

# Membership and Incentives in Network Alliances<sup>1</sup>

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# Membership and Incentives in Network Alliances

Index Terms: Network Alliance, Cannibalization, Incentives and Coordination, Exclusivity.

## Abstract

*We propose a general and precise model of a network alliance that addresses both the role of membership and the role of incentives in the coordination of actions and interactions of network alliance members. Using examples in such disparate industries as professional engineering, accounting services and commercial fueling as the basis of our model, we show that a commission fee chosen by the network provider can be combined with a classical exclusivity agreement - which does not restrict where members recruit customers while at the same time protecting the members' locations where customers are served - to motivate increases in member investment and, consequently, in network profits. We also show that the most profitable network size emerges naturally. That is, the most profitable network size restricts membership, and emerges as a consequence of the exclusivity agreement and the setting of the commission fee. Our results require that members' investments are more valuable with increases in other members' investments, that prospective members are sufficiently different that there is an adequate range in the business potential of members, and that the effect of other members' investments on a given member's business potential is moderately low.*

# 1 Introduction

In current theories of network organizations, the interactions within the organization - which are the building blocks of networks, are too easily taken as givens. When not taken as givens, the role of interactions in constructing coordination structures and carrying information between network members can be modeled [16]. A network theory that considers interactions should do either of two things. First, it should propose how adding or subtracting a particular interaction in a network organization will change the coordination among members in that network. Second, it should propose how a network structure enables or restricts interactions between parties [16, 21]. In this paper we propose a general and precise model of a horizontal network alliance that addresses both the role of membership and the role of incentives in the coordination of actions and interactions of network organization members. We concentrate on the role of interactions between members in improving the profitability of the alliance. The network alliances that are the subject of our paper are horizontal networks in which members are at the same level in the market channel. For example, mutually interdependent franchises with linkages between them have been described as horizontal networks [18].

In the recent 50-year retrospective of IEEE research in engineering management, several senior scholars highlighted the importance of networks along the lines of those described above to the engineering management profession. Tushman makes it clear that research on engineering management will be pushed to consider extended value nets where firms have to attend to interdependencies that are more interorganizational, understanding alternative architectures to gain benefits of synergy across business units [20]. Indeed, he states

*"... intense interdependencies outside the firm will increase the importance of building organizational forms, networks, and senior team capabilities to deal with the increased complexity of engineering/innovation management."* (p. 410, [20])

Roberts echoes Tushman's focus on network-oriented research as important to the future of engineering management. In his short review of work being conducted by junior colleagues, he includes networks of scientists in multi-site collaborative work groups, multi-firm alliances in industries, and networks of individuals developing and commercializing products like "open software" [15].

Versions of the alliance form we study can be found in referral networks in medicine, community mental health, and professional services such as consulting, investment banking, insurance, and accounting [5] as well as other industries. A specific illustration of the type of alliance we study is Koza and Lewin's example of a network of independent public accounting firms - Nexia International, formed to produce incremental income for members from cross-border referrals [5]. The three general features of Nexia embedded in our model are Nexia's commission fee for referral work, the sharing of referred work between members, and the exclusivity of membership within a country or market. The first two of these features are based on the network monitoring and measuring contributions, and using these to motivate member activity. The third feature is based on property rights and adds stability to membership by reducing conflict and competition.

This form also has potential in another group of professional service firms: engineering consulting. In 2002 engineers held 1.5 million jobs in the U.S. (Bureau of Labor Statistics), 354,000 of which were in the professional, scientific and technical service industry. These were primarily in architectural, engineering, and related services and in scientific research and development services. These services were provided on a contractual basis where, for example, firms designed construction projects or did other engineering work. Professional societies recognize over 25 major engineering specialties including aerospace, agricultural, biomedical, chemical, civil, electrical, industrial, and mechanical among others. Because a large engineering project covers multiple specialties, engineers in one specialty are often

required to work closely with engineers in other specialties, frequently collaborating with colleagues elsewhere in the country or abroad. This collaboration is mandated by the Code of Ethics for Engineers from the National Society of Professional Engineers, whereby engineers can only perform services in areas of their competence and qualification. This restriction extends to affixing signatures to official plans or documents.

Moreover, all 50 U.S. states require engineers that offer services to the public be licensed by the state, where the licensed engineers are called "Professional Engineers". The Code of Ethics also requires conformance with these state registration laws. Similar arrangements for the "professionalization" and licensing of engineers are in force in other jurisdictions such as the provinces in Canada. Thus, engineering projects that cross jurisdictional boundaries may also mandate collaboration between engineers that are licensed in the different jurisdictions.

Professional engineering firms can form networks to produce incremental income for members from cross-specialty and cross-jurisdictional referrals similar to professional accounting firms in Nexia International. Within a network, a commission fee for referral work can be implemented by monitoring and measuring contributions. And the exclusivity of membership flows naturally from professional licensing and ethics requirements - licensing by jurisdiction and by engineering specialty. In fact, this formal licensing already adds stability by ensuring qualification and reducing conflict. Research on this type of organizational form in engineering follows work on interdependencies and collaboration envisioned by Tushman and by Roberts in their 50-year retrospective articles for the IEEE-TEM [20],[15].

The horizontal network alliance form we examine is similar to franchising as it is a legal-based definition of a network organization, where the design problem is to find a set of Pareto-superior contracts - that is, legally binding agreements - between members [4]. Thus, this organization form is an agreed upon contracted mix of coordination mechanisms. Our alliance form differs from traditional franchising by not including mechanisms typical of

franchise agreements such as franchise fees, certain branding restrictions, resale price maintenance and quantity forcing. Rather, our form relies on classical exclusivity agreements and commission fees between members of the alliance based on between-member business. Koza and Lewin describe network alliances as an organization form involving multiple organizations with instrumental aims at the individual and/or collective level [5]. Such an organization with instrumental aims is consistent with Coleman who argues that rationally constructed organizations are the preeminent feature of modern industrial society [3].

One of the elements of our model is the application of information technology (IT) to coordinate the network alliance. Network organizations are characterized by pooled interdependence: each member in the organization contributes to the whole and each is supported by the whole [17]. A McKinsey study in the late 1990s suggested IT-driven reductions in costs of interaction would foster horizontal organization forms [2] - that is, through reductions in costs, IT-enabled coordination would substitute for alternative means of coordinating such as hierarchical organization forms.

Using IT to capture information that would have previously been infeasible, and using this information to embed sharing rules in the IT application to align incentives, Nault and Dexter proposed a new horizontal network organization form in the context of franchise networks where customers purchase across the network [12]. This new form was based on a case in commercial fueling - Pacific Pride [9]. Pacific Pride is a franchise network of electronically connected fuel stations, accessed through a dedicated ATM card. Purchases across franchise locations are perfectly monitored, keeping a full record of the transaction including when and where the transaction occurred, what and how much was purchased. A database uniquely identifies customers with franchises that recruited the customer to use the fueling network. The spillovers between franchisees result from customers that purchase in different locations - for example, motor carriers purchasing fuel.

Franchisees recruit customers, which are owners of vehicle fleets, to purchase from the network, and "own" the customers they recruit for purchases throughout the network. This is termed "ownership of customers" whereby customers recruited by one franchisee are identified with that franchisee on all network purchases [10]. The owning and serving franchisees divide profits proportionally on individual transactions - a transfer or commission fee. This division of profits mitigates the problem of underinvestment by franchisees in customer recruitment by rewarding franchisees that recruit customers with a portion of the profits made from those customers elsewhere in the network. This model has been generalized beyond franchising to network organizational forms where there is a single central authority (head office) and many members (branches).<sup>2</sup> In that form competition between members (e.g., franchisees) is controlled through two restrictions. The first is a classical exclusivity agreement where only one member is allowed to serve customers in a given geographical area. The second is exclusive rights of members to recruit customers residing in their territory - that is, no member could recruit customers from another member's territory.

The network alliance we study in this research is more general because we examine the effect of profit sharing between members under a classical exclusivity agreement only and where members retain financial and decision making autonomy in other dimensions. We believe this form is more common in industries such as in professional services where the location of the customer can be unrelated to where they are served, and an individual member's network of customer contacts may not follow easy to define boundaries. For example, Nexia's exclusivity agreement means that customers of a given member firm will have their work handled by the member firm in the country where the work is done [5]. In a network alliance of professional engineering firms the classical exclusivity is directly supported by li-

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<sup>2</sup>For example, [11] includes investment by a central authority that is complementary to local investments by members, and [14] allows the members rather than another entity, to own the network, resulting in a horizontal alliance.

censing that is defined by geographical boundaries (e.g., state) and by engineering specialty, therefore an exclusivity agreement means that customers of a member engineering firm will have their work handled by a member firm licensed in the jurisdiction and with the specialty needed to perform the work. Moreover, other organizations in the commercial fueling industry such as the Commercial Fueling Network (CFN) have classical exclusivity agreements but do not restrict where its members can recruit customers.

Classical exclusivity agreements based on the location of service only are important when it is not feasible to pre-assign customers to members. For many cases it may not be clear which customers are most likely to be recruited by which members until after they have been recruited. In CFN, for example, personal contacts are critical for members recruiting customers, and these personal contacts can be due to expertise in particular industries, school and family ties, etc., rather than proximity. Similarly, for professional services including engineering, personal contacts are paramount to getting business. A geographical assignment to only recruit residing customers would reduce the effectiveness of member investment. Furthermore, because personal contacts are rarely overlapping, there is little competition in customer recruitment that might otherwise dissipate member investment.

In a network alliance with members and a single network provider, we show that a commission fee chosen by the network provider can be combined with a classical exclusivity agreement to motivate increases in member investment and, consequently, in network profits. We also show that the most profitable network size emerges naturally. That is, the most profitable network size restricts membership, and emerges as a consequence of the exclusivity agreement and the setting of the commission fee. In order to obtain our results, we require that members' investments are complements, that there is sufficient range in the business potential of members, and that there is low cannibalization between members. Complementarity of member investment allows us to determine the effects of the commission



fee on equilibrium member investment. A sufficient range in business potential makes the commission fee more effective in increasing network size. Low cannibalization enhances the effects of increased member investment, reducing the negative effect of the commission fee on such investment.

Our formulation is related to the analytical research on franchising and specifically on underinvestment by franchisees. The classic example of underinvestment is territories that advertise at levels below those that would be optimal for an integrated firm because they do not receive the full benefits of their expenditures [7]. One approach to correcting this problem is franchisee monitoring by the franchisor to motivate service provision [6]. Another approach is to use profit sharing to provide investment incentives, typically through simple sharing rules [1]. The matching of customers with members that recruit them, the exclusivity agreements for service, and the commission fees for between-member business, combines these two approaches.

The remainder of the paper proceeds as follows. First we describe the horizontal network alliance structure of our model, our notation, the member demands, and our assumptions. Then we go through the analysis beginning with member investment, followed by network alliance participation, and then network provider profits. We conclude with a summary, generalizations, and possible extensions.

## 2 Network Alliance Structure

Following Nexia and CFN as examples, we consider the market for a single good (e.g., accounting services; fuel). A set of members, accounting firms or fueling stations, form a network of sites spanning a subset of geographical locations. Customers can be recruited by any member, but enter into an agreement with a single member to make purchases at any

location on the network. In other words, customers purchase from many different locations. Network members are governed by an exclusivity agreement whereby they are the only ones allowed to serve a particular location. Members make investments in effort to recruit customers to join and use the network of sites. Members know best how to employ their effort, that is, their investment, in order to recruit customers onto the network. Purchasing from all network members under a single agreement provides value to customers. Through Nexia members offer customers international reach, and further evidence of the member's credibility and reputation [5]. Through CFN members offer fleet management, consolidated billing, detailed purchase information, control over employees afforded by restricting the sites from which they can purchase, and special relationships between a member and its customers (see [13]). Thus, the price for services is determined by differentiated competition between members, with other networks, and with independents. This differentiated competition is the source of profits and no single member has sufficient power to set market prices.

The network provider's role is to set fees and track transactions between members. In its fee-setting role, the network provider sets a royalty that applies to purchases by customers recruited by one member making purchases from another member. For such purchases the network provider also sets a commission fee between the two members. In its tracking role, the network provider identifies transactions between members, ensuring that proper fees are exchanged. The network provider knows the range and distribution of members' business potential, but cannot identify individual members with their potential and therefore cannot implement fees strictly based on potential. IT is used to perfectly track transactions over the network. The network provider does not know the specific relationships between member investments and demands, and this is the information asymmetry. The member that recruits the customer bills the customer, and the commission fee is transferred from that member to the serving member. Our qualitative results are the same if the royalty is charged on all purchases, and if the member that bills the customer and the transfer of the commission fee

are reversed.

The network provider can be constituted in different ways. In Nexia the network provider is a corporation that acts as an umbrella organization. The management of Nexia is centralized in an International Secretariat, a Board of Directors, and a Council where each independent member firm, regardless of size, holds a single vote. Nexia International provides professional advisory bodies, an IT committee, publications, and a marketing and communications program. In contrast, CFN is owned and operated separately from its members, and its members are independent petroleum marketers. CFN provides an online, real-time network offering access to fueling.

There are two externalities between members. First is a membership (or network) externality where a network with more members directly increases purchases of customers recruited by one member from other members' locations and indirectly makes the network more attractive to customers, both because of the increase in the number of locations from which to purchase. Thus, this first externality is a classic network externality: a larger network is more valuable to all members. The second is an investment externality that occurs when one member's investment in customer recruitment benefits other members as these additional customers purchase across the network.

## **2.1 Member Demands**

In our model members differ in their potential to generate business through customer recruitment. These differences can be due to size of local population, number of contacts, brand name, etc. These differences could also be interpreted as the source of differentiation between members. For example, in professional engineering these differences could be due to where the firm's engineers are licensed and the specialties they cover. We treat these

differences generically as differences in size so that member size corresponds to a member's potential to recruit customers. We use the variable  $x$  to identify prospective members by their size so that prospective members are ordered between the largest potential member  $\bar{x}$  and the smallest potential member  $\underline{x}$ . Therefore, member size is defined as a real number between the smallest and largest prospective members,  $x \in [\underline{x}, \bar{x}]$ , and is distributed with the probability density  $g(x)$  which is positive only over the range of prospective member sizes. The network provider knows this distribution and can identify individual members. We use the variable  $y$  to represent the proportion of prospective members that participate in the network or network size. Network size as a proportion is defined as a real number between zero and one. The level of investment by member  $x$  is represented by the real number  $e_x$ , and  $e$  represents the vector of member investment over the range of member sizes, that is,  $e = (e_x, e_{\setminus x})$  where  $e_{\setminus x}$  is a vector of investments made by other members. We let each member's level of investment range from zero to some fixed maximum,  $e_x^{max}$ . We break the total demand faced by an individual member into three demand elements:

1. *Domestic demand*, denoted as  $d_D(x, y, e)$ , is demand from customers recruited by member  $x$  that purchase from member  $x$ .
2. *Exported demand*, denoted as  $d_E(x, y, e)$ , is demand from customers recruited by member  $x$  that purchase from members other than  $x$ .
3. *Imported demand*, denoted as  $d_I(x, y, e)$ , is demand from customers recruited by members other than  $x$  that purchase from member  $x$ .

Without the network there are no recorded cross-member demands so that exported and imported demands are zero.<sup>3</sup> Our demands capture the variety of effects that motivate

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<sup>3</sup>In order to concentrate on the reasoning in the analysis rather than on technical mathematical details, we assume that the functions we employ are continuously differentiable where necessary.

customers that require services to join the network. In this way we treat incentives for customers to join the network as exogenous - customers have requirements and can take advantage of the network's ability to offer convenience and an expanded range of services such as different engineering specialties and license coverage that comes from the alliance of firms. At the source, customers may have homogeneous or heterogeneous requirements. Thus, although we do not model customers' decisions directly, the result of their decisions is manifested in the structure of our demands.

The network provider does not know the relationship between member investment and demands, and therefore cannot offer a contract that specifies in detail the investments, for example, customer visits for recruiting, that members should make. That is, the network provider cannot determine the investments as effectively as the member, and cannot monitor whether contractually specified investments have been made. Moreover, the relationship between member investment and demand changes dynamically - customers change, grow and contract - so that members that are closest to their current and potential customers have the best knowledge. A contract designed to reveal a member's knowledge of the relationships between investment and demands would yield information that would be immediately out of date. Consequently, levels of member investment that would mirror those chosen in a fully informed and integrated firm are not contractible.

## **2.2 Assumptions**

To make clear the parts of our network alliance formulation, we make a specific set of assumptions that interrelate member size, network size, member investment, and profit. Other than the standard methods of setting up profit functions and deriving equilibrium levels of member investments and optimal levels of the royalty and commission fee, these assumptions are the only conditions we require to derive our conclusions.

**Assumption 1** (a) Domestic and exported demands are increasing in member size  $x$ . (b) All demands are increasing in network size  $y$ .

\*\*\* Put Table 1 about here \*\*\*

The mathematical forms of Assumption 1 are given in Table 1. The first two elements in Assumption 1 are consistent with our definition of member size - the effect of being larger is that there is greater potential to generate business through customer recruitment. That is, members with a larger potential set of customers have higher domestic and exported demands all other things equal. All demands increase in a larger network. Domestic demand increases because the network is more attractive, and exported and imported demands increase because a larger network increases cross-member demands for each type of demand. Assumption 1 is essentially an assumption about economies of scale. Domestic and exported demands are greater for larger members, and all demands are greater in a larger network.

For a given member,  $x$  measures the potential to recruit customers, and the location where customers purchase is predetermined. For Nexia, the locations where customers do business is not determined by the network, and for CFN, given a customer's routing, the locations at which the customer needs fuel are not determined by the network. If the potential to recruit customers is higher, then those additional customers recruited by a given member would not have necessarily otherwise been recruited by another member. Therefore, the increase in domestic demand from the first element of Assumption 1 would not have necessarily otherwise become imported demand:

$$\frac{\partial d_D(x, y, e)}{\partial x} \geq -\frac{\partial d_I(x, y, e)}{\partial x}. \quad (1)$$

**Assumption 2** (a) Domestic and exported demands are increasing in member investment  $e_x$ , and imported demand is nonincreasing in member investment  $e_x$ . (b) Domestic and

*exported demands are decreasing and imported demand is nondecreasing in investment by other members  $e_{\setminus x}$ .*

\*\*\* Put Table 2 about here \*\*\*

The mathematical forms of Assumption 2(a) are given in the first column of Table 2, and the forms of Assumption 2(b) are given in the second column of Table 2. The effects of a member's own investment on domestic and exported demands in Assumption 2(a) are that greater investment in customer recruiting yields more customers which in turn yield greater demand. Imported demand is weakly decreasing in member investment because increased domestic demand from additional member investment may have come from customers that would have otherwise become part of imported demand - in which case imported demand is decreasing in member investment, or from customers that may not have been successfully recruited by other network members - in which case imported demand is unaffected by member investment. Assumption 2(b) are the effects of investment by other members, and these effects are the reverse of Assumption 2(a): investment by other members takes away customers that might have otherwise been recruited, thereby reducing domestic and exported demand, and these customers may require service from  $x$ , thereby weakly increasing imported demand.

The magnitude of the effects depends on the degree to which members are recruiting the same customers. The directions of these effects are given in Assumption 2. In particular, the imported demand effect in Assumption 2(a), and the domestic and exported demand effect in Assumption 2(b) reflect the overlap in members' different potential customer bases. These effects in Assumption 2 are larger the more members' different potential customer bases overlap.

Again,  $x$  identifies a particular member through its potential to recruit customers. However, the location or members from which customers purchase is predetermined so that regardless of which member recruits a given customer, that customer has the same distribution of service needs amongst the members. Considering the customers that are sometimes served by  $x$ , investments by  $x$  in recruiting customers adds customers to its list of owned customers and they become domestic demand. This additional domestic demand would have either been imported demand if these customers had otherwise been successfully recruited by another member, or would not be purchasing from the network. Therefore, the additional domestic demand generated by increased investment by member  $x$  is no less than the decrease in imported demand caused by that same investment:

$$\frac{\partial d_D(x, y, e)}{\partial e_x} \geq -\frac{\partial d_I(x, y, e)}{\partial e_x} \quad (2)$$

**Assumption 3** *Members' investments,  $e_x$  and  $e_{\setminus x}$ , are weak complements for domestic and exported demands.*

The mathematical form of this assumption is given in the third column of Table 2. Weak complements means that the change in domestic and exported demand for member  $x$  from increased investment in recruiting customers - Assumption 2(a) - is increasing (or non-decreasing from "weak" complements) with increased investment by other members. Assumption 3 concerns how a change in domestic and exported demand from increased member investment - the marginal returns to investments in recruiting customers - changes with increased investment by other members. That is, Assumption 3 relates the effects of increases in multiple members' investments, and in this sense is a two-sided marginal investment externality: the returns to one member's investment is higher if other members' investments are higher. For members' investments to be weak complements, the marginal returns to a given member's investment must be (weakly) enhanced by the investments of other members.



This is likely to be the true if members' investments promote the benefits of the network whereby additional knowledge of the benefits of the network, regardless of which member this knowledge came from, increases a customer's likelihood of participating. This is also likely to be true if members' individual services, such as jurisdiction of licenses, engineering specialties, value added information in billing, maintenance, reputation, etc., are sufficiently differentiated. And finally, this is certain to be true in the weak complements sense if the sets of customers recruited by different members are separate.

Similar to (2) the effect of member investments being weak complements is greater on domestic demand than it is on imported demand:

$$\frac{\partial^2 d_D(x, y, e)}{\partial e_x \partial e_{\setminus x}} \geq -\frac{\partial^2 d_I(x, y, e)}{\partial e_x \partial e_{\setminus x}}. \quad (3)$$

If marginal losses on imported demand - the last part of Assumption 2(a) - are increasing with increases in other members' investments, then the right hand side of (3) is negative and the relation is true from Assumption 3. Alternatively, if marginal losses on imported demand are decreasing with increases in other members' investments so that the right hand side of (3) is positive, it is by no more than the increase in marginal returns on domestic demand from other members' investments. In other words, because increases in domestic demand - not only from member investment but also from members' investments as weak complements - come from customers that would either otherwise have been imported demand or not customers of the network at all, the effect of members' investments as weak complements on domestic demand is larger than the effect on imported demand. It is helpful to recognize that the cross effects on the right hand side of (3) are zero for customers unaffected by a given member's investment,  $e_x$ .

\*\*\* Put Table 3 about here \*\*\*

Table 3 organizes the investment responses captured in Assumption 2 and Assumption 3. A member's own investment in recruiting customers increases the demands that come directly from those customers - domestic and exported, and decrease the demand that would have otherwise come from other members' customers - imported demand. Investments by other members have the opposite effect - they reduce domestic and exported demand because other members' investments may take customers away that would otherwise be part of domestic and exported demand, and they increase imported demand because these same customers would still be served in member  $x$ 's territory. The effect of an increase in own and other members' investments simultaneously is to increase domestic and exported demand - members' investments as weak complements, or in other words, a two-sided marginal investment externality.

\*\*\* Put Table 4 about here \*\*\*

Table 4 summarizes the effects of member size - 1, a member's investment - 2, and multiple members' investments - 3, on domestic versus exported demand that follow from the structure of our setting. In essence, all three of these effects are greater on domestic demand than they are on imported demand.

**Assumption 4** (a) *The marginal increase in domestic and exported demands from member investment  $e_x$  is greater for larger members. The marginal increase in imported demands resulting from member investment  $e_x$  is no lesser for larger members.* (b) *For the smallest member in the network alliance, the total effect of member size  $x$  on profit is positive.* (c) *For the smallest member in the network alliance, imported demand is greater than exported demand.*

The mathematical form of Assumption 4(a) is given in the last column of Table 2. For domestic and exported demand, having greater potential to generate business through cus-

customer recruitment - that is, being a larger member - makes investment in customer recruiting more effective. The marginal imported demand losses due to investment increase with member size because larger members capture more customers for domestic demand that would otherwise have been imported demand than do smaller members.

Assumption 4(b) means that for the smallest member in the network alliance, the direct effect of a member's size is more important to customer demand than the indirect effect of network size. The positive direct effect from an increase in the size of the smallest member in the network alliance is from that member having a larger business potential. The negative indirect effect from an increase in the size of the smallest member in the network alliance is that the network is smaller and customers may be less eager to purchase from a smaller network because there are fewer members to purchase from. The direct result of this assumption is that all prospective members larger than the smallest member participate in the network alliance.

Assumption 4(c) makes use of the fact that member size measures potential business. The smallest member in the network alliance has the smallest amount of demand from customers it recruited. Therefore, Assumption 4(c) holds so long as the smallest member receives in aggregate a roughly even share of all other members' exported demands.<sup>4</sup>

The underlying basis of Assumption 4, like Assumption 1, is also related to economies of scale. Assumption 4(a) is that marginal returns are greater for larger members, Assumption 4(b) is that larger members are more profitable, and Assumption 4(c) is that larger members generate more customers.

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<sup>4</sup>As compared to [12] and [10], Assumption 2 requires an addition that accounts for the impact of investment by members on their imported demand whereby own investment will decrease, and others' investment will increase, imported demand. Assumption 4 (a) requires an addition of the cross effects of member investment and size on imported demand. Assumption 3 is new and specifies the cross effects of different member investments on the demands. There are assumptions in that prior work that are not required here. Details are available from the authors.

## 3 The Model

### 3.1 Member Investment

Member profits come from each of the demands. Each member's profit equation is

$$\pi(x, y, e) = p d_D(x, y, e) + [p - r - f] d_E(x, y, e) + f d_I(x, y, e) - e_x, \quad (4)$$

where  $p$  is the price per unit,  $r$  is the royalty,  $f$  is the commission fee, and  $e_x$  is the cost of investment measured in dollars. The first product on the right hand side is price times domestic demand. The second product is the margin - price less the royalty and the commission fee - times exported demand. The third is the commission fee times imported demand, and the last term is the cost of investment. We do not model price competition directly, treating price as determined by competition inside and outside of the network and equal across network members.<sup>5</sup> We also treat the price of the input (e.g., professional engineers, accountants; bulk fuel) as beyond the control of any single member and normalize it to zero so that our prices are net of costs to members. Maximizing the profit equation (4) by choice of level of own investment for each member defines an investment game between members. Using  $(\cdot)$  to represent the arguments in the profit and demand functions, the first-order condition for profit maximization by choice of investment level is

$$\frac{\partial \pi(\cdot)}{\partial e_x} = p \frac{\partial d_D(\cdot)}{\partial e_x} + [p - r - f] \frac{\partial d_E(\cdot)}{\partial e_x} + f \frac{\partial d_I(\cdot)}{\partial e_x} - 1 = 0. \quad (5)$$

The first term on the right hand side is the marginal return on domestic demand, and the second is the marginal return on exported demand. The third is the marginal loss on imported demand, and the last term is the marginal cost of investment. Assumption

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<sup>5</sup>Our results hold when price is increasing in member size so that price is member specific, as it would be if larger members could offer greater value added services such as multiple engineering specialties, integrated billing, maintenance schedules, etc. Our results also hold if price is increasing in network size, as it would be if having a larger network were more valuable to customers.

2(a) along with (2) ensure that the combined effects of investment on demands - domestic, exported and imported - are positive so that the first-order condition in (5) can equal zero. The first-order condition (5) for each of the members together defines a set of pure strategy Nash equilibria, and as we show in the Appendix, the investment game is "supermodular". A supermodular investment game - which essentially results from our weak complements assumption regarding members' investments - allows us to use special properties of such games in our analysis. One consequence of the member investment game being supermodular is that the set of equilibria can be ordered so that there is a largest Nash equilibrium where largest is defined by the members' levels of investments. We assume that this largest equilibrium is the one that would result as it would be the one preferred by the members. Because these investment levels depend on the royalty and the commission fee we denote this equilibrium as  $e(r, f)$ , or more simply as  $e(\cdot)$ .

As the investment game between members is supermodular, we are also able to determine how these equilibrium investments,  $e(\cdot)$ , change as a result of changes in the size of the member,  $x$ , the royalty,  $r$ , and the commission fee,  $f$ , in a straightforward manner: each of the effects take the sign of the cross partial derivative of member profit,  $\pi(\cdot)$ , with respect to member investment and the variable in question [19, 8].

First, member investment is increasing with increases in member size. Using (5), the cross partial derivative with respect to member investment and member size is

$$\frac{\partial^2 \pi(\cdot)}{\partial e_x \partial x} = p \frac{\partial^2 d_D(\cdot)}{\partial e_x \partial x} + [p - r - f] \frac{\partial^2 d_E(\cdot)}{\partial e_x \partial x} + f \frac{\partial^2 d_I(\cdot)}{\partial e_x \partial x} \quad (6)$$

in which each cross partial derivative is positive from Assumption 4(a). The intuition is that a larger member - one with greater potential business - has higher returns to investment and consequently has higher levels of investment:  $\partial e_x(\cdot)/\partial x > 0$ .

Second, member investment is decreasing with increases in the royalty. The cross partial

derivative with respect to member investment and the royalty is

$$\frac{\partial^2 \pi(\cdot)}{\partial e_x \partial r} = -\frac{\partial d_E(\cdot)}{\partial e_x} < 0, \quad (7)$$

where the sign comes directly from our Assumption 2. The intuition is that the network provider is taking money away from members through the royalty, leaving the member with a lower return to investment. Therefore, an increase in the royalty lowers the levels of member investment:  $\partial e_x(\cdot)/\partial r < 0$ .

Third, member investment is decreasing with increases in the commission fee. The cross partial derivative with respect to member investment and the commission fee is

$$\frac{\partial^2 \pi(\cdot)}{\partial e_x \partial f} = -\frac{\partial d_E(\cdot)}{\partial e_x} + \frac{\partial d_I(\cdot)}{\partial e_x} < 0, \quad (8)$$

where the sign again comes from our Assumption 2. An increase in the commission fee reduces return on investment from exported demand by transferring profits away from the member making the investment to the member collecting on imported demand that in turn is decreasing in investment, thereby inducing lower investment:  $\partial e_x(\cdot)/\partial f < 0$ . The commission fee has a greater impact than the royalty on member investment because in (8) the imported demand term - which is decreasing with increases in member investment - reinforces the exported demand term and the commission fee has a greater effect than the royalty in (7). Therefore,  $\partial e_x(\cdot)/\partial f < \partial e_x(\cdot)/\partial r < 0$ .

To summarize, our assumptions allow us to find an equilibrium in member investments. The equilibrium levels of member investments are higher for larger members, and are decreasing with increases in the royalty and commission fee. The effects of changes in the commission fee are larger than those of changes in the royalty because the former has effects through imported demand as well as exported demand.

### 3.2 Comparison to Exclusivity Agreements without Commission fees

If the network did not use commission fees, then members would receive the full price on domestic and imported demand, and profits would be a special case of (4) where the royalty is zero and the commission fee is equal to the price. Using the superscript "SC" to denote the special case of (4) where the royalty is zero and the commission fee is equal to the price,  $r = 0$  and  $f = p$ , gives the following member profit function:

$$\pi^{SC}(\cdot) = p d_D(\cdot) + p d_I(\cdot) - e_x.$$

In this case profit is made up of price times domestic demand and price times imported demand, less the cost of investment. This is the same profit function an independent member would face if it was the only member serving a given location. The first-order conditions that describe member investments are

$$\frac{\partial \pi^{SC}(\cdot)}{\partial e_x} = p \frac{\partial d_D(\cdot)}{\partial e_x} + p \frac{\partial d_I(\cdot)}{\partial e_x} - 1 = 0 \quad \forall x. \quad (9)$$

The first term on the right hand side is the marginal return on domestic demand, the second is the marginal loss on imported demand, and the third is the marginal cost of investment. The condition in (2) ensuring that the marginal return on domestic demand is larger than the marginal loss on imported demand is necessary for investment returns to be positive. This again is a supermodular game, and the supermodularity condition that gives the Nash equilibrium is provided in the appendix. As we showed in the previous section, an increase in the commission fee decreases member investment ( $\partial e_x(\cdot)/\partial f < 0$ ), meaning member investment is higher when the commission fee is less than the full price,  $f < p$ . Comparing the member profit function with commission fees,  $\pi(\cdot)$ , and its first-order conditions, (5), to profits and the first-order conditions without commission fees,  $\pi^{SC}(\cdot)$  and (9), the latter are

the special case when the royalty is zero and the commission fee equals the price,  $r = 0$  and  $f = p$ .

To summarize the result from this section, member investments are higher when the commission fee is less than the full price. The commission fee, which is a key element of our network structure, allows the member that recruited the customer to be compensated, increasing returns to member investment and thereby increasing member investment incentives.

### 3.3 Network Participation

Let the proportion of prospective members that participate in the network be defined as the set of members larger than the "smallest member":  $y(\tilde{x}) = \int_{\tilde{x}}^{\bar{x}} g(x)dx$ . We define this smallest member as the member that is indifferent between participating in the network and not - that is, the member that makes zero profit if it participates in the network:  $\tilde{x}(r, f, e(\cdot)) = \tilde{x}(\cdot) = \min\{x|\pi(x, y(x), e(\cdot)) = 0\}$ . Then we can define this smallest member implicitly through the zero profit condition as

$$\begin{aligned} \pi(\tilde{x}, y(\tilde{x}), e(\cdot)) &= \phi(r, f, e(\cdot), \tilde{x}) = \\ p d_D(\tilde{x}, y(\tilde{x}), e(\cdot)) &+ [p - r - f] d_E(\tilde{x}, y(\tilde{x}), e(\cdot)) + f d_I(\tilde{x}, y(\tilde{x}), e(\cdot)) - e_{\tilde{x}}(\cdot) = 0. \end{aligned} \tag{10}$$

In what follows we simplify some of the cumbersome notation by referring to  $\phi(r, t, e(\cdot), \tilde{x})$  as  $\phi(\cdot)$  and  $d_i(\tilde{x}, y(\tilde{x}), e(\cdot))$  as  $d_i(\cdot)$  for each of the three demands. Therefore, from Assumption 4(b) all prospective members with potential business larger than  $\tilde{x}(\cdot)$  find it profitable to participate in the network. Capturing the different variables that this smallest member depends on,  $(\cdot)$  in  $\tilde{x}(\cdot)$ , we can redefine the proportion of prospective members that participate



in the network as  $y(\tilde{x}(\cdot)) = \int_{\tilde{x}(\cdot)}^{\bar{x}} g(x)dx$  where the integration essentially sums the prospective members larger than  $\tilde{x}(\cdot)$ . It is clear that if the member that is indifferent between participating in the network and not is larger, then there are fewer prospective members larger than this indifferent member, and the size of the network decreases:

$$\frac{dy(\tilde{x}(\cdot))}{d\tilde{x}} = -g(\tilde{x}(\cdot)) < 0. \quad (11)$$

Exclusivity agreements for service allow members to compete only for customers rather than compete for customers and service. The intensity of this competition can be defined by cannibalization - the degree to which investments by other members reduce a given member's domestic and exported demand. We define cannibalization mathematically by

$$\left| \frac{\partial d_D(x, y, e)}{\partial e_{\setminus x}} + \frac{\partial d_E(x, y, e)}{\partial e_{\setminus x}} \right|,$$

where both elements are negative but the absolute value is used to focus on the magnitude of the effect. If cannibalization is low, then other members' investments have little effect on demand from a given member's own customers - that is, its domestic and exported demands. We can use the definition of cannibalization to show the effects of changes in other members' investments on the size of the smallest member - and by extension, on the size of the network.

We begin by showing the effects of changes in the royalty and commission fee on the size of the network. The effect of a change in the royalty on the size of the network follows the sign of  $\partial\phi(\cdot)/\partial r = -d_E(\tilde{\cdot}) < 0$ . Therefore, an increase in the royalty reduces the size of the network because it takes profits on exported demand away from members, thereby making every member less profitable. The mathematical details are in the Appendix.

The effect of a change in the commission fee on the size of the network follows the sign of

$$\frac{\partial\phi(\cdot)}{\partial f} = -d_E(\tilde{\cdot}) + d_I(\tilde{\cdot}) > 0, \quad (12)$$

where the effect is negative from Assumption 4(c). Therefore, an increase in the commission fee increases the size of the network. Recall that Assumption 4(c) is based on smaller members - those with a smaller potential customer base from which to recruit and therefore smaller potential for domestic and exported demand - having greater imported demand than exported demand. Because the commission fee goes from the member that recruited the customer (domestic and exported demand) to the member that served the customer (imported demand), an increase in the commission fee benefits smaller prospective members and increases the size of the network.

The effect of changes in other members' investment on the size of the network follows the sign of

$$\frac{\partial \phi(\cdot)}{\partial e_{\bar{x}}} = p \frac{\partial d_D(\cdot)}{\partial e_{\bar{x}}} + [p - r - f] \frac{\partial d_E(\cdot)}{\partial e_{\bar{x}}} + f \frac{\partial d_I(\cdot)}{\partial e_{\bar{x}}}. \quad (13)$$

If cannibalization is low, then the domestic and exported demand terms in (13) are small in magnitude. From Assumption 2(b) the imported demand term is positive and dominates the effect in (13). Therefore, additional investment by other members increases the size of the network. The intuition for this is that if cannibalization is low, then the sets of customers that different members target for their investment in recruitment - those that provide domestic and exported demands - are less overlapping. For a given member, investments by other members recruits customers that generate imported demands, which are exported demand for the other members, without taking customers that would otherwise have contributed towards the given member's domestic demand.

To summarize, we identified the smallest member participating in the network, and using features of this smallest member we found that network size decreases with increases in the royalty, but increases with increases in the commission fee. Defining cannibalization as the degree to which other members' investments affect domestic and exported demand, we showed that when cannibalization is low, increases in other members' investments increase

the size of the network.

### 3.4 Network Provider Profits

Either exported or imported demand can equally be used to define between-member volume. We use exported demand. Therefore, between-member volume is defined as the aggregate of exported demand over all members:  $q(\tilde{x}(\cdot), e(\cdot)) = q(\cdot) = \int_{\tilde{x}(\cdot)}^{\bar{x}} d_E(x, y(\tilde{x}(\cdot)), e(\cdot))g(x)dx$ . The network provider receives the royalty on between-member volume, and chooses the levels of the royalty and the commission fee. Profits for the network provider are the product of the royalty times between-member volume,  $\Pi(r, f) = r q(\cdot)$ . Maximizing profits, the two first-order conditions, one for the royalty and one for the commission fee, are

$$r \frac{dq(\cdot)}{dr} + q(\cdot) = 0 \quad \text{and} \quad r \frac{dq(\cdot)}{df} = 0.$$

The two first-order conditions require firstly that at the optimal value of the royalty, the change in between-member volume from a change in the royalty ( $dq(\cdot)/dr$ ) is negative. Secondly the first-order conditions require that at the optimal value of the commission fee, the change in between-member volume from a change in the commission fee ( $dq(\cdot)/df$ ) is zero. This first condition can always be satisfied as  $dq(\cdot)/dr = -q(\cdot)/r < 0$ , where the details are given in the Appendix.

When determining the commission fee, the network provider trades off the negative effect of a higher commission fee on member investment with the positive effect of a higher commission fee on network size. The optimal level of the commission fee requires that  $dq(\cdot)/df = 0$ .

The elements of the first-order condition for the commission fee are

$$\frac{dq(\cdot)}{df} = \frac{dq(\cdot)}{d\tilde{x}} \left[ \overbrace{\left[ \underbrace{\frac{\partial \tilde{x}(\cdot)}{\partial f}}_{Direct} + \underbrace{\frac{\partial \tilde{x}(\cdot)}{\partial e_{\setminus \tilde{x}}} \frac{\partial e_{\setminus \tilde{x}}(\cdot)}{\partial f}}_{Indirect} \right]}^{NetworkSize} \right] + \overbrace{\frac{\partial q(\cdot)}{\partial e} \frac{\partial e(\cdot)}{\partial f}}^{MemberInvestment} = 0. \quad (14)$$

The condition in (14) must hold if the commission fee is to be used at a level between zero and the full price. There are two elements comprising the total effect of a change in the commission fee on network volume in (14). The first is the direct effect of such a change on member investment which in turn affects network volume. The second is a combined effect through network size that is comprised of a direct effect of such a change on network size which in turn affects network volume, and an indirect effect of such a change in member investment which affects network size which in turn affects network volume. Again, the mathematical details of what follows are in the Appendix.

The last product in (14) is the direct effect of a change in the commission fee on network volume through member investment. If cannibalization is low, then other members' investments have little effect on demand from a given member's own customers, and therefore a member's own investment dominates the direct effect of member investment on network volume. Member investment is decreasing in the commission fee from our earlier results on the response of member investments to changes in the commission fee. Therefore, the direct effect of a change in the commission fee on network volume through member investment is the product of the positive effect of member investment on network volume and the negative effect of the commission fee on member investment, and this product is negative.

The remaining terms in (14) are the direct and indirect effects of a change in the commission fee on network volume through network size. The effect of network size on network volume is contained in the term outside the square brackets of (14). Recall that network size

is defined by the smallest member,  $\tilde{x}$ . The smaller the business potential of this member, the greater is the network size - directly from (11). Because the greater the business potential of the smallest member, the smaller is the network size, and the smaller the network size the smaller is the network volume, the term outside the square brackets of (14) is negative. The indirect effect of member investment on network size is contained in the product inside the square brackets of (14). The first term in the product is negative - increased investments by other members increases the profits of the smallest prospective members, decreasing the size of the smallest member. The second term in the product is also negative - an increase in the commission fee reduces member investment.

Therefore, there are two effects of a change in the commission fee on network volume: one a direct effect through member investment, and the other an indirect effect of member investment through network size. Both are negative. An interior solution - one where the commission fee is used at a level between zero and the full price - requires that the direct effect of a change in the commission fee on network size is positive and sufficiently large as to offset the other effects. In (14) this requires that the direct effect of a change in the commission fee on the smallest member is to decrease the size of the smallest member,  $\partial\tilde{x}(\cdot)/\partial f < 0$ , and is sufficiently large to dominate the remaining terms in (14). The magnitude of this decrease in the size of the smallest member depends on the difference between imported and exported demand for the smallest member,  $d_I(\tilde{\cdot}) - d_E(\tilde{\cdot})$  in (12). Consequently, if cannibalization is low and if imported demand is sufficiently greater than exported demand for the smallest member, then there is an interior solution to the commission fee.

To summarize, Assumption 4(c) assures that the difference between imported and exported demand for the smallest member is positive, and consequently,  $\partial\tilde{x}(\cdot)/\partial f < 0$  so that the commission fee increases network size. However, this difference alone is not sufficient to ensure an interior solution to the commission fee because the other impact of the com-

mission fee - decreased member investment - comes through a direct effect on investment (the last product in (14)) and an indirect effect on network size (last product in the square brackets of (14)). The difference between imported and exported demand for the smallest member reflects the range of members' business potential, and the broader that range, the more effective is the commission fee at directly increasing network size. Thus, this difference must be sufficiently large so that the positive direct effect of the commission fee on network size is greater than the indirect effect on size through investment, and counterbalances the direct effect of the commission fee on investment. The range of members' business potential reflected in the difference between imported and exported demand for the smallest member determines the sensitivity of the commission fee for increasing network size because if the range was narrow, then greater use of the commission fee would be necessary to increase network size and vice versa.

### 3.5 Comparison to an Integrated Firm

To replicate the investment levels that would be obtained in an integrated and fully informed firm requires that the network provider knows the relationship between individual members' investments and demands. The network provider could then offer prospective members the full price on domestic and exported demand and then charge a member-specific lump-sum fee for participating leaving each potential member indifferent between participating in the network or not. Recognizing that every member would participate so that  $y = 1$ , each member faces profits that are a special case of (4) where there is no royalty or commission fee.

As we show below, an integrated and fully informed firm would yield greater member investment than is obtained with the commission fee. The special case of (4) where the royalty and commission fee are zero,  $r = f = 0$ , and using  $L(x)$  to denote the member-specific

lump-sum participation fee, the first-best profit function for each member is  $\pi^{FB}(x, y, e) = pd_D(x, 1, e) + pd_E(x, 1, e) - e_x - L(x)$ . The superscript "FB" is used to denote first-best, that is, the integrated and fully informed firm. Profits are the product of price and both domestic and exported demand, less the cost of investment. The set of first-order conditions that describe member investments are

$$\frac{\partial \pi^{FB}(x, 1, e)}{\partial e_x} = p \frac{\partial d_D(x, 1, e)}{\partial e_x} + p \frac{\partial d_E(x, 1, e)}{\partial e_x} - 1 = 0 \quad \forall x, \quad (15)$$

where the first two terms are the marginal returns to investment through domestic and exported demand respectively, the last term is the marginal cost of investment, the member-specific lump-sum term drops out. The supermodularity condition is satisfied, and details are in the Appendix. The set of conditions (15) yield a pure strategy Nash equilibrium. From our earlier results the effect of an increase in the commission fee is to reduce member investment,  $\partial e_x(\cdot)/\partial f < 0$ , meaning investment is highest when there is no commission fee,  $f = 0$ .

To summarize, in our model the commission fee is used to increase network size, to the detriment of member investment. However, in an integrated and fully informed firm the maximum network size is assured and because all members receive the full price on domestic and exported demand already, setting the commission fee to zero maximizes profits from member investment.

### 3.6 Conclusion

We have shown that in a network alliance, where customers purchase from multiple members, a classical exclusivity agreement and an appropriate commission fee can increase member investment and network profits. The exclusivity agreement softens direct competition between members, and the commission fee motivates members to make investments in between-

member business. These two mechanisms - the exclusivity agreement and the commission fee - brings the performance of the network alliance closer to a fully integrated and informed firm without agency problems by aligning member incentives with network alliance incentives. We have also shown how the optimal network size restricts membership - not all prospective members should participate - and this optimal size emerges as a result of the exclusivity agreement and the commission fee. In this way we have articulated the role of incentives and the role of membership in the coordination of actions and interactions of members in network organizations. Our results are summarized in Table 5.

\*\*\* Put Table 5 about here \*\*\*

To obtain our results we have required that members' investments be more valuable with increases in other members' investments (weak complements), that prospective members are sufficiently different that there is an adequate range in the business potential of members, and that the effect of other members' investments on a given members business potential is moderately low (low cannibalization). In the process of our investigation, we have described how the analysis of this network alliance form can apply to very disparate industries: professional engineering, accounting services and commercial fueling. Thus, we can conclude that our qualitative results are highly generalizable within the constraints of our modeling methods.

There are limitations as a consequence of our analytical assumptions, the elements we include in our formulation, and underlying assumptions about how firms make decisions. Although we believe our analytical assumptions to be realistic, they do restrict the characteristics of demands and reactions of these demands to changes in member size and investment to which our analysis applies. At a more abstract level, our formulation makes several implicit assumptions about heterogeneity and which decisions are important to model. For example, our formulation collapses a multivariable set of features that differentiate members



into a single dimension we consider as size. In addition, we take customer behavior as being represented by our demands while not directly modeling customer decisions about joining the network - or even whether customers differ. These two aspects of our formulation rely on the set of features or customer behavior being adequately represented in "reduced form" (i.e., features summarized as size; customer behavior being summarized in our demands) as we use in our analysis. At an even more abstract level, we treat the network provider and members as rational profit maximizing entities, and assume there are no critical time elements that condition member investments and network provider fee-setting. These underlying assumptions about the business environment facing professional engineering firms, accounting firms, commercial fuelers, etc., are an approximation for what we believe to be at least the intended behavior of these firms.

It is important to note that the exclusivity agreement we employ does not restrict where members recruit customers while at the same time protecting the members' locations where customers are served. In some settings it is not possible to assign customers *a priori* - customers can only be identified with members after they have been recruited. In yet other settings there are no assignment algorithms that are consistent with members' knowledge of specific customers. Therefore, when implementing a network alliance the distinction between the classical exclusivity agreement - the one we use - and an agreement that also pre-assigns customers to members is important.

We also observe that new economy industries have been quick to employ aspects of the network alliance form. Amazon.com and Barnes & Noble each use a program that rewards alliance members for purchases made by customers they direct to Amazon or Barnes & Noble. The Amazon.com Associates Program (and Barnes & Noble Affiliates Program) awards a percentage of book sales in a given transaction to the site that referred the customer - a commission fee. The critical difference between this paper's model and these programs is

that while it is possible to determine which site referred the customer, an individual customer can be referred to Amazon.com (or Barnes & Noble) from different sites at different times. Thus, the identification of customers to members is transient, and perhaps more interestingly, network alliance membership is dynamic. Modifying this paper's model to answer questions about these programs is a fruitful avenue for future research.

## 4 Appendix

**Investment Equilibria: Exclusivity Agreements with Commission Fees** The cross effects of member investment and other members' investments,  $e_x$  and  $e_{\setminus x}$ , are

$$\frac{\partial^2 \pi(\cdot)}{\partial e_x \partial e_{\setminus x}} = p \frac{\partial^2 d_D(\cdot)}{\partial e_x \partial e_{\setminus x}} + [p - r - f] \frac{\partial^2 d_E(\cdot)}{\partial e_x \partial e_{\setminus x}} + f \frac{\partial^2 d_I(\cdot)}{\partial e_x \partial e_{\setminus x}} > 0. \quad (16)$$

The sign of (16) is determined by Assumption 3 and (3). Because the strategy intervals for  $e_x$  are non-empty, convex and compact, and the profits of each member are twice continuously differentiable in the strategies of all members, and the cross effects of member investment in (3) are positive, then the member investment game is supermodular. The set of conditions (5)  $\forall x \in [x, \bar{x}]$  define a set of pure strategy Nash equilibria and there exists a largest pure Nash equilibrium.

Under exclusivity agreements, (5) includes an imported demand term - the negative impact of a member's investment recruiting customers that may have been recruited by another member - but this term is more than offset by the increase in domestic demand in the first term. The equilibrium is assured because with the weak complements of investments from Assumption 3, the investment game between members is supermodular.

**Investment Equilibria: Exclusivity Agreements without Commission Fees** Each member's first-order condition for maximizing profit by choice of level of investment in exclusivity agreements without commission fees is (9). The supermodularity condition

$$\frac{\partial^2 \pi^{SC}(\cdot)}{\partial e_x \partial e_{\setminus x}} = p \frac{\partial^2 d_D(\cdot)}{\partial e_x \partial e_{\setminus x}} + p \frac{\partial^2 d_I(\cdot)}{\partial e_x \partial e_{\setminus x}} > 0$$

is satisfied by Assumption 3 and (3), and together with the properties of the strategy intervals and the continuity of profits in the members' strategies, (9) for all members yields the set of Nash equilibria.

**Investment Equilibria: Integrated Firm** The supermodularity condition that applies to (15) is

$$\frac{\partial^2 \pi(\cdot)}{\partial e_x \partial e_{\setminus x}} = p \frac{\partial^2 d_D(\cdot)}{\partial e_x \partial e_{\setminus x}} + p \frac{\partial^2 d_E(\cdot)}{\partial e_x \partial e_{\setminus x}} > 0$$

which is satisfied by Assumption 3. Together with the properties of the strategy intervals and the continuity of profits in the members' strategies, the set of conditions (15) yield a pure strategy Nash equilibrium.

**Effects on Network Size** The effects on network size are found using (10) and the implicit function rule. From the implicit function rule the three effects are

$$\frac{\partial \tilde{x}(\cdot)}{\partial r} = -\frac{\partial \phi(\cdot)/\partial r}{d\phi(\cdot)/d\tilde{x}}, \quad \frac{\partial \tilde{x}(\cdot)}{\partial f} = -\frac{\partial \phi(\cdot)/\partial t}{d\phi(\cdot)/d\tilde{x}}, \quad \text{and} \quad \frac{\partial \tilde{x}(\cdot)}{\partial e_{\setminus \tilde{x}}} = -\frac{\partial \phi(\cdot)/\partial e_{\setminus \tilde{x}}}{d\phi(\cdot)/d\tilde{x}},$$

where the denominators are written as total derivatives because the effect of  $\tilde{x}$  through  $y$  is needed as  $y$  is not an argument of  $\phi(\cdot)$ . Note that these effects are in terms of the smallest member that participates in the network, and the effects on network size are opposite to these from (11). From Assumption 4(b), profits of the smallest member are increasing in its size:

$$\frac{d\phi(\cdot)}{d\tilde{x}} = \frac{\partial \phi(\cdot)}{\partial \tilde{x}} + \frac{\partial \phi(\cdot)}{\partial y} \frac{dy(\tilde{x}(\cdot))}{d\tilde{x}} > 0.$$

Assumption 4(b) is needed because from (10), using Assumption 1 and (1),  $\partial\phi(\cdot)/\partial\tilde{x}$  and  $\partial\phi(\cdot)/\partial y$  are positive, and from (11) the last term on the right hand side is negative.

**Effects of Changes in the Royalty on Network Profit** The elements of this  $dq(\cdot)/dr$  are

$$\frac{dq(\cdot)}{dr} = \frac{dq(\cdot)}{d\tilde{x}} \left[ \frac{\partial\tilde{x}(\cdot)}{\partial r} + \frac{\partial\tilde{x}(\cdot)}{\partial e_{\backslash\tilde{x}}} \frac{\partial e_{\backslash\tilde{x}}(\cdot)}{\partial r} \right] + \frac{\partial q(\cdot)}{\partial e} \frac{\partial e(\cdot)}{\partial r} = -\frac{q(\cdot)}{r} < 0, \quad (17)$$

where the details of the last term are defined with the discussion of the commission fee below.

**Effects of Changes in the Commission Fee on Network Profit** There is a double use of notation in the last terms of (14) (and of (17)) where the member size index,  $x$ , is used both as an index of members and as an index of member's investment. We can clarify the last terms in (14) by employing a change of variable and rewriting them as

$$\frac{\partial q(\cdot)}{\partial e} \frac{\partial e(\cdot)}{\partial f} = \int_{\tilde{z}(\cdot)}^{\bar{z}} \left[ \int_{\tilde{x}(\cdot)}^{\bar{x}} \frac{\partial d_E(x, y(\tilde{x}(\cdot)), e(\cdot))}{\partial e_z} g(x) dx \right] \frac{\partial e_z(\cdot)}{\partial f} g(z) dz, \quad (18)$$

noting that  $\bar{z} = \bar{x}$  and  $\tilde{z}(\cdot) = \tilde{x}(\cdot)$ . (This can be done similarly for the last term of (17)). Thus, the term referred to as  $\partial q(\cdot)/\partial e$  in (14) is the quantity contained in the square brackets of (18). If cannibalization is low, then a member's own investment dominates, and the quantity in the square brackets of (18) is positive. From our earlier results on the response of member investments to changes in the commission fee, member investment is decreasing with increases in the commission fee. Therefore, the quantity in (18), and hence the last product in (14), is negative.

We can expand the derivative outside the square brackets of (14) and obtain

$$\frac{dq(\cdot)}{d\tilde{x}} = \frac{\partial q(\cdot)}{\partial \tilde{x}} + \frac{\partial q(\cdot)}{\partial y} \frac{dy(\tilde{x}(\cdot))}{d\tilde{x}}.$$

The first term is negative because  $\partial q(\cdot)/\partial \tilde{x} = -d_E(\tilde{x}, y(\tilde{x}(\cdot)), e(\cdot))g(\tilde{x}) < 0$  (Leibnitz's rule). From Assumption 1 whereby exported demand is increasing with increases in network size, the next term is positive,

$$\frac{\partial q(\cdot)}{\partial y} = \int_{\tilde{x}(\cdot)}^{\bar{x}} \frac{\partial d_E(x, y(\tilde{x}(\cdot)), e(\cdot))}{\partial y} g(x) dx > 0.$$

Together with (11), the last product is negative. Consequently,  $dq(\cdot)/d\tilde{x}$  is negative.

The product inside the square brackets of (14) is positive from our earlier results: the first term in the product is negative - the reverse of the effect of increased investment by other members on network size, and the second term in the product is also negative - the effect of an increase in the commission fee on member investment. Therefore, an interior solution requires that  $\partial \tilde{x}(\cdot)/\partial f < 0$ , the reverse of the effect of an increase in the commission fee on network size, is sufficiently large to dominate the remaining terms in (14). The magnitude of  $\partial \tilde{x}(\cdot)/\partial f < 0$  depends on the difference  $d_I(\tilde{\cdot}) - d_E(\tilde{\cdot})$  in (12).

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Demand	Member Size	Network Size
Domestic	$\frac{\partial d_D(x,y,e)}{\partial x} > 0$	$\frac{\partial d_D(x,y,e)}{\partial y} > 0$
Exported	$\frac{\partial d_E(x,y,e)}{\partial x} > 0$	$\frac{\partial d_E(x,y,e)}{\partial y} > 0$
Imported	-	$\frac{\partial d_I(x,y,e)}{\partial y} > 0$

Table 1: Demand Responses to Size



Demand	Own Investment	Other Investment	Other and Own Investment	Size and Own Investment	
Domestic	$\frac{\partial d_D(x,y,e)}{\partial e_x} > 0$	$\frac{\partial d_D(x,y,e)}{\partial e_{\setminus x}} < 0$	$\frac{\partial^2 d_D(x,y,e)}{\partial e_x \partial e_{\setminus x}} \geq 0$	$\frac{\partial^2 d_D(x,y,e)}{\partial e_x \partial x} > 0$	(A)
Exported	$\frac{\partial d_E(x,y,e)}{\partial e_x} > 0$	$\frac{\partial d_E(x,y,e)}{\partial e_{\setminus x}} < 0$	$\frac{\partial^2 d_E(x,y,e)}{\partial e_x \partial e_{\setminus x}} \geq 0$	$\frac{\partial^2 d_E(x,y,e)}{\partial e_x \partial x} > 0$	(B)
Imported	$\frac{\partial d_I(x,y,e)}{\partial e_x} \leq 0$	$\frac{\partial d_I(x,y,e)}{\partial e_{\setminus x}} \geq 0$	-	$\frac{\partial^2 d_I(x,y,e)}{\partial e_x \partial x} \geq 0$	(C)
Assumption	A2(a)	A2(b)	A3	A4(a)	

Table 2: Demand Responses to Investment

<b>Investment</b>		<b>Demand</b>	<b>Response to Investment</b>
Own	$\xrightarrow{(A)}$	Domestic	Increasing
	$\xrightarrow{(B)}$	Exported	Increasing
	$\xrightarrow{(C)}$	Imported	Decreasing
Other	$\xrightarrow{(A)}$	Domestic	Decreasing
	$\xrightarrow{(B)}$	Exported	Decreasing
	$\xrightarrow{(C)}$	Imported	Increasing
Own and Other	$\xrightarrow{(A)}$	Domestic	Increasing
	$\xrightarrow{(B)}$	Exported	Increasing
	$\xrightarrow{(C)}$	Imported	N/A

Table 3: **Investment Responses**

**Member Size**  $\frac{\partial d_D(x,y,e)}{\partial x} \geq -\frac{\partial d_I(x,y,e)}{\partial x}$  Equation (1)

**Member Investment**  $\frac{\partial d_D(x,y,e)}{\partial e_x} \geq -\frac{\partial d_I(x,y,e)}{\partial e_x}$  Equation (2)

**Members' Investment**  $\frac{\partial^2 d_D(x,y,e)}{\partial e_x \partial e_{\setminus x}} \geq -\frac{\partial^2 d_I(x,y,e)}{\partial e_x \partial e_{\setminus x}}$  Equation (3)

Table 4: **Effects on Domestic Versus Imported Demand**

<b>Section</b>	<b>Summary</b>
Section 3.1	<p>Finds equilibrium in member investments</p> <p>Member investments are increasing in member size and are decreasing in the royalty and commission fee</p> <p>The effects of member investment of changes in the commission fee are larger than those of the royalty</p>
Section 3.2	<p>Member investments are higher when the commission fee is less than the full price</p> <p>The commission fee compensates the member that recruits, increasing member incentives</p>
Section 3.3	<p>Network size increases in the commission fee and decreases in the royalty</p> <p>With low cannibalization, increases in other members' investment increase the size of the network</p>
Section 3.4	<p>The greater the difference in imported and exported demand, the more effective if the commission fee at directly increasing network size</p> <p>This direct effect of the commission fee on network size is critical to using the commission fee at a level between zero and the full price</p>
Section 3.5	<p>In an integrated and fully informed firm, the maximum network size is assured, and the commission fee can be used purely for incentives</p>

Table 5: **Summary by Section**