

IMAGES OF MATHEMATICS LEARNING REVEALED THROUGH STUDENTS' EXPERIENCES OF COLLABORATION

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This study focuses on students' images of mathematics learning and their relationships with mathematics. In this paper we consider how students described collaboration in mathematics classrooms, through the examination of students' autobiographical interviews and drawings. Our analysis revealed that many students considered mathematics learning mainly as an individualized and isolated process and did not perceive peer talk or collective exploration as meaningful. Our cross-analysis with students' feelings revealed that those who had positive feelings towards mathematics tended to find group work less helpful. Our findings illuminate a perceived gap between teachers' widespread use of group work as a teaching strategy and students' understanding and appreciation of the goals of such instruction.

PURPOSE OF THE STUDY AND LITERATURE REVIEW

The study from which the findings presented here are derived explores students' experiences of learning mathematics in Canadian schools and post-secondary institutions. This paper focuses specifically on how students perceive group work and collaboration in mathematics classrooms. Through students' descriptions of their experiences of collaboration in mathematics classrooms, we attempt to reveal their images of, and assumptions about, mathematics learning and how these relate to students' emotional relationships with mathematics.

Collaborative working has been implemented across disciplines as a tool for providing rich academic and social learning opportunities to students and group work is widely recommended as a teaching strategy in mathematics classrooms. For example, in its Principles and Standards for School Mathematics, the National Council of Teachers of Mathematics outlines the importance of group work for communicating, explaining, and justifying mathematical ideas among learners (National Council of Teachers of Mathematics, 2000). Collaboration, problem solving, and learning how to learn—essential components of the 21st century skills needed for navigating a rapidly changing society—can be developed through group work (Darling-Hammond et al., 2008; Trilling & Fadel, 2009). The kinds of learning that emerges from group work, however, cannot be taken for granted in mathematical classrooms. If the physical conditions and communication space for collaboration are not well prepared, learning by talking with peers cannot be guaranteed (Barron, 2003; Sfard & Kieran, 2001).

Collaboration and collective mathematical thinking are highly related to students' mathematical dispositions (Towers, Martin, & Heater, 2013). Over the past 30 years, researchers in the field of mathematics education and psychology have examined the interplay between the affective domain (beliefs, attitudes, and emotions) and teaching

and learning mathematics (Di Martino & Zan, 2011). Many of the studies investigating affect and mathematics in the field of cognitive psychology tend to focus on negative aspects, such as “math anxiety,” associated with mathematics (e.g., Ahmed, Minnaert, Kuyper, & van der Werf, 2012; Young, Wu, & Menon, 2012). Understanding a wider breadth of students’ emotional connections to mathematics is thus essential for designing mathematics instruction that enhances students’ dispositions for learning mathematics (Boaler, 2011). While various aspects of learning through group work has been researched in the mathematics education community (e.g., Barron, 2003; Esmonde, 2009; Ryve, Nilsson, & Pettersson, 2013; Webb, 1991; Yackel, Cobb, & Wood, 1991), little investigation has looked at the connection between students’ emotions, images of mathematics learning, and group work experiences. This research examines students’ emotional experiences and images of learning mathematics, in relation to the specific instructional context, group work.

THEORETICAL FRAMEWORK

This research is framed by enactivism, a theory of embodied cognition that emphasizes the interrelationship of cognition and emotion in learning (Maturana & Varela, 1992; Varela, Thompson, & Rosch, 1991). Enactivism recognizes human development and the surrounding environment as structurally coupled (Maturana & Varela, 1992) and therefore learning, in this frame, is seen as reciprocal activity. Students’ mathematical learning is not determined (solely) by the teacher or the learning environment, but is dependent on the kind of teaching experienced and the kind of mathematical milieu in which students are immersed. Enactivist thought reorients us to the significance of this mathematical milieu in shaping not only what students learn in school but also their emotional connections and relationships with the discipline. This enactivist frame, then, prompts us to seek to understand how students come to have particular relationships with mathematics, what being mathematical means to them, and the kinds of teaching and learning structures (such as group work) that are relevant as students develop particular dispositions for mathematics. Guided by enactivist thought, our investigation tries to understand how instructional contexts and the mathematical milieu in which students are immersed can influence students’ (emotional) relationships with mathematics learning.

RESEARCH DESIGN

The data on which we draw for this paper were gathered in the province of Alberta, which is located in Western Canada. The study’s participants are Kindergarten to Grade 12 students, post-secondary students, and members of the general public, but we focus here on data collected in the first phase of the study, which includes students from Kindergarten to Grade 9. Forms of data include semi-structured interviews, drawings (that represent participants’ ideas about what mathematics is, as well as their feelings when doing mathematics), and written and oral mathematics autobiographies (accounts of participants’ histories of learning mathematics).

To date, 94 interviews with Kindergarten to Grade 9 students (41 girls and 53 boys) have been conducted. We have also collected 95 mathematics autobiographies from post-secondary students and members of the general public through an online submission form.

All of the interviews were transcribed verbatim. In this paper, we mainly focus on elements of the transcripts that featured students' descriptions of group work and/or pair work in mathematics classrooms. In order to reveal students' images of mathematics learning, in relation to their experiences of group work, we also conducted thematic analyses of their drawings and associated descriptions of their feelings when doing mathematics.

FINDINGS: STUDENTS IMAGES FOR GROUP WORK AND MATHEMATICAL LEARNING

Across grades, group work or pair work was frequently reported as a classroom learning structure, although the ways in which, and the extent to which, group work was used varied. Students reported that they often worked with their desk partners (those sitting next to them in class), their friends, and project members. Tasks that were used for group work also varied. In some classes, group work was used only for projects. In other classes, group work or pair work was used regularly for completing a worksheet. However, no students reported working on tasks specifically tailored towards group work [such as group-worthy tasks described in Cohen and Lotan (2014)].

Overall, students' preferences were split: 37.3% of the students preferred individual work to group work and/or pair work and 29.4% of the students preferred group work and/or pair work to individual work. For 31 % of the students, their preference was mixed: it depended on types of tasks and peers working together for group work. There was only one student who reported to have no preference. While slightly more students talked negatively about group work in elementary grades, the difference across grades was not outstanding.

Through the cross-analysis focusing on students' feelings about math and group work preference, it was revealed that both positive and negative feelings towards mathematics could influence students' preferences for group work. Our analysis suggests that the students who had positive feelings towards mathematics tended not to find group work very helpful. Among students who preferred individualized learning to working with peers, 57.8 % (11 out of 19) were good at mathematics and 10.5 % (2 out of 19) had negative relationships with mathematics. Among the students who preferred group work to individualized learning, 26.6 % (4 out of 15) had positive relationships with mathematics and 26.6 % (4 out of 15) had negative relationships with mathematics.

Students preferred individual work for various reasons. For those who are good at mathematics, they felt group work was unnecessary and could be distracting. They said, for example, "(I prefer individual work) because I know how to do it and those

things like math,” or “Sometimes I see people copying and making noise and I can’t focus on what I’m doing.” When explaining a preference for individualized learning, a Grade 2 student said it was, “Because you have your own space and people can’t copy you.” A Grade 3 student described how he felt about being asked for help from peers as follows:

Interviewer: So do people ever come to you then and ask for help?

Student: Sometimes.

Interviewer: Sometimes, yeah. Do you like helping them or do you find that a bother?

Student: I don’t know what the word is, but yeah it just disturbs me while I’m trying to work independent.

Similarly, a Grade 1 student said, she preferred individual work “Because when I’m working with a friend they’re talking and I’m trying to work and I say ‘Please will you be quiet?’ and they keep talking.” A Grade 2 student said he would not like group work “Because in groups, it’s not so quiet.” In fact, some students perceived “talk” in the classroom as noise and distraction. For example, a Grade 5 student compared learning environments at home and at school and said: “Well, my mother is kind of strict of, um, getting it. That’s why I always get it right. Because I make up strategies and then school with my teacher I kind of, you know, have a lot of noise and that’s why I get sometimes slow in writing.” Another Grade 5 student said, “Sometimes when I don’t have noise around me I can focus and I like it a bit more but sometimes when it’s noisy I can’t focus and I can’t do it. But I usually like math when it’s quiet.” Similarly, when describing group work, a Grade 1 student said, “I sit over with a friend but sometimes I see people copying and making noise and I can’t focus on what I’m doing.” These students’ comments depict mathematics classrooms where learning and thinking are essentially individualized and thus talking with others (and others’ copying their work) is considered to be a distraction and disturbance.

While group work and pair work were used regularly in our respondents’ mathematics classrooms, students’ autobiographical interviews and drawings did not communicate an image of collaboration and collectivity for mathematics learning. In their drawings, most of the students represented isolated and individualistic images of classroom mathematics learning—predominantly with the drawings of a student sitting at a desk working alone (see Figure 1). Only a few early grade students drew drawings of collaborating with others.

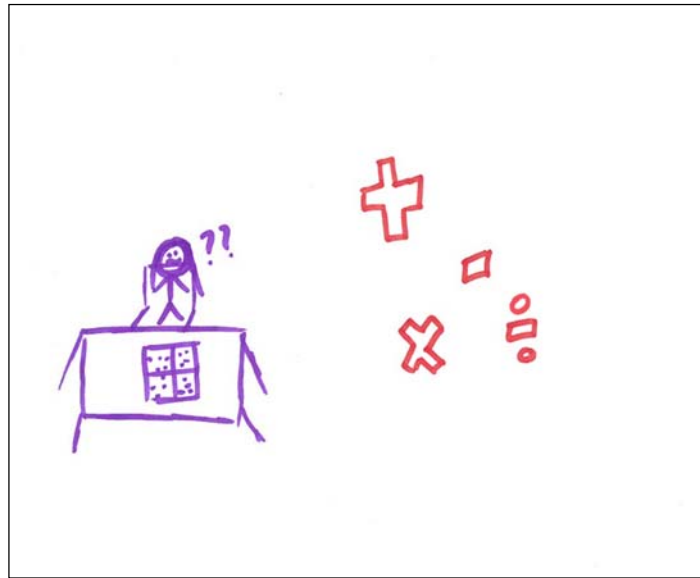


Figure 1: Typical student drawing of mathematics learning

For most of the students, when they got stuck on mathematics problems their strategy was to try to figure out the answer on their own, rather than collaborating with others. Many students said that they would sometimes seek help from a teacher or classmates but mostly they would try to work on their own. For example, a Grade 8 student said, “I usually like to work hard but in math it gets really hard. When it’s a hard stuff and I usually go up to the teacher several times, but he asks us to try and figure it out ourselves or ask friends and stuff.” Even when they were encouraged to ask their friends, many students across grades said they would still try to figure it out on their own. As students get older, they tend to rely more on themselves rather than seeking help from others, as represented by a quote from a Grade 8 student, “I developed the skill to always figure it out on my own until I could not.”

There was one exceptional but informative case wherein a Grade 5 student described how she liked to spend sufficient time to work on mathematics; and therefore she preferred working alone. This student enjoyed learning mathematics and working on problems. She said:

Normally I don’t like, really am a fan of working with someone else. When I work with other people they will want to do all the work and when I go up to the teacher answers will be wrong, and, but I take a lot of time. Once I took 25 minutes, um, to complete a math sheet that had 3 questions on it because I took my time.

In explaining why she likes to take time in mathematics, she said: “Well actually it’s quite fun, because the more actually slower you go the more better. Like in the hare and the turtle when they were racing the slower beat the faster.” This student’s description implies that, for her, collaboration and working with others are not compatible with spending time and exploring problems in depth.

In contrast to the above-introduced quotes, some students preferred group work because it helped them understand mathematics better by working with others. For example, a Grade 5 student said he preferred working with peers, “Just because if I don’t know something that they know then they can help me. Just, they don’t tell me the answer but they can tell me how to do it better.” A Grade 7 student said, “I prefer working in a group because it’s more fun and it just makes everything easier when there is more than one mind at work.” Similarly, a Grade 8 student said, “I can understand what they’re thinking and they can understand what I’m thinking and we can put that together and finish the question.” As these quotes indicate, these students recognized the benefits of group work and learning with peers. However, the number of students who recognized the benefits of working collectively with others was rather small (8.5%, 8 out of 94 students). Furthermore, most of the students perceived group work as a way of offering and/or receiving help for individualized tasks but not necessarily as an opportunity for creative collaboration. Our analysis shows that the majority of Kindergarten to Grade 9 students did not appreciate working with others and collaborating with others for deeper mathematics learning.

DISCUSSION AND EDUCATIONAL IMPLICATIONS

While collaborative learning and group work have been frequently used in mathematics classrooms in Canada and other countries, most of the students we interviewed still held images of mathematics learning that were mostly individualistic and isolated. Our analysis shows that merely experiencing group work does not convince all students of its usefulness. In our cross-analysis focusing on students’ feelings, it was revealed that those who had positive feelings towards mathematics tended not to find group work or pair work very helpful. The students who considered themselves to be adept at mathematics reported that group work and pair work were not beneficial because they mainly gave help to others but did not receive much in return. In fact, many students perceived the talk during group work as distracting and noisy.

Mathematics activities used during group work and pair work were characterized by the students as tasks in which finding a solution to the posed problems was the goal, rather than exploring multiple aspects of the problems and solutions. Because students considered mathematics mainly as an individualized and isolated process, many students did not perceive peer talk or collective exploration as meaningful, contrary to the perception of group work in other classrooms we have studied where the teacher deliberately structured mathematics learning through group activity (see, e.g., Towers et al., 2013).

As indicated in the interviews by some students, when they got stuck, trying to figure things out on their own was a commonly-observed solution. Even when they needed help and assistance in the very process of “figuring out,” they often did not have access to sufficient help or collaboration with others. Also, for those who think they are adept at mathematics, a lack of meanings for collaboration can deprive them of potential

opportunities to learn, because students can benefit from explaining and participating in discussions (Chizhik, 2001; Webb, 1985).

The picture of classroom mathematics learning we have described in this paper is problematic—especially given that some students who require help may hesitate to seek help in contexts where, despite the use of grouping in the classroom, value is placed more on individual competence and success. In our analysis focusing on immigrant students' mathematics learning experiences in Canadian schools, none of these students preferred group work over individualized work (Takeuchi & Towers, 2015). These students could not see the benefits of group work, even though newly-arrived immigrant students could have benefited from group work with peers who can draw out the expertise of immigrant students (Takeuchi, 2015).

Our research reminds us of the importance of creating a mathematics group work pedagogy that is deliberate, that embraces students' questions and dilemmas as a resource for meaningful mathematical learning, and that helps students to understand why they are being asked to work together and what they can learn from collaboration. Our findings suggest that there is a gap between teachers' use of group work in mathematics classrooms (which is widespread) and students' understanding of, and appreciation for, the potential benefits of this pedagogical approach. We see this as both a significant concern and a gap that is ripe for further study.

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