use surface features in their assessments of similarity. When asked to provide reasons for their judgements, experts again tended to cite principles of physics far more often than novices (93% and 33% of the time, respectively).

Hopkins, Campbell, and Peterson (1987) observed the same differences in the bases used by novices (veterinary students) and experts (a professor of veterinary science and the designer of a computer simulation of the cardiovascular system) in forming representations of the cardiovascular system. Subjects rated the predictability of 17 properties and variables of the cardiovascular system given each of the other properties and variables. Multi-dimensional scaling and clustering techniques were then applied to these ratings. It was found that novices organized the rating items according to two, essentially anatomic criteria: whether the items described the heart or the blood vessels and whether items described circulation through the lungs or through the rest of the body. In short, novices organized system properties and variables according to the physical structure of the system. Experts, on the other hand, organized these same items according to their measurability, the speed with which they change, and their independence of other characteristics. Hopkins et al. (1987, p. 5) characterize these criteria as "functional and systemic distinctions" and indicate that these are related to the deep structure of the cardiovascular system, while the criteria used by the novices are related to that system's surface features.

The functionality of the expert groupings obtained by Chi, Feltovich, and Glaser (1981) is supported by an additional experiment by these researchers in which new subjects, both expert and novice, were asked to indicate the basic approach they would take in solving the problems used in the original sorting task. The expert group used the same terms to identify solution methods that the previous group of experts had produced as category labels in the sorting task. In contrast, the novices produced either detailed solutions to the problems or highly general problem-solving heuristics.

Schiano (1986) also found strong evidence of the functionality of expert representations: skilled solvers of figural analogy problems were found to characterize problems using the transformational relations needed for their solution; less-skilled subjects used superficially available characteristics such as object position or shading, which are less related, or unrelated, to problem solution.

Even stronger support for the functionality of expert groupings is provided by Weiser and Shertz (1983), who had one group of novices and <u>two</u> groups of experts categorize computer programming problems. One expert group was composed of computer science graduates; the second, of former programmers who were managers in a large programming organization. Novices formed and labelled categories by application area, while the categorizations and labels of computing science graduates were based on the algorithms required for the problems' solution. Managers, however, used none of the organizational schemes incorporated in the problems. Interviews conducted after the experimental task revealed that subjects in this group sorted problems according to the type of programmer they would assign to the problem.

Smith (1990, 1992) also included 2 expert groups in a study of the categorization of genetics problems by novices, biology faculty members, and genetic counsellors, and found similar effects for both level and subdomain of expertise. Novices formed and labelled categories using surface features, faculty members used disciplinary principles, and genetics counsellors, like the managers in Weiser and Shertz (1983), used a principle-based scheme related to the type of task they would typically face in their professional work.

Two additional effects reported by Dufresne (1988) and Hardiman et al. (1989) must be noted for their qualification of the trends reported above. First, it was found that expert and novice judgements of similarity are not based entirely on one characteristic. Experts are more likely to identify problems as similar when they share surface features as well as a deep structure (Dufresne, 1988; Hardiman et al., 1989) and are less likely to select deep-structure matches when an alternative surface-feature match is also presented (Hardiman et al., 1989). Novices show the same pattern of behavior, with deep-structure and surface-feature characteristics reversed.

Adelson (1981) supports this ability of experts and novices to make use of both deep structure and surface features when forming representations. In this study, expert and novice programmers were presented with a short PPL program and a task related either to the program's algorithm (its deep structure) or its syntax (its surface features). Subjects were then asked a question either about the deep structure or about the surface features of that program. As could be expected, experts performed better on deep-structure questions, whereas novices performed better on surface-feature questions. However, expert performance on both types of questions improved when they had completed a surface-feature task prior to the presentation of the question and decreased when a deep-structure task was used. The presence of the surface-feature task apparently encouraged experts to incorporate surface features in their representation of the program, while the presence of the deep-structure task reinforced the experts' natural preference for that type of representation. The opposite pattern of results was observed in novices.

Dufresne (1988) and Hardiman et al. (1989) also found that novices' tendency to use surface features to judge similarity varied with problem-solving ability as measured by a problem-solving task. Novices with a high problem-solving ability tended to use deep structures more, and surface features less, than those with lower ability. Hardiman (1988) has obtained similar results using fraction word-problems both with undergraduate and with grade eight subjects. Again, this difference in the basis of similarity judgements is supported by the reasons that subjects provided for their assessments of problem similarity: more capable problem-solvers tended to refer to principles of physics in their justifications more often than less capable problem-solvers. Given the greater sensitivity of the more recent studies to such effects, and an assumption of the continuity of expertise, the observation of the two effects reported by Dufresne (1988) and Hardiman et al. (1989) in expert and novice groups is consistent -- indeed, would be predicted -- from their earlier observation in intermediate and advanced novice groups (Chi, Feltovich, & Glaser, 1981).

One apparent anomaly is reported by Dufresne (1988): In spite of the correlation between problem-solving skill and the probability of using deep structure to judge similarity, increasing novices' tendency to categorize problems on the basis of their deep structure did not produce a concomitant increase in problem-solving ability. In this study, novice subjects solved 25 practice problems using a computerized principle-based analysis tool, a computerized equation-based analysis tool, or a standard physics textbook. Pre- and post-tests of problem-solving ability, problem categorization, and explanations of the problem situation were administered. Only the group which used the principle-based analysis tool showed a significant increase in the use of deep structure in problem categorization and in explanations of the problem situation; however, subjects who received this treatment showed no more improvement in problem-solving ability than those using the standard physics textbook.

This pattern of results would seem to represent the development of a form of task-dependent expertise by the group using the principle-based analysis tool. Because of this software's emphasis on the identification of the physics principle required for a problem's solution (Dufresne, 1988, p. 5), subjects might develop considerable expertise at making such identifications, without acquiring all the skills and domain knowledge needed to actually apply those principles to problem solution. In this case, deep-structure categorization would lack the functionality it holds for expert problem-solvers, and would constitute a second skill unrelated to the solution of physics problems. Of course, under these circumstances, problem-solving ability could not be expected to increase with the ability to make such categorizations.

Given the different principles used by novices and experts in categorizing problems, it could be expected that experts would be able to organize problems into a more ordered, hierarchical structure than novices. Domain principles should provide more of a basis than surface features, both for identifying commonalities and for making discriminations between problems. Some evidence of this has been found. Chi, Glaser, and Rees (1981) encouraged subjects to subdivide and combine the categories into which they had initially sorted physics problems. Experts tended to form fewer initial groups containing more problems in each group, and included all of their initial groups in a limited number of superordinate categories. Novices formed smaller, more numerous categories, and substantial numbers of these were not subsumed within a superordinate category. Some novices also showed a limited ability to discriminate between problems, either failing to subdivide the initial categories, or creating a separate category for each problem. These results support experts' greater ability both to identify commonalities and to make discriminations between problems.

The possibility of some challenge to the above is raised by the inconsistent results obtained in studies which have used a one-pass problem-sorting task. Ideally, these studies would have found the pattern of results observed by Chi, Glaser, and Rees (1981) in subjects' initial problem sorts, with experts forming fewer and larger categories than novices. Weiser and Shertz (1983) did find that experts included more programming problems in their four largest categories than did novices; however, Smith (1990, 1992) observed the reverse in subjects' categorization of genetics problems, and Chi, Feltovich, and Glaser (1981), using physics problems, found no significant difference in the size or number of expert and novice categories. Smith (1992) does indicate that the genetics problems used in this study were selected to include a few problems each of a variety of problem types, and suggests that expert subjects were better able to discriminate between these types, thus forming more categories with fewer problems in each category. This explanation does not account for the lack of significant differences reported by Chi, Feltovich, and Glaser, and, it must be admitted, seems rather ad hoc. However, given the more detailed analysis performed by Chi, Glaser, and Rees (1981), and the lack of clearly contradictory results, it does not seem unreasonable to assert that experts do seem able to organize problems in a more ordered, hierarchical fashion than novices.

Differences in solution strategies

The observed differences between expert and novice problem representations predict a difference between the problem-solving strategies used by these two groups. The solution-specific organization of experts' representations and knowledge structures should, to a large extent, automatize the strategy selection stage of the problem-solving process. A fairly direct, forward-looking strategy can, therefore, be expected. Novice representations, on the other hand, lack such solution-specific organization and information; the selection of specific solution techniques should be a significantly effortful stage in the solution process. The use of intermediate strategies to organize this task is predicted.

These predictions are supported by the protocols obtained by Chi, Feltovich, and Glaser (1981) when subjects were asked to identify their basic approaches for solving specific physics problems. As has been discussed above, experts produced basic principles which could be used for solution, while novices produced general heuristics, or had to actually solve the problems to determine a solution method. More directly, both Larkin, McDermott, Simon, and Simon (1980a, 1980b) and Simon and Simon (1978) have analyzed protocols obtained from experts and novices during the solving of physics problems and observed that experts use a forward-looking strategy in which quantities given in the problem statement are used to determine previously unknown quantities until a solution is obtained. Novices first use means-ends analysis to work backward from the required solution, producing a sequence of sub-goals until a sub-goal is found which can be reached by using the values given in the problem statement. They then use a forward-looking strategy to work back through the sub-goals to solution.

This difference in problem-solving strategy has been incorporated in two versions of a production system developed by Simon and Simon (1978) and extended by Larkin et al. (1980a, 1980b) by varying the criterion used to select equations for use in problem solution. In the first, forward-looking version, equations are included in the problem-solving process if all but one of the variables in that equation are known, thereby allowing the equation to be solved for the single unknown quantity. In the second, backward-looking version, the model begins by selecting all equations which contain the unknown given in the problem, and then adds additional equations if they include unknown quantities included in previously selected equations. This process is continued until an equation is found which can be solved with the known quantities. The newly derived quantity is then substituted into preceding equations, and the process is continued until one of the originally selected equations can be solved for the unknown required by the problem.

The order in which equations are generated by these two models corresponds very closely to that observed in protocols of experts and novices solving the same problems: The model used by Simon and Simon (1978) produced the same equations in the same order as they were generated by experts and novices in 16 out of 19 or 84% of the problems solved; the model used by Larkin et al. (1980a, 1980b) produced identical orders in 18 of 19 or 94% of the problems solved by experts and in 17 of 19 or 89% of the problems solved by novices. Additional support for this difference in strategy use is provided by Sweller, Mawer, and Ward (1983), who observed a shift from means-ends analysis to forward-looking strategies with increasing expertise in mathematics problem solving, and by Anzai and Simon (1979), who identify the same strategy shift in the protocol of a subject learning to solve the Tower of Hanoi problem.

Priest and Lindsay (1992) did not observe this difference in strategy use, finding, instead, that both novices and experts used forward-looking strategies when solving physics problems. This study does present, however, two areas of methodological concern, either of which would seem capable of masking the indicated difference in strategy use. First, the protocols analyzed in this study consisted of the written work generated by the subjects during problem solution, rather than a verbal description of problem-solving activity. Such protocols would seem likely to exclude all indications of means-ends analysis, except where such analysis was conducted in the most formal fashion, as a process of explicit algebraic operations. This would seem inappropriately restrictive.

The second area of concern is the possibility that all of the subjects used were able to solve the experimental problems at the expert level. Although Priest and Lindsay (1992, p. 401) dismiss this as implausible, this possibility would seem to deserve serious consideration, given the rather stringent criterion used for inclusion in the final data analysis: Subjects were included in the protocol analysis only if they were able to solve all six of the experimental problems in the allotted time of 10 minutes per problem. While all of the subjects in the most expert group were able to meet this criterion, 33% of the subjects in the less expert groups were eliminated from the analysis; the novice subjects who were not excluded must actually have possessed considerable expertise. It would not seem implausible that such intermediate or advanced novices might dispense with the formal step-by-step analysis which could be detected by this design.

Schema theory

At present, the term "schema" is something of an umbrella term which subsumes a variety of models of experiential or episodic memory which attempt to explain how experience is structured and stored in memory in such a way that it can be used to guide future behavior in similar contexts. While these models differ in the mechanisms they propose for the acquisition, activation, and operation of these structures or schemata, certain elements of form and function do remain constant across the models. These elements constitute a general definition of schemata.

In the broadest sense, a schema is a memory structure which organizes a set of similar experiences as a set of constrained variables. Each variable in a schema represents an element common to all the encoded experiences, and, therefore, predictable in future experiences of the same type. A restaurant schema (see Figure 1), for example, would likely incorporate variables for "ordering a meal", "receiving a meal", "eating a meal", and "paying": For most people, these are the elements which they have encountered consistently, and thus expect, at a restaurant.

The variables in a schema are filled in or instantiated when the schema is activated by a particular context (see Figure 2). Each variable is instantiated by an element abstracted from the activating context to meet the constraints associated with that variable. These constraints may be fixed, or defined by the values used to instantiate other schema variables. In the restaurant schema, for example, the "paying" variable would be instantiated by the specific procedure which would be appropriate for the activating restaurant context. A fixed constraint on that variable would likely be that payment should be made in a medium of exchange such as cash or credit card, rather than through barter. The time of payment would likely be constrained by the instantiation of the "ordering a meal" variable. If ordering is done at a counter, as in a fast-food restaurant, payment would be made after ordering but before receiving the meal; if the meal is ordered from a waiter, payment after consumption of the meal

Figure 1
A simplified restaurant schema

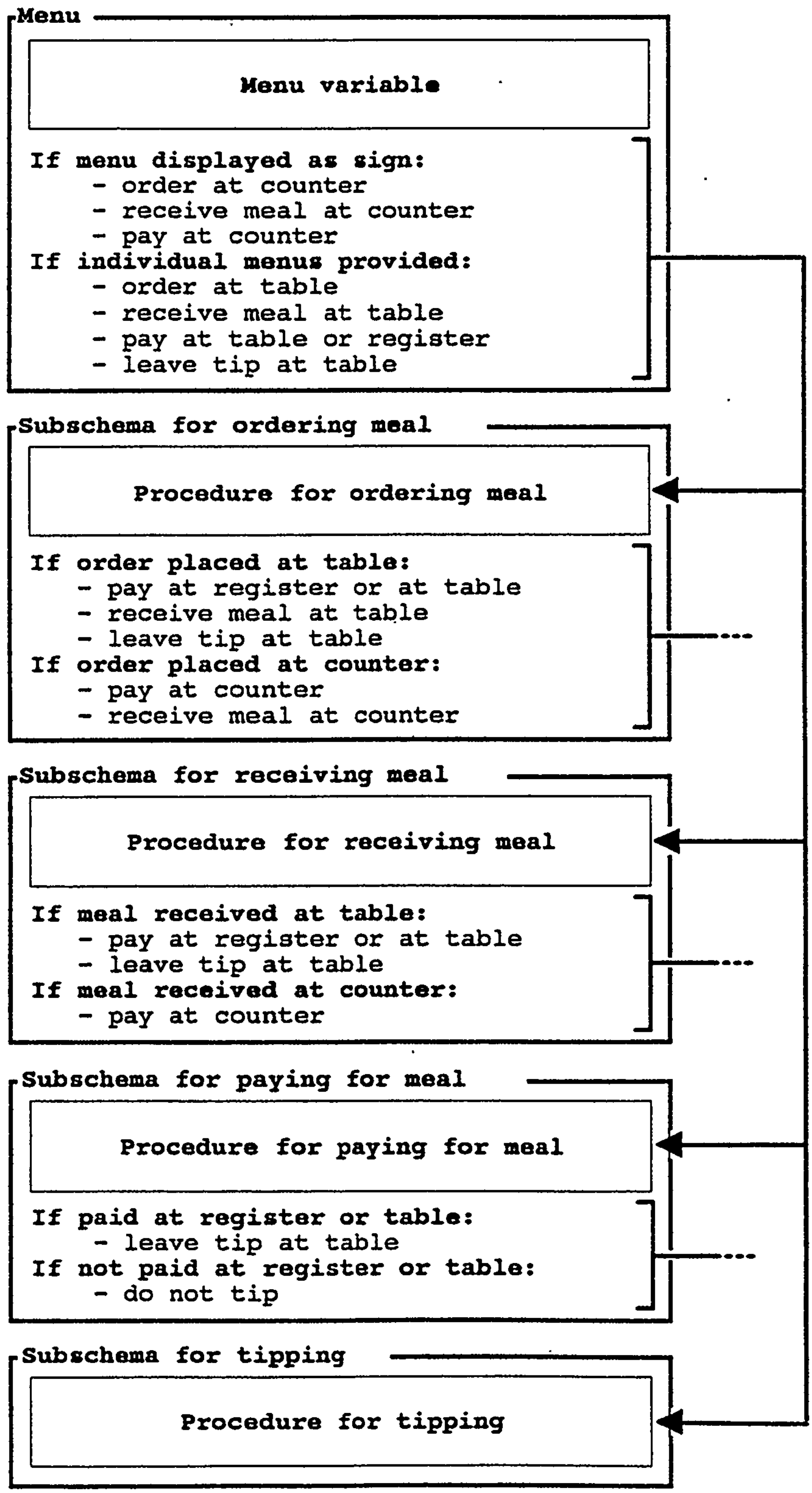
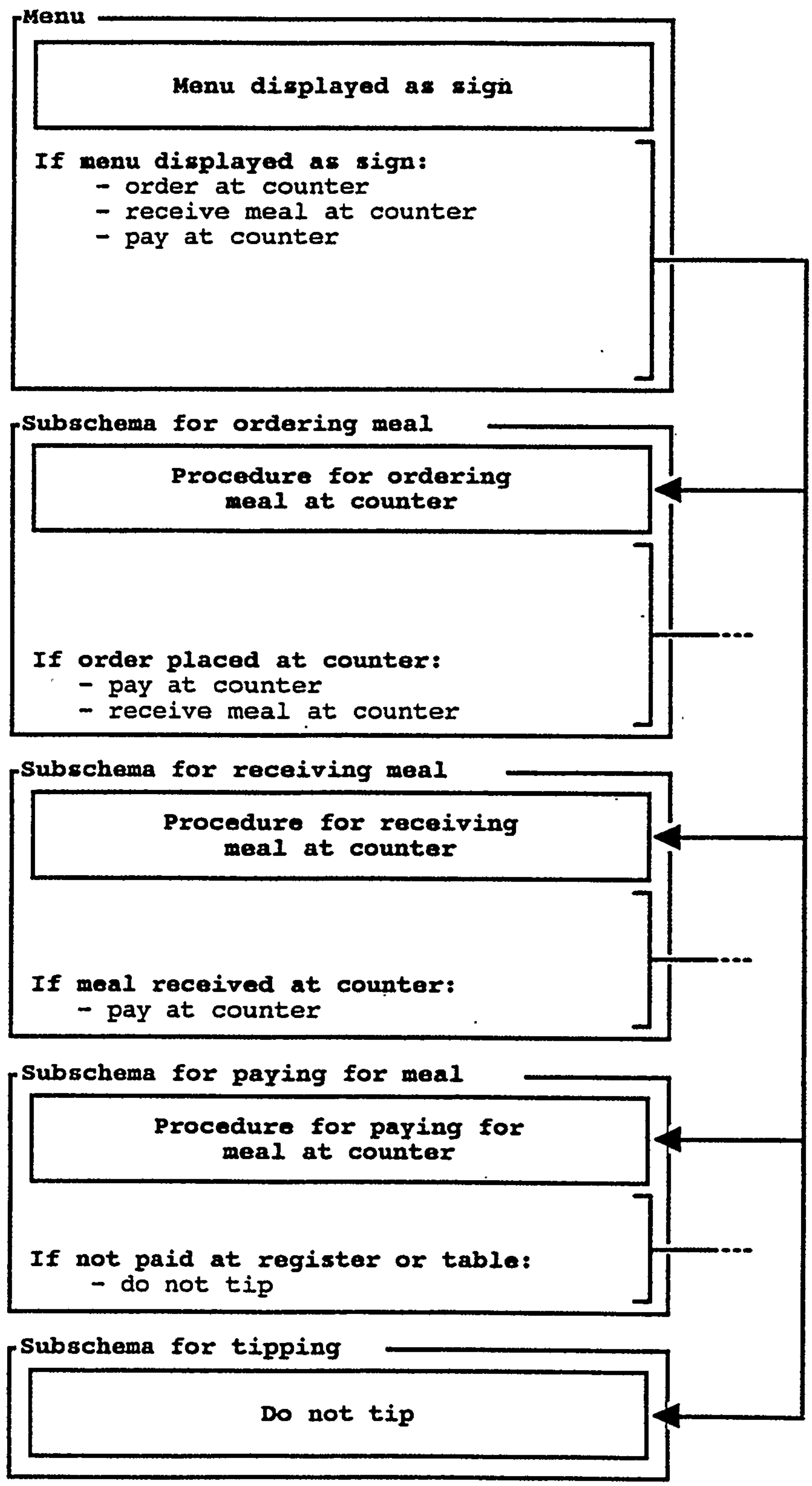


Figure 2
One possible instantiation of a restaurant schema



would be indicated. The procedure used to instantiate the "paying" variable would depend on the instantiation of the "ordering a meal" variable (and vice versa).

It can be hypothesized that, in problem-solving contexts, a schema may be associated with a particular class of problems. In this case, the solution of a problem of that class would actually depend on the instantiation of its associated schema. When a schema is fully instantiated -- when all of the relevant elements of the situation and the relationships between these elements are completely defined -- the process of obtaining a solution should be as straightforward as the problem of obtaining a meal in a familiar restaurant context. The problem should cease to be problematic.

Of course, the possession of a schema for a particular class of problems does not mean that all problems of that type can be solved easily, nor does it guarantee that the problem can and will be solved. Atypical problem contexts may inhibit or prevent the identification of the appropriate schema for that problem. Similarly, atypical or deliberately disguised elements of the problem may interfere with the instantiation of the schema variables. Instantiation may also become a problem if the pattern of mutual constraints is sufficiently complex that their resolution becomes non-trivial. If the schemata is not complete -- if variables are insufficiently constrained or if necessary subschemata are absent -- the final "solution" may actually consist of a number of equally likely alternatives.

Examples of the two types of problems -- those which could be solved automatically and effortlessly through schema activation versus those for which such a process would be highly effortful and possibly inadequate for the determination of a single, best solution -- are offered by the beginning and middle games of chess, respectively. In the beginning game, the number of piece positions which can be obtained through legal and strategically reasonable sequences of moves is relatively limited. These positions have been extensively analyzed and evaluated to determine the optimum move; learning these standard openings and their variations is recognized as part of the process of developing expertise in chess. Schema for these positions would be complete, with the move to be made rigidly constrained by the existing position. Schema solution of the problem represented by such positions should be quite effortless and automatic.

Because of the vast number of positions which might be encountered, the middle game of chess is not susceptible to the position-by-position analysis which has been devoted to the opening game. Consequently, the schemata which an expert might possess for this part of the game would be much less than complete. While fairly definite subschemata might exist for portions of a particular position, the constraints between these subschemata are likely to be more heuristic than algorithmic, producing alternatives rather than single solutions. In addition, the large number of chess relationships contained in a normal chess position would tend to produce a correspondingly complex system of mutual constraints between the variables and subschemata of the middle-game schema. The effortless resolution of so complex and indeterminate a system cannot be expected.

Under this hypothesis, expertise would be a function of (1) the number of problems for which schemata exist, (2) the organization of these schemata, and (3) the completeness of the problem schemata. The highest level of expertise -- that corresponding to the effortless, expert situation described above -- would be represented by a single domain schema, hierarchically organized into subschemata comprehending every type of problem which might be encountered in that domain. Once activated by the recognition that a particular problem belonged to the schematized domain, such a memory structure would be capable of managing the entire problem-solving process, from the selection of an appropriate schema, through the abstraction of relevant information for the instantiation of that schema's variables, to the determination of a final solution by the resolution of the constraints imposed by the instantiated schema variables.

Given the intended universality of schema theory, the lack of expertise under this hypothesis would be represented, not by the absence of any schema for a problem or problems in a domain, but by the incompleteness of the schemata possessed and by the lack of hierarchical organization for these schemata. In order to use previous experience as a guide for present problem-solving behavior, novices would have to have some sort of schema for solving problems in a given domain. However, the constraints incorporated in this schema would be general heuristics lacking the degree of constraint which would simplify, or ensure, their resolution. Such general schemata would also lack the hierarchical organization which would be typical of expert schemata and which would depend on domain knowledge. Not only the instantiation, but even the selection of an appropriate schema might be problematic. In short, a novice would be defined by his or her possession of general, heuristic, rather than domain-specific, problem schemata.

As has been mentioned above, the definition presented here is of schemata in their most general form. Far more detailed models have been developed by researchers such as Case (1985), Neves and Anderson (1981), Rumelhart and Norman (1976), and Schank and Abelson (1977). However, the available research on expertise is not sufficiently fine-grained to support the details of structure and operation included in these models. In fact, such details are not even necessary to explain the observed effects of expertise on problem-solving behavior, as these effects can be predicted from the basic model.

Schema theory and the effects of expertise

Differences in problem representation

To consider the observed differences in problem representation first, it must be remembered that, under the schema hypothesis, a problem would be represented as a partial instantiation of the schema for that problem class. In the case of the expert, the schemata for individual problem classes are incorporated into the domain schema through a hierarchical process of subschematization. A problem's class, and, therefore, its representation, is thus determined by the structure of the domain schema and is, in fact, a partial instantiation of that schema. As this structure also defines the solution process for a particular process, it can be expected (1) that expert problem representations would be directly related to their method of solution, (2) that experts would classify problems according to the techniques required for their solution and (3) that expert classification schemes would be hierarchically organized. It would also seem clear that (4) experts should be able to identify, quite specifically, the procedures they would use to solve a particular problem or class of problems.

As will be seen below, novices, with increasing experience, might develop domain-specific schemata for some types of problem; they might also develop reasonably constrained subschemata underneath their domain schema which would allow them to discriminate between broad classes of problems. However, the constraints between these upper and lower levels of organization would remain, by definition, heuristic. (The possession of a complete domain schema would, under this model, qualify an individual as an expert in that domain.) Thus, at some point, selection of the appropriate subschemata to complete the process of instantiation would become problematic.

This dependence on general heuristics would limit novices' ability to incorporate solution-related knowledge in their problem representations. The identification of the procedures needed for solution (i.e., of the subschemata to be used in instantiating the domain schema) would depend on the resolution of the general heuristics by which their selection was constrained and would, therefore, be part of the solution, not the representational process for that problem. Because of this, novices' problem representations can be expected to be data-driven, rather than goal-directed. In the absence of a clearly-defined solution procedure, the ability to judge the relevance of problem elements to solution would obviously be impaired; ready availability of information, rather than its strict necessity, is likely to determine the composition of the problem representation.

Again, novices' ability to classify problems according to the techniques required for their solution would depend on at least a partial solution of the problems being classified. Where this was prevented, it could be expected that novices would base their classifications on the information which was available: the surface features of the problem. Such features, which would be only incidentally related to the structure of the domain, could not be expected to provide the discriminatory power or the hierarchical organization afforded by domain principles. In summary, it could be expected (1) that novices' problem representations would rely heavily on information readily available from the problem statement, (2) that novices would classify problems according to their surface features, (3) that novice classification schemes would lack discriminatory power and hierarchical organization, and (4) that novices would not be able to provide a clear description of the approach they would use to solve a particular problem or class of problems.

Differences in solution strategy

From the above discussion, it should be clear that, while experts' domain schemata incorporate specific solution techniques as a hierarchy of subschemata and their constraints, novice schemata are characterized by their lack of well-defined solution procedures. Consequently, novices must identify specific solution techniques through the use of a general heuristic before they can proceed with the actual solution of a given problem. The use of a backward-looking strategy such as means-ends analysis is, therefore, predicted for this group. Experts, on the other hand, would have no need of such a strategy; the specific techniques required for the solution of a particular problem would already be available in their domain schemata. Thus, the use of a fairly direct, forward-looking strategy could be expected.

Differences in problem perception

The amount of information which must be abstracted from a problem to form its representation would vary from domain to domain. In some problems, such as physics, mathematics, or genetics word-problems, the problem statement would normally contain only a few pieces of information; the solution of such problems typically requires the identification of unknown quantities which can be inferred from the quantities given in the problem and from which the solution can, in turn, be inferred. In other domains, such as chess, bridge, go, or the comprehension of electronic schematics or computer programs, the problem "statement" may contain a large number of individual pieces of information, all of which must be considered in the problem representation. In these contexts, the problem which is posed is one of integration: these individual elements must be organized into a structure which can be comprehended and manipulated without the loss of significant information.

In this latter case, the subschemata included in the experts' domain schema would provide a powerful benefit. These would not only provide an available pattern for the organization of problem information in a form directly related to its use in problem solution, but would also significantly reduce the demands placed on short-term memory during the solution process. Because these subschemata are available in long-term memory, and because they constrain or specify the problem elements with which they may be instantiated, the elements included in a well-defined subschemata need not be represented individually. They may be replaced with an identifier for the subschemata, with, possibly, an indication of variations from the basic pattern defined by that subschemata. In short, these subschemata provide a powerful mechanism for chunking complex problem spaces in a fashion directly related to the solution of the problem involved.

Novices, lacking these well-defined subschemata, would, of course, also lack the mechanism these provide for organizing the problem space into functional chunks. As in the case of representation formation, the development of these functional units would be part of the process of solution rather than of representation. When given a task such as the short-term recall of a given problem, which required or would benefit from such chunking but in which solution was not possible, some other strategy would have to be adopted. The use of surface features with a concomitant loss of chunking power, could, again, be expected.

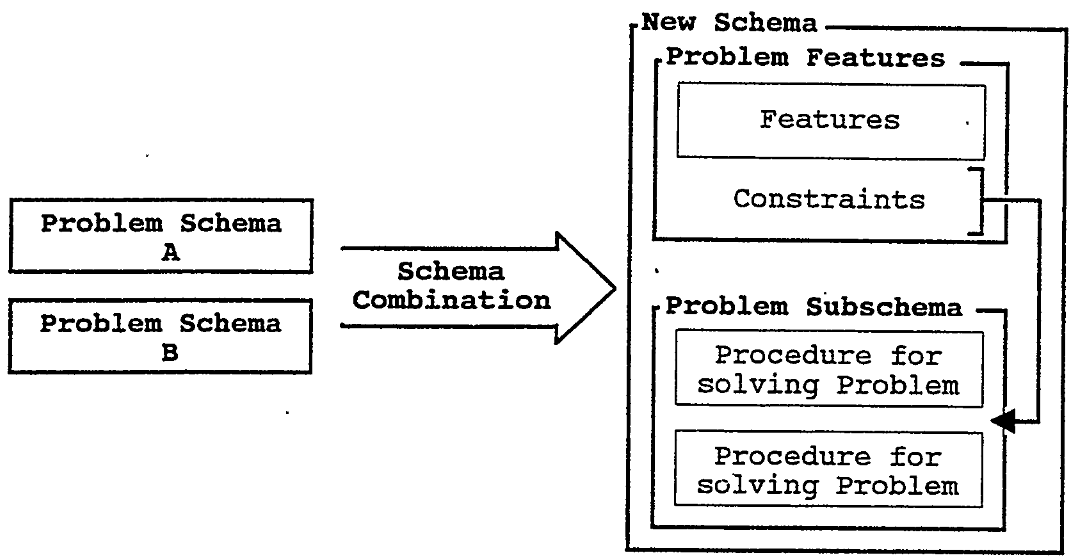
Of course, experts would not be expected to possess subschemata for problems whose organization was not congruent with the principles of the given domain. Where problems were randomly generated or constructed in contravention of domain principles, no benefit of expertise could be expected. Thus, in problem perception, as in problem representation and solution strategy, the differences in expert and novice problem-solving behaviours which have been discussed previously can be predicted (or at least deduced) from the basic principles of schema theory.

The development of expert schemata

Cooper and Sweller (1987) have suggested that the processes of schema acquisition and automatization provide a basic mechanism for the development of the hierarchical knowledge structures which guide expert problem solving. As novices gain experience with problem solving in a particular

Figure 3

Schema development through combination of existing schemata



domain, they begin to group problems that require the same or similar solution techniques (see Figure 3). With continuous use, these categories develop into schemata which define the criteria for inclusion in that category, the solution techniques to be applied, and the information required for solution. The problem-solving process is thus partially automatized, cognitive load is reduced, and cognitive resources are released for further schema acquisition or to process unique features of particular problem states (Sweller, 1988). More elaborated models of this mechanism, which emphasize the use of composition or coordination to combine schemata into more general structures, have been proposed by Neves and Anderson (1981) and, in a developmental context, by Case (1985).

Support for this mechanism is provided by Sweller, Mawer, and Ward (1983), who used a limited set of kinematics problems requiring only two different solution techniques. With repeated trials, it was found that subjects who shifted from means-ends analysis to the forward-looking strategies characteristic of expertise did so in a category-dependent fashion. Once a forward-looking strategy had been adopted for a problem from either category, that strategy was applied to subsequent problems of that type. This strategy shift did not occur simultaneously across problem types.

This mechanism and the available research into expert-novice differences emphasizes the upward development of hierarchical structures. Novices are seen as moving away from a reliance on the specific features of individual problems toward the use of general principles capable of organizing large groups of problems. Structure is gradually imposed by combining existing structural elements to form new, more generalized elements.

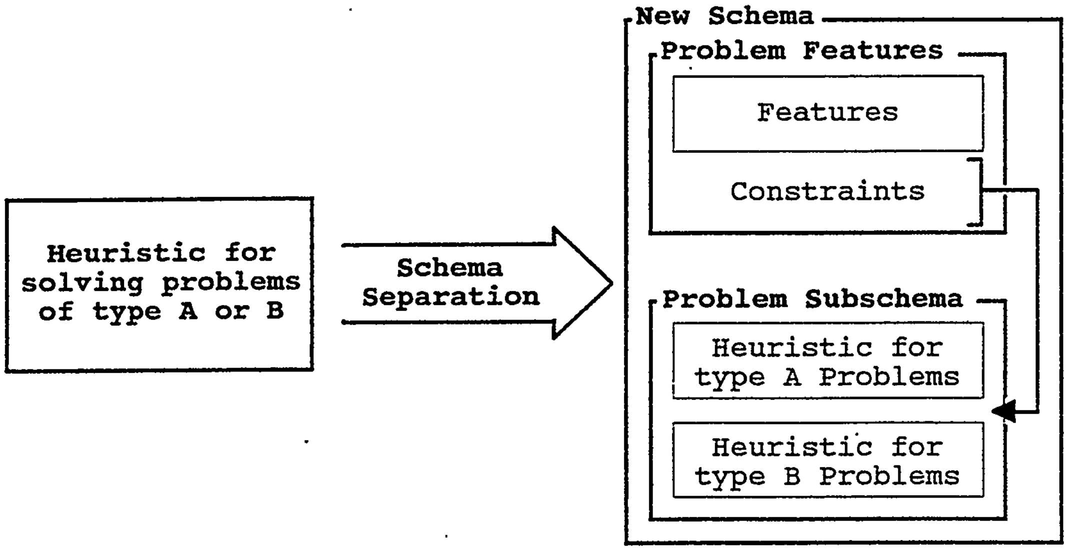
To a large extent, considerations of cognitive load support this emphasis. Novices are not trying to develop expert knowledge structures; they are attempting to find solutions for specific problems and problem types. Development of expertise is a secondary goal, and problem solution may leave few attentional resources available for this process (Sweller & Levine, 1982). Mechanisms which provide small, short-term, and successive reductions in cognitive load would thus seem more appropriate than those which might provide a larger facilitative effect but at a higher cognitive cost. Processes such as schema acquisition, composition, and automatization, which operate through the gradual extension of existing structures, would seem to offer this kind of immediate cognitive economy. As novices initially lack functionally defined structures other than the solution techniques for individual problems, an upward development of hierarchical structure can be expected, as the aforementioned processes are successively applied to these.

However, it must be noted that, within the general domain of problem-solving, the recognition of a specific domain provides a second level of organization available to, and used by, novices. While the same general heuristics may be employed across domains, the application of these can be expected to differ strongly between domains. Means-ends analysis might be employed, for example, in the solution of both chess and physics problems; however, the form of those analyses, the sub-goals they produce, and the features of the problem space which are considered can be expected to show few similarities between the domains. A novice's recognition of a particular problem-solving domain and the expectations which guide his behavior in that domain would seem, in effect, to constitute a highly general subschema within the problem-solving schema.

The existence of this second level of structure and the two-stage solution process used by novices suggest the possibility that a second, downward mechanism might be involved in the development of expert-like knowledge structures (see Figure 4). This would operate through the process of schema separation and the partial automatization of sub-goal formation in means-ends analysis. As novices gain experience with a domain, they may learn to differentiate sub-domains which require the application of fairly discrete groups of solution techniques. Separating the original domain schema into subschemata based on these groups would limit the breadth of search required during

Figure 4

Schema development through separation of existing schemata



means-ends analysis, thus reducing cognitive load and making further schema separation possible. Successive applications of this process would result in a gradual downward growth of hierarchical structure which would complement the upward growth available through schema acquisition, composition, and automatization.

Some support for this hypothesis is offered by casual observation. As has been noted above, novice problem-solving behaviours vary widely across such domains as chess and physics. Even when operating in domains such as Euclidean geometry and trigonometry, where surface features may be highly similar, novices seem able to identify discrete systems of relevant operations.

Two pieces of experimental evidence can also be found for this hypothesis in the study performed by Chi, Feltovich, and Glaser (1981). In the categorization tasks discussed above, 5 of the 20 categories identified were shared by both experts and novices. These accounted for 40% and 31% of the problems categorized by novices and experts, respectively. In the elaboration task, the two category labels which occupy the second level of the given expert schema -- "Conservation of Energy" and "Newton's Force Laws" -- also appear in the novice schema. Principles which appear lower in that portion of the expert schema concerned with general principles do not appear in the novice schema. This pattern of results suggests that some structuring of the uppermost levels of the novice knowledge structure has occurred.

Additional support for this hypothesis is provided by Smith (1990, 1992) who observed that the "sex-linked" label, which was one of the categories most commonly used by faculty members when classifying genetics problems, was also commonly used by novices, accounting for 29% of the category labels produced by novices. Smith notes that, in novices, this categorization did not seem to be supported by the procedural knowledge necessary for successful solution. This suggests that, while novices were able to recognize the existence of a general category of sex-linked problems, solution of problems within this category still depended on the use of a general heuristic.

Summary of proposed developmental models

Two models for the development of expert schemata can thus be proposed. In the first, these knowledge structures are developed by the repeated application of a single operation: the solution procedures for individual problems are combined through a process of composition, coordination, or accretion into successively more general structures until, finally, a domain-wide schema is obtained. In the second model, this combinatorial process is supplemented by a process of schema separation, in which novices learn to recognize broad classes of problems within a domain, thus establishing subschemata within the highly general, heuristic domain schema they initially possess. These subschemata, although still heuristic, would provide an additional degree of constraint which would benefit means-ends analysis and might, in turn, develop subschemata of their own. This process would continue until the most specific subschemata developed by this process could be completed by the most general structures developed by the complementary combinatorial process -- until, as it were, the two ends met in the middle.

Experimental comparison of the developmental models

These two models can be distinguished experimentally by comparing the actual ability of novices to judge the relevance or irrelevance of specific equations to the solution of physics word problems with their ability as predicted by each of the two models. In those cases where the problems to be considered have been incorporated into a schema through the operation of the combinatorial process described above, both models make the same prediction: As the required information would be included in the schema for that problem, the task should be a trivial one; the number of equations identified as possibly relevant to the solution of such a schematized problem should approach or equal the number actually required for that problem's solution.

In the case of unschematized problems, however, different behaviours are predicted by each of the two models. In the absence of means-ends analysis, and with the exceptions discussed below, the combinatorial process common to both models does not provide a mechanism for determining the relevance or irrelevance of particular equations to problems for which no schema exists. If this is the sole process involved in the development of expert schemata, the number of equations identified as possibly relevant to an unschematized problem should approach or equal the number of equations available for classification. It should be noted that this pattern of classification would not change with increasing expertise, which would reduce the probability of encountering an unschematized problem, but would not affect the manner in which such problems were processed.

If, on the other hand, this combinatorial process is supplemented by a process of schema separation, the ability to discriminate between broad classes of problems should appear at fairly low skill levels, with the first separation of the problem-solving domain into subschemata. As expertise increases, and the problem-solving domain is separated into progressively finer subschemata, the discriminative power afforded by the developing structure should also increase. The number of equations which need to be considered as potentially relevant to the solution of a particular problem should decrease with each separation of the subschema which includes that problem, until the problem is incorporated into a completed schema through the operation of the combinatorial process. Thus, novices should be able to identify a significant number of equations as irrelevant to the solution of unschematized problems, and this number should increase with increasing expertise; the number of equations identified as possibly relevant to such problems should show a concomitant decrease. It might also be noted that the number of equations identified as probably -- as opposed to possibly -- relevant to an unschematized problem would not be expected to increase with expertise, as the discriminative power provided by the developing knowledge structure would be wholly exclusionary.

The above discussion has assumed that all problems are either wholly schematized or wholly unschematized, with no intermediate cases. In actuality, partial schematization could occur in two ways: either the structures being combined, or the information needed to combine these structures might be incomplete. In the first case, the partial schema would provide two pieces of information not available for wholly unschematized problems, identifying both a relevant, if somewhat heuristic, subschemata for solving problems of the given type, and a set of related subschemata which could be used to solve problems of a similar, but not identical, type. Under these circumstances, it can be expected that the set of related subschemata would be used to provide partial guidance during the solution of the given problem. Strategic information in these subschemata could be used to abridge the process of means-ends analysis by supplying a series of appropriate subgoals and solution procedures to replace portions of that analysis; the problem solver should also be able to identify, with a high degree of confidence, the probable relevance to the given problem of the equations included in these related subschemata. In the absence of means-ends analysis, however, the set of related subschemata would not identify the modifications to their solution strategies which would be needed to produce a viable solution strategy for the given problem. Further means-ends analysis would be needed to identify the equations which must be added to, or substituted into, the existing strategies. In the absence of such analysis, the problem solver would still lack the means of assessing the probable relevance of equations not included in the set of related subschemata. Thus, even where this type of partial schematization has occurred, the combinatorial process would still not provide a mechanism for identifying equations as irrelevant to solution. If this is the sole process involved in the formation of expert schemata, it would be expected that the number of equations identified as possibly relevant to such partially schematized problems would approach or equal the number available for classification.

As mentioned above, a second type of partial schematization is possible, in which the schemata being combined are complete, but the information needed to combine these into a more general, fully constrained schema is not. In this case, the partial schema would identify a set of subschemata which could be used to solve the given problem, but would not identify the specific subschema to be used for that problem. This form of partial schematization would allow the problem solver to identify a significant number of equations as irrelevant to the solution of a problem, because it would separate the set of available equations into two discrete categories: those which were included in one of the subschemata of the incomplete schema and which might, therefore, be relevant to solution, and those which were not included in any of these subschemata and which would, therefore, be clearly irrelevant to solution. Thus both of the proposed models of development predict that the number of equations identified as relevant to the solution of such partially schematized problems would be significantly less than the number of equations available for classification. This value can not, therefore, be used to discriminate between the two models.

However, the effect of increasing expertise on the number of equations identified as possibly relevant to the solution of a given problem can be used to make this discrimination. As noted above, the two-process model of schema development predicts that the number of equations identified as possibly relevant to the solution of an unschematized problem would decrease with increasing expertise, as the developing knowledge structure allowed successively finer discriminations between problem classes. The one-process model predicts that the number of equations identified as possibly relevant to wholly unschematized problems would remain constant, at or near the number of equations available for classification; this prediction, as has been seen, would also apply in those cases where schematization was incomplete because information was missing from one or more of the subschemata being combined.

In the last type of partial schematization which has been described, where schematization is incomplete because information needed to select the appropriate subschemata is missing, the number of equations identified as possibly relevant to the solution of the given problem would depend on the number of equations included in the set of possibly appropriate subschemata. As expertise increases, so, too, will the complexity and size of the structures being combined to form new schemata, and the number of equations these structures contain. Thus, the one-process model predicts that, in such cases of partial schematization, the number of equations classified as potentially relevant should also increase.

It should also be noted that, under both models, the number of partial schemata should be relatively small. Both models are based on the premise that the cognitive resources required for the development of new schemata are made available by the partial automatization of the problem-solving process provided by existing schemata. The existence of large numbers of incomplete schemata would not be consistent with this premise. Thus, the increase, with expertise, in the number of equations rated as relevant to partially schematized problems should not be sufficient to mask the decrease predicted by the two-process model.

Experimental hypothesis

Thus, the two models which have been proposed can be tested experimentally in the domain of physics by (1) presenting subjects at different levels of expertise with a series of physics word problems and a set of equations which might be used to solve those problems, (2) having the subjects rate the apparent relevance of the individual equations to each problem, and (3) examining the effect of expertise on the number of equations identified as possibly relevant to unschematized problems. If the combinatorial process described above is the sole process involved in the development of expert schemata, this number should remain constant or increase with increasing expertise. If, on the other hand, a process of schema separation is also involved in the development of expert schemata, this number should decrease with increasing expertise.

Implementational considerations

In implementing the above, two additional questions need to be considered. First, subjects must be prevented from solving or performing means-ends analysis on the problems while performing the rating task. The preferred -- because least intrusive -- solution to this problem would be to advise subjects, in the instructions for the experimental task, that they should not attempt to solve the given problems or subject them to detailed analysis, but should provide ratings of probable relevance based on their first impression of the given problems. If this approach should prove inadequate, the final sentence of each problem, which identifies the quantity to be obtained as a solution could be removed; Sweller, Mawer, and Ward (1983) have shown that this is effective in preventing means-ends analysis of word problems.

The second implementational question which needs to be considered is the identification of schematized problems. Both of the proposed models predict that the number of schematized problems will increase with expertise, and that a minimum number of equations will be identified as possibly relevant to the solution of such problems. Thus, if schematized problems are not excluded from the analysis, both models would predict that the number of equations identified as possibly relevant to the combined set of schematized and unschematized problems would decrease with increasing expertise. The presence or absence of this effect could not, then, be used to discriminate between the models.

As has been indicated above, the equations identified as possibly relevant to the solution of a schematized problem should be those actually required for that problem's solution; in the case of unschematized problems, both models predict that a significant number of additional equations would be so identified. Thus, the number of equations required to solve a particular problem could be used as a criterion for identifying cases where subjects possess complete schemata for particular problems: Those cases for which the number of equations identified as possibly relevant was equal to the number of equations required for solution would be excluded from the analysis; all other cases would be retained.

However, this criterion would seem to be inappropriately stringent, as the existence of any unidentified factor which would elevate the number of equations identified as possibly relevant to the solution of schematized problems would result in the inclusion of significant numbers of such cases in the data analysis, thereby invalidating the use of this measure to discriminate between the proposed models. The use of this criterion would be particularly questionable if it were found necessary to remove that part of each problem which identified the quantity to be obtained as a solution in order to prevent means-ends analysis. It would seem likely that this information would be used in selecting the appropriate schema for a given problem. Its elimination, therefore, could produce a form of induced partial schematization: subjects would be able to identify a set of complete problem schemata or subschemata which might be used to solve a given problem, but would be unable to identify the specific schema required.

Earlier, it was said that this type of partial schematization would not prevent the use of the indicated measure to discriminate between the two models which have been proposed. That conclusion, however, was based on the premise that the number of such partial schematizations which would be produced in the normal course of schema development would be limited, and that there is no reason to suppose that this number would vary with expertise. This premise would not be valid where this condition was produced for wholly schematized problems by the removal of information from the problem statement. As noted above, the number of wholly schematized problems and, therefore, the number of such "partial unschematizations" would increase with expertise, and could be highly significant even at moderate skill levels. Thus, the inclusion of a significant number of such problems in the data analysis could produce a significant increase in the number of equations identified as possibly relevant, confounding the effect being used to discriminate between the proposed models of schema development.

Given the above, it would seem more appropriate to use the characteristics of the data set, rather than the number of equations actually required for the solution of individual problems, to establish a criterion for classifying problems as schematized or unschematized. It should be possible to identify such a criterion through an inspection of the distribution of results for individual problems or for the combined set of problems. If necessary, separate criteria could be established for subjects from each level of expertise included in the study; however, the possibility of introducing experimenter bias into such a procedure must be carefully considered. In performing such a procedure, preference should be given to the exclusion, rather than the retention of doubtful cases. While this might result in the exclusion of some cases where partial schematization has occurred as part of the normal process of schema development, the number of such cases should, as noted above, be minimal. At worst, this might slightly exaggerate an increase, with increasing expertise, in the number of equations identified as relevant to partially schematized problems, thus favoring the one-process model over the two-process model.

CHAPTER III

METHODOLOGY

Subjects

Two groups of novices (Novices and Advanced Novices) and one group of intermediate (Intermediates) subjects were used. These subjects were recruited from physics courses being taught at the University of Calgary. The two novice groups were recruited from the two half-courses in introductory mechanics required for all physics majors at this university. Successful completion of the first of these courses is a prerequisite for enrolment in the second. Novices were drawn from the first of these courses; Advanced Novices, from the second. The intermediate group was recruited from a course in advanced mechanics intended for third- and fourth-year students majoring in physics. All subjects were paid volunteers.

Recruitment was conducted through classroom presentations by the experimenter, in which the purpose of the research and the nature of the experimental task were briefly explained, and volunteers were enlisted. Experimental sessions for all subjects were conducted within two weeks of a subject's enrolment in this study, and were scheduled at the subjects' convenience. For both novice groups, these presentations were given, and experimental sessions were conducted, in the last three weeks of the subjects' enrolment in the indicated courses. Scheduling of the recruitment and experimental sessions for Intermediate subjects varied, although sessions for all subjects were conducted during the fall and winter terms.

A total of 26 subjects -- 10 Novices, 10 Advanced Novices, and 6 Intermediates -- participated in this study. As noted below, one Novice subject was excluded from the data analysis. These group sizes might be considered small by some standards; they are, however, typical of those used in investigations of expertise, where the need for skilled or expert subjects makes the use of large sample sizes impractical. Subject data on major field of study, program year, age, gender, number of post-secondary physics courses previously completed, and number of physics courses currently being taken was collected from all subjects (see Table 1).

Materials

Sixteen mechanics problems, each requiring a different combination of equations for solution, were prepared by the experimenter, with the assistance of members of the department of physics at the University of Calgary. These problems were modelled after exercises and practice problems included in the textbook (Gettys, Keller, & Skove, 1989)

Table 1
Subject Data

	Novice	Advanced Novice	Intermediate
n	9	10	6
Major			
Physics	3	2	6
Science	1	5	0
Math/Engineering	3	0	0
Other	2	3	0
Year of Program			
1	7	6	0
2	0	1	1
3	0	0	2
4	1	0	3
Unknown	1	2	0
Previous physics courses			
0	5	0	0
1	3	10	0
2	1	0	0
...	.	.	.
7	0	0	2
8	0	0	2
9	0	0	0
10	0	0	1
11	0	0	1
Current physics courses			
0	0	10	1
1	6	0	1
2	2	0	4
3	0	0	0
4	0	0	0
5	1	0	0
Age			
Min. (yrs.-mos.)	16-11	17-11	20-1
Max. (yrs.-mos.)	22-6	20-0	25-8
Mean (yrs.-mos.)	19-2.3	18-8.5	22-5.8
Gender			
Male	8	5	6
Female	1	5	0

used for the two mechanics courses from which Novice and Advanced Novice subjects were obtained. None of the model problems were used during classroom lectures, or assigned as part of the required work for these courses.

The problems used as models for the experimental problems were drawn from 4 chapters (3, 4, 7, and 8) of Gettys et al.; these chapters discuss four different concept areas: one-dimensional motion, two-dimensional motion, force, and work, respectively. These concepts are introduced, both in the textbook and in the introductory mechanics course, in the order indicated and, presumably, represent successively more complex concepts. All of the indicated concepts are taught in the first of the two mechanics courses from which Novice and Advanced Novice subjects were taken. Four experimental problems were constructed for each concept area. In three of the concept areas -- two-dimensional motion, force, and work -- care was taken to use objects and situations which seemed to be typical of those used in the corresponding chapter of Gettys et al.. Problems in the fourth group, one-dimensional motion, were deliberately constructed to have surface features typical of problems in two-dimensional motion. In each concept area, 2 of the 4 problems could be solved by the successive application of two separate equations; the other 2 problems required the use of 3 different equations to obtain a solution. Thus, a 4 x 2 (Concept Area x Level of Difficulty), fully-crossed design, with 2 problems in each group, was used for the experimental problems.

Two problems were included in each problem group to allow the results to be checked for the effects of fatigue or task unfamiliarity. As described below, for each subject, one of the two problems of each type was randomly assigned to the first or second half of the problem set. Thus, each subject rated two sets of problems, each of which contained one problem of each of the 8 problem types.

To discourage subjects from attempting to solve the experimental problems, numeric quantities in all problems were blocked out; however, the units of measure associated with those quantities were retained. Initially, that part of each problem which identified the quantity to be obtained as a solution was included in the problem as it was seen by the subjects. However, during the first experimental session with 1 Novice subject, it became evident that this subject was subjecting the problems to detailed analysis, in spite of directions to the contrary. This subject was excluded from the study, and, in all future sessions, this portion of each problem was blocked out.

All problems were checked by a faculty member of the Department of Physics, University of Calgary to ensure that they were reasonably constructed and that the use of the indicated equations was the simplest and shortest solution possible for each problem. The 16 problems used in this study are presented in Appendix B.

A list of the 13 equations needed to solve all 16 of the test problems was compiled and used as the set of equations to be rated for their probable relevance to the solution of the test problems. These equations were presented in the form, and using the notation, employed by Gettys et al. (1989). This list was alphabetized according to the variables and alphabetic subscripts they included; mathematical symbols, numbers, and Greek letters were ignored. The same list, in the same order, was used for all subjects and problems. (This list may be seen in Appendix D, which presents a sample problem page.) All equations were checked by a faculty member of the Department of Physics, University of Calgary.

Order of presentation for the test problems was controlled by an 8-item rotational schedule, with the initial position of each of the 8 problem types randomly assigned. Problem order for individual subjects was determined by block assignment according to this schedule. Separate schedules were maintained for each of the 3 subject groups; the same order of problem types was used for all 3 subject groups.

For each subject, 2 of the 8 possible problem orders were randomly selected from the unused orders available for that subject group. These were designated, in the order of their selection, as the problem orders to be used for the first and second half of the problem set. One of the two problems of each of the 8 problem types was then randomly selected for use in the first half of the problem set; the other problem was used for the second half of the problem set. The problems in each half were then arranged by type according to the selected order.

A test booklet for each subject was prepared by compiling (1) a cover page containing the subject number, (2) a page for the entry of subject data, (3) two pages of instructions, and (4) the two halves of the problem set. Each problem was presented on a separate page containing, from top to bottom, (1) brief directions for the task, (2) an explanation of the rating scale to be used, (3) the test problem, and (4) the 13 equations to be ordered. Because the two models differ in the degree (possible vs. probable) as well as the direction (relevant vs. not relevant) of the ratings they predict, a 4-point rating scale was used: (1) <u>Probably not needed</u>, (2) <u>Might not be needed</u>, (3) <u>Might be needed</u>, and (4) <u>Probably needed</u>. Equations were presented in two columns, with each equation preceded by 4 checkboxes to be used for rating that equation. (The two pages of instructions, and a sample problem page have been attached as Appendices C and D, respectively.)

Procedure

Subjects were tested individually or in small groups, with each subject working individually with a separate copy of the test materials. Sessions were held in a small conference room used by the Department of Educational Psychology at the University of Calgary, with subjects seated at a large conference table. No materials related to physics or mathematics were visible in the test area.

At the beginning of the experimental session, consent forms and test booklets were distributed to the subjects. The consent form (see Appendix A) was then reviewed by the experimenter. The experimenter briefly described the experimental task, and cautioned subjects against performing a detailed analysis of the test problems. Subjects were also advised that their test booklets contained detailed instructions for the experimental task. (These directions included (1) a description of the rating scale, with an example of a problem and the ratings given for three equations, (2) an indication that subjects should not return to a problem once it had been completed, and (3) a reminder that subjects should base their ratings on their first impression to a problem.) Subjects then read, completed, and signed consent forms; completed the page of subject data; read the detailed instructions for the experimental task; and completed the experimental task at their own pace.

Subjects who arrived late were given, individually, the same directions which had been given at the beginning of the experimental session. As each subject completed the task, his test booklet and consent form were collected and he was paid the 5 dollar (Cdn) honorarium. Subjects were free to leave after their booklets and consent forms had been collected. The experimenter remained in the room until all subjects had completed the task.

Experimental sessions were not timed; all subjects completed the task in less than 1 hour and no subject required less than 20 minutes to complete the task.

CHAPTER IV

RESULTS AND ANALYSES

Preliminary analysis

Rating tabulation

The total number of equations given each of the 4 possible ratings were tabulated by subject, level of expertise, concept area, level of difficulty, and test half. One additional measure -- the number of problems rated either 3 (might be needed) or 4 (probably needed) -- was also calculated and tabulated. Because the complementary measure -- the number of problems rated either 2 (might not be needed) or 1 (probably not needed) -- is simply 13 - (the number rated as 3 or 4), it was not calculated and does not appear in any of the analyses reported here. The means, standard deviations, and modes of the 5 tabulated measures are reported in Table 2.

Analyses of variance

As an exploratory analysis, a separate ANOVA was performed on each of the five tabulated measures. In these analyses, both level of difficulty and concept area were included as within-subjects variables; level of expertise and test half were included as between-subjects variables. Although test half might also have been specified as a

Table 2

Ratings: Descriptive statistics

Rating	Mean	Standard Deviation	Mode
1	5.41	2.75	7 (1)
2	1.38	1.60	0
3	2.27	2.27	0
4	3.92	2.34	2
3/4	6.19	2.55	7

within-subjects variable, the random assignment of problems to test half for each subject would have made both the analyses and their interpretation considerably more complicated. It was, therefore, decided to treat this as a between-subjects variable, at the cost of some power in the analyses. Because sample sizes for the three levels of expertise were not equal, a regression method was used for the calculation of the sums of squares. Multiple comparisons were conducted using the significance of the differences between means as predictors of the regression line calculated for each analysis of variance.

A multivariate analysis was not performed because the arithmetic relationship between the five measures would have violated the assumption of a joint normal distribution. To compensate for the effect of multiple tests on confidence levels, an alpha level of .01 was selected, with the reservation that effects or interactions with $.01 < p <= .05$ would be considered, if these appeared in more than one of the five analyses. The use of these significance levels for individual analyses ensured that the combined alpha level for all five analyses was less than or equal to .05 in all cases.

No main effects for level of expertise or test half were found for any of the five measures.

A significant main effect for concept area was found in all five measures (see Table 3 and Figures 5, 6, and 7).

Table 3

Mean number of ratings: Rating by concept area

			Concept Area			
Rating	F(3,42)	p	1	2	3	4
1	4.72	.006	6.08	4.11	4.57	6.86
2	3.18	.033	1.36	1.48	1.56	1.12
3	32.85	.001	1.86	2.61	2.63	1.99
4	15.58	.001	3.68	4.75	4.22	3.02
3/4	29.99	.001	5.54	7.36	6.85	5.01

Figure 5

Number of equations rated 1 or 4 by concept area

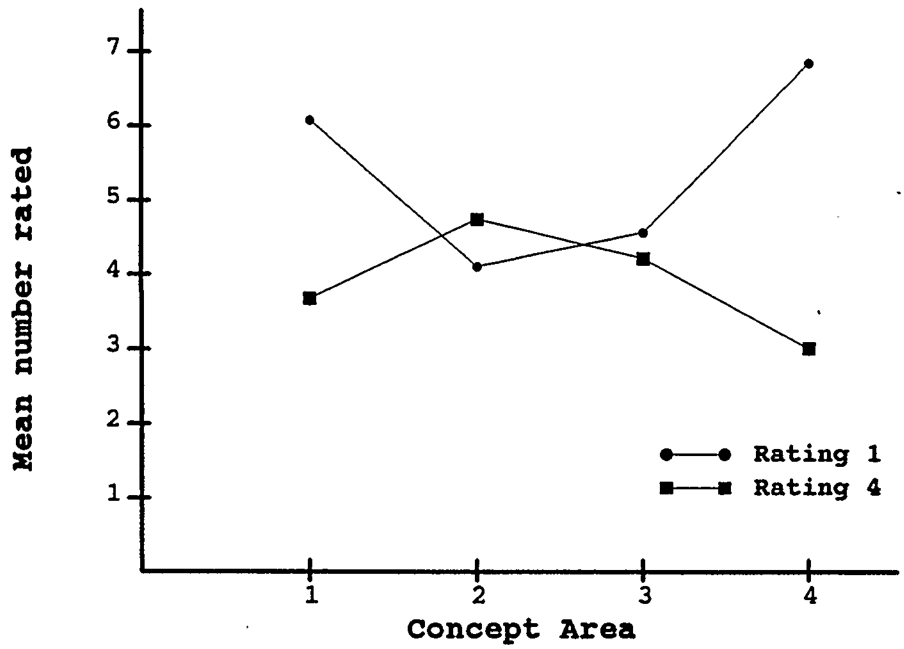


Figure 6

Number of equations rated 2 or 3 by concept area

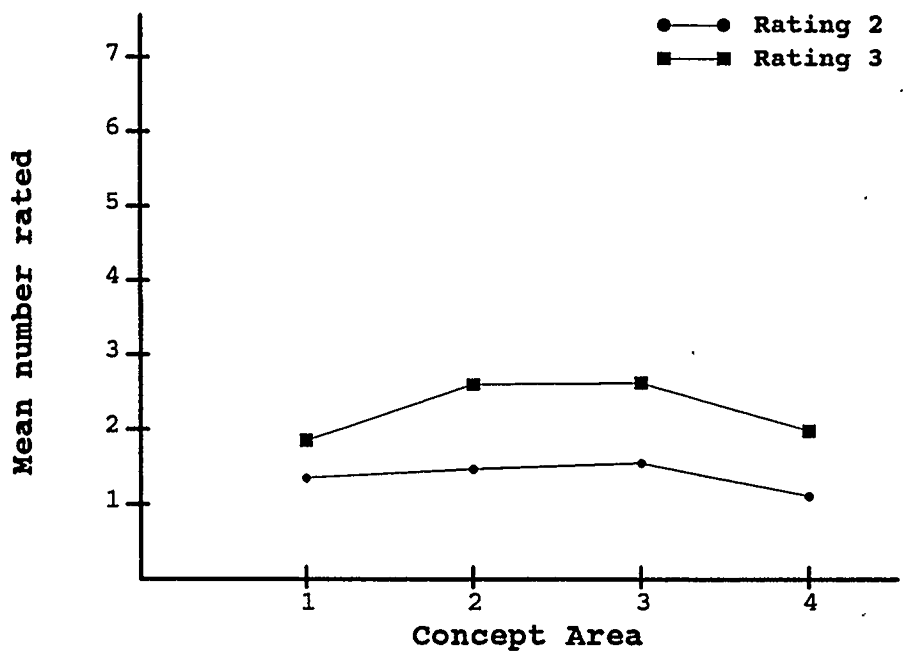
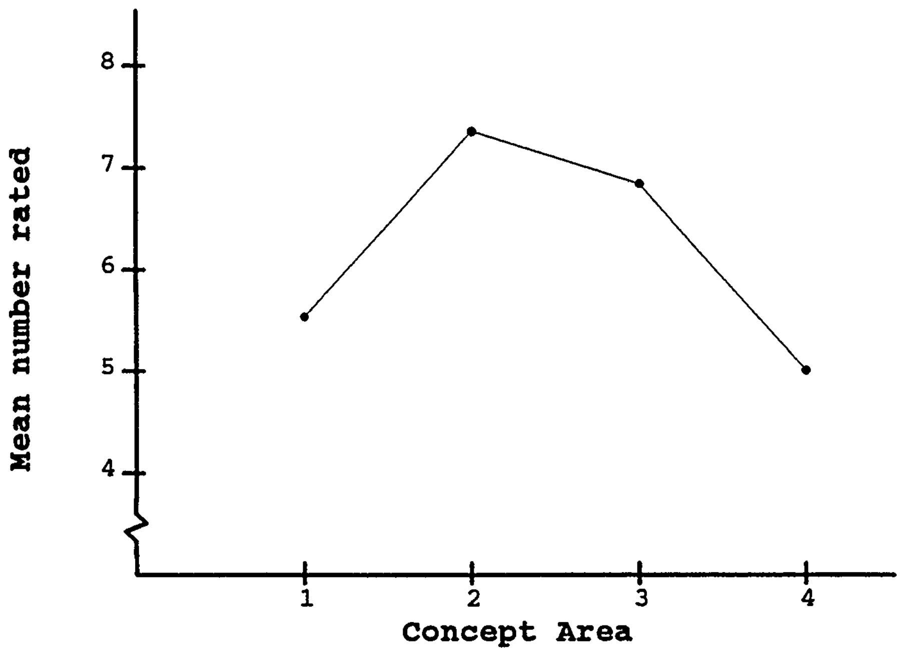


Figure 7:

Number of equations rated 3/4 by concept area.



Concept Area 1 (one-dimensional motion) and Concept Area 4 (work) problems had more equations rated 1 than Concept Area 3 (force) or Concept Area 2 (two-dimensional motion) problems ($F(1,44) = 14.68$, $p < .001$); Concept Area 2 problems had fewer equations rated 1 than Concept Areas 1, 3, or 4 ($F(1,44) = 7.77$, $p = .008$). The pattern of results for Rating 4 was the reverse of this: Problems from Concept Areas 2 and 3 had more equations rated 4 than problems from Concept Areas 1 and 4 ($F(1,44) = 33.93$, $p < .001$); fewer equations were given 4 ratings for problems from Concept Area 4 than for problems from Concept Areas 1, 2, or 3 ($F(1,44) = 35.44$, $p < .001$). The effect for Rating 3/4 was similar to that for Concept Area 4: problems from Concept Area 2 had more equations rated 3 or 4 than problems from Concept Areas 1, 3, or 4 ($F(1,44) = 60.76$, $p < .001$) and problems from Concept Area 4 had fewer problems rated 3 or 4 than problems from Concept Areas 1, 2, or 3. Multiple comparisons revealed no significant between-level differences for Ratings 2 or 3. The significant main effects found in the analyses of variance for these measures would indicate that more equations were given 2 ratings for problems from Concept Area 3 than for problems from Concept Area 4 and that more equations were given 3 ratings for Concept Area 3 problems than for Concept Area 1 problems.

A significant main effect for difficulty was found for Ratings 1 ($F(1,44) = 25.53$, $p < .001$), 3 ($F(1,44) = 6.21$, p

Table 4

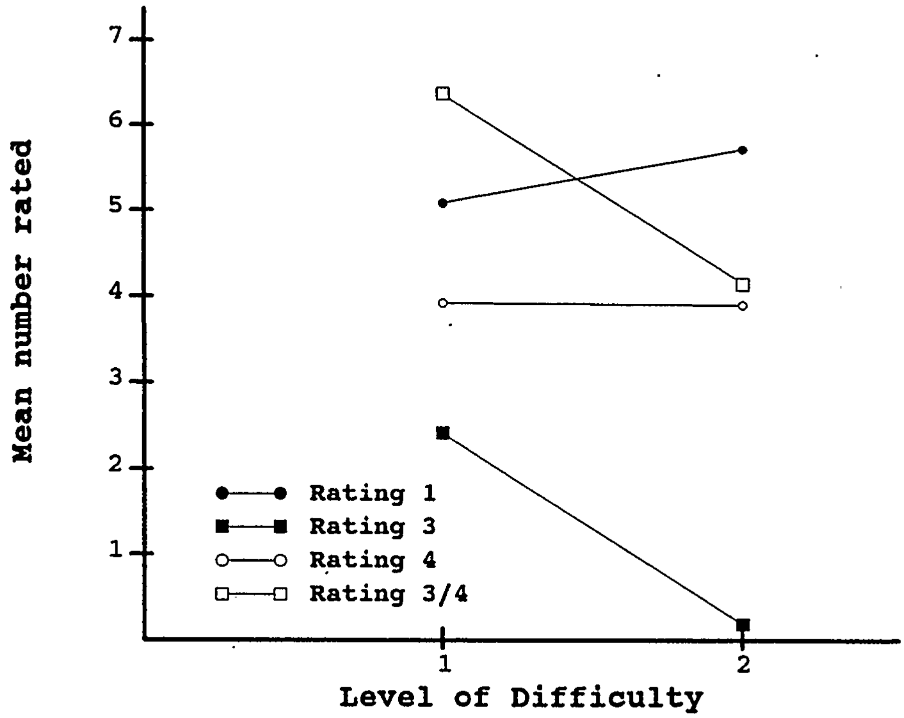
Mean number of ratings by difficulty

	Difficulty	
Rating	1	2
1	5.09	5.72
3	2.44	0.21
4	3.93	3.91
3/4	6.37	4.12

Figure 8

Number of equations rated 1, 3, 4, and 3/4

by level of difficulty



= .017), 4 ($F(1,44) = 122.53$, $p < .001$), and 3/4 ($F(1,44) = 46.82$, $p < .001$) (see Table 4 and Figure 8). Subjects rated more equations 3 or 4 when 2 equations were required to solve the problem (Difficulty Level 1) than they did when 3 equations were required to solve the problem (Difficulty Level 2). Subjects rated far more equations as 3 (possibly relevant) for problems of Difficulty Level 1 than they did for problems of Difficulty Level 2. Subjects gave more 1 ratings for problems of Difficulty Level 2 than they did for Difficulty Level 1 problems. There was also a slight, but statistically significant tendency to give more 4 ratings for problems of Difficulty Level 1 than for problems of Difficulty Level 2. As would be expected from the results for Ratings 3 and 4, subjects rated far more equations as 3 or 4 when problems were from Difficulty Level 1 than when problems were from Difficulty Level 2.

A significant interaction for Concept Area x Difficulty Level was found for Ratings 1 ($F(3,42) = 40.73$, $p < .001$), 3 ($F(3,42) = 19.82$, $p < .001$), 4 ($F(3,42) = 49.29$, $p < .001$), and 3/4 ($F(3,42) = 37.67$, $p < .001$) (see Figures 9 to 12). Pairwise comparisons were not performed; however, it would appear thatn in Concept Areas 1 and 4, fewer equations were rated as 1 for Difficulty Level 1 problems than for Difficulty Level 2 problems; this difference was not observed for Concept Areas 2 and 3. Fewer equations were rated as 1 for Concept Area 2 and 3 problems than for

Figure 9

Number of equations rated 1 by difficulty and concept

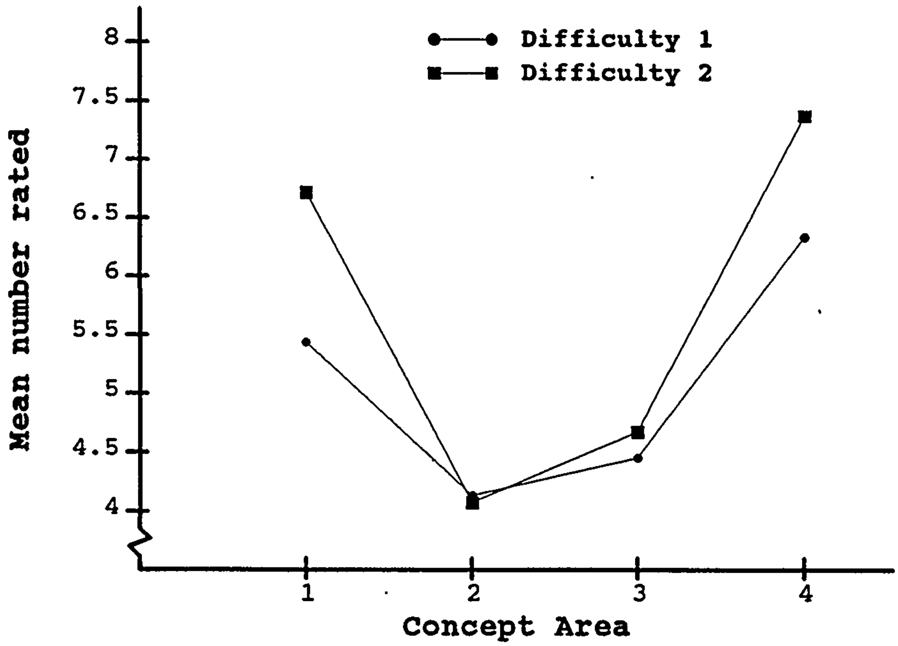


Figure 10

Number of equations rated 3 by difficulty and concept area

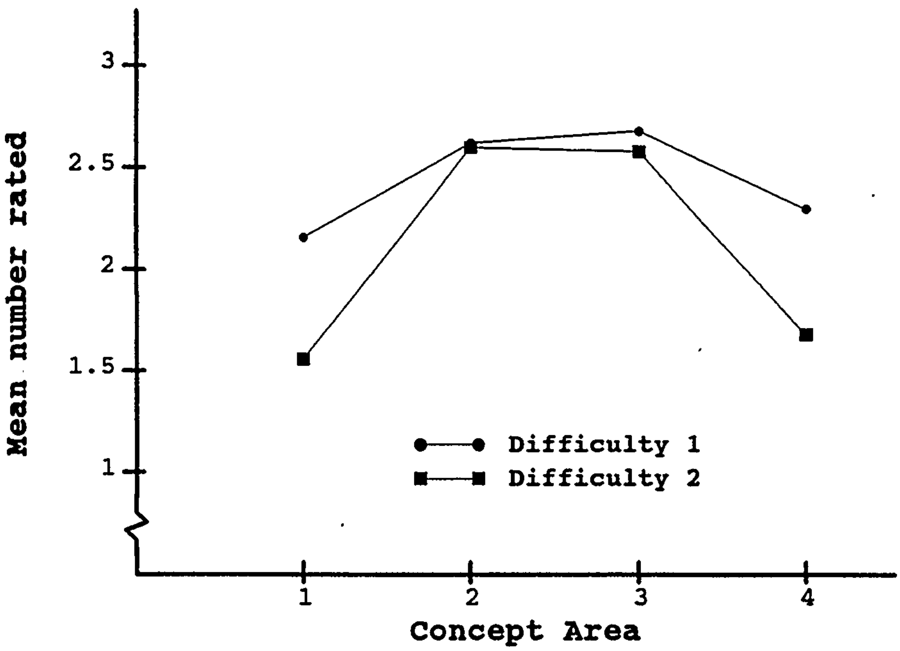


Figure 11

Number of equations rated 4 by difficulty and concept area

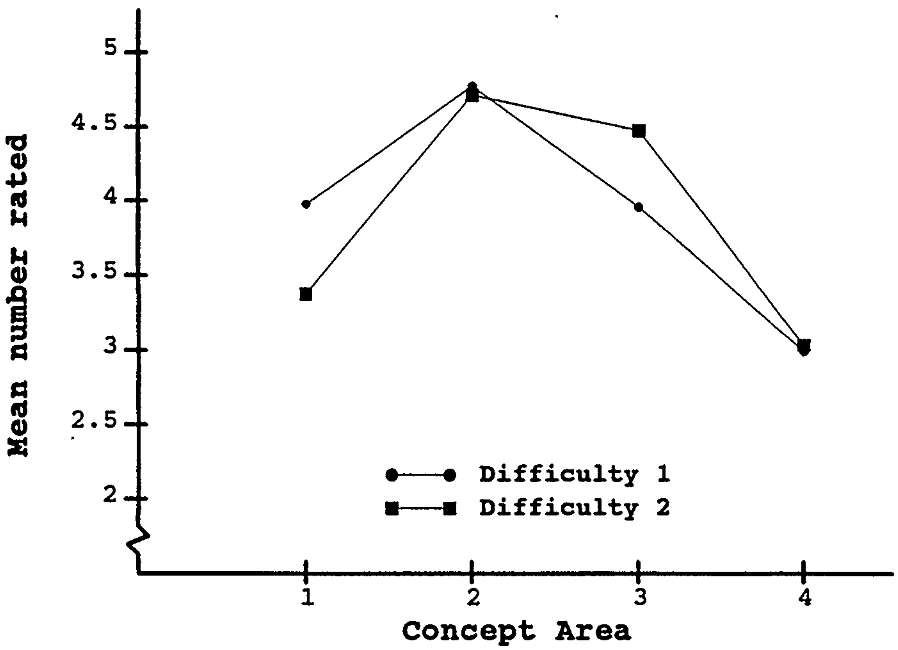
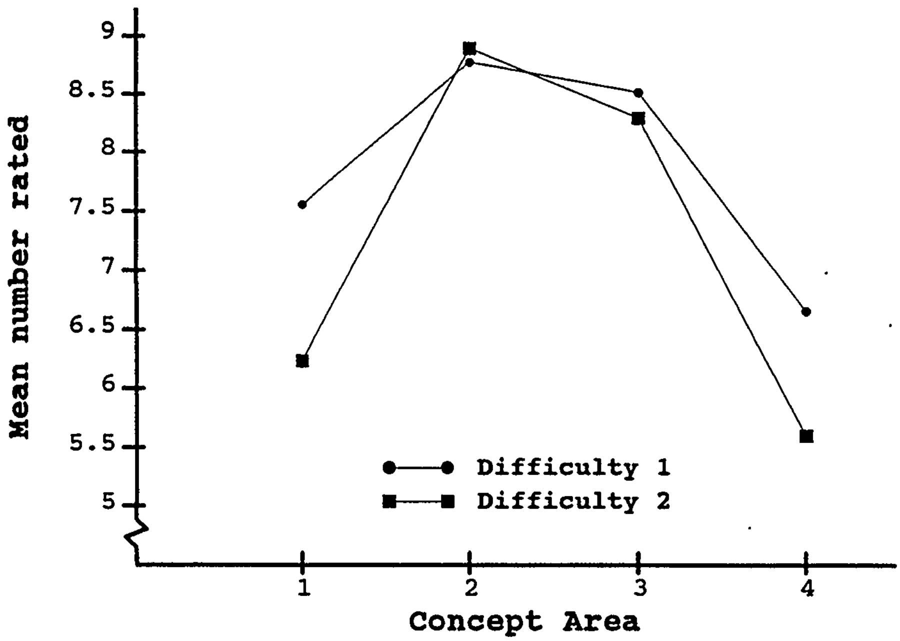


Figure 12

Number of equations rated 3/4 by difficulty and concept area



Concept Area 1 and 4 problems for both levels of difficulty. A reverse interaction was found for Rating 3: in concept Areas 1 and 4, more equations seem to have been rated as 2 for Difficulty Level 1 problems than for Difficulty Level 2 problems; more equations seem to have been rated 3 for problems from Concept Areas 2 and 3 than for problems from Concept Areas 1 and 4. The interaction for Rating 4 is similar to that for Rating 3 for Concept Areas 1 and 2 and for Difficulty Level 2. For problems of Difficulty Level 1 from Concept Areas 3 and 4, the number of equations rated as 4 was lower, relative to other combinations, than was observed for rating 3 or the combined 3/4 rating. In Concept Area 3, subjects actually rated fewer equations as 3 for problems of Difficulty Level 1 than for problems of Difficulty Level 2; in Concept Area 4, Difficulty Level had no effect on the number of equations so rated.

A significant Level of Expertise x Concept Area was found for ratings 1 ($F(6,86) = 2.32$, $p = .04$) and 2 ($F(6,86) = 3.57$, $p = .003$) (see Figures 13 and 14). No readily interpretable pattern of results can be seen, here. However, it might be noted that, when the number of equations rated as 1 is considered, the interaction seems to be caused by the different sensitivity of the three levels of expertise to the presence of problems of Concept Area 2 or 3, with Novices and Advanced Novices reducing the number of equations rated as 1 for these two concept areas. The

Figure 13

Number of equations rated 1 by concept area and level of expertise

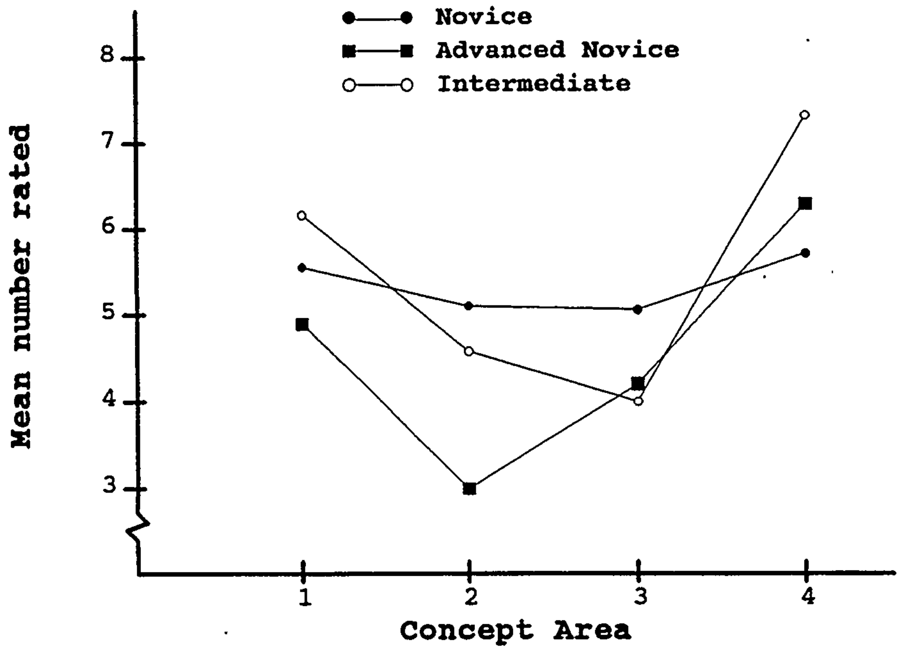
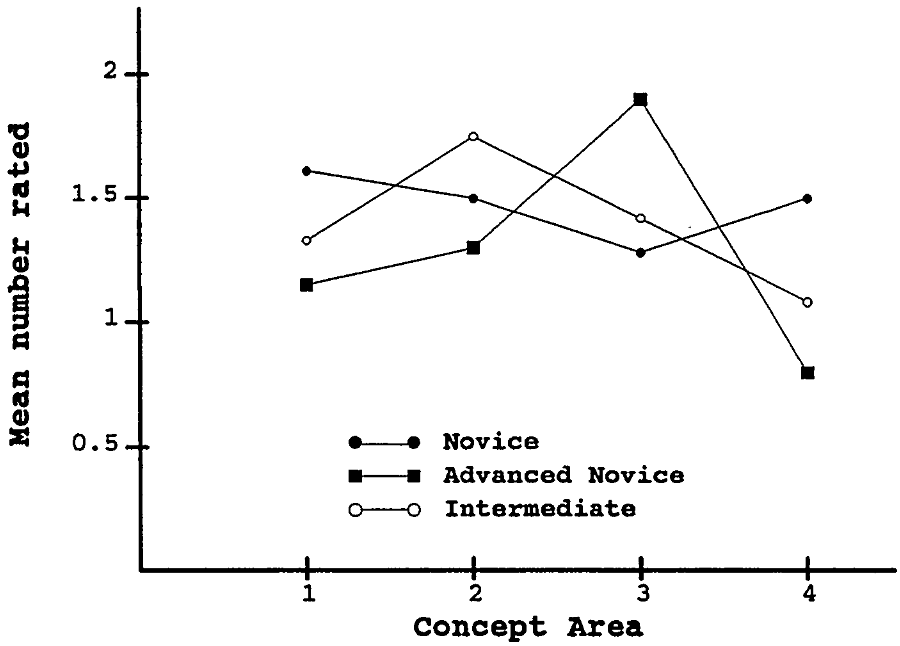


Figure 14

Number of equations rated 2 by concept area and level of expertise



number of 1 ratings given by Intermediates was affected much less by concept area.

An interaction was found for Level of Expertise by Level of Difficulty for 1 ratings; however, as this had a marginal significance level ($p = .04$) and was not found for any other rating, it will not be considered here.

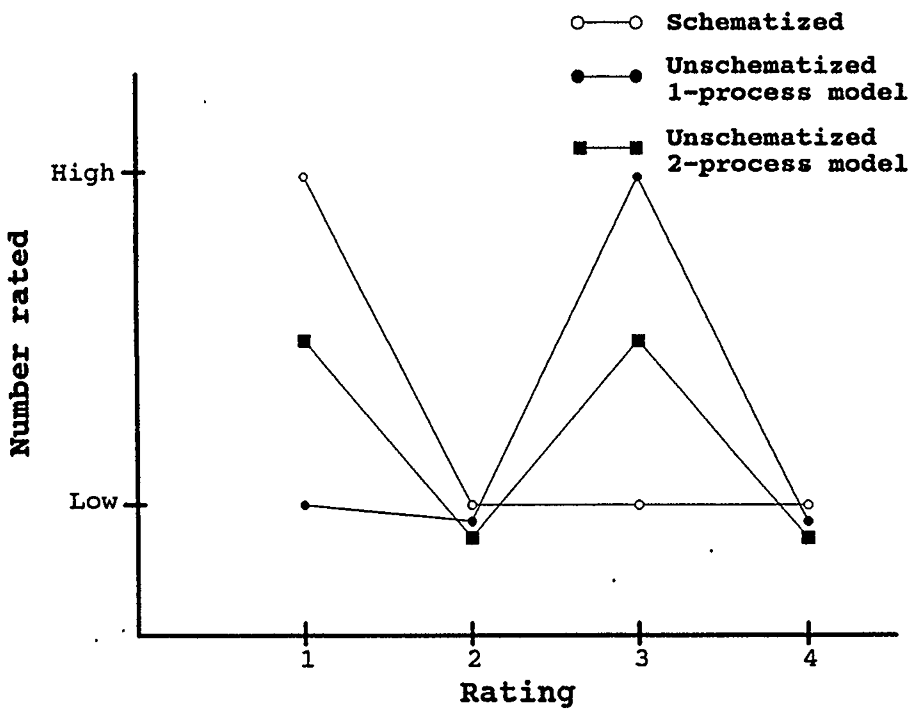
Separation of schematized and unschematized problems

In selecting a rating to be used to separate schematized and unschematized problems for further analysis, the predictions made by both models for both types of problems were compared to find a measure that would maximize the difference between the two problem types (see Figure 15). In the case of schematized problems, the prediction for both models was the same: Because subjects should be able to identify the specific equations needed for problem solution, the number of equations rated as probably relevant should approach or equal the number required for solution; all other equations should be rated as probably not needed. Thus, the mean for Rating 1 should be high, and the mean for Ratings 2, 3, and 4 should be low. For unschematized problems, the one-process model would not provide any mechanism for assessing the probable relevance of equations. All equations should, therefore, be given a rating of 3 (possibly needed). Under the two-process model subjects

Figure 15

Predicted number of ratings

by model and schematization



would be able to identify a significant number of equations as probably irrelevant to the solution of unschematized problems; all other equations should be rated as possibly relevant. Thus, both Rating 1 and Rating 3 should be moderately high. All other ratings should be low.

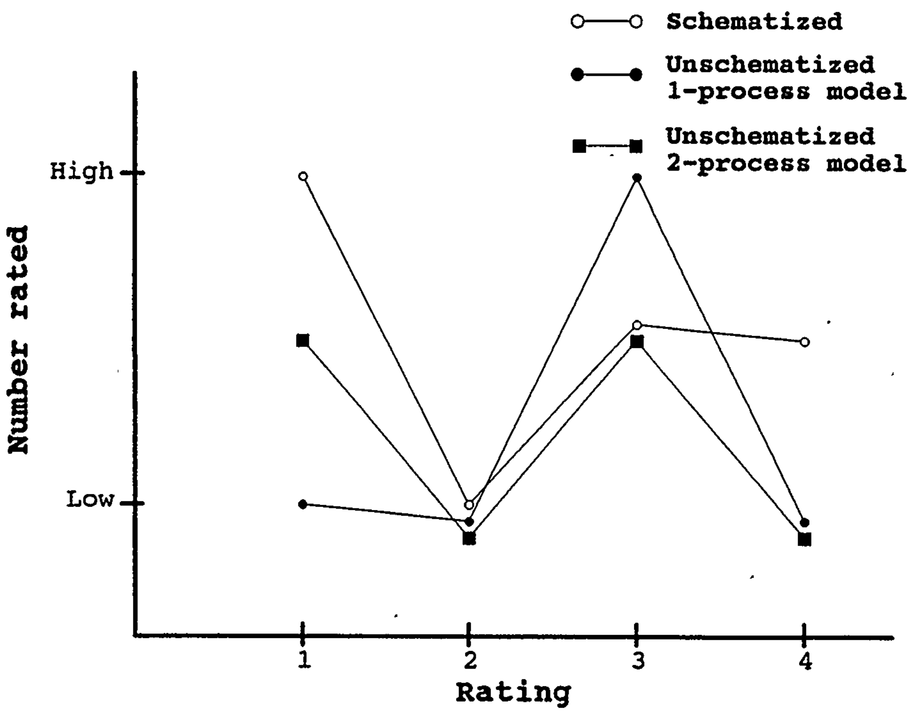
Based on the above, either Rating 1 or Rating 3 might be used to separate schematized and unschematized problems. However, it was felt that one additional effect needed to be considered. As discussed earlier, the removal of problem statements from the test problems could be expected to increase the number of equations rated as probably relevant to schematized problems. This would tend to decrease the number of equations rated 1. However, its major effect might be on the number of equations rated 3 and 4. As the number of equations which would normally be rated as 4 increased, there might be a tendency to view these equations as possibly, rather than probably, relevant, increasing the number of 3 ratings and decreasing the number of 4 ratings (see Figure 16). Thus, Rating 1 was chosen as the measure most likely to be useful in separating schematized and unschematized problems.

An inspection of the distributions for the 5 ratings confirmed this prediction: the distribution of Rating 1 showed a clear tendency to bimodality, which was not evident in the distributions for other ratings. However, the two modes, which occurred at 7 and 1, were not sufficiently

Figure 16

Predicted number of ratings

by model and schematization (modified)



separated to clearly discriminate between the two types of problems. Combined scores designed to capture predicted differences between schematized and unschematized problems across ratings (Rating 1 + Rating 4, Rating 1 - Rating 3, Rating 1 + Rating 4 - Rating 3) did not improve the separation between the two modes. The distributions of Rating 1 for each category of the four dependent variables (concept area, test half, level of difficulty and level of expertise) were then examined; it was found that the distributions of Rating 1 for Concept Area 2 showed a reasonable separation of the two modes, and that a bimodal distribution was visible when Rating 1 was examined across test half. Separating the distribution by both concept area and test half did not improve the separation of the two modes in Concept Areas 2 and 3 and still did not allow any separation of problems from Concept Area 4. However, it did improve the separation of the modes in Concept Area 1.

Based on this inspection, separate criteria for excluding schematized problems were established for problems from Concept Area 2, Concept Area 3, Concept Area 1/Test Half 1, and Concept Area 1/Test Half 2. Because no reasonable criterion for separating problems from Concept Area 4 could be found, these problems were excluded from the test of the two models as possibly schematized. The criteria used for separating schematized and unschematized problems can be found in Table 5; the distributions on which

Table 5

Inclusion criteria

Concept Area	Rating 1 <=
1 (Half 2)	2
1 (Half 1)	5
2	1
3	3
4	-

Figure 17

Distribution of problems by number of equations rated 1 for Concept Area 1, Test Half 1

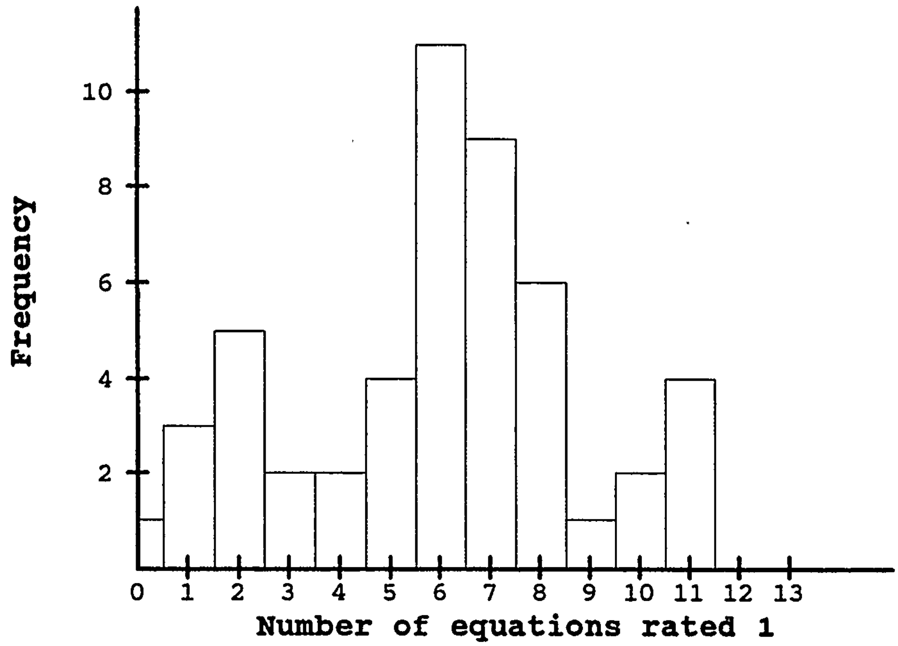


Figure 18

Distribution of problems by number of equations rated 1 for Concept Area 1, Test Half 2

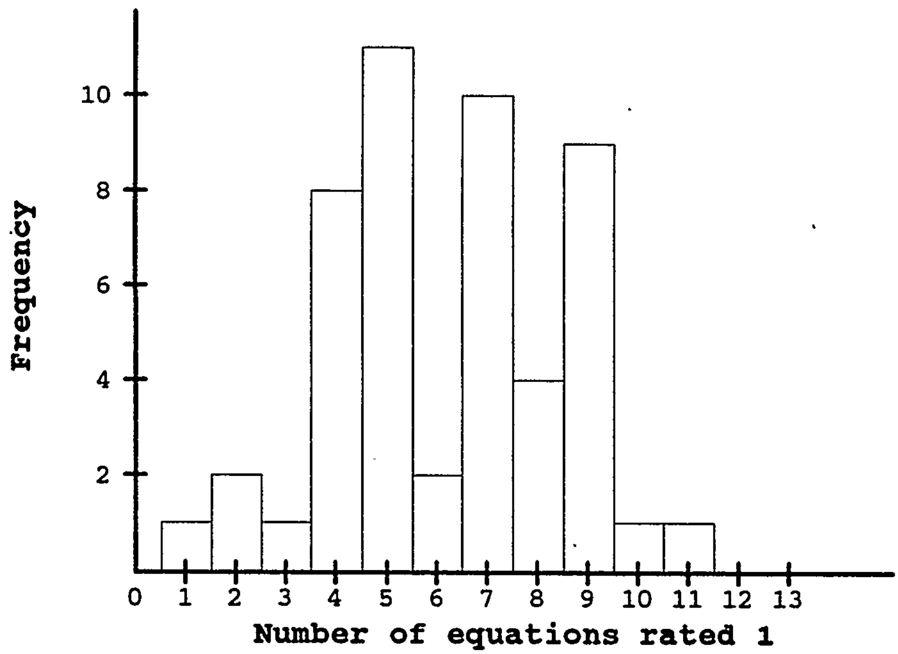


Figure 19

Distribution of problems by number of equations rated 1 for Concept Area 2

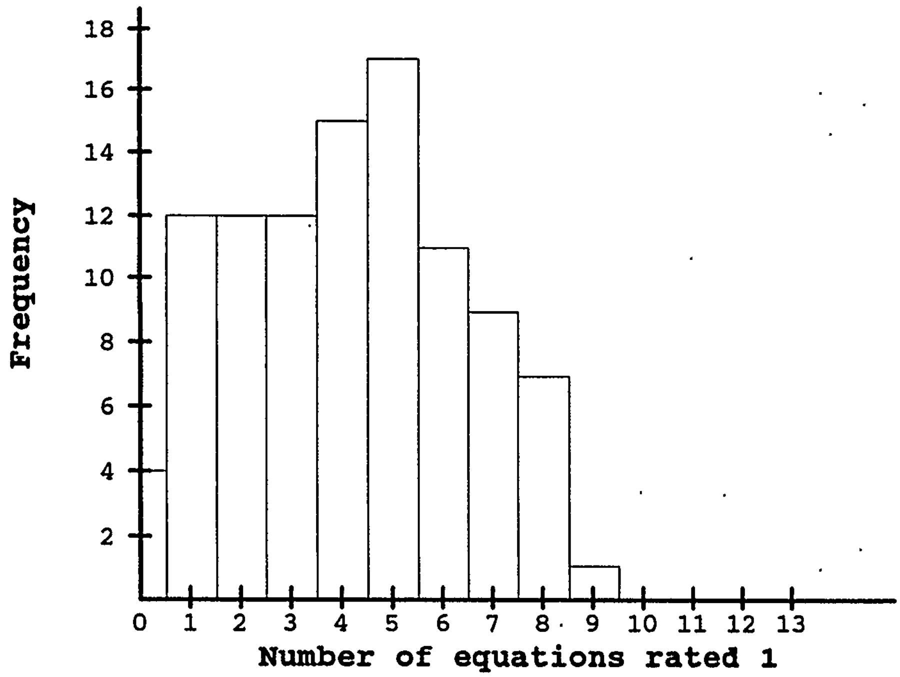


Figure 20

Distribution of problems by number of equations rated 1 for Concept Area 3

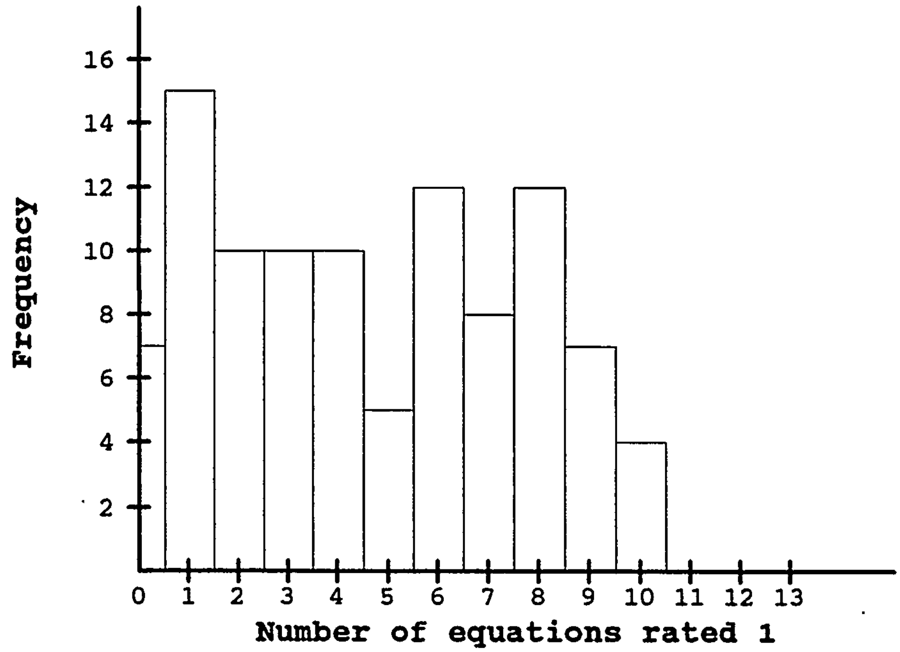


Figure 21

Distribution of problems by number of equations rated 1 for Concept Area 4

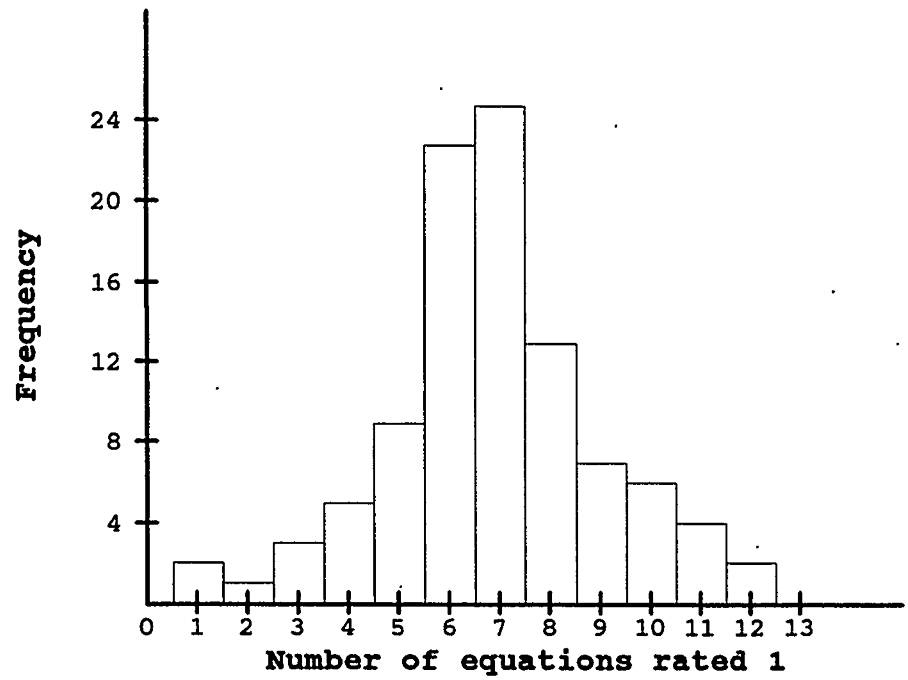


Table 6

Cell counts for problems included in test of main hypothesis

	Level of Expertise			
	1	2	3	
Concept Area 1				
Difficulty 1	5	11	3	19
Difficulty 2	3	9	1	13
Concept Area 2				
Difficulty 1	2	5	2	9
Difficulty 2	1	4	2	7
Concept Area 3				
Difficulty 1	8	10	6	24
Difficulty 2	8	7	3	18
Concept Area 4				
Difficulty 1	0	0	0	0
Difficulty 2	0	0	0	0
	27	46	17	

these are based are presented in Figures 17 to 21. Cell counts for the problems selected for inclusion in the main hypothesis are presented in Table 6.

Before proceeding it must be noted that removing the problem statement from the test problems had a greater effect than expected. As discussed in Chapter 2, it was predicted that this might increase the number of equations rated as probably needed, with a concomitant decrease in the number rated as probably not needed. A downward shift in the mode representing schematized problems, which would otherwise occur at the high end of the distribution for Rating 1, was, therefore, expected. It was not, however, anticipated that this shift might be sufficiently large to shift this mode into the center of the range of possible scores. As this part of the range is precisely the area where the effects predicted by the two-process model would occur, the exclusion of problems with Rating 1 scores in this area from the test of the main hypothesis could be expected to seriously impair the ability of any subsequent analysis to detect these effects.

Test of the main hypothesis

An ANOVA was performed on the number of equations rated as either 3 or 4 for the problems selected as unschematized on the basis of the criteria which had been established.

Because of the inequality of cell sizes and the possibility of empty cells, a regression technique was used for calculating the sums of squares. For similar reasons, no variables were included as within-subject variables; all variables were treated as between-subject variables. Including all four of the possible dependent variables in the design produced an unacceptably large number of low cell sizes. Consequently, a preliminary ANOVA was run to identify variables which might be excluded. This analysis showed that Level of Difficulty did not have a significant main effect and was not included in any significant interaction. This variable was, therefore, excluded from the final design, which included Level of Expertise, Concept Area, and Test Half as dependent variables.

This analysis showed no significant main effect for Level of Expertise or Test Half. A significant main effect for Concept Area was found ($F(2,57) = 5.56$, $p = .014$). Scheffe comparisons showed that more equations were rated as 3 or 4 when problems from the selected group were from Concept Area 2 than from Concept Area 1 ($p < .05$) (see Table 7).

Supplemental analysis

An additional ANOVA was performed on the number of equations rated as 3 or 4 using the set of problems which

Table 7

Mean number of equations rated 3 or 4 by concept area (unschematized problems)

	Concept Area		
	1	2	3
Mean	7.63	9.88	9.05
SD	1.98	1.71	2.62

had been excluded from the test of the main hypothesis as possibly schematized. Because of the larger number of problems included in this analysis (310 vs 90 for the test of the main hypothesis) all 4 possible dependent variables were included in the analysis. As in the test of the main hypothesis, a regression technique was used for calculating the sums of squares, and all variables were included as between-subjects variables.

This analysis showed no significant main effects for level of expertise, test half, or level of difficulty. A significant main effect for Concept Area was found ($F(3,263) = 18.53$, $p < .001$). Scheffe tests showed that more equations were rated as 3 or 4 when problems from the schematized group were from Concept Area 2 than when they were from Concept Areas 4 or 1 ($p < .05$) (see Table 8 and Figure 22).

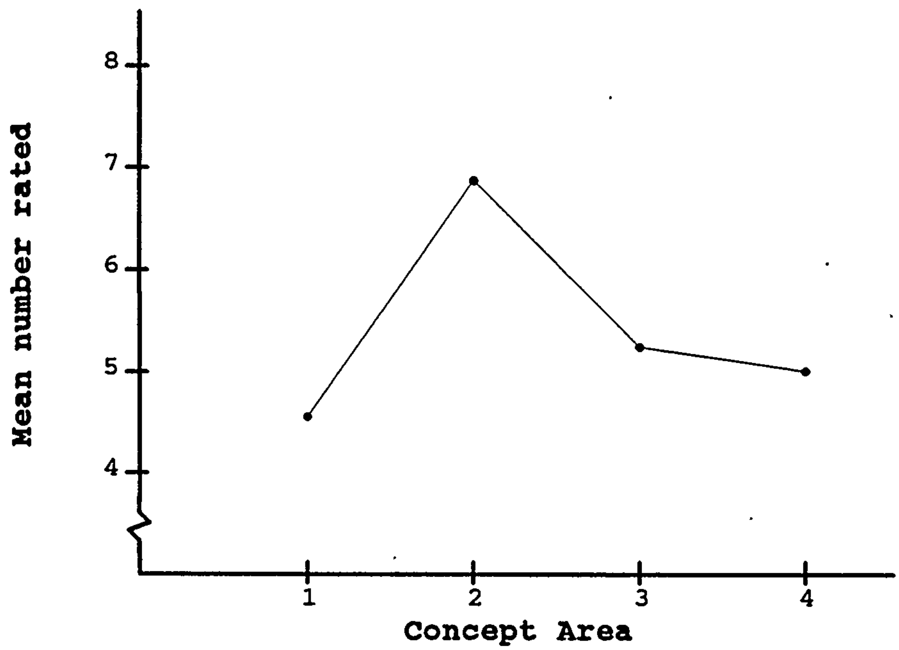
Table 8

Mean number of equations rated 3 or 4 by concept area (schematized problems)

	Concept Area			
	1	2	3	4
Mean	4.56	6.88	5.25	5.01
SD	1.86	1.80	1.85	2.05

Figure 22

Number of equations rated 3 or 4 for schematized problems by concept area



CHAPTER V

DISCUSSION

Test of the main hypothesis

Level of expertise was found not to have a significant effect on the perceived relevance of equations to the solution of those test problems selected as unschematized. According to the experimental hypothesis of this study, this finding would support a model in which expert schemata were developed solely through a process of schema combination, over a model which included a complementary process of schema separation.

However, as indicated in the previous chapter, this finding should not be interpreted as supporting either of the two models, because of the failure to find appropriate criteria for identifying schematized problems for exclusion from the test of the main hypothesis. While a set of criteria was adopted for this purpose, these were clearly too exclusive for the purpose for which they were intended. If schema separation does occur, it should permit the identification of all equations not included in the subschemata for a problem category as irrelevant to the solution of problems from that class; the number so identified could be expected to be far higher than most of the criteria used here. Thus, it is likely that not only schematized problems, but also unschematized problems which might show the effect of schema separation were excluded from the set of problems used to test the main hypothesis.

The use of such stringent criteria was dictated by the need to ensure the exclusion of schematized problems from the set of test problems, and by the location of the mode representing such problems in the distribution of the number of equations selected as probably irrelevant to the solution of the test problems. Both of the proposed models predicted that this mode would occur at or near the upper end of the range of possible values, but instead, this mode occurred in the center of that range. It had been expected that the removal, from each problem, of the sentence which identified the quantity to be obtained as a solution might cause a downward displacement of this mode; however, it had not been anticipated that the size of this displacement would be as great as was observed. While larger than expected, the observed displacement is not necessarily inconsistent with the proposed models, nor is there any reason to suppose the operation of some unrecognized factor. It would appear that the effect of modifying the test problems was simply underestimated.

It would seem likely that the problems which were included in the test of the main hypothesis were those for which class information was missing or for which the available class information was insufficient to identify, as irrelevant to the solution of a given problem, the equations included in the experimental task. To some extent, the large number of problems which satisfied the excessively stringent criteria for inclusion (90 of 400, or 22.5% of the total number of problems) supports the one-process over the two-process model. Under the one-process model, as discussed in Chapter II, class information would be available only for schematized problems; the 90 problems selected for the test of the main hypothesis would, therefore, represent the entire set of unschematized problems encountered by subjects during the experimental task. Under the two-process model, the selected problems would represent only a subset of the unschematized problems encountered; the actual number of such problems would be considerably higher.

It must be noted, however that two methodological factors may have contributed to the large number of problems for which class information was insufficient to identify equations as irrelevant to the solution of a particular problem. First, as indicated above, the modifications which were made to the test problems in order to prevent means-ends analysis had an unexpectedly powerful, adverse effect on subjects' ability to make such identifications. It is quite possible that, for some subjects and some problems, this effect was strong enough to produce the low number of such identifications needed to satisfy the inclusion criteria. Second, it must be noted that the problems employed in this study constitute a small and quite closely-related subset within the set of physics problems. It would not, for example, be difficult to construct a problem whose solution would require the application of equations from all four of the concept areas employed in this study. This relationship between problem categories would tend to reduce the number of equations which could be identified as probably irrelevant to the solution of the test problems. A different pattern of results might be observed if the set of test problems, or the set of equations to be classified, included some items from other areas of physics.

Incidental support for the proposed models

As indicated above, the results obtained in this experiment cannot be used to discriminate conclusively between the two models which were proposed for the development of expertise. However, in at least one instance, these results do conform to the pattern predicted by both models and thus provide some general support for schema combination as the process primarily responsible for the development of expertise. First, the means of the distributions of the number of equations given each of the four possible ratings (see Table 2, p. 74) follow the pattern predicted for a subdomain in which a moderately high degree of schematization had occurred. In such a subdomain, both models would predict that a relatively large number of equations would be identified as probably irrelevant, and that a very small number of equations would be identified as possibly irrelevant, to the solution of problems in that subdomain (see Figures 15, p. 92 and 16, p. 94, and the attendant discussion on the selection of criteria for separating schematized and unschematized problems, p. 91). In addition, if the effect of the modification to the test problems is considered, it could be expected that moderately large numbers of equations would be identified as probably or possibly relevant to solution. As can be seen in Table 2, this is the pattern of results that was obtained.

It might be noted, here, that the relatively modest number of equations identified as possibly relevant (Rating 3) would tend to support the two-process model, which predicts more ratings of probable, than of possible, relevance, over the one-process model, which predicts the reverse. However, it must also be noted that the obtained value would be affected by (1) the proportion of test problems which had been schematized, (2) the effect of the modifications to the test problems, and (3) the possible effect of the latter on subjects' perception of the rating scale. As the contribution of these factors to the obtained value cannot be determined, the support which this provides must be considered as strictly limited.

Other effects

Concept area was found to have a very robust effect on subjects' performance on the rating task. Concept area was found to have a main effect on the number of equations rated for all four ratings when the full set of test problems was considered, and on the total number of equations identified as possibly or probably relevant when the two sets of problems identified as unschematized and possibly schematized were considered separately. In short, concept area had a main effect on every rating measure analyzed. In addition, when the full set of problems was considered, significant Concept Area x Level of Difficulty interactions were found for all four ratings, and significant Concept Area x Level of Expertise interactions were found for Ratings 1 and 2.

The most consistent feature of these effects and interactions is their separation of the four concept areas into two pairs, with one pair composed of Concept Areas 1 and 4, and the other, of Concept Areas 2 and 3. Results within each pair tend to be similar; results observed across pairs are noticeably different. Concept Areas 1 and 4 are characterized by the large number of <u>probably not needed</u>, and the small number of <u>probably needed</u>, ratings which were observed. Ratings for Concept Areas 2 and 3, and ratings of possibly not needed and possibly needed across all four concept areas, tend to lie between these two extremes and to be approximately equal. When the full set of test problems is considered, these differences are more pronounced for problems requiring three equations (Difficulty Level 2) than for those requiring only two equations (Difficulty Level 1) for their solution, and for the Advanced Novice and Intermediate groups. This pattern of results would seem to indicate that subjects were able to classify problems from Concept Areas 1 and 4 more precisely than those from Concept Areas 2 and 3.

No clear explanation for these results is readily apparent, although a number of partial explanations based on the proposed models can be advanced when only the main effect of concept area is considered. Since Concept Area 1 is the simplest and earliest learned of the four areas, it is likely that schemata have been formed for a greater proportion of the problems in this area than in other areas. Similarly, the schemata for individual problem types in this area may have been combined to form a more integrated hierarchical structure than exists in other areas. More schema separation may also have occurred in this area, providing a more precise mechanism for the identification of problem class -- and, possibly, one less vulnerable to the effects of missing information in the test problems. In the case of Concept Area 4, it is possible that problems in this area, as the most advanced and most recently learned, form a more recognizable and more independent problem class than do problems from other area. The difficulty with any of these explanations is that they would not seem to be consistent with the finding that these effects increase with problem difficulty. Thus, in the absence of additional data regarding these effects, it must be suspected that they are an artifact of the particular types of problems used or of their construction.

Summary

Although it did not prove possible to complete the comparison of the two models proposed for the development of expertise, this study did provide some incidental support for the process of schema combination as the primary process in the development of expertise. In particular, this study demonstrated the utility of such models in predicting the performance of physics students in rating the perceived relevance of equations to problems from four concept areas.

The major factor in this study's failure to obtain more conclusive results was clearly a modification of the test problems designed to prevent subjects from performing means-ends analysis of the problems during the rating task. However, a persistent effect of concept area on performance in the experimental task which was not predicted, and could not be explained, by either of the proposed models suggests that unidentified factors of problem type or construction may also represent possible confounds of the effect chosen to discriminate between the two models which were proposed.

Given the possible implications of this research for educational practice and the evidence that current models for the development of expertise may be incomplete, further research on this question would seem indicated. In pursuing such research, the evidence that unknown factors of problem type or structure may represent possible confounds cannot be overlooked, and the use of some experimental task other than that employed here is recommended. If such an alternate task cannot be devised or is deemed inappropriate then it is clearly imperative that some technique for preventing means-ends analysis which would be less intrusive than the modification of the test problems used in this study should be employed.

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APPENDIX A

CONSENT FORM

CONSENT FOR RESEARCH PARTICIPATION

I hereby consent to participate as a subject in the research project entitled "The development of expert-like knowledge structures: A two-process model" conducted by Darcy Rollack under the supervision of Professor John Mueller of the Department of Educational Psychology at the University of Calgary. I understand that the study will involve the following general procedures: I will be asked to provide basic personal information on my major field of study, the physics courses I have previously completed, my age, and my gender. I will then be selecting, from a list of equations, those which I think might be relevant or irrelevant to the solution of a sequence of physics problems. The research project is expected to help improve techniques of instruction in educational settings by extending our basic understanding of how expertise develops.

I understand that my participation is completely voluntary, and I am free to withdraw from the study at any time I choose, without penalty.

The general plan of this study has been outlined to me, including any possible known risks. I understand that this project is not expected to involve risks of harm any greater than those ordinarily encountered in daily life. I also understand that it is not possible to identify all potential risks in any procedure, but that all reasonable safeguards have been taken to minimize the potential risks.

I understand that the results of this project will be coded in such a way that my identity will not be physically attached to the final data that I produce. The key listing my identity and the group-subject code number will be kept separate from the data in a locked area accessible only to the project director, and it will be physically destroyed at the conclusion of the project.

I understand that the results of this research may be published or reported to government agencies, funding agencies, or scientific groups, but my name will not be associated in any way with any published results.

I understand that the project director will be contacting the Registrar's office to obtain a copy of my academic record.

I understand that if at any time I have questions, I can contact the project director at 220-3874.

______________________ (Date)

______________________ (Signature)

______________________ (Participant's name, printed)

______________________ (Student Number)

APPENDIX B

TEST PROBLEMS

<u>Concept Area 1, Level of Difficulty 1</u>

A billiard ball is rolling down a plank. Between [redacted] s and [redacted] s after starting, its horizontal velocity increases by [redacted] m/s. The ball continues rolling for a total of [redacted] s. What will its displacement in the direction of motion be after that time?

A baseball is thrown directly upwards. It reaches a maximum height of [redacted] m above the point of release. It was observed [redacted] s after it was released. What was its height at that time?

<u>Concept Area 1, Level of Difficulty 2</u>

An artillery shell is fired on an unknown planet. It loses [redacted] m/s or vertical velocity in the first [redacted] s of flight. A second shell reaches its maximum height [redacted] s after being fired. What is the maximum height the second object will attain?

A projectile fired from the Earth's equator reaches a maximum height of [redacted] m. It is observed [redacted] s before it reaches its maximum height. What was its height at that time?

<u>Concept Area 2, Level of Difficulty 1</u>

A football is thrown at an angle of [redacted] degrees with the horizon. Its initial speed is [redacted] m/s. It is observed after [redacted] s. What was its horizontal displacement at that time?

An air gun is fired at an angle of [redacted] degrees with the horizon. When its pellet is first observed, its vertical velocity is [redacted] m/s and it is decelerating at [redacted] m/s^2. It is observed for [redacted] s. What will its horizontal velocity be at the end of that time?

<u>Concept Area 2, Level of Difficulty 2</u>

A baseball is thrown at an angle of [redacted] degrees from the horizon. Its initial speed is [redacted] m/s. It is observed after [redacted] s. What will its vertical displacement be at that time?

A golf ball leaves the tee with a speed of [redacted] m/s and a horizontal velocity of [redacted] m/s. It is observed after [redacted] s. What will its vertical displacement be at that time?

<u>Concept Area 3, Level of Difficulty 1</u>

A tennis ball weighs [redacted] kg. During a typical serve, a tennis racket exerts [redacted] N of force for a period of [redacted] s. What will the velocity of the ball be at the end of that time?

A cart weighs [redacted] kg. It is accelerated from rest to a velocity of [redacted] m/s in a time of [redacted] s. What force was acting on it to create this acceleration?

<u>Concept Area 3, Level of Difficulty 2</u>

A [redacted] kg bullet is accelerated in a rifle barrel at [redacted] degrees from the horizon by a net force of ■ N acting for ■ s. What is its vertical velocity at the end of that time?

A softball weighing ■ kg is thrown at an angle of ■ degrees from the horizon. During the pitch, it is accelerated by a net force of ■ N for ■ s. What is the maximum height it will reach?

<u>Concept Area 4, Level of Difficulty 1</u>

The velocity of a sled increases at a rate of ■ m/s^2 while it travels a distance of ■ m. The sled weighs ■ kg. How much work was done to create this acceleration?

A crate travelling at a constant velocity has ■ N of kinetic energy. It travels ■ m in ■ s. What is its mass?

<u>Concept Area 4, Level of Difficulty 2</u>

A force of ■ N is applied to a ■ kg block initially at rest. The force remains constant for ■ s. How much work will have been done?

A spring expends ■ J to accelerate a marble at a constant rate of ■ m/s^2 while it travels ■ m. At one point, the marble has a kinetic energy of ■ n. How fast is the marble travelling at that point?

APPENDIX C

DIRECTIONS FOR EXPERIMENTAL TASK

Read the following problem. Below the problem are 13 equations. For each equation, indicate how likely it is that this equation will be needed to solve the problem.

Use the following rating scale:

1 - Probably not needed.
2 - Might not be needed.
3 - Might be needed.
4 - Probably needed.

> A baseball is thrown directly upwards. It reaches a maximum height of [redacted] m above the point of release. It was observed [redacted] s after it was released.
> [redacted]

1	2	3	4	
☐	☐	☐	☐	$a = \frac{dv}{dt}$
☐	☐	☐	☐	$h_m = \frac{v_0^2}{2g}$
☐	☐	☐	☐	$h_m = \frac{(v_0 \sin\theta)^2}{2g}$
☐	☐	☐	☐	$\Sigma F = ma$
☐	☐	☐	☐	$K = \frac{1}{2} mv^2$
☐	☐	☐	☐	$t_m = \frac{v_0}{g}$
☐	☐	☐	☐	$v_x = \frac{dx}{dt}$

1	2	3	4	
☐	☐	☐	☐	$v_x(t) = v_{x_0} + a_x t$
☐	☐	☐	☐	$v_{x_0} = v\cos\theta$
☐	☐	☐	☐	$v_{y_0} = v\sin\theta$
☐	☐	☐	☐	$x(t) = x_0 + v_{x_0} t + \frac{1}{2} a_x t^2$
☐	☐	☐	☐	$y(t) = y_0 + v_{y_0} t + \frac{1}{2} a_y t^2$
☐	☐	☐	☐	$W = F\lambda\cos\theta$

When you have rated all the equations for one problem, go on to the next problem. Please do not go back to a problem once you have started on the next one.

You should not try to solve the problems or to identify the exact equations you would need to solve that problem. When you are reading a problem, you might think you see exactly how it would have been solved. If this happens, simply mark the necessary equations as "Probably needed", mark all the other equations as "Probably not needed", and go on to the next problem.

Do not spend too long on any one problem or equation. We are most interested in you first reactions.

If you have any questions about these instructions, please ask the experimenter for clarification before you begin. If you feel that you clearly understand these instructions, you may turn the page and begin the first problem.

APPENDIX D

SAMPLE PROBLEM PAGE

Read the following problem. Below the problem are 13 equations.
For each equation, indicate how likely it is that this equation
will be needed to solve the problem.

Use the following rating scale:

1 - Probably not needed.
2 - Might not be needed.
3 - Might be needed.
4 - Probably needed.

> A baseball is thrown directly upwards. It reaches a maximum height of [redacted] m above the point of release. It was observed [redacted] s after it was released.
> [redacted]

1	2	3	4	Equation
☐	☐	☐	☐	$a = \frac{dv}{dt}$
☐	☐	☐	☐	$h_m = \frac{v_0^2}{2g}$
☐	☐	☐	☐	$h_m = \frac{(v_0 \sin\theta)^2}{2g}$
☐	☐	☐	☐	$\Sigma F = ma$
☐	☐	☐	☐	$K = \frac{1}{2} mv^2$
☐	☐	☐	☐	$t_m = \frac{v_0}{g}$
☐	☐	☐	☐	$v_x = \frac{dx}{dt}$

1	2	3	4	Equation
☐	☐	☐	☐	$v_x(t) = v_{x_0} + a_x t$
☐	☐	☐	☐	$v_{x_0} = v\cos\theta$
☐	☐	☐	☐	$v_{y_0} = v\sin\theta$
☐	☐	☐	☐	$x(t) = x_0 + v_{x_0} t + \frac{1}{2} a_x t^2$
☐	☐	☐	☐	$y(t) = y_0 + v_{y_0} t + \frac{1}{2} a_y t^2$
☐	☐	☐	☐	$W = F\lambda\cos\theta$