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JUXTAPOSING MATHEMATICAL EXTENSIONS WITH COGNITIVELY LOADED QUESTIONS IN THE MATHEMATICS CLASSROOM

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Providing mathematical extensions (i.e. bonus questions) intended to evoke deep mathematical thinking after students complete assigned tasks is challenging for teachers. In this paper, we use the Variation Theory of Learning to challenge a common misconception that mathematical extensions should include many interrelated elements and impose a high cognitive load to promote deeper thinking. We present an analysis of observed extensions and provide alternative routes. Pedagogical implications for the design of mathematical extensions are presented.

Keywords: Mathematical knowledge for teaching; mathematical extensions

INTRODUCTION AND PURPOSE

In the field of mathematics education, the fact that students learn at different paces is well established. One explanation for this phenomenon is that different students require various durations of learning time to attain the same degree of mastery (Bloom, 1981; Carroll, 1963). As a result, mathematics educators have developed strategies to enrich learning for those early finishers. For instance, Guskey (2010) suggested offering enrichment or extension activities to those students who have mastered the instructed concept. Similarly, Mighton (2011) proposed the

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idea of bonuses, which are in-class activities, intended to extend and deepen students' mathematical understanding.

From the onset in the Math Minds initiative – a five-year partnership between a funder, a large school district, a mathematical charity, a children's support group, and a university education faculty – we encouraged teachers to provide students with lesson-focused extensions or bonus questions (see Preciado Babb, Metz, Sabbaghan, & Davis, 2015). However, we did not precisely define what bonus questions were. Instead, we presented strategies outlining possibilities for development that drew from the resource used in the initiative. An analysis of documented classroom observations and teacher interviews indicated that teachers perceived bonus questions as extra challenges, and indicated that the purpose of such extensions was to deepen students' mathematical understanding (Preciado-Babb, et al., 2015). Furthermore, our inquiry revealed that many of the bonus questions offered in class demanded higher levels of intrinsic cognitive load in comparison to those activities offered in the resource. After two years of studying various forms of bonus questions we now describe them as *a thread of appropriate variation that comes after the lesson focus has become an explicit object for the learner*. With the intention to survey whether tasks that impose higher cognitive load also induce deeper mathematical thinking, we present a case where a teacher introduced two bonus questions – one with high cognitive load and one with low cognitive load – and offer considerations regarding how questions with low cognitive load can promote novel mathematical thinking. We also suggest variations aimed to extend the lesson objective based on our new definition.

BACKGROUND

Before moving forward, we would like to provide a concise definition of the *object of learning* and offer a brief summary of *intrinsic cognitive load*. We would also like to outline the features of the resource used in the Math Minds initiative as they relate to the topic of this paper.

The object of learning is an important component of Marton's (2015) Variation Theory of Learning. Variation Theory proposes that new understanding involves experiencing variance against a background of invariance. For example, in order to fully understand the concept of a ripe tomato (the object of learning), you need to understand that all ripe tomatoes are red. If you cannot see the redness of a tomato, you will not be able to discern whether a tomato is ripe. In this case, the redness of a tomato is a critical feature to understand the object of learning (ripe tomato). Applying principles of structured variation, teaching this concept involves a sequence of tasks where learners would experience variation in the dimension of colour so as to discern redness. This might be a sequence where we show several parings where in each instance one tomato is red and the other is not red (and saying that the ripe tomato is the red one) allowing us to structure the learners' awareness to experience the phenomenon of a ripe tomato in the intended way.

Cognitive Load Theory stipulates that in order to facilitate learning, cognitive load must be kept at a manageable level. While there are three sources of cognitive load (intrinsic, extraneous, and germane), this study focuses on intrinsic cognitive load. The main generator of intrinsic cognitive load is *element interactivity* (Sweller & Chandler, 1991). When learners are required to draw upon multiple elements of information and integrate them to complete a task, it is very likely that they would perceive the task as complicated. For example, learning to memorise a mathematical formula, such as the area of a circle ($A = \pi r^2$), would impose a low intrinsic load. To process this information, students simply need to consider the five elements of this information in isolation (without knowing what they are), as if they were memorizing a five-digit number. However, asking students to apply the formula ($A = \pi r^2$) to a mathematical problem requires the learner to relate and compare elements of the formula with other learning elements in the problem. This task is more complicated than memorizing the formula and imposes a higher cognitive load.

JUMP Math – the resource used in the Math Minds initiative – includes student practice books, teachers' guides, and prepared SmartBoard slides. The pedagogical structure often observed in the resource was first characterized as micro-level scaffolding (Sabbaghan, Preciado Babb, Metz, & Davis, 2015), which involves "stepping-back" to a point where the teacher is sure that all students can apply their knowledge correctly and independently and reformulating instruction to address students' specific difficulties. We later added the notion of micro-discernment to the pedagogical structure, which involves breaking concepts and skills into small, carefully sequenced steps, where new pieces of information are highlighted by being the only element of change. Teachers are expected to both assess whether students have successfully discerned the critical features of the object of learning and to be responsive to students' needs before new variations are introduced. When the majority of the students demonstrate that they can apply the newly instructed micro-concepts independently and correctly, the teacher assigns practice activities so as to consolidate the concept.

OBSERVED LESSON

We observed a lesson in the fourth grade on pictographs, where the object of learning was data representation by grouping. The teacher began the lesson by showing a slide (see Figure 1) defining *scale*. Next, he asked students to write the number of books that the two circles represented (each representing five) on their portable whiteboards. Students wrote their answers and showed them to the teacher. Here, we did not observe any incorrect responses. Next, the teacher showed a slide with four circles, then six circles, each time asking for the numerical value. For each activity, students wrote down their answers and showed them to the teacher. No incorrect responses were observed.

The teacher then changed the scale so that each circle represented three books. He asked students to show him the number of books for five circles. No incorrect responses were observed.

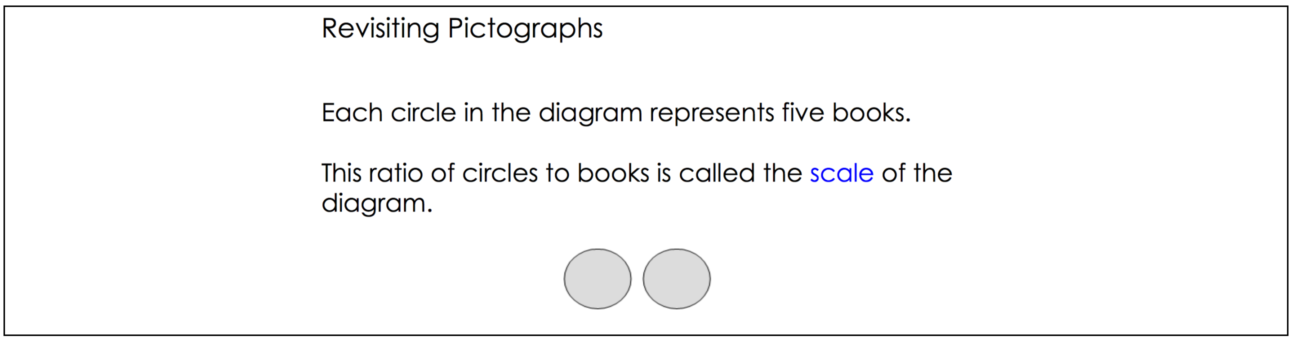


Figure 1: First slide introducing scale (JUMP Math, 2012; PDM 4-4, Slide 1).

In the next slide (see Figure 2), *tallies* were introduced as a pre-step for representing numbers as pictographs. The teacher continued the lesson by asking students to show him the tallies for two circles.

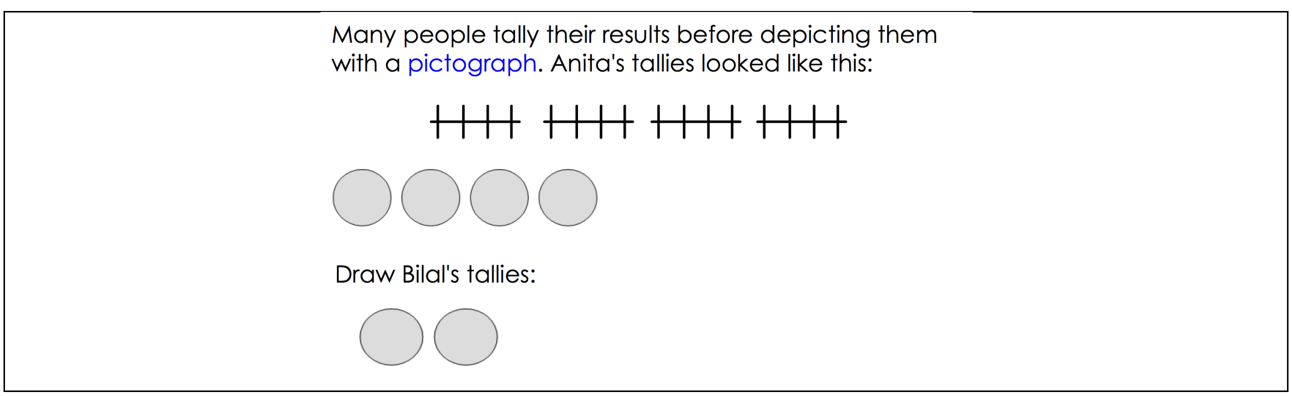
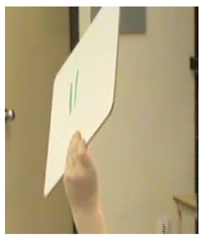


Figure 2: Second slide utilizing tallies (JUMP Math, 2012; PDM 4-4, Slide 4).

Here, we noticed that some of the students did not answer the question correctly, as shown in Section A in Figure 3. The teacher responded by asking students who did not answer correctly to state how many books each circle represented. When students responded by saying the correct answer (i.e., 5), the teacher asked students to imagine that the tallies were books and asked the question again. This time, everyone answered correctly as seen in Section B in Figure 3.

Section A



Section B

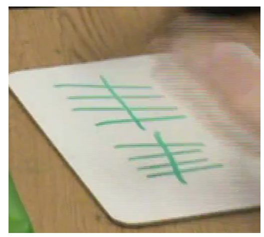


Figure 3: Example of incorrect, and self-corrected answer.

The teacher then proceeded to change the scale, indicating that each circle now represented three books. He asked students to show him the number of circles for the tallies in Figure 5.

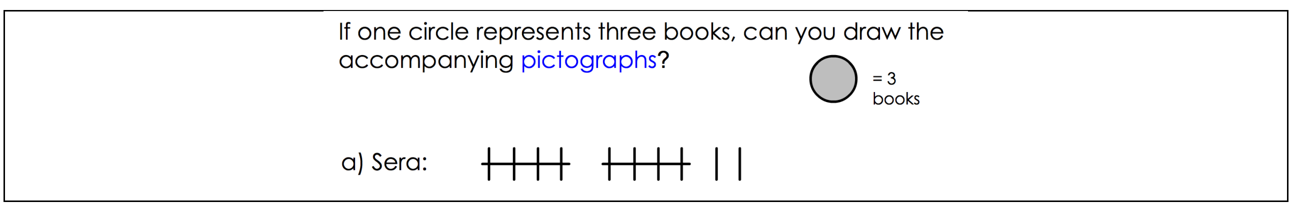


Figure 5: Task requiring regrouping tallies (JUMP Math, 2012; PDM 4-4, Slide 6).

Some students initially had difficulty answering this question. However, when the teacher reminded the students that the scale was three and not five, all students answered the question correctly. Finally, the teacher gave the bonus question (henceforth B1) displayed in Figure 6.

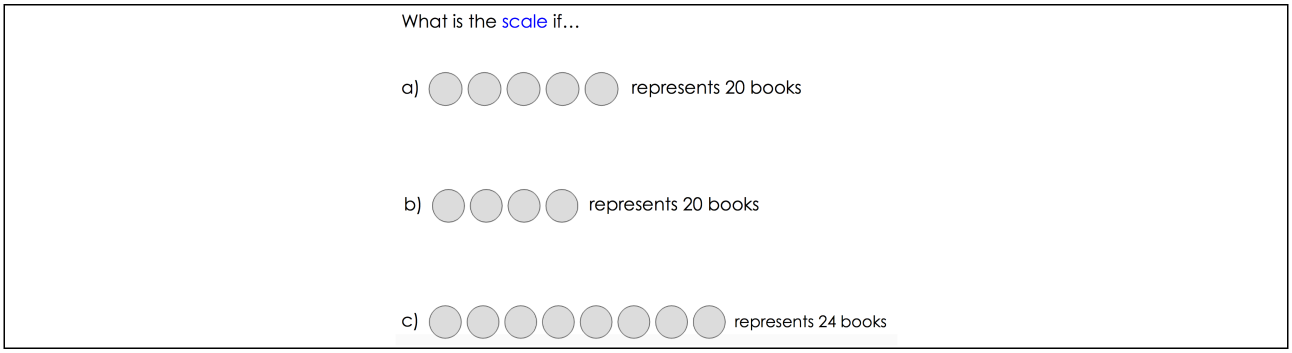


Figure 6: First bonus question (B1; JUMP Math, 2012; PDM 4-4, Slide 10).

The response from some students was immediate. Although other students eventually provided the correct answer, we observed that some students did not engage with the question. The teacher then provided the answers for B1 and explained how they were obtained. Next, he assigned activities in the resource's practice book corresponding with the lesson on pictographs. Then, he began monitoring students' progress and answered individual questions. After about 10 minutes, when a group of students had completed the assigned activities, the teacher revealed a class bonus question (henceforth B2, see Figure 7). He explained what students were to do and returned to answering questions for students who had not yet finished. Some students successfully completed the task and were praised. However, additional extensions were not provided.

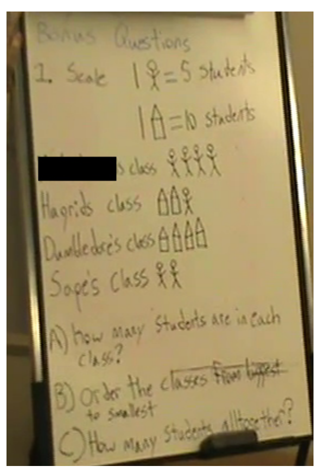


Figure 7: Second bonus question (B2).

DISCUSSION

Pictographs are a form of data representation, that require the application of grouping. Therefore, in order to achieve the object of learning, had to discern critical features related to grouping dimensions of various scales. In the lesson we observed, it seems that bonus questions were presented when the teacher was convinced that his students had realized this object of learning.

Prior to the introduction of B1, all the elements of each independent section of the question (i.e. the circles, scale, total) had been previously introduced and the students had demonstrated that they knew how to apply the critical features correctly and independently. However, what B1 required was different from what had been practiced with the students during the lesson. In all the previous questions, the *scale* and the number of symbols were given, and students were asked to find the total. In B1, the *total* and number of symbols is given, and the question asks for the scale. As the procedure to answer such a question was not practiced, engagement with it is likely to promote deeper mathematical thinking, because the question requires a novel application of the critical features.

The second bonus question, B2, includes different elements. As with B1, all elements in the problem were already presented. Although the items of the problem are not interdependent, answering them requires representing the various combinations of symbols in numeric form. Once this step is competed, students can proceed to provide answers to the three subsequent questions. The number of elements and steps that students need to attend to in B2 is significantly more than in B1. Therefore, B2 imposes a higher intrinsic cognitive load.

Intrinsic cognitive load aside, the procedure required to solve B2 had already been practiced in class. Therefore, B2 does not seem to promote any novel modes of thinking related to grouping as a mathematical construct. In other words, due to the high intrinsic cognitive load, students may perceive B2 as difficult and complicated. However, as novel modes of thinking are not required here, successfully completing B2 does not particularly expand students' mathematical horizon on the topic of grouping.

Highlighting the object of learning, one appropriate trajectory of variation after B1 could be to ask students to find two ways of grouping 30, which requires the identification of the scale and the number of symbols in the process (B3). Thirty has eight factors (i.e., 1, 2, 3, 5, 6, 10, 15), so whatever groupings the students create, we can then ask for another scale (up to six more). Again, every element in this question (scale, symbols, and total) had been previously introduced and practiced. Furthermore, the question does not impose a high cognitive load, as the number of interacting elements is limited. However, approaching this question requires novel modes of thinking regarding grouping, because the procedure to obtain the answer was not presented in this lesson. Finally, when/if students are able to discern all eight scales, we could provoke even deeper thinking by asking why they cannot produce other scales without using partial symbols.

If students are able to completely and independently answer the variations posed in B3, a possible next step would be to go to a higher level of abstraction. For example, we could ask students to identify a number that can be grouped using five different scales (B4, the answer is 16). Students offered this question would have demonstrated mastery in using the elements required in B4 by completing B3. However, B4 requires new modes of engagement with the object of learning as none of the elements in the question is given. Therefore, we feel that B4 can potentially take students to the edge of their mathematical competence on grouping.

CONCLUSION

We used Variation Theory of Learning to challenge a common misconception that mathematical extensions should include many interrelated elements and impose a high cognitive load to promote deeper thinking. Our results showed that in many cases, teachers associate deeper mathematical thinking with higher intrinsic cognitive load. Our inquiry into bonus questions has revealed that high intrinsic cognitive load does not necessarily evoke deep understanding of the object of learning. In fact, we suggest that bonus questions imposing low cognitive load could engage students in novel modes of thinking and extend their mathematical understanding.

References

Bloom, B. S. (1981). *All our children learning: A primer for parents, teachers, and other educators*. New York: McGraw-Hill.

Carroll, J. (1963). A model of school learning. *The Teachers College Record, 64*(8), 723-723.

Guskey, T. R. (2010). Lessons of mastery learning. *Educational Leadership, 68*(2), 52-57.

Marton, F. (2015). *Necessary conditions of learning*. New York: Routledge

Mighton, J. (2011). *The end of ignorance: Multiplying our human potential*. Toronto: Vintage Canada.

Preciado Babb, A. P., Metz, M., Sabbaghan, S., & Davis, B. (2015). Insights on the relationships between mathematics knowledge for teachers and curricular material. In T. G. Bartell, K. N. Bieda, R. T. Putnam, K. Bradfield & H. Dominguez (Eds.), *Proceedings of the 37th annual meeting of the North American chapter of the international group for the Psychology of Mathematics Education,* pp. 796 – 803. Lansing, MI: Michigan State University.

Sabbaghan, S., Preciado Babb, A. P., Metz, M., & Davis, B. (2015). Dynamic responsive pedagogy: Implications of micro-level scaffolding. In A. P. Preciado Babb, M. Takeuchi, and J. Lock (Eds.) *Proceedings of the IDEAS: Designing Responsive Pedagogy Conference*, pp. 198 - 207. Werklund School of the Education, University of Calgary.

Sweller, J., & Chandler, P. (1991). Evidence for cognitive load theory. *Cognition and Instruction, 8*(4), 351-362.