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Vortex Formation and Shedding in the Wake of Surface-Mounted Finite Aspect-Ratio Square Cylinders

Sattari, Pooria

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Vortex Formation and Shedding in the Wake of Surface-Mounted Finite Aspect-Ratio Square Cylinders

by

Pooria Sattari

A THESIS
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ABSTRACT

Bluff-body vortex shedding with focus on the turbulent wake of finite surface-mounted square cylinders at moderate Reynolds numbers is studied experimentally using particle image velocimetry (PIV) and hotwire anemometry (HWA). Emphasis is put on understanding the formation and growth process of vortices, their interaction and separation from the feeding shear layer.

The conventionally described vortex shedding process is reassessed. It is concluded that contrary to the traditionally proposed mechanism, the mutual interaction between counter-rotating vortices is not the cause for the limit to vortex growth. Rather, the vortex growth is naturally limited and the feedback from the shed vortices serves to lock-in the shedding frequency. Furthermore, based on observation of the shedding of an isolated two-dimensional vortex, a length scale defined as the distance between the shear-layer edge and vortex center at the streamwise location of maximum vortex circulation is shown to result in a collapse of non-dimensional circulation. On the ground of scaling principles, this scale is shown to also result in a collapse of the shedding frequency.

This work also resolves an apparent discrepancy in the literature with respect to the existence of symmetric and anti-symmetric shedding regimes. Using spatial cross-correlation, instantaneous phase relationships, and phase-averaged velocity data obtained from PIV and HWA, it is shown that the shedding in the wake of surface-mounted finite square cylinders is predominantly anti-symmetric and thus consistent with the von Kármán process. What had been interpreted as symmetric shedding appears to be a distortion of the regular shedding process. During periods of low-amplitude fluctuations, two counter-rotating vortices exist concurrently in the base region. However, counter-rotating vortices are still shed alternately.
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Chapter One: Introduction

1.1 MOTIVATION

Separated turbulent flow around surface-mounted finite-aspect-ratio cylinders is a domain of fluid mechanics of multi-fold significance yet beyond the reach of theoretical analysis. The complexity of this family of flows arises due to the inherent unsteadiness and three-dimensionality of the problem for even moderate Reynolds numbers. Generally, these flows are characterised by the periodic accumulation of circulation into a concentrated-vorticity region, i.e. the base vortex, which is then convected downstream in the form of coherent structures. These structures are initially formed parallel to the obstacle but might reorient or reconnect travelling downstream. In order to properly characterize the wake flow, an understanding of the evolution of quasi-periodically shed structures into the wake is required, bearing in mind that these structures dictate the mean flow topology and turbulent transport characteristics. Understanding the wake flow is important as it controls mass and momentum transport, vorticity generation and the turbulence energy budget.

The scope of the present thesis is to gain a deeper understanding about the vortex shedding phenomenon with special focus on finite surface-mounted square cylinders. Emphasis will be on understanding the evolution process of vortices, their interaction and the separation process of the vortex from the feeding shear layer.
From the application point-of-view, the quasi-periodic flow generated downstream of circular and square cross-section obstacles may be considered as a simplified, heuristic model for a wide range of industrial and environmental flows. Free standing structures such as buildings, smoke stacks, wind turbines, as well as electronic cooling devices and mountains are some typical examples. Therefore, understanding the properties of this category of flows is sought in many applications, where forces exerted on structures, as well as downstream transport of heat and mass are to be estimated or controlled. High levels of velocity and pressure fluctuations associated with flow separation upstream and on the faces of obstacles are important factors to be considered in the design of buildings and wind turbines (Sakamoto and Oiwa, 1984). Furthermore, high-turbulence mixing zones in the wake strongly influence heat and mass transfer, which is of relevance to design of efficient heat exchangers and placement of smoke stacks and wind turbines (Vincent, 1977; Igarashi, 1985; Wang et al., 2009). The present experiments were carried out at a Reynolds number of 12,000 based on the obstacle width and free-stream velocity. Due to the fixed separation point at the obstacle front edge, as will be shown in Chapter two, the flow features are independent of the Reynolds number. However, the turbulence intensity has been shown.

An experimental approach to the problem is well justified by considering the shortcomings of commonly used computational techniques: Large Eddy Simulation (LES) and Direct Numerical Simulation (DNS). Due to the spectral separation

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1 Note that each chapter of the thesis has its own reference list.
assumption inherent to LES, small-scale flow features close to solid boundaries cannot be directly solved for, and should hence be modeled. Techniques to model the small structures of the flow are not well-developed yet and need to be verified by experimental data. Yet these features are critical to the generation of circulation, which governs the formation and downstream evolution of large-scale vortical structures. Limitations on computational resources and extreme sensitivity to oncoming boundary conditions limit DNS implementation to relatively low Reynolds numbers, significantly smaller than those occurring in most practical situations.

1.2 BACKGROUND AND OBJECTIVES

In this section, the main objectives and expected outcomes of this thesis will be presented. To provide the reader with an overview of current state-of-the-art, several of the more relevant studies will be reviewed. Furthermore, the shortcomings identified in previous studies and as yet-unresolved issues will be highlighted. The present thesis is structured in a “paper-based format” including three articles attached in the following chapters. Therefore, the literature review in the “Introduction” section will be abridged in order to focus on the thesis purpose and to avoid repetition, since each chapter introduces a detailed review on that particular subject.

Over the past century, various aspects of the shedding phenomenon in the wake of two-dimensional (2D) or “infinite” circular cylinders, and to a lesser extent square cylinders, have been the subject of hundreds of papers, whereby a large archive of
information in the literature has been collected. A review on vortex shedding behind circular cylinders can be found in Morkovin (1964), Oertel (1990) Coutanceau and Defaye (1991), Williamson (1996), Zdravkovich (1997, 2003), and Sumer and Fredøse (1997). In contrast, despite the engineering practicality and importance to fundamental fluid mechanics knowledge, noticeably less research has been carried out on finite bluff bodies mounted on a flat surface. This is partly due to shortcomings of experimental tools and limitations on the numerical simulation front in terms of spatial and temporal resolution in properly resolving the flow features.

1.2.1 Symmetric/anti-symmetric shedding

One of the observed phenomena in previous studies with respect to vortex shedding behind finite surface-mounted obstacles is what had been referred to in the literature as “symmetric shedding”. For 2D square and circular cylinders, it is well established that shortly after the inception of vortex shedding above a critical Reynolds number (Re ~ 60-90), the flow is characterized by periodic, alternate (anti-symmetric) shedding of vortices known as Kármán-type shedding (Okajima, 1982; Williamson, 1996). In this flow, vortices are created at the back of the body and detach periodically from either side of the body. In contrast, for finite surface-mounted obstacles, the possibility of symmetric shedding of vortices at higher Reynolds numbers under certain conditions was reported (Sakamoto and Arie, 1983; Wang and Zhou, 2009). In such an arrangement, two symmetric vortices would form in the obstacle base region, shed simultaneously and travel downstream together. However, previous conclusions on
symmetric shedding (both visualizations and PIV measurements) have to a large extent relied on single snapshots of the flow field (Fig. 1-1), whereas in order to gain a true understanding of the shedding process, tracking individual vortices over a certain time period seems a more objective approach. So far, no direct evidence of symmetric shedding of vortices has been provided for finite surface-mounted cylinders at moderate Reynolds numbers. For example hotwire anemometry measurements, a relatively easy-access, yet very helpful technique for determining instantaneous phase relationships, have never been shown in the literature for the finite cylinder cases. Moreover, the conclusions were to a large extent drawn based on visualizations at low Reynolds numbers. The validity of extrapolating results from low to high Reynolds numbers still needs to be verified. In fact, the experimental study of Sakamoto and Haniu (1990) and Direct Numerical Simulations of Mittal (1998) on spheres show that the shedding behaviour is indeed Reynolds-number-dependent. In both studies symmetric shedding in the wake of spheres was observed only for a Reynolds numbers below 400. For a circular cylinder, Green and Gerrard (1993) observed lower levels of cross-flow in the near-wake at very low Reynolds numbers (Re = 73). The shedding cycle at low Re was characterised by vortex splitting and higher shear stresses.
Fig. 1-1: Top row: Schematic of proposed vortex structure in “symmetric” (left columns) and anti-symmetric shedding (right columns) modes proposed by Sakamoto and Arie (1983). Middle and bottom rows: Sample low-Reynolds-number visualization snapshots (Sakamoto and Arie, 1983 (middle row); Wang and Zhou, 2009 (bottom row)) representing the two shedding modes. Single flow snapshots can be misleading when used for the characterisation of the shedding mode.

In order to study the vortex formation and shedding from finite surface-mounted cylinders, it is essential to first clarify the issue of symmetric versus anti-symmetric shedding through a systematic analysis. The existence of anti-symmetric and symmetric shedding regimes would imply that the flow properties and aerodynamic forces could vary to a large extent from one regime to the other. Anti-symmetry would be accompanied by high-amplitude fluctuations in the velocity and pressure fields, and a stable vortex skeleton travelling downstream with little changes, whereas symmetric
shedding would result in low amplitude fluctuations and formation of rings, unstable in nature and likely to rearrange as they are convected downstream. Once the discrepancy regarding the shedding mode (i.e. symmetric versus anti-symmetric shedding) is clarified, it is possible to proceed with a more in-depth study of vortex structure evolution and the shedding mechanism in general: the sequence of events resulting in the separation of the vortex from the feeding shear layer.

1.2.2 Vortex growth and separation from a feeding shear layer

The conventional description for bluff-body vortex shedding (see for example Gerrard, 1966) is that initially the shear layer on one side of the obstacle rolls up into a vortex, as schematically shown in Fig. 1-2. The induction from this growing vortex draws the opposing shear layer across the wake. The connection between the vortex and the feeding shear layer breaks off due to the interference of the opposing-side shear layer, whereby the vortex is ultimately shed. This process is then repeated on the opposite side. The conventional explanation would imply that the vortex would grow without limitation if the opposite shear layer were not interfering. However, the present observations suggest that the vortex growth in the base region is naturally limited. Therefore, a reassessment of conventionally described vortex shedding process is sought in the present thesis. To this end, a better insight into the concept of vortex formation from a feeding shear layer in general is necessary.
Vortex formation from a feeding shear layer has been studied recently for various vortex-generation configurations. One of the major purposes of these studies has been to understand mechanisms through which the growth of a vortex may be limited. In particular, effort has been put on searching for time or length scales associated with the limit to vortex growth. Although these previously studied configurations mostly involve unsteady vortex formation, they are similar to quasi-periodic shedding in the sense that every vortex shedding cycle may be considered as a new formation process from a starting shear layer.

Gharib et al. (1998) studied the vortex-ring formation through the ejection of fluid from a circular orifice and observed a limit to the vortex-ring growth (in terms of vortex strength or circulation) for piston-stroke-to-diameter ratios of approximately $L/D \geq 4$, where $L$ is the piston stroke and $D$ is the orifice diameter. Further circulation did not enter the vortex ring, but was observed to be accumulated in
Kelvin-Helmholtz-type vortices formed in a trailing shear layer behind the vortex ring. This interruption of the feeding mechanism is termed the “pinch-off”. Gharib et al. showed that the pinch-off occurred at a critical dimensionless time, $T^* = \bar{U}_p t / D \approx 4$, where $\bar{U}_p$ is the average piston velocity and $t$ represents time elapsed from the start of vortex formation. This critical formation time was termed the formation number. The limitation of vortex growth has been observed also in other configurations similar to bluff bodies. Jeon and Gharib (2004) observed a saturation of the vortex circulation in the wake of a circular cylinder accelerated from rest. A similar behavior was observed for plunging airfoils (Rival et al., 2009), finite aspect ratio plates accelerating from rest (Ringuette et al., 2007), and for a plate rotating about one of the plate edges (DeVoria and Ringuette, 2011).

Shusser and Gharib (2000) explained the observed pinch-off in terms of the translational velocity of the vortex ring exceeding the shear-layer velocity due to self induction. However, for other vortex-generation configurations no explicit physical explanations for the separation of the vortex from the feeding shear layer has been provided so far. This lack of consensus is in part due to the existence of multiple potential parameters, which could cause the separation. These parameters include three-dimensionalities of flow, vortex interaction with other vortices or solid surfaces, and background flows in the form of co-flows and counter flows. Therefore, to better understand the process for a more simplified case with a reduced number of potential parameters, vortex formation in isolation from a 2D shear layer starting from rest was
considered in the present research. Vortex evolution in this configuration is not linked to interactions with other vortical structures or walls that could potentially influence vortex growth. The insights and conclusions of this study will be extrapolated in the context of bluff-body shedding, which is inherently a more complex flow due to interaction of multiple vortices with each other as well as with the obstacle. Furthermore, a model for vortex growth (in terms of size and strength) due to influx of circulation-containing mass from a two-dimensional shear layer will be proposed. This model provides a relationship for vortex growth in terms of shear-layer velocity distribution and thickness and might possibly be extended to bluff-body vortex growth and other vortex generation systems involving a growing vortex fed by a shear layer.

1.2.3 Bluff-body vortex shedding process - Scaling considerations

As mentioned in the previous section, some of the present observations from phase-averaged PIV data with respect to vortex growth in the wake of surface-mounted square cylinders do not entirely match with the conventional phenomenological description of bluff-body vortex shedding. Therefore, a reassessment of the conventional vortex shedding model is pursued in the present thesis. To this end, an alternative hypothesis for bluff-body vortex shedding will be proposed and verified. Briefly, based on this hypothesis, the vortex reaches its growth limit independent of the mutual interaction of wake vortices. The mutual interaction only serves as a mechanism through which the shedding frequency locks in with the periodic activity in the wake.
Recent analysis of the 3D reconstruction of a flow field from phase-averaged PIV measurements within the bluff-body aerodynamics group at the University of Calgary suggests that the three-dimensional structure of shed vortices behind finite obstacles may change depending on the obstacle aspect ratio and the boundary-layer thickness. More specifically, for aspect ratio 4 (height-to-width ratio, $AR = h/d = 4$) obstacles, vortical structures form half loops, whereas for an aspect ratio 8 ($AR = 8$) obstacle placed in a thick boundary layer, vortices are shed in the form of full-loops. Briefly, the half-loop structure is identified by an arc-shaped vortical structure consisting of the principal vortex core and a streamwise connector strand as described in detail in Bourgeois et al. (2011). Surprisingly, the Strouhal number remains constant despite these structural changes. Furthermore, despite the appearance of connecting strands in the case of finite aspect ratio surface-mounted square cylinders, and the connection close to the free end, the shedding pattern in the mid-height planes looks similar to infinite cylinders. The observed similarities suggest that one might be able to explore a generalized model for bluff-body vortex shedding, applicable to both two-dimensional and three-dimensional flows. Based on the generalized model, scaling parameters to obtain a collapse in the non-dimensional circulation may be sought.

The search for relevant length scales associated with bluff-body vortex shedding has been the subject of a number of studies in the context of search for universal shedding Strouhal numbers. Roshko (1954) attempted to find a universal Strouhal number, based on the idea that there must be a characteristic width that scales with the lateral vortex spacing in von Kármán’s shedding model. He used notched-hodography
to characterize the relationship between the base pressure coefficient and the wake width for three geometries (circular cylinders, flat plates and wedges). It was shown that the wake width decreases with a decrease in the base pressure. The proposed scaling scheme also suggests that the shedding frequency is lower for higher base pressure (i.e. larger wake width and higher-velocity shear layer). Lower base pressure results in higher pressure differential between the front and the back of the obstacle, and consequently a larger drag coefficient. Roshko’s calculations, however, are based on von Kármán’s first-order stability analysis, which predicts a vortex spacing ratio of $b/a = 0.281$, where “$b$” and “$a$” are the lateral and longitudinal spacing of wake vortices, respectively. As pointed out by Wille (1960), any array of vortices is unstable to any order of disturbances higher than first, which suggests that the assumption of $b/a = 0.281$ should be used cautiously. In fact, various authors (e.g. Berger, 1964) have measured values of $b/a$ between 0.20 and 0.40 with $b/a$ increasing with distance along the wake. Bearman (1967) was able to collapse the shedding frequency in configurations with splitter plates and base bleed by using the Kronauer criterion to obtain the lateral spacing of vortices. This criterion states that the vortex street adjusts itself into the configuration giving minimum vortex street drag. Kronauer’s criterion is not based on direct experimental evidence (Bearman, 1967). Furthermore, when using more recent experimental observations on prismatic obstacles (present study), this approach does not satisfactorily collapse the shedding frequency data. Griffin (1978) used the wake width at the end of the formation region (defined as the streamwise location of maximum velocity fluctuations) as the characteristic length to obtain a
universal Strouhal number for vortex shedding from vibrating cylinders. Different definitions of formation region in the literature are provided in Griffin (1995). As will be discussed in Chapter four of the thesis, a better collapse in the non-dimensional frequency can be obtained by taking into account the lateral location of the vortex core in the characteristic length scale.

Based on the proposed hypothesis for vortex shedding in this thesis, it is expected that the characteristic length associated with vortex growth should scale with the length scale to obtain a universal shedding frequency. In the present research, it is intended to search for appropriate length and velocity scales to obtain a collapse of non-dimensional circulation and shedding frequency for different obstacles.

1.3 CONTENT & LAYOUT OF THE THESIS

The present thesis will be structured in a “paper-based format”. A summary of the main objectives and motivation of the thesis was described in the Introduction. In the following three chapters (Chapters 2-4), three papers published in or submitted to peer-reviewed journals will be included. Each of these three papers focuses on a specific topic related to the formation and shedding of vortices. In Chapter 2, two distinct vortex shedding regimes (regimes A and B) for an aspect ratio 4 surface-mounted square cylinder are described. Based on detailed analysis, the existing discrepancy in the literature with respect to anti-symmetric versus symmetric vortex shedding is reconciled. In Chapter 3, the evolution of an isolated line vortex generated by a two-dimensional jet
is studied. A model for vortex growth is proposed, and the separation mechanism of the vortex from the shear layer is discussed. Chapter 4 focuses on the vortex shedding process in the wake of bluff bodies. More specifically, a hypothesis related to the synchronization between vortex growth and periodic wake activity is proposed and tested. Also, the existence of possible time and length scales associated with vortex growth is investigated. The synthesis, i.e. connection between the papers, is made in Chapter 5. Finally, major conclusions and recommendations for future work will be presented in Chapter 6.

An additional paper is included as appendix A. This paper (peer-reviewed conference paper) contains analysis of PIV measurements for an AR = 4 square cylinder as well as comparison of hotwire measurements and data analysis for square and circular cylinders of aspect ratios from 0.5 to 8. More in-depth analysis of flow around an AR = 4 square cylinder is repeated in Chapter 2. However, the section related to the influence of aspect ratio on shedding in Appendix A (not published elsewhere) helps to support the arguments in this thesis.
1.4 REFERENCES


Chapter Two: On the vortex dynamics in the wake of a finite surface-mounted square cylinder

This Chapter is a paper published as:


Department of Mechanical and Manufacturing Engineering, University of Calgary, Calgary, AB T2N 1N4, Canada

Author Contribution: Pooria Sattari and Robert Martinuzzi developed the idea of the paper. Pooria Sattari analyzed instantaneous flow fields, conducted hotwire and PIV measurements, and phase and spectral analysis. Jason Bourgeois contributed to PIV measurements, boundary-layer profile measurements, and performed wavelet transform and phase-averaged data analysis. Robert Martinuzzi supervised the project.

Some modifications have been made to the originally published paper in response to queries raised by candidate’s Ph.D. examination committee.
2.1 ABSTRACT

The shedding process in the near wake of a surface-mounted, square cross-section cylinder of height-to-width aspect ratio 4 at a Reynolds number of 12,000 based on free stream velocity and the obstacle width was investigated. The boundary layer thickness was 0.18 obstacle heights based on 99% free stream velocity. The study is performed using planar high frame-rate particle image velocimetry synchronized with pressure measurements and hot-wire anemometry. Spatial cross-correlation, instantaneous phase relationships and phase-averaged velocity data are reported. Two dominant vortex shedding regimes are observed. During intervals of high-amplitude pressure fluctuations on the obstacle side faces, alternate formation and shedding of vortices is observed (regime A) similar to the von Kármán process. Regime B is characterized by two co-existing vortices in the obstacle lee throughout the shedding cycle and is observed within low-amplitude pressure fluctuation intervals. Despite the coexisting vortices in the base region, opposite sign vorticity is still shed out-of-phase downstream of this vortex-pair giving rise to a staggered arrangement of counter-rotating vortices downstream. While the probability of occurrence of Regime B increases towards the free end, the amplitude modulation remains coherent along the obstacle height. Conditionally phase-averaged reconstructions of the flow field are consistent with the spatial distribution of the phase relationships and their probability density function. Earlier observations are reconciled showing that the symmetric shedding of vortices is a rare occurrence.
2.2 INTRODUCTION

The quasi-periodic flow generated downstream of surface-mounted, vertical prismatic cylinders of finite height, $h$, may be considered as a simplified, heuristic model for understanding flow properties and control of a wide range of industrial and environmental flows. Induced loading associated with high levels of velocity and pressure fluctuations due to vortex structures originating from flow separation upstream and on the faces of obstacles are important factors to be considered in the design of free-standing structures such as buildings and wind turbines (Sakamoto and Oiwa, 1984). The high-turbulence mixing zones in the wake strongly influence heat and mass transfer which is of relevance to the design of more efficient heat exchangers or the placement of smoke stacks and wind turbines (Vincent, 1977; Igarashi, 1985; Wang et al., 2009). These transport and mixing processes are an expression of the influence of the wake dynamics.

A characterisation of quasi-periodic wake dynamics requires an understanding of vortex formation and shedding since the resulting instantaneous flow topology plays an important role in determining coherent and turbulent fluctuation levels. Sakamoto and Arie (1983) studied the influence of boundary layer thickness, $\delta$, and obstacle aspect ratio ($h/d$, where $d$ is the obstacle width) on the vortex shedding in the turbulent wake of vertically mounted, finite square-section cylinders mounted with one face normal to the flow. Tests were conducted for Reynolds numbers, based on $d$ and the free-stream velocity ($U_\infty$), up to $Re_d \sim 150,000$. They reported that, for a fixed $Re_d$ and relative
boundary layer thickness, \( \delta h \), two shedding regimes could be identified by plotting the shedding frequency as a function of the aspect ratio. It was shown that data collapsed on two different curves for shorter and taller obstacles. The critical aspect ratio for the transition between regimes depended on \( \delta h \). Largely based on low \( Re_d \) (270-730) visualisations, for which the wake remained laminar, they concluded that for all high aspect ratio obstacles anti-symmetric (von Kármán-type) shedding occurred while for low aspect ratios, counter-rotating vortices were shed symmetrically.

Wang and co-workers conducted detailed investigations of the wake flow for square-section cylinders of aspect ratio 5 and \( Re_d = 11,500 \) (Wang et al., 2006), and aspect ratios 3 to 7 for \( Re_d = 9,300 \) (Wang and Zhou, 2009) for different \( \delta h \). Comprehensive Laser Doppler Velocimetry (LDV) measurements provided a detailed map of the mean turbulent field and selected Hotwire Anemometry (HWA) measurements showed that periodicity in the tip region remained strong and the coherence function well defined about a single (shedding) frequency, \( f_s \). To assess the relative probability of symmetric versus anti-symmetric shedding a histogram of the relative location of the vortex cores was constructed for the case \( h/d = 5 \). If the vortex centres between two counter-rotating vortices was less than half the spacing between the nearest co-rotating core, these were said to be shed symmetrically. The location of the vortex centres were estimated from randomly sampled, isolated Particle Image Velocimetry (PIV) snapshots (sampled at 15 Hz when compared to \( f_s \sim 37 \) Hz). The probability of anti-symmetric shedding was thus assessed higher near the plate-obstacle
junction and lower (as low as 20%) near the free-end. For an aspect ratio 7 cylinder, the results were similar, although the probability of anti-symmetric shedding was slightly lower at the plate-body junction than at the mid span. A model was proposed for two possible flow structures with either symmetric or anti-symmetric counter-rotating vortex arrangements. Further conditional analysis or other methods to resolve the shedding cycle are needed to test or possibly refine the proposed models in terms of topological completeness.

Studies for other finite aspect ratio geometries show some similarities but also important differences to the aforementioned observations. The study by Okamoto and Sunabashiri (1992) on finite surface-mounted circular cylinders showed the co-existence of alternating shedding along most of the body height and in-phase shedding near the free-end (tip). In contrast to the square cylinder, significant spectral broadening and a reduction in the cross-correlation coefficient between the plate-body junction and tip regions were reported. These findings are similar to those found in several other studies on circular geometries. The near free-end behaviour is generally attributed to interactions between streamwise tip vortices (cf. Ayoub and Karamcheti, 1982; Szepessy and Bearman, 1992; Sumner and Heseltine, 2008; Park and Lee, 2002, 2003) and/or vortex splitting (Gerich and Eckelmann, 1982; Eisenlohr and Eckelmann, 1989).

Changes in shedding behaviour along the height has been reported for low aspect ratio, surface-mounted vertical tapered plates (Vosper et al., 1999; Castro and Rogers, 2002; Castro and Watson, 2004) and square-based pyramids (Martinuzzi and AbuOmar,
2003; Martinuzzi 2008) in turbulent wakes and thin oncoming boundary layers. Results from these studies suggest in-phase shedding of structures near the sharp tip and alternating (von Kármán-type) shedding over most of the obstacle, with increasing strength toward the plate-body junction. Spectra of the near-wake velocity field and surface pressures on the side faces of the tapered geometry show that the shedding frequency associated with the two modes are slightly different, leading to the appearance of harmonic and sub-harmonic modes (cf. Castro and Watson, 2004; Martinuzzi, 2008) and broadening of the spectral peak about the principal frequency. The phase relationship between fluctuations on opposing sides shows a transition from out-of-phase to in-phase shedding and the cross-correlation coefficient changes sign over a relatively small vertical displacement. While this behaviour has similarities to that observed for the finite circular cylinders, it has not been reported in the square cylinder studies. Important details with regard to the shedding mode for the square cylinder geometry were obtained at a much lower $Re_d$ (<730 for Sakamoto and Arie, 1983; 221 for Wang and Zhou, 2009) and thus perturbations of the separated shear layer are much less significant than for the fully turbulent wakes and may result in some modification in the behaviour. For example, for the two-dimensional circular cylinder, three-dimensional wake instabilities appear at $Re_d \sim 180$ and grow as $Re_d$ increases. Shear layer transition ($Re_d > 1000$) leads to significant modifications to the base formation region (cf. Williamson, 1996). It has also been shown that for vertical rectangular and elliptical plates (Kiya et al., 2001), hairpin vortices are shed symmetrically at low $Re_d$ and increasingly asymmetrically as $Re_d$ is
increased. In the case of square prisms, transition has been observed in the Reynolds number range 150-200 by Saha et al. (2003) and Luo et al. (2003).

Except for low $Re_d$, below which three-dimensional instabilities appear, disruptions or modifications of the shedding process are inherent to wakes for two-dimensional circular cylinders (Tritton, 1959; Williamson, 1992), normal flat plates (Najjar and Balachandar, 1988; Miau et al., 2004; Wu et al., 2005) and square cylinders (Bearman and Obasaju, 1982; Bailey et al., 2002). For these geometries events of disruption in the von Kármán shedding process leading to low-frequency modulation in the velocity/pressure fluctuation amplitude, are associated with local dislocation/split in vortex tubes. These events are characterised as bursts of lower velocity fluctuation amplitude, loss of shedding coherence and strong three-dimensional effects. While the mutual influence of tip and wake dynamics is generally accepted (cf. Eckelmann and co-workers), the nature of the interactions appears geometry dependent.

From the aforementioned studies, the observation of symmetric and anti-symmetric shedding for square cylinders appears distinct from the tip interactions observed for finite circular cylinders, which show more similarity to observations for tapered geometries. Observations of symmetric shedding have to a large extent relied on low Reynolds number visualizations and single snapshots of the flow field. A better understanding of the shedding characteristics might be gained by resolving the formation and shedding process. To this end, detailed investigations of the vortex formation and shedding process in the leeward region of a vertical surface-mounted square cylinder of
aspect ratio 4 are conducted. Sequences of instantaneous high-frame rate PIV flow fields are presented, from which two shedding regimes are identified. The behaviour of the fluctuating pressure field on the surface of the obstacles and the fluctuating velocity from multiple hotwire anemometry (HWA) measurements in the wake are related to these shedding regimes. It is shown that the pressure fluctuations along the sides of the obstacle are coherent, allowing for a conditionally sampled (phase-averaged) reconstruction for each regime. Details of the coherent structure are discussed and reconciled with earlier observations.

2.3 EXPERIMENTAL SETUP

Measurements were conducted in a suction-type open-test-section wind tunnel, as shown schematically in Fig. 2-1. The tunnel inlet-area inlet diameter of the nozzle is 3 m. The flow passes through a set of three 20 mesh metal grids, a 24 and a 30 mesh metal grid, and one 80 mesh nylon screen before entering a 36:1 contraction leading to the 0.5 m diameter inlet to the working section.

A flat plate with a 15° sharp leading edge shown schematically in the upper detail of Fig. 2-1 was placed inside the working section. The flow exits the working section through a bell mouth and a diffuser section with a half-angle of 2.5°. The flow is driven by two counter-rotating fans with separate power units.

The obstacle was mounted on a 1.2-m long, 0.8-m wide and 0.01-m thick reinforced aluminum plate with a sharp leading edge. The plate surface was machined flat
to within 30 µm. The plate support structure was anchored horizontally and vertically. The resonant frequency of the plate assembly was measured using accelerometers to be approximately 7 kHz. When operated, the vibration amplitude of the assembly was less than the transducer uncertainty of ±10 µm and was imperceptible to the touch.

The machined aluminum square-section obstacle has width of $d = 0.0127$ m and height $h = 0.0508$ m. It was instrumented with 0.3-mm diameter pressure taps on the opposing side faces at vertical locations $0.25h$, $0.50h$ and $0.75h$ above the ground plate and $0.25d$, $0.50d$ and $0.75d$ from the leading edges, respectively. The obstacles were mounted with a non-instrumented face normal to the on-coming flow. The obstacle geometry, flow and co-ordinate system nomenclature are also shown in Fig. 2-1.

![Wind tunnel schematic and nomenclature. $\varphi_1$ and $\varphi_2$ represent wind tunnel inlet diameter and working section inlet diameter, respectively.](image)
A base turbulent boundary layer was generated on the sharp leading edge flat plate. Measurements were taken at a free-stream velocity, \( U_\infty = 15 \text{ m/s} \), corresponding to a Reynolds number \( Re_d = U_\infty d/v = 12,000 \) where \( v \) is the kinematic viscosity of air. The free-stream turbulence intensity is 0.8%. The characteristics of the undisturbed boundary layer were measured using a single-probe hot wire anemometer. The boundary layer thickness based on 99% free stream velocity at the mounting location of the cylinder (\( x = 200\text{mm} \approx 16d \) from the leading edge of the plate) was \( \delta/h = 0.18 \; (\delta/D = 0.72) \). Figure 2-2 shows the normalized mean velocity and turbulence intensity profiles. The boundary layer is turbulent, but developing. It was verified that the profile did not vary in the spanwise direction in the range \(-0.15 \text{m} < y < 0.15 \text{m} \) \((-12 < y/d < 12)\). Symmetry about the plane \( y = 0 \) in the measured mean velocity and Reynolds stress fields was used to verify the orientation of the obstacle.
Fig. 2-2: Undisturbed boundary layer mean and rms velocity profiles at the obstacle location (obstacle removed).

Since the obstacle leading edges are sharp, the shedding process is not expected to be sensitive to the Reynolds number over the range tested, which is corroborated in Fig. 2-3. The shedding frequency, $f_s$, is directly proportional to $U_\infty$ yielding a Strouhal number, $St = f_s d/U_\infty$ of $0.100\pm0.003$, which agrees with those values reported by Sakamoto and Arie (1983) in a fully developed, thin turbulent boundary layer. It is thus expected that the shedding process in the formation region and near-wake region will not be significantly different despite the fact that the boundary layer is still developing.
Figure 2-3: Shedding frequency vs $Re_d$.

Frequency spectra and phase relationships between different points in the flow field were obtained using single-probe hotwire anemometers, HWA (TSI 1210-20, bridge model TSI IFA-100), operated in constant-temperature mode. The hotwire thickness and resistance were 5 $\mu$m and 3.5 $\Omega$ (at 20°C), respectively. The overheat ratio was set at 1.8. Amplitude amplification and phase lag of a hotwire with the above characteristics are negligible for frequencies below 3000 Hz in constant-temperature mode (Bruun, 1995). Therefore, hotwire anemometry is suitable for the current measurements of shedding occurring at 120 Hz. The hotwires were calibrated against a Pitot-static tube (using an inclined manometer) in the free-stream in the range $3 \text{ m/s} < U_x < 35 \text{ m/s}$. Frequency compensation was conducted following the manufacturer’s instructions. Data from different HWA probes were acquired simultaneously using two 5-channel National
Instruments acquisition boards (NI 9227, 24-bit resolution) and processed using LabVIEW. Data were sampled at rates of 10.24 kHz over 10 s sampling windows (corresponding to at least 1150 shedding cycles at $U_\infty = 15$ m/s). An anti-aliasing filter was applied. Presented spectra and cross-correlation data have been averaged for typically 10 sampling windows.

For all HWA measurements, one probe was used to monitor the free-stream velocity. Two configurations were used. Two hotwires were placed on opposing sides of the plane $y = 0$ at $y/d = \pm 1.2$ or $y/d = \pm 2$ and traversed together in the horizontal direction. Traverses were made for $x/d = 0.5$, 1, 2 and 4 using a three-axis computer-controlled traverse unit (position accuracy 10 µm). It was verified based on the probability density function of the velocity, that the wires were placed well outside the formation region, where the streamwise velocity component is negative or near-zero. To avoid influence of velocity component in the $z$-direction, the wires were mounted with their axis along the $z$-direction. In the second configuration, one probe was held at the fixed location: $x/d = 0.5$, $y/d = -1.2$, $z/d = 0.5$, while the second probe was traversed in the $z$-direction at $x/d = 0.5$, 1, 2 and 4 and $y/d = 1.2$ or $y/d = 2$.

The experimental uncertainty within a 95% confidence interval for the instantaneous phase difference is estimated to be $\pm 5^\circ$, based on the elemental instrument uncertainty, process standard deviation and repeatability characteristics (Coleman and Steele, 1999). The uncertainty sources include peak determination of shedding frequency.
in the spectra, free-stream velocity variations, and the phase-lag due to tubing from the pressure sensor to the obstacle.

A LaVision high frame-rate particle imaging velocimeter was used for planar velocity field measurements in three horizontal planes parallel to the free-end ($z/h = 0.25$, 0.5 and 0.75) and in the vertical plane $y/d = 0$. A Photonics Industries 527 nm Nd-YLF laser light sheet with an aperture angle of $12^\circ$ was used to illuminate olive oil particles of approximately 1µm generated by a Laskin nozzle particle nebulizer. Data were acquired with a HighSpeedStar 5 CMOS camera (1024×1024 pixels) in double-pulse mode with a pulse separation of 50 µs between the images in an image pair. Image pairs were captured at rates of 500 Hz or 1000 Hz, or approximately 5 to 10 data points per shedding cycle. Five to ten independent sequences of 1000 vector fields were acquired so that statistics were calculated over 500 to 2000 shedding cycles. Interrogation windows of 32×32 pixels were used in the wake using a frame-straddled arrangement, resulting in a spatial resolution of the data within a given PIV velocity field of approximately 3mm, or $d/4$. Where greater resolution was necessary, interrogation windows of 16×16 pixels were used, yielding a resolution of $d/8$. With this configuration, the estimated uncertainty on individual vector measurements (see Westerweel, 2000) is $\Delta u/U_\infty = \pm0.025$ while the estimated statistical uncertainty on the phase-averaged vector fields, which is a function of the standard deviation at a given position, ranges from $\Delta<u>/U_\infty = \pm0.002$ to a maximum of $\pm0.027$. 
The fluctuating surface pressure was measured with pressure transducers (AllSensors Corp, model 1INCH-D1-4VMINI) with taps of diameter 0.3 mm on either side of the cylinder. These measurements are analyzed in order to find the phase of the shedding cycle for each PIV vector field. Pressure tube lengths were kept as short as possible. The frequency response of the tapping-tubing-transducer system was determined to be flat to approximately 250 Hz, ensuring that the output voltage phase and amplitude distortion of the pressure transducers was minimized in the frequency range of the vortex shedding. The sampling rate for the reference surface pressure was 10.24 kHz and was synchronised with the PIV measurements using a TTL trigger sent from the PIV system at the start of a measurement.

2.3.1 Methodology of phase analysis & phase averaging

The instantaneous flow variables of a quasi-periodic flow can be decomposed as the sum of a coherent (phase averaged) part and an incoherent (random) part. For the velocity these are

\[
\mathbf{u}(\mathbf{x}, \phi(t), t) = <\mathbf{U}> (\mathbf{x}, \phi(t)) + \mathbf{u}''(\mathbf{x}, t)
\]  

(2-1)

where bold font indicate vectors, \( \mathbf{u} \) is the instantaneous velocity field, \( <\mathbf{U}> \) the phase averaged field, \( \mathbf{u}'' \) is the fluctuation away from the phase average, and \( \phi(t) \) is a given phase reference of the shedding cycle.

To determine the instantaneous phase of the pressure signal, it was first smoothed and local maxima were determined. Signal smoothing was performed by reconstruction of the signal from a truncated Fourier series. Two thousand Fourier modes of the FFT for
a 100,000 point sample were used, corresponding to modes up to approximately 200 Hz, nearly twice the shedding frequency (120 Hz). The phase, $\phi(t) \in [0, 2\pi)$, was then determined by interpolation. For comparison, a Hilbert transform or a wavelet transform with a Morlet of centre-frequency equal to that of the vortex shedding frequency were used to determine the phase. These methods resulted in very similar phase averaged results. A sample time trace of pressure fluctuations ($\Delta p = p - \bar{p}$, where $\bar{p}$ is the time-averaged pressure) normalized by the r.m.s. of pressure fluctuations ($p_{\text{RMS}}$), together with the smoothed truncated Fourier series and the corresponding instantaneous phase of the pressure fluctuation, is shown in Fig. 2-4. For phase averaging, the shedding period was discretized into twenty phase steps, $\phi_n$, $n = 1, \ldots, 20$ ($\Delta \phi = \pi/10$). For each phase step, the ensemble average was undertaken such that any PIV vector field falling within $\phi_n - \Delta \phi/2 < \phi_n < \phi_n + \Delta \phi/2$ was used to obtain statistics (mean and root-mean-square values) for the discrete phase $\phi_n$.

To verify consistency of the phase-averaged results, pressure taps at $z/h = 0.50$ and $z/h = 0.75$ were sampled simultaneously with the reference measurement at $z/h = 0.25$ and showed a high level of coherence, although lower fluctuating energy towards the free-end, along the height of the cylinder.
Fig. 2-4: The phase value for each PIV vector field is determined by interpolating the phase as found from the surface pressure fluctuation ($\Delta p = p - \bar{p}$) signal at $z/h = 0.25$ smoothed using a truncated Fourier series representation with 2000 Fourier modes. $p_{RMS}$ is the r.m.s of pressure fluctuations.

2.4 Results

Velocity and pressure measurement results in the obstacle base region and wake are presented to characterize the vortex shedding process. An overview of the mean flow field is provided first to establish the context. From the instantaneous PIV data, two distinct vortex configurations are identified in the base region. These are then related to the behaviour of the fluctuations of the obstacle surface pressure and wake velocity. A
description of the shedding process consistent with observations in the base and wake regions is developed from a phase-averaged analysis of the velocimetry data.

2.4.1 Mean velocity field

The time-averaged velocity field is presented in terms of the vector components, together with sectional streamlines, in three horizontal planes \( z/h = 0.25, 0.5 \) and 0.75 \( z/h \) and the vertical symmetry plane \( y/d = 0 \) in Fig. 2-5. In Fig. 2-5 (and other figures depicting flow streamlines), the locations of critical points and streamlines were selected manually in Tecplot. In Fig. 2-5, no assumptions of symmetry were made. Asymmetry (defined as the difference in velocity-vector-angle with free stream on opposite wake sides) is less than 3%. The mean flow in the planes \( z/h = 0.25 \) and \( z/h = 0.50 \) are similar. These results are qualitatively similar to the mean flow description provided by Sakamoto and Arie (1983), Wang et al. (2006) or Wang and Zhou (2009) for square section cylinders with similar aspect ratios protruding the boundary layer. Two vortex cores exist in the base region. Downstream of the mean recirculation zone, a positive bifurcation line runs along the obstacle plane of symmetry, \( y/d = 0 \), as indicated by the diverging sectional streamlines (Chong et al., 1990). In the plane \( z/h = 0.75 \), the recirculation length and distance between vortex cores are reduced. An important qualitative difference, relative to lower planes, is that the sectional streamlines converge along \( y/d = 0 \), indicating a negative bifurcation line.

In the vertical plane of symmetry, \( y/d = 0 \), the mean separation streamline from the upper leading edge does not attach (i.e. the recirculation zone is topologically open).
A negative bifurcation line extends from a saddle point and separates regions of up- and downwash.

Fig. 2-5: Mean velocity vectors (normalized by $U_\infty$) and sectional stream lines in the planes a) $z/h = 0.25$, b) $z/h = 0.50$, c) $z/h = 0.75$, d) $y/d = 0$ (the vertical plane of symmetry). For clarification, every second horizontal vector and every third vertical vector is shown, respectively. Blanked area is in the shadow or reflection of the laser light sheet. No assumptions of symmetry were made. Asymmetry (defined as the difference in velocity-vector-angle with free stream on opposite wake sides) is less than 3%.

### 2.4.2 Identification of the two shedding regimes

Approximately eight PIV images per shedding cycle were captured to track vortices in the obstacle base region during the shedding cycle. Upon inspection of PIV snapshots, two configurations for the forming and shedding of base vortices (denoted as regimes A and B) are predominantly observed. Figures 2-6 and 2-7 provide typical samples for the two regimes in the form of sectional streamlines in the stationary
reference frame in the plane \( z/h = 0.25 \) obtained from successive PIV snapshots. Vorticity contours, non-dimensionlized by free-stream velocity and obstacle width \( (\omega^* = \alpha d / U_\infty) \), are shown in colour to highlight the convection of the vortices.

The occurrence of these regimes is closely related to changes in the amplitude of the pressure fluctuations on the obstacle side faces. Time traces of the surface pressure coefficient, \( C_p = (p - p_\infty) / (\gamma \rho U_\infty^2) \), recorded simultaneously on opposing side faces of the obstacle \( (x/d = 0, y/d = \pm 0.50, z/h = 0.25) \) corresponding to PIV sequences in Figs. 2-6 and 2-7 are shown in Fig. 2-8a together with their instantaneous phase difference (Fig. 2-8b) as determined using the wavelet transform (Details of the wavelet transform described in Appendix A). The air density was calculated based on the ambient temperature and pressure readings prior to start of each experiment set. Typically, prolonged periods of high-amplitude fluctuations are observed interspersed with bursts of low-amplitude activity. The occurrence and duration of the low-amplitude intervals vary randomly.

Figure 2-6 illustrates regime A, which is observed throughout periods of high-amplitude fluctuations (HAF). Vortices of opposite sign vorticity are formed and shed alternately in similar fashion to the classical von Kármán process. Briefly, the vorticity from the upper shear layer feeds the nascent clockwise rotating vortex, identified by a focus directly behind the obstacle in Fig. 2-6a. This vortex (N2) grows until it interferes with the shear layer on the opposing side. A saddle point, S, appears as the circulation flux is interrupted to the lee counter-clockwise rotating vortex (P2) which is then shed,
Fig. 2-6b-c. A new counter-clockwise vortex (P$_3$) is formed behind the lower trailing corner and the process is repeated, completing a full shedding cycle. The period of fluctuations is regular and the phase difference between opposing sides is approximately 180° (Fig. 2-8), consistent with well-organized, alternate shedding of counter-rotating vortices.
Fig. 2-6: Samples of PIV sectional streamlines in the stationary frame of reference in the plane $z/h = 0.25$ superimposed on non-dimensional vorticity ($\omega^* = \omega d / U_\infty$) contours representing anti-symmetric formation of vortices (regime A) in the base region. Snapshots separated by $\Delta t = 0.001s$ (approximately one eighth of the period). Vortices with positive and negative sense of rotation are labeled ‘P’ and ‘N’, respectively. Blanked area is in the shadow or reflection of the laser light sheet.
Fig. 2-7: Samples of PIV sectional streamlines in the stationary frame of reference in the plane $z/h = 0.25$ superimposed on non-dimensional vorticity ($\omega^* = \omega d / U_\infty$) contours representing co-existing vortices in the base region (regime B). Snapshots separated by $\Delta t = 0.001\text{s}$ (approximately one eighth of the period). Vortices with positive and negative sense of rotation are labeled ‘P’ and ‘N’, respectively. Blanked area is in the shadow or reflection of the laser light sheet.
Regime B is observed during the side pressure low amplitude fluctuation periods (LAF), and is characterised by the co-existence of two counter-rotating vortices throughout the shedding cycle as illustrated in Fig. 2-7. Upon closer inspection, it is observed that counter-rotating vorticity “patches” detach alternately from the base region, giving rise to an anti-symmetric vortex street as is evidenced in Fig. 2-7c-f. The clockwise vortex (N2) for instance undergoes a partial shedding as it is split into two smaller vortices, N2a and N2b. N2a is then shed out-of-phase with counter-clockwise vortex P2a while N2b and P2b remain in the formation region.

With higher phase-jitter (as evident in Fig. 2-8b), vortices in regime B are shed close to the nominal shedding frequency, f_s. When regime B is present, the phase difference distribution is broader but remains within ±30° of 180°. This process is not regarded as symmetric shedding as the two counter-rotating base vortices are not shed simultaneously.

An approximate quantitative comparison of the shedding process during regimes A and B can be obtained by estimating the circulation content in the formation region. For this purpose, two rectangular areas in the base region were considered as shown in Fig. 2-9a. In each area (box) the magnitude of the circulation was calculated by integrating the vorticity \( \left| \mathbf{\Gamma} \right| = \frac{1}{U_w d} \sum \omega_{i,j} |\Delta A_{i,j}| \) associated with a vortex \( \lambda_2 < 0 \) criterion of Jeong and Hussain, 1995), where \( \omega_{i,j} \) and \( \Delta A_{i,j} \) are the vorticity level and the
area of PIV interrogation windows, respectively. The streamwise extent of the two boxes was chosen as $1.5 < x/d < 2.5$ and $0 < y/d < \pm 2.5$ to cover the formation region.

An illustrative example is given in Fig. 2-9b for a case in the $z/h = 0.5$ plane together with the pressure time traces at $z/h = 0.25$. Similar behaviour is observed at other elevations. Regime A shedding is characterized by higher amplitude out-of-phase fluctuations in circulation with minima close to zero. This behaviour is consistent with the alternate formation and shedding of vortices from the base region. Unlike regime A, the circulation minima in regime B stay at a higher than zero as would be expected for the continuous presence of two vortices in the formation region. The lower amplitude (when compared to regime A) out-of-phase fluctuations thus correspond to the alternate shedding of vorticity patches as described for regime B. Note that a reduction in circulation fluctuation amplitude is consistent with an increase in the cycle-averaged pressure on the side faces since a smaller pressure differential between the windward (front) face and the back face of the cylinder implies a reduction in the circulation flux during the shedding cycle (Roshko, 1954).
Fig. 2-8: a) Pressure traces corresponding to PIV snapshots in Figs. 2-6 and 2-7. Solid and dashed lines represent values on $y > 0$ and $y < 0$ sides, respectively. b) Phase difference, $\Delta \theta$, for the same signals for the total window, HAF and LAF intervals. Vertical bars indicate the range in which PIV measurements in Figs. 2-6 and 2-7 were presented. Solid and dashed lines represent values on $y > 0$ and $y < 0$ sides, respectively.
Fig. 2-9: a) Schematic of the selected boxes in the formation region. b) Magnitude of circulation inside the boxes in Fig. 2-9a together with pressure coefficients on the side face of the cylinder at \( z/h = 0.25 \). Solid and dashed lines represent values on \( y > 0 \) and \( y < 0 \) sides, respectively. Circulation is normalized by \( U_\infty \) and \( d \).
2.4.3 Shedding behaviour outside the formation region

Several earlier studies (Sakamoto and Arie, 1983; Wang and co-workers) discussed the existence of symmetric (in-phase) shedding for surface-mounted square cylinders of similar aspect ratio, noting that the probability of in-phase events increased towards the free-end. These observations were quantified by comparing the relative location of vortex cores of opposite sign vorticity in single, isolated PIV realisations of randomly sampled cycles. Comparing the base vortex configurations at different times during the same shedding cycle of regime B, however, suggests that the true shedding phase is difficult to distinguish in the absence of cycle-resolved sequences. In viewing single snapshots, the vortex configuration in Fig. 2-7a or 2-7f, for example, could be interpreted as out-of-phase while those in Fig. 2-7e or 2-7d as in-phase events. Figures 2-7c and 2-7d show, however, that the counter-rotating vortex pair is observed even as a vortex is shed from the end of the formation region.

The behaviour observed in the velocity fluctuations, obtained in the obstacle wake with HWA, is consistent with the interpretation that vortices of opposite-sign vorticity are shed alternately in both regimes. Consider the juxtaposition of the simultaneous surface pressure and velocity fluctuations, as obtained at locations y/d = ±2, z/h = 0.4 adjacent to the base formation region, x/d = 1 in Fig. 2-10a, and downstream at x/d = 4 in Fig. 2-10b. The time traces of the velocity fluctuations in the vicinity of the obstacle are qualitatively similar to those of the side face pressure fluctuations. The downstream velocity fluctuations, however, show clear periodicity even during low-amplitude events and the
amplitude of the fluctuations is less modulated. For example, during the LAF interval observed in the surface pressure fluctuations (\( \tau_b, 2.09 \text{ s} < t < 2.13 \text{ s} \)) the wake velocity fluctuation amplitude remains similar to that observed during the high amplitude events. During this interval, regime B shedding is mainly observed where those vortices which are shed alternately form behind the base vortex pair result in a clear periodic signature at downstream stations. A similar behaviour is observed near the free end of the obstacle. Figure 2-10c shows sample time traces of velocity at \( z/h = 0.75 \) together with pressure fluctuations at \( z/h = 0.25 \). Within LAF intervals (1.92 s < \( t < 1.94 \text{ s} \)) the periodicity in the pressure signal on the obstacle face is weakened while the periodic shedding activity is still observed at \( x/d = 4 \). In the obstacle base region, the presence of two co-existing vortices throughout the cycle confines the lateral motion of the shear layer such that periodic fluctuations induced by downstream events are masked by local random fluctuations. During the high-amplitude intervals, vortices are formed and shed from the base region impressing a periodic signature on the obstacle faces and wake measurement stations.

While regime B is observed during the LAF intervals, rare events of disorganised activity are also observed. For example, during the interval \( \tau_b, 2.01 \text{ s} < t < 2.02 \text{ s} \), in-phase type events ostensibly occur and the wake velocity fluctuations at \( x/d = 4 \) also display lower amplitude and the phase relationship between opposing sides is altered. These events seem to correspond to a breakdown of the shedding process.
Fig. 2-10: Simultaneous fluctuating surface pressure coefficient and velocity. For all three cases, pressures measured on obstacle side faces at $x/d = 0$, $y/d = \pm 0.5$, $z/h = 0.25$. Velocities measured at $y/d = \pm 2$ and a) $x/d = 1$, $z/h = 0.4$; b) $x/d = 4$, $z/h = 0.4$; c) $x/d = 4$, $z/h = 0.80$. Solid and dashed lines represent values on $y > 0$ and $y < 0$ sides, respectively.
2.4.4 Shedding characteristics along the obstacle height

The instantaneous phase difference of the velocity fluctuations measured with HWA at the same elevation, but on opposite sides of the plane of symmetry \((y/d = 0)\) along the height of the obstacle were measured. Figure 2-11 shows the normalized probability density function, pdf, of the phase difference, \(\Delta \theta\), at \(x/d = 1\), \(x/d = 2\) and \(x/d = 4\). Near the formation region, for example at \(x/d = 1\) in Fig. 2-11a, \(\Delta \theta\) broadens with increasing height. This broadening of \(\Delta \theta\) correlates with an increase in the probability of occurrence of regime B as the free end of the obstacle is approached. The statistics of occurrence of regime B at three heights along the obstacle was estimated in two ways: first by associating sequence events where the circulation fluctuation amplitude was reduced (see Fig. 2-9b) and, second, by visual inspection of 5000 PIV event frames. The statistics agreed well between both approaches. Regime B occurred approximately 6%, 11% and 14% of the total observation time at \(z/h = 0.25\), 0.50 and 0.75, respectively. Note that LAF events represent 15% of the total observation window. Hence, there is a distinction between HAF/LAF periods and regimes A/B. During HAF intervals, only regime A is present whereas during LAF, both regimes A and B can be observed.

The pdfs also provide further evidence that counter-rotating vortices are mainly shed alternately (i.e. out-of-phase) in both regimes as the mean phase difference remains around 180° at all elevations and downstream locations. Downstream of the formation region, \(\Delta \theta\) changes little with elevation and the probability of in-phase (\(\Delta \theta < 90^\circ\) or \(\Delta \theta > 270^\circ\)) events is insignificant (Fig. 2-11c). For \(z/h > 0.5\), the occurrence of in-phase
events is no longer negligible, but remains small never exceeding 2% of the total observations.

Fig. 2-11: Normalized probability density functions (pdf) of $\Delta \theta$ along the obstacle height a) $x/d = 1$, b) $x/d = 2$, c) $x/d = 4$. Dashed vertical line represents 180°.

Power spectral density functions (psdf) of the velocity fluctuations, obtained in the wake at $x/d = 1$, $x/d = 2$ and $x/d = 4$ using a single-wire HWA, are shown in Fig. 2-12. These spectra are averaged over at least 10 realizations each 10 s long. The frequency at the psdf peak, which corresponds to $St = 0.100\pm0.003$, is associated with the periodic shedding of vortices. The shape of the psdf changes little with elevation. There is little evidence of spectral broadening at higher elevations as would be expected if the shedding
regime were altered or cellular-type shedding were to occur, suggesting that the shedding process over much of the obstacle height remains coherent.

Fig. 2-12: Power spectral density functions for hotwire measurements at \( y/d = 1.2 \). a) \( x/d = 1 \), b) \( x/d = 2 \), c) \( x/d = 4 \). Spectra are off-set by constant factor for clarity.

Further analysis of the shedding process during LAF or HAF intervals would benefit from conditional averaging. To determine a suitable criterion for synchronizing the velocity field measurements at different planes, the fluctuating surface pressure on the side faces of the obstacle at different heights is considered. Figure 2-13 shows fluctuating pressure time traces at three positions (\( x/d = 0; \) \( z/h = 0.25, 0.5 \) and 0.75) along the obstacle height. Generally HAF and LAF intervals coincide suggesting that the changes that characterize LAF and HAF intervals are coherent over the obstacle height.
Closer consideration of the surface pressure fluctuations, \( C'_p = (p - \bar{p})/\left(\frac{1}{2} \rho U^2 \right) \), of Fig. 2-13 also shows that the median pressure (cycle-average about which the pressure fluctuates) is consistently higher during LAF than HAF intervals. Transition in the median pressure levels are also strongly correlated over the obstacle height on both side faces. Test results also showed that an equivalent separation of the HAF and LAF intervals could be achieved by applying a peak-to-peak amplitude threshold to the signals at \( z/h = 0.25 \) or 0.50. It is thus possible to separate the HAF and LAF intervals using a single reference.

Fig. 2-13: a) Simultaneous surface pressure traces at \( z/h = 0.25, 0.5, \) and 0.75. \( C'_p \) values are off-set by 0.51\( C'_p \) for clarity. Moving average over ~3 cycles shown. b) Juxtaposition of moving averages at \( z/h = 0.25, 0.50, \) and 0.75.
2.4.5 Phase-averaged flow

A phase-averaged decomposition of the velocity field during HAF and LAF intervals was chosen to represent the average shedding process. To separate high and low amplitude events, an amplitude based criterion was used. The peak-to-peak $C_p$ fluctuation amplitude during LAF is typically below 0.17 (20 Pa), while for HAF the peak-to-peak fluctuation amplitude varies from 0.5 – 1.2 (60 – 140 Pa). The fluctuation peak-to-peak amplitude was determined for each cycle of the pressure fluctuations at $z/h = 0.25$. The threshold amplitude was set in a range over which there was little variation in the number of cycles associated with the high amplitude fluctuations. Approximately 85% of the cycles were associated with HAF.

The Fourier spectra shown in Fig. 2-12 are insufficient to determine if periodicity is sustained during the LAF intervals. To this end, wavelet spectra during HAF and LAF intervals are shown in Fig. 2-14. During the LAF intervals, the psdf shows an increase of the fluctuation energy levels about the same shedding frequency, $f_s$, as HAF events, although the energy levels are lower. These results are consistent with the pressure fluctuation observations, showing lower fluctuation amplitude during regime B events (Fig. 2-8). Also to note is the energy accumulation at approximately $f_s/10$. Intervals of low-amplitude fluctuations (LAF) typically last between 10 and 20 nominal shedding periods ($1/f_s$). Hence, this energy accumulation is associated with the envelope of the pressure amplitude modulation. Similar accumulations have been observed in the wake of
two-dimensional obstacles and have been associated with vortex dislocations (Wu et al., 2005).

Fig. 2-14: Wavelet spectra during HAF and LAF intervals for surface pressure signals at $z/h = 0.25$. A complex Morlet was used with a mother wavelet of bandpass frequency 1.5 and a centre frequency of 1.

Phase-averaged results of PIV measurements in three horizontal planes and in the vertical plane of symmetry are presented in this section to characterize the vortex formation and shedding process during high and low-amplitude fluctuation intervals. The averages were constructed from PIV measurements collected over 500 to 2000 shedding cycles and distributed over 20 equal phase bins per shedding cycle as described in the experimental section. Ensemble averages were obtained in each bin. Results are presented in the stationary and in the convective reference frames.

Figure 2-15 shows sectional streamlines in the stationary reference frame superimposed on vorticity contours ($\omega^* = \omega D / U_\infty$) in the planes $z/h = 0.25, 0.50$ and
0.75 for HAF at different phases during the shedding cycle. As expected, the phase-averaged representation of the vortex formation and shedding process in the stationary frame resembles the von Kármán process. Once shed, the vortices convect quickly such that their passage can not be gauged (through the appearance of saddles and nodes in the streamline pattern) in the stationary frame. Rather, in the stationary frame, the vortex passage can only be distinguished from the meandering in the velocity pattern in the wake. However, when viewed in the convective reference frame (Fig. 2-16), the streamlines and vorticity cores align. The convective velocity was taken to be the velocity at the centre of a vortex as identified using the minimum $\lambda_2$-criterion (Jeong and Hussein, 1995). The celerity of the shed vortices was also calculated using the centroid of conditional vorticity contours and Taylor’s hypothesis of “frozen flow field convection” and corroborated the value obtained using the minimum $\lambda_2$-criterion. The celerity asymptotically approaches a constant value of $u_c \approx 0.80 U_\infty$ (Fig. 2-17) which is similar to previously reported for convective velocity of vortices for two-dimensional square cylinder ($u_c \approx 0.78 U_\infty$ for Lyn et al., 1995; $u_c \approx 0.76 U_\infty$ in Bearman and Trueman, 1972) and slightly lower than those for the circular cylinder ($u_c \approx 0.83 U_\infty$ in Cantwell and Coles, 1983; $u_c \approx 0.87 U_\infty$ in Zhou and Antonia, 1992).

Significant differences are observed when comparing the different horizontal planes. In the plane $z/h = 0.25$, counter-rotating vortices are convected in a classical staggered arrangement. Three-dimensional effects are evidenced in the higher planes. For $z/h = 0.50$, two co-rotating vortex cores are observed (both moving at $u_c$), while in the
plane $z/h = 0.75$ the vortex cores cannot be identified although the vortex induced meandering of the bifurcation line is easily recognised.

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**Fig. 2-15**: Phase averaged non-dimensional vorticity ($\omega^* = \omega d / U_\infty$) field superimposed on sectional streamlines over one shedding cycle for high amplitude fluctuations (HAF). Streamlines in stationary frame of reference. Blanked area is in the shadow or reflection of the laser light sheet.
Fig. 2-16: Phase averaged non-dimensional vorticity ($\omega^* = \omega d / U_\infty$) field superimposed on sectional streamlines over one shedding cycle for high amplitude fluctuations (HAF). Streamlines in convective frame of reference ($u_c = 0.8$). Blanked area is in the shadow or reflection of the laser light sheet.
Fig. 2-17: Calculated convective velocity at $z/h = 0.25$ and 0.50 planes from PIV measurements.

The behaviour observed in the plane $z/h = 0.75$ is related to a reorientation of the initially flow-normal vortex structures resulting in the tip of these structures tilting upstream immediately downstream of the obstacle. Comparing the location of the phase-averaged vortex cores in the horizontal planes at different elevations provides further indication that the shed structures are inclined. Figure 2-18 shows contours of the normal component of vorticity and the $\lambda_2 = 0$ iso-levels from which the location of the vortex cores can be estimated in three horizontal planes at the same shedding phase. The relative location of the cores in the obstacle base region is vertically aligned, but once shed the vortices in the upper planes lag those in the lower planes.
Fig. 2-18: Non-dimensional contours of the z-component vorticity of phase averaged measurements at \( z/h = 0.25, 0.5, \) and 0.75 (from top to bottom). Bold lines represent \( \lambda_2 = 0 \) boundaries. Contour increments are 0.2. Blanked area is in the shadow or reflection of the laser light sheet. All the measurements are taken from a common relative phase of \( \phi = \pi/20 \).
Further evidence of the tilting of the shed structures is provided by considering the average phase difference relative to a fixed reference location \((x/d = 0.5, y/d = -1.2, z/h = 0.5)\), as obtained from HWA and shown in Fig. 2-19 as a function of elevation for several downstream locations along the plane \(y/d = 1.2\). For any given downstream location \(x/d\), the phase difference remains nearly constant up to \(z/h \approx 0.4\), above which there is an increasing lag as the tip region is approached. The estimated phase lag from the phase-averaged fields (Fig. 2-18) matches, within the experimental uncertainty of \(\pm 5^\circ\), those observed from HWA.

Fig. 2-19: Downstream phase difference relative to a fixed hotwire located at \(x/d = 0.5, y/d = -1.2, z/h = 0.5\) locations. \(Re_d = 6400\) shown with solid lines and closed symbols and \(Re_d = 12,000\) with broken lines and open symbols.

The observations drawn from Fig. 2-19 are consistent with the view that the shed vortices remain coherent along the height and, after an initial tilting/deformation in the formation region, the structures are convected at a constant rate downstream, \(u_c\), as
supported by the results shown in Fig. 2-17. In the formation region \((x/d = 0.5 \text{ and } 1)\), the phase does not change in the streamwise direction, suggesting that there is little motion of the vortices. Downstream of the formation region, however, the phase-curves are uniformly off-set and directly proportional to the streamwise location. Towards the free end, shed vortices cannot be seen since the vortices shed tend to be convected downwards, out of view of planes above \(0.75h\) such that defining a convective velocity in this plane may not be physical.

In the plane \(z/h = 0.5\), the appearance of a secondary core (\(V2\) in Fig. 2-18) is related to the existence of vorticity strands interconnecting opposing shed vortices. This three-dimensional nature of the shedding process is illustrated in the phase-averaged sequence in the plane \(y/d = 0\) of Fig. 2-20. The tilting of the shed vortices is observed as cross-stream vorticity in the lee of the free end. According to Helmholtz’s vortex theorem, vortex lines must either extend to the boundaries of the fluid or form a closed loop. To satisfy the solenoidal condition on the vorticity field, the cross-stream vorticity lines interconnect the opposing shed structures, effectively forming strands crossing the plane \(y/d = 0\). These strands travel downstream and drop towards the plate along the separated shear layer. Over one cycle, two connecting strands with the same sense of cross-stream vorticity are observed in \(y/d = 0\), one strand for each vortex shed with opposing sign of vorticity normal to the ground plate (Bourgeois et al., 2011).
Fig. 2-20: Phase-averaged contours of cross-stream vorticity component in the vertical symmetry plane $y/d = 0$. Blanked area is in the shadow or reflection of the laser light sheet.

The passage of these strands downstream of the recirculation region provides an additional coherent fluctuation contribution and modifies the wake structure. Evidence of this three-dimensional effect can be observed, for example, in the wake distribution of the lateral velocity fluctuation components, $\sqrt{v'^2}$, shown for three horizontal planes in Fig. 2-21a-c. The location of the maximum for $\sqrt{v'^2}$ is generally associated with the end of the formation region in two-dimensional flows (Cantwell and Coles, 1983; Griffin, 1995) and is typically located in the downstream proximity of the end of the mean recirculation region along $y/d = 0$ (cf. Cantwell and Coles, 1983). In the planes $z/h = 0.25$ and $0.75$ the maxima for $\sqrt{v'^2}$ generally occur at the end of the mean recirculation zone. In contrast, for $z/h = 0.5$, the maximum for $\sqrt{v'^2}$ is shifted significantly downstream. Note that the
core of the strands remain above the bifurcation line in the plane $y/d = 0$ (Fig. 2-5d) such that these are not seen in the plane $z/h = 0.25$. At $z/h = 0.25$ the shedding process is thus nearly two-dimensional and the maximum of $\sqrt{\nu^2}$ occurs at the end of the recirculation region.
Fig. 2-21: Contours of non-dimensionalized $\sqrt{\nu'^2}$ at $z/h = 0.25$, 0.50 and 0.75 (contour increment of 0.02). The position of the end of the recirculation zone is shown with separatrices of the wake saddle point in the contour plots and with an arrow in the graph of the distribution of $\sqrt{\nu'^2}$ along the mean symmetry plane ($y/d = 0$). Assumption of flow symmetry along $y/h=0$ was made to reduce measurement uncertainty.
The sectional streamlines and coloured vorticity contours for the phase-averaged sequence during the LAF intervals in the planes $z/h = 0.25, 0.50$ and $0.75$ are shown in Fig. 2-22 in the stationary frame of reference. In the plane $z/h = 0.25$, the alternate formation and shedding of counter-rotating vortices is similar to that observed during HAF intervals (Fig. 2-15a and 2-16a). In contrast, at higher elevations (Fig. 22b, c), two vortex cores always co-exist in the formation region and opposite sign vorticity is shed alternately from the end of the formation region. To identify vortex arrangement in terms of saddles and nodes in the flow field, sectional streamlines in the convective frame of reference are presented in Fig. 23. In the convective reference frame, a staggered arrangement of counter-rotating vortices is again observed in the planes $z/h = 0.25$ and $0.50$ (for $z/h = 0.75$ the flow pattern is similar to Fig. 2-16c and is not shown for brevity). The estimated celerity of the vortices does not differ significantly from that observed during HAF intervals. The phase-averaged representation during the LAF intervals is thus consistent with regime B observations, noting the increased probability of observing co-existing vortex cores towards the free-end.
Fig. 2-22: Phase averaged non-dimensional vorticity field \( \omega^* = \omega d / U_\infty \) superimposed on sectional streamlines over one shedding cycle for low amplitude fluctuations (LAF). Streamlines in stationary frame of reference. Blanked area is in the shadow or reflection of the laser light sheet.
Fig. 2-23: Phase averaged non-dimensional vorticity field ($\omega^* = \omega d / U_\infty$) superimposed on sectional streamlines over one shedding cycle for low amplitude fluctuations (LAF). Streamlines in convective frame of reference ($u_c=0.8$). Blanked area is in the shadow or reflection of the laser light sheet.

For the turbulent vortex shedding behind a two-dimensional plate normal to the flow, Wu et al. (2005) associated LAF intervals with a disruption of the regular shedding process. This earlier work also consolidates observations from several studies on two-dimensional obstacles and attributes bursts of low amplitude activity correlated over a large span of the obstacle to vortex dislocations. Typically, the abrupt dislocation,
characterized by a highly three-dimensional wake, is followed by a readjustment period lasting 10 to 20 shedding cycles. This interpretation is consistent with present observations, where the dislocation is a short lived (and thus rare) event during which multiple vortices may be randomly shed, followed by a LAF readjustment period.

Thus, by extension, during a readjustment of the vortex shedding during the LAF intervals, it would be expected that the vortex shedding would be weaker. This was already suggested from the circulation evolution in the base region in Fig. 2-9. A more robust statistical verification of this statement can be obtained from the normalized circulation of the phase averaged field at $z/h = 0.25$, shown in Fig. 2-24. The vortex strength was estimated from the surface integral of vorticity inside the vortex area defined by $\lambda_2 < 0$. Generally, the vortex strength is observed to be lower for LAF than for HAF at all streamwise locations as would be expected from the partial shedding of vorticity during regime B as illustrated in Fig. 2-9. The downstream decay of vortex strength is expected to be due to the turbulent dissipation of vorticity as observed in phase-averaged measurements of Lyn et al. (1995) in the wake of a square cylinder.
Fig. 2-24: Phase-averaged vortex strength (\( \int \omega_z dA \), where \( A \) vortex area defined by \( \lambda_2 < 0 \)) at \( z/h = 0.25 \) in HAF and LAF normalized by free-stream velocity and obstacle width. \( |\Gamma| \) is normalized by \( U_\infty \) and \( d \).

As described by Noack et al. (2003), the LAF intervals may be viewed as a transition or shift between two states of the mean field, a disrupted state and a shedding state. The disrupted state would be associated with a longer recirculation zone and an increased base pressure compared with the shedding state. This is precisely what is found for the recirculation length in the conditional averaged fields during LAF and HAF intervals shown in Fig. 2-25 and the higher cycle-averaged pressure in Fig. 2-13.
Fig. 2-25: Time averaged recirculation zone at $z/h = 0.25, 0.50$ and $0.75$ in HAF (left) and LAF (right).

2.5 CONCLUDING REMARKS

The vortex formation and shedding process in the turbulent wake of a surface-mounted, square-section vertical cylinder of aspect ratio 4, placed in a thin, developing boundary layer of thickness $\delta \approx 0.18h$ was investigated experimentally. The surface pressure fluctuation signals on the side faces of the obstacle and velocity fluctuations adjacent to the formation region are characterised by extended periods of high-amplitude fluctuations (HAF) interspersed by bursts of randomly occurring intervals of low-amplitude fluctuations (LAF) lasting between 10 and 20 nominal shedding periods $(1/f_s)$. Two prevalent shedding regimes in the wake are observed. Regime A is akin to the von
Kármán shedding process. In regime B, a co-existing pair of counter-rotating vortices is observed in the formation region throughout the shedding cycle, while counter-rotating vortices are alternately shed from the end of the formation region. The vortices are shed at, or very close to, the nominal shedding frequency, but an increase in the phase jitter is also observed.

The transition between HAF and LAF intervals is strongly correlated over the obstacle height and HAF intervals are observed 85% of the time. During HAF, regime A is exclusively observed and the phase difference between surface pressure and velocity fluctuations on opposing sides of the geometric plane of symmetry remains nearly 180° (out-of-phase shedding). In the LAF intervals, brief periods of disorganised wake activity are observed in which the shedding process appears interrupted. These periods are associated with large excursions from 180° (more than ~30°). While these periods are observed with greater probability as the free-end is approached, they remain rare occurring less than 2% of the total observation window. During the remainder of the LAF intervals, both regime A and B can be observed. However, regime B occurs with increasing probability at higher planes. Near the free end, regime A is rarely observed during LAF intervals. The phase relation between pressure or velocity fluctuations on opposing sides of the obstacle vary within ± 30° around 180°, during regime B and is associated with the broadening of the phase difference pdf as the free-end is approached.

Several earlier studies (Wang and Zhou, 2009; Wang et al, 2006) have reported the occurrence of symmetric shedding events, with increasing probability towards the
free-end, largely based on low Reynolds number visualisations and isolated PIV snapshots. It was also suggested that the more periodic (regular) nature of the velocity fluctuations in the wake was due to a reorganisation of the symmetrically shed vortices. The co-existing vortex pairs during regime B can explain the aforementioned observations. Furthermore, the present results show that the velocity fluctuations are quite periodic immediately downstream of the formation region, which is consistent with the alternate shedding of vortices observed in regime B, rather than a gradual downstream adjustment.

It is suspected that a triggering mechanism originating from the obstacle free end interferes with the regular von Kármán-type shedding (regime A) resulting in a low-frequency modulation in the shedding amplitude and a modification in the vortex configuration in the base region in regime B. Whether this mechanism is akin to vortex splitting/dislocation reported for 2-D geometries (Williamson, 1992; Najjar and Balachandar, 1988) or other distinct phenomena specific to surface-mounted finite cylinders requires further clarification and is the subject of on-going work.
2.6 REFERENCES


Chapter Three: Growth and separation of a start-up vortex from a two-dimensional shear layer

The experiments reported in this Chapter were carried out by Pooria Sattari at the Institute for Fluid Mechanics and Aerodynamics at the Technische Universität Darmstadt. This Chapter is a paper published as:


1Department of Mechanical and Manufacturing Engineering, University of Calgary, Calgary, AB T2N 1N4, Canada
2Institute for Fluid Mechanics and Aerodynamics, Center of Smart Interfaces, Technische Universität Darmstadt, 64287 Darmstadt, Germany

Some modifications have been made to the originally published paper in response to queries raised by candidate’s Ph.D. examination committee.
3.1 ABSTRACT

The evolution of an isolated line vortex generated by a starting two-dimensional jet is studied experimentally using time-resolved particle image velocimetry. The vortex growth in this current configuration is not linked to any externally imposed length scales or interactions with other vortical structures or walls that could potentially influence vortex growth. A model for the early-stage vortex growth, based on the transport of circulation from the shear layer into the vortex, is proposed and found to agree well with experimental data. The model provides a scaling scheme for vortex growth using shear-layer characteristic velocity and shear-layer thickness. The vortex growth is limited through a gradual separation of the vortex from the feeding shear layer, arising from decreased shear-layer curvature. This phenomenon is linked to a competition between the shear-layer tendency to remain in the streamwise direction and the induced velocity from the vortex on the shear layer. Finally, a dimensionless number representing this competition is introduced, which in turn is able to describe the gradual separation of the vortex from the shear layer.

3.2 INTRODUCTION

The impulsive start of a shear layer typically initiates roll-up leading to the formation of a vortex. The vortex will grow in size and strength as long as there is a circulation flux into the vortex. Reaching higher levels of vortex circulation on lifting surfaces can be advantageous for lift and thrust production [1-2]. For instance, Milano
and Gharib [3] showed that for a two-degree-of-freedom flapping plate, maximum average lift was generated for flapping kinematics, which produced leading-edge vortices of maximum circulation. In another example, a cylinder-piston apparatus was used by Krueger [4] to conclude that by maximizing the size of the vortex ring, the efficiency of momentum transport could be optimized.

In practice vortex growth is limited. Gharib et al. [5] studied the vortex-ring formation through the ejection of fluid from a circular orifice and observed a limit to the vortex-ring growth for piston-stroke-to-diameter ratios of approximately $L/D \geq 4$. Further circulation was observed to be rejected, which accumulated in Kelvin-Helmholtz-type vortices formed in a trailing shear layer behind the vortex ring [5-6]. This interruption of the feeding mechanism is termed the “pinch-off” and has been related to the Kelvin-Benjamin variational principle [5], which states that a steady axis-touching vortex ring has maximum energy compared to other vorticity arrangements with the same impulse. Using a slug model to quantify the shear-layer energy, Gharib et al. showed that the pinch-off occurred at a critical dimensionless time, $T^* = \frac{t}{D} \approx 4$, where $U_p$ is the average piston velocity and $t$ represents time elapsed from the start of the vortex formation. This critical formation time was termed the formation number. Shusser and Gharib [7] showed that the pinch-off can be explained equivalently in terms of the translational velocity of the vortex ring exceeding the shear-layer velocity due to self induction. Based on this latter explanation, strategies to delay the pinch-off were proposed by temporarily varying the piston exit diameter [8-9].
The limitation of vortex growth has been observed also in configurations involving moving objects either from rest or with periodic motion. Jeon and Gharib [10] observed a saturation of the vortex circulation in the wake of a circular cylinder starting from rest. The saturation occurred after the cylinder had moved four cylinder diameters ($d$), i.e. a formation time of $U \theta / d = 4$, where $U$ is the average cylinder velocity. A similar behavior was observed for finite-aspect-ratio plates moving from rest [11] and for a plate rotating around one of the plate edges [12]. In the latter case, formation times on the order of 0.4-0.9 were reported, smaller than observed in previous studies [10-11].

In the search for a unifying principle, Dabiri proposed a formation number as $\hat{T} = C \Gamma / UD \approx 4$, where $C$ is a configuration-based constant, $\Gamma$ is the vortex strength, and $U$ and $D$ are the shear-layer feeding velocity and length scale, respectively [1]. The proposed formation number, however, is ambiguous since the constant $C$ is configuration-dependent and not known for different vortex-generating systems. No explanation was proposed by Dabiri as to how $C$ may be calculated. Furthermore, several recent studies on identical vortex-generation configurations (where $C$ is presumably constant) indicate a need for a reassessment of the concept of universal formation number. For the case of a plunging airfoil Rival et al. [2] observed a saturation of the leading-edge vortex at a formation number of $\hat{T} = 4$. However, higher formation numbers, up to $\hat{T} = 6$, were observed for plunging airfoils in tandem configuration [13].

The notion of a universal formation number implies that for a configuration of length scale $D$, the vortex is unable to grow beyond a given limit. The length scale $D$ is typically geometry-related, e.g. nozzle opening, cylinder diameter, plate width or airfoil...
chord. We refer to these length scales associated with the geometry of the vortex generator as “natural” length scales.

In the studies by Pedrizzetti [14] and Domenichini [15], the vortex formation from a two-dimensional orifice was investigated. Interestingly, a pinch-off process was not observed, but rather the vortex remained attached to the orifice edge by a shear layer whose velocity was larger than the convective velocity of the vortex. The same behavior was observed by Afanasyev for a two-dimensional stratified flow generated by a finite-size planar nozzle [16].

Although in the studies by Pedrizzetti and Domenichini no comment on the relationship between pinch-off and length scales of any kind is made, their results suggest that the existence of a natural length scale—in this case nozzle size— is not a sufficient condition for pinch-off. However, the question as to whether the existence of a natural length scale is a necessary condition for a limit to vortex growth remains unanswered. Thus, the purpose of the present work is to investigate the formation process of a two-dimensional line vortex in isolation in which there are no length scales externally imposed by the vortex-generation apparatus. An approximation to the two-dimensional vortex generation was simulated through the start up of a planar two-dimensional jet. The jet flow is generated by a dielectric barrier discharge (DBD) plasma actuator [17]. The plasma actuator was convenient due to its ability to provide a rapid shear layer on demand. In contrast to complex, three-dimensional vortex formation associated with flapping flight, e.g. spanwise influence on leading-edge vortex formation as studied by
Beem et al. [18], the current study focuses on a two-dimensional vortex-formation process with no wall (wing) or vortex interaction (tip/trailing edge).

A model is proposed for the early-stage vortex formation. The model is based on Kaden’s [19] vortex growth formulation and takes into account non-steady flow variations. This growth model will be shown to be consistent with experimental data obtained with particle image velocimetry (PIV). In addition to investigating the growth process of the vortex, the separation mechanism of the vortex from the shear layer is also discussed.

### 3.3 Experimental Setup & Methodology

**3.3.1 Vortex generation**

The two-dimensional planar jet is generated using a dielectric barrier discharge plasma actuator. The actuator consists of two electrodes separated by a dielectric material taped on the surface of a base plate. When a high alternating voltage is applied between the electrodes, a charge build-up on the dielectric surface occurs and causes ionization of the surrounding fluid molecules. The charged molecules are accelerated in the electromagnetic field and by collision with neutral molecules they transfer momentum into the fluid. In quiescent air, this leads to a starting jet flow with a maximum velocity above the wall surface rolling up into a vortex (Fig. 3-1), as also observed by Kotsonis et al. [20]. By adjusting the high voltage of the plasma actuator, it is therefore possible to adjust the maximum velocity of the shear layer.
Figure 3-1 shows a schematic of the plasma actuator. The electrodes were installed as close as possible, e.g. 16 mm (this distance was unavoidable due to the width of electrodes) to the trailing edge of a flat plate in order to avoid wall influence on the vortex generation. The trailing edge of the plate was sharpened to 10 degrees to fix flow separation at the plate trailing edge. The base plate material was black acrylic glass (Plexiglas) to eliminate laser-light reflections. The plasma actuator spanned the entire 0.15 m plate width. A large electrode aspect ratio (30:1) was chosen to reduce three-dimensional effects due to the actuator-end effects. At the actuator ends, the vortex lines may reorient to satisfy Helmholtz’s vorticity theorem, which states that vortex lines must either extend to the boundaries of the fluid or form a closed loop. The flow twodimensionality was confirmed by evaluating the two-dimensional continuity equation \( \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \), which was found to be negligible (close to zero) in the domain of the present experiments. Further details regarding the electrical specifications of the plasma actuator are described in Kriegseis et al. [17]. The dimensions of the plasma actuator and the electrical specifications were selected according to the existing experience within the Aerodynamics research group at TU Darmstadt for optimized performance of plasma actuators. The flat plate with plasma actuator was mounted on a three-axis manual traverse and placed in a 1.1 m × 0.5 m × 0.5 m sealed container made out of transparent acrylic glass, as shown in Fig. 3-2a. The tank width was three times larger than the width of the plasma actuator and the tank length was twenty times larger than the length of the
shear layer at maximum growth, to eliminate any confinement and background-flow effects in the tank.

The evolution of an isolated line vortex generated by a starting two-dimensional jet is studied experimentally using time-resolved particle image velocimetry. The vortex growth in this current configuration is not linked to any externally-imposed length scales or interactions with other vortical structures or walls that could potentially influence vortex growth. A model for the early-stage vortex growth, based on the transport of circulation from the shear layer into the vortex, is proposed and found to agree well with experimental data. The model provides a scaling scheme for vortex growth using shear-layer characteristic velocity and shear-layer thickness. The vortex growth is limited through a gradual separation of the vortex from the feeding shear layer, arising from decreased shear-layer curvature. This phenomenon is linked to a competition between the shear-layer tendency to remain in the streamwise direction and the induced velocity from the vortex on the shear layer. Finally, a dimensionless number representing this competition is introduced, which in turn is able to describe the gradual separation of the vortex from the shear layer.

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A model is proposed for the early-stage vortex formation. The model is based on Kaden’s [19] vortex growth formulation and takes into account non-steady flow variations. This growth model will be shown to be consistent with experimental data obtained with particle image velocimetry (PIV). In addition to investigating the growth process of the vortex, the separation mechanism of the vortex from the shear layer is also discussed.

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Figure 3-1 shows a schematic of the plasma actuator. The electrodes were installed very close to the trailing edge of a flat plate in order to avoid wall influence on
the vortex generation. The trailing edge of the plate was sharpened to 10 degrees to fix flow separation at the plate trailing edge. The base plate material was black acrylic glass to eliminate laser-light reflections. The plasma actuator spanned the entire 0.15 m plate width. The electrode aspect ratio was chosen as 30:1 to ensure two-dimensionality. Further details regarding the electrical specifications of the plasma actuator are described in Kriegseis et al. [17]. The flat plate with plasma actuator was mounted on a three-axis manual traverse and placed in a 0.5m × 0.5m × 1.1m sealed container made out of transparent acrylic glass, as shown in Fig. 3-2a.

Fig. 3-1: Layout of plasma actuator for the impulsive generation of an isolated two-dimensional line vortex. Note wall-jet and free-jet profiles shown for sake of clarification.

As summarized in Table 3-1, the shear-layer maximum velocity was controlled by the output voltage and ranged from 3.1m/s to 5.0m/s. The operating frequency of the plasma actuator was kept constant at 11 kHz.
Table 3-1. Plasma actuator voltage and shear-layer characteristics for different test cases.

<table>
<thead>
<tr>
<th>Case</th>
<th>Output Voltage, kV</th>
<th>shear-layer velocity, m/s</th>
<th>shear-layer thickness, mm</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>mean</td>
<td>STD</td>
<td>mean</td>
</tr>
<tr>
<td>A</td>
<td>12.27</td>
<td>0.05</td>
<td>3.1</td>
</tr>
<tr>
<td></td>
<td>0.96</td>
<td>0.12</td>
<td></td>
</tr>
<tr>
<td>B</td>
<td>13.41</td>
<td>0.05</td>
<td>3.5</td>
</tr>
<tr>
<td></td>
<td>0.99</td>
<td>0.13</td>
<td></td>
</tr>
<tr>
<td>C</td>
<td>14.65</td>
<td>0.05</td>
<td>4.2</td>
</tr>
<tr>
<td></td>
<td>1.06</td>
<td>0.15</td>
<td></td>
</tr>
<tr>
<td>D</td>
<td>16.24</td>
<td>0.05</td>
<td>5.0</td>
</tr>
<tr>
<td></td>
<td>1.10</td>
<td>0.22</td>
<td></td>
</tr>
</tbody>
</table>

3.5.2 Particle image velocimetry

Time-resolved PIV was used to study the evolution process of the vortex. Of special interest was the characterization of the vortex growth and shear layer. Since shear-layer thickness was typically one order of magnitude smaller than the vortex core, two cameras were used simultaneously on opposite sides of container: one camera with a larger field of view (FOV=80 mm × 50 mm) was employed for vortex growth characterization; a second camera with a smaller field of view (FOV=20 mm × 12 mm) was focused on the shear layer to resolve its velocity profile directly. The camera arrangement and relative locations of the fields of view are shown in Figs. 3-2a and 3-2b, respectively. The camera FOVs were aligned with the x-y axes.

A Dantec Dynamics A/S time-resolved PIV system was used for velocity field measurements. A Nd:YLF Litron dual-cavity laser (LDY303) with a maximum power of 70 W and a maximum output energy of approximately 10 mJ per pulse was operated at 3 kHz providing a 527-nm laser light sheet with a thickness of approximately 2 mm. Images were captured by two Phantom V12.1 CMOS cameras at 3 kHz in the double-frame mode with \( dt = 50 \mu s \). To allow for larger particle displacement every first image of
the image pairs was used to calculate velocities in the larger field of view. This effectively provided $dt = 333\mu s$ ($1/3000$ s) for the zoomed-out field of view. The cameras had a $1280\times800$ pixel resolution (full sensor used). Micro Nikkor 105-mm lenses were used for both cameras, although at different distances from the laser sheet. Image correlations were carried out using DynamicStudio v3.14 software. The vector fields in both fields of view were calculated using an adaptive correlation (with two refinement steps starting from $128\times128$ pixel) and a final $32\times32$ pixel interrogation window size with $50\%$ overlap. This resulted in vector separations of $0.3$ mm and $1$ mm in the smaller and larger FOVs, respectively. Calculations of maximum shear-layer velocity were repeated using a $16$ by $16$ pixel interrogation window. The difference with original calculations (from $32$ by $32$ pixel window) was approximately $2\%$. Therefore, the current spatial resolution sufficiently resolves the velocity gradients inside the shear layer. The adaptive correlation method calculates the velocity vectors with an initial interrogation area (IA) of the size four times the size of the final IA and uses the intermediary results as information for the next IA of smaller size, until the final IA size is reached. A local neighbourhood validation method with a $3\times3$ filter was used to lightly smooth the vector fields in order to more clearly define the vortex core. In this method, individual vectors are compared to the local vectors in the neighborhood vector area. If a spurious vector is detected, it is removed and replaced by a vector, which is calculated by local interpolation of the vectors present in the $3\times3$ area. Interpolation was performed using moving average methodology and a $0.15$ velocity gradient acceptance factor. Typically,
5% of the vectors in the larger FOV and 2% of the vectors in the smaller FOV were replaced.

The accuracy of measured velocity field was estimated to lie below 1% of the maximum recorded velocity, assuming a maximum sub-pixel interpolation accuracy of 0.2 pixel, see Raffel et al. [21]. Subsequently, random errors in the vorticity and circulation due to PIV resolution could be estimated to be \( \Delta \omega u_{\text{max}} / r = \pm 0.01 \) and \( \Delta \Gamma / u_{\text{max}} r = \pm 0.025 \), respectively, where \( r \) is the average vortex core radius, \( u_{\text{max}} \) is the maximum shear-layer velocity and \( \Gamma \) is the vortex circulation. A central differencing scheme with second-order accuracy was used to estimate the velocity gradients. This lead to a systematic error in circulation and shear-layer thickness which is estimated to be less than \( \pm 5\% \) in both cases. To ensure repeatability, the experiments were repeated four times for case A. The overall uncertainties in vortex radius and circulation (95% confidence level corresponding to student t function, \( t (3) = 2.353 \)) were lower than \( \pm 14\% \) for radius and \( \pm 12\% \) for circulation among these repeated tests.

A Dantec Dynamics A/S high-volume liquid seeding generator (model 10F03) was used to generate seeding particles at approximately 1\( \mu \)m in diameter from the DEHS (Di-Ethyl-Hexyl-Sebacat) solvent. The particle image size was approximately 4 pixels in the zoomed-in field of view and ~1 pixel for the zoomed-out field of view. An even distribution of particle sub-pixel displacement around zero was checked so as to ensure the data was not affected by peak locking. The PIV system and plasma actuator were triggered to start simultaneously using a digital signal output from LabVIEW through a NI USB-6210 data acquisition system.
Fig. 3-2: a) Experimental setup showing the laser light plane, two high-speed cameras and the plasma actuator mounted on a manual traverse unit placed inside acrylic glass container. The flow is from left to right.

b) Schematic of the two fields of view (figure to scale). The larger field of view of (80 mm × 50 mm) was used to characterize the vortex growth while the smaller field of view (20 mm × 12 mm) was used to resolve the high velocity gradients inside the shear layer.

3.5.3 Measurement of shear-layer thickness and velocity

The shear-layer thickness, $D$, was characterized by the vorticity thickness given by Brown and Roshko [22] as:

$$D(t) = \frac{u_{\text{max}}(t) - u_o(t)}{\left(\frac{\partial u}{\partial y}\right)_{\text{max}}},$$

(3-1)
where $u_{\text{max}}$ is the maximum velocity in the shear layer and $u_0$ is the velocity at the outside edge of the shear layer. The shear-layer maximum velocity and thickness were obtained from the velocity profile at the streamwise location of the vortex center at each instant in time (along the dashed line in Fig. 3-1). Since in the present experimental arrangement the shear-layer velocity and thickness do not vary significantly in time, representative quantifications of $D(t)$ and $u_{\text{max}}$ are given by the time-averaged value, as presented in Table 3-1 together with their standard deviations for different test cases (statistics obtained from over 30 realizations).

### 3.5.4 Measurement of vortex circulation and vortex size

The vortex circulation was calculated using the surface integral of vorticity normal to the measurement plane as:

$$
\Gamma = \sum_{i,j} \omega_{i,j} \Delta A_{i,j},
$$

(3-2)

where $\omega_{i,j}$ and $\Delta A_{i,j}$ are the vorticity and quarter-area of each PIV interrogation area (note the 50% overlap in PIV correlations). While for instance the $\lambda_2$ criterion is an objective vortex identification scheme, it had the effect of excluding parts of the vortex resulting in underestimation of the total vortex circulation. To obtain an accurate estimate of vortex size and strength, a threshold-based algorithm was used instead. The vorticity threshold was set at 0.1 of the peak vorticity measured at each operating plasma actuator voltage. This threshold proved suitable to exclude the shear-layer vorticity in the calculation of vortex-core strength. Since the plasma actuator remained on during a given
test, some of the vorticity in the flow field was associated with the shear layer. To exclude these vorticity “patches” detached from the vortex core, the threshold criterion was used to accept only the simply connected vortex patch about the center of rotation in the calculation of vortex-core strength. The conclusions remained unaffected when the calculations were repeated using other vorticity thresholds in the range $0.1 \pm 0.03$ of the peak vorticity. The vortex size was obtained by calculating the area of the simply connected vorticity region. The radius of the vortex was then taken as the radius of a circle with an area equal to that of the vortex.

### 3.6 Early-Stage Vortex Development

Figure 3-3 shows a smoke visualization sequence of vortex evolution in time for case D. In the current experiments, the plasma actuator continued to operate during the entire measurement sequence. The start-up phase of the plasma actuator is one order of magnitude faster (less than 3 ms) than the time scale associated with the growth of the vortex (~20 ms to reach maximum growth). Due to the presence of the plate in the start-up phase of the jet, initially the shear layer rolls up only on one side forming one vortex growing in size and circulation. As shown in the last image of the visualization sequence of Fig. 3-3, a second weaker vortex with clockwise rotation is generated further downstream of the plate trailing edge. This vortex forms well after the evolution phase of the counter-clockwise vortex of interest and therefore is not further studied.
Fig. 3-3: Sample smoke visualization of vortex evolution for case D. As the vortex evolves, it conveys downstream in the horizontal direction as well as away from the shear layer in the vertical direction. Flow is from left to right. The masked region marks the base plate.
Figure 3-4 shows a typical $u$-component velocity distribution through the center of the vortex along the line A-A’ at time $t = 22\text{ms}$ and for case C. For subsequent analysis, the center of the vortex was chosen as the point of maximum vorticity inside the vortex core. Velocity profiles are shown for each of the two fields of view in Fig. 3-4. Note that when using the larger field of view, the velocities in the high-velocity gradient zone of the shear layer are underestimated due to the lower spatial resolution.

The jet consists of two parallel shear layers with opposite signs of vorticity. The vortex is fed by the circulation flux from the upper shear layer with positive (counter-clockwise) vorticity. On the upper side the velocity distribution exhibits four regions marked in Fig. 3-4. In the first region (zone 1) the velocity decreases from $u_{\text{max}}$ to a value $u_o$. The velocity stays relatively constant at $u_o$ in the second zone between the shear layer and the core of the vortex, resulting in a zone of low vorticity (zone 2). As presented by the statistics in Table 3-2, it was experimentally observed that: $u_o / u_{\text{max}} = 0.5 \pm 0.04$. The third zone is the core of the vortex (zone 3) followed by a zone in which a transition towards the outer quiescent fluid takes place (zone 4).

**Table 3-2 Statistics of $u_o / u_{\text{max}}$ for different test cases.**

<table>
<thead>
<tr>
<th>Case</th>
<th>$u_o / u_{\text{max}}$ mean</th>
<th>STD</th>
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</thead>
<tbody>
<tr>
<td>A</td>
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<tr>
<td>B</td>
<td>0.53</td>
<td>0.04</td>
</tr>
<tr>
<td>C</td>
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<td>0.05</td>
</tr>
<tr>
<td>D</td>
<td>0.49</td>
<td>0.05</td>
</tr>
</tbody>
</table>
On the lower side of the jet the vorticity is clockwise as the velocity rapidly decreases from $u_{\text{max}}$ to zero (zone 5). This velocity gradient is due to the no-slip boundary condition upstream of the plate trailing edge and due to quiescent outer flow downstream of the trailing edge. The two opposite-signed shear layers (zones 1 and 5) are evident in the sample vorticity contour of Fig. 3-4b obtained from the zoomed-in field of view. The absolute vorticity level is larger within the lower (clockwise) shear layer as the velocity gradient occurs within a rather narrow distance. The flow structure after the passage of the vortex is akin to typical wall jets consisting of two layers of opposite-signed vorticity. Accurate characterization of the mean flow field is presented in Moreau [23].
Chapter Three

Fig. 3-4: a) An example of velocity distribution inside the vortex with feeding shear layer (along line A-A’ as shown in the lower left figure) for case C at $t = 22$ms. The measured velocities from both fields of view (black triangles for FOV=80 mm $\times$ 50 mm and open circles for FOV = 20 mm $\times$ 12 mm) are compared. The velocity distribution exhibits four regions (i.e. zones 1-4) on the upper side of the shear layer. b) Sample vorticity field, $\omega$, (out of plane component) of the jet flow exhibiting upper (counter-clockwise) and lower (clockwise) shear layers.

As shown in the visualization sequence in Fig. 3-3, as the vortex evolves, it convects downstream in the horizontal direction as well as away from the shear layer in
the vertical direction. Figure 3-5 shows the $x$-location of the center of the vortex as a function of time for different test cases. Note that the stepwise behavior is due to the limited PIV spatial resolution for the large field of view. The average convective velocity of the vortex in the streamwise direction ($u_c$) was estimated by a linear regression through the recorded positions of the vortex center as a function of time (dashed lines in Fig. 3-5). As indicated in Fig. 3-5, for each test case, this velocity was found to scale with the maximum shear-layer velocity as $u_c = (0.12 \pm 0.01)u_{\text{max}}$. The convection behavior of the vortex can be explained through a flow model by considering induced velocities of the two opposite-signed vortex sheets of the shear layer. According to the Biot-Savart relation, the counter-clockwise vorticity of the shear layer tends to induce a velocity on the vortex in the negative $x$-direction. However, the lower half of the shear layer (clockwise rotation) induces a velocity on the vortex in the positive $x$-direction. Since the vorticity is higher on the lower half of the shear layer the net induced velocity is positive. To examine this argument, the convective velocity of the vortex due to the presence of the two parallel shear layers with a vorticity distribution similar to the present experiment was calculated using the Biot-Savart relation,

$$u_c = \frac{1}{4\pi} \oint \frac{\vec{\omega} \times \vec{\zeta}}{|\vec{\zeta}|^3} \, dV,$$

(3-3)

where $\vec{\omega}$ is the vorticity vector, $\vec{\zeta}$ is the vector pointing from the vortex-sheet segment towards the vortex center as depicted in the upper left corner of Fig. 3-5 and $dV$ is the differential element of volume. Note that the requirement for using the Biot-Savart relation is that the shear layer be separated from the vortex core by an inviscid region. As
discussed in the text surrounding Fig. 3-4, there is indeed a region of negligible vorticity (zone 2 in Fig. 3-4) between the shear layer and the vortex core. Therefore, the use of Biot-Savart relation to estimate the induced velocity on the vortex core in this study is justified.

The convective velocities obtained from Biot-Savart are represented by the slopes of the solid lines in Fig. 3-5. The origin of the lines is chosen arbitrarily (without loss of generality). At earlier stages of vortex formation, the convective speed of the vortex is slightly over-estimated by Biot-Savart relation, but later the predicted speed is very close to experimental data.

Fig. 3-5: The x-position of the vortex core as a function of time. The slope of the solid lines represents the mean convective speed of the vortex \( (u_c) \) obtained using the Biot-Savart relation.
3.6.1 Model for early-stage vortex development

Two main effects contribute to the vortex growth: transport of vorticity from the shear layer via mass flux into the vortex, as suggested by Kaden [19], as well as viscous diffusion. Following Kaden [19], i.e. assuming the diffusion to be negligible, the size of the vortex may be estimated by the time integral of mass flow rate into the vortex (Fig. 3-6). Given that the flow is two-dimensional and incompressible, the volume flow rate per unit width, $\dot{Q}$, can be related to the vortex radius $r$, where

$$\int_0^t \dot{Q}(t) dt = \pi r^2(t) . \quad (3-4)$$

Considering the convective velocity of the vortex, $u_c$, the volume flow rate per unit width entering the vortex from the counter-clockwise vorticity shear layer may be written in terms of a velocity distribution:

$$\dot{Q}(t) = \int_0^{D(t)} [u(t,y) - u_c(t)] dy . \quad (3-5)$$

Fig. 3-6: Model of vortex growth based on Kaden [19], with growth due to the inflow of circulation-containing mass from the shear layer with thickness $D$ and a maximum velocity $u_{max}$. The vortex with radius $r$, and circulation $\Gamma$, travels with a velocity $u_c$. 
Equations (3.4) and (3.5) are time-dependent and are presented in general form. In the present case, however, the shear-layer velocity distribution and thickness vary negligibly in time, allowing equations (3.4) and (3.5) to be re-written in terms of time-averaged values. Considering that the mean shear-layer velocity can be approximated by the average of the maximum shear-layer velocity, $u_{\text{max}}$, and with the velocity outside the shear layer, $u_o$, Equations (3.4) and (3.5) become:

\[
\bar{Q}t \approx \pi r^2(t),
\]

\[
\bar{Q} \approx \left[\bar{u}(t,y) - \bar{u}_e(t)\right]D(t) \approx \left(\frac{\bar{u}_{\text{max}} + \bar{u}_o}{2} - \bar{u}_e\right)D.
\]

Furthermore, based on the experimental observations, it is possible to scale $u_o$ and $u_e$ with $u_{\text{max}}$:

\[
\bar{u}_o \approx 0.5\bar{u}_{\text{max}},
\]

\[
\bar{u}_e \approx 0.12\bar{u}_{\text{max}}.
\]

Finally, by substituting equations (3.7)-(3.9) into equation (3.6), an expression for the growth of the vortex radius in time is obtained:

\[
r(t) = \sqrt{0.63 \frac{\bar{u}_{\text{max}} D}{\pi} t}.
\]

Equation (3.10) suggests that the vortex grows with the square root of time and that the vortex size is a function of thickness and velocity of the shear layer. Having obtained the radius, the vortex strength is determined as the line integral of the velocity around the core of the vortex, i.e. $\Gamma = \int \bar{u} \cdot d\bar{l}$, where $d\bar{l}$ is the vector representing the
differential length around the vortex core. By approximating the vortex area to be of circular shape and assuming (as an approximation) a uniform-magnitude velocity distribution around the vortex core in the frame of reference moving with the vortex, the line integral may be written in a more simplified form:

\[ \Gamma(t) \approx 2\pi r(t)u_r(t), \quad (3-11) \]

where \( u_r \) is the rotational velocity at the edge of vortex core in the frame of reference moving with the vortex. Since a uniform velocity distribution around the vortex core is assumed, it is sufficient to have \( u_r \) at only one point around the vortex core. The validity of this approximation is justified when considering the velocity profile of Fig. 3-4 as an example. In Fig. 3-4, the velocities on the lower (zone 2) and upper (zone 3) vortex sides in the fixed frame of reference of laboratory are 2.0 and 1.1, respectively. Considering the calculated convective velocity of 0.5 m/s for case C (see Fig. 3-5), the velocities on the lower and upper vortex sides in the frame of reference of vortex are calculated as 1.5 m/s and 1.6 m/s, respectively. This observation justifies the assumption of uniform velocity around the vortex in the convective frame of reference.

In the present setup, the velocity in the region between the vortex core and the shear layer (zone 2 in Fig. 3-4), \( u_o \), can be taken as the velocity at the edge of the vortex core. However, as depicted in Fig. 3-4, to transform this velocity from the stationary frame of reference into the frame of reference of the vortex, \( u_r \) should be considered as:

\[ u_r = u_o - u_c, \quad (3-12) \]
After substitution of \( r(t) \) from equation (3-10) into equation (3-11), and applying equations (3-8), (3-9) and (3-12), an expression for the vortex circulation in time as a function of shear-layer characteristics is obtained:

\[
\Gamma(t) \approx 2\pi r(t)(\overline{u}_o - \overline{u}_c) \approx 0.76\pi \overline{u}_{\text{max}} \sqrt{\frac{0.63 \overline{u}_{\text{max}} D}{\pi}} t .
\]  

(3-13)

### 3.6.2 Scaling of vortex growth

Considering the relationship for circulation from equation (3-13), it is possible to obtain a scaling scheme for vortex growth based on shear-layer characteristics by scaling both sides of equation (3-13) by \( u_{\text{max}} D \) to obtain:

\[
\tilde{\Gamma} = \frac{\Gamma(t)}{\overline{u}_{\text{max}} D} \approx 1.07 \sqrt{\frac{\overline{u}_{\text{max}} t}{D}} = 1.07 \tilde{t} ,
\]  

(3-14)

which is a linear relationship between dimensionless circulation, \( \tilde{\Gamma} \), and dimensionless time \( \tilde{t} \), independent of geometric length scales of the problem. This scaling approach is in line with Dabiri’s formulation of the formation time using shear-layer characteristics [1].

### 3.7 RESULTS

Figure 3-7 illustrates the estimated vortex radius as a function of time for the test cases summarized in Table 3-1. The growth predictions from the proposed model (equation 3-10) are also shown and agree with experiments in the early vortex growth stage. The diffusion rate, \( \sqrt{4\nu t} \), typical of, for instance, a Lamb-Oseen vortex with constant circulation, has been plotted for comparison, suggesting that the growth rate
associated with viscous diffusion is much slower compared to the influx from the shear layer. The strong agreement between the model and the experimentally-measured vortex radius also indicates that entrainment of quiescent fluid outside the vortex core into the vortex does not significantly contribute to the growth of the vortex.

Generally, the vortex growth is faster when the maximum velocity in the shear layer is increased. However, the vortex size is observed to asymptotically reach a plateau, thus deviating from the model. Due to this asymptotic behavior it is not possible to specify a precise time associated with vortex growth limit. However, the approximate onset of the plateau in vortex size determined qualitatively for each case has been marked by a dashed line in Fig. 3-7.

Fig. 3-7: Vortex radius for different test cases together with predictions by the proposed growth model (solid black lines). The growth rate due to viscous diffusion, $\sqrt{4\nu t}$, (Lamb-Oseen model) is included for comparison. The dashed line marks the approximate onset of the vortex-growth plateau. Error bars represent the uncertainty in vortex radius for case D (every 5th bar shown for clarity).
A very similar behavior is observed for the growth of circulation (Fig. 3-8). The start of the plateau in the vortex strength coincides with that for the vortex size. This plateau indicates that the flux of circulation into the vortex decreases through a gradual process.

Fig. 3-8: Vortex strength for different test cases together with predictions by the proposed growth model (solid lines). The dashed line marks the approximate onset of the vortex-growth plateau and is the same as in Fig. 3-7. Error bars represent the uncertainty in circulation for case D (every 5th bar shown for clarity).

3.7.1 Separation of vortex from shear layer

The limit in vortex growth shown in Figs. 3-7 and 3-8 indicates that the circulation flux from the shear layer into the vortex is eventually terminated. It is hypothesized that this limit is related to the change in the shear-layer curvature, or
“flattening” of the shear layer away from the vortex in time, leading to a gradual separation of the vortex from the shear layer. This phenomenon is shown schematically in Fig. 3-9a. Flow patterns and vorticity contours at two time steps corresponding to early-stage vortex formation (left) and a time after the vortex separation from the shear layer (right) are presented in Fig. 3-9b and 3-9c, respectively. The increased shear-layer radius of curvature ($\kappa$) is evident from the change in the streamline patterns. Prior to the separation, the streamlines are directed towards the core of the vortex, whereas after separation, some of the streamlines are directed away from the core of the vortex.
Fig. 3-9: (a) Schematic representing the separation mechanism of the vortex from the feeding shear layer, where $\kappa$ is the radius of curvature of the shear layer and $+$ represents the center of shear-layer curvature. The separation is linked to a competition between the shear layer’s tendency to remain in the streamwise direction and the induced velocity from the vortex on the shear layer. (b) Sample planar streamlines in the stationary frame of reference before (left) and after (right) the separation of vortex from the shear layer. (c) Vorticity contours corresponding to velocity fields in (b).

The “flattening” of the shear layer is due to the competition between the tendency of fluid particles inside the shear layer to maintain an inertial trajectory in the streamwise direction and the induced velocity (upwards) by the vortex. The induced velocity by the vortex on the shear layer is a function of vortex strength, $\Gamma$, and is also inversely related to the distance, $S$, between the core of the vortex and the shear layer. The shear-layer
maximum velocity, \( u_{\text{max}} \), can be considered as the characteristic parameter associated with the shear layer’s tendency to remain in the streamwise direction. This competition can be therefore expressed in terms of a dimensionless number,

\[
\Gamma^* = \frac{\Gamma}{u_{\text{max}} S^*}.
\]  

(3-15)

As long as the induction from the vortex is sufficiently large compared to \( u_{\text{max}} \), i.e. large \( \Gamma^* \), the shear-layer curvature will remain close to that of the vortex core radius. At the same time the circulation-containing fluid from the shear layer will also be drawn towards the vortex core. In the current configuration, however, the vortex core travels away from the shear layer with a faster rate than the circulation growth. This process leads to a gradual decrease in the contribution of vortex-induced velocity on the shear layer, i.e. a decrease in \( \Gamma^* \), hence a “flattening” of the shear layer and a separation of the vortex from the feeding shear layer. Figure 3-10 presents \( \Gamma^* \) as a function of time for the four test cases showing the decrease of \( \Gamma^* \) in time. The separation occurs at approximately \( \Gamma^* \approx 1.5 \) for all four test cases. \( S \) was calculated by measuring the distance between the vortex center (point of maximum vorticity) and the point of maximum shear-layer velocity. The uncertainty in \( S \) due to PIV resolution is estimated to be \( \pm 0.66 \) mm (half of the sum of interrogation window sizes in small and large FOVs). Considering the uncertainty in \( \Gamma \), the overall uncertainty in \( \Gamma^* \) is estimated to be \( \pm 20\% \). The absolute value of this quantity, however, does not influence the present arguments as the trend is of interest.
Fig. 3-10: $\Gamma^*$ is a measure of the competition between the induced velocity from the vortex and the tendency of fluid particles in the shear layer to move in the streamwise direction. $\Gamma^*$ decreases in time due to the reduced contribution of vortex-induced velocity leading to a flattening of the shear-layer. The dashed line marks the approximate separation time of the vortex from the shear layer, as seen in Fig. 3-7 and 3-8.

Based on the above argument the following relationship describing the connection between the radius of curvature of the shear layer ($\kappa$), vortex radius ($r$) and dimensionless number $\Gamma^*$ may be proposed based on scaling principles:

$$\frac{r}{\kappa} \propto \frac{\Gamma}{u_{\text{max}} S}.$$  \hspace{1cm} (3-16)
In the current experimental arrangement, the distance $S$ grows faster than the vortex size, $r$, and continues to increase when the vortex size has already reached a plateau. This is evident from the plot of $r/S$ as a function of time in Fig. 3-11 and indicates that $S$ depends on the global flow field rather than directly on vortex growth itself. The uncertainty in $r/S$ is estimated to be ±22%. The absolute value of this quantity, however, does not influence the present arguments as the trend is of interest. The dependence of spacing $S$ on the flow field provides the possibility of delaying the separation of the vortex from the shear layer by manipulating the distance $S$ through a change in the global flow conditions. The significance of distance $S$ on vortex growth has also been observed in the numerical study by Mohseni et al. [8], where a delay in vortex ring pinch-off was achieved by gradually increasing the orifice diameter so that the vortex ring would grow away from the symmetry axis. In other configurations involving vortex growth, such as the case of a plunging airfoil, the free-stream velocity may have an impact on the variations in distance $S$ with a favorable/unfavorable influence on vortex growth [2]. Alternatively, in the present setup the separation may potentially be delayed by decreasing the shear-layer velocity in time so that the induced velocity from the vortex remains competitive with the shear-layer velocity for a longer time period.
Fig. 3-11: The drop in the vortex radius-to-spacing ratio suggests that $S$ depends on the overall flow field rather than the growth of the vortex itself. The dashed line marks the approximate separation time of the vortex from the shear layer, as seen in Fig. 3-7 and 3-8.

Since the radius of curvature is difficult to determine directly from the measurements, an approximate measure of shear-layer “flattening” can be obtained by considering the geometric parameter $S/H$, where $H$ is the distance between the center of the vortex and the shear layer in the horizontal direction, as depicted in the upper-right corner of Fig. 3-12. The $S/H$ ratio for test cases A-D is presented in Fig. 3-12, where for clarity cases B-D have been shifted vertically by $S/H = 1$, 2 and 3, respectively. As expected, a drop in $S/H$ is observed, which coincides with the plateau in vortex size and
circulation (Figs. 3-7 and 3-8). The approximate time of the drop in $S/H$ is marked by the dashed line in Fig. 3-12.

Fig. 3-12: Parameter $S/H$ provides a measure of the shear-layer curvature. The drop in the shear-layer curvature corresponding to the separation of the shear layer from the vortex is marked by the dashed line (the same line as in Figs. 3-7 and 3-8). The drop occurs at the same approximate time of the plateau in vortex growth. For clarity, cases B-D have been plotted with a vertical offset of $S/H = 1, 2$ and $3$, respectively.
3.8 CONCLUDING REMARKS

The evolution of an isolated line vortex generated by a starting two-dimensional jet was studied experimentally using time-resolved particle image velocimetry. Based on Kaden’s [19] vortex-growth formulation, a model was proposed for the early-stage vortex formation and was shown to be consistent with the experimental results. Through this model, a scaling scheme for the growth stage of the vortex using the shear-layer characteristic velocity and the shear-layer thickness was obtained. The vortex growth was, however, observed to be limited. The limitation was found to be linked to a competition between the shear-layer tendency to remain in the streamwise direction and the induced velocity from the vortex on the shear layer itself. It was proposed that this competition is a function of the vortex strength, \( \Gamma \), spacing between the shear layer and the vortex, \( S \), and the shear-layer characteristic velocity, \( u_{\text{max}} \), and therefore can be described using a dimensionless number, \( \Gamma^* = \Gamma/u_{\text{max}} S \). As the contribution of the velocity induced by the vortex relative to the shear-layer characteristic velocity is reduced, the shear layer “flattens” and consequently separates from the vortex through a gradual process. It is observed that separation occurs at \( \Gamma^* \approx 1.5 \). We hypothesize that the separation process may be hindered by maintaining the vortex-induced velocity comparable with the shear-layer characteristic velocity through (i) controlling the spacing \( S \) by external means such as a co-flow or (ii) a temporal decrease in the shear-layer velocity. Pedrizzetti’s [14] simulation of a two-dimensional vortex dipole may in fact be a confirmation of this hypothesis. Unlike the present study, in Pedrizzetti [14] the
induction from the opposite-signed mirror vortex (through Biot-Savart) causes the distance between the shear layer and the vortex to remain relatively constant. Data extracted from Pedrizzetti [14] for the low Reynolds number case suggests that the non-dimensional circulation remains larger than $\Gamma^* = 10$. Despite the uncertainty in data extraction, these approximations of $\Gamma^*$ are much higher than in the present study at the time of plateau ($\approx 1.5$). This might explain the continuation of attachment of the vortex to the shear layer in the numerical studies of Pedrizzetti and Domenichini as well.

In addition to $\Gamma^*$, turbulence is also expected to play a role in the separation process. Unlike previous studies of Pedrizzetti and Afanasyev, the flow in the present study appears to be turbulent (Re $>$1500 based on shear-layer velocity and mean vortex diameter). In turbulent flows instabilities are more easily amplified. At sufficiently low $\Gamma^*$ the connection of the vortex to the shear layer is susceptible to perturbations and ready to break off. Hence, these perturbations can accelerate the disconnection of the vortex from the shear layer. A more detailed analysis of the influence of turbulence on vortex evolution is subject to future studies.
3.9 ACKNOWLEDGEMENTS

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3.10 REFERENCES


Chapter Four: On the vortex growth pattern in bluff-body shedding: scaling considerations

This Chapter will be submitted to Physics of Fluids as:


Department of Mechanical and Manufacturing Engineering, University of Calgary, Calgary, AB T2N 1N4, Canada

4.1 ABSTRACT

The vortex shedding process in the turbulent wake of finite aspect ratio, surface-mounted square section cylinders is investigated and compared to the process for two-dimensional square and circular cylinders. Vortex growth scaling parameters are developed and it is shown that these parameters can be used to scale the shedding frequency for both two- and three-dimensional geometries. The results support the view that the vortex growth in the base region is naturally limited and that feedback from the shed vortices serves to lock-in the shedding frequency.

4.2 INTRODUCTION

The periodic shedding of staggered rows of counter-rotating vortices in the wake of elongated cylindrical or prismatic bluff bodies is a well-documented phenomenon. Typically, the advection speed of the vortices changes very slowly downstream of the
obstacle base region and the non-dimensional shedding frequency (the Strouhal number) is nearly constant or varies mildly over a broad range of Reynolds numbers. Considering the stability to infinitesimal perturbations of a wake model consisting of point vortices, von Kármán [1] showed that only one staggered arrangement of vortices was stable, which was characterized by a specific vortex spacing ratio. Saffman and co-workers [2] further showed that for finite (viscous) core vortices, the vortex arrangement was stable for finite perturbations, although the critical spacing ratio was slightly modified.

A common description of the process leading to periodic bluff-body vortex shedding links the interaction of opposing shear layers in the obstacle base region [3]. Initially, the shear layer on one side of the obstacle rolls up into a vortex. The induction from this growing vortex draws the opposing shear layer across the wake along the obstacle leeward face. As the opposing shear layer penetrates deeper into the base region, it eventually interferes with the feeding shear layer, thereby interrupting the flow of circulation to the growing vortex and causing it to shed. This process is then repeated on the opposite side. While this phenomenological model adequately explains the cyclical formation of staggered rows of alternating sign vortices, it is based on local arguments and it is not immediately clear that it will result in a stable arrangement in the wake.

The interaction between opposing shear layers is not a necessary condition for shedding of vortices. Examples include bluff bodies with a splitter plate placed in the wake [3,4] and flow in a cavity [5]. Rowley et al. observed nearly-periodic shedding of vortices in cavities when the length and/or depth of the cavity was large compared to the upstream boundary-layer thickness [5]. This distinct flow regime was referred to as a
“wake mode” by Gharib and Roshko [6]. The shedding frequency, \( f_s \), of vortices in the wake mode (based on depth of cavity, \( b_c \)) was approximately \( St = f_s b_c / U_e = 0.06 \), where \( U_e \) is the mean velocity outside the cavity. Interestingly, this Strouhal number is close to that associated with vortex shedding in the wake of square cylinders if \( 2b_c \), analogous to the width of a square cylinder, is used instead as a length scale. Rowley et al. associated the periodic shedding behaviour to absolute instabilities in the flow. A recent study by Sattari et al. [7], considering the formation and shedding of an isolated line vortex, also suggested the growth of the vortex was limited by local time scales. Similarly, both Triantafyllou et al. [8] as well as Monkewitz and Nguyen [9], performed instability analyses for families of mean wake velocity profiles. Monkewitz and Nguyen noted that, while the initial separated shear layer is convectively unstable, much of the base flow region (to the end of the mean recirculation) is absolutely unstable. Further downstream, the flow is again convectively unstable. If a small-amplitude perturbation grows in place at the location of its generation, the flow is called absolutely unstable. However, if the perturbation is convected downstream and ultimately leaves the flow at the location of its generation undisturbed, the flow is termed convectively unstable. Monkewitz and Nguyen identified parameters for which the flow is absolutely unstable locally, i.e. for which instability waves amplifying in the upstream direction exist. They also interpreted the von Kármán street to be an intrinsic response of the system, determined by the resonance between upstream and downstream instability waves, from which shedding frequencies to within 10% of experimental data were estimated. These
observations suggest that the mutual interaction between opposing vortices is not the mechanism for vortex shedding. Rather, the mutual interaction is the mechanism by which the shedding frequency and an inherent instability in the base region become locked-in.

Historically, several studies have focused on the existence of a “universal” Strouhal number suggested by von Kármán’s analysis. Monkewitz and Nguyen [9] argued that the shedding frequency is expected to scale with some integral scale describing the wake since the stability branch points scale with the vorticity thickness (used as a measure of the wake width). Roshko [10] proposed a universal Strouhal number based on the assumption that the distance between the shear layers is equal to the vortex core separation from von Kármán’s shedding model. He modeled the shear layer as equal vortex sheets and determined their separation using notched-hodography based on the mean base pressure coefficient, 
\[ C_{pb} = \left( p_b - p_{\infty} \right) \left( \frac{U_{\infty}}{2 \rho U_{\infty}^2} \right) \], where \( p_b \) is the mean based pressure; \( p_{\infty} \) and \( U_{\infty} \) are the free-stream pressure and velocity. Roshko, however, observed that when a splitter plate was placed in the wake of circular cylinders, the shedding did not collapse onto a single “universal” Strouhal number. Bearman [4] was able to collapse the shedding frequency in configurations with splitter plates and base bleed by using the lateral spacing of vortices, \( b \), as the length scale. To obtain wake properties he used Kronauer’s criterion. This criterion states that for a given vortex velocity, the vortex street spacing ratio adjusts to the configuration giving minimum vortex street drag. Kronauer’s criterion is not based on direct experimental evidence. Furthermore, when using more recent experimental observations on prismatic obstacles,
this approach does not satisfactorily collapse the shedding frequency data. Williamson and Brown [11] showed that the shedding Strouhal number could be collapsed for a circular cylinder in the Reynolds number range of 50-140,000 using $d + 2\delta_w$, where $d$ is the cylinder diameter and $\delta_w$ is the vorticity thickness at a downstream location $x/d = 1$. No justifications were given as to why the $x/d = 1$ location was considered.

These studies suggest that a suitable scaling of the shedding frequency should account for the formation process in the obstacle base region and the downstream distribution of the vortex street. In this study, it is hypothesized that the growth in circulation of the base-region vortex is naturally limited. As the maximum circulation of the vortex is asymptotically approached, the circulation flow from the feeding shear layer to the forming vortex tends to zero and becomes more susceptible to interruption from local disturbances. The principle source of disturbances arises from induction by the downstream vortex street, thereby locking-in the vortex shedding frequency.

This paper is organized as follows. The growth of the base vortex is characterized using experimental observations. A phase-averaged representation of the shedding process is used to relate the base flow instability to the growth limitation of the forming vortex by showing that the strength of the shed vortices scales locally. It is shown that at the location of maximum streamwise mean fluctuation, a length scale based on the vorticity thickness varies little through the shedding cycle and is thus well represented by the mean flow field. These scales are then related to the mean field and applied to the scaling of the vortex shedding frequency in the wake of surface-mounted, square cross-section cylinders, as examples of three-dimensional bluff bodies. This scaling is also
tested with the well-documented studies on the square cylinder (Lyn et al. [12]) and a two-dimensional circular cylinder (Cantwell and Coles [13]).

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</table>

### 4.3 Experimental Apparatus

In this section, an overview of the experimental setup is presented. For a more detailed description of the setup, the reader is directed to Sattari et al. [14]. Experiments were conducted in a suction-type, open-test-section wind tunnel, as shown schematically in Fig. 4-1. The obstacle was mounted on a 1.2-m long, 0.8-m wide and 0.01-m thick aluminum flat plate with a sharp leading edge. The machined aluminum square-section obstacles had a width of d = 0.0127 m and heights of h = 0.0508 m or 0.116 m. The obstacles were instrumented with pressure taps on opposing faces at vertical locations 0.25h, 0.50h and 0.75h above the ground plate and 0.25d, 0.50d and 0.75d from the leading edges, respectively. The obstacles were mounted with a face normal to the oncoming flow. The obstacle geometry, flow and co-ordinate system nomenclature are also shown in Fig. 4-1 for the aspect ratio 4 obstacle.
Vortex shedding was investigated for two obstacles \((h/d = 4\) and \(8\)) for a thin and a thick boundary layer. Measurements were taken at a free-stream velocity, \(U_\infty = 15\, \text{m/s}\), corresponding to a Reynolds number \(\text{Re}_d = U_\infty d / \nu = 12,000\), where \(\nu\) is the kinematic viscosity of air. The free-stream turbulence intensity was 0.8\% for all tests. The thin boundary layer developed naturally from the sharp leading edge. The thick boundary layer was generated by placing a 3 mm trip rod at the leading edge. The characteristics of the undisturbed boundary layers were measured using a single-probe hot wire anemometer. The boundary-layer thickness based on 99\% free-stream velocity at the
mounting location of the cylinder \( (x = 200\text{mm} \approx 16d \text{ from the leading edge of the plate}) \) was \( \delta / d = 0.72 \) (\( \delta / h = 0.18 \) and 0.09 for aspect ratio 4 and 8 obstacles, respectively) in the case of thin boundary layer and was \( \delta / d = 2.4 \) (\( \delta / h = 0.60 \) and 0.30 for \( h/d = 4 \) and \( h/d = 8 \) obstacles, respectively) in the case of thick boundary layer. The boundary layers are turbulent but developing.

A LaVision high-frame-rate particle image velocimetry (PIV) system was used for planar velocity field measurements in three horizontal planes parallel to the free end \((z/h = 0.25, 0.5 \text{ and } 0.75)\) and in the vertical plane \(y/d = 0\). A Photonics Industries 527 nm Nd-YLF laser light sheet was used for illumination. Data were acquired with a HighSpeedStar 5 CMOS camera (1024×1024 pixels) in double-pulse mode with a pulse separation of 50 µs between the image pairs. Image pairs were captured at rates of 500 Hz or 1000 Hz, equivalent to approximately 5 to 10 data points per shedding cycle. Five to ten independent sequences of 1000 vector fields were acquired so that statistics were calculated over 500 to 2000 shedding cycles. Interrogation windows of 32×32 pixels were used in the wake using a frame-straddling arrangement, resulting in a spatial resolution of the data within a given PIV velocity field of approximately 3 mm or \( d/4 \). Where greater resolution was necessary, interrogation windows of 16×16 pixels were used, yielding a resolution of \( d/8 \). With this configuration, the estimated uncertainty on individual vector measurements (see Westerweel, 2000) is \( \Delta u/U_\infty = \pm 0.025 \), while the estimated statistical uncertainty on the phase-averaged vector fields, which is a function of the standard deviation at a given position, ranges from \( \Delta < u > /U_\infty = \pm 0.002 \) to a maximum of \( \pm 0.027 \).
The fluctuating surface pressure was measured with pressure transducers (AllSensors Corp, model 1INCH-D1-4VMINI). These measurements were analyzed in order to find the phase of the shedding cycle for each PIV vector field. The sampling rate for the reference surface pressure was 10.24 kHz and was synchronised with the PIV measurements using a TTL trigger sent from the PIV system at the start of a measurement. The uncertainty for the instantaneous phase determination is estimated to be ± 5°, which is based on the elemental instrument uncertainty, process standard deviation and repeatability characteristics, within a 95% confidence interval (Coleman and Steele, 1999).

4.3.1 Methodology of phase analysis and phase averaging

The instantaneous flow variables of a quasi-periodic flow can be decomposed as the sum of a coherent (phase-averaged) part and an incoherent (random) part. For the velocity field these components are:

\[ \mathbf{u}(\mathbf{x}, \Phi(t), t) = \langle \mathbf{U} \rangle (\mathbf{x}, \Phi(t)) + \mathbf{u}'(\mathbf{x}, t), \]  

(4-1)

where bold font denotes vectors, \( \mathbf{u} \) is the instantaneous velocity field, \( \langle \mathbf{U} \rangle \) the phase-averaged field, \( \mathbf{u}' \) the fluctuation away from the phase average, and \( \Phi(t) \) a given phase reference of the shedding cycle. For this work, the surface pressure signal at \( x/d = 0 \) and \( z/h = 0.25 \) is chosen as a reference signal. To determine the instantaneous phase of the pressure signal, the fluctuating component of the pressure signal was reconstructed from a truncated Fourier series. Two thousand Fourier modes of the FFT for a 100,000 point sample were used, corresponding to modes up to approximately 200 Hz, nearly twice the
shedding frequency (~120 Hz). A wavelet transform with a complex Morlet of centre-frequency equal to that of the vortex shedding frequency was used to determine the phase. For the double decomposition ensemble averaging, the shedding period was discretized into twenty phase steps, $\Phi_n$, $n = 1, ..., 20$ ($\Delta \Phi = \pi/10$). For each phase step, the ensemble average was undertaken such that any PIV vector field falling within $\Phi_n - \Delta \Phi / 2 < \Phi < \Phi_n + \Delta \Phi / 2$ was used to obtain statistics (mean and root-mean-square values) for the discrete phase $\Phi_n$.

### 4.4 RESULTS AND DISCUSSIONS

Vortex formation and shedding in the base region of a finite aspect ratio cylinder is briefly reviewed, based on phase-averaged data, in part to establish the similarity to the shedding process in two-dimensional geometries. The vortex growth is characterized and a scaling argument is developed in terms of the phase-averaged and mean flow fields. Consideration is then given to the scaling of the shedding frequency.

#### 4.4.1 Vortex/shear-layer separation process

The phase-averaged representation of the vortex formation and shedding process in the base region is illustrated in Fig. 4-2 for the obstacle $h/d = 4$ in the plane $z/h = 0.25$ (case A1). Non-dimensional vorticity contours, $\omega^* = \alpha d / U_\infty$ (non-dimensionalized by free-stream velocity and obstacle width), are shown for four phases of the shedding cycle. The regions denoting a vortex ($\lambda_2 < 0$) are bound by solid lines. The location of the vortex centroid as a function of the shedding phase is shown in Fig. 4-3.
The shedding process depicted in Figs. 4-2 and 4-3 is similar to that observed for all the geometries investigated in this study (cases A, B, C) as well as for the two-dimensional cases (D [12], E [13]). In Fig. 4-2, A, B denote the forming vortices over the upper (clockwise vortex) and lower (counter-clockwise vortex) faces, respectively, while A', B' denote vortices formed in the previous cycle. At the phase Φ₁, the vortex B' has shed and the forming vortex B has entered the base region, interfering with the upper shear layer. The vortex A' is still connected to the upper (separated) shear layer (continuous vorticity contours) while a new vortex, A, is forming over the upper obstacle face. The vortex A' has shed in phase Φ₂, noting that the vorticity contours no longer connect to A'. The vortex A grows in strength, entering the base region displacing vortex B, as seen for phase Φ₃. The vortex A then grows in the base region and interferes with the lower shear layer, causing the appearance of a new vortex Bₙ and the shedding of vortex B in phase Φ₄. As shown in phase Φ₅ the process is then repeated.

The position of the centroid of vortex A as a function of the shedding phase is shown in Fig. 4-3. The centroid position is shown for other cases as well. While the absolute positions differ between cases – expected since the extent of the backflow in the base region differs significantly between cases – the general evolution is similar. Figure 4-4 shows the circulation of vortex A as estimated from the integration of the plane-normal component of phase-averaged vorticity over the area inside the closed curve bounding the region λ₂ < 0.

The vortex A centroid from its inception over the obstacle upper face to Φ ~ 300° (approximately when B sheds) moves at a nearly constant rate in the streamwise
direction. It penetrates the base region and reaches a minimum distance, $y_{\text{min}}$, from the centre-plane ($y = 0$) at $\Phi \sim 300^\circ$. The $y$-displacement is more likely induced by the circulation of vortex B. From Fig. 4-4, the strength (circulation) of vortex A grows to the point when vortex B sheds ($\Phi \sim 300^\circ$).

![Contour plots](image)

**Fig. 4-2:** Contours of non-dimensional vorticity ($\omega^* = \omega d / U_\infty$) at five shedding phases for case $A_1$. The regions denoting a vortex ($\lambda_2 < 0$) are bound by solid lines.
Fig. 4-3: Vortex centroid in the a) streamwise and b) spanwise directions as a function of shedding phase $\Phi$ for different test cases. Vortex A moves at a nearly constant rate in the streamwise direction up to $\Phi \sim 300^\circ$ (approximately when B sheds). Vortex A reaches a minimum distance, $y_{min}$, from the centre-plane ($y = 0$) at $\Phi \sim 300^\circ$. After vortex B sheds, the rate of streamwise displacement of vortex A increases and the centroid moves away from the centre-plane.

After vortex B sheds, the rate of streamwise displacement of vortex A increases and the centroid moves away from the centre-plane until the vortex sheds at $\Phi \sim 460^\circ$.

Mutual induction between the newly-formed vortex $B_n$ and $A$ appears to be the primary mechanism for the newly-formed vortex to enter the base region and for vortex $A$ to move away from the centre-plane. After vortex B sheds, the strength of $A$ does not increase, even though the uninterrupted vorticity contours suggest that the vortex is still connected to the upper shear layer.

Relating the mean flow field to the shedding process may be beneficial in establishing the scaling arguments developed in the subsequent sections. Figure 4-5 shows the iso-contours of the streamwise, $\bar{u}$, and spanwise, $\bar{v}$, root-mean-square fluctuations. Also shown are the broken lines indicating the streamwise centroid location.
corresponding to the shed vortex at its maximum strength from Fig. 4-4. The magnitude, streamwise location and spanwise location of maximum $\overline{u}$ ($\overline{u}_{\text{max}}$) for different cases are summarized in Table 4-2. It is observed that the position of maximum vortex strength, $x_{r,\text{max}}$, consistently occurs in the vicinity of the maximum $\overline{u}$. Griffin [15] synthesized the results of several studies of the circular cylinder wake at low Reynolds numbers ($< 350$) and concluded that the maximum $\overline{u}$ location coincided with the position where the forming vortex reaches its maximum circulation. Upon closer inspection of the results for the two-dimensional square cylinder [12] and circular cylinder [13], together with the present results, it can be inferred that this association holds at higher (turbulent) Reynolds numbers as well.

Fig. 4-4: Vortex circulation as a function of shedding phase, $\Phi$, at various horizontal planes. After vortex B sheds, the strength of A does not increase, even though the vortex is still connected to the shear layer.
Fig. 4-5: Contours of $\overline{u}$ and $\overline{v}$. Broken line represents the approximate streamwise location of maximum vortex strength indicating that the maximum strength occurs at the approximate streamwise location of $\overline{u}_{\text{max}}$. Note case E taken from Cantwell and Coles [13].
Table 4-2. Value and location of mean streamwise Reynolds normal stress ($\overline{u}_{\text{max}}$), vortex position closest to centerline ($y_{\text{min}}$) and separation point of vortex ($x_s/d$).

<table>
<thead>
<tr>
<th>Case</th>
<th>A₁</th>
<th>A₂</th>
<th>B₁</th>
<th>B₂</th>
<th>C</th>
<th>D</th>
<th>E</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\overline{u}_{\text{max}}$</td>
<td>1.4</td>
<td>1.02</td>
<td>0.52</td>
<td>0.71</td>
<td>0.86</td>
<td>---</td>
<td>0.28</td>
</tr>
<tr>
<td>$x(\overline{u}_{\text{max}})/d$</td>
<td>1.7</td>
<td>1.8</td>
<td>3.6</td>
<td>3.6</td>
<td>3</td>
<td>~1.75</td>
<td>1</td>
</tr>
<tr>
<td>$y(\overline{u}_{\text{max}})/d$</td>
<td>0.83</td>
<td>1.02</td>
<td>1.20</td>
<td>1.15</td>
<td>1.13</td>
<td>---</td>
<td>~0.38</td>
</tr>
<tr>
<td>$x_s/d$</td>
<td>2.2-2.5</td>
<td>2.2-2.5</td>
<td>3.3</td>
<td>3.3</td>
<td>3.3</td>
<td>2.5-3</td>
<td>1.5-2</td>
</tr>
<tr>
<td>$y_{\text{min}}/d$</td>
<td>0.54</td>
<td>0.58</td>
<td>0.78</td>
<td>0.83</td>
<td>0.72</td>
<td>0.12</td>
<td>(center, centroid not available)</td>
</tr>
<tr>
<td>$x(y_{\text{min}})/d$</td>
<td>1.5</td>
<td>2</td>
<td>2.6-2.9</td>
<td>2.8-3.3</td>
<td>2.4-2.8</td>
<td>2</td>
<td>2</td>
</tr>
</tbody>
</table>

Closer inspection of the phase-averaged velocity profiles in the base region up to $x_{c_{\text{max}}}$, indicates that, although these velocities fluctuate significantly during the shedding cycle, the $y$-location of the maximum $u$-component velocity surprisingly little. In fact, the $y$-location, $S_o$, of the maximum mean streamwise velocity component, $u$, is found to be in agreement with the $y$-location of the vortex when it reaches its maximum circulation. Hence, $S_o$ can be used to approximate the wake width at $x_{c_{\text{max}}}$.

In several earlier studies [9, 11], the vorticity thickness, $\delta_w = (u_{\text{max}} - u_{\text{min}}) / (\partial u/\partial y)_{\text{max}}$, has been proposed as a scaling parameter for the shedding frequency. It is found that for the cases studied here, $\delta_w$ varies little during the shedding cycle, as is shown in Fig. 4-6.
Fig. 4-6: Ratio of the wake width ($S_w = \delta_w + d/2$) obtained from mean and phase-averaged fields at the streamwise location of $\overline{u_{\text{max}}}$.

4.4.2 Vortex growth pattern

As shown in Fig. 4-2, the vortex grows and reaches a maximum approximately when the opposing vortex is shed. However, vorticity strands still extend to the vortex along the separated shear layer suggesting that the vortex remains attached to the shear layer (Fig. 4-2, $\Phi_1$ and $\Phi_3$). The process leading to the detachment of the vortex from the shear layer appears gradual. The centroid of the vortex moves away from the centre-plane and the rate of displacement increases.

Given the similarity of this process for the different two-dimensional and three-dimensional geometries considered here, it is reasonable to suppose that the growth characteristics share a common scaling. A suitable velocity scale would be the
characteristic velocity of the feeding shear layer, \( u_s \). The length scale would be associated with the characteristic shear-layer length at the time of maximum growth (i.e. half the shedding cycle) or \( u_s / 2f_s \), so that the maximum circulation, \( \gamma \), should scale with \( u_s^2 / 2f_s \). Roshko [10] presented the same scaling based on circulation flux arguments, defining the circulation shed to be:

\[
\frac{\Gamma_o}{U_o D} = \frac{1}{2St} \left( \frac{u_s}{U_o} \right)^2 ; \quad \frac{u_s}{U_o} = \sqrt{1 - \frac{C_{pb}}{\gamma}} , \quad (4-2)
\]

where \( C_{pb} \) is the mean base pressure, such that \( \frac{\gamma}{\Gamma_o} = \frac{\gamma}{u_s^2 / 2f_s} \).

As seen from Table 4-3, \( \gamma / \Gamma_o \) varies little for the test cases considered including those for two-dimensional square and circular cylinders [12,13], especially when considering the relatively high uncertainty in estimating the circulation.

The growth pattern and delayed shedding behavior is similar to that observed by Sattari et al. [7] for the formation and shedding of an isolated line vortex. In the latter study, it was proposed that the maximum circulation scaled with the velocity at the edge of the shear layer and the distance between the vortex centroid and shear layer edge, approximated by \( S_o - y_{min} \).

The maximum circulation strength, \( \gamma \), scaled with \( u_s \) and \( S_o - y_{min} \), is shown in Fig. 4-7. For comparative purposes, Fig. 4-7 includes results with three additional length scales. The scaling with \( S_o \) corresponds to the wake half-width as defined by Roshko [10] for scaling the shedding process. The scale \( S_w = d / 2 + \delta_w \) is that used by
Williamson and Brown [11] to scale the shedding frequency in circular cylinder wakes. The scale $S_w - y_{\min}$ is included for completeness. It is observed that a satisfactory scaling is obtained using $S_o - y_{\min}$ for all cases considered. The use of $S_o$ does not appear to satisfactorily collapse two-dimensional and three-dimensional results. The scales $S_w$ and $S_w - y_{\min}$ appear less satisfactory than with $S_o - y_{\min}$, but remain plausible as the collapse remains marginally within the uncertainty bounds.

<table>
<thead>
<tr>
<th>Case</th>
<th>A_1</th>
<th>A_2</th>
<th>B_1</th>
<th>B_2</th>
<th>C</th>
<th>D</th>
<th>E</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\gamma / \Gamma_o$</td>
<td>0.39</td>
<td>0.41</td>
<td>0.47</td>
<td>0.40</td>
<td>0.50</td>
<td>0.44</td>
<td>0.41</td>
</tr>
</tbody>
</table>

### 4.4.3 Scaling of the shedding frequency

Observing that the maximum vortex strength, $\gamma$, scales with $u_\gamma$ and the shedding frequency, $f_s$, and locally with $u_\gamma$ and a vortex formation length scale, a scaling of $f_s$ with $u_\gamma$ and the local length scale is a reasonable expectation. Monkewitz and Nguyen [9], for example, have argued that the periodic shedding is a resonant phenomenon, whereby the wake convective instability excites the global instability in the base region. Table 4-4 provides a summary of the Strouhal number obtained using different length scales. Also included is the classical definition, $St = f_s d / U_\infty$, and using the wake width scale, $b$, using Kronauer’s criterion. Immediately, it is seen that $St_{\gamma y}$, based on the vortex
formation length-scale, provides a tightly bound collapse. Scales based on the wake width, $S_o$ or $b$, are less satisfactory in collapsing the three-dimensional obstacle data with those for two-dimensional geometries. This observation also holds for scaling based on the vorticity, $\delta_w$, alone. However, using the scale $d/2 + \delta_w$, proposed for circular cylinders [11], results in a comparable collapse to that using $S_o - y_{min}$. Note that in [11], the scales were selected at the end of the mean recirculation region, which for the circular cylinder is close to the location of $\bar{u}_{max}$.

Fig. 4-7: Scaling of circulation for different test cases using shear-layer maximum velocity ($u_f/U_\infty = \sqrt{1-c_{pb}}$) and $S_w$, $S_w - y_{min}$, $S_o$ and $S_o - y_{min}$ as the characteristic length.
Table 4-4. Non-dimensional shedding frequency using varying length scales.

<table>
<thead>
<tr>
<th>Case</th>
<th>$A_1$</th>
<th>$A_2$</th>
<th>$B_1$</th>
<th>$B_2$</th>
<th>$C$</th>
<th>$D$</th>
<th>$E$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C_{pb}$</td>
<td>-0.61</td>
<td>-0.59</td>
<td>-0.57</td>
<td>-0.56</td>
<td>-0.48</td>
<td>-1.6</td>
<td>-1.21</td>
</tr>
<tr>
<td>$u_s/U_w = \sqrt{1 - C_{pb}}$</td>
<td>1.27</td>
<td>1.26</td>
<td>1.25</td>
<td>1.25</td>
<td>1.22</td>
<td>1.61</td>
<td>1.49</td>
</tr>
<tr>
<td>$St = f_s d / U_{\infty}$</td>
<td>0.100</td>
<td>0.100</td>
<td>0.102</td>
<td>0.102</td>
<td>0.102</td>
<td>0.132</td>
<td>0.179</td>
</tr>
<tr>
<td>$St_o = f_s S_o / u_s$</td>
<td>0.13</td>
<td>0.12</td>
<td>0.17</td>
<td>0.16</td>
<td>0.15</td>
<td>0.10</td>
<td>0.10</td>
</tr>
<tr>
<td>$St_{oy} = f_s (S_o - y_{min}) / u_s$</td>
<td>0.082</td>
<td>0.076</td>
<td>0.103</td>
<td>0.092</td>
<td>0.088</td>
<td>0.088</td>
<td>0.077</td>
</tr>
<tr>
<td>$St_w = f_s S_w / u_s$</td>
<td>0.13</td>
<td>0.12</td>
<td>0.15</td>
<td>0.14</td>
<td>0.14</td>
<td>0.14</td>
<td>0.12</td>
</tr>
<tr>
<td>$S_w = d / 2 + \delta_w$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$St_{wy} = f_s (S_w - y_{min}) / u_s$</td>
<td>0.088</td>
<td>0.072</td>
<td>0.085</td>
<td>0.068</td>
<td>0.078</td>
<td>0.110</td>
<td>0.105</td>
</tr>
<tr>
<td>$St_\delta = f_s \delta_w / u_s$</td>
<td>0.091</td>
<td>0.080</td>
<td>0.109</td>
<td>0.099</td>
<td>0.098</td>
<td>0.079</td>
<td>0.060</td>
</tr>
<tr>
<td>$St_b = f_s b / u_s$</td>
<td>0.205</td>
<td>0.206</td>
<td>0.208</td>
<td>0.208</td>
<td>0.214</td>
<td>0.164</td>
<td>0.170</td>
</tr>
</tbody>
</table>

4.5 **CONCLUSIONS**

The vortex formation and shedding process in the wake of finite aspect ratio, surface-mounted square cylinders was investigated and compared to that for square and circular cross-section two-dimensional cylinders. It is shown that the maximum strength (circulation) of the forming vortex and the shedding frequency scale with the initial shear-layer intensity, characterized through $u_s = \sqrt{1 - C_{pb}}$ and the distance between the shear layer edge and vortex centroid ($S_o - y_{min}$), at the shedding phase corresponding to the maximum vortex growth.
It is observed that the maximum vortex strength is reached roughly half-way through the shedding cycle. Similarities to the growth pattern and scaling for the formation and shedding of an isolated two-dimensional vortex [7] suggest that the growth of the forming vortex scales locally and is naturally limited. Once fully formed, the motility of the vortex (convective speed of the vortex characterized by the rate of displacement of the centroid) increases until the vortex finally sheds. These observations are consistent with the resonance mechanism inferred by Monkewitz and Nguyen [9]. When the vortex has fully formed, the base region is unstable and shedding is triggered by the disturbances induced by the wake flow (primarily induction from the previously shed vortices), thus locking in the shedding period.

The use of a shedding phase dependent quantity, $y_{\text{min}}$, is inconvenient. However, it was shown that the downstream location of $\tilde{u}_{\text{max}}$ approximates the downstream location of the centroid at the point when the vortex reaches its maximum circulation. At this location, the edge of the shear layer and the vorticity thickness when the vortex is fully formed are well approximated by the mean values, $S_o$ and $\delta_w$, such that a scaling with $d/2 + \delta_w$ provide a reasonable collapse for both two-dimensional and three-dimensional geometries.
4.6 REFERENCES


Chapter Five: Synthesis

In Chapters 2-4, three papers each focusing on a specific topic related to vortex formation and shedding were presented. The purpose of this chapter is to synthesise the ideas introduced in the three papers. A summary of the main findings of the papers will be presented and the connection between the ideas will be described. In the next chapter, recommendations for future work will be suggested.

5.1 HYPOTHESIS

The goal of this research is to gain a more in-depth understanding of bluff-body shedding mechanism with focus on finite surface-mounted obstacles and to clarify the observed discrepancies in the literature. One of the issues pertaining to bluff-body vortex shedding is the process through which the vortices reach maximum growth and detach from the shear layer. The conventional description for bluff-body vortex shedding (see for example Gerrard, 1966) implies that the vortex would grow without limitation if the opposite shear layer were not interfering. However, our measurements of vortex circulation for an AR=4 square cylinder along with some recent studies on pitching airfoils (e.g. DeVoria and Ringuette, 2011) support the view that the forming vortex gradually reaches a maximum growth independent of the opposite-sign vortex. This observation is in disagreement with the earlier descriptions of vortex shedding process and indicates the need for their reassessment. In particular, the immediate questions to
address are: if the vortex shedding is not caused by the interference of opposite-side shear layer, what alternative mechanism may cause the growing vortex to separate from the feeding shear layer? How does the vortex growth pattern look like within a shedding cycle? What factors interfere with the flux of circulation into the vortex? Is there a natural universal limit to the growth of the vortex as proposed by Dabiri (2009) regardless of vortex-generating configuration? Is there a link between the proposed universal Strouhal numbers and the limit to vortex growth?

Towards answering these questions, a hypothesis for the shedding process was proposed and tested. According to this hypothesis, a distinction between the process through which the vortex reaches maximum growth and periodic shedding in the wake should be made. More specifically, the mutual interaction between opposing vortices is not the mechanism for the limit to vortex growth. Rather, the mutual interaction is the mechanism by which the shedding frequency gets locked-in with the wake periodicity.

Based on this hypothesis, the vortex grows asymptotically and reaches a plateau. The circulation flux is determined at the shear layer separation point on the obstacle and is thus expected to remain nearly constant. Hence, the existence of a growth-plateau implies that increasingly less circulation is fed to the growing vortex. Thus, approximately at the time the plateau is reached, the connection between the vortex and the feeding shear layer is weak and susceptible to disturbances and ready to break off. The precise shedding time is imposed by disturbances or fluctuations due to the advecting vortex street in the wake. Through Biot-Savart induction, the forming vortex moves away from the symmetry axis, consequently causing the feeding shear layer to deflect and eventually detach from the
vortex. It is therefore expected that the deflection of the opposite-side shear layer and formation of a vortex on the opposing side are in fact a consequence of the shedding process, but not its cause.

For this mechanism to work, a synchronization between the shedding frequency and the time scale associated with the growth of the vortex should hold. It is thus expected that the scales associated with vortex growth should match with the scales of periodic downstream activity. To test this argument, shedding in the wake of different test cases of surface-mounted finite square cylinders (AR=4, thin boundary layer, AR=8 thin boundary layer, AR=8 thick boundary layer) is examined. Depending on the aspect ratio and boundary-layer thickness, significant structural changes may occur in the flow. For an aspect ratio 4 obstacle placed in a thin boundary layer, the structures are shed in the form of half-loops as described in detail in Bourgeois et al. (2011). The same “half-loop” structure was observed in recent studies (Bourgeois et al., in publication process) for an aspect ratio 8 obstacle in a thin boundary layer. However, for an aspect ratio 8 obstacle placed in a thick boundary layer, the structure changes from half-loop to full-loop. Interestingly, as shown in Appendix A, despite the altered structure of shed vortices, the shedding Strouhal number remained unaltered for all the three aforementioned test cases. The constant Strouhal number suggests that the scaling scheme might be independent of the three-dimensionality in the flow and hence applicable to both 2D and 3D obstacles. To verify this, available data from the literature on 2D square and circular cylinders are analyzed and compared in support of the arguments.
In order to verify the above-described hypothesis, a number of steps towards understanding the shedding process were necessary. Briefly, these steps are:

i) Verifying that periodic vortex shedding is indeed of the Kármán-type (i.e. similar to 2D case) for finite cylinders. This step is necessary since reports in the literature suggested the existence of an alternative, periodic “symmetric shedding” regime. The mechanisms leading to Kármán-type and “symmetric” periodic shedding are inherently different. Hence, resolving the type of shedding serves to focus the subsequent investigations.

ii) Study the evolution of an isolated 2D vortex. These experiments aim at understanding the formation and shedding of vortices in a simplified configuration to allow insight into the parameters governing formation and shedding.

iii) Applying the gained knowledge to the more complex bluff-body shedding process and identify appropriate scaling.

5.1.1 Reconciliation of symmetric/anti-symmetric shedding

One of the necessary steps towards studying the vortex shedding process of finite surface-mounted cylinders was to clarify the discrepancy in the literature with respect to anti-symmetric versus symmetric shedding. The importance of reconciling this topic lies in the fact that the existence of a symmetric shedding mode would imply a fundamental change in the interaction of vortices compared to the anti-symmetric mode. As discussed in detail in Chapter 4, the shedding mechanism is tightly related to downstream oscillations of the flow field due to the out-of phase passage of vortices. An in-phase
shedding mode would, however, imply minimal downstream fluctuations, lack of a triggering activity required for vortex detachment from the feeding shear layer, and consequently an unstable vortex arrangement in the wake.

The reconciliation was accomplished through a series of systematic analysis described in detail in Chapter 2. Using spatial cross-correlation, instantaneous phase relationships, and phase-averaged velocity data obtained from PIV and hotwire anemometry measurements, it was shown that the shedding in the wake of surface-mounted finite square cylinders was predominantly anti-symmetric (regime A). What had been referred to as symmetric shedding in the past, based on low-Reynolds number visualizations and single-snapshot PIV measurements, was in fact a distortion of the regular shedding process accompanied with low amplitude pressure and velocity fluctuations in the base region (regime B). In this mode, two vortices can exist simultaneously in the base region, however, counter-rotating vortices are still shed alternately from opposing sides. Furthermore, it was found that the Strouhal number remained constant over the entire height of the obstacle. These patterns were observed for all tested aspect ratios between 1 and 8.

Since the shedding frequency in both regimes A and B are nearly the same, it is possible to characterise the flow field in terms of a shedding phase. This feature was used to obtain the phase-averaged flow field in a grid of horizontal and vertical planes. The data were then used to verify the proposed hypothesis for the shedding process described earlier in this chapter.
5.1.2 Evolution of an isolated 2D vortex

The process of formation and shedding of a vortex associated with bluff bodies is a rather complex phenomenon due to the presence of the obstacle, the flapping of the shear layer, and interaction of multiple vortices. To gain a better understanding on vortex formation in a simpler configuration, vortex generation in an isolated environment with no wall or vortex interaction or reorientation of the vortex was desired. Such conditions can effectively be achieved through the generation of a line vortex from a starting two-dimensional shear layer in an isolated environment. The conclusions for this simpler configuration will then be generalized to the case of bluff-body shedding.

An approximation to the two-dimensional vortex generation was simulated through the start up of a planar two-dimensional jet produced by a plasma actuator. The vortex growth in such a configuration was observed to be limited through a gradual separation of the vortex from the feeding shear layer, arising from decreased shear-layer curvature. This phenomenon was linked to a competition between the shear-layer tendency to remain in the streamwise direction and the induced velocity from the vortex on the shear layer. As long as the induction from the vortex is sufficiently large compared to shear-layer velocity, the shear-layer curvature remains close to that of the vortex core radius. At the same time, the circulation-containing fluid from the shear layer will also be drawn towards the vortex core. However, due to the global flow features in this specific configuration, the vortex core travels away from the shear layer with a faster rate than the circulation growth. This process leads to a gradual decrease in the contribution of vortex-
induced velocity on the shear layer, hence a “flattening” of the shear layer, and eventually a separation of the vortex from the feeding shear layer.

In addition to investigating the separation mechanism, a model for the early-stage vortex growth, based on the transport of circulation from the shear layer into the vortex was developed. The model is applicable to any vortex generation system from a nominally 2D shear layer. Based on this model, the rate of vortex growth in time is not linear in time, but rather increases with the square-root of time. This suggests that only a fraction of the produced circulation in the shear layer is convected into the vortex, whereas a linear growth with a slope equal to $0.5u_s^2$ ($u_s$ is the shear-layer maximum velocity) would imply that all the produced circulation is transferred into the vortex. Vortex growth predicted by this model is in line with earlier observations reporting that only a portion of the generated vorticity is convected in the individual vortices downstream of bluff bodies (e.g. Fage and Johansen, 1927).

The above-described experiment along with some of previous investigations indicate that the cause of detachment of the vortex from the shear layer is in fact configuration-dependent. Therefore, the idea of a “universal” formation number proposed by Dabiri (2009) may fail in certain configurations depending on the global flow field. In the case of vortex rings for instance, the separation occurs due to an increased vortex ring convective velocity compared to the feeding shear layer. In the case of the present experiments with the isolated vortex, however, the detachment occurred due to the competition between the shear-layer tendency to remain in the streamwise direction and the induced velocity from the vortex on the shear layer. Pedrizzeti’s (2011) simulations of
a dipole created from a two-dimensional orifice is a good example of conditions where the shear layer remains attached to the vortex, and therefore an unlimited vortex growth is achieved. With respect to bluff-body shedding, it is expected that the detachment of the growing vortex from the feeding shear layer should be caused by the induction (Biot-Savart) from downstream vortices. The mechanism for this interaction was the subject of the third article of this thesis and will be reiterated in the following sections.

5.1.3 Bluff-body shedding mechanism

To test the proposed hypothesis, phase-averaged velocity and vorticity fields for different cases were considered. These cases included results obtained from the present experiments as well as data from well-documented previous studies on a two-dimensional square cylinder (Lyn et al., 1995) and a two-dimensional circular cylinder (Cantwell and Coles, 1983). In the present experiments, vortex shedding in the wake of three distinct flows was investigated: aspect ratio 4 and 8 surface-mounted square cylinder placed in a thin boundary layer, and an aspect ratio 8 surface-mounted square cylinder placed in a thick boundary layer. Furthermore, to compare the influence of flow three-dimensionality, where possible, results in more than one plane along the obstacle height were investigated.

The vortex was observed to asymptotically reach a plateau within the shedding cycle. Once maximum growth was reached, the vortex remained attached to the shear layer. In line with the proposed hypothesis, the reduced flux of circulation into the vortex is an indication of the weak connection between the vortex and the shear layer, which could easily break off by any disturbances. It was shown that the break-off is triggered by
the Biot-Savart induction of the shed vortices, which tend to move the vortex away from
the symmetry axis. Towards the end of the shedding cycle, the vortex reaches a minimum \( y \)-
distance (\( y_{\text{min}} \)) from the centre-plane (\( y = 0 \)) and is afterwards moved away from the
centre-plane until the vortex sheds.

The ratio of maximum vortex circulation, \( \gamma \), to the total circulation generated at
the obstacle leading edge (\( \Gamma_a = 1 - \overline{C_{pb}} / 2St \)) was noted to be nearly constant for all test
cases including those reported by Lyn et al. (1995) and Cantwell and Coles (1983),
indicating that only a fraction of the circulation from the feeding shear layer is convected
into the vortex core. Two phenomena are expected to be at play for this phenomenon. A
part of the shear-layer circulation is accumulated in regions of flow field outside the
vortex core. These regions are basically the zones in which the contribution of the shear
strain-rate is larger than the rotational part in the vorticity term. Additionally, some of the
circulation is cancelled by opposite-sign circulation generated in the obstacle base region.
Interestingly, the asymptotic growth of vortex strength is in line with the observation of
Sattari et al. (2012) for the growth of an isolated vortex from a starting two-dimensional
shear layer.

Although the vortex growth is naturally limited, the mutual interaction of wake
vortices does play a role in the shedding process, as it results in a lock-in of vortex
shedding with wake periodicity. The synchronization between the vortex growth in the
base region and the wake periodicity requires that the scales for the vortex to reach
maximum growth should match with the scales associated with the periodic wake
activity. As for the velocity scale, the vortex circulation should naturally scale with the
shear-layer characteristic velocity. The shear-layer velocity is calculated to good approximation from the base pressure coefficient as $u_s/U_\infty = \sqrt{1-C_{pb}}$. This is in line with previously used velocity scales by Roshko (1954) and Bearman (1967) among others. As for the characteristic length, a length scale defined as the distance between the shear-layer edge and vortex center at the approximate streamwise location of maximum vortex strength was considered and shown to result in a collapse of non-dimensional circulation for all test cases. It was also verified that at the location of maximum streamwise velocity fluctuations, the $\gamma$-location of the shear-layer edge changes surprisingly little and is thus well represented by the mean flow field. Therefore, the length scale could be obtained from the mean field rather than requiring the phase-dependent velocity information. This facilitates similar calculations where phase-averaged data are not available.
5.2 REFERENCES


Chapter Six: Conclusions & recommendations for future studies

6.1 CONCLUSIONS

The goal of the present thesis was to gain a deeper understanding about the vortex shedding phenomenon with special focus on finite surface-mounted square cylinders. Emphasis was put on understanding the formation and growth process of vortices, their interaction and the mechanism through which the vortex is detached from the feeding shear layer.

As a first step, the discrepancy with respect to symmetric versus anti-symmetric vortex shedding was resolved. Using spatial instantaneous phase relationships, cross-correlation, and phase-averaged velocity data, it was shown that anti-symmetric shedding (regime A) is the dominant shedding regime in the wake of surface-mounted finite square cylinders. What was referred to as symmetric shedding in previous studies based on low-Reynolds number visualizations and single-snapshot PIV measurements was, in fact, a distortion of the regular shedding accompanied with low amplitude pressure and velocity fluctuations in the base region (regime B). In this mode, two vortices are formed in the base region. However, vortices are still shed alternately with the same shedding frequency as in regime A. In addition to contributing to the physical understanding, this study was also a prerequisite for further study of surface-mounted bluff-body shedding. A phase-averaged investigation of the quasi-periodic activity would not have been justified
in the presence of symmetric shedding mode with a distinct frequency, different from that associated with Kármán-type shedding.

With the purpose of elucidating the observed discrepancies in the literature, a reassessment of the conventionally described vortex shedding process was pursued by introducing an alternative hypothesis for bluff-body vortex shedding. Results showed that the circulation flux from the feeding shear layer into the vortex was reduced in time such that the vortex growth was asymptotic. This observation was very similar to the trend for the growth of an isolated two-dimensional vortex from a starting shear layer (Sattari et al., 2012). It was concluded that the vortex growth was naturally limited resulting in a weak, ready-to-break off, connection between the vortex and the shear layer towards the end of the shedding cycle. Biot-Savart induction from the wake vortices was suggested to act as a disturbance and eventually trigger the disconnection of the vortex from the shear layer. This study contributes to the understanding of vortex shedding as it clarifies that the vortex growth is naturally limited, rather than being terminated due to the mutual interaction of opposing vortices, as implied in previous studies. Rather, the mutual vortex interaction is the cause for the shedding frequency to lock in with the wake periodicity.

The present study on bluff-body vortex shedding was carried out at a free stream turbulence intensity of 0.8%. Previous studies have shown that higher turbulence level results in an increased base pressure (i.e. reduced drag), increased mixing, and weakened vortex shedding (Lee, 1975; Nakamura and Ohya, 1984). The existing studies in the literature are, however, limited to statistical investigations. The extent of the influence of turbulence intensity on the shed structures requires further future investigation.
6.2 RECOMMENDATIONS FOR FUTURE WORK

The observations made in the present thesis raise further interesting questions. Answering these questions will advance understanding on vortex formation and bluff-body vortex shedding. In the following section these questions will be discussed and strategies towards tackling them in the future will be recommended.

1. As discussed in detail in Chapter 4, the strength of the growing vortex in the wake of bluff bodies asymptotically reaches a plateau within the shedding cycle. This trend was specifically evident from Fig. 4-4 of Chapter 4, where the trend of vortex circulation as a function of time was presented. Although annihilation has been argued in the literature to be responsible for this behaviour, a detailed analysis demonstrating this argument has not been provided so far. It is hypothesized that the reduced vortex circulation occurs due to two phenomena. A fraction of circulation contained in the shear-layer is accumulated in regions of flow field outside the vortex core. In these regions, the contribution of the shear strain-rate is larger than the rotational part in the vorticity term. Additionally, some of the circulation is also cancelled by opposite-sign circulation generated in the obstacle base region. As illustrated in Fig. 6-1, due to the no-slip condition at the obstacle back face, this region is a source of counter-clockwise circulation, opposite to the clockwise circulation of the shear layer. As the vortex grows in time, the rate of production of opposite-sign circulation also increases, which eventually becomes significant compared to the circulation transferred from the feeding shear layer into the vortex. To accurately assess the contribution of each
of these two phenomena, considering a circulation budget is recommended. To do so, the circulation flux into the vortex and the rate of the opposite-sign circulation production at the obstacle rear face should be directly measured as a function of time. To this end, high-resolution measurements within the shear layer and the base region are necessary in order to properly resolve the velocity gradients, in particular inside the shear layer.

![Diagram](image)

Fig. 6-1: Illustration of proposed hypothesis for vorticity cancellation mechanism in the base region of bluff bodies. Circulation with an opposite sign compared to the counter-clockwise shear-layer vorticity is generated due to no-slip condition at the obstacle back surface. Clockwise circulation is expected to be produced with a higher rate as the vortex strength increases.

2. In Chapter 3, the evolution of an isolated line vortex generated by a starting two-dimensional jet was studied. The separation of the vortex from the shear layer was linked to the traveling of the vortex away from the shear layer, which resulted in reduced vortex induction on the shear layer. The reduced induction, in turn, caused the shear layer to gradually “flatten”, whereby the shear layer was eventually detached from the vortex. It is expected that if the global flow field could be altered such to prevent the vortex from traveling away from the shear
layer, the vortex/shear-layer connection could be maintained. Considering Biot-Savart, such a change in the flow field can be achieved by generating a dipole as illustrated in Fig. 6-2, rather than a single vortex. Under these conditions, the opposite-sign mirror vortex will induce an additional velocity component on the vortex, such that the net convective velocity of the vortex \( u_{net} = u_1 + u_2 \) in Fig. 6-2) away from the shear layer is reduced. Consequently, the rate of drop in \( \Gamma^* \) (as defined in Chapter 3) will decrease, and the vortex will remain attached to the shear layer for an extended time period. This would result in a stronger vortex in terms of size and circulation. It is therefore recommended to repeat the experiments using a double-actuator as shown in Fig. 6-2. For stronger mutual induction of the mirror vortices, it is recommended to keep the thickness of the base plate as small as possible and the electrodes as close as possible to the edge of the plate. If the above-described methodology for an extended growth of the vortex proves to be functional, guidelines for achieving stronger vortices by changing the global flow field can be proposed.
Fig. 6-2: Proposed double-actuator setup for the generation of a dipole. It is expected that under these conditions the vortex will remain attached to the shear layer for an extend time period resulting in a stronger vortex.

Furthermore, in Chapter 3, the growth and separation of the vortex generated by a starting shear-layer was described in terms of kinematic arguments. Alternatively, a dynamic approach for the evolution process can be considered, as the vortex growth can be attributed to the flux of momentum and kinetic energy from the shear layer into the vortex. Interestingly, Shusser and Gharib (2000) showed that in the case of vortex rings forming in a cylinder-piston configuration, the two approaches result in similar conclusions. It is suggested that a comparison between kinematic and dynamic interpretations can provide further insight into the physics of the vortex evolution process.
6.3 REFERENCES


Appendix A: Turbulent wake of surface-mounted finite aspect-ratio bluff bodies: Effect of aspect ratio and cross-section shape

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Author Contribution: Pooria Sattari and Robert Martinuzzi developed the idea of the paper. Pooria Sattari analyzed instantaneous flow fields, conducted hotwire and PIV measurements, and phase and spectral analysis. Jason Bourgeois contributed to PIV measurements, and performed wavelet transform analysis. Robert Martinuzzi supervised the project.

Some modifications have been made to the originally published paper in response to queries raised by candidate’s Ph.D. examination committee.


A.1 ABSTRACT

Dynamic shedding characteristics in the wake of vertical, surface-mounted square and circular cylinders of height-to-width aspect ratio \((H/D)\) 0.5, 2, 4 and 8 are investigated using high frame-rate particle image velocimetry and thermal anemometry for Reynolds numbers, based on the obstacle width/diameter ranging between \(6 \times 10^3\) and \(10^5\). The wavelet transform was used to determine the instantaneous phase relationship for either pressure or velocity fluctuations on opposite sides of the obstacles. Except for the smallest aspect ratio, two shedding modes with different vortex configurations are observed in the formation region: i) Alternate formation of vortices accompanied by high-amplitude fluctuation; ii) Events of simultaneous formation of two vortices resulting in low-amplitude activity and varying phase relationship between opposing sides. These counter-rotating coexisting vortices are generally shed alternately. In the case of the aspect ratio 0.5 prism, periodic activity is irregular and the shedding behavior is different from higher-aspect-ratio bodies.

A.2 INTRODUCTION

Periodic vortex shedding in the turbulent wake of surface-mounted finite-aspect-ratio bluff bodies is a complex three-dimensional phenomenon which involves the interaction of non-steady vortical structures of different scales, orientations and intensities. The three main coherent structures associated with the flow around these bodies include the horseshoe vortices generated by the adverse pressure gradient
upstream of the obstacle at the wall-body junction, the Kármán-like shed structures in the mid-span, and the tip vortices. One expects the contribution of each of these structures to the overall flow field to vary with obstacle shape and aspect ratio \((AR = H/D)\). Yet, there are few comprehensive comparative parametric studies.

The behavior of the wake velocity and obstacle surface pressure is characterized by nearly constant frequency oscillations with intervals of high-amplitude fluctuations interspersed with low-amplitude activity of varying length depending on the aspect ratio and cross-sectional shape. Generally, the phase relationship between fluctuations on opposing sides of the wake may vary along the height and the distance downstream of the obstacle.

Mainly based on visualizations at low Reynolds numbers, it has been observed [1-2] that below a critical aspect ratio (varying between 1 and 7 depending on boundary layer characteristics and obstacle geometry [3]), the shedding regime changes from anti-symmetric (Kármán-type) to symmetric (arch-type). It was also suggested that for higher aspect ratio circular cylinders, towards the free end, the shedding becomes symmetric [2].

Wang and Zhou [4] and Wang et al. [5] used a Q-criterion method to detect vortex cores in the flow field based on low-frame rate (15 fps) PIV measurements in planes parallel to the free end along body’s height. They found that as the free end is approached, the probability of symmetric arrangements of shed vortices increased relative to anti-symmetric arrangements in the wake for aspect ratio 5 and 7 prisms at Reynolds numbers ~10,000.
The observed symmetry has been attributed to the suppression of the Kármán-type shedding process due to the downwash generated by tip vortices near the free end. However, the formation mechanism of these tip vortices and how these interact with the formation region in the obstacle lee are still not well understood [4, 6-8]. Whether the same mechanism exists for both circular and square sections and whether the formation of the tip vortices is inherently periodic or modulated by the downstream shedding remains unclear.

In the present work we aim to investigate in greater detail the dynamic shedding behavior for square and circular cylinders of aspect ratios 0.5, 2, 4 and 8. High frame-rate particle imaging velocimetry (HFPIV) and thermal anemometry were used as experimental tools. A complex wavelet transform technique was implemented for the instantaneous determination of the shedding phase from opposite sides of these obstacles along their height. Surface shear stress field patterns deduced from oil film visualizations were used to examine the effect of wake dynamics on the mean flow pattern.

### A.3 Experimental Setup

The experiments were carried out in an open-section wind tunnel at the University of Calgary with a 0.5-m-diameter circular working section. Seven turbulence-reduction mesh screens are installed upstream of the contraction channel (area contraction ratio 16:1) resulting in a free stream turbulence intensity below 0.8% at the free-stream velocity used for the experiments ($U_w \approx 15\text{ m/s}$).
Circular and square section cylinders with aspect ratios 0.5, 2, 4 and 8 were machined from aluminum. The height of all the obstacles was 50.8 mm. Reynolds numbers based on obstacle width, \( D \), and free stream velocity, \( U_\infty \) ranged between \( 6 \times 10^3 \) and \( 10^5 \). Pressure taps with 0.35-mm diameter were drilled 12.7 mm (\( z/H = 0.25 \)) from the base on both side faces of \( AR = 4 \) prism for pressure measurements. The obstacle and coordinate system nomenclature are summarized in Fig. A-1.

![Diagram showing obstacle and coordinate system nomenclature](image)

**Fig. A-1:** Schematic of the obstacle and coordinate system nomenclature. \( H \): obstacle height; \( D \): obstacle width/diameter; \( \delta \): boundary layer thickness; \( x \), \( y \) and \( z \): coordinate axes, where \( x \) is the streamwise direction.

A flat plate machined from aluminum with a sharp leading edge was used to generate the boundary layer flow. All obstacles were mounted 0.2 m from the plate leading edge where the boundary layer thickness was approximately 8 mm (\( \delta/H = 0.18 \)). The boundary layer mean and rms (root-mean-square) streamwise profiles at this location (with the obstacle removed) are presented in Fig. A-2.
Fig. A-2: Boundary layer -●- mean \((U/U_\infty)\) -▲- rms \((u_{rms}/U_\infty)\) profiles at the obstacle location (obstacle removed).

A LaVision high frame rate particle image velocimetry (PIV) system was used for planar velocity field measurements. The laser sheet illumination was generated by a high power 527 nm Nd-Yag pulsed laser. A HighSpeedStar 5 camera (1024×1024 pixels) was used to capture images in the double frame mode \((dt = 50\mu s)\) at a 1000-fps sampling rate. The camera field of view was approximately 195 mm×195 mm. A 32 × 32 pixel interrogation window (6 mm×6 mm) with 50% overlap in both directions resulting in a vector separation of 3mm was used. Two Laskin particle generators placed outside (upstream) of the tunnel were used to generate seeding olive oil particles (mean diameter of approximately 1 µm).

Differential pressure transducers (All Sensors Corp, model 1 INCH-D1-4V-MINI) were used to measure pressure fluctuations simultaneously on both sides of the
aspect ratio 4 prism. The pressure transducer and PIV data acquisition were synchronized through TTL trigger signals. The differential pressure transducers had a range of 0-250 Pa and a resolution of 0.26 Pa. The pressure transducers were calibrated against an inclined manometer using a known pressure source. The pressure sensing system dynamic response was estimated against a flush-mounted transducer. Pressure data were sampled at 10240 Hz for 10 s.

For velocity measurements and spectral analysis, single hotwire anemometers (probe model TSI 1210-20, bridge model TSI IFA-100) operated in constant-temperature mode were used. Two wires were placed at either y/D = ±1.2 or y/D = ±2. A three-axis motorized computer-controlled traverse unit was used to position the wires at streamwise locations x/D = 0.5, 1, 2 and 4 along desired heights. One additional wire measured the free stream velocity. Hotwires were calibrated against a Pitot tube. Data were sampled simultaneously with pressure data at 10240 Hz for a record length of 100,000 samples.

A 24-bit National Instruments (NI-9227) data acquisition system with built-in anti-aliasing filter and a LabVIEW program were used to record hotwire and pressure data.

A mixture of oil, kerosene and graphite was used for the oil film visualizations to obtain the mean surface flow patterns. The tunnel was operated after the mixture had been applied and until the mixture dried.
A.4 Wavelet Transform Method

To investigate the phase angle of vortex shedding in the wake of the obstacles, the instantaneous velocities at \( y/D = \pm 1.2 \) and \( y/D = \pm 2 \) were measured simultaneously using two hotwire probes. Complex wavelet transforms were used to extract the instantaneous shedding phase difference between two signals [9]. The complex wavelet transform of a velocity signal \( u(t) \) with a scale \( \lambda \) (proportional to the inverse of frequency) and time lag \( \tau \) is defined as

\[
W(\lambda, \tau) = \int_{-\infty}^{\infty} u(t) \psi^{*}_{\lambda, \tau}(t) dt.
\]  

where the asterisk denotes the complex conjugate, and the wavelet function for a given scale and time lag is

\[
\psi_{\lambda, \tau}(t) = \frac{1}{\sqrt{\lambda}} \psi\left(\frac{t - \tau}{\lambda}\right).
\]

The analyzing wavelet used is the complex Morlet wavelet. It is constructed from a harmonic oscillation with an amplitude that has a Gaussian envelope. The Morlet wavelet with bandwidth, \( f_b \), and center frequency, \( f_c \), is defined as

\[
\psi(t) = \frac{1}{\sqrt{\pi f_b}} e^{i 2\pi f_c t} e^{-t^2 / f_b^2}.
\]

The phase of the vortex shedding cycle is determined by \( \phi = \tan^{-1} \text{Im}(W)/R(W) \), where \( W \) is the complex value of the wavelet transform at the scale associated with the frequency of vortex shedding as taken from the peak of the Fourier power spectra. The mother wavelet (which is the basic wavelet which is subsequently scaled and shifted in
time) used had \( f_b = 1.5 \text{Hz} \) and \( f_c = 1 \text{Hz} \) and is shown in Fig. A-3. The Mother wavelet is then scaled to have a center frequency equal to the frequency peak in the Fourier power spectra and the convolution integral (1) is taken.

Fig. A-3: Complex Morlet used for analysis.

**A.5 RESULTS AND DISCUSSIONS**

In this section synchronized PIV/pressure measurements are presented for the geometry \( AR = 4 \) square cylinder in order to investigate the nature of vortex formation in the base region and the shedding of the vortices. All presented PIV measurements are from a horizontal plane at \( z/H = 0.5 \).

Approximately 8 PIV images per shedding cycle were captured enabling to track the evolution of vortices (shedding frequency was \( f_s = 122 \text{Hz} \) corresponding to \( St = 0.10 \)). By inspecting the PIV snapshots for different shedding cycles, two typical vortex configurations at the lee of the obstacle are distinguished. One configuration corresponds
to shedding periods during which vortices of opposite sign circulation are formed and shed alternatively in a fashion similar to the classical two-dimensional von Kármán process. A typical sequence of such a configuration is illustrated in Fig. A-4 using sectional streamlines in the stationary frame of reference. These snapshots are separated by one quarter of the nominal shedding period ($\Delta t^* = \Delta t \times f_s = 0.25$, $\Delta t$: PIV sampling interval).

In the second configuration, two counter-rotating vortices exist simultaneously in the base region throughout the shedding cycle as shown in Fig. A-5.

![Fig. A-4: Sectional streamlines from PIV snapshots ($z/H = 0.5$) representing alternate formation of vortices in the base region (snapshots separated by $\Delta t = 0.002 \, s$, $\Delta t^* = 0.25$).](image)
Fig. A-5: Sectional streamlines from PIV snapshots (z/H = 0.5) representing coexistence of two vortices in the base region (snapshots separated by Δt = 0.002 s, Δt' = 0.25).

Pressure traces on both side faces of the obstacle (relative to free stream static pressure) corresponding to the PIV snapshots in Figs. A-4 and A-5 (PIV and pressure measurements were synchronized) are presented in Fig. A-6a. It is noted that the first vortex formation mode (i.e. alternate formation) is accompanied by high-amplitude well-organized fluctuations. In contrast, during the time intervals when two counter-rotating vortices coexist in the formation region, the pressure fluctuations on the obstacle side faces have much lower amplitude.

Figure A-6b illustrates the phase relationship between pressure fluctuations on opposite sides for the same time interval. During high-amplitude intervals, the phase difference is nearly constant (180° out-of-phase), whereas during low-amplitude
Appendix A

intervals, the phase relationship is less regular and changes significantly. A similar behavior is observed in the velocity time series measured close to obstacle in the vicinity of the formation region ($x/D < 2$).

![Graph showing pressure and phase difference over time](image)

Fig. A-6: a) Pressure from obstacle’s opposing sides synchronized with PIV snapshots in figures 4 and 5, b) corresponding phase difference. -A-: alternate vortex formation, -C-: coexistence of two base vortices.

Figure A-7 shows a sample of simultaneously recorded side face pressures and downstream velocities ($x/D = 4$, $y/D = \pm 2$, $z/H = 0.5$). As expected, during high-amplitude intervals, both pressure and downstream velocity fluctuations on opposing sides are both $\sim 180^\circ$ out-of-phase. During the low-amplitude intervals (observed in the pressure signals), however, the behavior observed in the formation region differs from
that observed downstream. Whereas the phase relationship between the pressure fluctuation on opposing faces varies significantly and is often in-phase, the amplitude of the downstream velocity fluctuations is less modulated, the periodicity more easily recognized and the fluctuations on opposing sides are again out-of-phase (~180°).

![Diagram](image)

Fig. A-7: Simultaneous velocity and pressures for the AR = 4 prism. Velocities measured at \( x/D = 4, \ y/D = 2, \ z/H = 0.5 \), \( p1 \& p2, \ u1 \& u2 \): pressures and velocities on opposing sides, respectively).

Closer inspection of the shedding sequence during the low-amplitude intervals suggests that, at a given instant during the shedding cycle, only a portion of the circulation from one of the coexisting vortices is shed (cf. Fig. A-5b). Generally, this partial shedding process occurs alternately from opposing base vortices. However, the time interval between the shedding of opposite circulation vortices varies randomly from cycle-to-cycle. As a result, the spacing between vortex pairs immediately downstream of
the formation region varies greatly, giving rise to large variations in the phase differences as observed in Fig. A-6b (intervals marked C). Nevertheless, as will be shown later, the probability of in-phase shedding remains small. Since only a portion of the circulation in the base region is transferred to the shed vortices, these are weaker than those observed during the alternating events of the first configuration, which may account for the lower fluctuation amplitudes.

**A.5.1 Effect of aspect ratio and shape**

This section focuses on the differences observed in the wake dynamics between square and circular section cylinders at different aspect ratios.

Sample time series of the fluctuation velocity, $u'$, for the square section cylinders of AR = 2, 4 and 8 for two elevations are shown in Fig. A-8. For AR = 4 and AR = 8 prisms, as the free end is approached the modulation in the signal becomes stronger and low-amplitude activity is more pronounced. This observation is consistent with observations made previously [2, 4]. These weak fluctuations in the free-end region occur much more frequently for AR = 8 compared to AR = 4. In the case of AR = 2, however, no noticeable differences in the fluctuation amplitude are observed between the free end and wall-junction regions. Hence, for this case, the influence of the tip flow appears to extend over the entire height of the obstacle. Unexpectedly, perhaps, the low-amplitude activity occurs less frequently than higher aspect ratio obstacles. For AR = 0.5, the amplitude is randomly modulated and the shedding activity differs significantly from that observed for higher aspect ratio obstacles.
Appendix A

Fig. A-8: Velocity time series for AR = 8, 4 and 2 square cylinders measured at x/D = 1, y/D = 1.2.
Figure A-9 illustrates the probability density function (PDF) of instantaneous phase differences at $x/D = 1, y/D = \pm 1.2$ for prisms of different aspect ratios. The peak in all of the PDFs occurs in the proximity of $180^\circ$. The PDF distribution, except for $AR = 0.5$, shows that in-phase events are very rare. These observations confirm the domination of out-of-phase shedding behavior observed in PIV snapshots, and velocity and pressure time series. At $z/H = 0.8$, the PDF is broader for $AR = 4$ and $8$ obstacles due to the more pronounced low-amplitude activity in that region, whereby the variations in the instantaneous phase manifest themselves in a broader PDF. Nevertheless, in-phase events remain rare. Thus even during the low-amplitude intervals, vortices are shed mostly in an alternating fashion even though two counter-rotating vortices exist in the base region. For $AR = 2$, the PDFs are very similar to those observed for larger aspect ratio obstacles near the free end. For $AR = 0.5$, the PDF is much broader than the other cases. For this case, probability of in-phase behavior is non-negligible, suggesting a distinct shedding pattern, different from higher aspect ratio geometries.
The standard deviation of the relative phase may be used as a measure to quantify the broadness of the PDF. An increase in the standard deviation may be associated with an increase in the low-amplitude fluctuations (mode with coexisting base vortices). The standard deviations for both circular and square section cylinders are presented in Fig. A-10 at two streamwise locations ($x/D = 1$ and $x/D = 4$). For large aspect ratio square cylinders, around the formation region ($x/D = 1$), the standard deviation increases with height, consistent with the PDF trends observed in Fig. A-9. Downstream ($x/D = 4$), the standard deviation increases less with height suggesting a rearrangement of the vortices shed during the low-amplitude intervals as described in relation to Fig. A-5. The
reduction in the standard deviation (as a measure of the broadness of the PDF) is consistent with the increase in “out-of-phase” behavior observed downstream as shown in Fig. A-7.
Fig. A-10: Standard deviation of PDF for square and circular cylinders at $x/D = 1$ and 4, $y/D = \pm 1.2$.

For AR = 2, however, the standard deviation does not change with height or downstream location, but is at levels comparable to those found near the free end for
larger aspect ratios, again suggesting that the influence of the tip flow extends over the entire height. For the AR = 0.5 square cylinder, the standard deviation (not shown for brevity) is of the order of 100° and does not change with location.

For the circular cylinders, the interpretation of the results obtained is less clear. Considering points for $z/H > 0.4$, the trends in the standard deviation along the height and in the streamwise direction are similar to those observed for the square cylinder. The standard deviation for circular cylinders is higher than for square geometries, probably, due to the randomness associated with the motion of the separation line on the obstacle faces. Note that the shedding was very weak at $x/D = 1$ for AR = 8, and therefore no reliable wavelet analysis could be made. Near the wall-obstacle junction (at $x/D = 1$), however, the standard deviation for the circular cylinder is much larger than above. This effect is not observed at $x/D = 4$ or for the square geometry. This effect may be related to the nature of the location of the flow separation line on the obstacle face. Near the junction, the presence of the horseshoe vortex induces pressure gradients along the faces which may interfere with both the location of the separation line and the circulation flux generated at the separation point. As the obstacle aspect ratio is further reduced to one (or less), periodicity is no longer observed for the circular geometry. For the square geometry, the location of flow separation is fixed by the sharp edge and thus less sensitivity to the horseshoe vortex would be expected. While similar explanations to account for the differences between rectangular and circular cylinder geometries have been offered before [1-3], it is noted that this disruption of the junction region formation
process has consequences on the downstream flow and vortex topology which still have not been satisfactorily resolved.

Figure A-11 illustrates power spectra at different heights for square and circular cylinders. For the square cylinder geometries, the spectra for AR = 2, 4 and 8 show strong, distinctive peaks with little loss of power as the tip is approached. For AR = 0.5, the spectra are broad with accumulations of energy about the nominal shedding frequency. This weak periodicity is confirmed from the velocity correlation functions shown in Fig. A-12a for AR = 0.5 at $x/D = 1$, $y/D = \pm 1.2$, $z/H = 0.8$. The cross-correlation function shows that the mean phase difference is close to $180^\circ$, as indicated in Fig. A-9d, but true in-phase events occur with higher probability than for other obstacles as suggested by the time series in Fig. A-12b and the PDF of Fig. A-9d.

For circular cylinders, also shown in Fig. A-11, the spectral peaks are generally broader, probably due to the non-fixed nature of the separation region. An important difference to the behavior noted for the square cylinders is a loss of power near the free end and in the wall junction region. The latter observation can be related to the interference with the horseshoe vortex as stated previously. However, the loss of fluctuation energy near the free end suggests that the nature of the tip flow may also change. Differences in the strength of vortices at the tip of circular cylinders have been reported [7-8], but there seems to be a lack of consensus in reported results. For example down and upwash (i.e. difference in sense of rotation) have both been reported. These differences may be related to the effect of the boundary layer thickness.
Fig. A-11: Power spectra for square and circular cylinders at $z/H = 0.2, 0.4, 0.6$ and $0.8$, $x/D = 1$, $y/D = 1.2$ (graphs shifted vertically for clarification).
Fig. A-12: a) Velocity correlations on opposing sides, b) velocity time series on opposing sides of AR = 0.5 prism at x/D = 1, y/D = ±1.2, z/H = 0.8.

In order to examine the influence of aspect ratio on the mean flow field, oil film visualizations were carried out. Oil film visualization is a useful tool in identifying the footprint of large-scale structures in the flow field. Representative images for two aspect ratios are shown in Fig. A-13 for the square and circular geometry. Clearly, a deeper penetration of the flow into the wake is distinguished for AR = 4 by considering the stronger necking in the wake. Furthermore, the circulation zones and saddle points become closer and are merged together in the case of AR = 4 compared to AR = 2 obstacles. It is noted that the surface stress lines associated with the horseshoe vortex tend to be closer to the circular than the square obstacle, which is consistent with the
observation that the streamwise structure is more likely to interfere with the vortex formation region in the junction region. These differences show a significant change in the mean flow pattern topology and indicate that important differences in the vortex structure in the wake exist for these two geometries.

Fig. A-13: Oil film visualizations (top view) a) AR = 2 square, b) AR = 2 circular, c) AR = 4 square, d) AR = 4 circular.

Further comparison of the results obtained for the square and circular geometries is complicated by the nature of the separation line on the obstacle faces, which is critical in determining the rate at which vorticity is generated on the surface and ultimately affects the relative strength (circulation) of vortices formed and interacting in the wake.
For the square cylinder, the separation is fixed at the sharp leading edges and the influence of the Reynolds number and upstream boundary layer transition is minimal over the range considered in this study. For the circular cylinder, however, the location and curvature of the separation line depends strongly on both the Reynolds number and the nature of the on-coming boundary layer. The spectra in Fig. A-11 suggest that the strength of the shed vortices are weaker for the circular than the square cylinder wake. Since the vorticity flux from the boundary layer, for example, is similar in both cases, the disruption of the shedding mechanism near the wall-obstacle junction is expected to be stronger for the circular than the square cylinder, as supported from Fig. A-10.

**A.6 Conclusions**

The dynamics of vortex formation and vortex shedding in the wake of AR = 0.5, 2, 4 and 8 square and circular cylinders was investigated. Using synchronized pressure/PIV and thermal anemometry measurements, two competing vortex formation modes in the formation region were distinguished: (i) A mode similar to the classical shedding behind two-dimensional cylinders, i.e. alternate vortex formation on opposing sides accompanied by high-amplitude, out-of-phase pressure and velocity fluctuations; (ii) A second mode characterized by the coexistence of a pair of counter-rotating vortices in the formation region accompanied by low-amplitude fluctuations in the wake with varying phase. These latter types of structures tend to shed anti-symmetrically as they are convected downstream. The low-amplitude events are observed more frequently as the free end is approached.
Generally the same behavior is observed for both circular and square cylinders, although the shedding behind circular cylinders is less regular due to the non-fixed nature of the separation line on the obstacle face. However, it is observed that the shedding process is different near the junction and free end region for circular cylinders when compared to square geometries. At the junction, the formation process is disrupted, which can be surmised to occur due to interference of the horseshoe vortex with the surface separation process. The square geometry appears to be less sensitive due to the sharp edges (fixed separation line) and the observation that the horseshoe structure appears further away from the obstacle. However, the differences observed at the obstacle tip are more difficult to explain and require further investigation. Results of the oil film visualizations suggest that differences in the vortex shedding dynamics are accompanied by noticeable changes in the mean wake structure, which is expected on topological grounds.

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A.8 REFERENCES


