

PETROPHYSICAL ROCK MODELING

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ABSTRACT

Generation, migration, trapping, and discovery of petroleum, as well as its primary and enhanced recovery, depend to a large extent upon the geometry of the porous microstructure. The determination of petrophysical rock properties is difficult because of their complexity ranging from apparent random structures to configurations with heterogeneities at different scales. We present a method of visualizing a realistic 3D representation of these structures. While rock descriptions always contain many deterministic elements and features, some elements are most conveniently generated by stochastic models.

The primary purpose of this study was to survey ways in which stochastic modeling might be used to create rock images, obtain realistic representation of rock textures, and provide basic model control and visualization capability.

We use an "octree" data structure to represent rock models of arbitrary precision. Objects created using this technique are collections of subvolumes of variable sizes each having the same physical properties. A reconstruction of porous media results in a pore network where the total porosity is the same as that of the actual rock.

The model and techniques presented have been designed in an extensible fashion to enable the development of algorithms for a simulation of geometrical models, along with other simulated properties of the rock models such as electrical resistivity, fluid flow, permeability, and acoustic velocity.

The developed methodology may be used to do: quick evaluation of rock parameters, calibration, teaching, determination of the estimation sensitivity, and testing of the computer software for image analysis.

KEYWORDS: Rock Modeling, Stochastic Modeling, Computer Graphics, Solid Texturing, Computational Geometry.

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INTRODUCTION

The area of fluid accumulation and transport in porous media has been a matter of investigation for many years because of its practical and scientific importance in petroleum engineering. The physical properties in porous media are strongly influenced by the actual microstructure of the pore space.

Computer generated models can help in developing methods to estimate electromagnetic, electric, and sonic responses in rocks saturated with a variety of fluids in static and dynamic conditions (e.g. simulate logging tool responses). We present a simulation technique for porous media, which enables us to produce a realistic visualization both in 3D and in 2D.

The model we have developed will also enable us to account for properties such as those listed below:

- (1) Chemical composition.
- (2) Grain shape, sorting, orientation, and size distribution.
- (3) Pore size distribution.
- (4) Porosity.
- (5) Saturation.

and enable us to make estimates of resultant properties such as:

- (1) Permeability.
- (2) Electrical conductivity.
- (3) Sonic velocity.
- (4) Dielectric Constant.

The rock description is a combination of observations, knowledge, and scientific "guessing" (the stochastic component). A stochastic phenomenon or variable is characterized by the property that a given set of circumstances does not always lead to the same outcome (therefore there is no deterministic regularity), but to different outcomes in such a way that there is statistical regularity. An appropriate model can be developed by sampling the phenomenon of interest. Then, through educated guesses, the model builder would select a known distribution form, make an estimate of the parameter(s) of this distribution, and then test to see how good a fit has been obtained.

The variables we seek to describe with stochastic techniques are those that influence the amount, position accessibility, and flow through rocks. The dynamic properties of the porous media are related to the permeability (conductivity) of a pore (microscopic scale) or of the total porous media (macroscopic scale). Porous media have been subjected to numerous models on both scales [Fatt56], [Waxm68], [Hard90], in an attempt to predict the behaviour of the system (fluid transport in the medium).

Random Packing and Tube Models

The first models were made of regular arrangements of sphere packings and allowed the calculation of the permeability of these media [Viss72], [Peac77]. An expansion of the regular pack of spherical models to random packing leads to a local arrangement of the spheres which could "vary" or intersect by overgrowing [Schw87]).

Network Approach

The percolation theory and network models constitute another approach to modeling. A discussion on a simple model made of cylindrical tubes as a geometrical component of pore structure and on the network of tubes was presented in three papers [Fatt56]. Some of the network models describe permeability on the basis of the electrical resistor analogue [Kirk73], others were concerned with the relation between capillary pressure and fluid properties [Dull76]. The connecting concept was linked to the idea of pore neck and pore body [Ward78]. Percolation process modeling deals with penetration of fluids into porous materials. The pore space is considered to be a random medium composed of conducting channels (a lattice), some of which permit flow through them and others which do not. This model is attractive from the point of view of describing the capillary phenomena in porous media [Macd86], [Diaz87]. In such cases a three dimensional network (lattice) is comprised of pore bodies linked by pore throats.

In lattice models different mechanisms are used to define pore system for any given random point. The most common approach has each point represent a center of a pore unit which is given a selected shape, orientation and connectivity to its neighbours. A model of pore size in three dimensions is generated by randomly placing shapes in space, where their size and connecting elements are controlled by distribution functions. In [Spea91] a porous media is being modeled as a three-dimensional array of cubes joined by a network of cylinders along x,y,z axes.

Image Analysis and Statistical Modeling

Problems of physical models proposed in connection with the network and percolation approach is not to account for the actual distribution of the pore size. A more realistic approach involves the measurement of the size of pores from thin sections. This method involves impregnation techniques of the porous medium followed by preparation of thin or thick sections which can be photographed or processed directly under a microscope. The image can be analyzed manually or by automatic digitizing systems. The information derived from the cross section in terms of digitized data is interpreted statistically as the distribution or autocorrelations of the measured property [Ehrl84], [Etri88], [Tomi90]. Any of these measurements taken on a section has to be transformed in some way into a three-dimensional model that describes the pore shape and volume. This cannot be done without assumptions and simplifications involving simple geometrical models.

Reconstruction of porous media

The idea that pore geometry and topology can be measured and modeled in three dimensions has been behind serial sectioning of the porous material [Kauf83] and a computer 3D reconstruction [Bria84]. The serial sections of impregnated porous media are photographed under the microscope and the image is digitized and processed. The result is a three dimensional matrix containing the enhanced digitized image.

Much work has been done in order to apply percolation concepts to flow phenomena and in the area of three dimensional modeling of pore space and topology. What is missing is the strong connection between the distribution of pore bodies and throats from different kinds of measurement techniques and a modeling technique which allows the recreation of the actual geometry and topology of the pore space network. Combined with such information, the percolation theory concepts will provide a powerful tool in the investigation of the dynamics and statics of fluids in porous media.

WORK OVERVIEW

In this paper we describe the results of an effort to devise a framework for a new 3D geometric modeling scheme and associated algorithms for modeling porous rocks. Rock objects of arbitrary complexity can be encoded, manipulated, analyzed, and displayed in three dimensions. An implementation of the scheme allows the modeling of porous rock in three dimensions based on pore and grain size distributions (see Fig. 1). We assume that pore space in a rock can be described by the following parameters:

- (1) Pore body size distribution.
- (2) Pore throat size distribution.
- (3) Grain size distribution.

An adaptive space subdivision method and the octree data structure have been chosen to realize these goals. A tutorial survey of the octree and related hierarchical data structures is presented in [Same84] and [Same90]. An octree is a hierarchical data structure which recursively subdivides the space into eight blocks (octants) based on some decomposition rules.

The model we have implemented treats both the pore and the pore throat as objects built of the same basic units but of different size. The model of the network of pores and throats is built of empty cubic elements where the throat is built of the smallest size cubes. The same approach is applied to modeling grains or rock matrix (everything that is not empty space). Combined with the octree data structure this approach allows a unified and simple approach for the porous media modeling.

Starting with an initial cube the first level of the octree represents eight sub-cubes (or voxels) found by dividing space along the three planes (xy,xz,yz). At lower level a voxel

may be divided or not depending on the stochastic process. The pseudo random generation of the voxels at each octree level, based on the laboratory obtained distributions, should constrain the porosity of the model to be the same as that of the actual porous media.

The model uses a three dimensional joint distribution function $f(x,y,z)$ of the rock space, placed in the orthogonal coordinate system. The function is derived from a known continuous function, describing regular pore/grain distributions which were obtained from digitization of thin sections and sieve analysis of rock. A reconstruction of the porous media gives a network of empty/solid cubes of corresponding sizes where the porosity distribution corresponds to the actual rock.

The system allows us to build both: 3D images of rock models, and 2D images (cross-sections which are slices through the tree-dimensional model). The viewing part of the system displays these images on two graphics windows. One of the windows is used for 3D image, while the second window allows a sequential visualizing of 2D images-slices.

MODELING AND VISUALIZATION

The software system we have designed allows a digital model of some rock sample to be built and processed. The processing provides certain data as well as views of the model. In this work we concentrate on describing the graphical output from the model.

There are two output forms required for visualization of the model:

- (1) Perspective views of rock models.
- (2) Slices through the rock sample.

Two major steps are required in producing these images:

- (1) Build a model.
- (2) Visualize the model.

MODELING

In the modeling system we have implemented the following points were considered:

- (1) Finding pore/grain size distributions and the volume associated with these distributions.
- (2) Statistical representation of the above parameters by random sampling.
- (3) Computer data structure allowing description of rock information.
- (4) A computer system which produces the above information in a form allowing later visualization of our models.

Fig. 2 shows a general scheme for tuning properties of the modeling system based on the octree as the selected data structure.

Pore and Grain Size distributions

Reservoir rocks possess variations in their properties on several length scales. The study of such a system requires quantitative description of the rock properties on all spatial scales. The three dimensional pore and pore throat space geometry can be defined by the distribution function of length. This function is the characteristic of the size of a pore body and a pore throat (neck), respectively, along with the introduction of a geometric model of pore shape. This three dimensional pore topology defines the connectivity of a pore network. A combined frequency distribution or histogram of pore and throat size is used to identify the function shape.

This project does not distinguish between a pore body or a pore throat. Both are just an empty space randomly placed, based on the true size distribution. Clusters of different size pores create larger and more complicated pores and, if connected, they make porous medium permeable. The model does not require any calculation of probability of finding a pore near a grain and vice versa. Each decision of a pore-grain classification is statistically independent and is not limited to only these two events. However other approaches may be included in the model.

Statistical (Stochastic) Representation

To infer a large quantity of data from a limited input we require a compact method describing complex rock objects. Such an approach is provided by the use of random stochastic models [Nayl66]. The simplest stochastic model assigns to each octant (rock partition) a value, drawn independently at random from a sample space with a probability given by a specified histogram as described above.

Presently, two types of random number generators have been designed. They produce random numbers in the range of 0-255 (typical range for the number of grey levels in the image analysis) and represent uniform and normal distribution respectively. The requested distribution parameters of the random number generators are supplied as arguments passed to the functions.

Computer Representation of Rock Grain and Pore Systems

The octree as a hierarchical tree structure has been chosen, where nodes represent cubes of exponentially decreasing size. Each node in the tree corresponds to a region of the modeled rock, we represent the node geometrically as a cube or "voxel". If the node completely describes the region of the rock (one type of the space - grain/pore), it is a leaf node. If not, the node points to the eight children that represent the eight octants or subregions of the parent node.

There are several advantages to this data structure. First, there is a single primitive shape, a cube. An arbitrary complex rock-object can be represented to the precision of the smallest cube. Second, only a single set of manipulation and algorithms is required for all rock types, regardless of their complexity or shape sophistication. Third, we have the ability to trade off computation against modeling precision. This means, that a coarse image can be generated very quickly with a low fidelity of details.

Operations for creating three dimensional graphical representations, such as the hidden surface removal, can be simplified because the object parts are kept spatially pre-sorted at all times. By traversing the tree in the proper sequence regions of the rock space will be visited in a uniform direction. Because of the hierarchical structure, the root node represents the entire rock object. Nodes at a level together with the higher nodes completely describe the entire rock to the resolution of that level.

The main disadvantage of the octree data structure is the large memory requirement. According to Samet [Same90] it is on average in order of the object's total surface. However the prime motivation for octree development is to reduce the amount of space to store data through the use of homogenous blocks. It is not the best solution for models corresponding to a checkerboard in 2D space, or objects where leaf nodes start to appear on the lowest levels. In such a degenerate situation, the octree structure would require the leaf nodes in a number equivalent to the 3D array representation. In addition, the extra nodes connecting from the root node and pointers in every leaf node would be required.

In the model presented here the root node describes a cube of unit size located in the origin of the three dimensional coordinate system, such that one of its vertices is located in the point (0,0,0). The root node is of type "grey" and is subject to the recursive subdivision into octants of the same type until the level reaches the starting level for leaf nodes. The root level is enumerated with the highest level number and subdivision stops after reaching level zero. At the leaf starting level and below it (based on the grey levels generated by the random number generator) a decision is made and each octant is assigned a value (the octant code) white/black/grey which corresponds to empty/rock/mixture.

Grey octants are further subdivided, but cannot be created at the zero level. The octant code can be a more complex description and can be extended to account for different types of material, colour, density, electrical/ thermal/fluid conductivity, etc. It would complicate the decision making but would not affect any of the developed algorithms. The octant enumeration is based on the input pore/grain distribution of the laboratory measured rocks, and is a function of the octree level. This process is shown in Fig. 3 and Fig. 4.

A COMPUTER SYSTEM FOR ROCK VISUALIZATION

In order to generate these images the following steps must be taken:

- (1) Building the octree to some chosen level.
- (2) Turning the model into a set of polygons ("Polygonization").
- (3) Prototyping.
- (4) Rendering required images.
- (5) Visualizing the model in the controlled environment.

A detailed description of the computer system used and the efficiency of the process is described in a paper currently in preparation [Fede91]. What follows is an overview of the processes involved.

Building the Octree

The octree is built as described above. The criterion for stopping subdivision is simple; the user designates the number of subdivisions. We have found that six or more subdivisions are required to produce useful images. However to find reasonable views of the rock sample using our interactive system (lighting, camera position, colours, etc.) we must first prototype the sample at a low level of subdivision. Fortunately this is easily done with the current octree data structure. When the viewing and rendering parameters are found the model is subdivided to a higher level for production of the final images.

Polygonization

In order to produce a shaded image of the rock (the rendering process) the cubic voxels first have to be reduced to their constituent polygons. This simply means finding three faces whose surface normals are towards the viewer.

At this stage the information on surface properties such as colour, diffusion, reflection, refraction is included with the polygons. The polygonized description of the rock model together with the environment information is stored in a file ready to be passed to the renderer.

Colour

Voxels representing rock material are displayed in a "grey" colour (70% of red, green, and blue), while pore spaces are shown in black (or the background colour). The colour scheme can be altered to show other properties such as pore topology, e.g. to create a "cast" of pore spaces. In such a case the pores may be displayed in some colour other than background (we have found red to be effective). The system may be extended to model more than one rock type and the method is easily extended to user supplied colour schemes.

Prototyping

In the order of a million or more polygons may be generated to produce the desired result. Such large data volumes cannot be viewed interactively on current hardware. However the user can control the complexity very simply by controlling the depth of the octree. Data volume is traded for complexity. If the distribution is kept consistent at each subdivision level then the model has property of self similarity (One of the properties of a fractal process [Mand77]). Using this simplified prototype of the model, various parameters can be chosen interactively. These include:

- (1) Viewing Parameters (These include the position of the eye or "camera" and the type of lens [Newm79], [Foll82]).
- (2) Colour as discussed above.
- (3) Surface properties, including, specular reflection and refraction diffuse constant. (These viewing parameters and surface properties are discussed in [Roge85]).

Rendering

An intermediate file which contains a complete description of an object (rock), its attributes and the scene environment together with polygons describing the scene, is converted into an image file. The polygonized file is rendered to get a full colour shaded image with only the visible surfaces being processed (many commercial packages exist for rendering polygon data). The renderer projects and shades the polygons and also provides translucency effects in the final image.

Visualization

A 3D digital image may be thought of as composed of a series of "slices" parallel to the XY-plane. Each slice image is a set of voxels-squares (whose "z" components are identical) with a colour representing the colour of the original nodes (cubes). The visualization of rock images is performed with a program developed for an interactive workstation (Silicon Graphics Personal Iris). It is a menu driven system allowing the user to view both 3D images and their 2D slices in timely fashion. The program displays images stored already in raster files containing rendered images of rock models. The user controls all manipulations by a mouse selected menu and mouse clicking. A large number of images may be displayed. There are two graphics windows: one for the 3D image and the second for the 2D slices. A menu window contains three buttons: Program name, Next, and Stop. The Next button, or the left and right mouse buttons allow the user to flip through the 2D images in a circular fashion. Slices are rendered with exactly the same visualization parameters and spatial alignments as were applied to the corresponding 3D image. It allows sequential traversal through a 3D object during slicing and makes this interactive visualization process more realistic.

RESULTS

During the experimental part, efforts were made to create a variety of images corresponding to the most common type of rocks in petroleum reservoirs. Most images were created using 512*512 resolution, but 1024*1024 images were created as well to get a better image quality. Rock creation modeling techniques were tested using the random number generators with variable thresholds on each octree level. On the last (zero) level a pore threshold has to be equal to the grain threshold and this must be obeyed to prevent creating non-leaf nodes. The three dimensional images were obtained for models with the number of subdivision levels ranging between 2 and 7, with the leaf nodes starting between 1 and 6.

Images on Plate 1 show how more complicated external shapes together with the internal variability may be created using different node distributions. Plate 2 shows samples of the 2D slices taken from rock models.

Shape and internal structure of modeled objects

The overall shape of the object can be controlled by the top few levels of the subdivision and the internal structure can be controlled by deeper levels. Most of the six and seven octree level models were created using a high probability of empty spaces (white nodes) close to the root level. It resulted in large holes in objects (external shape variability). At the same time the subdivision of low levels allowed to create a microstructure variability.

Evaluation of Results

The result evaluation has been based mainly on the visual analysis of the images obtained. A few of the 2D images were analyzed using the Image Analysis System Quantimet Q520. The total porosity from the image analysis was a function of thresholds applied to the image by the Quantimet and varied 10% in both direction from the true value. The 3D images were, in general, not rich enough in the microtexture variability characteristic to rocks, but still of good quality. Nevertheless results obtained have shown promising agreement with laboratory Scanning Electron Microscope images at the required quality.

IMPLEMENTATION

The system has been implemented on a Silicon Graphics Personal Iris workstation. The programs were built using the GRAPHICSLAND facilities described already in [Wyvi86].

The user interface has been built around the JOY[McPh89] Library developed at University of Calgary. All programs were created using the C language for programming.

Performance and Efficiency

The efficiency of the implementation is detailed in [Fede91]. A brief discussion follows. The approach presented in this work does not produce the most efficient algorithms. It was designed to offer the opportunity to try the new approach quickly in the existing hardware/software environment of Graphicsland.

Performance analysis shows that the computation is not evenly distributed among the modeling and rendering. In particular, the cost of generating complex, structured models does not dominate the computation. Instead, the expense arises from the need to process a vast amount of three-dimensional details during rendering in order to create the desired visual results.

Both modeling and rendering time is a function of the number of nodes in the tree. It may take only a portion of a minute to run both programs for simplest models. Unfortunately creating realistic images requires hours of computing time in some extreme cases.

It is difficult to compare this performance with the cost of creating synthetic images using different techniques. According to [Schw90] modeling of 2D slices based on numerical filters applied to a 3D grid of rock/pore numbers may take hours on the SUN4 Workstation. By using a cube as a modeling primitive and the recursion depth to control the model resolution, we achieve two important benefits. The process never runs out of details, since it can always generate more data. It can never produce too much detail than necessary. Therefore the computational effort is always rewarded with the model complexity. The future extensions should benefit from the model simplicity.

FUTURE EXTENSIONS and POSSIBLE APPLICATIONS

The techniques described may be used to build geometrical models of porous rocks. Future development include algorithms for simulation of fluid flows, with electrical and sonic properties of these models. This will result in a synthetic data base of rock properties. It will contain 3D and 2D images and corresponding geometrical, flow, electric, and sonic properties.

Such a data base will help to evaluate the potential of reservoirs and monitoring recovery processes. If rock types in the analyzed reservoir are similar to those in the data base, a user can infer all parameters without having to take long and detailed measurements or tests, thus saving time and money (see Fig.5).

CONCLUSIONS

A new technique for the modeling of porous media has been designed and tested. The results obtained show that it is possible to create realistic rock images using solid modeling and stochastic techniques. The octree data structure has been used to build models. The advantage of this method is that it allows the user to generate a variety of rock images by specifying a few simple parameters.

Further, the information derived during modeling is used to calculate the true model porosity and grain size distribution at each level. The system allows the user to build 3D images and 2D "section views" which can be viewed interactively. This approach allows us to create the real-world phenomena that would be difficult or impossible to generate with other methods.

The images developed closely reflect real ones, they also have a similar pore continuity and shape as those in real images. However, some differences can be seen and they are functionally dependent on the octree depth, and light of the scene.

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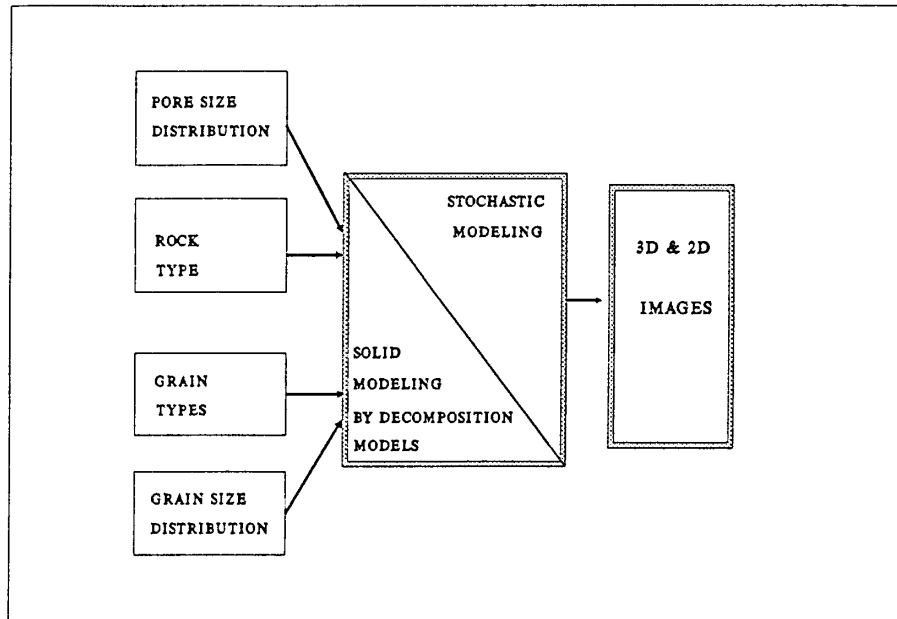


Fig. 1 Techniques Used for Porous Rock Modeling

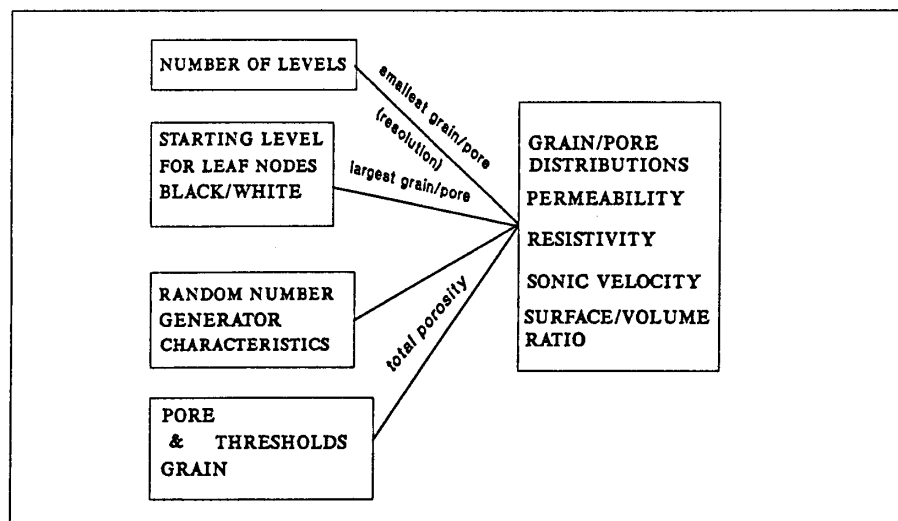


Fig. 2 Rock Tuning Properties Through Octrees

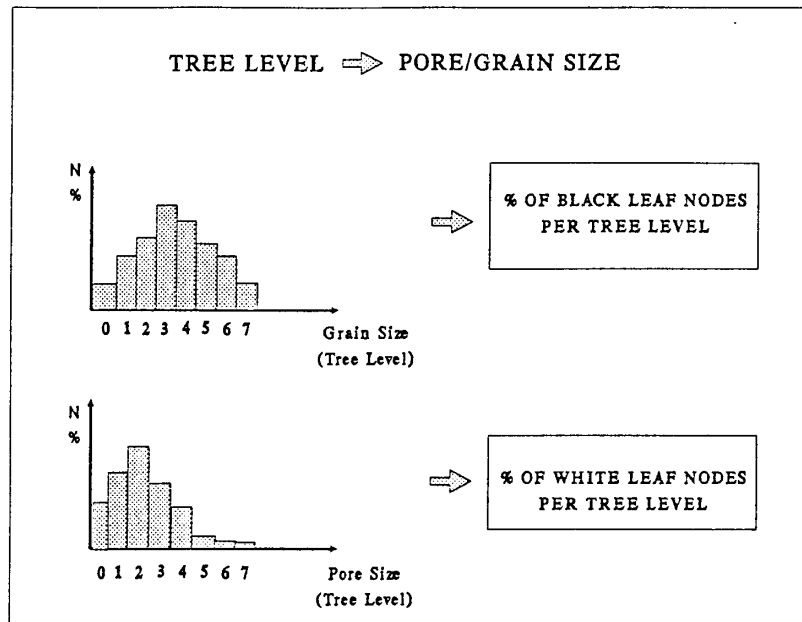


Fig. 3 Adaptive Space Subdivision Approach

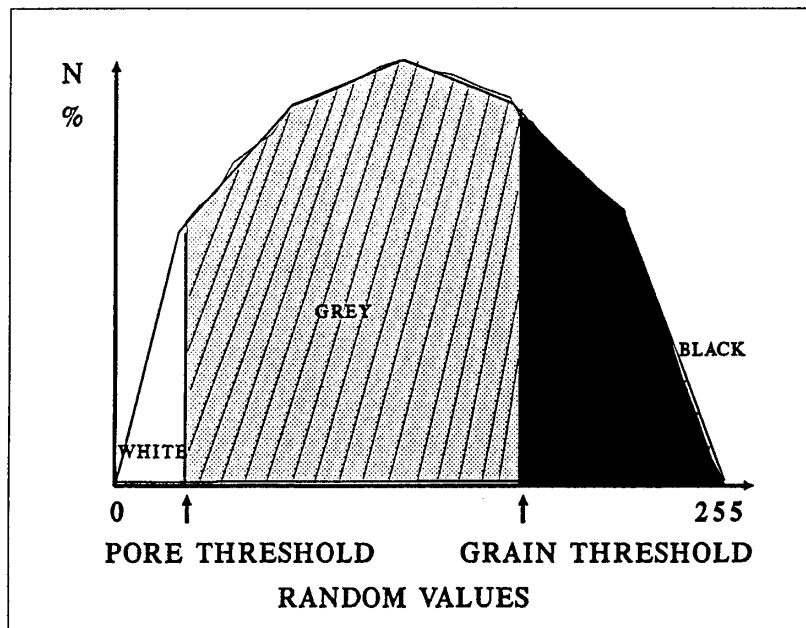


Fig. 4 Octant Type Generation

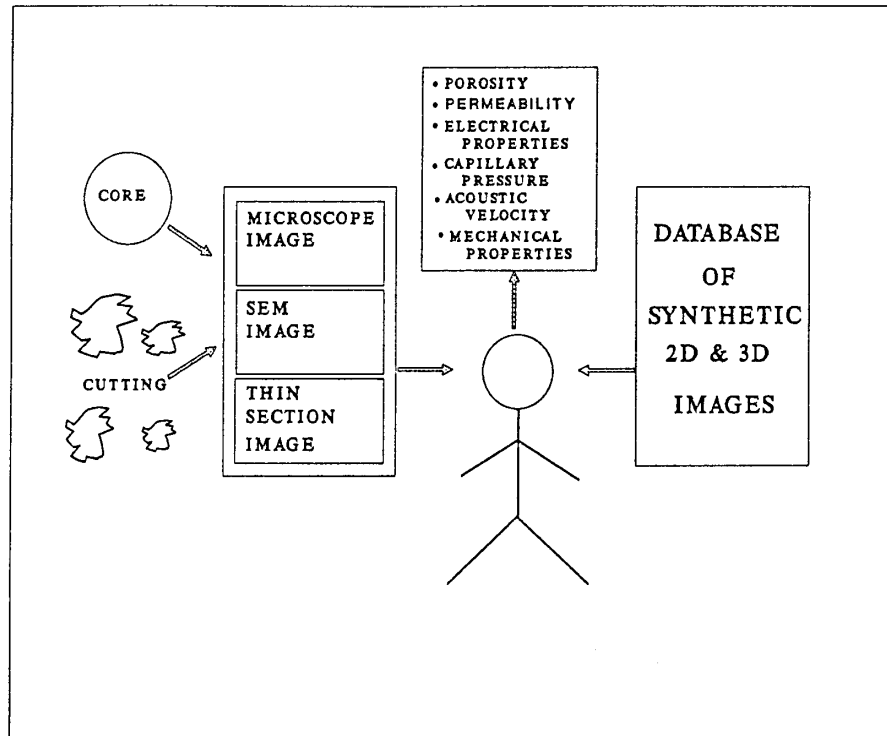


Fig. 5 Rock Analysis Using Synthetic Images

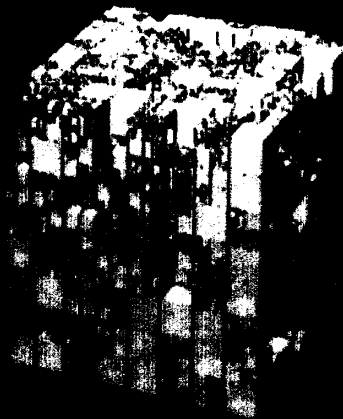
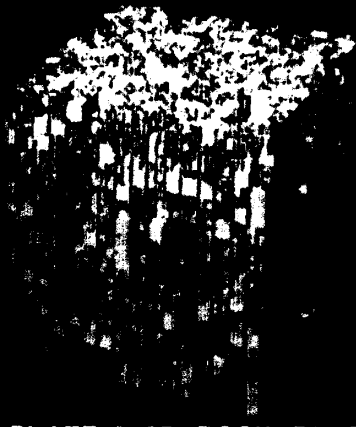
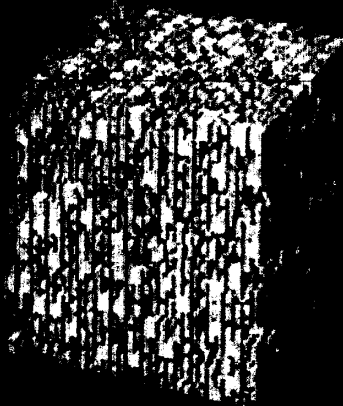
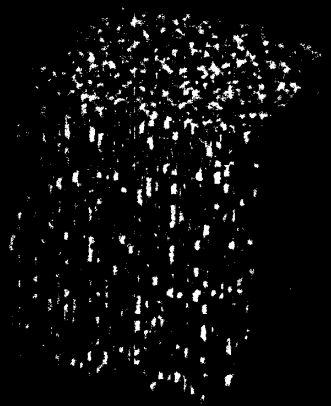


PLATE 1: 3D ROCK PRESENTATIONS

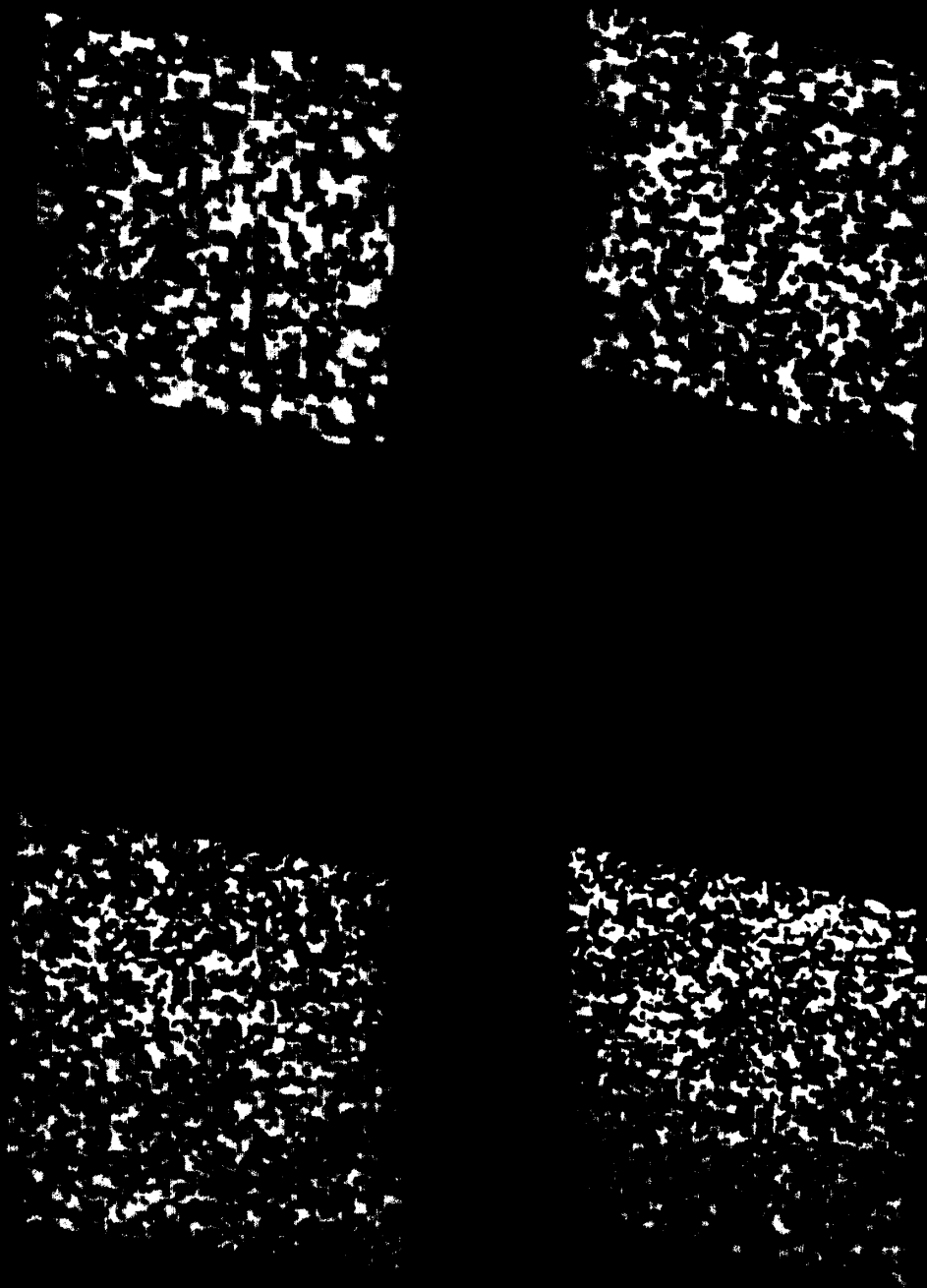


PLATE 2: 2D CROSS-SECTIONS THROUGH ROCKS