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Viscoelasticity of Articular Cartilage and Ligament: Constitutive Modeling and Experiments

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Viscoelasticity of Articular Cartilage and Ligament:
Constitutive Modeling and Experiments

by

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A THESIS
SUBMITTED TO THE FACULTY OF GRADUATE STUDIES
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Articular cartilage and ligament are soft fibrous connective tissues with apparent viscoelastic behavior. An anisotropic visco-hyperelastic constitutive model for these tissues has been proposed in this study based on the short-term and long-term internal variables. The constitutive model was particularized for both tissues, numerically implemented into the finite element software package ABAQUS and the material parameters were identified using the available experimental data. The constitutive model was able to capture both the short-term and long-term time-dependent response of these tissues with less difficulty in material characterization process. Due to the lack of the desired tensile experimental results on articular cartilage, the mechanical behavior of this tissue was also examined experimentally. The tensile stiffness of articular cartilage was found to be rate-dependent. It has been shown by numerical simulations that the strain-rate dependent tensile stiffness of collagen fibers can also contribute to the highly rate-dependent compressive response of articular cartilage, besides the fluid-driven viscoelasticity.
Acknowledgements

I would like to express my gratitude to my supervisor, Dr. LePing Li, for providing me the opportunity to work on this interesting area of research and his guidance and constructive feedbacks in the course of my project.

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<td>F</td>
<td>deformation gradient</td>
</tr>
<tr>
<td>C</td>
<td>right Cauchy-Green deformation tensor</td>
</tr>
<tr>
<td>(\dot{C})</td>
<td>time derivative of the right Cauchy-Green deformation tensor</td>
</tr>
<tr>
<td>b</td>
<td>left Cauchy-Green deformation tensor</td>
</tr>
<tr>
<td>D</td>
<td>spatial rate of deformation tensor</td>
</tr>
<tr>
<td>L</td>
<td>spatial velocity gradient</td>
</tr>
<tr>
<td>E</td>
<td>Lagrange or Green strain tensor</td>
</tr>
<tr>
<td>e</td>
<td>Euler or Almansi strain tensor</td>
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<tr>
<td>(\Psi)</td>
<td>Helmholtz free energy function</td>
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<td>S</td>
<td>second Piola-Kirchhoff stress</td>
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<td>(\sigma)</td>
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<td>Structure tensor in the reference configuration</td>
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<td>(\lambda)</td>
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<tr>
<td>(I_n)</td>
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<tr>
<td>(F_{vol})</td>
<td>volumetric part of the deformation gradient</td>
</tr>
<tr>
<td>Symbol</td>
<td>Description</td>
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<td>$\tilde{F}$</td>
<td>deviatoric part of the deformation gradient</td>
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<td>$I_n$</td>
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<td>$J$</td>
<td>determinant of the deformation gradient</td>
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<tr>
<td>$\ddot{F}$</td>
<td>deviatoric part of the deformation gradient</td>
</tr>
<tr>
<td>$\mathcal{P}$</td>
<td>fourth-order projection tensor</td>
</tr>
<tr>
<td>$\mathbb{I}$</td>
<td>fourth-order identity tensor</td>
</tr>
<tr>
<td>$\mathcal{I}$</td>
<td>supersymmetric fourth-order identity tensor</td>
</tr>
<tr>
<td>$\Upsilon$</td>
<td>Long-term viscous stress</td>
</tr>
<tr>
<td>$\mathcal{L}_v$</td>
<td>Lie derivative operator</td>
</tr>
<tr>
<td>$\tau$</td>
<td>Kirchhoff stress</td>
</tr>
<tr>
<td>$\mathcal{C}'$</td>
<td>Jaumann tangent moduli</td>
</tr>
<tr>
<td>$\mathcal{W}$</td>
<td>Skew-symmetric part of the spatial velocity gradient</td>
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<tr>
<td>QLV</td>
<td>Quasi-linear viscoelasticity</td>
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<td>OA</td>
<td>Osteoarthritis</td>
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Chapter 1

Introduction

Most soft biological tissues exhibit time-dependent behavior and are considered to be viscoelastic. Ligament and articular cartilage specifically show apparent viscoelastic behavior. Both tissues are composed of a solid matrix which absorbs a considerable amount of water. The solid matrix consists of non-fibrillar matrix and fiber network. Although the fluid movement and exudation in the tissues is a source of viscoelasticity, the solid matrix itself is also viscoelastic. The inherent viscoelasticity may arise from the hierarchical structure of the solid matrix constituents and their relative motion. The total time-dependent behavior of the tissue is governed by the interplay of the aforementioned mechanisms, i.e., fluid-driven and inherent viscoelasticity. However, the degree to which a specific mechanism is dominant depends on the loading configuration. Generally, the fluid-driven viscoelasticity is the major cause of the time-dependent response under compression. However, in tension, fluid takes little part in the load bearing mechanism and the inherent viscoelasticity of the tissue becomes dominant.

Articular cartilage mostly sustains compression suggesting that fluid pressurization is a dominant mechanism in the apparent time-dependent behavior. Nevertheless, the solid-matrix of cartilage is also composed of collagen fibers (mainly type II) orienting in the lateral direction that limit the expansion due to Poisson’s effect. Although most of the load and viscoelastic response is contributed by the fluid, collagen fibers also play a significant role by enhancing the fluid pressurization. The higher the stiffness of the fibers are, the less the lateral expansion of the tissue is. Consequently, this leads to higher fluid pressurization and load support. Ligament, on the other hand, is mainly composed of collagen fibers (mainly type I) with high tensile stiffness. This tissue mostly experiences tension and fluid-driven
Figure 1.1: Articular cartilage and ligaments are the crucial components of the knee joint (adopted from Makris et al. (2011) with permission).

Viscoelasticity does not seem to be a key player in its overall response.

Viscoelastic materials are time- and rate-dependent. Therefore, they show different stiffnesses at a specified strain level when loaded under different strain-rates. The rate-sensitivity of the stiffness, however, depends on the viscoelastic properties of the material. While the viscoelastic properties of ligament have been studied with reference to strain-rate (Pioletti, 1997), the strain-rate sensitivity of the tensile response of cartilage has not been examined in a wide range of strain-rates. Therefore, the role of the possible rate-dependent stiffness of collagen fibers in the overall compressive behavior of the tissue is not fully understood.

Ligament and articular cartilage are crucial components of the knee joint and together sustain different loading regimes enabling the joint to function effectively. The function of each tissue in the joint can be affected by damage that compromises the mechanical
properties of the tissue. Osteoarthritis (OA) is a joint disease that is commonly observed in the knee joint. In some cases, it starts with degeneration of cartilage in the superficial layer at some zones and advances to deeper layers and reaches the underlying bone. In the advanced states, it may lead to pain and total disability. There are many factors contributing to the initiation and progression of this disease. However, the mechanical environment and loading that cartilage is experiencing have been shown to be important. Mechanical loading can affect the tissue health in two ways. At smaller scales, the mechanical environment of cells determines their activities in proliferation, production of extracellular matrix, suppression or death (Zuscik et al., 2008; Lei and Guo, 2005; Zamli and Sharif, 2011) which affects the overall tissue quality and may play a role in initiation and progression of OA (Kerin et al., 2002). At macroscale, mechanical loadings can cause damage to tissues which compromises the mechanical properties of the whole tissue or some of its constituents. This, in turn, can cause an overload to other tissue components and introduce a possibility of damage to them as well. The cartilage defects (Ding et al., 2007; Janakiramanan et al., 2006) as well as the injury in ligaments (Lohmander et al., 2007; Fleming et al., 2005) may lead to osteoarthritis. Damages to tissues usually occur due to large strain or strain-rates. High strain-rate incidents are uncontrolled or less controlled and in extreme cases are impacts.

The mechanical environment of the tissue is mostly determined by the loads experienced in the tissue during daily activities. The frequency of the cyclic loading of the knee joint in physiological condition during walking and running is normally in the range of a second (Oberg et al., 1993). Moreover, damages usually occur at high strain-rates. This necessitates the study of the tissues in the knee joint at higher strain-rates. For this purpose, a model of articular cartilage and ligament is needed which is capable of capturing the high strain-rate phenomena.

The mechanical behavior of articular cartilage, ligament, and other soft tissues, can be described by continuum mechanics constitutive laws. These mathematical equations can be
Figure 1.2: This table shows the major types of the constitutive models proposed for articular cartilage and ligament. Only a few references are mentioned here; however, more detailed description of the models for both tissues will be presented in Chapter 4.

The models are highly nonlinear and complex that describe the large deformation behavior of the tissues, or can be linearized to predict the small deformation response. In the past, numerous models were proposed for articular cartilage and ligament. Monophasic models for cartilage were mostly phenomenological and able to capture the equilibrium, instantaneous elastic or some time-dependent responses of the tissue, if the viscoelasticity was formulated. Biphasic models expressed a more precise description of the tissue composition that were able to shed light to the role of interstitial fluid flow in the time-dependent response. Fiber-reinforced biphasic models later introduced for cartilage brought about more success in revealing the crucial role of collagen fibers in the high ratio of the transient to the equilibrium response observed in

<table>
<thead>
<tr>
<th>Articular Cartilage</th>
<th>Ligament</th>
</tr>
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<tbody>
<tr>
<td>• Monophasic models</td>
<td>• Elastic:</td>
</tr>
<tr>
<td>• Elastic [1]</td>
<td>• Phenomenological [10]</td>
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<td>• Biphasic models</td>
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<td>• Biphasic elastic [3]</td>
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References:
articular cartilage compressive loading. Similarly for ligament, both phenomenological and structurally-motivated models were introduced. As mentioned previously, ligament is mostly in tension and fluid pressurization may not play a major role in the viscoelastic response of the tissue. Therefore, most of the models introduced for ligament hitherto were monophasic that fall into two categories of elastic and viscoelastic. The major models for these tissues are summarized in Fig. 1. More detailed description of the models, however, will be discussed in Chapter 4. The mechanical models of the tissues can be implemented into finite element programs to study the mechanical response of the tissue under various loading and boundary conditions. In the past, most of the models were limited to simple axisymmetric geometries. More recently, finite element knee joint models have been introduced that are able to account for the time dependent response of the tissues as well (Gu and Li, 2011). This brings the opportunity to study the tissues in a more realistic environment including the effect of each tissue on others in the joint. However, the first step is to develop a constitutive model that is able to capture the desired mechanical behavior of articular cartilage and ligament.

1.1 Objectives

1- The inherent viscoelasticity of biological tissues has been modeled by integral type and differential type constitutive models. The integral type models describe the current state of deformation by taking into account the history of the past deformation through a hereditary integral. These models are suitable to predict the long-term or relaxation behavior of viscoelastic materials. On the other hand, differential type models include a measure of strain-rate by which the rate-dependent response of viscoelastic materials is obtained. This class of models predicts the transient response with an appropriate strain-rate sensitivity, however, they fail to account for long-term phenomena. Integral type models produce a less strain-rate sensitive transient response and characterization of the material parameters for both ramp and relaxation phases involves some difficulties. Therefore, one objective of the
current study was to develop a visco-hyperelastic model that is able to employ the advantages of both models while accounting for both short-term and long-term response of the tissue with higher predictive capability.

2- While the tensile mechanical behavior of ligament has been experimentally investigated at high strain-rates, the tensile response of cartilage in tension in accordance with strain-rate has not been fully understood. Therefore, another objective of this thesis was to perform tensile tests on articular cartilage to characterize the transient response of the tissue at various strain-rates. The results are to be used for identifying the material parameters of the proposed constitutive model.

3- The secondary motivation of the tensile experiments on cartilage was to examine the inherent hysteresis or damping characteristics of the tissue with reference to strain-rate. It has been thought that the overall hysteresis of the tissue arise from the fluid exudation during loading (Mak, 1986; Setton et al., 1993). The theory of poroelasticity then predicts that the hysteresis of the tissue should decrease at higher strain-rates, i.e., the tissue should become more elastic. This is also shown in experiments at frequencies near physiological condition (Johnson et al., 1977; Higginson and Snaith, 1979). However, a recent study on articular cartilage has shown that the hysteresis at impact and high loading rates increases (Edelsten et al., 2010). Therefore, It was hypothesized that this hysteresis arises from the inherent properties of the solid matrix. The superficial zone of articular cartilage was thought to be mostly affected at very high loading rates such as impact. Therefore, it was further hypothesized that the collagen fibers in the superficial layer play a role in the increased hysteresis at very high strain-rates. Tensile experiments were then designed in a way that can reveal the inherent hysteresis of cartilage at high strain-rates.

4- Another objective of the current study was to numerically implement the visco-hyperelastic constitutive model into the finite element software package ABAQUS (Hibbitt, Karlsson & Sorensen Inc., Providence, RI, USA) by a UMAT user subroutine in FORTRAN. The model
was to be used to investigate the local stresses, strains and pore pressure distribution in the depth of the tissue as well as the total reaction force under compression. Specifically, this objective was to integrate the strain-rate-dependent stiffness of cartilage obtained from experiments and the proposed constitutive model and implement it in a cartilage finite element model. This model then may explain if the high ratio of the peak (short-term) to equilibrium response of cartilage under compression at high strain-rates is also contributed by the tensile strain-rate stiffening of collagen fibers in the lateral direction.

1.2 Contributions

1- A novel visco-hyperelastic constitutive model has been proposed in this study based on the short-term and long-term internal variables. Moreover, the constitutive model was particularized for articular cartilage and ligament. The model is specifically capable of predicting the high strain-rate phenomena.

2- An inhomogeneous anisotropic poro-visco-hyperelastic model of articular cartilage with depth-dependent fiber orientation and material properties was developed and numerically implemented into ABAQUS. Such a model for articular cartilage has not been reported in the literature before. The analytical derivation of the elasticity and viscosity tensors were also reported and numerically implemented which led to the fastest possible convergence rate.

3- This study experimentally investigate the tensile behavior of articular cartilage under various strains and strain-rates. The tensile transient response as well as the inherent hysteresis of articular cartilage had not been investigated under these ranges of strains and strain-rates.

4- This study shows the possible mechanism of strain-rate dependency of the compressive response of articular explained by the strain-rate dependent tensile stiffness of collagen fibers.
1.3 Thesis Overview

As previously mentioned, a fiber-reinforced viscoelastic model is developed in this project. The constitutive model is particularized for articular cartilage and ligament and coded into ABAQUS via a UMAT subroutine. Therefore, it can be used for finite element analysis on articular cartilage and ligament under different types of loading. The material parameters in the constitutive models are needed to be determined using experimental data. This can be achieved by fitting the constitutive model to the data obtained from experiments. For ligament, some experimental data is available from literature. However, the tensile test re-
sults on articular cartilage under the desired strains and strain-rates have not been reported before. Therefore, tensile tests on articular cartilage were also done as a part of this project to be used for characterizing the constitutive model. The first 5 chapters are introductory chapters discussing the continuum mechanics preliminaries, structure and function of the tissues, a literature review of the constitutive models proposed for these tissues and standard mechanical tests. The methods and the results are discussed in chapter 6 and 7 respectively followed by the discussion and conclusion. A more detailed description of each chapter (except the current chapter which is an introduction) is as follows:

**Chapter 2:** This chapter is a brief introduction to continuum mechanics, theory of hyperelasticity and viscoelasticity. It helps the readers without a background in continuum mechanics to understand the notation and the theories used in Chapter 6 to describe the methods of constitutive modeling. This chapter starts with the basic definition of variables in continuum mechanics including strain and stress measures. It is followed by an introduction to the theory of hyperelasticity as well as different approaches in modeling viscoelastic materials.

**Chapter 3:** In this chapter, the structure and function of articular cartilage and ligament are discussed. Both tissues are well-hydrated and consist of a non-fibrillar and fibrillar matrix. A more detailed description of the composition of the tissues including inhomogeneities and anisotropies as well as the presence of different phases is reviewed in this chapter. Understanding the complex structure of these tissues facilitates the understanding of the notion behind different classes of constitutive models proposed for articular cartilage and ligament. The function of both tissues in the knee joint is also discussed to emphasize the significance of the studies done on the mechanics of these tissues.

**Chapter 4:** This chapter is a literature review of the previously published constitutive models for articular cartilage and ligament. The evolution of the constitutive models for these tissues can be appreciated while addressing the advantages and drawbacks associated
with different models. Also, the current state of development in this area is identified as a departing point for the constitutive model introduced in this project.

Chapter 5: In this chapter, the standard mechanical tests commonly used for articular cartilage and ligament are described. The results if these tests can be used for characterization of the material and the corresponding constitutive model.

Chapter 6: This chapter includes the methods used for constitutive modeling, numerical implementation, finite element analysis, material characterization and experiments. The chapter starts by describing the constitutive model developed in this study; the constitutive model is then particularized for articular cartilage and ligament. The chapter continues by discussing the numerical implementation procedures used for coding the proposed constitutive model into the finite element software package ABAQUS. Also, two finite element models constructed for finite element analysis are described in this subsection. The next step is to describe the methods used for material characterization of the constitutive model with experimental data, i.e., the optimization techniques. As mentioned previously, tensile tests of articular cartilage were also done in this study to be used for material characterization. Therefore, the methods for the mechanical tests including the sample preparation process and the loading protocol are also described in this section.

Chapter 7: This chapter presents the results obtained from constitutive modeling, experiments and finite element analysis. The first two sections contain the material parameters for ligament and articular cartilage found from fitting the constitutive models into the available experimental data from literature. They are followed by a comparison of the proposed constitutive model with the past traditional approaches to viscoelasticity. Next, the experimental results of articular cartilage tensile tests are presented. The results are divided into two subsections discussing the transient response during ramp loading and the hysteresis response. Finally, the results obtained from the finite element modeling of articular cartilage are presented. These findings mainly highlight the effect of collagen fibers viscoelasticity for-
mulated by the proposed constitutive model on the overall compressive response of articular cartilage.

**Chapter 8:** This chapter includes the discussion, conclusion and future directions. First, advantages and limitations of the proposed constitutive model is discussed. In the next section, the experimental results of articular cartilage in tension is analyzed. The numerical results of articular cartilage obtained from ABAQUS were also further discussed. This chapter ends with the concluding remarks and identifying the areas that need future work.
Chapter 2

Continuum Mechanics Preliminaries

Throughout this thesis, we assume $\Omega_0$ to be our reference or undeformed configuration and $\Omega$ be the deformed or spatial configuration. The mapping $\chi : \Omega_0 \rightarrow \mathbb{R}^3$ which takes the point $X \in \Omega_0$ from the reference configuration to the point $x \in \Omega$ in the deformed configuration can be considered as a function that describes the deformation of a body. The gradient of the deformation is then defined as $F(X) = \frac{\partial \chi(X)}{\partial X}$ which maps the vectors in the tangent space of point $X$ in the reference configuration to the tangent space of point $x$ in the deformed configuration. $F$ is called deformation gradient and maps the vectors from the undeformed to the deformed configuration, unlike $\chi$ which maps the points. It should be noted that the deformation gradient, $F$, is a two-point tensor and consequently unsymmetric. This is also clear from its definition. The determinant of the deformation gradient will also be denoted by $J$ throughout this thesis. The spatial velocity gradient $L$ is also defined as:

$$L = \frac{\partial}{\partial x} v(x, t) = \dot{F} \cdot F^{-1}. \quad (2.1)$$

The spatial velocity gradient can be written as the summation of its symmetric and skew-symmetric parts. The symmetric part, also called the spatial rate of deformation tensor, is denoted by $D$:

$$D = \frac{1}{2}(L + L^T), \quad (2.2)$$

whereas the skew-symmetric part is denoted by $W$:

$$W = \frac{1}{2}(L - L^T). \quad (2.3)$$
2.1 Different Measures of Strain

For describing the deformation of bodies, different strain measures have been introduced other than the deformation gradient. First, assume $\vec{W}_1$ and $\vec{W}_2$ are two vectors in the tangent space of point $X$ and $\vec{w}_1$ and $\vec{w}_2$ are two tangent vectors of point $x$. Let us start by considering the inner product of $\vec{w}_1$ and $\vec{w}_2$:

$$w_1 . w_2 = (F \vec{W}_1) . (F \vec{W}_2)$$
$$= \vec{W}_1 . (F^T F) \vec{W}_2.$$  \hspace{1cm} (2.4)

The right Cauchy-Green deformation tensor is defined as:

$$C = F^T F,$$  \hspace{1cm} (2.5)

by which the inner product of two vectors $\vec{w}_1$ and $\vec{w}_2$ can be written as:

$$\vec{w}_1 . \vec{w}_2 = \vec{W}_1 . C \vec{W}_2.$$  \hspace{1cm} (2.6)

Similarly, by considering the inner product of $\vec{W}_1$ and $\vec{W}_2$:

$$\vec{W}_1 . \vec{W}_2 = (F^{-1} \vec{w}_1) . (F^{-1} \vec{w}_2)$$
$$= \vec{w}_1 . (F^{-T} F^{-1}) \vec{w}_2$$
$$= \vec{w}_1 . (FF^T)^{-1} \vec{w}_2,$$  \hspace{1cm} (2.7)

and introducing the left Cauchy-Green deformation tensor as:

$$b = FF^T,$$  \hspace{1cm} (2.8)

the inner product of two vectors $\vec{W}_1$ and $\vec{W}_2$ can be written as:

$$\vec{W}_1 . \vec{W}_2 = \vec{w}_1 . b^{-1} \vec{w}_2.$$  \hspace{1cm} (2.9)

The right Cauchy-Green deformation tensor $C$ is a material tensor, whereas the left Cauchy-Green deformation tensor $b$ is a spatial tensor. They are two main measures of strain.
commonly used in constitutive modeling of solids and fluids. As can be seen, they describe the change of the length of vectors along with their relative angles between configurations.

We can further consider the difference of the inner products of tangent vectors in the deformed and undeformed configurations to arrive at more measures of strain:

\[
\vec{w}_1 . \vec{w}_2 - \vec{W}_1 . \vec{W}_2 = \vec{w}_1 . C\vec{w}_2 - \vec{W}_1 . I\vec{W}_2 \\
= \vec{w}_1 . I\vec{w}_2 - \vec{w}_1 . b^{-1} \vec{w}_2. \tag{2.10}
\]

Motivated by the aforementioned concept, the Lagrange or Green strain and Euler or Almansi strain are defined respectively as:

\[
E = \frac{1}{2} (C - I), \tag{2.11}
\]

and

\[
e = \frac{1}{2} (I - b^{-1}). \tag{2.12}
\]

Thus, the differences of the squared norms can be written as:

\[
\frac{1}{2} (\vec{w}_1 . \vec{w}_2 - \vec{W}_1 . \vec{W}_2) = \vec{W}_1 . E\vec{W}_2 = \vec{w}_1 . e\vec{w}_2. \tag{2.13}
\]

2.2 Hyperelastic Materials

The material for which a Helmholtz free energy function or stored energy function (shortly called energy function) exists is called hyperelastic or Green elastic material (Ogden, 1984). For hyperelastic materials, the Helmholtz free energy function is of the form

\[
\Psi = \Psi(F(X, t), \theta(X, t), X), \tag{2.14}
\]

in which \(F(X, t)\) is the deformation gradient, \(\theta(X, t)\) is temperature and \(X\) is the position. However, for isothermal and homogeneous hyperelastic materials, the Helmholtz free energy function is

\[
\Psi = \Psi(F(X, t)). \tag{2.15}
\]
The Helmholtz free energy function can also be written as a function of other deformation measures:
\[ \Psi = \Psi(C(X, t)) = \bar{\Psi}(E(X, t)). \] (2.16)

The second Piola-Kirchhoff stress for hyperelastic materials is obtained from the Helmholtz free energy function as:
\[ S = 2 \frac{\partial \Psi}{\partial \bar{C}}. \] (2.17)

This measure of stress is completely in material form. However, pushing forward the second Piola-Kirchhoff stress leads to the Cauchy stress which is in the deformed configuration (Marsden and Hughes, 1994):
\[ \sigma = J^{-1} \chi_\ast [S] = J^{-1} FSF^T. \] (2.18)

Another measure of stress that is useful specially when comparing the stress results to experimental data is the first Piola-Kirchhoff stress which can be defined as the current force per undeformed area. This measure of stress can be obtained from second Piola-Kirchhoff stress or Cauchy stress by means of a Piola transformation:
\[ P = J \sigma F^{-T} = FS. \] (2.19)

Elasticity tensor is a fourth order tensor that can be obtained from the second Piola-Kirchhoff stress:
\[ C = 2 \frac{\partial S}{\partial \bar{C}}. \] (2.20)

In the case of hyperelastic materials in which the stress has been obtained from a Helmholtz free energy function, the elasticity tensor can be obtained from the energy function as well:
\[ C = 4 \frac{\partial^2 \psi}{\partial \bar{C}^2}. \] (2.21)

Likewise, this elasticity tensor is in material form. In order to obtain the elasticity tensor in the deformed configuration, a push-forward operation has to be performed (Marsden and Hughes, 1994):
\[ c = J^{-1} FF \bar{C} F^T F^T. \] (2.22)
2.3 Theory of Viscoelasticity

As can be observed from the previous section on hyperelasticity, the energy function and consequently the stress of a hyperelastic material does not depend on time or the history of the deformation. Unlike hyperelastic materials or more generally elastic materials, the response of the viscoelastic materials depends on the deformation history. The time dependent response of viscoelastic materials is usually characterized by hysteresis, stress relaxation and creep (Fig. 2.1). Hysteresis phenomena can be observed when a material is under loading and unloading process. The energy dissipated during this process determined the damping characteristics of the material. Stress relaxation is the decrease in the stress at constant strain, whilst creep is the increase in strain when stress is held constant. Several formulations and approaches have been used for describing the time dependent behavior of viscoelastic materials. These approaches can be categorized into three main groups which will be discussed in the following.

2.3.1 Integral Type Viscoelasticity

The integral type formulations are the most commonly used constitutive models for describing the time-dependent behavior of viscoelastic materials. The notion behind the integral type formulations is taking the summation of the history of the deformation of the material into account. In the following, a few of them will be shortly discussed. More detailed
descriptions can be found in Wineman (2009).

Green and Rivlin (1957) introduced one of the first viscoelastic models which was described by a series of integrals:

\[
S[E(t-s)] = \int_{-\infty}^{t} K_1(t-s_1) dE(s_1)
\]
\[
+ \int_{-\infty}^{t} \int_{-\infty}^{t} K_2(t-s_1, t-s_2) dE(s_1) dE(s_2)
\]
\[
+ \int_{-\infty}^{t} \int_{-\infty}^{t} \int_{-\infty}^{t} K_3(t-s_1, t-s_2, t-s_3) dE(s_1) dE(s_2) dE(s_3)
\]
\[
+ \cdots ,
\]

(2.23)

in which \(K_i\)'s are tensor-valued kernels. These kernels are describing the relaxation mechanism of the material. Having proposed the concept of fading memory, Coleman and Noll (1961) introduced a new formulation which is called finite linear viscoelasticity:

\[
S(t) = k[C(t)] + \int_{-\infty}^{t} K[C(t), t-s][C(s) - I] ds.
\]

(2.24)

As the aforementioned formula is linear, Pipkin and Rogers (1968) later suggested a nonlinear constitutive theory:

\[
S(t) = K[C(t)] + \int_{0}^{t} \frac{\partial}{\partial(t-s)} K[C(s), t-s] ds.
\]

(2.25)

When \(K[C, s]\) is separable, the quasi-linear viscoelastic (QLV) formula will be obtained which was proposed by Fung (1993):

\[
S(t) = K[C(t)] + \int_{0}^{t} K[C(s)] \frac{\partial G(t-s)}{\partial(t-s)} ds.
\]

(2.26)

The quasi-linear viscoelasticity theory is one of the most well-known constitutive models in the field of biomechanics widely used for describing the time dependent mechanical behavior of soft biological tissues (Mak, 1986b; Funk et al., 2000; Woo et al., 1980, 1981; Drapaca et al., 2006; Huyghe et al., 1991).
2.3.2 Differential Type Viscoelasticity

This theory can be applicable to viscoelastic and visco-hyperelastic materials; the latter uses a free energy function comparable to the definition of hyperelastic materials. We discuss this theory in the context of visco-hyperelasticity in this section to provide the foundation for the constitutive model introduced in this thesis.

Similar to the concept of hyperelasticity, a Helmholtz free energy function can be defined for the time-dependent materials. This energy function, in the case of strain-rate dependent (differential type) viscohyperelasticity, depends on the right Cauchy-Green deformation tensor as well as its time derivative for describing the rate dependence of the material. The energy function is normally decomposed additively into hyperelastic and viscous terms (Limbert and Middleton, 2004):

$$\Psi(C, \dot{C}) = \Psi^e(C) + \Psi^v(C, \dot{C}).$$

(2.27)

The total stress then can be derived as the summation of hyperelastic and viscous stresses from the free energy function:

$$S = S^e + S^v,$$

(2.28)

$$S = \frac{\partial \Psi}{\partial C} + \frac{\partial \Psi}{\partial \dot{C}}.$$  

(2.29)

This method has been used by Pioletti and Rakotomanana (2000) and Limbert and Middleton (2006) for describing the time dependent behavior of soft biological tissues for the case of isotropy and transversely isotropy respectively.

2.3.3 Theory of Viscoelasticity with Internal Variables

The Helmholtz free energy function in this theory depends on external and internal variables. The external variables describe the motion and deformation of the body at the current state of deformation, whereas the internal variables represent the history of the deformation governed by the dissipation mechanisms in the material (Holzapfel, 2000).
The Helmholtz free energy function based on \( n \) internal variables can be written as:

\[
\Psi = \Psi(C, \Xi_n),
\]

(2.30)

in which \( \Xi_n \) denotes the internal variable. The internal variables are obtained from the corresponding evolution equations. In general, an evolution equation can be written in the form of a differential equation (Reese and Govindjee, 1998):

\[
\dot{\Xi}_n = G(C, \Xi_1, \Xi_2, \cdots, \Xi_n),
\]

(2.31)

in which \( G \) is a tensor-valued function of internal and external variables.

2.4 Transversely Isotropy

Anisotropic materials in general possess random directional-dependent mechanical properties. A sub-class of anisotropic materials are transversely isotropic materials with one set of mechanical properties along one direction. In the plane normal to that direction, the mechanical properties are the same in all directions.

We assume \( n_0 \) to be the unit vector in the preferred direction in the undeformed or reference configuration. Furthermore, \( n \) is considered to be the unit vector of the preferred direction in the deformed configuration. The relationship between the unit vectors in the reference and current configuration is obtained as:

\[
F.n_0 = \lambda_n n,
\]

(2.32)

in which \( \lambda_n \) is the stretch ratio in the preferred direction and can be calculated as follows:

\[
F.n_0 = \tilde{n} \quad \Rightarrow \quad \lambda_n = \frac{\tilde{n}}{|\tilde{n}|}.
\]

(2.33)

For defining the Helmholtz free energy function for anisotropic hyperelastic or viscohyperelastic materials, the structure tensor should be introduced as well. The structure tensor
incorporates the directional-dependent mechanical properties into the Helmholtz free energy function. The structure tensors in the reference and current configurations are defined as:

$$N_0 = n_0 \otimes n_0,$$  \hspace{1cm} (2.34)

$$N = n \otimes n.$$  \hspace{1cm} (2.35)

Thus, the hyperelastic and viscous Helmholtz free energy functions in the case of anisotropy can be written with the aid of the structure tensor as:

$$\Psi^e = \Psi^e(C, N_0),$$  \hspace{1cm} (2.36)

$$\Psi^v = \Psi^v(C, \dot{C}, N_0).$$  \hspace{1cm} (2.37)

It should be noted that as the structure tensor in the energy function is defined in the reference configuration, the Helmholtz free energy function remains objective.

2.5 Invariant-based Formulations of Hyperelasticity

The Helmholtz free energy function is invariant with respect to the rotations of the reference configuration. Therefore, it can be expressed in terms of the invariants of the tensorial variables. In the case of isotropic materials, the Helmholtz free energy function is dependent on $C$ only. Thus, the three invariants of $C$:

$$I_1 = tr\{C\}, \quad I_2 = tr\{C^2\}, \quad I_3 = tr\{C^3\}$$  \hspace{1cm} (2.38)

suffice to form the energy function.

However, the structure tensor as a new tensorial variable has been incorporated to the energy function in anisotropic materials. As a result, further invariants should be considered to account for the structure tensor $N_0$. These new invariants or semi-invariants (Holzapfel, 2000) are as follows:

$$I_4 = n_0 \otimes C.n_0 = N_0 : C, \quad I_5 = n_0 \otimes C^2.n_0 = N_0 : C^2.$$  \hspace{1cm} (2.39)
The Helmholtz free energy function for anisotropic hyperelastic materials then can be written in terms of the aforementioned invariants:

\[ \Psi^e = \Psi^e(I_1, I_2, I_3, I_4, I_5) \] (2.40)

The second Piola-Kirchoff stress is obtained according to the equation 2.17 using the chain rule:

\[ \mathbf{S} = 2 \left[ \sum_{i=1}^{5} \left( \frac{\partial \Psi^e}{\partial I_i} \frac{\partial I_i}{\partial \mathbf{C}} \right) \right] \]. (2.41)

Similar to the hyperelastic energy function, the viscous function can also be expressed in terms of the invariants of the tensorial variables. The invariants for the integrity basis of \( \{ \mathbf{C}, \dot{\mathbf{C}}, \mathbf{N}_0 \} \) are defined as (Boehler, 1987):

\[ J_1 = 1 : \dot{\mathbf{C}}, \quad J_2 = \frac{1}{2} \left( 1 : \dot{\mathbf{C}}^2 \right), \quad J_3 = \text{det}(\dot{\mathbf{C}}), \] (2.42)

\[ J_4 = \mathbf{N}_0 : \dot{\mathbf{C}}, \quad J_5 = \mathbf{N}_0 : \dot{\mathbf{C}}^2, \] (2.43)

\[ J_6 = 1 : (\mathbf{C} \cdot \dot{\mathbf{C}}), \quad J_7 = 1 : (\mathbf{C} \cdot \dot{\mathbf{C}}^2), \]
\[ J_8 = 1 : (\mathbf{C}^2 \cdot \dot{\mathbf{C}}), \quad J_9 = 1 : (\mathbf{C}^2 \cdot \dot{\mathbf{C}}^2), \]
\[ J_{10} = 1 : (\mathbf{N}_0 \cdot \mathbf{C} \cdot \dot{\mathbf{C}}), \quad J_{11} = 1 : (\mathbf{N}_0 \cdot \mathbf{C} \cdot \dot{\mathbf{C}}^2), \]
\[ J_{12} = 1 : (\mathbf{N}_0 \cdot \mathbf{C}^2 \cdot \dot{\mathbf{C}}). \]

Finally, the viscous energy function and the viscous second Piola-Kirchhoff stress are obtained respectively as:

\[ \Psi^v = \Psi^v(J_1, J_2, J_3, ..., J_{12}), \] (2.44)

\[ \mathbf{S}^v = 2 \left[ \sum_{i=1}^{12} \left( \frac{\partial \Psi^v}{\partial J_i} \frac{\partial J_i}{\partial \mathbf{C}} \right) \right] \]. (2.45)

### 2.6 Volumetric-isochoric decomposition

Splitting the Helmholtz energy function into volumetric (dilatation) and isochoric (deviatoric, distortional or volume-preserving) parts is a common practice in treating the compressible
and nearly incompressible materials in continuum and computational mechanics (Lubliner, 1985; Simo and Taylor, 1991; Ogden, 1984; Lu and Pister, 1975). Multiplicative decomposition of the deformation gradient into volumetric and deviatoric parts is one approach that has been used for establishing the free energy function decomposition. The volumetric part of the deformation gradient is assumed to produce change of volume without causing change of shape, whereas the isochoric part leads to the change in shape while the volume remains constant. By defining the volumetric part of the deformation gradient as:

\[ \mathbf{F}_{\text{vol}} = J^{1/3} \mathbf{1}, \]

where \( \mathbf{1} \) is the identity tensor, the multiplicative split is obtained:

\[ \mathbf{F} = \mathbf{F}_{\text{vol}} \mathbf{F}_{\text{dev}}, \quad \mathbf{F}_{\text{dev}} = J^{-1/3} \mathbf{F}. \] (2.47)

In the equation above, \( \mathbf{F}_{\text{dev}} \) is the isochoric or deviatoric part of the deformation gradient. Similarly, the deviatoric or modified right Cauchy-Green deformation tensor is defined as:

\[ \bar{\mathbf{C}} = \mathbf{F}^T \mathbf{F} = J^{-2/3} \mathbf{C} \] (2.48)

The other isochoric tensorial variables can be defined in terms of the modified deformation gradient in an analogous manner.

The free energy function can also be expressed in terms of the new decomposed tensorial variables:

\[ \Psi(\mathbf{C}) = \bar{\Psi}(J, \bar{\mathbf{C}}). \] (2.49)

It is usually more convenient to assume that the volumetric and deviatoric parts are additively decoupled:

\[ \Psi(\mathbf{C}) = \Psi_{\text{vol}}(J) + \Psi_{\text{dev}}(\bar{\mathbf{C}}). \] (2.50)

The free energy function can also be written in terms of the invariants:

\[ \Psi(I_1, I_2, I_3) = \Psi_{\text{vol}}(I_3) + \Psi_{\text{dev}}(\bar{I}_1, \bar{I}_2) \] (2.51)
in which $I_i$’s are invariants of $\bar{C}$. This approach has been extended to nearly incompressible anisotropic materials and the decomposition has been applied on the other invariants as well (Holzapfel and Gasser, 2001; Limbert et al., 2003):

$$\Psi(I_1, I_2, I_3) = \Psi_{\text{vol}}(J) + \Psi_{\text{dev}}(\bar{I}_1, \bar{I}_2, \bar{I}_4, \bar{I}_5)$$ (2.52)

The aforementioned decomposition was also applied on anisotropic compressible materials (Zhurov et al., 2007). However, this approach for decomposition of the free energy of anisotropic materials has been challenged by Sansour (2008) and Guo et al. (2008) as it may lead to physically inaccurate results. Considering equation 2.52, if a spherical state of stress is applied on a body, the second term which only contains the deviatoric variables would be zero. This means that a hydrostatic state of deformation has been obtained which is not the case for anisotropic materials. Therefore, it is suggested that the directional-dependent invariants, i.e., $I_4$ and $I_5$ remains coupled with $J$ or $\sqrt{I_3}$. Consequently, the Helmholtz free energy for an anisotropic material is considered to be of the following form in this study:

$$\Psi(I_1, I_2, I_3) = \Psi_{\text{vol}}(J) + \Psi_{\text{dev}}(\bar{I}_1, \bar{I}_2) + \Psi_f(I_4, I_5)$$ (2.53)

It should be noted that the decomposition of directional dependent invariants is acceptable for nearly incompressible materials (Sansour, 2008).

2.6.1 Stress derivation

Following the volumetric and deviatoric split, the stress derivation procedure should be modified using the chain rule. Considering the equation 2.41 as a start point, the stress can be obtained as:

$$\mathbf{S} = 2 \left[ \sum_{i=1}^{3} \left( \frac{\partial \Psi^e}{\partial \bar{I}_i} \frac{\partial \bar{C}}{\partial \bar{C}} \right) \right].$$ (2.54)

The stress derivation procedure for the anisotropic part of the Helmholtz free energy function remains the same, because the corresponding invariants have not been modified.
For deriving the stresses, some preliminaries regarding the derivatives of scalar- and tensor-valued functions are required. The derivative of the determinant of the deformation gradient, \( J \), with respect to the right Cauchy-Green deformation tensor is obtained as:

\[
\frac{\partial J}{\partial C} = \frac{J}{2} C^{-1}.
\]  

(2.55)

Moreover, the derivative of the modified right Cauchy-Green deformation tensor with respect to the non-modified one is required for stress calculation:

\[
\frac{\partial \bar{C}}{\partial C} = J^{-2/3}(I - \frac{1}{3} C \otimes C^{-1}) = J^{-2/3}P^T,
\]  

(2.56)

in which \( P \) is the projection tensor (Holzapfel, 2000):

\[
P = I - \frac{1}{3} C^{-1} \otimes C,
\]  

(2.57)

and \( I \) is the fourth order identity tensor. This fourth order tensor is symmetric but not supersymmetric and is defined as follows (Itskov, 2007):

\[
I = 1 \otimes 1.
\]  

(2.58)
3.1 Structure and Function of Articular Cartilage

Articular cartilage is a soft connective tissue mostly found in diarthrodial joints. It is also called hyaline cartilage because of its glass-like appearance. The other two types of cartilages are elastic cartilage and fibrocartilage. Cartilage consists of a porous extracellular matrix saturated with a fluid. The extracellular matrix is composed of proteoglycans and collagen fibres. It also contains elastin fibres except for articular cartilage. The elastic cartilage possesses higher amounts of elastin fibres, whereas fibrocartilage contains more collagen fibres which are stiffer than elastin fibres. Articular cartilage consists of less collagen fibres and more proteoglycans as compared to fibrocartilage. Articular cartilage absorbs fluid into its porous matrix due to the negatively charged proteoglycans. The only cells in articular cartilage are chondrocytes which are surrounded by the extracellular matrix. Articular cartilage is avascular in nature, i.e., there is no blood supply in the tissue, which limits the cartilage capabilities in self-repairing when damaged. The mechanical loading that the cells are experiencing largely determines their activity in production or degradation of extracellular matrix (Grodzinsky et al., 2000). Articular cartilage is highly inhomogeneous and anisotropic. First, the mechanical properties vary along the tissue depth, often characterized by three distinct zones, superficial, middle and deep zones (Muehleman et al., 2004). The depth of the superficial zone is reported to be 10%-20% of the total thickness of the tissue; the collagen fibres are arranged tangentially to the articular surface in this zone. The middle zone usually occupies 40%-60% of the tissue thickness with randomly oriented collagen fibres. The deep zone is adjacent to the underlying bone, which makes up 20%-50% of cartilage thickness, with fibres being oriented perpendicular to the cartilage-bone interface (Mow and Huiskis,
Figure 3.1: A schematic picture of articular cartilage with its main constituents namely, cartilage cells (chondrocytes), collagen fibers and proteoglycans.

2005) (Fig. 3.1). The compressive modulus of articular cartilage is small in the superficial zone and increases through the depth (Schinagl et al., 1997). The percentage of collagen network, proteoglycans and water content are also variable with depth (Xia, 2008; Yin et al., 2011). Second, the site-specific tissue thickness and fibre orientation also contribute to tissue inhomogeneity and anisotropy. The fibres are oriented according to the split-line pattern throughout the joint (Below et al., 2002). Articular cartilage can be divided into high-weight and low-weight bearing regions (Walker and Hajek, 1972) with variable thicknesses in the regions (Cohen et al., 1999). Although the fibres are parallel to the surface in all regions, the concentration and organization of them along the depth are different in high-weight and low-weight bearing regions (Gomez et al., 2000).

Presence of negatively charged groups on proteoglycans also contributes to the mechanical properties of articular cartilage. The fixed charge density (FCD) attracts the interstitial fluid with positive mobile ions into the tissue (Wan et al., 2004). The ion concentration results in a swelling pressure which is sustained by the collagen fibres in tension (Maroudas, 1976). This effect causes a state of pre-stress in the tissue even when not externally loaded. The variation of FCD with depth adds to the inhomogeneity of articular cartilage which affects the swelling,
tensile and compressive response of the tissue (Korhonen and Jurvelin, 2010; Wan et al., 2010; Che). The mechanical behaviour of the tissue has been observed to be nonlinear in both compression and tension. The properties of the collagen fibres predominantly determine the nonlinear response of the tissue in tension (Pins et al., 1997; Charlebois et al., 2004; Woo et al., 1976). The fibre nonlinearity in tension will in turn cause the nonlinear compressive response of the tissue in the direction perpendicular to the fibre orientation. In other words, when articular cartilage is compressed in the thickness direction, the collagen fibres tangential to the articular surface will resist the lateral expansion associated with the Poisson’s effect, resulting in nonlinear compressive response because of the nonlinear resistance from the fibres. A fast compression would enhance the compressive nonlinearity when the resulting higher fluid pressure escalates the lateral expansion (Li and Herzog, 2004a). Furthermore, the dependence of the permeability on volume strain is another source of nonlinearity in the tissue (Lai et al., 1981; Mansour and Mow, 1976).

Articular cartilage covers the ends of bones in diarthrodial joints. It acts as a load-bearing tissue while facilitating the movements of the joints by providing lubrication. The load bearing mechanism of articular cartilage is determined by all the constituents of the tissue, as well as the interplay between them (Oloyede and Broom, 1993; Wayne, 1995; Mukherjee and Wayne, 1998). Articular cartilage is known to be viscoelastic in both compression and tension. This property is reflected in the stress relaxation and creep behaviour of the tissue. The compressive viscoelasticity of cartilage arises from the fluid pressurization and stretch of collagen fibres. An instantaneous compression of articular cartilage is largely sustained by fluid pressurization with less compressive stress in the solid matrix. The fluid pressure is in turn sustained by the tensile resistance of the collagen fibres, which are of high tensile stiffness. The fluid then starts to diffuse out of the tissue and therefore, the compressive stress in the solid matrix increases. This pressurization mechanism provides sufficiently high compressive stiffness at fast knee compression, and low stiffness to facilitate joint motion (Li
3.2 Structure and Function of Ligament

Ligaments are collagenous tissues found in different joints in the human body that connect the bones to bones. Ligament has a hierarchical structure that starts with collagens at small...
Figure 3.3: There are four major ligaments in human knee joint. The anterior cruciate ligament and posterior cruciate ligament are in the joint capsule. However, the medial collateral ligament and lateral collateral ligament are outside the joint capsule (adopted from Makris et al. (2011) with permission).

scale and moving upward to form fibrils, fibers and fascicles (Kastelic et al., 1978) (Fig. 3.2). Elastic fibers are also present in the tissue in a smaller quantity and 1% of the dry weight is associated with a non-fibrillar or ground matrix. 60% to 80% of the wet weight of ligaments is occupied by water while the collagen fibers (mainly type I) make up to 70% of the dry weight of the tissue (Frank, 2004). The alignment of the fibers primarily in one direction gives the tissue strong transversely isotropic characteristics. The fibers resist tensile loadings that provide the main function of ligaments. The tensile behavior of ligaments is viscoelastic arising from the hierarchical structure of fibers and their interaction with the ground matrix. Although ligaments are quite hydrated, the fluid-driven viscoelasticity is not likely to be the
dominant source of this time-dependent behavior in tension.

There are four major types of ligaments present in the human knee joint, namely, anterior cruciate ligament (ACL), posterior cruciate ligament (PCL), medial collateral ligament (MCL) and lateral collateral ligament (LCL). ACL and PCL are located inside the synovial capsule and the two others are outside the capsule. ACL is connected from one side to the posterior side of the lateral femoral condyle and from the other side to the anterior intercondylar fossa of the tibia. PCL connects the anterior side of intercondylar notch to the posterior intercondylar fossa of the tibia. The MCL and LCL connect the bones on the medial and lateral side of the joints respectively (Gray and Lewis, 1918) (Fig. 3.2).

The ligaments in the knee joint are the ones more prone to injury. In addition, injured ligaments do not recover soon (Frank et al., 1983) and may impose extra loading to be sustained by other tissues in the joint. Specifically, osteoarthritis has been observed in patients with previous injury in their ligaments (Lohmander et al., 2007; Fleming et al., 2005). The injury to these tissues can occur as a result of extreme strain. These high strains are usually applied at high strain-rates during impact in sports and other incidents. The viscoelastic nature of the tissue provides higher stiffness at higher strain-rates that enable the tissue to tolerate more loading in these situations.
Constitutive Models for Articular Cartilage and Ligament

4.1 Constitutive Models for Articular Cartilage

From a mechanical point of view, articular cartilage can be considered as a multiphasic fluid-saturated fibre-reinforced composite, which is highly inhomogeneous and anisotropic with nonlinear and viscoelastic properties. In the last two decades, many attempts have been made to take the complexities of the tissue into account in a single model. It should be noted that it is virtually impossible to model the exact characteristics of the tissue. A degree of simplification has to be made in any model to avoid some formulation and numerical complexities. The early studies on cartilage mechanics were based on the theory of linear elasticity (Hayes et al., 1972). Obviously, these models were not able to predict the time-dependent response of the tissue. In the meanwhile, viscoelastic monophasic models were also introduced (Hayes and Mockros, 1971). These phenomenological models describe the time-dependent stress and strain of the tissue without explicit consideration of the interstitial fluid flow. The most commonly used theory for biological tissues is the quasi-linear viscoelastic theory (Fung, 1993), often referred to as QLV theory, in which the viscoelastic stress is determined by a hereditary integral

\[ \sigma(t) = \int_{-\infty}^{t} G(t - \tau) \dot{\epsilon} d\tau \]  

(4.1)

where \( \sigma \) is the elastic stress at equilibrium. The integral accounts for the effect of the history of the deformation, which weakens with time at a rate determined by the reduced relaxation function, \( G(t) \).

The biphasic and fibril-reinforced models were later introduced for cartilage which brought about certain success in predicting the time-dependent mechanical response of the tissue.
These models will be discussed in the following subsections.

4.1.1 Biphasic and Triphasic Models of Articular Cartilage

As the time-dependence and the viscoelastic nature of the tissue were further experimentally investigated, the need for implementing the fluid phase became necessary. The poroelasticity was first used in bone mechanics four decades ago (Nowinski and Davis, 1970; Nowinski, 1971, 1972). The theory was first implemented for articular cartilage a few years later (Higginson et al., 1976). The original biphasic theory proposed particularly for articular cartilage was introduced soon after that (Mow et al., 1980). In its first version, the solid matrix of cartilage was linearly elastic, but later extended to account for large deformation as well (Kwan et al., 1990). The total stress in a biphasic model, $\sigma$, is the sum of the stress in the fluid, $\sigma^f$, and the stress in the solid matrix, $\sigma^m$:

$$\sigma^f = -\phi^f p 1,$$

$$\sigma^s = -\phi^s p 1 + \lambda tr(E) 1 + 2\mu E,$$

$$\sigma = \sigma^s + \sigma^f.$$  \hspace{1cm} (4.2)

where $p$ is the fluid pressure, or pore pressure. $\phi^f$ and $\phi^s$ are the volume fractions of the fluid and solid respectively. $\lambda$ and $\mu$ are Lam constants of the solid, and $E$ is the Green strain tensor. Equation 4.2 may be easier to interpret using the concept of effective stress:

$$\sigma^{eff} = \lambda tr(E) 1 + 2\mu E,$$

$$\sigma = -p 1 + \sigma^{eff}.$$  \hspace{1cm} (4.3)

The effective stress is used in the commercial finite element software ABAQUS (Simulia, Providence, RI, United States), which is widely used to simulate the mechanical response of biological tissues.

The solid-fluid interaction is described by Darcy’s law

$$\phi^f (v^f_i - v^s_i) = K_{ij} \frac{\partial p}{\partial x_j}$$  \hspace{1cm} (4.4)
in which \( v_{fi} \) and \( v_{si} \) are velocities of fluid and solid part in \( x_i \) direction respectively and \( K_{ij} \) is the orthotropic hydraulic permeability.

It is believed that the solid matrix of articular cartilage is also viscoelastic (Hayes and Bodine, 1978). The biphasic theory can be extended to account for the inherent viscoelasticity of the tissue to form poroviscoelastic models, when the effective stress is replaced by a hereditary integral such as equation 4.1.

The equilibrium compressive response of articular cartilage is mainly supported by the proteoglycans rather than the collagen fibres. The compressive modulus of the proteoglycan-depleted cartilage matrix was found to be approximately 2% of the normal tissue modulus in unconfined compression (Canal Guterl et al., 2010). This observation, however, did not mean that the PG matrix supports 98% of the equilibrium loading in normal cartilage. The fibres in the normal tissue restrain the lateral expansion, and thus contribute to the compressive load support in the perpendicular direction. This fibril-reinforcement, however, is minimized at equilibrium (Li et al., 1999). Therefore, the compressive load response at equilibrium should be predominantly supported by the PG matrix. Quantitatively, two thirds of the equilibrium modulus contributed by PG molecules arose from the electrostatic effects (Canal Guterl et al., 2010). The electrostatic contribution is caused by FCD which induces the Donnan osmotic pressure. The electric charges are explicitly accounted for in the triphasic model (Lai et al., 1991):

\[
\sigma = -(p + T_c)1 + \lambda tr(E)1 + 2\mu E,
\]
\[
\sigma^f = \sigma^w + \sigma^+ + \sigma^-.
\]  

(4.5)

in which \( T_c \) is called the chemical-expansion stress resulting from the repulsive forces of the negative charges on proteoglycans in the solid matrix. \( \sigma^w \) is the hydraulic pressure, and \( \sigma^+ \) and \( \sigma^- \) are resulted from the electrolytes in the interstitium. Using the effective stress, the total stress is

\[
\sigma = -(p + T_c)1 + \sigma^{\text{eff}}
\]  

(4.6)
4.1.2 Fiber-reinforced Models for Articular Cartilage

Another significant characteristic of articular cartilage is its anisotropy arising from the fibre-network in the tissue. The fibres alone are not able to sustain compression due to their slenderness, but are strong and highly nonlinear in tension. From this perspective, articular cartilage may be best modelled as a fibril-reinforced fluid-saturated composite, which somewhat mimics the microstructure of the tissue. The effective stress in the solid matrix can then be considered as the sum of the stresses in the nonfibrillar and fibrillar matrices. Therefore, the constitutive equations can be obtained after the effective stress is modified

\[
\sigma^{\text{eff}} = \lambda tr(E) \mathbf{1} + 2\mu E + \sigma^f
\]  

(4.7)

The fibrillar stress is neglected if it is in compression in the fibre direction. For simplicity, the shear stresses in the fibrillar matrix may also be ignored; the shear is assumed to be sustained by the nonfibrillar matrix. Thus, the only nonzero components in \( \sigma^f \) are the tensile stresses. Both linear (Soulhat et al., 1999) and nonlinear (Li et al., 1999) were proposed for the collagen fibers. The nonlinear representation of collagen fibers were determined to be a key factor in predicting the high fluid pressurization and the highly nonlinear load response of the tissue.

Collagen fibers are also viscoelastic. In order to account for collagen viscoelasticity, a QLV type integral for the determination of the tensile stress in the fibrillar matrix can be used (Li and Herzog, 2004b). In another fibril-reinforced modelling, collagen viscoelasticity is introduced with a spring-dashpot system (Wilson et al., 2004).

The orientation of fibres in articular cartilage may also influence the fluid flow and pressurization in the tissue. The fibre orientation is three dimensional, although it was taken to be axisymmetric in the simulation of unconfined compression testing in order to simplify the computation (Soulhat et al., 1999; Li et al., 2000). The arcade model is another mathematical simplification of the actual fibre structure (Benninghoff, 1925), which was implemented into
a fibril-reinforced model (Wilson et al., 2004). There is no major difficulty to implement the variation of fibre orientations by rotating the local axis. In the case of continuous variation of fibre direction, a statistical function can be introduced to describe the fibre orientation, as it was first introduced for skin and other fibrous tissues (Lanir, 1979, 1983). In doing this, a probability density function or angular distribution function, \( R(\theta, \phi) \), is used to represent the possibility of fibre alignment in a given direction, specified by two angles \( \theta \) and \( \phi \) in the spherical coordinate system. The stresses can then be written in terms of this probability function as:

\[
\sigma = \int_{\Omega} R(\theta, \phi) \bar{\sigma}(n(\theta, \phi)) \ d\Omega \tag{4.8}
\]

in which \( n \) is the unit vector in the direction of fibres. This approach has been used for other soft biological tissues (Hurschler et al., 1997; Gasser et al., 2006) and later adopted in the biphasic models of articular cartilage as well (Ateshian, 2009; Lei and Szeri, 2006). Swelling can also be incorporated into the fibril-reinforced models by adding the osmotic pressure difference \( \Delta \pi \) and the stress due to chemical expansion (Wilson et al., 2005):

\[
\sigma = \sigma^s_m + \sigma^s_f - (\Delta \pi + \mu_f + T_c) \mathbf{1} \tag{4.9}
\]

where \( \mu_f \) is the chemical potential. \( \sigma^s_m \) must be considered as the effective stress of the nonfibrillar matrix. The difference of the internal and external osmotic pressures, \( \Delta \pi \), is also referred to as Donnan swelling pressure gradient. The osmotic pressure and chemical expansion can be calculated based on the ion concentrations (Huyghe and Janssen, 1997).

4.2 Constitutive models for ligament

Most of the constitutive models for ligaments fall within two general groups of elastic and viscoelastic models. Although ligament includes a considerable amount of water, most models have assumed the tissue to be monophasic. Apparently, the elastic models are only able to predict the equilibrium response of the tissue or the instantaneous response upon using the
instantaneous modulus in the formulation. However, viscoelastic models are able to account for the time-dependent response of the tissue as well.

4.2.1 Elastic models

The elastic models of ligament focused on capturing the nonlinear tensile behavior of the tissue arising from the stiffening of the fibers with stretch. This stiffening is postulated to be caused by gradual recruitment of crimped fibers in tensile loading. This recruitment was first described structurally by employing systems of springs and dashpots forming a rheological model (Viidik, 1968; Frisn et al., 1969). This microstructure was also described by using segments connected rigidly making a zigzag pattern (Diamant et al., 1972; Stouffer et al., 1985), or sinusoidal crimping pattern (Comninou and Yannas, 1976). Other models introduced a specified stretch ratio in which the fibers become uncrimped and active, therefore, their stiffness increases (Decraemer et al., 1980). In other studies, the total response of the tissue was divided into two linear portions with different stiffnesses before and after the uncrimping (Kwan and Woo, 1989). The nonlinear behavior can also be described phenomenologically by using an exponential function for the stress-stretch relationship (Fung, 1967).

All the aforementioned models were for the one-dimensional case. The continuum-based models can bring the advantage of describing the behavior of the tissue in 3D with more predictive capability. They also have the advantage of being implemented in finite element procedures to investigate more complicated geometries and boundary conditions. The composite materials mechanics theory with linear fiber reinforcement was used in some studies (Ault and Hoffman, 1992a,b). Statistical fiber distribution in the tissue were also considered later (Hurschler et al., 1997), similar to the work of Lanir (1983). Incompressible transversely isotropic models were also introduced and implemented in finite element packages (Weiss et al., 1996).

The recruitment of fibers through the cross-sectional area of the tissue were also con-
sidered (Thornton et al., 1997, 2001). It was suggested that this consideration enables the model to capture both the creep and relaxation response of ligaments. Multiscale methods were also used more recently in describing the behavior of collagenous tissues (Maceri et al., 2010). The multiscale models can start at nanoscale and scale up through micro to macroscale. The aforementioned model was found capable of predicting the experimental data very well.

4.2.2 Viscoelastic models

Various approaches have been used for modeling the viscoelasticity of ligaments. Integral-type formulations including the quasi-linear viscoelasticity theory (Fung, 1993) and other single integral models (Johnson et al., 1996) are used which predict both the transient and relaxation behavior of the tissue. Among integral type models, QLV theory is the most common approach to modeling viscoelasticity. In this theory, the time dependent response of the material is decoupled from the elastic response. The elastic response can be nonlinear, however, the time-dependent response does not change at different strain levels. The inaccuracy of the model for ligaments at larger strains is widely known (DeFrata and Li, 2007) and therefore, it was suggested that fully nonlinear viscoelastic models are required for ligaments (Provenzano et al., 2002; Hingorani et al., 2004).

On the other hand, other types of viscoelasticity formulations classified as differential type models were also used for ligaments. The time-dependent response is described by a function with dependence on a measure of strain-rate which is able to account for the viscoelastic response during loading. This formulation of viscoelasticity was discussed in section 2.3.2. While these models were shown to be successful in describing the viscoelastic response of ligament specially at high strain-rates, they fail to account for the stress relaxation. This class of models was initially introduced for the isotropic case (Pioletti et al., 1998; Pioletti and Rakotomanana, 2000). However, they were extended to be transversely isotropic by including the collagen fibers later (Limbert and Taylor, 2002; Limbert and Middleton, 2004;
Zhurov et al., 2007).
Mechanical experiments help gain an understanding of the behavior of materials. They further can be used for material characterization and determining the material parameters in the constitutive model suggested for the material under consideration. The choice of the mechanical experiments depends on the specific mechanical properties aimed to be studied, for instance, tensile, compressive or shear modulus. Articular cartilage shares some similarities with soils. Therefore, some experiments previously used for soil characterization have been adopted for articular cartilage as well. In the following, the most important types of experiments used for articular cartilage and ligament will be briefly discussed.

5.1 Unconfined Compression

The unconfined compression test is normally done on cylindrical specimens of cartilage extracted from the joint (Fig. 5.1 a). The specimen is placed in a chamber filled with PBS (Phosphate Buffer Saline) solution. The uniaxial compression is applied in the cylinder axis direction while the specimen is free to expand laterally. The compressive force is sustained by the solid matrix as well as the fluid which is trapped in the tissue due to the drag force. After the fluid drainage, all the load will be tolerated by the solid matrix. Due to the lateral expansion, the fibers are in tension and the proteoglycans sustain the compressive forces in the axial direction. The Poisson’s ratio and compressive elastic modulus as well as the permeability of the tissue can be obtained from this test.
Figure 5.1: Standard mechanical tests that are commonly used for characterization of articular cartilage. Cartilage specimens are kept in a PBS or saline solution during the test to stay hydrated.

5.2 Confined Compression

In the confined compression geometry, the cylindrical specimen is placed in a confining chamber constraining it from any expansion and fluid exudation from the sides (Fig. 5.1 b). On the top or the bottom, a porous filter is usually used that allows the out-flow of the interstitial fluid. In this testing configuration, the fluid flow is almost one-dimensional. Therefore, some material properties can be calculated analytically considering cartilage as an isotropic homogeneous material. The hydraulic permeability as well as the aggregate modulus can be obtained from this test (Mow and Guo, 2002). The aggregate modulus is a term used in the realm of cartilage mechanics which is also called P-wave modulus in solid mechanics. It can be defined in terms of the elastic modulus, $E$, and Poisson’s ratio, $\nu$, as
follows:

\[ H_A = \frac{E(1 - \nu)}{(1 + \nu)(1 - 2\nu)}. \]  

(5.1)

Due to the constraint on the lateral expansion of the tissue, the collagen fiber network does not play a major role in confined compression test.

5.3 Indentation

The indentation test is another type of mechanical test in which the surface of cartilage is indented by a spherical or cylindrical indenter (Fig. 5.1 c). The stress fields produced in this case are more complex and numerical calculations are required in order to obtain the material properties. However, this test makes in-situ and even in-vivo testing possible. The Poisson’s ratio, aggregate modulus and permeability can be derived from the results of this test.

5.4 Tension

The collagen fibers in articular cartilage can only sustain tension. Therefore, their mechanical behavior can be explicitly revealed in tensile tests. Although collagen fibers are also active during unconfined compression, the compressive modulus is obtained by the contribution of the non-fibrillar matrix and the fiber network together. However, the tensile test can reveal the tensile stiffness of the fibrillar network if done in the direction of collagen fibers. The mechanical behavior of ligament can also be examined in tensile testing as this tissue mostly experiences tensile loads in-vivo.
Chapter 6

Methods

This chapter describes the methods used in the constitutive modeling, numerical implementation of the model into the finite element software package ABAQUS, finite element models and characterization of the material parameters as well as the methods used for the tensile tests of articular cartilage. The first section includes the description of the viscoelastic constitutive model developed in this study. The constitutive model is further particularized for articular cartilage and ligament discussed in Sections 6.2 and 6.3. The numerical implementation procedure and the finite element models used in this study are described in details in Section 6.4. The material parameters in the constitutive models need to be characterized with the experimental data as well. The curve fitting process used for this purpose is described in Section 6.5. Finally, the experimental methods for the tensile tests of articular cartilage including the material preparation process and the loading protocol are explained in Section 6.6.

6.1 Constitutive Modeling

As discussed in section 2.3 on the theories of viscoelasticity, the approaches that have been used for modeling viscoelastic materials fall into three groups. Integral type formulations and quasi-linear viscoelasticity in particular have been widely used for describing the time dependent response of soft biological tissues. Nevertheless, the integral type formulations have been developed for step strain conditions and mostly describe the relaxation of the material (Fung, 1993). Therefore, different methods were later introduced for characterizing the QLV formulation under finite ramp times (Abramowitch and Woo, 2004). In addition, it has been reported that integral type equations do not provide good results at high strain
rates (Pioletti et al., 1998; Limbert and Middleton, 2004; Roan and Vemaganti, 2011). These authors, alternatively, used constitutive models with an explicit dependence on the time derivative of the right Cauchy-Green deformation tensor also known as differential type models, as discussed in section 2.3.2. Unlike the QLV theory, these equations enjoy more sensitivity to strain rate and were successful in capturing the time dependent response of ligaments under higher loading rates. This sensitivity to strain rate makes these models favorable for some applications. Care should be taken in using this constitutive equation where the viscoelastic response of the material is not sharply dependent on the strain rate, unless they are appropriately modified. In addition, these models fail to predict the stress relaxation response and consequently, they are only suitable for materials with “short term memories” (Roan and Vemaganti, 2011). On the other hand, the other approach is based on the concept of internal variables which was discussed in section 2.3.3. This approach originated from thermodynamics with internal variables and is the generalization to the theory of viscoelasticity. Furthermore, it assigns a free energy function to the material that brings thermodynamics consistency to the theory as well. The internal variables represent different internal and dissipating mechanisms in the material. Several internal variables are allowed to be employed when different dissipation mechanisms are present in the material. However, solving the evolution equations for internal variables finally leads to an integral-based formulation suffering from the same limitations as mentioned above, for instance, for the QLV theory. Considering the evolution equation to be a linear differential equation (Holzapfel, 2000):

\[
\dot{\Upsilon}_n + \frac{\Upsilon_n}{\tau_n} = g_n \dot{S}^e,
\]

(6.1)

the non-equilibrium or viscous stress, \(\Upsilon\), will be obtained as:

\[
\Upsilon(t) = g_n \int_{-\infty}^{t} e^{-(t-s)/\tau_n} \dot{S}^e ds.
\]

(6.2)

\(g_n\) represents the relative stiffness of the viscous to elastic body and \(\tau_n\) is the time constant. Apparently, the stress response is similar to the quasi-linear viscoelasticity as the time con-
stant, \( \tau_n \), in the evolution equation has been considered to be independent of the strain. By considering the time constant to be a function of strain, a fully nonlinear integral-based formulation for viscous response can be obtained (Peña et al., 2008a).

**General formulation of the constitutive model:**

Taking into account the advantages and drawbacks associated with the aforementioned approaches, a new constitutive model for viscoelastic materials will be developed based on the short-term and long-term internal variables. Employing such a concept in the constitutive model ensures that the rate sensitivity of the materials under loading (if applicable) is captured while the stress relaxation response is also predicted. This model can be applied on anisotropic polymers, composites and soft biological tissues. As discussed in sections 2.2, 2.3.2 and 2.3.3, a Helmoltz free energy function can be defined for a material based on a set of tensorial variables that describe the necessary thermomechanical characteristics of the specified material. We assume that the free energy function is of the form:

\[
\Psi = \Psi(C, \Gamma, N_0),
\]

(6.3)

in which \( \Gamma \) is the internal variable, \( C \) is the right Cauchy-Green deformation tensor and \( N \) is the structure tensor previously introduced (section 2.1 and 2.4). The internal variable spans over the entire time scale in which the deformation is in progress. However, we introduce short-term and long-term internal variables that account for the viscous responses according to their time scales. The short-term internal variable can be chosen to be the time derivative of the right Cauchy-Green deformation tensor, \( \dot{C} \). This variable characterizes the viscous response during the loading process which also can be identified as the transient response. The long-term internal variable, \( \Xi \), however, determines the viscoelastic behavior of the material at larger time scales, for instance stress relaxation. Therefore, the Helmholtz free energy function can be written in terms of the short-term and long-term internal variables:

\[
\Psi = \Psi(C, \dot{C}, \Xi, N_0)
\]

(6.4)
It is further assumed that the hyperelastic and viscous functions are decoupled following the previous studies (Holzapfel, 2000; Pioletti et al., 1998; Limbert and Middleton, 2004):

$$\Psi(C, \dot{C}, \Xi, N_0) = \Psi^e(C, N_0) + \Psi^v_s(C, \dot{C}, N_0) + \Psi^v_l(C, \Xi, N_0).$$ (6.5)

The superscript $e$ and $v$ shows the elastic and viscous functions respectively, whereas subscript $s$ represents the short term viscous response and subscript $l$ is used for the long term viscous function.

Describing the stress relaxation response of viscoelastic materials using only one time constant may not be always possible. Therefore, several internal variables associated with each time constant can be used together to account for the full relaxation response. The relaxation behavior is determined by a combination of different mechanisms represented by the corresponding internal variables. The effects of all long-term mechanisms are present during the relaxation phase. During this process, each mechanism contributes to the long-term response based on its time dependent weight. The weight of each mechanism is determined by the time and proportionality constants associated with each dissipation process. The stress relaxation response can then be described by a set of evolutionary mechanisms defined in strain or stress space. It should be noted that the short-term and long-term internal variables are assumed not to overlap. The transient viscoelastic response during the loading process will be obtained solely by the short-term internal variable. The time derivative of the right Cauchy-Green deformation tensor as a measure of strain-rate is denoted as an internal variable as it includes the history of the deformation although very small. This situation is the case especially at high rates of loading. Introducing the long term integral variables requires additional constitutive equations also known as evolution equations from which the long-term viscous stress or strain response can be obtained (Holzapfel and Gasser, 2001; Reese and Govindjee, 1998). Here, we aim at deriving an evolution equation that results in a decay of the short-term viscous response that finally reaches the elastic or equilibrium stress. For this purpose, the decomposition of the deformation gradient into its elastic and
viscous components should be considered:

$$F = F^e.F^v,$$  \hspace{1cm} (6.6)

from which the viscous right Cauchy-Green deformation tensor is obtained: $C^v = F^{vT}.F^v$.

The multiplicative decomposition of the deformation gradient in this manner was introduced and used in other studies as well (Reese and Govindjee, 1998; Huber and Tsakmakis, 2000).

The evolution equation first can be described in the strain space:

$$\dot{C}^v + \frac{C^v}{\tau} = \dot{C}$$  \hspace{1cm} (6.7)

and then multiplied by the viscous stiffness, $C^v$, to obtain the viscous response in the stress form:

$$\dot{S}^v_l + \frac{S^v_l}{\tau} = S^v_s.$$  \hspace{1cm} (6.8)

This equation differs from other linear evolution equations (Reese and Govindjee, 1998) (equation 6.1) in a sense that the stiffness of the viscous body is not proportional to the stiffness of the elastic body (Gasser and Forsell, 2011). This form of evolution equation also provides the ground for incorporating the short- and long-term internal variables together.

Upon solving this equation, the long-term viscous stress of the material is obtained as:

$$S^v_l(t) = \int_{\delta}^{t} e^{-(t-T)/\tau} S^v_s dT,$$  \hspace{1cm} (6.9)

where $\delta$ is the time in which the loading is applied. From $t = 0$ to $t = \delta$, the viscous response is only determined by $S^v_s$, while the integral-based formulation (equation 6.9) express the relaxation behavior of the material afterwards. In summary, the stress response of the material can be written as the summation of the hyperelastic, short-term and long-term viscous stresses:

$$S(t) = S^e(t) + S^v_s(t) + \sum_{i=1}^{N} g_i \int_{\delta}^{t} e^{-(t-T)/\tau_i} S^v_s dT$$  \hspace{1cm} (6.10)

The long-term response is constructed from multiple mechanisms with different time constants ($\tau_i$) that contribute to the viscous response according to their weights ($g_i$) at time $t$. 

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The time constant $\tau$ in the evolution equation (equation 6.8) has been considered to be independent of strain. Therefore, a quasi-linear form of viscoelasticity has been obtained. The evolution equation can be further extended to accommodate strain-dependent time constants that leads to a fully nonlinear description of viscoelasticity (Peña et al., 2008b).

6.2 Applications to Ligament

Ligament is a soft tissue mostly consisting of collagen fibers aligned in the longitudinal direction (section 3.2). Although the tissue is well hydrated, it has been modeled as an incompressible monophasic material in the literature as discussed in section 4.2. The free energy function developed in section 6.1 will be particularized here for ligament. Ligament will be considered as an incompressible transversely isotropic viscoelastic material. The ground matrix (non-fibrillar matrix) is considered to be hyperelastic that sustains both compression and tension while the fibers are modeled as viscoelastic that sustain tension only. Assuming that the hyperelastic and viscous parts as well as the ground matrix and fibers contributions are additively decoupled, the energy function for ligament can be written as follows:

$$\Psi(C, \dot{C}, \Xi, N_0) = \Psi^e_m(C) + \Psi^v_f(C, N_0) + \Psi^v_s(C, \dot{C}, N_0) + \Psi^v_l(C, \Xi, N_0).$$

In the above equation, the subscripts $m$ and $f$ represent the matrix and fibers respectively. The energy function can be also expressed in terms of the invariants of the corresponding tensorial variables:

$$\Psi(I_n, J_n, \Xi) = \Psi^e_m(I_1, I_2, I_3) + \Psi^v_f(I_4, I_5) + \Psi^v_s(I_4, I_5, J_4, ..., J_{12})$$
$$+ \Psi^v_l(I_4, I_5, J_4, ..., J_{12}, \Xi).$$

(6.12)

The different parts of the energy function can now be particularized for ligament. The isotropic part representing the ground matrix is considered to be Neo-Hookean:

$$\Psi^e_m = a_1(I_1 - 3).$$

(6.13)
An exponential function that can account for the fiber stiffening with stretch is adopted for the elastic parts of the fibers (Limbert and Taylor, 2002):

$$\Psi_f^e = \frac{a_2}{2a_3} e^{a_3(I_4-1)^2}.$$  \hspace{1cm} (6.14)

The tension-compression nonlinearity in the tissue requires the fibers to be active only under tension. This has been considered in the model by employing a piecewise function that determines whether the collagen fibers contribute to the overall response based on their stretch state:

$$\Psi^e = \begin{cases} 
    a_1(I_1 - 3) & \text{if } I_4 \leq 1 \\
    a_1(I_1 - 3) + \frac{a_2}{2a_3} e^{a_3(I_4-1)^2} & \text{if } I_4 > 1 
\end{cases} \hspace{1cm} (6.15)$$

The invariant $I_4$ represents the squared stretch ratio of fibers. The stretch ratio greater than 1 indicates that the fibers are in tension.

For the viscous part representing the short-term time-dependent response of the material, the following energy function is proposed:

$$\Psi^v_s = \begin{cases} 
    0 & \text{if } I_4 \leq 1 \\
    a_4(I_4 - 1)e^{a_5(I_4-1)^2}J_5 & \text{if } I_4 > 1 
\end{cases} \hspace{1cm} (6.16)$$

as can be observed, the ground matrix is hyperelastic and therefore, the viscous response under compression ($I_4 \leq 1$) is zero.

6.2.1 Stress Derivation

The hyperelastic second Piola-Kirchhoff stress can be obtained from the Helmholtz free energy introduced for elastic part of the ground matrix and fibers (equation 2.41). As mentioned previously, the ligament is modeled as an incompressible material. In this section, the strict incompressibility is enforced by introducing a Lagrange multiplier, $p$, into the equation which applies this constraint. Therefore, the stresses of the ground matrix and fibers are obtained as follow respectively:

$$S^e_m = pC^{-1} + 2a_1I,$$  \hspace{1cm} (6.17)
\[ S_f^v = 2a_2 e^{a_3(I_4-1)^2} (I_4 - 1)N_0. \] (6.18)

The short-term viscous second Piola-Kirchhoff stress of the fibers is also derived from the Helmholtz free energy function according to equation 2.45:

\[ S_s^v = a_4 (I_4 - 1) e^{a_5(I_4-1)^2} (N_0 \dot{C} + \dot{C}N_0). \] (6.19)

The long term viscous stress is obtained from a hereditary integral which includes the short-term viscous stress:

\[ S_l^v = \int_0^t e^{-(t-T)/\tau} S_s^v dT. \] (6.20)

The total long term response of the material, however, is characterized by a combination of dissipating mechanisms that play a role in the process according to their time constants. If we consider the aforementioned integral represents a dissipation mechanism, and \( g \) is the weight by which it contributes to the response, the overall long term viscous stress can be obtained as the summation of the dissipative mechanisms:

\[ S_l^v = \sum_{i=1}^{N} g_i \int_0^t e^{-(t-T)/\tau_i} S_s^v dT. \] (6.21)

Normally, using three terms (N=3 in the equation above) has been shown to be sufficient to capture the long-term response of biological materials (Suh and Disilvestro, 1999; Vena et al., 2006). By merging all the components of the stresses obtained earlier, the total viscoelastic second Piola-Kirchhoff stress of the ligament can be written in the following form:

\[
S(t) = \begin{cases} 
  pC^{-1} + 2a_11 & \text{if } I_4 \leq 1 \\
  pC^{-1} + 2a_11 + 2a_2 e^{a_3(I_4-1)^2} (I_4 - 1)N_0 \\
  + a_4(I_4 - 1)e^{a_5(I_4-1)^2} (N_0 \dot{C} + \dot{C}N_0) & \text{if } I_4 > 1 \\
  + \sum_{i=1}^{3} g_i \int_0^t e^{-(t-T)/\tau_i} S_s^v dT
\end{cases}
\] (6.22)
6.2.2 Stresses in Uniaxial Tension

As one of the objectives of the study was to use the proposed constitutive model to fit the experimental data, the second Piola-Kirchhoff stress obtained in the previous section should be converted to the first Piola-Kirchhoff or nominal stress which is measured in experiments. Also, as the experimental data considered in this study are in uniaxial tension, first let us start with writing the stresses by substituting the tensile deformation state in the equations. Assuming that the stretch ratio \( \lambda \) is applied in the axial direction, the deformation gradient for uniaxial loading will be of the following form for incompressible materials:

\[
F = \begin{bmatrix}
\lambda & 0 & 0 \\
0 & \frac{1}{\sqrt{\lambda}} & 0 \\
0 & 0 & \frac{1}{\sqrt{\lambda}}
\end{bmatrix}.
\] (6.23)

The right Cauchy-Green deformation tensor along with its inverse then can be obtained from the deformation gradient:

\[
C = \begin{bmatrix}
\lambda^2 & 0 & 0 \\
0 & \frac{1}{\lambda} & 0 \\
0 & 0 & \frac{1}{\lambda}
\end{bmatrix},
\] (6.24)

\[
C^{-1} = \begin{bmatrix}
\frac{1}{\lambda^2} & 0 & 0 \\
0 & \lambda & 0 \\
0 & 0 & \lambda
\end{bmatrix}.
\] (6.25)

In the lack of shear deformation, only diagonal elements of the stress tensor are non-zero. By substituting the aforementioned matrices in equations 6.17 and 6.18, the hyperelastic second Piola-Kirchhoff stress for ligament under uniaxial tension can be written as:

\[
S_{11}^e = 2a_1 + p\left(\frac{1}{\lambda^2}\right) + 2a_2(\lambda^2 - 1)e^{\alpha_3(\lambda^2 - 1)^2},
\] (6.26)

\[
S_{22}^e = S_{33}^e = 2a_1 + p\lambda.
\] (6.27)
The Lagrange multiplier, $p$, can be obtained by enforcing the boundary conditions. Knowing that the load is applied in the direction 1, the stresses in the other two directions are zero:

$$S_{22}^e = S_{33}^e = 0.$$  \hspace{1cm} (6.28)

Therefore, the Lagrange multiplier, $p$, can be calculated from the above constraint:

$$p = -\frac{2a_1}{\lambda}.$$  \hspace{1cm} (6.29)

Having obtained the Lagrange multiplier, the hyperelastic stresses in the direction of loading can be obtained:

$$S_{11}^e = 2a_1(1 - \frac{1}{\lambda^3}) + 2a_2(\lambda^2 - 1)e^{a_3(\lambda^2-1)^2}.$$  \hspace{1cm} (6.30)

The derivative of the right Cauchy-Green deformation tensor with respect to time, $\dot{C}$, is also required for calculation of the short-term viscous stress:

$$\dot{C} = \begin{bmatrix} 2\lambda \ddot{\lambda} & 0 & 0 \\ 0 & -\frac{\dot{\lambda}}{\lambda^2} & 0 \\ 0 & 0 & -\frac{\ddot{\lambda}}{\lambda^2} \end{bmatrix}.$$  \hspace{1cm} (6.31)

The short-term viscous stress can be obtained similarly by substituting the corresponding matrices in equation 6.19:

$$S_{11}^v = 4a_4(\lambda^2 - 1)\lambda e^{a_5(\lambda^2-1)^2}\ddot{\lambda}.$$  \hspace{1cm} (6.32)

The long-term viscous stress in the direction of loading expressed by the hereditary integral is also in terms of the short-term viscous stress. The second Piola-Kirchoff stresses obtained above are transformed to the first Piola-Kirchhoff stress by applying the Piola transformation
The total nominal stress is then as follows:

\[
P_{11}(t) = \begin{cases} 
2a_1(\lambda - \frac{1}{\lambda^2}) & \text{if } I_4 \leq 1 \\
2a_1(\lambda - \frac{1}{\lambda^2}) + 2a_2\lambda(\lambda^2 - 1)e^{a_3(\lambda^2 - 1)^2} \\
+4a_4(\lambda^2 - 1)\lambda^2e^{a_5(\lambda^2 - 1)^2}\dot{\lambda} & \text{if } I_4 > 1 \\
+\sum_{i=1}^{3} g_i \int_{0}^{t} e^{-(t-T)/\tau_i} P_{s,11}^v dT 
\end{cases} \quad (6.33)
\]

6.2.3 Volumetric-deviatoric Decomposition

The stresses discussed in the previous section are obtained for strictly incompressible materials which can be used for material characterization. However, enforcing incompressibility in finite element procedures is associated with some difficulties. For instance, the volumetric constraint can lead to mesh or volumetric locking (Spilker and Munir, 1980; Donea and Belytschko, 1992). Therefore, a slight compressibility should be introduced in the constitutive model and the material simulated as nearly incompressible. For compressible materials, it is usually more convenient to decouple the volumetric and deviatoric responses. Therefore, the free energy function of the material should be decomposed into volumetric and deviatoric parts as well:

\[
\Psi(C, \dot{C}, \Xi, N_0) = \Psi_{\text{vol}}(J) + \Psi^{e}_{m,\text{dev}}(\bar{C}) + \Psi^{e}_f(C, N_0) + \Psi^{v}_s(C, \dot{C}, N_0) + \Psi^{v}_v(C, \Xi, N_0) \quad (6.34)
\]

It should be noted that the deviatoric-volumetric decomposition has not been applied on the anisotropic part of the free energy. Doing so will result in a hydrostatic state of deformation under spherical loading which is not valid for anisotropic materials (Sansour, 2008). This issue was also discussed in section 2.6. The volumetric free energy function is expressed in the form of a polynomial (Bonet and Wood, 2008):

\[
\Psi_{\text{vol}} = \frac{1}{2} \kappa (J - 1)^2 \quad (6.35)
\]
The isotropic part of the free energy is rewritten in terms of the deviatoric invariants:

\[ \Psi^e_{m,\text{dev}} = a_1(I_1 - 3) \]  

(6.36)

The volumetric and isotropic second Piola-Kirchhoff stresses are derived from the energy functions according to the equation 2.54 respectively:

\[ S_{\text{vol}} = 2 \frac{\partial \Psi_{\text{vol}}(J)}{\partial J} \frac{\partial J}{\partial C} = \kappa J(J - 1)C^{-1}, \]  

(6.37)

\[ S^e_{m,\text{dev}} = 2 \frac{\partial \Psi_m(I_1)}{\partial I_1} \frac{\partial I_1}{\partial C} = 2J^{-2/3} \left( a_11 - \frac{1}{3}(a_1I_1)C^{-1} \right). \]  

(6.38)

These stresses along with anisotropic visco-hyperelastic stresses derived in the previous section will be used in the numerical implementation in the finite element program.

6.3 Applications to Articular Cartilage

Articular cartilage can be modeled as a porous fiber-reinforced composite material. Consequently, the strain energy can be decoupled for the matrix and fibers. This is a common approach in the constitutive modeling of composites and anisotropic soft biological tissues (Pioletti et al., 1998; Holzapfel and Gasser, 2001; Ateshian, 2009). The isotropic part of the energy models the non-fibrillar matrix in articular cartilage, whereas the fibers are represented by the anisotropic part. The role of collagen fibers is well-known in the fluid pressurization in the tissue and their viscoelastic behavior has been experimentally investigated (Woo et al., 1980; Charlebois et al., 2004). The stiffness of fibers is normally one order of magnitude greater than that of the non-fibrillar matrix (Li et al., 1999). Also, the fibers stiffen considerably by stretch. Measuring the viscoelastic properties of the non-fibrillar matrix is experimentally difficult as it may not be possible to separate the fluid-driven and inherent viscoelasticity of the non-fibrillar matrix in compression tests. Even in nanoindentation experiments, the effect of proteoglycans on the viscoelastic behavior was not dominant (Han et al., 2011). In the shear tests, the viscoelastic behavior of cartilage was detected
to depend on the collagen fiber content as well (Hayes and Bodine, 1978). Therefore, the non-fibrillar matrix of articular cartilage is considered to be hyperelastic in this study and the fibers are considered to be viscoelastic. The energy function for articular cartilage can also be expressed in terms of the same equilibrium and non-equilibrium tensorial variables as discussed for ligaments (section 6.2):

$$\Psi(C, \dot{C}, \Xi, N_0) = \Psi^e_m(C) + \Psi^e_f(C, N_0) + \Psi^v_s(C, \dot{C}, N_0) + \Psi^v_l(C, \Xi, N_0)$$ (6.39)

Similarly, the energy function can be expressed in terms of the invariants as well (equation 6.12). According to the experimental data and previous constitutive models for articular cartilage, a Neo-Hookean energy function can represent the mechanical behavior of the non-fibrillar matrix of the tissue:

$$\Psi^e_m = b_1(I_1 - 3).$$ (6.40)

The following Helmholtz free energy function is proposed for the fiber network of articular cartilage which phenomenologically can account for the self-stiffening of the collagen fibers with stretch:

$$\Psi^e_f = \frac{1}{2}b_2(I_4 - 1)^2 + \frac{1}{3}b_3(I_4 - 1)^3.$$ (6.41)

Therefore, the total hyperelastic energy function of articular cartilage can be written as a piecewise function:

$$\Psi^e = \begin{cases} b_1(I_1 - 3) & \text{if } I_4 \leq 1 \\
 b_1(I_1 - 3) + \frac{1}{2}b_2(I_4 - 1)^2 + \frac{1}{3}b_3(I_4 - 1)^3 & \text{if } I_4 > 1 \end{cases}$$ (6.42)

This piecewise function ensures that the fibers are only active under tension.

The short-term Viscous Helmholtz free energy has been chosen based on the current available experimental data on the transient response of cartilage in tension:

$$\Psi^v_s = \begin{cases} 0 & \text{if } I_4 \leq 1 \\
 b_4 \ln I_4(I_4 - 1)J_5 & \text{if } I_4 > 1 \end{cases}$$ (6.43)
6.3.1 Stress Derivation

The constitutive model of articular cartilage shares a similarity in the isotropic part of the energy function with that of the ligament. Also, the procedure of deriving the stresses has already been discussed in section 6.2.1. Therefore, only the derivation of the anisotropic hyperelastic and short-term viscous terms are presented here. The second Piola-Kirchhoff stress of the fibers can be derived as:

\[
S_e^f = (b_2(I_4 - 1) + b_3(I_4 - 1)^2)N_0.
\]

(6.44)

The short term viscous function will be obtained by the taking the derivative of the viscous Helmholtz free energy function with respect to \( \dot{C} \):

\[
S_v^{s,f} = 2b_4 \ln I_4(I_4 - 1)(N_0 \dot{C} + \dot{C}N_0).
\]

(6.45)

Finally, considering that the incompressibility condition has been applied as well, the total second Piola-Kirchhoff stress for articular cartilage is as follows:

\[
S(t) = \begin{cases} 
  pC^{-1} + 2b_1 & \text{if } I_4 \leq 1 \\
  pC^{-1} + 2b_1 + (b_2(I_4 - 1) + b_3(I_4 - 1)^2)N_0 \\
  + 2b_4 \ln I_4(I_4 - 1)(N_0 \dot{C} + \dot{C}N_0) & \text{if } I_4 > 1 \\
  + \sum_{i=1}^{3} g_i \int_0^t e^{-(t-T)/\tau_i} S_s^v dT
\end{cases}
\]

(6.46)

6.3.2 Stresses in Uniaxial Tension

The constitutive model of articular cartilage should also be characterized by experiments. The inherent viscoelasticity of the fibers can be revealed only in tension. Therefore, the nominal or first Piola-Kirchhoff stress should be derived for cartilage. The deformation gradient as well as the right Cauchy-Green deformation tensor along with its time derivative for the case of uniaxial tension were mentioned in section 6.2.2. These matrices are used here to obtain the stress component in the direction of loading. The second Piola-Kirchhoff stress
was derived first, then transformed to the first Piola-Kirchhoff stress; Both are mentioned here respectively:

\[
S_{11} = \begin{cases} 
2b_1(1 - \frac{1}{\lambda^3}) & \text{if } I_4 \leq 1 \\
2b_1(1 - \frac{1}{\lambda^3}) + b_2(\lambda^2 - 1) + b_3(\lambda^2 - 1)^2 
+ 8b_4\lambda \ln \lambda^2(\lambda^2 - 1) \dot{\lambda} & \text{if } I_4 > 1 \\
+ \sum_{i=1}^{3} g_i \int_{0}^{t} e^{-(t-T)/\tau_i} S_{s,11}^w dT 
\end{cases}
\]

(6.47)

\[
P_{11} = \begin{cases} 
2b_1(\lambda - \frac{1}{\lambda^3}) & \text{if } I_4 \leq 1 \\
2b_1(\lambda - \frac{1}{\lambda^3}) + b_2\lambda(\lambda^2 - 1) + b_3\lambda(\lambda^2 - 1)^2 
+ 8b_4\lambda^2 \ln \lambda^2(\lambda^2 - 1) \dot{\lambda} & \text{if } I_4 > 1 \\
+ \sum_{i=1}^{3} g_i \int_{0}^{t} e^{-(t-T)/\tau_i} P_{s,11}^w dT 
\end{cases}
\]

(6.48)

6.3.3 Volumetric-Deviatoric Decomposition

For the same reason discussed in section 6.2.3, the constitutive model has to be implemented as a nearly incompressible material in the finite element procedure. Therefore, the Helmholtz free energy function of articular cartilage should be decomposed into the volumetric and deviatoric parts to more conveniently enforce the nearly incompressibility condition. However, the decomposition was applied only on the isotropic hyperelastic term which is identical to the one chosen for ligament. The decomposed stresses and the procedure is similar to what was mentioned in section 6.2.3 and will be skipped here for the sake of brevity.

6.4 Numerical Implementation

ABAQUS (Hibbitt, Karlsson & Sorensen Inc., Providence, RI, USA) is a general purpose finite element package that has been very commonly used in the field of biomechanics. One
of the reasons for that is its ability in handling nonlinear materials as well as its solver for porous media mechanics. ABAQUS offers a variety of material constitutive models built into the software for isotropic and anisotropic cases. However, more complicated and customized constitutive models developed for specific materials are not available in the software. In order to implement the proposed constitutive model into ABAQUS, a user-material subroutine (UMAT subroutine) should be used. UMAT is coded in Fortran and can be linked to ABAQUS to be used for solving a variety of problems with different geometries and boundary conditions with the specified material constitutive law.

6.4.1 UMAT Subroutine

The UMAT subroutine is called for all integration points at each time step. Each UMAT subroutine has multiple inputs and outputs. ABAQUS passes the information such as the deformation gradient (DFGRD0, DFGRD1), strain (DSTRAN), the time step (DTIME) as well as some state variables (STATEV) into the UMAT subroutine. The stresses as well as the Jacobian matrix should then be updated at the end of each time step by the UMAT subroutine. Cauchy stress is the measure of stress used in ABAQUS and UMAT subroutine. The Jacobian matrix is obtained from the Jaumann tangent stiffness corresponding to the Jaumann objective rate of Cauchy stress. The process of implementing both the stress and Jacobian matrices into the UMAT subroutine will be explained in the following sections, and the numerical schemes will be discussed.

6.4.2 Stress Update

- Hyperelastic stress:

The hyperelastic stress is updated at each increment based on the information on the deformation gradient at the beginning and end of the increment passed to the user subroutine by ABAQUS. Based on the input deformation gradient, the right Cauchy-Green deformation tensor and its invariants will be calculated and used for updating the stress. Obviously, no
time discretization is needed for computing the hyperelastic stress.

• Short term viscous stress:
The short-term viscous stress is expressed in terms of the time derivative of the right Cauchy-Green deformation tensor and its invariants. As the deformation gradient is known in both the beginning and end of each increment as well as the time increment, the time derivative of the right Cauchy-Green deformation tensor can be approximated. Similarly, the stress is updated at the end of each increment.

• Long term viscous stress:
The long term viscous stress is in the form of a hereditary integral. There have been two different types of integral-based theories that were numerically implemented. The first one is the QLV theory and the second one is the hereditary integral that only represent the long-term response of the material. For both cases, a recurrence formula is used for updating the viscous stress. Let us start with the QLV theory stress update, assuming that the stress at time $t + \Delta t$ is expressed as:

$$S(t + \Delta t) = \int_0^{t+\Delta t} G(t + \Delta t - T) \frac{\partial S}{\partial T} dT.$$  \hspace{1cm} (6.49)

This equation can be divided into two parts from 0 to $t$ and $t$ to $t + \Delta t$:

$$S(t + \Delta t) = \int_0^t G(t + \Delta t - T) \frac{\partial S}{\partial T} dT + \int_t^{t+\Delta t} G(t + \Delta t - T) \frac{\partial S}{\partial T} dT.$$  \hspace{1cm} (6.50)

When the reduced relaxation function, $G(t)$, is expressed by an exponential function:

$$G(t) = g e^{-t/\tau},$$  \hspace{1cm} (6.51)

equation 6.50 can be written in the following form:

$$S(t + \Delta t) = g \int_0^t e^{-(t + \Delta t - T)/\tau} \frac{\partial S}{\partial T} dT + g \int_t^{t+\Delta t} e^{-(t + \Delta t - T)/\tau} \frac{\partial S}{\partial T} dT.$$  \hspace{1cm} (6.52)

In the first part of equation 6.52, the terms not dependent on time $t$ can be taken out of the integral:

$$g \int_0^t e^{-(t + \Delta t - T)/\tau} \frac{\partial S}{\partial T} dT = g e^{-\Delta t/\tau} \int_0^{t+\Delta t} e^{-(t - T)/\tau} \frac{\partial S}{\partial T} dT = e^{-\Delta t/\tau} S(t).$$  \hspace{1cm} (6.53)
The second part of the equation 6.52 can be calculated analytically:

\[
g \int_t^{t+\Delta t} e^{-(t+\Delta t-T)/\tau} \frac{\partial S}{\partial T}dT = g\tau(1-e^{-\Delta t/\tau}) \frac{S_e(\Delta t + t) - S^e(t)}{\Delta t}.
\]  

(6.54)

Thus, finally, an algorithm has been reached that updates the stress at \( t + \Delta t \), having the stress at \( t \):

\[
S(t + \Delta t) = e^{-\Delta t/\tau}S(t) + g\tau(1-e^{-\Delta t/\tau}) \frac{S_e(\Delta t + t) - S^e(t)}{\Delta t}.
\]

(6.55)

Similar procedures have been used by Puso and Weiss (1998) and Vena et al. (2006) for numerically implementing the quasi-linear viscoelasticity. The long-term hereditary integral is similar to the equation 6.49 with the difference that the integral starts at \( \delta \) (time of ramp loading) instead of zero and the time derivative of the elastic response is substituted by the short-term viscous stress. The long-term viscous stress can be defined at time \( t + \Delta t \) as follows:

\[
S^v(t + \Delta t) = \int_{\delta}^{t+\Delta t} G(t + \Delta t - T)S^v(\delta)dT,
\]

(6.56)

The stress update scheme for the long-term viscous response then can be obtained through the same procedure:

\[
S^v(t + \Delta t) = e^{-\Delta t/\tau}S^v(t) + g\tau(1-e^{-\Delta t/\tau})S^v(\delta), \quad t > \delta.
\]

(6.57)

6.4.3 Jacobian Matrix Update

In addition to the updated stress, ABAQUS requires the updated Jacobian matrix at the end of each increment to be used in the Newton-type method in the implicit finite element solver. The Jacobian matrix can be either approximated numerically or derived analytically. The analytical Jacobian matrix, upon being used in the numerical implementations, leads to the most rapid convergence possible. However, the analytical derivation of the Jacobian matrix for anisotropic nonlinear materials is mathematically complex. The approximation of the material Jacobian can be done more easily, nevertheless, the quadratic rate of convergence may be lost. Therefore, the analytical derivation has been done in this study to achieve
the best convergence rate possible. Different tangent moduli have been defined based on
the type of the objective stress rate used. ABAQUS has adopted the Jaumann rate of the
Cauchy stress and consequently, the corresponding Jacobian matrix should be defined in the
ABAQUS UMAT subroutine. The elasticity tensor defined in 2.20 is in relationship with the
convected or Oldroyd rate of the Kirchhoff stress (Sun et al., 2008; Miehe, 1996):

\[
\mathcal{L}_v \tau = \dot{\tau} - L \tau - \tau L^T = \varepsilon : D,
\]

(6.58)

whereas the Jaumann rate of Kirchhoff stress is:

\[
\tau^J = \dot{\tau} - W \tau - \tau W^T = C^J : D.
\]

(6.59)

The Jaumann tangent moduli can be obtained in terms of the elasticity tensor following the
procedure by Prot and Skallerud (2009):

\[
\mathcal{L}_v \tau = \tau^J - (L - W) \tau - \tau (L - W)^T = J \varepsilon : D
\]

(6.60)

\[
= \tau^J - D \tau - \tau D,
\]

\[
J \varepsilon : D = C^J : D - J C' : D,
\]

(6.61)

\[
C' = J (\varepsilon + C'),
\]

(6.62)

where

\[
C' : D = D \sigma + \sigma D.
\]

(6.63)

The stiffness matrix, \( C' \), can be written using indicial notation:

\[
C'_{ijkl} = \frac{1}{2} (\delta_{ik} \sigma_{jl} + \delta_{il} \sigma_{jk} + \delta_{jk} \sigma_{il} + \delta_{jl} \sigma_{ik}).
\]

(6.64)

Finally, the Jacobian matrix in ABAQUS which is defined as \( \frac{1}{J} C' \) to be consistent with the
Cauchy stress will be obtained as:

\[
\varepsilon_{ijkl}^{AB AQUS} = \varepsilon_{ijkl} + \frac{1}{2} (\delta_{ik} \sigma_{jl} + \delta_{il} \sigma_{jk} + \delta_{jk} \sigma_{il} + \delta_{jl} \sigma_{ik}).
\]

(6.65)
In the following, the elasticity and viscosity tensors ($c_{ijkl}$ in the equation above) for different contributions of the stress will be derived for both constitutive models of articular cartilage and ligament.

- **Volumetric elasticity tensor:**
  Both constitutive models of ligament and articular cartilage share the same volumetric stress. The volumetric elasticity tensor is obtained from the volumetric stress (equation 6.37) by applying equation 2.20:
  \[
  C^{vol} = 2\kappa (J^2 - J) \frac{\partial C^{-1}}{\partial C} + 2\kappa (J^2 - \frac{J}{2}) C^{-1} \otimes C^{-1}
  \]
  which can be expressed in the deformed state by the applying the push-forward operation 2.22:
  \[
  c^{vol} = -2\kappa (J - 1) I + 2\kappa (J - \frac{1}{2}) 1 \otimes 1,
  \]
  where $I$ is the supersymmetric identity tensor (Itskov, 2007).

- **Isotropic elasticity tensor:**
  The elasticity tensor for the isotropic hyperelastic part for cartilage and ligament is as follows:
  \[
  C^{iso}_e = -\frac{4}{3} J^{-2/3} c_1 (C^{-1} \otimes 1 + 1 \otimes C^{-1} - \frac{1}{3} I_1 C^{-1} \otimes C^{-1} + I_1 \frac{\partial C^{-1}}{\partial C}),
  \]
  where
  \[
  \frac{\partial C^{-1}}{\partial C} = -\frac{1}{2} (C^{-1} \otimes C^{-1} + C^{-1} \otimes C^{-1}).
  \]
  The spatial elasticity tensor is obtained by pushing the material tensor forward:
  \[
  c^{iso}_e = \frac{4}{3} J^{-5/3} c_1 (I_1 I + \frac{1}{3} I_1 1 \otimes 1 - 1 \otimes b - b \otimes 1).
  \]

- **Anisotropic elasticity tensor:**
  The elasticity tensor of the hyperelastic anisotropic part of ligament model is obtained by taking the derivative of equation 6.18:
  \[
  C^e_f = 2c_2 e^{c_3(I_4-1)^2} \left(1 + 2c_3(I_4-1)^2\right) N_0 \otimes N_0,
  \]
and can be expressed in the spatial form as:

\[ e^e_f = 2c_2e^{(I_4 - 1)^2} \left(1 + 2c_3(I_4 - 1)^2\right) I_4^2 N \otimes N. \]  

(6.72)

Articular cartilage hyperelastic anisotropic elasticity tensors are of the following form in the reference and deformed configurations respectively:

\[ C^e_f = 2(b_2 + b_2(I_4 - 1))N_0 \otimes N_0, \]  

(6.73)

\[ e^e_f = 2I_4^2(b_2 + b_2(I_4 - 1))N \otimes N. \]  

(6.74)

- Short-term viscosity tensor:

The material and spatial viscosity tensors for the viscous part of the constitutive model of ligament are obtained from equation 2.29 respectively:

\[ C^v_s = 2a_4(I_4 - 1)e^{\alpha(I_4 - 1)^2}[1 \otimes N_0 + N_0 \otimes 1], \]  

(6.75)

\[ e^v_s = 2a_4 I_4 e^{\alpha(I_4 - 1)^2}[b \otimes N + N \otimes b]. \]  

(6.76)

Similarly, the viscosity tensors for articular cartilage are as follows:

\[ C^v_s = 4b_4 \ln I_4(I_4 - 1)[1 \otimes N_0 + N_0 \otimes 1], \]  

(6.77)

\[ e^v_s = 4b_4 \ln I_4(I_4^2 - I_4)[b \otimes N + N \otimes b]. \]  

(6.78)

- Long-term viscosity tensor:

By taking the derivative of equation 6.55 with respect to \( C(t + \Delta t) \), the material and spatial viscosity tensors for the QLV viscous contribution are as follows:

\[ C^{QLV} = \sum_{k=1}^{N} g_k \tau_k \frac{1 - e^{\Delta t/\tau_k}}{\Delta t} \times C^e, \]  

(6.79)

\[ e^{QLV} = \sum_{k=1}^{N} g_k \tau_k \frac{1 - e^{\Delta t/\tau_k}}{\Delta t} \times e^e. \]  

(6.80)

For the long-term viscous response, the material and spatial viscosity tensors can be obtained by the same analogy:

\[ C^{L} = \sum_{k=1}^{N} g_k \tau_k \frac{1 - e^{\Delta t/\tau_k}}{\Delta t} \times C^v, \]  

(6.81)
By substituting the obtained elasticity and viscosity tensors in equation 6.65, the Jacobian matrix needed for the UMAT subroutine will be obtained.

\[ \varepsilon''_i = \sum_{k=1}^{N} \frac{g_k \tau_k}{\Delta t} \frac{1 - e^{\Delta t/\tau_k}}{\Delta t} \times \varepsilon''_s. \quad (6.82) \]

By substituting the obtained elasticity and viscosity tensors in equation 6.65, the Jacobian matrix needed for the UMAT subroutine will be obtained.

6.4.4 Finite Element Model

The proposed constitutive models for articular cartilage and ligament that were numerically implemented in the finite element software package ABAQUS should initially be tested to verify the numerical solution. Later, they can be used for solving problems involving complex geometries and boundary conditions. A finite element model of a simple geometry was constructed to be tested in tension. Although the model is able to capture 3D phenomena, the FE model is considered to be axisymmetric to save computational time. In addition to tensile testing, the model of articular cartilage was implemented to an unconfined compression and indentation test geometry. The proposed constitutive model describes the behavior of
Figure 6.2: The finite element mesh for the axisymmetric geometry used for indentation test simulations in ABAQUS. The mesh is finer near the indentation area where the sample experiences more severe deformations. The fibers’ orientation through the depth was chosen so as to reflect the true tissue structure.

The solid matrix of cartilage. The solid matrix was considered to be porous. The porous media properties including the void ratio of 3.5 and permeability of 0.002 $mm^4/Ns$ and 0.001 $mm^4/Ns$ in the direction and perpendicular to the direction of fibers were assigned to the mesh respectively (Gu and Li, 2011). Therefore, they form a fiber-reinforced poro-visco-hyperelastic model of cartilage that accurately mimics the structure of the tissue. The direction of the fibers as well as the mechanical properties of the solid matrix were considered to be depth-dependent reflecting the true structure of articular cartilage. The axisymmetric FE model was constructed with 8-node porous continuum elements including quadratic displacement and bilinear pore pressure interpolations: CAX8P in ABAQUS (Fig. 6.1 and 6.2). For the unconfined compression geometry, the fluid is free to exude out from
the sides. This is true for the indentation test while the fluid can exude from the tissue on the top regions that are not in contact with the indenter as well. Therefore, zero pore pressure was chosen as the boundary condition at $r = R$ and some regions on the top. A vertical displacement was applied on the top downward. For unconfined compression, considering low friction of cartilage specimens, it was assumed that cartilage is open to horizontal displacements at the top ($z = h$), while at the bottom, both the vertical and horizontal displacements were constrained. The fluid cannot exude out either from the top or the bottom regions as the platens and indenter are impermeable.

6.5 Identification of Material Properties

The experimental results can be used to determine the material parameters in the constitutive models proposed in this study for articular cartilage and ligament. There are a number of parameters in the model associated with the hyperelastic (equilibrium) and viscous (non-equilibrium) or time-dependent response of the tissue. The experimental data should be chosen in such a way as to characterize the parameters in a step-by-step fashion that minimizes the correlation of parameters. For instance, choosing time-dependent experimental data such as stress relaxation at the first step requires the elastic and viscous parameters to be evaluated simultaneously. As the contribution of each phase in the total response is not known initially, this may lead to a wrong combination of parameters that satisfies the objective, while giving a non-physical meaning. Also, this combination of parameters may not be unique. Therefore, a choice of experimental data that only reflects the effect of a specified component of the model is crucial in the material characterization process.

The model then should be fit to the suitable experimental data to find the parameters that satisfy this condition. This process is also known as curve fitting which is normally done through a minimization process. The least-square method was used in the current study. In this method, the summation of the square of the differences between the experimental and
model results should be minimized. Therefore, the objective function can be defined as:

$$f_{obj} = \sum_{i=1}^{N} (P(a_n, \lambda_i) - P_{exp}(\lambda_i))^2.$$  \hspace{1cm} (6.83)

$N$ is the number of data points in which the experimental results were recorded, $P$ is the nominal stress and $\lambda$ is stretch. $a_n$ represents the material parameters that minimize the objective function. There are also some constraints on the material parameters determined by the requirements of the problem of interest. The constraints may be in the form of linear or nonlinear equalities and inequalities. In the current study, the parameters of the proposed constitutive equations should be positive to ensure the convexity of the energy function and positive-definiteness of the elasticity tensors. In the viscoelastic formulations, the proportionality constants in the long-term response should also obey the following constraint:

$$\sum_{i=1}^{3} g_i = 1.$$  \hspace{1cm} (6.84)

This constraint ensures that the reduced relaxation function $G(t)$ at $t = \delta$, i.e., the end of ramp loading is 1 and vanishes at infinity:

$$G(\delta) = 1, \quad G(\infty) = 0 \quad (6.85)$$

6.6 Mechanical Tests

The compressive stiffness of cartilage is contributed by the interplay between the solid matrix and the interstitial fluid. Collagen fibers play a crucial role in the overall response of the tissue by limiting the lateral expansion which results in higher fluid pressurization. Collagen fibers are identified as viscoelastic materials that sustain tensile loads with one order of magnitude higher stiffness than that of the non-fibrillar matrix. However, as a viscoelastic material, the strain-rate sensitivity of the stiffness and consequently, the transient response of this constituent are not fully understood. Therefore, tensile experimental tests were planned to be done as a part of this study. The experimental results were used to characterize the
Figure 6.3: Femoral condyles of bovine stifle joint. The samples were extracted from the lateral and medial condyles (1) and the grooves (2).

proposed constitutive equation for cartilage. Moreover, the strain-rate sensitive stiffness of cartilage was used in numerical simulation of cartilage under unconfined compressive loading to help reveal the possible mechanism behind the high ratio of the transient to equilibrium response observed in articular cartilage.

6.6.1 Sample Preparation

Bovine knee joint was chosen for this study based on ease of availability and the large area of cartilage on the joint. Unlike compressive experiments, the samples extracted for tensile loading should initially be extracted from locations with the least amount of surface curvature. Therefore, larger joints such as bovine knee joint make it more convenient to harvest samples that allow later processing with less difficulty.

37 samples were extracted from the femoral condyles and grooves of 12 bovine knee joints (Fig. 6.3). The tensile tests were to be done in the direction of collagen fibers. Therefore, the fibers’ orientation on the surface of the tissue should be identified before or during extraction. The pin-pricking technique with India ink (Below et al., 2002) was used on 2 bovine knee joints to reveal the split-line pattern over the joint surface. The orientation was recorded and later used to identify the fibers’ direction on the extracted samples. Two method for extraction of samples were used. Initially, the samples were cut by
Figure 6.4: Cylindrical samples consisted of articular cartilage and the underlying bone attached. They were extracted by a diamond core drill with an inner diameter of 0.5".

A hand saw. The shapes of the samples extracted using this method were mostly rectangular with approximate dimensions of 15mm × 15mm. Although this method provides more flexibility in accessing different areas of the joint, the extraction process was time-consuming. Moreover, the dimension of the samples could not be controlled. Therefore, the remaining samples were extracted using a drill machine with a diamond core bit. The samples were cylindrical with diameter of 0.5" including cartilage with the underlying bone attached (Fig. 6.4).

In the second step, a specimen from the surface of the cartilage which includes the collagen fibers in parallel to the surface should be obtained. The desired thickness of the superficial zone should be 250µ approximately. In order to cut a specimen with this amount of accuracy, a rotary microtome (Leica RM2125 RTS) was used. The samples were gripped in the device clamp in such a way that the surface of cartilage was parallel to the blade (Fig. 6.5). Great care was exercised while performing this process by orienting the 2-axis movable clamp of the device. When the blade was adjusted parallel to and touching the surface, a 250µ forward step movement was applied and the sample was sectioned from the surface (Fig. 6.6).

Finally, dumbbell-shaped specimens were cut from the circular ones by using a plastic template of the dumbbell shape and a razor blade. The thickness and dimensions of each
Figure 6.5: The extracted cylindrical samples were fixed on a rotary microtome. The cartilage specimens from the superficial zone of the tissue were cut by adjusting the blade of the microtome parallel to the surface of cartilage.

sample was measured with a digital caliper before tensile testing. This measurement, which was done in the middle of the samples, resulted in the thickness of $271 \pm 48 \mu m$ (mean±SD, n=37) for all samples.

6.6.2 Experimental Set-up

A Bose ElectroForce® 3200 machine was used for tensile testing of the cartilage specimens. This kind of Bose machines has been used for high frequency loading protocols and is able to capture the expected range of loads in this experiment. A 45 N load cell (force transducer) was used in the experiments which provided good accuracy and resolution for the range of expected forces. Two clamps were used to grip the samples: One was mounted on top of
Figure 6.6: The circular specimens that were cut from the surface of the articular cartilage cylindrical samples using the microtome. These specimens were further cut by a blade into the shape of a dumbbell more suitable for tensile tests.

the load cell and the other one on the mover actuated by the motor. Cartilage samples were very thin and slippery as well. Gripping the tissue in the clamps tightly adjusted results in the tear and damage to the portion of the tissue under the clamp. On the other hand, if the clamp does not get fixed tight enough, that may result in the slippage of the tissue during testing. To overcome this problem, sandpapers were also used on the clamps to increase the friction and preventing the cartilage samples from slipping. The sand paper 1500 with small grain sizes was chosen as larger grain sizes has been shown to damage the tissue (Charlebois et al., 2004). In order to minimize the possibility of slippage of samples, a very thin layer of glue was also used between the sand papers and the cartilage samples.

6.6.3 Loading Protocol

In order to effectively satisfy the objective of the experimental study, i.e., examining the effect of strain-rate and strain separately on the viscoelastic response of the tissue, the following loading protocol was used. Three levels of strain at 2%, 5% and 10% were chosen. At each group, the strain was applied under seven different strain-rates of 0.1%/s, 1%/s, 10%/s, 25%/s, 40%/s, 50%/s and 80%/s. The tests consisted of both loading and unloading under constant strain-rate as opposed to sinusoidal loading where strain-rate varies by time. This
loading protocol decouples the effect of strain and strain-rate on the transient response and allows the results to be interpreted independently. Furthermore, the hysteresis measured during the loading-unloading process reveals the nonlinear effects of strain and strain-rate separately on the inherent damping characteristics of articular cartilage. Before the samples were tested under the aforementioned loading protocol, they were preconditioned under sinusoidal loading. The loading magnitude was the same as the strain aimed to be applied later on the tissue and the frequency was chosen to be $1 \text{ Hz}$. 30 cycles of loading were performed after which no considerable change in the reaction force was observed. A tare load of 0.1 $N$ was applied before each test to ensure the tissues stayed in tension.

6.6.4 Data Acquisition and Processing

The experimental data acquisition was done using the Wintest® software provided with the Bose machine. The raw data included the time and the force recorded for each specimen under the specified loading conditions. The data then were transferred to MATLAB version 7.12 (MathWorks Inc., Natick, MA, USA) for further processing. The results of the tests for each specimen along with the dimension of the specimen and applied displacements were stored in different files within one folder. Multiple programs written in MATLAB environment were then employed which automatically read the data based on the name of the file associated with the applied strain and strain-rate. The programs then calculated the stress v.s. strain, peak stress and hysteresis and finally, plotted the results.
Chapter 7

Results

This chapter presents the obtained material parameters for ligament and articular cartilage, the experimental tensile tests and the numerical results of articular cartilage. In the first two sections, the material properties of the anterior cruciate ligament and articular cartilage are presented. The material parameters were obtained by fitting the constitutive model to the available experimental data in the literature. The experimental results of the tensile tests of articular cartilage will be presented in Section 7.4. The inherent hysteresis and the transient response of articular cartilage under different strain-rates are discussed in this section. This results were used to evaluate the rate-dependent transient material parameters of articular cartilage as well. Finally, the numerical results of articular cartilage simulations in indentation and unconfined compression tests are presented in Section 7.5. The rate-dependent constitutive model characterized by the experiments were employed in the numerical simulations. The results show the effect of the rate-dependent fiber-reinforcement on the overall compressive response of articular cartilage.

7.1 Material Properties of Ligaments

The uniaxial tensile testing results of the anterior cruciate ligament were chosen to evaluate the material parameters of the constitutive model. The experimental data from the work of Pioletti (1997) were used for this purpose. These tests consisted of ramp-loading at equilibrium and three strain-rates of 25%/s, 38%/s and 50%/s while 13% to 16% strain was applied. In addition, stress relaxation of ligaments was reported separately after different levels of applied strain. The elastic constitutive parameters \((a_1, a_2, a_3)\) were found by fitting the model to the equilibrium data. For characterizing the short-term response, only the
Figure 7.1: The elastic (equilibrium) and viscoelastic behavior of the anterior cruciate ligament under uniaxial tension shown under different strain-rates. The constitutive model (shown by the lines) is fitted to the experimental data to obtain the elastic and short-term viscous material parameters. The material properties are summarized in Table 7.1.

data during ramp loading was considered. The tensile result under 25%/s loading rate was used to determine the short-term viscous parameters \((a_4, a_5)\) (Fig. 7.1). Interestingly, the constitutive model is able to predict the tensile results very well at the higher strain-rates of 38%/s and 50%/s without the need to optimize the parameters for all strain-rates. This fact shows the suitability of the short-term viscous constitutive model in terms of its strain-rate sensitivity at this range of strain-rates. The stress relaxation response was also used for characterizing the long-term viscous parameters \((\tau_1, \tau_2, \tau_3, g_1, g_2, g_3)\) (Fig. 7.1). The stress was normalized to the peak stress showing the decay of stress by time with respect to the maximum stress upon ramp loading. This fact is consistent with the assumption of quasi-linearity as the long-term viscous parameters \((\tau_i \text{ and } g_i)\) are independent of strain.
Figure 7.2: The stress relaxation of the anterior cruciate ligament obtained after 16% ramp loading (Pioletti, 1997). The transient stress response was normalized to the peak stress. The long-term viscous component of the constitutive model was fitted to the experimental data to evaluate the long-term viscous parameters ($\tau_i$’s and $g_i$’s).

The evaluated parameters of the constitutive equations are summarized in Table 7.1. By employing the model parameters obtained separately from the equilibrium, transient and relaxation responses, the constitutive model is then able to account for both the ramp loading and relaxation phases at different strain-rates (Fig. 7.3). The results show that the full ramp and relaxation response can be predicted with the proposed constitutive model while enjoying more sensitivity to the applied strain-rate. Besides, separating the ramp loading and relaxation responses and performing the material characterization process independently for each phase facilitated the optimization process.

Table 7.1: The viscoelastic material properties of the ligament constitutive model were obtained from ramp loading (Fig. 7.1) and stress relaxation tests (Fig. 7.1).

<table>
<thead>
<tr>
<th>Elastic</th>
<th>Short-term Viscous</th>
<th>Long-term Viscous</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a_2 = 2.213E6$</td>
<td>$a_4 = 0.3653E6$</td>
<td>$\tau_1 = 3.77$, $\tau_2 = 148.24$, $\tau_3 = 10987.98$</td>
</tr>
<tr>
<td>$a_3 = 3.879$</td>
<td>$a_5 = 0.652$</td>
<td>$g_1 = 0.154$, $g_2 = 0.161$, $g_3 = 0.682$</td>
</tr>
</tbody>
</table>
Figure 7.3: The previously evaluated parameters enable the constitutive model to account for the viscoelastic behavior of the anterior cruciate ligament both in the ramp loading and stress relaxation at the same time. 10% strain applied under 25%/s, 38%/s and 50%/s strain-rates.

7.2 Material Parameters of Articular Cartilage

The constitutive model was fit to the multi-step ramp loading and relaxation uniaxial tensile experimental data (Charlebois et al., 2004) to determine the model parameters. The foregoing test is composed of 5 steps of ramp loading with 2% strain under 0.15%/s strain-rate. Each loading step is followed by a relaxation period which was chosen long enough for the tissue to reach equilibrium completely. Besides, the confined compression test result of articular cartilage (Ateshian et al., 1997) was used to evaluate the stiffness of the non-fibrillar matrix of the tissue ($b_1$) (Fig. 7.4). During this test, the fibers are confined in the lateral direction and their contribution to the overall response is relatively negligible. Having obtained $b_1$, the stiffness of the collagen fibers ($b_2$) is determined using the equilibrium result of uniaxial tension (Fig. 7.5). The ramp loading and stress relaxation phases have been used separately in order to evaluate the short-term ($b_3$) and long-term viscous parameters ($\tau_1, \tau_2, \tau_3, g_1, g_2, g_3$) respectively. The material parameters are summarized in Table 7.2. The whole deformation process is shown and compared to the experimental results in Fig. 7.6.
Figure 7.4: The experimental results of articular cartilage in confined compression under equilibrium condition (Ateshian et al., 1997) was used to characterize the non-fibrillar matrix. The compressive test was done until 50% strain level.

The optimization process was found considerably less intricate compared to when only the single integral viscoelasticity model was used. Another advantage that the proposed constitutive model provides for material characterization is the viscoelastic behavior during ramp loading. This is more pronounced especially at multiple-step ramp and relaxation testing protocols. The proposed evolution equation in this study (equation 6.8) eliminates the necessity for the stiffness of the viscous body to be proportional to the stiffness of the elastic body. According to the experimental results, the stiffness of articular cartilage is changing with strain other than with strain-rate. This suggests that a constant stiffness proportion-

<table>
<thead>
<tr>
<th>Elastic</th>
<th>Short-term Viscous</th>
<th>Long-term Viscous</th>
</tr>
</thead>
<tbody>
<tr>
<td>$b_1 = 0.425E5$</td>
<td>$b_4 = 190E6$</td>
<td>$\tau_1 = 141.65$, $\tau_2 = 3.55$, $\tau_3 = 14303.43$</td>
</tr>
<tr>
<td>$b_2 = 0.5E5$, $b_3 = 14.2E6$</td>
<td></td>
<td>$g_1 = 0.346$, $g_2 = 0.0709$, $g_3 = 0.582$</td>
</tr>
</tbody>
</table>

Table 7.2: The viscoelastic material parameters of the articular cartilage constitutive model were found using the compressive (Fig. 7.4) and tensile experimental tests (Fig. 7.6).
Figure 7.5: The tensile elastic stiffness of collagen fibers in articular cartilage was determined by the uniaxial tensile experimental results (Charlebois et al., 2004).

...ality parameter cannot predict the initial viscoelastic response during ramp loading for all levels of strains. This limitation is one of the reasons the QLV model alone deviates from the experimental results (Li and Herzog, 2004a).

7.3 Comparison of the QLV and the proposed constitutive model

One of the main objectives of introducing the viscoelastic constitutive model was to obtain a transient response with a greater or adjustable degree of strain-rate sensitivity. In this section, the quasi-linear theory and the proposed constitutive model are compared at different strain-rates. The material properties obtained from ligaments (Table 7.1) were used in the proposed constitutive model. The material properties of the quasi-linear theory were adopted from literature (Li and Herzog, 2004a). These material properties were obtained from the multiple-ramp and relaxation test of cartilage presented in section 7.2. The results were obtained from the finite element model in uniaxial tension. 10% strain was applied in both cases at strain-rates of 1%/s, 10%/s, 25%/s and 50%/s. The stress response was
Figure 7.6: The experimental results of multiple ramp and relaxation of cartilage in tension was used to fully characterize the constitutive model. The foregoing test consisted of 5 steps of 2% ramp loading under 0.15%/s followed by stress relaxation. The experimental results during loading at each step were used to determine the short-term viscous parameters while, the relaxation phases were used to evaluate the long term viscous parameters. Having been furnished by the model parameters found separately from equilibrium, short-term and long-term responses of cartilage, the constitutive model is able to predict the total behavior of cartilage in tension.

normalized to the elastic equilibrium stress. Therefore, the peak stresses in both models can be compared at different strain-rates. The peak stress predicted by the QLV model shows a minimal sensitivity to the applied strain-rate. The peak stress is almost constant in all cases (Fig. 7.8). On the other hand, the proposed constitutive model produced rate-sensitive results (Fig. 7.7). The peak stress at the highest strain-rate is approximately three times greater than the one at the slowest strain-rate. The proposed constitutive model is ideal for materials that show a considerable amount of rate-sensitivity in their viscoelastic response.
Figure 7.7: The ramp loading and relaxation numerical results of the proposed constitutive model obtained from ABAQUS under 10\% strain and 1%/s, 10%/s, 25%/s and 50%/s strain-rates. The results show a considerable strain-rate sensitivity of the peak stress to the strain-rate. The ligament material parameters (Table 7.1) were used in the current comparison.

Figure 7.8: The numerical results of the QLV formulation obtained from ABAQUS shows a minimal sensitivity to strain-rate. Articular cartilage was considered for this case and 10\% tensile strain was applied under 1%/s, 10%/s, 25%/s and 50%/s strain-rates. The predicted peak stress is almost constant for all strain-rates. The material properties were adopted from the literature (Li and Herzog, 2004a).
7.4 Experimental Results

The experimental results obtained from tensile testing of articular cartilage with the method described in Section 6.6 are presented in this section. The test consisted of loading and unloading. Therefore, in addition to the transient response, the inherent hysteresis of articular cartilage was also obtained. The results of hysteresis will be presented first, followed by the transient response and characterization of the tissue under different strain-rates.

7.4.1 Hysteresis

The loading-unloading force versus deformation response of articular cartilage was obtained at three levels of strains: 2%, 5% and 10% and 7 different strain-rates: 0.1%/s, 1%/s, 10%/s, 25%/s, 40%/s, 50%/s, 80%/s (Fig. 7.11, 7.10 and 7.9). The area under the loading process represents the input energy, whereas the area under the unloading process represents the recovered energy. The difference between these energies shows the total energy absorption or the energy dissipated during the loading-unloading process. In order to make these hysteresis results comparable to each other, the hysteresis area was normalized to the input energy. Therefore, the normalized results demonstrate what portion of the input energy is dissipated (Fig. 7.12). The results show that the energy dissipation decreases when moving from slow (0.1%/s) to moderate strain-rates (10%/s). However, it starts to increase at higher strain-rates and reaches its maximum at the highest strain-rate of 80%/s. This dissipation is also considerably higher than the initial hysteresis at the slowest strain-rate (0.1%/s). Interestingly, this pattern of change of hysteresis with strain-rate was observed in all three levels of strains. The normalized hysteresis also shows an increase when moving to higher strains while comparing specified strain-rates.
Figure 7.9: The hysteresis shown for 15% strain level under different strain-rates

Figure 7.10: The hysteresis shown for 8% strain level under different strain-rates
Figure 7.11: The hysteresis shown for 3% strain level under different strain-rates.

Figure 7.12: The normalized hysteresis area shown for different strains and strain-rates. At each level of strain, the pattern of the change of the hysteresis with strain-rate is similar.
7.4.2 Transient Response

The loading process of the tensile testing was considered in order to investigate the transient response of the tissue under various strain-rates. The transient response showed a considerable amount of strain-rate sensitivity. The maximum stress produced at the end of loading is increasing by strain-rate (Fig. 7.16). This increase is sharper at higher strains with the maximum relative stress of 1.9 at the highest strain-rate with respect to the slow strain-rate of 0.1%/s. Besides comparing the maximum stress, the instantaneous stiffness was also characterized with reference to strain-rate (Fig. 7.13 and 7.14). Two constitutive models were used in the characterization process. These models are quadratic and exponential similar to the elastic part of the constitutive model of articular cartilage (section 6.3) and ligament (section 6.2) respectively. Using both models provides the chance to interpret the results from different points of view. The first Piola-Kirchhoff or nominal stress is the measure of stress that corresponds to the experimental measurements. The quadratic and exponential nominal stresses then are as follow respectively:

\[
P_{11}^e = c_1 \lambda (\lambda^2 - 1) + c_2 \lambda (\lambda^2 - 1)^2, \quad (7.1)
\]

\[
P_{11}^e = 2c_3 \lambda (\lambda^2 - 1) e^{c_4 (\lambda^2 - 1)^2}. \quad (7.2)
\]

Both models are able to fit the experimental data very well (Fig. 7.13 and 7.14). In the quadratic formulation (equation 7.1), the parameter \(c_1\) can be interpreted as the elastic modulus at zero strain and \(c_2 (\lambda^2 - 1)\) is the elastic modulus that changes with strain. On the other hand, in the exponential function, the parameters \(c_3\) characterizes the tensile stiffness while the nonlinearity of the response is determined by the parameter \(c_4\). The parameters found from each model are summarized in Tables 7.3 and 7.4.

In order to make it possible for the stiffness to be used in linear constitutive models as well, the elastic modulus at 5% strain, \(E_{5\%}\), was also derived (Fig. 7.15). This elastic modulus is defined as the tangent to the stress-strain curves at 5% strain. The increase of \(E_{5\%}\) is more noticeable from 0.1%/s until 25%/s. However, the elastic modulus does not
Figure 7.13: The viscoelastic response of articular cartilage under various loading rates was used to characterize the instantaneous stiffness of the tissue using quadratic and exponential constitutive models. In this figure, only experiments under 0.1%/s, 1%/s, 25%/s and 50%/s strain-rates are shown.

Figure 7.14: The viscoelastic response of articular cartilage under various strain-rates was used to characterize the instantaneous stiffness of the tissue using quadratic and exponential constitutive models. In this figure, only experiments under 10%/s, 40%/s and 80%/s strain-rates are shown.
Figure 7.15: The elastic modulus at 5% strain was obtained as the tangent to the stress vs. strain curves. The increase of this elastic modulus is more pronounced between 0.1%/s and 25%/s strain-rates.

Figure 7.16: The maximum (peak) stress obtained in tensile testing of articular cartilage in tension under various strains- and strain-rates. The peak stress is not considerably sensitive to strain-rate at smaller strains (3%). However, the higher the strain is, the more sensitive the peak stress becomes.
Table 7.3: Instantaneous material properties of the articular cartilage quadratic constitutive model under various strain-rates. The material parameters were obtained by fitting the quadratic formulation of stress (equation 7.1) to the experimental data (Fig. 7.13 and 7.14).

<table>
<thead>
<tr>
<th></th>
<th>0.1%/s</th>
<th>1%/s</th>
<th>10%/s</th>
<th>25%/s</th>
<th>40%/s</th>
<th>50%/s</th>
<th>80%/s</th>
</tr>
</thead>
<tbody>
<tr>
<td>$c_1$ (MPa)</td>
<td>6.13</td>
<td>6.21</td>
<td>6.23</td>
<td>8.958</td>
<td>12.922</td>
<td>12.912</td>
<td>18.951</td>
</tr>
<tr>
<td>$c_2$ (MPa)</td>
<td>47.484</td>
<td>66.999</td>
<td>79.998</td>
<td>80.369</td>
<td>55.295</td>
<td>60.116</td>
<td>27.82</td>
</tr>
</tbody>
</table>

Table 7.4: Instantaneous material properties of the articular cartilage exponential constitutive model under various strain-rates. The material parameters were obtained by fitting the exponential formulation of stress (equation 7.2) to the experimental data (Fig. 7.13 and 7.14).

<table>
<thead>
<tr>
<th></th>
<th>0.1%/s</th>
<th>1%/s</th>
<th>10%/s</th>
<th>25%/s</th>
<th>40%/s</th>
<th>50%/s</th>
<th>80%/s</th>
</tr>
</thead>
<tbody>
<tr>
<td>$c_3$ (MPa)</td>
<td>7.19</td>
<td>8.901</td>
<td>11.999</td>
<td>13.075</td>
<td>14.004</td>
<td>15.527</td>
<td>19.103</td>
</tr>
<tr>
<td>$c_4$</td>
<td>4.078</td>
<td>4.011</td>
<td>2.87</td>
<td>2.816</td>
<td>2.698</td>
<td>2.271</td>
<td>1.270</td>
</tr>
</tbody>
</table>

change considerably at higher strain-rates. However, due to higher initial modulus, higher maximum stresses were produced at higher strain-rates.

### 7.5 Numerical Results

The results obtained from numerical simulation in ABAQUS will be presented in this section. The finite element model of articular cartilage discussed in section 6.4.4 was used in the modeling. Firstly, the pore pressure distribution within articular cartilage will be presented and the effect of depth-dependent material properties and fiber orientation will be discussed. Furthermore, the strain-rate dependent compressive response of articular cartilage will be investigated. For this purpose, finite element models of cartilage in unconfined and indentation configurations were used (section 6.4.4).
7.5.1 Depth-dependent Fluid Pressurization

The finite element model of articular cartilage furnished with depth-dependent material properties and anisotropies is employed to be tested in indentation testing (Fig. 6.2). The indenter applied a 10% strain downward with the rate of 0.2%/s. The thickness of the superficial, middle and deep zones were considered to be 20%, 50% and 30% of the whole tissue depth. The pore pressure contour throughout the tissue is shown in Fig. 7.17. In order to examine the effect of the thickness of each layer through the depth of articular cartilage on the fluid pressurization, another case was also considered. The same finite element model was chosen with the same dimensions and material properties. However, in this model, the thickness of the superficial, middle and deep zones was set to be 10%, 40% and 50% of the depth respectively. The pore pressure distribution for this case is shown in Fig. 7.18. As can be observed, the model with a smaller portion of the superficial layer shows larger pore pressure especially in the upper part of the tissue compared to the model with more extended thickness assigned to the superficial zone. The maximum pressure shows a 13% increase in the second case which emphasizes the importance of not only the depth-dependent fiber orientation but the portion of the thickness of each layer throughout the joint.

7.5.2 Transient Response

The effect of the strain-rate dependent self-stiffening of collagen fibers is examined on the rate-dependent compressive response of articular cartilage in unconfined compression. The strain-rate dependent stiffening of collagen fibers in tensile loading was found in experiments and presented in section 7.4.2. The constitutive model that was characterized by the experimental results was used as to represent the transient response of collagen fibers in unconfined compression. 5% compression was applied under three different strain-rates of 0.1%/s, 1%/s and 10%/s. This range of strain-rates with two orders of magnitude difference from the slowest and the fastest loading rate suffices to capture the strain-rate stiffening of cartilage.
Figure 7.17: Pore pressure (POR in MPa) distribution throughout the depth of articular cartilage in indentation testing in an axisymmetric geometry. The tissue was indented at 10% strain with the rate of 0.2%/s. In this case, the thickness of the superficial, middle and deep zones was considered to be 20%, 50% and 30% of the total depth respectively.

Figure 7.18: Pore pressure (POR in MPa) distribution throughout the depth of articular cartilage in indentation testing in an axisymmetric geometry. The tissue was indented at 10% strain with the rate of 0.2%/s. In this case, the thickness of the superficial, middle and deep zones was considered to be 10%, 40% and 50% of the total depth respectively.
Figure 7.19: The instantaneous reaction force response of articular cartilage under unconstrained compression increases with strain-rates (blue bars). In this case, the quasi-linear theory (QLV) was used to represent the viscoelastic behavior of collagen fibers. When the constitutive model of fibers is substituted by the strain-rate sensitive model, the increase of the reaction force by strain-rate accentuates (red bars).

In order to determine the effect of fiber stiffening on the overall compressive reaction force, two cases were considered. In the first model, the collagen fibers are viscoelastic represented by the QLV theory. This formulation was shown not to be strain-rate-sensitive in the transient response (Section 7.3). The other model uses the formulation characterized for the transient response of cartilage. The results show that the peak reaction force is increasing at higher strain-rates (Fig. 7.19). However, when collagen fibers are considered to be strain-rate-sensitive, this increase of the reaction force with strain-rate is accentuated. The relative increase of reaction force in the instantaneous model with respect to the QLV model in strain-rate of 10%/s is 34%. This ratio of increase in the transient response is higher than the relative increase of 11% at strain-rate of 1%/s. This fact suggests that the increase of the stiffness of collagen fibers is also amplified by the fluid pressurization.
Chapter 8
Discussion, Conclusion and Future Work

8.1 Constitutive Model

The model described in this study is able to predict the viscoelastic behavior of materials in a wide range of strain-rates. The strain-rate sensitivity of the transient response depends on the choice of the short-term viscous energy function. Therefore, the rate-dependent mechanical behavior of the material in consideration should be experimentally known prior to adopting such a formulation. Based on the available experimental results, a suitable short-term energy function can be chosen able to capture the transient response in the strain-rate range of interest. The examples presented in the current study are among biological tissues. However, this model may be equally applicable to other viscoelastic materials such as polymers, hydrogels and in particular to fiber-reinforced and anisotropic materials.

This model shows capabilities in predicting the behavior of viscoelastic materials during loading, in particular at high strain-rates while at the same time accounting for the stress relaxation response as well. The differential type models have been commonly used for soft tissues such as the anterior cruciate ligament (Pioletti et al., 1998; Limbert and Middleton, 2006; De Vita and Slaughter, 2006), liver (Roan and Vemaganti, 2011) and periodontal ligament (Zhurov et al., 2007). These models, however, fail to predict the relaxation response and suggested to be used for materials with “short-time memory” (Roan and Vemaganti, 2011). The current model removes this limitation by introducing the short-term and long-term internal variables. The proposed evolution equation furnishes the ground for conjunction of the short-term and long-term phases which makes it advantageous to similar approaches published hitherto (Pioletti and Rakotomanana, 2000).

The results discussed on ligament mechanics (section 7.1) focused on the effect of strain-
rate on the peak stress. The experimental data on ligaments (Pioletti, 1997) suggests strong strain-rate dependent behavior of ligament at high strain-rates. Therefore, the proposed constitutive model is suitable to be used for this tissue. In fact, the constitutive model provided a very good fit to the experimental data. The rate-dependent behavior of articular cartilage had not been investigated comprehensively in tension. There was only a single study that reports a considerable increase of the stiffness of cartilage at the high strain-rate of 70%/s (Verteramo and Seedhom, 2004). The experiments done as a part of this thesis showed the rate-dependent tensile behavior of cartilage as well (section 7.4.2). However, the equilibrium result is also needed in order to propose a short-term viscous function for cartilage that is valid under a wider range of loading rates. The current short-term viscous function for articular cartilage has been proposed based on the available experimental data. This formulation is able to capture the nonlinearities associated with strain as it can capture the nonlinear short-term response (ramp loading) under different levels of strain (Fig. 7.6). In terms of strain-rate sensitivity, this function can accommodate the possible changes in case of availability of more experimental results. These probable changes, however, do not affect the results and conclusions made in this study.

The transient response of the material governed by the short-term viscous function is nonlinear in terms of strain. This is opposed to the QLV theory employed in other studies (Li and Herzog, 2004a) which uses fixed time and proportionality constants ($\tau, g$ in equation 6.2). Therefore, in addition to the relaxation response, the nonlinearities associated with strain in the transient response also cannot be captured in the QLV formulation. Failing to predict the correct peak stress during loading would, in turn, result in difficulties and perhaps failure in the optimization process aimed to fit the experimental data in the relaxation period. This is especially the case for multiple ramp and relaxation tests such as the tensile test of articular cartilage discussed in section 7.2. In this case, the viscoelastic nonlinearities in both loading and stress relaxation phases are needed to be taken into account using one set of
Figure 8.1: QLV theory used to fit the same experimental data of tensile test on articular cartilage from literature (Li and Herzog, 2004b). Comparison of the predicted stress by the QLV theory in this figure with the results obtained in this study (Fig. 7.6) highlights the improvement made in the capabilities of the proposed model to account for the nonlinearities.

parameters in a single constitutive model. The current model addresses this issue by using a nonlinear short-term function which led to a correct prediction of the transient response. A comparison of the same experimental data fitted by the QLV theory (Fig. 8.1) shows a considerable improvement in the fit provided by the proposed constitutive model in this study (Fig. 7.6).

The evolution equation used in this study utilizes fixed time constants. This equation leads to quasi-linearity of the stress relaxation response. In other words, the short-term (transient) response is fully nonlinear, however, the long term (stress relaxation) response is quasi-linear. This limitation can be easily overcome by introducing strain-dependent time constants. However, the numerical implementation of the quasi-linear theory is more straight-forward compared to fully nonlinear cases and was considered in this study for simplicity. Thus, the concept of using short-term and long-term internal variables could be
explained clearly without adding more complexities to the formulation.

The quasi-linearity of the long-term response can be a limitation as most biomaterials show nonlinear stress relaxation. This is also true for ligament and articular cartilage considered in this study. The QLV theory was used in some studies on ligament (Pioletti, 1997; Abramowitch et al., 2004; Vena et al., 2006) while, Provenzano et al. (2002) showed that the nonlinear viscoelastic theories can better predict the relaxation response of this tissue. Later, the quasi-linear theory was assessed against the experimental data of Pioletti (1997) by DeFrate and Li (2007). They also concluded that there are some mismatches when using QLV theory at large deformations. Similarly, articular cartilage was modeled using the QLV theory in some studies (Woo et al., 1980; Simon et al., 1984; Li and Herzog, 2004b). However, nonlinear theories were shown to be more successful in fitting the experimental data at different strain-levels (Park and Ateshian, 2006). It should be noted that the suitability of the QLV theory should be assessed using experiments on different levels of loading such as multi-step ramp and stress relaxation (Li and Herzog, 2004b). The quasilinear theory may provide an excellent fit to the data when a single level of strain, although large, is considered. This is because the reduced relaxation function, $G(t)$, which dictates the speed of the dissipation of the transient stress can be a nonlinear function of time. However, without any dependency on strain, it may fail to account for the strain-dependent nonlinear phenomena.

Knowing that articular cartilage and ligament along with many other biomaterials exhibit nonlinear relaxation behavior, the evolution equation should be equipped with time-dependent time constants. In the current study, ligament was investigated under different strain-rates but a constant strain for all cases. Therefore, the assumption of quasi-linearity does not affect the results. Articular cartilage was, however, studied under multiple ramp loading and relaxation protocol. Although the obtained results for stress relaxation are satisfactory, fully nonlinear models can improve the fit to the experimental data.

The two cases discussed for articular cartilage and ligament in this thesis showed a strain-
rate dependent behavior, especially at high strain-rates. On the other hand, there may exist viscoelastic materials whose transient behavior is minimally sensitive to the applied strain-rate. The concept of pseudo-elasticity was proposed to be used for such materials (Fung, 1993). However, this method only simulates the transient response of viscoelastic material, while failing to predict the stress relaxation. Zhang et al. (2007) introduced a rate-insensitive viscoelastic model for soft tissues, however, their model was limited to linear materials. Within the framework of the proposed constitutive model in this study, the short term energy function can be replaced with a pseudo-elastic energy function without any dependency or with a weak dependency on strain-rate. This approach predicts the transient viscoelastic response while at the same time accounting for the stress relaxation. Therefore, the proposed constitutive model can be applicable to rate-insensitive viscoelastic materials as well.

8.2 Articular Cartilage Experimental Results

The tensile experiments on articular cartilage revealed different degrees of strain-rate sensitivity at different strain levels. The peak stress produced under 15% loading showed the most dramatic sensitivity to strain-rate. The ratio of the peak stresses at the highest and lowest strain-rate was observed to be 1.95. This ratio was higher than the one at 8% strain which was measured to be approximately 1.6. The strain-rate sensitivity was not considerable at smaller strains in the range of 3%. This means that one can use the QLV theory or elastic formulation with instantaneous modulus for collagen fibers as long as the analysis is limited to small deformations.

The role of fluid-driven viscoelasticity is judged negligible in the tensile experiments on articular cartilage in this study. Although the fluid considerably takes part in the load bearing mechanism in compression (Oloyede and Broom, 1993), it has been shown numerically that the interstitial fluid does not play a major role in tension (Li et al., 2005). The nu-
merical simulations in the aforementioned study were performed under smaller strain-rates compared to the rates of loading in the current study. However, the increase of strain-rate is not likely to cause the fluid-driven mechanism to become dominant as the fluid can only sustain hydrostatic compression.

According to the obtained material properties of the proposed constitutive models (equation 7.1 and 7.2), the behavior of cartilage gradually becomes more linear from lower to higher strain-rates. The behavior of cartilage at equilibrium is highly nonlinear (Fig. 7.5). The nonlinear stress-stretch relationships were also found at low strain-rates (Woo et al., 1976, 1979). This fact is also observed in the current study and the linearization of the response can be confirmed by analyzing the material parameters of the two proposed constitutive formulations. In the quadratic formulation, the initial stiffness, $c_1$ increases by strain-rate and the ratio of $c_2$ to $c_1$ ($\frac{c_2}{c_1}$) decreases at higher rates of loading. The trend of relative change of the parameters of the exponential formulation shows this fact as well. The parameter $c_4$ which determines the degree of the nonlinearity of the response decreases at higher strain-rates. This linearization of the stress response compared to the highly nonlinear equilibrium response of articular cartilage indicates the increase of the viscous contribution by strain-rate.

The inherent hysteresis of articular cartilage also changes nonlinearly with strain-rate. The trend of change of hysteresis with strain-rate is similar in all levels of strains (Fig. 7.12). This suggests that this behavior is likely to be caused by a similar mechanism irrespective of the loading magnitude. It should be noted that the hysteresis is increasing by strain, however, only the trend of its change with loading rate is similar in different strains. As the tensile tests were done in the direction of fibers, it can be concluded that this behavior arises either from the fibers themselves or their interaction with the non-fibrillar matrix. The hysteresis testing on single collagen fibers also showed a similar trend in change of hysteresis with strain-rate (Poissant and Barthelat, 2012). The aforementioned study highlights the
role of collagen fibers in the inherent hysteresis of cartilage, i.e., the effect of collagen fibers may be of greater significance on the damping characteristics of the tissue compared to the cross-linking in the fiber network. In addition, it confirms the findings of the current study.

The high hysteresis observed at higher strain-rates (25%/s-80%/s) is thought to be caused by an unfolding mechanism in collagen fibers. The collagen fibers are naturally in a wavy configuration when not loaded. This structure can also be compared to a helix. The uncrimping process of this structure results in extra stiffness of the fibers. This stiffening was previously observed at equilibrium (Akizuki et al., 1986). The results of the current study further shows the possible rate-dependent uncrimping of collagen fibers. At higher strain-rates, the uncrimping or unfolding of the helical structure of collagen fibers may be boosted. This extra unfolding then, in turn, results in the lower stiffness of the fibers in the unloading process. Another possibility is the rate of the recovery of the tissue from uncrimping. From this point of view, the higher hysteresis at high strain-rates can be explained to be caused by the same uncrimping at lower strain-rates which is not immediately recoverable. There was observed another increase of hysteresis at lower strain-rates. This fact shows that the effect of softening and relaxation of the material becomes dominant at lower strain-rates.

The elevated hysteresis at high strain-rates observed in the current tensile experiments on articular cartilage provides support for the increased hysteresis of cartilage under compressive impact loading (Edelsten et al., 2010). As the theory of poroelasticity suggests that the hysteresis should decrease at higher strain-rates, the possible damping mechanism arisen from the collagen fibers in the superficial layer of the tissue may be the reason for the aforementioned observations.

The loading magnitude also affects the hysteresis of articular cartilage. This phenomenon can also be explained by the uncrimping of collagen fibers, i.e., the unfolding of collagen fibers becomes less recoverable at higher stretches. This effect is accentuated at higher strain-rates which is in agreement with the aforementioned mechanism. Some studies on other molecules
such as titin shows an elevated hysteresis when the loading is applied over a specified stretch (Herzog et al., 2012). This suggests a threshold over which the unfolding of the molecule is less recoverable.

As can be observed in the hysteresis loops at 8% and 15% strains (Fig. 7.10 and 7.9), the elastic modulus defined as the tangent to the stress-stretch curves, decreases when the stretch is approaching its maximum. This loss of stiffness occurs only at strain-rates higher than 25%/s under 15% loading and 40%/s under 8% loading conditions respectively. The loss of stiffness can be explained from three main points of view. The first scenario is the slippage between the tissue sample and the clamps. However, the sand paper in combination with the glue used in fixing the cartilage specimens minimized the probability of the slippage occurring. Moreover, the observations by the naked eye after doing the test did not indicate any misplacement of the samples in the clamps. This leads us to the possibility of non-recoverable deformations in cartilage at large deformations. These non-recoverable changes can be either time-dependent, i.e., viscoelastic or permanent, i.e., plastic. The plastic deformation in the context of articular cartilage can be interpreted as damage to the tissue. The loss of stiffness at large deformations justifies the possibility of damage occurring rather than a viscoelastic effect. More importantly, the loss of stiffness at high strain-rates demonstrates a rate-dependent damage mechanism in the tensile behavior of articular cartilage.

The gauge-to-gauge measure of strain was used in this study. This method of strain measurement was used in tensile testing of articular cartilage in some studies (Charlebois et al., 2004; Akizuki et al., 1986), whereas others employed optical techniques to measure the strain in the center of the specimen (Woo et al., 1979; Roth and Mow, 1980). Although the optical measurement can provide more accurate results, it can be accompanied by some difficulties in the presence of fluid flow or bathing solution on the tissue (Akizuki et al., 1986), or when the experiments are done under high frequencies (Park and Ateshian, 2006). The optical methods can also be used in measuring the cross-sectional area of specimens. While
the optical technique can improve the absolute value of the measurement for each test, the relative results both in terms of strain and strain-rate variability will not be affected. This can be also justified considering the main objective of the current experimental study which was to investigate the strain-rate sensitivity of articular cartilage in tension. Therefore, the conclusions made from these experiments remain unaffected.

8.3 Numerical Results

The numerical results show the importance of the depth-dependent material properties. The effect of the depth-dependent properties were investigated previously (Li et al., 2000). However, this study also shows the effect of the depth of each layer specifically in fluid pressurization. The depth of each layer of cartilage is reported to be within an approximated range discussed in section 3.1. As the exact choice of the thickness of each layer is affecting the pore pressure distribution, the obtained results suggest that this factor should be taken into consideration in numerical simulations of cartilage. In axisymmetric geometry, the distribution of pore pressure or pattern of pressure gradient irrespective of its magnitude is only slightly modified. However, this effect is likely to become consequential in 3D simulations of the joint considering the magnitude of the fluid pressurization. The thickness of articular cartilage in the knee joint varies by location. On the other hand, the portion of the total depth assigned to each layer might be changing as well in different locations. Therefore, the adjacent zones in the joint with layers of different thickness experience various pore fluid pressure. This pressure gradient, in turn, may affect the flow of the fluid within the tissue in the joint level as well.

The fluid pressurization in the tissue is also highly dependent on collagen fibers. The stiffer the collagen fibers are, the less lateral expansion in the tissue occurs. The dramatic effect of strain-rate on compressive response of articular cartilage had been experimentally observed (Oloyede et al., 1992). The rate-dependent compressive behavior of this tissue
were later numerically investigated as well (Li et al., 2003; Li and Herzog, 2004a). Also, an interplay was found between the collagen fiber-reinforcement and fluid pressurization. Higher fluid pressurization leads to more stiffening of fibers and this, in turn, causes higher fluid pressure. However, the fiber-reinforcement was either elastic (Li et al., 2003; Li and Herzog, 2004a) or viscoelastic with QLV theory (Li et al., 2005, 2008) and the simulations were limited to smaller strain-rates. Therefore, the source of the high transient reaction force at high strain-rates remained unanswered, as the previously published constitutive models were unable to capture this response. In this study, the rate-dependent tensile stiffness of collagen fibers is shown to elevate the fluid pressurization and consequently, greatly enhances the reaction force. Therefore, it suggests that the viscoelasticity of collagen fibers is a determining factor in the mechanical function of articular cartilage. It should be emphasized that the viscoelasticity of collagen fibers can only show its role when the tissue is simulated under various loading rates and a suitable formulation of viscoelasticity is chosen with enough degree of strain-rate sensitivity. The QLV formulation of viscoelasticity that was applied for collagen fibers of cartilage can be effective only in a limited range of strain-rates. However, as concluded in Section 8.2, the strain-rate sensitivity is less noticeable at small strains. Therefore, the QLV formulation can still be effective in cartilage simulations under small strain loading. It should be noted, however, that according to the fluid-driven fiber-reinforcement of collagens (Li et al., 2008), even small changes to the stiffness of the fibers in small strain regime may be accentuated under high-rate compression loadings.

8.4 Conclusion

An anisotropic visco-hyperelastic constitutive model was developed in this study based on short-term and long-term internal variables. This constitutive model enjoys more strain-rate sensitivity in the short-term response while at the same time can account for the mid-term stress relaxation behavior of the material. The constitutive model can be divided into two
components of viscous functions according to their time scales. This facilitates the material characterization process and leads to more accurate prediction of the experimental data. The model is constructed upon an invariant-based energy function ensuring the thermodynamic consistency. The constitutive model is particularized for articular cartilage and ligament as two biological tissues with apparent viscoelastic properties. The model is characterized for both tissues based on the available experimental data. The proposed constitutive model provides a very good fit to the experimental tests which could not be achieved with the traditional integral type formulations of viscoelasticity.

Moreover, the tensile behavior of articular cartilage was experimentally investigated. The inherent transient response and hysteresis of the tissue was found to depend on both strain and strain-rate. The strain-rate sensitivity of the mechanical response of cartilage was examined and the constitutive model for cartilage was characterized with the obtained experimental results. The results also show non-recoverable deformations in the tensile testing of articular cartilage, especially when loaded up to 15% strain. The non-recoverable deformations are likely to be caused mainly by damage to the tissue at high strain-rates. The elevated hysteresis observed at high strain-rates in collagen fibers may be a part of the mechanism that contributes to the high hysteresis observed in impact loading of articular cartilage.

The proposed constitutive model was also numerically implemented into the finite element software package ABAQUS with a UMAT user subroutine coded in Fortran. The constitutive formulation was used to build a depth-dependent fiber-reinforced poro-visco-hyperelastic model of cartilage. The model was employed to study the depth-dependent fluid pressurization in the tissue as well as the reaction force in indentation and unconfined compression configurations respectively. The choice of the thickness of each layer through cartilage depth was found to affect the fluid pressurization. In addition, the rate-dependent stiffening of the collagen fibers observed in experiments was shown to have a crucial effect
on the rate-dependent compressive response of the tissue.

8.5 Future Work

The proposed constitutive model is furnished with an evolution equation with fixed time constants. Three time constants were used to predict the whole relaxation process. However, the time constants do not currently depend on the loading magnitude. Therefore, the evolution equation leads to a quasi-linear description of viscoelasticity. Considering that many viscoelastic biomaterials exhibit nonlinear behavior, by introducing a strain-dependent time constant, the model will become fully nonlinear and able to be used in a wider range of strains.

The finite element model of cartilage that uses the aforementioned constitutive model should also be tested against more experiments at different strain-rates. This leads to a better characterization of the material as well as an understanding of the effect of each constituent and its material properties on the rate-dependent viscoelastic response.

The constitutive model is able to capture high strain-rate phenomena. Therefore, it can be implemented in 3D patient-specific finite element models of knee joint to study the mechanisms that may lead to damage to knee components. Also, it can be used at physiological loading-rates during walking and running.

An interesting trend in the hysteresis of articular cartilage was observed during tensile experiments. Therefore, a hysteresis model can also be developed based upon the current constitutive model. The elevated hysteresis of collagen fibers at high strains and strain-rates can be formulated in the finite element model of cartilage to investigate its interaction with fluid-driven hysteresis of cartilage.

The tensile mechanical experiments on articular cartilage revealed new characteristics of cartilage. However, more experiments are needed in order to understand the questions remain unanswered. For instance, recording the experiments by a high speed camera may
help disclose whether any slippage occurs during testing. Also, observation of the samples after experiments under a microscope can also reveal some possible damages and micro-tears that might lead to the loss of stiffness. In addition, optical measurements for the thickness and deformation of the samples may be beneficial in producing more accurate results.
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