

Contrast and Contrastivism: The Logic of Contrastive Knowledge
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I. Introduction

Since its conception in the early 1950's, epistemic logic has undergone few changes. At base, epistemic logic is an adaptation of modal logic designed to treat knowledge rather than necessity. One of the major features of modal logic that epistemic logic retains is its propositional nature; what is known is some proposition. Because of this, epistemic logic's knowledge operator implies a certain knowledge relation; specifically a binary relation between a subject and a proposition (Ksp). Epistemic logic, then, is fine so long as the correct account of the knowledge relation is one in which that relation is binary.

Recently, however, there has been some debate as to whether or not the knowledge relation is indeed binary. Jonathan Schaffer's 2004 and 2008 papers *From Contextualism to Contrastivism* and *The Contrast-Sensitivity of Knowledge Ascriptions* argue for a novel epistemological theory, contrastivism, the upshot of which is that the knowledge relation isn't binary but instead ternary. Under this theory knowledge isn't a relation that holds between a subject and a proposition, but rather a relation that holds between a subject, a proposition, and a contrast proposition. Knowledge isn't of propositions *simpliciter* but rather of propositions in light of some relevant alternative proposition(s); some proposition *rather than* some other proposition.

Broadly this paper is about exploring the compatibility between epistemic logic and contrastivism and the possible need for, and structure of, a contrastivist logic. I'll argue that if contrastivism is true then the following are true

1. Epistemic logic is unsound.
2. Epistemic logic cannot represent contrastive knowledge.
3. Epistemic logic must be replaced by a contrastive logic with a binary knowledge operator.

And if contrastivism is false then the following are true:

1. Epistemic logic is fine.
2. Epistemic logic can represent contrastive knowledge.
3. No changes need to be made to epistemic logic.

Implied here is that the truth of contrastivism determines what it is for something to count as contrastive knowledge. If contrastivism is true then contrastive knowledge is a relatively strong concept that cannot be captured by epistemic logic, but if false contrastive knowledge is a fairly weak concept that can be captured by epistemic logic. A contrastivist logic is needed in general if contrastivism is true, not needed at all if contrastivism is false, and desired at all times by the contrastivist who takes his theory to be true.

There's nothing really surprising about these results or clever about the arguments that generate them; they're straightforwardly implied by the contrastivist account. What I take as interesting about this is, as far as I can tell, no published work on contrastivism makes any mention of a contrastivist logic or the logical representation of contrastive knowledge. Although some of the published works talk about contrast propositions in terms of possible worlds none make the leap to discussing contrastive knowledge in modal logic terms.

As suggested by the title, this paper will be split into three distinct sections. The first section is dedicated to discussing the structure and possible logical representation of contrastive sentences in the context of knowledge. The second section will offer some expository notes on contrastivism and outline why the logic from the first section will not satisfy the contrastivist. Inspired by the first two sections, the third section will be dedicated to a discussion of the logic of contrastive knowledge. It turns out that many of the features of contrastivism that necessitate a contrastivist logic likewise make one very difficult to offer. The second problem with offering a contrastivist logic is that there's a certain amount of ambiguity in Schaffer's account that makes determining what would count as acceptable semantics quite difficult. Although I'll offer an outline of a contrastivist logic (CL) and briefly discuss possible axioms for it I'm fairly certain that they'll both be inadequate or false because of the factors just mentioned.

II. Contrast

A. Contrastive Sentences

What are contrastive sentences? To answer somewhat glibly, a contrastive sentence (CS herein) is a sentence in which two propositions are held in contrast with one another. Put another way, a contrastive sentence is any sentence that establishes a contrastive relation between two (or more) propositions. Following Schaffer, I'll focus on the use of 'rather than' (RT herein) as the main way to indicate that a sentence is establishing a contrastive relation between its component propositions.

Contrastive Sentence: Any sentence in which two propositions, p and q , are connected by 'rather than'; ' p rather than q '.

So, what do we mean when we say ' p rather than q '? Given that the nature of p and q (as propositions) is well-understood, uncertainty about the meaning of contrastive sentences is almost certainly due to uncertainty about the meaning of RT. Consider the following contrastive sentences:

CS1: Calgary is in Canada rather than the United States

CS2: Calgary is in the United States rather than Canada

CS3: Calgary is in the United States rather than Mexico

CS4: Calgary is in Canada rather than Scotland

These sentences exhaust the truth-value combinations of p and q ; in CS1 p is true and q is false, in CS2 p is false and q is true, in CS3 both p and q are false, and in CS4 (fun fact) both p and q are true. What, if anything, can we say about the truth-values of CS1-CS4 on the basis of the truth-values of their respective p 's and q 's? It turns out that we actually can say surprisingly little, because the truth value of CS's aren't wholly determined by the truth-values of their composite propositions. Although I take it as intuitively correct that CS1 is true, it isn't true *just because* Calgary actually is in Canada and actually isn't in the United States. Likewise although CS2 seems intuitively incorrect, it isn't *just because* Calgary isn't actually in the United States and is actually in Canada. In the case of CS1, p being true and q being false are necessary for CS1's truth but not sufficient. On the other hand, p being false is sufficient for CS2 being false.

What about CS3 and CS4? If p being false was sufficient to make CS2 false then CS3 is likewise false. CS4 isn't obviously false; if we were ignorant about there being a Calgary in Scotland we might well affirm that CS4 is absolutely true. Although Schaffer might want to rule this sentence as un-evaluable I have no problem admitting that it's most likely false as I think we have good reason to hold q 's falsity as necessary for the truth of this type of sentence. To highlight why Schaffer hesitates to conclude that this sentence is false (or true) consider the following sentence:

CS5: Calgary is in Canada rather than $1+1=3$

What are we to make of this sentence? Presumably p is true and q is false but we'd hardly want to conclude that CS5 is true. This is because p and q in this case aren't mutually exclusive (like they weren't in CS4). Schaffer argues that to evaluate contrastive sentences the two propositions must be mutually exclusive, that is, it can't be the case that both p and q (it must be the case that $\neg(p \wedge q)$). We need to be careful here as the exclusivity we require is more than just $\neg(p \wedge q)$ as a matter of fact, because p being true and q being false trivially imply this. We need the truth of p and q to be negatively correlated; we need their truth-values to be dependent in a specific way. The obvious way to formalize this condition is $(p \rightarrow \neg q)$ (and by contraposition, $(q \rightarrow \neg p)$), but aren't these just logically equivalent to $\neg(p \wedge q)$? This is an odd situation where our non-logical vocabulary and our logical vocabulary don't exactly match up. In the case of CS5, p being true and q being false imply that $\neg(p \wedge q)$ but we'd want to deny that $(p \rightarrow \neg q)$ despite this being logically contradictory. I believe the disconnect here is that although $\neg(p \wedge q)$ may be true in this case, we can easily imagine a case in which $(p \wedge q)$; what we need is for it to be the case that $\Box\neg(p \wedge q)$. In *Contrastive Knowledge*, Schaffer implicitly recognizes this difficulty and formulates his requirement of mutual exclusivity as $\{P\} \cap \{Q\} = \emptyset$; loosely that there are no worlds in which p and q simultaneously.

As far as semantics for RT goes I believe that it's sufficient for p and q to be dependent in the $(p \rightarrow \neg q)$ sense as this implies that p and q are mutually exclusive. Given this we can outline some tentative truth-conditions for RT sentences:

A contrastive sentence ' p rather than q ' is true iff...

1. p is true
2. q is false
3. It's the case that $(p \rightarrow \neg q)$

As an aside, I earlier called RT a 'linguistic construct' but didn't elaborate on what kind of linguistic construct it is. Is it a sentential operator? I'm honestly not sure. Note how I defined 'contrastive sentence' as any sentence with two propositions connected by RT and then went on to offer no contrastive sentences fitting that schema. This is because my concern is with the logical representation rather than the linguistic reality of contrastive sentence. What I mean by this is that a sentence of the form 'p rather than q' seems blatantly ungrammatical but logically meaningful. Consider:

CS1Expanded: Calgary is in Canada rather than Calgary is in the United States

I take this sentence as equivalent to CS1, but as before, I don't believe that it's completely grammatical; we almost certainly understand its meaning but its form is wrong. Considering the expanded CS5 might actually suggest a better definition for a contrastive sentence:

CS5Expanded: Calgary is in Canada rather than Calgary is in $1+1=3$

It's now more obvious why CS5 is un-evaluable; its expanded form is meaningless. That is, we might deny that CS5 is a contrastive sentence because it can't be expanded into a meaningful 'p rather than q' sentence. We would likewise consider CS2-CS4 false just because they unpack into meaningful but false sentences. On this basis we might offer the revised definition of a contrastive sentence:

CS*: A contrastive sentence is any grammatical sentence that can be expanded into a meaningful construction of the form 'p rather than q'

I hesitate to use sentence due to the ungrammatical nature of these expanded rather than 'sentences', so I use construction instead. Schaffer uses the term construction himself, so it's not without precedent in this context. Taking this as our definition of contrastive sentences actually does some work to ensure that only relevant alternatives to p saturate the q spot. If q is an irrelevant alternative in the short-form sentence then the long-form sentence will likely be meaningless.

B. Knowledge of Contrastive Sentences

Assume that we're interested in representing knowledge of contrastive statements. What does this look like? The obvious answer here is that to know a contrastive sentence is just to know its truth conditions. Here's how we might represent knowledge of CS's in epistemic logic:

$K(p \text{ RT } q) \text{ iff } K(p) \text{ and } K(\neg q) \text{ and } K(p \rightarrow \neg q)$

If this is acceptable then there seems to be no real incompatibility between epistemic logic and contrastive knowledge as contrastive knowledge can easily be represented within epistemic logic.

Although I do indeed believe this to be an adequate epistemic logic representation of knowledge of contrastive sentences, despite the fact that I've equivocated between the two, I don't believe that is nearly an adequate representation of contrastive knowledge. To be clearer, for the non-contrastivist contrastive knowledge *just is* knowledge of contrastive sentences; the non-contrastivist has no reason to need more than the reductive account of contrastive knowledge offered above. For the contrastivist, however, contrastive knowledge *isn't* reducible to knowledge of contrastive sentences. Although both the non-contrastivist and contrastivist might be interested in the logical representation of contrastive knowledge what counts as an adequate logic to the non-contrastivist will necessarily be seen as insufficient by the contrastivist due to his additional epistemological commitments. Although the non-contrastivist *might* be happy with an epistemic logic representation of contrastive sentences the contrastivist *can't* be happy with such a representation. To show why this is the case I'll first need to discuss...

III. Contrastivism

A. Historical Context

As mentioned in the introduction, contrastivism as a theory deals with the nature of knowledge, specifically the structure of the knowledge relation. Knowledge, as a concept, can be seen as a relation between a subject and, so as not to use any philosophically loaded terms, *something*. Specifically, 'knows' is meant to capture something like the notion of epistemic certainty with regards to some state of affairs, fact, or proposition. The immediate questions that may be asked of this relation (and the questions of which most epistemological literature is devoted to answering) are (Blaauw, 2007):

1. What is the nature of the knowledge relation?
2. What is the nature of the relata?

Answering these two questions (in some form or another) has been the traditional task of epistemologists, and has produced the majority of the material taught in contemporary epistemology classes. Participants in the debate, although they might object, can largely be classified into two groups; the dogmatists, who hold knowledge as obtainable, and the skeptics, who believe that we can't possibly meet the high standards that knowledge require. Recently, however, much of the debate has been focused on attempts to reconcile the two positions; to produce a conception of knowledge that gives adequate deference to the valid concerns of the skeptics while still making knowledge* available for everyday use.

Those pursuing this type of program can, to use the term *very* loosely, be considered epistemic pragmatists. If there's any theory that we can consider truly pragmatic in this sense it would surely be contextualism. Contextualism is catch-all phrase for theories that hold the truth-value of knowledge attributions as context sensitive. That is, what 'knows' means or denotes depends on the context in which it's being uttered. Contextualists argue that 'knows' shifty denotation might satisfy both the dogmatist and the skeptic by allowing for true knowledge attributions in everyday situations but still deny the truth of knowledge attributions when the epistemic stakes are higher (when skeptical scenarios are salient). Or, to paraphrase Schaffer (2004), contextualism allows us to know in the courtroom but not in the classroom.

Needless to say, contextualism isn't without problems. Chief among them (from a logical perspective, at least) is that it seems to imply a great many qualitatively different knowledge relations; one for each distinct context with no obvious way to trade between them. Although many contextualists have offered arguments against this claim many critics remain unconvinced, believing contextualism to be a deeply flawed theory.

Inspired by the treatment of knowledge in Fred Dretske's *Epistemic Operators*, Schaffer's contrastivism can be thought of as a response to the perceived failings of contextualism. Schaffer argues that contrastivism, more so than contextualism, can serve as a bridge between the dogmatist and the skeptic. I mentioned Blaauw's two questions earlier to outline exactly why contrastivism is unique as far as epistemological theories go (and why

contrastivism, rather than any of the other theories might demand a revision of epistemic logic). Contrastivism doesn't directly answer either of those two questions, rather it answers a third question (Blaauw, 2007):

3. What is the adicity (arity) of the knowledge relation?

B. Contrastivism

The vast majority of traditional epistemology holds the knowledge relation to have an adicity of 2; there are two distinct relata, a subject and a proposition. This view generates the knowledge attribution schema 'S knows that P', where S and P are replaced with suitable candidates. Contrastivism, negatively, is the thesis that there are no true sentences of the form 'S knows that P'. Positively, contrastivism is the thesis that knowledge is a three-place relation taking a subject, a proposition, and a (or set of) contrast propositions (relevant alternatives). In Schaffer's own words, knowledge is a "...a *ternary, contrastive* relation..." rather than a "...*binary, categorical* relation..." (Schaffer, 2004).

As an answer to Blaauw's third question rather than his first and second, contrastivism isn't exactly an account of knowledge in the traditional epistemological sense. Although Schaffer has his preferred necessary and sufficient conditions for knowledge, contrastivism as a theory remains neutral with regards to such conditions; it "...is compatible with virtually any analysis of knowledge (even none at all)" (Schaffer, 2004). This gives contrastivism considerable

appeal, as if it's the case that binarity is an unquestioned epistemological assumption (that is, lacking explicit argumentation) then there ought to be no real reason to prefer it over ternicity, for which there is explicit positive argumentation.

Returning to the negative aspect of contrastivism, in Schaffer's own words:

"The traditional epistemologist supposes that there is one Ksp relation, while the contextualist suggests that there are many. The contrastivist says there is none – the assumption that knowledge is a binary relation is an error due to the seductive pull of the surface grammar of a special form of utterance." (Schaffer, 2004)

Schaffer is clear in his opposition to *any* knowledge of propositions *simpliciter*; no proposition is known except with regards to some relevant alternative, all non-contrastive knowledge-ascriptions are false. As a stark example, Schaffer would claim even that 'Agent s knows that 2+2=4' is false; although there is something we can know about '2+2=4' it's going to be something like 'Agent s knows that 2+2=4 rather than {1, 2, 3, 5, 6... n}'. In this case the *q* in the contrastive sentence is a set of rejected relevant alternatives. This simply abbreviates the (perhaps infinite) conjunction ($K_{s4,1} \wedge K_{s4,2} \dots \wedge K_{s4,n}$).

Schaffer acknowledges that rejecting the binary knowledge relation wholesale might be seen as unnecessarily radical. The proponent of this view admits that although some knowledge ascriptions have overtly contrastive form there are still true binary knowledge ascriptions; why

not just augment the binary relation with a separate ternary relation to handle contrastive knowledge? Schaffer responds with the following:

“Why not let knowledge come in both binary and contrastive forms? Because (i) this would require an ambiguity in knows that the evidence does not support (ii)... the contrastive form fits all our knowledge ascriptions, and (iii)... the binary form is paradoxical.” (Schaffer, 2004)

To understand this response it's necessary to understand the motivations behind contrastivism. As already mentioned, contrastivism can be seen as a response to the perceived failings of contextualism. Schaffer suggests that contrastivism overcomes those failings by “...employing a better linguistic model.” (Schaffer, 2004). In general, Schaffer's arguments for contrastivism are based on linguistic evidence; he argues that contrastivism is the correct account of the knowledge relation because our actual use of 'knows' refutes binarity and suggests ternicity.

Ternicity: 'Knows' denotes a three-place relation $Kspq$

Specifically, Schaffer endorses the view that “'knows' is lexically ternary. Since lexical structure projects onto syntactic structure (Chomsky, 1977), this implies that all knowledge ascriptions contain a syntactically real contrast variable q in their logical form.” (Schaffer, 2004). Trivially, if it's true that all knowledge ascriptions contain a syntactically real contrast variable

then it's easy to imagine why Schaffer denies the truth of any knowledge ascriptions without such a thing. Likewise, if the evidence suggests that 'knows' denotes a three-place relation then allowing for binary knowledge is allowing for un-evidenced ambiguity in 'knows' for it would imply a second denotation for 'knows' (denoting a two-place relation in addition to the three-place contrastive relation).

How does the contrastive form fit all our knowledge ascriptions? The contrastive account of knowledge...

"...links knowledge to inquiry and to discrimination. There is no such thing as inquiring into p, unless one specifies: as opposed to what? There is no such thing as discriminating that p, unless one adds: from what? And likewise I will argue that there is no such thing as knowing that p, unless one clarifies: rather than what?" (Schaffer, 2004)

To Schaffer, knowledge ascriptions are used to certify that some agent can "...answer the question..." (P.236). Consider the following case (adapted from *Contrastive Knowledge*):

The world's largest sapphire has been stolen. Sherlock Holmes has been hired to determine who the thief was. There are three suspects; Mary, Peter, and Paul. After a thorough investigation, Holmes discovers Mary's fingerprints at the scene of the crime and declares "Aha! I know who stole the sapphire!"

Here Holmes is certifying that he can answer the question 'Who stole the sapphire?'. Mary's fingerprints at the scene of the crime establish that it was her who stole the sapphire rather than Peter or Paul, so Holmes knows that Mary stole the sapphire rather than Peter or Paul. This seems just to be begging the question in the contrastivist's favor however; what's wrong with simply saying that Holmes knows that Mary stole the sapphire? Presumably 'Mary stole the sapphire' is indeed an answer to the question 'Who stole the sapphire?', but it's also an answer to the question 'What did Mary steal?'. The problem here is that Mary's fingerprints at the scene of the crime don't establish that Mary stole the sapphire (rather than the diamond, or whatever) only that if it was anyone that stole something it was her. The binary knowledge relation doesn't allow for the kind of partial knowledge that the details of this case seem to imply. On the basis of fingerprint evidence alone Holmes ought not to know the answer to 'What did Mary steal?', only to 'Who stole the sapphire?', but binarity seems to imply that Holmes knows the answer to both. If the purpose of knowledge is to score progress in inquiry then ternicity fares far better than binarity.

It does seem as if there are some knowledge ascriptions that cannot rightly be made without being of overtly contrastive form, but is it the case that all knowledge ascriptions have an overtly contrastive form? Although 'all' questions are somewhat difficult to answer, I'd venture that it's plausible that the contrastive form fits all our knowledge ascriptions. Consider again the case where agent *s* knows that $2+2=4$. Does this have an overtly contrastive form? Yes, and it depends on what it is about $2+2=4$ that *s* knows. Consider the following contrastive variants of *s*'s knowledge:

- a) s knows that $2+2=4$ rather than $\{1, 2, 3, 5, \text{etc}\}$ (anything but 4)
- b) s knows that $2+2=4$ rather than $2+3=4$ (or $2+1$, or $2+4$, etc)
- c) s knows that $2+2=4$ rather than $3+2=4$ (or $1+2$, or $4+2$, etc)
- d) s knows that $2+2=4$ is true rather than $2+2=4$ is false
- e) s knows that $2+2=4$ is implied by the true arithmetic statement $1+1=2$ rather than the true arithmetic statement $\pi = 3.1415\dots$

To be quite honest I'm not sure what the contrastivist would say about these examples. What does seem clear is that each of (a)-(e) say something different about what we can know about $2+2=4$ that the binary knowledge relation can't exactly capture. It's conceivable that an agent with limited mathematical knowledge might know only one (or some combination of) (a)-(e), so not only does ' s knows that $2+2=4$ ' have an overtly contrastive form, it has multiple overtly contrastive forms that have the ability to express s 's knowledge far more precisely than their non-contrastive counterpart.

Schaffer's third argument against the binary knowledge relation is that it succumbs to certain paradoxes that the ternary relation doesn't. Specifically, he argues that contrastivism resolves the closure paradox that seems to undermine the binary knowledge relation. Consider (Schaffer, 2004):

(C1) Moore knows that he has hands.

(C2) Moore doesn't know that he is not a brain-in-a-vat

(C3) If Moore doesn't know that he is not a brain-in-a-vat, then he doesn't know that he has hands.

These premises are apparently all plausible but contradictory when taken together. This is an example of a simple closure paradox generated by deducing contradictory knowledge. Binary responses to this paradox vary, but the strategies either involve denying the truth of one of the claims or denying that the three claims are indeed contradictory. The skeptic, for example, denies C1, whereas the dogmatist might deny C2. The contextualist would deny that C2 and C3, although they entail $\neg C1$ by *modus ponens*, don't contradict C1 because the "...denotation of "knows" shifts, rendering the argument equivocal..." (Schaffer, 2004).

One of contrastivism's main features is that it supposedly can reconcile ordinary knowledge and skeptical doubt because claims of each type involve different contrasts. The contrastivist re-writes the paradox like this (Schaffer, 2004):

(C1) Moore knows that he has hands rather than stumps

(C2) Moore doesn't know that he is handed rather than envatted

(C3) If Moore doesn't know that he's not envatted rather than envatted then he doesn't know that he's handed rather than envatted.

There doesn't seem to be any obvious contradiction in this scenario. Contrastivism can overcome this paradox because it limits closure schemas to those that will "...preserve discrimination of truth...". In Schaffer's own words "In general, for any subject s and proposition p , s is in a position to know that p rather than q for any proposition q within s 's discriminatory range. Whereas for any q that extends beyond s 's discriminatory range, $\neg Kspq$." (2004). The two valid closure schemas Schaffer offers are as follows:

Expand-p: if $p_1 \rightarrow p_2$ then $Ksp_1q \rightarrow Ksp_2q$

Contract-q: if $q_2 \rightarrow q_1$ then $Kspq_1 \rightarrow Kspq_2$

Whereas the following schemas are invalid:

Contract-p: if $p_2 \rightarrow p_1$ then $Ksp_1q \rightarrow Ksp_2q$

Expand-q: if $q_1 \rightarrow q_2$ then $Kspq_1 \rightarrow Kspq_2$

Replace-p: $Ksp_1q \rightarrow Ksp_2q$

Replace-q: $Kspq_1 \rightarrow Kspq_2$

Ex-p is meant to capture the notion that if I can discriminate p from q then I can likewise discriminate something implied by p from q . *Con-q* attempts to capture the notion that if I can discriminate p from q then I can also discriminate p from something that implies q . In both cases we're limited to only weaker knowledge by implication. The closure paradox arises from

an inappropriate use of *Rep-q*; Moore knowing that he has hands rather than stumps cannot be used to infer that Moore knows that he has hands rather than vat-images of hands.

Before I move on, a couple of notes about these closure schemas. They're of obvious interest to a logic of contrastive knowledge so I'll be discussing them in more detail later (as possible axioms for a contrastive logic). For now though, I'd like to raise a concern I have regarding these schemas. In my discussion of RT and as discussed by Schaffer (2004), contrastive sentences require propositions that are mutually exclusive and contrast propositions that are appropriately relevant. Neither of Schaffer's two valid schemas preserve mutual exclusivity or guarantee that the contrast proposition will remain a relevant alternative to p . Say that I know that the sky is blue rather than green. Presumably if it's true that the sky is blue then, trivially, blue is a color ($\text{SkyBlue} \rightarrow \text{BlueColor}$) but then all of a sudden I also know that blue is a color rather than the sky is green. Blue being a color and the sky being green are presumably not mutually exclusive and the color of sky hardly seems like a relevant alternative to a claim about the existence of a certain color. Returning to the notion of knowledge as certifying that one can answer the question, it's easy to see what question "the sky is blue rather than green" answers but what question does "blue is a color rather than the sky is green" answer? Even further, knowledge by implications seems as if it should only hold if we have knowledge of that implication's truth, but by Schaffer's own account we can't know anything of the form ' $s \rightarrow t$ ' because it's a single proposition. This last concern can be remedied by contrastivizing the implication so that it can be the subject of knowledge, which I'll discuss in the upcoming section.

C. Contrastivism and Epistemic Logic

The goal of this paper is to establish that a contrastivist logic is necessary if contrastivism is true. Working under the assumption that we want the ability to logically represent knowledge, the need for a contrastivist logic is established depending on the truth of the following statements:

1. Contrastive knowledge cannot be represented in epistemic logic.
2. If contrastivism is true then epistemic logic is unsound.
3. Contrastivism is true.

If (1) is true then epistemic logic is adequate as far as the logical representation of knowledge is concerned, but if we're still interested in representing contrastive knowledge (the stronger version of knowledge of contrastive sentences) then we still need a modified epistemic logic for that purpose (call this a contrastivist logic). If (2) and (3) are true then epistemic logic says nothing true about knowledge and therefore cannot be held as representing it; it needs to be replaced altogether with a contrastivist logic.

Representation

(1) is false if contrastive knowledge can be represented in epistemic logic. In essence this is a reduction strategy that attempts to define the ternary relation ($Kspq$) in binary relation

(Ksp) terms, or what should be a binary knowledge operator (Cpq , p is contrastively known rather than q) in terms of a unary knowledge operator (EL's Kp). This is the kind of strategy I considered in section 1 that I believe would only satisfy the non-contrastivist. Here's a generalization of the strategy and a couple of examples from the literature (Schaffer, 2008):

ELRep: Explain the contrast by holding it as some kind of logical embellishment added to the known proposition. Cpq is $K(p \text{ PL } q)$. *CondRep* and *ConjRep* are examples of this.

CondRep: Cpq is simply $K((p \vee q) \rightarrow p)$

ConjRep: Cpq is simply $K(p \wedge \neg q)$

TCRep: Cpq is simply $K(p \rightarrow \neg q)$ and $K(p)$ and $K(\neg q)$ (Knowing the truth-conditions of a RT sentence)

I take it as established that the non-contrastivist will likely be satisfied by knowledge of contrastive sentences (from section 1, *TCRep* here). I mentioned then that knowledge of contrastive sentences and contrastive knowledge are different things and that the latter is not reducible to the former. I use contrastive knowledge here to refer to the knowledge of contrastivism, so any reduction attempt of this sort must not violate any tenets of contrastivism if it is to satisfy the contrastivist. To be clear, this section assumes contrastivism's truth to argue that the contrastivist cannot be satisfied with an epistemic logic representation of their account of knowledge; the arguments here are just arguments against this type of strategy by contrastivists.

In response to this strategy I quote Schaffer (Schaffer, 2008):

“There are two main problems with the conditionals [conjunctions, disjunctions] strategy. The first problem is the problem of linguistic plausibility. In general, the best semantic treatment of question, “rather than” clauses, clefts, foci, and presuppositions is in terms of contrasts. Semantic sensitivity to these features is sensitivity to the contrasts. There are no conditionals [conjunctions, disjunctions] in these constructions. The thought that contrastive constructions can be analysed via conditionals seems to be a pure invention, fabricated solely to fit the knowledge ascription data onto the Procrustean bed of Ksp .”

The point here is that the main purpose of the contrast proposition in contrastive knowledge ascriptions is to vary the truth-value of the knowledge ascription in different contexts (conversational, answering different questions, etc). Reducing contrast to propositional logic embellishment undermines much of the appeal of holding the contrast proposition as a distinct argument in that its ability to vary the truth value of the knowledge proposition decreases drastically.

Although *TCRep* looks similar to the other strategies, it's meant to represent the notion that Cpq can be reduced to Kp where p is the proposition “ p rather than q ”; in this case I consider the proposition “ p rather than q ” to just be its truth-conditions. The problem with this strategy is that it “...treats the contrast as a mere adjunct of what is known, so that “ s knows

that p rather than q is treated as; Ksp_{adj} , where p_{adj} is the proposition that p rather than q (Schaffer, 2008). According to Schaffer an adjunct is a semantically optional element whereas contrast clauses are syntactically real arguments that require saturation to generate coherent statements (of knowledge). *TCRep* almost certainly treats q as semantically optional as the logic still allows for knowledge of p without reference to q . Epistemic logic seems to lack the ability to require a contrast q for every proposition p so contrast will almost necessarily be treated as an adjunct in our EL representation of contrastive knowledge.

More generally, it's hard to imagine how, if the ternary knowledge relation isn't reducible to a binary knowledge relation that an adequate logic for it could possibly have a unary operator. The contrastivist insists that contrast proposition slot in the knowledge relation is syntactically real and requires saturation for the knowledge relation to hold but epistemic logic's unary operator has no syntactic opening for that contrast proposition and epistemic logic has no real mechanism to ensure that even a simulated q *must* be filled.

D. Compatibility

(2) suggests that contrastivism and epistemic logic are incompatible, and that if contrastivism is true then epistemic logic is unsound. What do I mean by unsound? Simply that epistemic logic falsely attributes knowledge. This unsoundness is a trivial product of contrastivism's negative characterization (that no sentences of the form " s knows that p " are true). Everywhere epistemic logic ascribes knowledge it does so non-contrastively; every

sentence of epistemic logic using the knowledge operator is false. This implies that all reduction strategies will necessarily fail to satisfy the contrastivist. This is the reason I take contrastive knowledge as irreducible to knowledge of contrastive sentences. To the non-contrastivist, contrastive sentences are just propositions and there's absolutely no problem with knowing propositions *simpliciter*; there is no difference between contrastive knowledge and knowledge of contrastive sentences. To the contrastivist, contrastive sentences are just propositions and propositions *can't* be known *simpliciter*; contrastive knowledge doesn't and can't consist in knowing propositions representing contrastive sentences.

E. Is Contrastivism True?

This paper is meant to establish the need for a contrastivist logic and I believe that, with regards to the contrastivists, that task is largely complete. For non-contrastivists the need for a contrastivist logic depends entirely on the truth of contrastivism, which I've yet to offer explicit argumentation for. In the interest of thoroughness I'll briefly outline some of the arguments for contrastivism here.

1. Linguistic data justifies the contrastivist thesis.

This is Schaffer's position and it's convincingly argued (I believe) across the excellent *Contrastive Knowledge, The Contrast-Sensitivity of Knowledge Ascriptions, and From*

Contextualism to Contrastivism. Although some of the argumentation is along traditional epistemological veins the large majority is, in some sense, empirical.

2. Contrastivism can likewise be justified without appeal to linguistic data.

Walter Sinnott-Armstrong, in *A Contrastivist Manifesto*, argues that a binary knowledge relation is conceptually inferior to a ternary knowledge relation and deserves replacing. Sinnott-Armstrong's contrastivism is stronger than Schaffer's in that it holds that all claims of reasons are relative to contrast classes. Because of its generality, Sinnott-Armstrong argues for its application to many non-epistemological problems such as explanation, free will, and reasons for actions. The argumentation in *Manifesto* is more traditionally philosophical and demonstrates that although contrastivism might *best* be justified by linguistic data it isn't *only* justified by linguistic data. Additional argumentation of this sort can be found in Adam Morton and Anti Karjalainen's 2003 paper *Contrastive Knowledge*.

3. Results from recent studies in experimental epistemology suggest that people's intuitions about knowledge in cases match contrastivism rather than contextualism or invariantism's predictions.

In their forthcoming paper *Contrastivism Surveyed*, Jonathan Schaffer and Joshua Knobe survey some of the recent studies of intuitions about knowledge in cases. These studies are generally performed on a large scale (over 1000 participants) and ask the participants to

determine whether the agent in the cases they're given to read should be considered to know given the details of the case. The purpose of these studies is to measure the effects of factors such as stakes, context, and salience on intuitions about knowledge. Schaffer and Knobe identified three such factors (contrast of the ascriber, salience for the ascriber, stakes for the subject) and determined how each account of knowledge (contrastivism, contextualism, subject-sensitive invariantism, classical invariantism) predicted people's intuitions would change with these factors. Experimental evidence suggests that participants willingness to ascribe knowledge shifts with contrast and salience for the ascriber but not with stakes for the subject. This is exactly what contrastivism predicts, so Jonathan Schaffer (with reservations) takes this as partial vindication of the contrastive model.

What now?

In previous versions of this paper I included far more argumentation against epistemic logic from the contrastivist standpoint. I quickly realized that such a thing was largely a waste of time and effort because an epistemic logic representation of contrastive knowledge is a complete non-starter for the contrastivist; a point I hope I've made somewhat clear here. The takeaway from this section is that a contrastivist logic is needed (and likely desired) by contrastivists regardless of the truth of their theory, required in general if their theory is true, and not required at all if their theory is false. This seems more than sufficient to motivate a discussion of such a logic.

IV. Section 3 – The Logic of Contrastive Knowledge

A. Necessary Features

Although I've done contrastivism as a whole little justice, I believe that I've mostly picked out its logically interesting features (both in my exposition of the theory and in the arguments against a unary operator). Here are some of those features:

- 1) Single propositions *cannot and must not be known*. Ever.
- 2) p and q must be in a certain relation with regards to one another.
 - a) They ought to be mutually exclusive.
 - b) Their truth-values ought to be dependent.
- 3) Contrastive knowledge is factive, $Kspq$ only if p .
- 4) Contrastive knowledge is closed under modified logical consequence.
- 5) The knowledge operator will be called C
- 6) The knowledge operator must have an arity of 2; Cpq .
 - a) The p operand takes the proposition to be known.
 - b) The q operand takes the contrast proposition
 - c) From 2, only certain p and q pairs can occupy p and q
 - d) C can subscript agents like K, $C_Apq = \text{"A contrastively knows } p \text{ rather than } q\text{"}$

Given that this is a modal logic, let's consider the obvious semantics for Cpq first:

CS1: Cpq iff pq at all accessible worlds

This is the direct analogue of the semantics for K in epistemic logic. Although this isn't outright implausible I doubt that the contrastivist would be satisfied by this explanation, as requiring the contrastive statement to be true at all accessible worlds completely contradicts the purpose of the contrast proposition in the first place. Some of the accessible worlds will be worlds in which skeptical hypotheses are true; what q does is allow us to ignore those worlds by establishing a modal context (a subset of all possible worlds) in which we can more easily evaluate p .

*CS2: Cpq iff pq in some subset of all the *a priori* possible worlds*

Although this may be more accurate it isn't much clearer than *CS1*. What determines the subset of possible worlds in which we evaluate pq ? It seems obvious that the subset needs to be definable from p and q as that's really all the information we have at hand when evaluating a specific contrastive knowledge attribution.

CS3: Cpq iff pq in the subset of all possible worlds defined by $\neg(\neg p \wedge \neg q)$

Here we consider just the worlds in which not neither p nor q , which is just a clumsy way of saying that we only consider the worlds in which one or the other are true. What is it for pq

to be the case in each of these worlds? We might fall back on the truth-conditions for rather than sentences from section 1 and modify the semantics for Cpq accordingly.

$CS4:Cpq$ iff $(p \rightarrow \neg q)$, p , and $\neg q$ at all worlds defined by $(p \vee q)$

Immediately evident here is that Cpq just if there are no q worlds in the set of worlds defined by $(p \vee q)$. Consider the following sentences:

Easy: Moore knows that he has hands rather than stumps

Hard: Moore knows that he has hands rather than vat-images of hands

For the sake of argument assume that *easy* is true and *hard* is false. $CS4$ seems to imply that the reason *easy* is true is because there are no possible worlds in which 'stumps' and the reason that *hard* is false is because there are possible worlds in which 'vat-images', which I believe misses the mark somewhat. Indeed, pq isn't really meant to be evaluated at all. In the case of contrastive knowledge ascriptions, q plays a couple of different roles:

- 1) q is that from which p must be discriminated
- 2) q indicates which question is being answered
- 3) q focuses our epistemic attention to a limited set of epistemic alternatives
- 4) q is that which must be ruled out to Cpq

Does (1) suggest any possible changes to *CS4*? The ability to discriminate between p and q certainly seems relevant to our ability to Cpq , and does seem like it might be the relevant difference between *easy* and *hard*.

CS5: Cpq iff p can be discriminated from q at all worlds defined by $(p \vee q)$

I'm not exactly sure to interpret this. Revisiting the Moore example, it seems to suggest that Moore, if he is in a world in which he has stumps or a world in which he has hands, can determine which of those things he has. It seems important to limit the worlds to those worlds on the discrimination account because of the case where Moore is in a world where he has vat-images of hands. In such a world Moore obviously can't discriminate between hands and stumps, so if such worlds are included then the knowledge attribution 'hands rather than stumps' is false.

How does (2) fare? In *easy*, the question is something like "What does Moore have on the end of his arms?" to which Moore can give the correct answer "hands rather than stumps". Worlds in which 'vat-images' are eliminated from consideration here, as vat-images aren't something that one has on the ends of their arms; they cease to be a relevant alternative. Moore's ability to answer this question suggests that, a la (4), he is able to rule out stumps.

CS6: Cpq iff $\neg q$ and $(\neg q \rightarrow p)$ at all accessible worlds in which $(p \vee q)$

This is intended to represent the scenario when a multiple choice question with two mutually exclusive answers is posed. If one of the answers can be ruled out you can conclude that the other answer is the case; you know that the un-ruled out answer rather than the ruled out answer. Despite being similar in form to *CS4* I believe that this is somewhat more plausible as the added emphasis on the contrast proposition over *CS4* seems to better represent the way we might actually go about answering questions.

There's one proposal that I'd like to consider similar to *CS6*, based on (5):

5) q is that for which if negative proof can be given, p can be concluded

CS7: Cpq iff $\Box_{pq}\neg q$ where $\Box_{pq}\neg q$ is true when $\neg q$ at all $(p \vee q)$ worlds.

Here I use the \Box operator to denote proof; in this case the subscript determines a disjunction that determines the set of worlds at which $\neg q$ must hold for $\Box_{pq}\neg q$ to be true. Although this is actually the weakest of all the proposals it's meant to show that the notion of evaluating only certain worlds can be naturally accounted for (either this way, or via subsets of states). This isn't meant to stand alone but might serve as a necessary condition for Cpq in another definition.

Although I don't find any of *CS1-CS7* to be exceptional I believe that *CS6* is probably the most plausible. As such the semantics offered in the following section are meant to reflect *CS6*.

B. Contrastivist Logic (CL)

This is a modal logic, so we immediately have a set of epistemic alternatives (worlds) and an accessibility relation for that set. These are defined as follows:

S = the set of all epistemic alternatives.

\mathcal{P} = the accessibility relation for S

We consider propositions relative to subsets of S so a formal treatment of those subsets couldn't hurt. We also have:

S_{pq} = the set of S worlds in which $(p \vee q)$ is true ($S_{pq} \subseteq S$)

\mathcal{P}_{pq} = the accessibility relation for S_{pq}

As per CS6 we arrive at contrastive knowledge by a process of ruling out alternatives that imply the answer to the question, so:

Cpq iff $\neg q$ and $(\neg q \rightarrow p)$ in all S_{pq} worlds.

Adding a truth-assignment gives us a CL model \mathbb{M} :

$\mathbb{M} = (S, \pi, S_{pq} \dots S_{xy}, \mathcal{P}, \mathcal{P}_{pq} \dots \mathcal{P}_{xy})$

S = set of states.

π = truth-assignment to each proposition per state.

S_{pq} = The subset of S worlds in which $(p \vee q)$ is true ($S_{pq} \subseteq S$)

\mathcal{P} = The accessibility relation for S

\mathcal{P}_{pq} = the accessibility relation for S_{pq} (Would this be different?)

With this we can define truth in a model for K as follows:

$(\mathbb{M}, s) \models Cpq$ iff $(\mathbb{M}, t) \models \neg q$ and $\mathbb{M}, t \models (\neg q \rightarrow p)$ for all t such that $\mathcal{P}_{pq}(s, t)$

Which just says that if $\neg q$ and $(\neg q \rightarrow p)$ is the case in all states t accessible via \mathcal{P}_{pq} from s (all $t \in S_{pq}$) then we can conclude that Cpq . Likewise if $\neg p$ is the case in S_{pq} then Cpq . It follows from the semantics that $\neg Cpq$ iff q for some $t \in S_{pq}$. From this we also have an account of something like contrastive belief in the dual of K :

$(\mathbb{M}, s) \models Bpq$ iff $((\mathbb{M}, t) \models \neg q$ and $\mathbb{M}, t \models (\neg q \rightarrow p)$ for some t such that $\mathcal{P}_{pq}(s, t)$

Which just says that if $\neg q$ is the case in some state t accessible via \mathcal{P}_{pq} from s (some $t \in S_{pq}$) then we can conclude that Bpq .

Axioms for CL

(C_1) - $C(\phi \rightarrow \psi, \phi \rightarrow \neg\psi) \rightarrow (C\phi \rightarrow C\psi)$

(C₂) - $C(\gamma \rightarrow \rho, \gamma \rightarrow \neg\rho) \rightarrow (C\phi\rho \rightarrow C\phi\gamma)$

(C₁₂) - $C(\phi \rightarrow \Psi, \phi \rightarrow \neg\Psi) \rightarrow C(\gamma \rightarrow \rho, \gamma \rightarrow \neg\rho) \rightarrow (C\phi\rho \rightarrow C\Psi\gamma)$

These axioms are straight ports of Schaffer's *Expand-p* and *Contract-q* also argued for by Jonathan Kvanvig in *Contextualism and Closure* (2008). Because all we have is a binary operator and we require knowledge of the *Exp/Con* implications to deduce knowledge on their behalf I've had to contrastivize these axioms somewhat. $C(\phi \rightarrow \Psi, \phi \rightarrow \neg\Psi)$, for example, is read as "It's contrastively known that $\phi \rightarrow \Psi$ rather than $\phi \rightarrow \neg\Psi$ ". C₁₂ is simply both *Expand-p* and *Contract-q* combined into one axiom.

(D) $C\phi\psi \rightarrow \neg C\psi\phi$

(T) $C\phi\psi \rightarrow \phi\psi$

(T*) $C\phi\psi \rightarrow \phi$

(4) $C\phi\psi \rightarrow C(C\phi\psi, C\psi\phi)$

(5) $\neg C\phi\psi \rightarrow C(\neg C\phi\psi, C\psi\phi)$

(.2) $\neg C(\neg C\phi\psi, C\phi\psi) \rightarrow C(\neg C\psi\phi, C\phi\psi)$

These are contrastivized versions of the important axioms of epistemic logic. We now have two factivity axioms to denote that contrastive knowledge implies the factivity of both the contrastive statement and of just the knowledge proposition. It's interesting to note that although I haven't tested these axioms in CL most of them have intuitively correct readings although it's likely that some will mistakenly formalize some of Schaffer's disallowed schemas.

V. Conclusion

Despite the difficulty in producing one, I do genuinely believe that there's a need for an adequate contrastivist logic, whatever that might look like. What I've outlined in the preceding section might charitably be considered first steps towards a thing. Despite the fact that I could easily argue against my own contrastivist logic, I take the fact that it can be offered while not violating any of the 6-odd necessary conditions I outlined as a promising sign for the prospects of such a program. It's somewhat discouraging that many of the same epistemological commitments that necessitate, given the truth of contrastivism, a contrastivist logic also make that logic more complex than traditional epistemic logic. There is still some hope that the clearing up of some ambiguity surrounding the contrastive knowledge relation will reveal an intuitive and elegant logic.

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