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COMPOSITE MEASURES FOR THE EVALUATION OF INVESTMENT PERFORMANCE

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I. Introduction

The composite measures of investment performance: the reward-to-variability index, by Sharpe ([29], [30]) and Lintner [23], and the reward-to-volatility index, by Treynor [33], were developed after Markowitz ([24], [25]) and Tobin [32] popularized the mean-variance framework of analyzing the problems of certain investments. Since these are ex ante measures they are not directly applicable to the evaluation of ex post performance. A theoretical basis for doing so has been provided by Jensen ([17], [18]) who also developed another composite performance measure, the predictability index. In practice, these composite measures have been found to have problems. Foremost, they have been observed to exhibit systematic biases. Various causes of the biases have been proposed. These are: the existence of unequal lending and borrowing rates, the failure to consider higher moments of return distributions, and the elusive "true" holding period.

The purpose of this paper is to examine if the biases could be caused by the deficiency of not considering asymmetry of return distributions and the inability to specify the correct holding period. In Section II the literature related to the problems is reviewed. In Section III we develop a composite performance measure based on recent advances in Capital Market theory. Section IV presents the methodology and results of empirical research. We summarize our findings and conclusions in Section V.

II. Problems With Composite Performance Measures

The evaluation of investment performance has traditionally been based on returns alone. The realization that the capital market is risk-averse necessitated the comparison of risk measures along with returns. However, simultaneous consideration of risk and return faces a trade-off problem that requires knowledge about investors' preferences to resolve. The mean-variance composite performance measures: Sharpe and

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Lintner's reward-to-variability index, Treynor's reward-to-volatility index and Jensen's predictability index eliminated this trade-off problem. It is no wonder that they have found wide appeal, especially in the evaluation of the performance of mutual funds ([7], [12], [17], [18], [19], [26], [27], [30], [31], [33]).

However, in practice, a serious problem has been found in the use of these composite measures. This is the systematic bias demonstrated by Friend and Blume [11] and Gaumnitz [12]. They showed that historically, all three composite measures exhibited systematically biased relationship with the risk measures. It was concluded in [11] that this is due to the invalidity of an assumption used in the development of the composite performance measures; the assumption being that of the existence of equal lending and borrowing rates and opportunities for all investors.

Ever since the composite performance measures were proposed, some researchers ([1], [2], [3], [6], [15], [16], [28]) have argued that the performance measures may be inadequate because of the failure to consider higher moments of the distributions of investment returns. Theoretical efforts ([2], [14], [15], [16]) to include higher moments have been made. Regrettably, these efforts have produced multiple criteria for performance evaluation. They have therefore reintroduced a trade-off problem that requires knowledge about investors' preferences to resolve. Empirically, an attempt [19] has been made to arrive at a composite measure that takes the possible asymmetrical distribution of returns into consideration by following Markowitz's [25] suggestion of using the semi-variance as the measure of risk, thereby resulting in a reward-to-semivariance index. However, the paper committed the error of designating as semivariance a measure that is more appropriately referred to as "half-variance."[1] Therefore, the results do not hold for semivariance. Markowitz's

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[1] Designating the two in symbols, we have:

$$\text{(Half-variance } HV_i = \frac{1}{n} \sum_{t=1}^{n} (R_{it} - \bar{R}_i)^2 ,$$

$$\text{where: } R_{it} - \bar{R}_i = \begin{cases} 0 & \text{if } R_{it} - \bar{R}_i > 0, \\ R_{it} - \bar{R}_i & \text{if } R_{it} - \bar{R}_i < 0. \end{cases}$$

$$\text{(Semi-variance) } SV_i = \frac{1}{n} \sum_{t=1}^{n} (R_{it} - h)^2 ,$$

$$\text{where: } R_{it} - h = \begin{cases} 0 & \text{if } R_{it} - h > 0, \\ R_{it} - h & \text{if } R_{it} - h < 0; \end{cases}$$

suggestion of using semivariance as the measure to take account of asymmetrical return distributions must therefore be considered as yet untested.

As pointed out by Hogan and Warren [13], semivariance, which permits decision makers to measure risk from some fixed point of reference, may be more consistent with investors' intuitive feel of risk as the failure to earn a target return. Even if return distributions are symmetrical, the semivariance still yields information different from the variance, because the target return is unique and may therefore be below or above the mean of the return distributions being considered. As implied in footnote 1, the half-variance yields information similar to the variance, when return distributions are symmetrical.

While the use of semivariance as a measure of risk is intuitively appealing, it is, in a sense, ad hoc because of the absence of a theoretical basis. Therefore, there is a need to develop a theoretically rigorous composite measure that takes higher moments into account. Section III presents a new measure, ER_i, that takes the third moment into account. This evaluation measure, similar to Jensen's, is better than the reward-to-semivariance index in two ways. First, its use is based on a rigorously developed and tested model. Second, a simultaneous test of statistical significance of performance differences is undertaken in the form of the t-statistic for ER_i. Statistical test of significance for performance comparisons using the reward-to-semivariance index is not

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[1]Continued

where: n = number of observations;

R_{it} = rate of return from asset i for the tth observation;

R_i = means return from asset i;

h = required or minimum acceptable return.

A preference ordering consistent in the Von Neumann-Morgenstern sense requires a utility function U(R) such that the expected utility,

$$E_i[U(R)] = \int_{-\infty}^{\infty} U(R)\, dF(R_i),$$

can be used to rank probability distributions of returns. U(R), which may be interpreted as the weighing function, must be the same for all the probability distributions being ranked.

Ranking by expected return and semivariance may be represented as investors having the following utility function:

$$U(R) = a + bR + c\,[\min(R-h), 0)]^2.$$

A consistent ranking by semivariance requires that h be independent of the probability distribution being ranked. Otherwise, the ranking will be using different weighting schemes for different distributions. Since ranking by the half-variance using the above utility function requires that h be equal to each of the means of the probability distributions being ranked, it allows consistent ranking only if all the means are equal. And only in that rare case will ranking by the half-variance be the same as ranking by the semivariance.

Actually, even half-variance is a misnomer because, empirically, the half-variance is not equal to the variance divided by two. Of course, statistically, the half-variance is the maximum-likelihood and hence consistent estimator of one-half the variance of a symmetric distribution.

possible until the distribution for the semivariance estimate, which is not Chi-square, is known.

The other inadequacy of composite measures examined in this study concerns the holding period that must be assumed before the performance measures can be calculated. Levy [22] has suggested that another cause of the systematic bias may be the assumed holding period used in calculating the performance measures. He showed, analytically, that even if the mean-variance capital market model on which the composite performance measures are based is correct, calculating the measures with an assumed holding period that is not equal to the true holding period will create a systematic bias. Jensen [18] has examined the holding period question and recommended a solution. Cheng and Deets [8] have argued that Jensen's solution to the holding period question is in error and have, in turn, recommended a different solution. More recently, Lee [21] has shown that if rates of return are lognormally distributed, the market has an instantaneous holding period and investors make their decisions on the basis of means and variances, then the Jensen and Cheng-Deets solutions are the same and correct. Since recent evidence ([20], [28]) seems to indicate that investors consider skewness, in addition to mean and variance, in portfolio decisions, the holding period question must be considered unsolved.

III. A Mean-Variance-Skewness Composite Performance Measure

Assume that in the capital market (1) there exists a risk-free rate (R_F) at which all investors are able to lend and borrow; (2) there is homogeneity of probability beliefs among investors; (3) investors make one-period investment decisions based on mean, variance, and skewness; and (4) investors prefer higher means and skewness but lower variance. Then, at equilibrium, Kraus and Litzenberger [20] have shown that given the market return, the risk-free rate, the systematic risk, and the systematic skewness of investment i, there will be a linear relationship between the expected return on the security or portfolio with the systematic risk and the systematic skewness. In symbols:

(1) $$\bar{R}_i = R_F + b_1\beta_i + b_2\gamma_i \qquad i = 1, \ldots\ldots, N$$

where

$\bar{R}_i$ = *ex ante* required or equilibrium expected return from security i given the market return, R_F, β_i, and γ_i;

R_F = risk-free rate;

β_i = systematic risk of security i,

= $Cov(R_i, R_m)/Var(R_M)$;

γ_i = systematic skewness

= $E\ [R_i - R_i)\ (R_M - R_M)^2]/E(R_M - R_M)^3]$;

$\bar{R}_M$ = expected market return;

$b_1 + b_2$ = $\bar{R}_M - R_F$, which follows directly from equation (1) since β_M and γ_M are equal to one.

The relationship states that if the expected market return, the systematic risk, and the systematic skewness of investment i are known, then there is a rate of return required from investment i. This required rate of return is the one calculated from equation (1). Therefore, if ex ante measures of return, risk, and skewness are available, the required rate of return can be used as a benchmark to evaluate the expected rate of return from investment i. If the expected return from investment i is lower (higher) than the required return, investment i is an inferior (superior) opportunity.

Ex ante measures are, of course, not available. Therefore, evaluations, by necessity, have to be based on ex post measures. Since equation (1) is an ex ante relationship, it is not applicable. An ex post required rate of return or benchmark is needed. The benchmark to be developed is in the form of the ex post market premiums for risk and skewness.

The basis for using the market portfolio as a benchmark lies in the results of Evans and Archer [9] showing that portfolios constructed with randomly selected securities easily approximate the market portfolio. It is argued that if an investor always has the alternative of being paid the market return, then the market return must be the minimum return that should be earned.

In the world of mean, variance, and skewness, the reasoning is similar, i.e., if an investor always has the alternative of being paid the market premiums for risk and skewness, then these premiums plus the risk-free rate must be the minimum return that should be earned. However, with mean, variance, and skewness, the required rate of return can be set only with, in addition to the market portfolio, an investment opportunity with zero systematic risk and nonzero systematic skewness. The zero systematic risk investment opportunity has been invoked before by other researchers ([4], [5]); therefore, it is not new. The new condition is that this investment opportunity must have nonzero systematic skewness.

It is worthwhile to note, intuitively, the necessity of the zero systematic risk but nonzero systematic skewness investment, in addition to the risk-free asset, or vice-versa. It is obvious that a particular systematic risk can be obtained by the proper combination of the risk-free asset and the market portfolio. However, when the combination is set to achieve a specific systematic risk, a particular systematic skewness will also be obtained for the resulting combination. Similarly, the investor can obtain a specific systematic skewness by the proper combination of the risk-free asset and the market portfolio but will lose control over the systematic risk. Therefore, with only the risk-free asset and the market portfolio as basic investments, the investor cannot achieve both the target systematic risk and systematic skewness. With the addition of the zero systematic risk but nonzero systematic skewness portfolio, both systematic risk and systematic skewness may then be set to specific levels. It is easy to imagine the alternative case of how the risk-free asset is also necessary for the purpose of achieving both desired systematic risk and systematic skewness.

One question remains. This is the question of whether the risk-free asset and the zero systematic risk but nonzero systematic skewness portfolio can co-exist. In the mean-variance market, they cannot. If the zero beta portfolio has a return higher than the risk-free rate, then the risk-free rate will not need to exist and vice-versa. If the returns are equal, again one is superfluous. However, in the mean-variance-skewness market, the zero-beta portfolio does not dominate the risk-free rate by a higher return; nor is it necessarily dominated by the risk-free asset if it has a lower return. It must both have a positive skewness and a return higher than or equal to the risk-free rate to dominate the latter and have both a negative skewness and return lower than or equal to the risk-free rate to be dominated. Therefore, the risk-free asset and the zero-beta, nonzero systematic skewness portfolio can co-exist as long as the portfolio has a negative skewness and higher return or a postive skewness and lower return. Whether the portfolio has the former or latter combination of skewness and return is an empirical question that is beyond the scope of this study.

The development of the required return follows the following steps. First, assume that we observe the rate of return of investment i over a past period. Simultaneously, we observe its systematic risk and systematic skewness. If it had been possible to uniquely combine the risk-free asset, the market portfolio, and the zero systematic risk, nonzero systematic skewness investment in such a way that the systematic risk and systematic skewness of the combination equaled those of investment i, then the return on the combination over the same past period must have been obtainable by investor. The return on the combination may, therefore, serve as the benchmark or minimum rate of return for all investment opportunities with the same systematic risk and systematic skewness over the past period.

The objectives of the analytical development that follow are, therefore, to determine the unique proportions of the risk-free asset, the market portfolio, and investment Z needed to set the required return; and to show that these proportions are a priori achievable. The benchmark return is also obtained as part of the development.

Let the return, R_i, the systematic risk, β_i, and the systematic skewness, γ_i, for a past period be given. Further, let the return from the risk-free asset, R_F; the return, R_Z, from Black's zero beta portfolio Z which has zero systematic risk and nonzero systematic skewness, γ_Z; and the return from the market portfolio, R_M also be given. An investment that is a combination of the risk-free asset, the zero beta portfolio Z, and the market portfolio in the proportions X_F, S_Z, and $(1-X_Z-X_F)$, respectively, should have earned a return defined as:

(2) $$R = x_F R_F + x_Z R_Z + (1-X_Z-X_F)\ R_M$$

To determine what this return should be if the systematic risk and systematic skewness were equal to those of investment i, we set the systematic risk and systematic skewness of the portfolio in equation (2) to β_i and γ_i, respectively. Then we solve the equations for X_F and X_Z.

By definition,

$$(3) \qquad \beta = \mathrm{Cov}(R,\ R_M)/\mathrm{Var}(R_M).$$

Substituting equation (2) into (3) and setting β equal to β_i,

$$(4) \qquad \beta_i = \mathrm{Cov}[x_F R_F + x_Z R_Z + (1-x_F-x_Z)R_M, R_M]/\mathrm{Var}(R_M).$$

Since R_F is riskless, we have

$$(5) \qquad \beta_i = \mathrm{Cov}[x_Z R_Z + (1-x_F-x_Z)R_M, R_M]/\mathrm{Var}(R_M).$$

By our assumption that portfolio Z has zero systematic risk, $\mathrm{Cov}(R_Z, R_M) = 0$. Therefore,

$$(6) \qquad \beta_i = (1-x_F-x_Z).$$

Also by definition,

$$(7) \qquad \gamma = E[R-\bar{R})(R_M-\bar{R}_M)^2]/E[(R_M-\bar{R}_M)^3].$$

Again substituting equation (2), observing that R_F is riskless, and setting γ equal to γ_i, we have

$$(8) \qquad \gamma_i = (1-x_Z-x_F) + E[x_Z(R_Z-\bar{R}_Z)\ (R_M-\bar{R}_M)^2]/E[(R_M-\bar{R}_M)^3].$$

This may be expressed simply as

$$(9) \qquad \gamma_i = x_Z\gamma_Z + (1-x_Z-x_F),$$

where $\gamma_Z = E[(R_Z-\bar{R}_Z)(R_M-\bar{R}_M)^2]$ and is nonzero by our assumption.

Solving (6) and (9) simultaneously for x_F and x_Z, we have:

$$(10) \qquad x_Z = \frac{\gamma_i-\beta_i}{\gamma_Z}$$

$$(11) \qquad x_F = 1-\beta_i - \frac{\gamma_i-\beta_i}{\gamma_Z}$$

This shows that the mixture of the risk-free asset, the zero beta portfolio, and the market portfolio to achieve any combination of systematic risk and systematic skewness is unique. Moreover, it shows that once the systematic skewness of the zero beta portfolio is known, investors can set the unique mixture by simply setting their desired systematic risk and systematic skewness levels. Substituting equations (10) and (11) into (2), we get:

$$(12) \qquad R = (1-\beta_i - \frac{\gamma_i-\beta_i}{\gamma_Z})R_F + (\frac{\gamma_i-\beta_i}{\gamma_Z})R_Z + \beta_i R_M .$$

Collecting terms, we have the benchmark or minimum return,

$$(13) \qquad R = R_F + [R_M - R_F - \frac{(R_Z-R_F)}{\gamma_Z}]\ \beta_i + \frac{(R_Z-R_F)}{\gamma_Z}\gamma_i .$$

It is interesting to note that if γ_i is zero, equation (13) does not reduce to the mean-variance security market line. In fact, the required return is higher than that of the mean-variance model. This is so because (R_Z-R_F) and γ_Z should always have opposite signs, as long as skewness is preferred. Intuitively, this should be obvious because if the market values skewness, then an investment without skewness has to be required to yield a higher return than if the market is indifferent to skewness.

Equation (13) shows an <u>ex post</u> linear relationship among R_i, β_i, and γ_i. This defines a rate of return that an investor can attain and therefore can be used as the required return on investments with systematic risk and systematic skewness equal to those of investment i. Hence, we can define a measure of performance, which we shall call an Excess Return index, for an investment as:

$$(14) \qquad ER = R\ (\text{actual}) - R\ (\text{required})$$

where R (required) is as defined in equation (13).

Conceptually, this performance measure requires more information to apply than that derived from the mean-variance market model by Jensen ([17], [18]). In addition to information about the market portfolio and the risk-free asset, it requires knowledge on the part of the investor about the return and systematic skewness of a zero systematic risk and nonzero investment opportunity. However, in practice, similar to Jensen's predictability index, ER is simply the intercept of the characteristic equation corresponding to the mean-variance-skewness market line (1). It has been shown in [20] that the characteristic equation is:

$$(15) \qquad (R_i - R_F) = ER_i + C_{1i}(R_M - R_F) + C_{2i}\,(R_M - \bar{R}_M)^2 + e_i.$$

Using this Excess Return index wil therefore require information about R_i, R_M, and R_F only, as in the mean-variance case.

The validity of this performance evaluation measure depends on how well equation (1) describes the market. Kraus and Litzenberger [20] have presented empirical results to show that equation (1) may be a better description of the market than the mean-variance model. A further test of the validity of this measure will be whether the measure exhibits systematic bias as the mean-variance composite measures do. This test has been performed and the results are presented in Section IV.

IV. Empirical Research Methodology

Empirical tests were conducted to examine two things. First, the composite performance measures: Sharpe and Lintner's reward-to-variability index, Treynor's reward-to-volatility index, Jensen's predictability index, reward-to-semivariance index, the excess return index developed here, and for completeness, the reward-to-half-variance index used in [19], were tested for systematic biases. The results were to indicate if the failure to consider the possible asymmetry of return distributions is the sole cause of the systematic biases. Next, we examined how the holding period length used in calculating the measures affected the systematic biases.

The sample used consisted of all the funds actively traded and whose prices were reported in Barron's from January 1955 to January 1974. In total, 111 funds were used. The prices of each fund on Monday, in the first week of each quarter from January 1955 on were collected. Seventy-seven such observations were collected for the period studied. The prices were adjusted for stock dividends and stock splits. Rate of return on each fund for each quarter were computed as:

$$(16) \qquad R_{it} = \frac{P_{i,t+1} + D_{it} - P_{it}}{P_{it}} \qquad \begin{array}{l} i = 1,\ldots\ldots, 111 \\ t = 1,\ldots\ldots, \ 76 \end{array}$$

where P_{it} is the price of the ith fund at the beginning of the tth period and D_{it} is the combined dividend and capital gains from transactions. Standard & Poor's 500 Composite Stock Index was used as the market portfolio. The yield on 3-month Treasury Bills was used as the risk-free rate.

The composite measures tested were the following:

Sharpe and Lintner's reward-to-variability index:

$$(17) \qquad S_i = \frac{\bar{R}_i - R_F}{\sigma_i}$$

where σ_i was the standard deviation for fund i's return.

Treynor's reward-to-volatility index:

$$(18) \qquad T_i = \frac{\bar{R}_i - R_F}{\beta_i},$$

Jensen's predictability index:

(19) J_i = Average ($R_i - R_F$) - (β_i) x Average ($R_m - R_F$),

where J_i and β_i were obtained from the characteristic equation:

(20) $(R_{it} - R_{Ft}) = J_i + \beta_i (R_{Mt} - R_{Ft}) + e_{it}$

with classical assumptions about the distributions of e_{it}, i = 1,....,111.

Reward-to-half-variance index:

(21) $$RHV_i = \frac{\bar{R}_i - R_F}{\sqrt{HV_i}},$$

where HV_i was defined as in footnote 1.

Reward-to-semivariance index:

(22) $$RSV_i = \frac{\bar{R}_i - R_F}{\sqrt{SV_i}}$$

where SV_i was as defined in footnote 1 with R_F = h.

Excess return index:

(23) ER_i = Average ($R_{it} - R_{Ft}$) - C_{1i} Average ($R_{Mt} - R_{Ft}$) - C_{2i} Average $(R_{Mt} - \bar{R}_M)^2$,

where C_{1i} and C_{2i} were obtained from the characteristic equation (15).

All six composite performance measures were calculated for the 111 mutual funds and for the S&P 500 index for eight different holding periods: One quarter to eight quarters. The different holding period returns were computed as follows:

(24) $$R^N_{iT} = \sum_{t = (T-1)xN + 1}^{(T-1)xN + N} (1 + R^1_{it}) \qquad T = 1,....., \text{integer } \frac{76}{N}$$

where R^N_{iT} is the N-quarter rate of return from mutual fund i for period T and R^1_{it} is the quarterly rate of return from mutual fund i for period t.

Then, for each holding period length, regressions of each composite performance on the corresponding risk measure were run cross-sectionally. To illustrate: the 111 Sharpe-Lintner indexes were regressed against the 111 standard deviations; the Treynor indexes against the systematic risks; Jensen's indexes against the systematic risks; the reward-to-half-variance against the half-standard deviation; the reward-to-semivariance against the semi-standard deviation. To facilitate comparison, the excess return indexes were regressed against the systematic risks alone. In addition, the excess return indexes were also regressed against the systematic skewnesses and the systematic risks.[2]

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[2]The systematic risks and systematic skewnesses were determined as follows:

$$\beta_i = C_{1i} + C_{2i}\,(M^3_M/\sigma^2_M) \qquad i = 1,\ldots, 111$$

$$\gamma_i = C_{1i} + C_{2i} \quad [k^4_M - (\sigma^2_M)^2]/M^3_M$$

where:

β_i = systematic risk for fund i;

γ_i = systematic skewness for fund i;

C_{1i} = regression coefficient for the linear term of the characteristic equation for fund i;

C_{2i} = regression coefficient for the quadratic term of the characteristic equation for fund i;

M^3_M = $E[(RM - \bar{R}_M)^3]$;

σ^2_M = $E[(RM - \bar{R}_M)^2]$;

k^4_M = $E[(RM - \bar{R}_M)^4]$.

The rationale for the above measurements are as follows:

Given a portfolio or security in the mean-variance-skewness market, then

$$R_i - R_F = ER_i + C_{1i}(R_M - R_F) + C_{2i}(R_M - \bar{R}_M)^2 + e_i.$$

Taking the expected values, we have

$$\bar{R}_i - R_F = ER_i + C_{1i}(\bar{R}_M - R_F) + C_{2i}\,\sigma_M{}^2 + e_i.$$

By the classical assumptions in regression, $\bar{e}_i = 0$. In an efficient market, $\overline{ER}_i$ should also be zero. Therefore, expressing returns in terms of deviation from the mean,

$$R_i - \bar{R}_i = ER_i + C_{1i}(R_M - \bar{R}_M) + C_{2i}[(R_M - \bar{R}_M)^2 - \sigma_M{}^2] + e_i.$$

Multiplying both sides of the above deviation form equation by $(R_M - \bar{R}_M)$, taking the expected values, and dividing by $\sigma_M{}^2$, we have

$$\beta_i = \frac{E[(R_i - \bar{R}_i)(R_M - \bar{R}_M)]}{\sigma_M{}^2}$$

$$= E[(ER_i)(R_M - \bar{R}_M)] + C_{1i} + \frac{C_{2i}M^3{}_M}{\sigma_M{}^2} .$$

In an efficient market, there should be no systematic relationship between the market and excess returns. Therefore, the first term is equal to zero and

$$\beta_i = C_{1i} + \frac{C_{2i}M^3{}_M}{\sigma_M{}^2} .$$

Similarly, the systematic skewness is derived by multiplying both sides of the deviation form equation by $(R_M - \bar{R}_M)^2$, taking expected values and dividing both sides by M^3_M.

The results of the evaluation of quarterly performance are presented in Table 1. The results for the other holding periods are not presented but are summarized in Table 2. Tables 3 and 4 summarize the results of the tests for systematic biases.

Tables 1 and 2 show that, as expected by Arditti [2], taking skewness into consideration does increase the number of mutual funds judged superior. However, as a whole, the mutual fund industry does not offer better investment opportunities than the market portfolio. For example, Table 2 shows that, even in the best case of seven-quarter holding periods, only approximately 20 percent of the total number of funds were judged superior. Our results therefore confirm the findings of earlier researchers ([17], [18], [30], [31], [33]) that mutual fund managers have not been able to "beat" the market.

The main concern of this study is the systematic bias the composite performance measures exhibit. Table 3 confirms other researchers' ([11], [12]) results and showed systematic bias when Sharpe's index, Treynor's index, and Jensen's index were used to evaluate performances. Moreover, our results show that when holding periods were lengthened, the biases remained significant.

It shows that the reward-to-half-variance index also exhibited systematic bias for all the holding periods tested. This is contrary to the results in [19] where the reward-to-half-variance index was shown to eliminate the systematic bias. Since this study used a larger sample of mutual funds (111 versus 40) over a longer time period (76 quarters encompassing the 24 quarters), it may be concluded that the results shown in [19] are not valid. Moreover, since the use of the half-variance commits a theoretical error, the performance measure should not be used.

The results for the composite measures that take into account the possible asymmetrical distribution of rates of return show that for holding periods from one to four quarters, the measures still exhibited systematic bias. With a holding period of five quarters, the reward-to-semivariance index no longer exhibited a bias although the excess return index still did. With holding period lengths of six and seven quarters, neither measure exhibited a bias. With holding period length equal to eight quarters, the excess return index still showed no bias, although the reward-to-semivariance started to show it again. It would have been interesting to continue to lengthen the holding period, but due to limitations imposed by the data, this could not be done. However, these results do point out a very strong case for taking account of skewness in the evaluation of investment performance. They also point out that selecting the holding period is a crucial step. The excess return index was also regressed against the systematic risk and systematic skewness simultaneously. The results, as presented in Table 4, are similar to those presented in Table 3.

The differences in behavior between the mean-variance composite measures and the measures that take into account asymmetrical distributions are even more striking when Figure 1 is examined The biases of the mean-variance composite measures showed either no trend or increasing trends.[3] The bias of Sharpe's index has shown no definite

[3] Readers who are familiar with Levy's article [20] will find that the effects of holding period length on the composite performance measures, as determined empirically here, did not conform with his theoretical results. However, this has no implication for Levy's results which were based on the assumption that we have a mean-variance market.

trend with the increase of holding period length. Those of Treynor's index and Jensen's index both actually increased with increasing holding period length.[4]

On the other hand, the biases of the two measures that take into account possible asymmetrical return distributions showed dramatic drops in magnitude at holding period lengths of 6, 7, and 8 quarters. The results therefore, show that only the 6, 7, and 8 quarter performance evaluations using the excess return index and the 5, 6, and 7 quarter performance evaluations using the reward-to-semivariance index are interpretable, during the period from January 1955 to January 1974. These results also indicate that both the consideration of asymmetry of return distributions and the use of the proper holding period are important.

While the excess return index and the RSV may be used to take account of the asymmetry of the return distributions, the proper holding period is more difficult to find. It is recommended here that an analysis like this study be performed to choose the proper holding period length for evaluations. The holding period length should be chosen such that no systematic bias is exhibited by the performance measures. This gives the full benefit of the doubt to mutual fund managers. Although our results seem to show that at the holding periods when the systematic bias disappeared, more mutual funds were judged to have superior performance, it is not known if this will always be so.

V. Summary and Conclusions

We have examined the mean-variance composite measures and found them to be unsatisfactory, specifically because they have been found to exhibit systematic bias. The literature has also suggested that these composite measures may be inadequate because of their inherent inability to take the possible asymmetry of return distributions into consideration. Furthermore, it has been suggested that one possible cause of the systematic bias may be the failure to identify the appropriate holding period.

This study first developed an Excess Return index as a performance measure based on a recently published mean-variance-skewness market model. Then this index and the other composite measures were tested for systematic bias. Additionally, the effect of changing the holding period length on the systematic biases was studied.

Results showed that the performance measures that also considered the asymmetry of return distributions, in addition to the mean and variance, were better. This conclusion was arrived at by observing that the systematic biases could be removed through changing the holding period length for the excess return index and the reward-to-semivariance index while they could not be removed for the mean-variance composite measures.

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[4] The fact that the systematic biases of the Jensen and Treynor indexes increase in value should not be interpreted to mean that at shorter holding periods, the systematic biases will disappear. This is so because there is no apparent tendency for the t-statistics of the biases to approach insignificant levels. Friend and Blume [11] have shown that biases exist for holding periods equal to one month. Moreover, the typically less than once-per-year average portfolio turnover rate of mutual funds would indicate the reasonableness of longer holding periods.

Although the tests for the effects of holding period length seemed to indicate that, for the time period tested, the systematic biases were removed when the assumed holding period was between five and eight quarters, we cannot conclude that the elusive "true" holding period is found in that range. This is so because investors probably change their holding period lengths in response to perceived uncertainty.

A by-product of this study is the presentation of seven portfolio measures for 111 mutual funds over 76 quarters from January 1955 to January 1974. The results supported earlier evidence that mutual fund managers have not been able to "beat" the market.

TABLE 1

RESULTS OF MUTUAL FUND EVALUATION WITH HP=1 QTRS

MF NO.	SHARPE	TREYNOR	JENSEN	RSV	RHV	EXCESS RETURNS
1	-0.0196	-0.0106	-0.0031	-0.0236	-0.0238	-0.0170
2	-0.1053	-0.0080	-0.0108	-0.1341	-0.1451	-0.0102
3	-0.3209	-0.0359	-0.0163	-0.3464	-0.4318	-0.0179
4	-0.1082	-0.0081	-0.0122	-0.1304	-0.1397	-0.0102
5	-0.1723	-0.0133	-0.0164	-0.1998	-0.2239	-0.0161
6	-0.1305	-0.0118	-0.0152	-0.1600	-0.1749	-0.0155
7	-0.1851	-0.0180	-0.0179	-0.2074	-0.2300	-0.0149
8	-0.0234	-0.0021	-0.0071	-0.0338	-0.0345	-0.0080
9	-0.1568	-0.0122	-0.0153	-0.1842	-0.2044	-0.0140
10	-0.1955	-0.0169	-0.0209	-0.2041	-0.2198	-0.0031
11	-0.0551	-0.0041	-0.0081	-0.0697	-0.0723	-0.0066
12	-0.0830	-0.0060	-0.0109	-0.1055	-0.1114	-0.0099
13	-0.0410	-0.0042	-0.0100	-0.0486	-0.0496	-0.0136
14	-0.0484	-0.0042	-0.0070	-0.0652	-0.0674	-0.0051
15	-0.0416	-0.0038	-0.0101	-0.0564	-0.0582	-0.0060
16	0.0062	0.0008	-0.0037	0.0080	0.0079	0.0010*
17	-0.0977	-0.0076	-0.0105	-0.1194	-0.1271	-0.0073
18	-0.1911	-0.0146	-0.0132	-0.2217	-0.2515	-0.0128
19	-0.0821	-0.0076	-0.0139	-0.0949	-0.0990	-0.0139
20	-0.1853	-0.0159	-0.0155	-0.2190	-0.2467	-0.0167
21	-0.1431	-0.0116	-0.0141	-0.1731	-0.1900	-0.0137
22	-0.1522	-0.0141	-0.0203	-0.1829	-0.2017	-0.0134
23	-0.0980	-0.0088	-0.0139	-0.1310	-0.1412	-0.0099
24	-0.0560	-0.0048	-0.0119	-0.0733	-0.0762	-0.0136
25	-0.0214	-0.0017	-0.0098	-0.0275	-0.0279	-0.0052
26	-0.2394	-0.0187	-0.0171	-0.2753	-0.3257	-0.0180
27	-0.0431	-0.0035	-0.0091	-0.0597	-0.0614	-0.0093
28	0.0934	0.0076*	0.0014*	0.1293	0.1214	0.0048*
29	-0.1965	-0.0156	-0.0123	-0.2270	-0.2590	-0.0111
30	-0.0381	-0.0032	-0.0080	-0.0524	-0.0539	-0.0027
31	-0.0420	-0.0037	-0.0097	-0.0528	-0.0541	-0.0097
32	-0.1387	-0.0164	-0.0163	-0.1538	-0.1645	-0.0069
33	-0.0522	-0.0045	-0.0114	-0.0679	-0.0702	-0.0054
34	0.0244	0.0019	-0.0055	0.0281	0.0278	0.0140*
35	-0.0305	-0.0037	-0.0062	-0.0420	-0.0429	0.0094*

* Performance Superior to the market

MF NO.	SHARPE	TREYNOR	JENSEN	RSV	RHV	EXCESS RETURNS
36	-0.2081	-0.0158	-0.0140	-0.2391	-0.2750	-0.0138
37	-0.1703	-0.0164	-0.0224	-0.2113	-0.2397	-0.0168
38	-0.1334	-0.0111	-0.0124	-0.1668	-0.1840	-0.0119
39	-0.2367	-0.0216	-0.0139	-0.2712	-0.3201	-0.0135
40	-0.0141	-0.0011	-0.0075	-0.0180	-0.0181	-0.0011
41	-0.0352	-0.0027	-0.0075	-0.0476	-0.0489	-0.0078
42	-0.2061	-0.0185	-0.0148	-0.2333	-0.2660	-0.0134
43	-0.1737	-0.0130	-0.0121	-0.2095	-0.2363	-0.0141
44	-0.0362	-0.0029	-0.0094	-0.0440	-0.0450	-0.0045
45	-0.1412	-0.0110	-0.0115	-0.1694	-0.1861	-0.0117
46	-0.1673	-0.0127	-0.0131	-0.1999	-0.2237	-0.0127
47	-0.1107	-0.0104	-0.0200	-0.1274	-0.1352	-0.0163
48	-0.0315	-0.0023	-0.0078	-0.0397	-0.0405	-0.0040
49	-0.0498	-0.0035	-0.0098	-0.0619	-0.0639	-0.0077
50	0.0293	0.0025	-0.0044	0.0408	0.0401	-0.0074
51	-0.1808	-0.0133	-0.0117	-0.2071	-0.2320	-0.0089
52	-0.4785	-0.0690	-0.0120	-0.4804	-0.6571	-0.0136
53	-0.0205	-0.0016	-0.0065	-0.0258	-0.0261	-0.0025
54	0.0642	0.0050	-0.0010	0.0852	0.0820	0.0044*
55	-0.6722	-0.2545	-0.0141	-0.5847	-0.9313	-0.0146
56	-0.5470	-0.0641	-0.0142	-0.5178	-0.7750	-0.0133
57	-0.3428	-0.0338	-0.0163	-0.3604	-0.4408	-0.0154
58	-0.2993	-0.0226	-0.0164	-0.3257	-0.3969	-0.0160
59	-0.0019	-0.0001	-0.0072	-0.0024	-0.0024	-0.0042
60	-0.0509	-0.0044	-0.0084	-0.0640	-0.0662	-0.0021
61	-0.1085	-0.0098	-0.0131	-0.1335	-0.1426	-0.0082
62	-0.0313	-0.0033	-0.0130	-0.0393	-0.0400	-0.0051
63	0.0017	0.0001	-0.0087	0.0022	0.0022	0.0001*
64	-0.1192	-0.0132	-0.0202	-0.1347	-0.1426	-0.0173
65	-0.1190	-0.0110	-0.0127	-0.1539	-0.1679	-0.0134
66	-0.2919	-0.0239	-0.0156	-0.3274	-0.4021	-0.0147
67	-0.0357	-0.0040	-0.0109	-0.0511	-0.0524	-0.0022
68	-0.1449	-0.0122	-0.0117	-0.1792	-0.1970	-0.0100
69	-0.2918	-0.0277	-0.0271	-0.3059	-0.3601	-0.0268
70	0.0806	0.0102*	0.0040*	0.1204	0.1136	0.0101*
71	-0.0627	-0.0047	-0.0104	-0.0770	-0.0800	0.0056
72	0.0915	0.0073*	0.0010	0.1274	0.1202	0.0034*
73	-0.0964	-0.0075	-0.0086	-0.1188	-0.1262	-0.0094
74	-0.1307	-0.0106	-0.0147	-0.1581	-0.1723	-0.0151

* Performance superior to market.

MF NO.	SHARPE	TREYNOR	JENSEN	RSV	RHV	EXCESS RETURNS
75	-0.0954	-0.0090	-0.0112	-0.1213	-0.1292	-0.0090
76	-0.1546	-0.0127	-0.0112	-0.1991	-0.2255	-0.0180
77	0.0930	0.0070*	0.0008*	0.1245	0.1178	0.0061*
78	-0.2469	0.0200	-0.0164	-0.2721	-0.3154	-0.0152
79	-0.3714	-0.0375	-0.0200	-0.3705	-0.4512	-0.0174
80	-0.0370	-0.0029	-0.0103	-0.0473	-0.0484	-0.0060
81	-0.2862	-0.0263	-0.0170	-0.3201	-0.3851	-0.0162
82	-0.1400	-0.0122	-0.0127	-0.1814	-0.2014	-0.0105
83	-0.1472	-0.0122	-0.0154	-0.1786	-0.1977	-0.0152
84	-0.1139	-0.0091	-0.0137	-0.1390	-0.1505	-0.0122
85	-0.2547	-0.0207	-0.0159	-0.2864	-0.3397	-0.0178
86	-0.0069	-0.0005	-0.0072	-0.0091	-0.0091	-0.0074
87	-0.1000	-0.0075	-0.0127	-0.1225	-0.1308	-0.0097
88	0.0350	0.0027	-0.0028	0.0484	0.0472	0.0001*
89	-0.0484	-0.0037	-0.0076	-0.0641	-0.0664	-0.0068
90	-0.0698	-0.0064	-0.0088	-0.0864	-0.0900	-0.0053
91	0.0084	0.0008	-0.0058	0.0098	0.0098	-0.0037
92	-0.0306	-0.0031	-0.0078	-0.0389	-0.0396	-0.0006
93	-0.1166	-0.0096	-0.0149	-0.1381	-0.1482	-0.0107
94	-0.0144	-0.0013	-0.0076	-0.0201	-0.0203	-0.0065
95	-0.1360	-0.0097	-0.0128	-0.1678	-0.1846	-0.0124
96	-0.0713	-0.0059	-0.0090	-0.0911	-0.0956	-0.0084
97	-0.0816	-0.0065	-0.0111	-0.0999	-0.1054	-0.0092
98	-0.0314	-0.0025	-0.0063	-0.0405	-0.0413	-0.0044
99	-0.0527	-0.0045	-0.0101	-0.0711	-0.0738	-0.0088
100	-0.0557	-0.0045	-0.0092	-0.0699	-0.0724	-0.0035
101	-0.0304	-0.0022	-0.0079	-0.0391	-0.0398	-0.0060
102	-0.0469	-0.0032	-0.0084	-0.0597	-0.0616	-0.0074
103	0.0242	0.0028	-0.0038	0.0386	0.0380	0.0124*
104	0.0894	0.0093*	0.0026*	0.1491*	0.1385*	0.0056*
105	-0.0682	-0.0066	-0.0164	-0.0913	-0.0960	-0.0189
106	-0.1772	-0.0141	-0.0165	-0.2185	-0.2493	-0.0195
107	-0.0598	-0.0046	-0.0091	-0.0748	-0.0776	-0.0073
108	-0.0042	-0.0003	-0.0061	-0.0054	-0.0054	-0.0030
109	-0.2149	-0.0168	-0.0143	-0.2539	-0.2964	-0.0146
110	-0.1220	-0.0100	-0.0101	-0.1458	-0.1569	-0.0070
111	-0.0993	-0.0083	-0.0115	-0.1279	-0.1373	-0.0086
Market	0.0959	0.0062	0.0000	0.1338	0.1254	0.0000

* Performance superior to the market.

TABLE 2

THE NUMBER OF MUTUAL FUNDS THAT OUTPERFORMED THE MARKET (JANUARY 1955 TO DECEMBER 1974) ACCORDING TO THE DIFFERENT COMPOSITE PERFORMANCE MEASURES

Holding period length (qtrs.)	Performance Measure					
	Sharpe	Treynor	Jensen	Excess return	RSV	RHV
1	0	5	5	10	1	1
2	0	5	5	9	3	2
3	0	3	3	9	0	0
4	0	5	5	6	1	2
5	0	4	4	9	1	1
6	5	7	7	4	5	5
7	4	7	7	21	5	5
8	3	7	7	10	4	4

TABLE 3

REGRESSION RESULTS FOR PERFORMANCE MEASURES VERSUS CORRESPONDING RISK MEASURES

	Holding Period (quarters)							
	1	2	3	4	5	6	7	8
1. Sharpe's index versus standard deviation:								
slope	3.5	3.7	3.6	3.5	3.8	3.3	3.5	3.4
	(8.6)	(9.1)	(8.6)	(8.5)	(8.3)	(7.6)	(7.2)	(5.9)
intercept	-0.36	-0.50	-0.65	-0.72	-0.84	-0.73	-0.80	-0.81
	(-11.7)	(-11.8)	(-11.4)	(-11.6)	(-12.2)	(-9.2)	(-9.5)	(-8.0)
R^2	0.60	0.57	0.60	0.60	0.62	0.66	0.68	(0.76)
2. Treynor's index versus systematic risk:								
slope	0.06	0.11	0.11	0.20	0.21	0.24	0.19	0.31
	(7.3)	(9.1)	(9.5)	(7.8)	(11.0)	(7.5)	(9.4)	(7.8)
intercept	-0.06	-0.11	-0.12	-0.21	-0.23	-0.24	-0.21	-0.31
	(-8.6)	(-10.6)	(-11.6)	(-9.3)	(-14.1)	(-8.6)	(-11.3)	(-9.2)
R^2	0.67	0.57	0.55	0.65	0.47	0.64	0.55	0.64
3. Jensen's index versus systematic risk:								
slope	0.005	0.012	0.016	0.022	0.025	0.027	0.039	0.023
	(2.4)	(3.0)	(2.7)	(2.7)	(2.6)	(2.2)	(2.8)	(1.9)
intercept	-0.02	-0.03	-0.05	-0.07	-0.08	-0.09	-0.11	-0.10
	(-8.4)	(-9.2)	(-9.4)	(-9.2)	(-9.9)	(-8.5)	(-8.4)	(-6.9)
R^2	0.95	0.93	0.94	0.94	0.94	0.96	0.94	0.99
4. Reward-to-half-variance versus half-standard deviation:								
slope	5.94	6.42	6.91	6.62	6.62	6.25	6.87	6.96
	(8.25)	(8.97)	(8.57)	(8.30)	(7.72)	(6.74)	(7.37)	(6.39)
intercept	-0.46	-0.71	-0.91	-0.99	-1.10	-0.97	-1.16	-1.20
	(-11.3)	(-11.7)	(-11.3)	(-11.4)	(-11.7)	(-8.3)	(-9.4)	(-8.5)
R^2	0.38	0.42	0.40	0.39	0.35	0.29	0.33	0.27

( ) indicates t-values

* insignificant even at 10%

TABLE 3 (Cont'd)

5. Reward-to-semi-variance versus semi-standard deviation:

slope	3.80	3.31	2.47	1.48	0.72*	-1.19*	-0.99*	-2.98
	(5.7)	(5.3)	(3.7)	(2.2)	(1.0)	(-1.4)	(-1.1)	(-3.1)
intercept	-0.34	-0.44	-0.45	-0.39	-0.37	0.04*	-0.03*	0.26
	(-8.6)	(-7.8)	(-6.1)	(-4.7)	(-4.1)	(0.3)	(-0.2)	(1.8)
R^2	0.23	0.20	0.11	0.04	0.01	0.02	0.01	0.08

6. Excess return index versus systematic risk:

slope	0.013	0.025	0.037	0.036	0.032	0.004*	0.024*	-0.037*
	(4.8)	(4.5)	(4.8)	(3.6)	(2.6)	(0.3)	(1.0)	(-1.5)
intercept	-0.019	-0.040	-0.064	-0.070	-0.076	-0.073	-0.063	-0.027*
	(-8.5)	(-8.2)	(-8.6)	(-7.9)	(-7.2)	(-5.6)	(-2.7)	(-1.3)
R^2	0.17	0.16	0.17	0.10	0.06	0.00	0.01	0.02

( ) indicates t-values.

* insignificant even at 10%

TABLE 4

REGRESSION RESULTS FOR THE EXCESS RETURN INDEX VERSUS SYSTEMATIC RISK AND SYSTEMATIC SKEWNESS

	Holding Period (quarters)							
	1	2	3	4	5	6	7	8
Coefficient for risk	0.009 (1.9)	0.012 (1.9)	0.034 (2.0)	0.028* (1.4)	-0.012* (-0.76)	0.023* (0.82)	-0.071* (-1.9)	-0.059 (-1.8)
Coefficient for skewness	0.05* (1.1)	0.104* (1.2)	0.026* (0.2)	0.047* (0.5)	0.313 (3.8)	-0.100* (-0.8)	0.529 (3.2)	0.139* (1.0)
Intercept	-0.019 (-8.5)	-0.039 (-8.2)	-0.064 (-8.5)	-0.070 (-7.9)	-0.084 (-8.3)	-0.071 (-5.3)	-0.065 (-2.9)	-0.032 (-1.5)
R^2	0.18	0.17	0.17	0.11	0.17	0.01	0.10	0.03

( ) indicates t-values.

* insignificant at 10%.

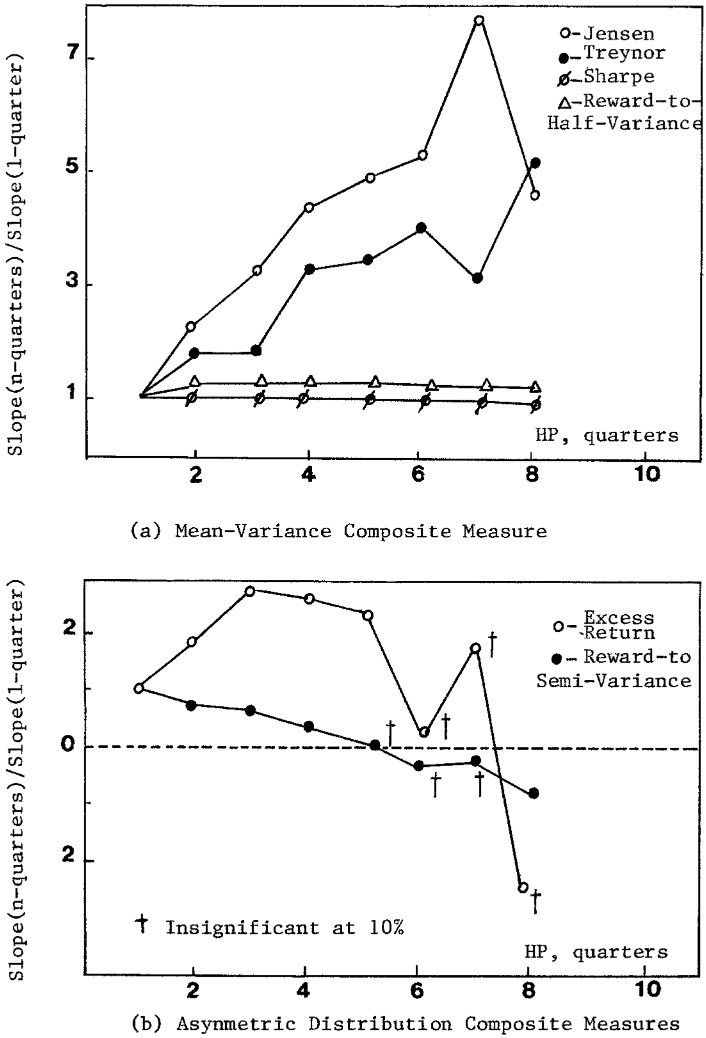

Figure 1. The Response of the systematic biases of the composite measures to increased assumed holding period.

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