UNIVERSITY OF CALGARY

Double-Head Studs as Shear Reinforcement in Concrete I-Beams

by

Ramez Botros Gayed

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ABSTRACT

This research aims to explore the feasibility of use of double-head studs as replacement of conventional stirrups in beams. Six reinforced concrete beams having I-shape cross-sections and heights ranging from 300 to 600 mm have been produced. The shear reinforcement in half the length of each beam is in the form of single leg stirrups with 90° and 135° bends at the ends. In the other half of the beam, double-head studs are used as shear reinforcement and fabricated from the same stock of deformed steel bars. The beams have been tested under single point load spaced 250 mm from the mid-point of the 3.2 m beams’ span. Results have shown that the half-beams reinforced transversely with double-head studs exhibited 6-12 percent higher shear strength and 5-11 percent more shear ductility than the other halves with single-leg hooked stirrups.
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TO MY FATHER, MOTHER AND BROTHER

WITH ALL MY LOVE AND RESPECT
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LIST OF SYMBOLS

All symbols are defined where they are first appeared. The following list contains the most frequently used symbols:

\[ \begin{align*}
\text{a} &= \text{shear span} \\
A_s &= \text{area of flexural tensile reinforcement} \\
A'_s &= \text{area of longitudinal reinforcement placed in the compression zone} \\
A_v &= \text{cross-sectional area of all the shear reinforcement branches within a spacing \( s \)} \\
A_w &= \text{effective web area in resisting shear} \\
b_{fl} &= \text{width of beam upper or lower flange} \\
b_w &= \text{web width of beam} \\
c_b &= \text{clear cover to shear reinforcement at the bottom of beam} \\
c_t &= \text{clear cover to shear reinforcement at the top of beam} \\
C &= \text{compression force acting on the cross section} \\
COV &= \text{coefficient of variation of statistical data} \\
d &= \text{effective depth of beam measured from extreme compression fibre to centroid of tension reinforcement} \\
d' &= \text{depth of the reinforcement placed in the compression zone measured from extreme compression fibre to centroid of reinforcement} \\
da_a &= \text{nominal maximum coarse aggregate size in the concrete mixture} \\
d_b &= \text{nominal diameter of steel bar} \\
d_{ev} &= \text{effective height of shear reinforcement} \\
d_v &= \text{distance between resultants of compressive and tensile forces due to a moment acting on the cross section, according to CSA A23.3-94} \\
E_c &= \text{concrete modulus of elasticity} \\
E_s &= \text{steel modulus of elasticity} \\
f_1 &= \text{principal tensile stress} \\
f_2 &= \text{principal compressive stress} \\
f'_c &= \text{concrete uniaxial compressive strength} \\
f_{cr} &= \text{concrete cracking stress} \\
f'_t &= \text{concrete tensile strength} \\
f_u &= \text{ultimate tensile strength of steel reinforcement} \\
f_y &= \text{reinforcement yield strength} \\
f_{yv} &= \text{yield strength of transverse shear reinforcement} \\
h &= \text{total height of the beam cross section} \\
h_f &= \text{thickness of flange in I- or T-beams} \\
j &= \text{ratio between flexural internal lever arm and effective depth of the cross section}
\end{align*} \]
The document contains a list of variables and their definitions related to structural engineering. Here is the plain text representation:

\[ l_{cr} = \text{effective length of concrete crack tooth, according to Kani 1964} \]

\[ m = \text{number of shear reinforcements crossed by a shear crack} \]

\[ M = \text{bending moment} \]

\[ M_{flex} = \text{flexural moment capacity of the cross section} \]

\[ N = \text{axial force on section} \]

\[ P = \text{applied vertical load to the beam} \]

\[ P_{cr} = \text{vertical load at the start of shear cracking} \]

\[ P_{u} = \text{highest recorded load value of the concentrated load applied} \]

\[ P_{yy} = \text{vertical load at yielding of a stirrup or a stud} \]

\[ s = \text{spacing of shear reinforcement measured parallel to the beam axis} \]

\[ t_{h} = \text{thickness of the steel circular head at both ends of a stud} \]

\[ T = \text{tensile force in the longitudinal tensile reinforcement} \]

\[ v = \text{shear stress} \]

\[ v_{c} = \text{shear stress provided by concrete} \]

\[ v_{y} = \text{factored shear stress, according to CSA A23.3-94} \]

\[ v_{u} = \text{failure (ultimate) shear stress} \]

\[ V = \text{shearing force} \]

\[ V_{45} = \text{shearing force resisted by concrete on a 45° plane, according to the shear friction model} \]

\[ V_{ay} = \text{shearing force resisted by the aggregate interlock} \]

\[ V_{c} = \text{shear force resisted by concrete} \]

\[ V_{cr} = \text{shearing force at the start of shear cracking} \]

\[ V_{cz} = \text{shearing force resisted by the compression zone} \]

\[ V_{d} = \text{shearing force resisted by the dowel action of the tensile reinforcement} \]

\[ V_{r} = \text{design shear strength calculated by a shear strength equation} \]

\[ V_{s} = \text{shearing force carried by shear reinforcement} \]

\[ V_{sl} = \text{shearing force carried by one bar of the shear reinforcement} \]

\[ V_{u} = \text{shearing force at failure} \]

\[ w = \text{width of shear crack} \]

\[ \alpha = \text{global justification factor for both depth and concrete strength of a beam, according to the shear friction model} \]

\[ \alpha_{1} = \text{ratio of the average stress in the equivalent rectangular compression block to the specified concrete strength, according to CSA A23.3-94} \]

\[ \alpha_{\text{depth}} = \text{factor for beam heights other than 500 mm, according to the shear friction model} \]

\[ \alpha_{\text{strength}} = \text{factor for beam concrete strengths other than 30 MPa, according to the shear friction model} \]

\[ \beta = \text{a factor that accounts for shear resistance of cracked concrete, according to the MCFT} \]

\[ \beta_{1} = \text{ratio of depth of the equivalent rectangular compression block to the depth of the neutral axis, according to CSA A23.3-94} \]

\[ \beta_{v} = \text{a calibration coefficient, according to the shear friction theory, Loov (2001)} \]

\[ \Delta_{cr} = \text{deflection at the start of shear cracking} \]
\[ \Delta_u = \text{deflection at the highest recorded load value} \]
\[ \Delta_{u80} = \text{deflection after the strength drops to 80 percent of the highest recorded load value} \]
\[ \Delta_{y\text{ideal}} = \text{deflection at yield in an idealized bilinear load-deflection graph} \]
\[ \Delta_{yy} = \text{deflection at which yielding starts in a stirrup or a stud} \]
\[ \Delta_x = \text{spacing of shear cracks, according to Kani 1964} \]
\[ \varepsilon_1 = \text{principal tensile strain} \]
\[ \varepsilon_2 = \text{principal compressive strain} \]
\[ \varepsilon_x = \text{longitudinal strain of flexural tensile reinforcement, according to CSA A23.3-94} \]
\[ \varepsilon_y = \text{yield strain of flexural tensile reinforcement} \]
\[ \varepsilon_{yy} = \text{yield strain of shear reinforcement} \]
\[ \theta = \text{inclination angle of diagonal compressive stresses with respect to the longitudinal axis of the beam, according to CSA A23.3-94} \]
\[ \lambda = \text{a factor that accounts for the concrete density, according to CSA A23.3-94} \]
\[ \mu = \text{shear ductility parameter} \]
\[ \rho_b = \text{balanced steel ratio} \]
\[ \rho_l = \text{longitudinal tensile reinforcement ratio} \]
\[ \rho_t = \text{transverse shear reinforcement ratio} \]
\[ \phi_c = \text{concrete resistance factor that equals to 0.6, according to CSA A23.3-94} \]
\[ \phi_s = \text{steel resistance factor that equals to 0.85, according to CSA A23.3-94} \]
CHAPTER 1
INTRODUCTION

1.1 General

Concrete beams having I-shape cross-sections are common products of the precasting industry because of their relatively small weight and efficiency in resisting bending moments. The thickness of the web of precast I-beams is often governed by constructibility, rather than by the need for strength. The web is commonly provided with shear reinforcement in the form of a mesh of bars at each of the two surfaces of the web. The vertical bars of the meshes are the vertical legs of the shear reinforcement that are commonly designed to provide the shear strength. To prevent reinforcement congestion and for ease of casting concrete, minimum web widths are limited. Therefore, if one row of well-anchored vertical bars is placed in the middle surface of the web to replace the vertical legs of the conventional meshes, smaller web thicknesses and thus more reduction in weights will be possible.

Shear reinforcement has an important role in reinforced concrete beams. Many investigations have been conducted to explore this role and the factors affecting it. Early investigations have revealed four main advantages of the shear reinforcement including:

1- Resisting shear stresses that cause diagonal cracks. Tensile forces develop in the transverse shear reinforcement that intercept the diagonal cracks.

2- Adequate shear reinforcement can increase the load carrying capacity of beams to attain their flexural strength.

3- Shear reinforcement controls the widths of the inclined cracks.

4- In columns, transverse reinforcement confines the concrete core, increases their axial and shear capacities and substantially enhances their ductility.

Attaining the flexural capacity of reinforced concrete beams is of particular importance in regions with high seismic activities. Flexural failure is more ductile than shear failure since the latter happens suddenly with small deformations. Design codes specify transverse reinforcement spacing at connections between columns and beams to prevent premature shear failure in plastic hinges, which develop at the beams’ ends.
1.2 Historical review of shear design development

During the last century, many investigators wrote about shear in beams. Karim Rebeiz (1999) divided the period starting from the turn of the last century until nowadays in time intervals that showed the development in understanding the shear phenomenon. The following brief review has been based mainly on his survey.

From 1900-1920

Prior to 1900, it was erroneously thought that shear failure in concrete beams was a pure shear phenomenon similar to what would occur in steel or wooden members. Vertical shear reinforcement was also thought to act as shear keys resisting only horizontal shear stresses in a similar way to shear connectors in composite girders.

In the early 1900s, truss models were used as conceptual tools in the analysis and design of reinforced concrete beams. In 1899, the concepts of diagonal tension and truss analogy were presented by Ritter. It was assumed that after inclined shear cracking takes place in a reinforced concrete beam, the cracked beam could be idealized as a parallel chord truss. In this truss model, the concrete compression zone is represented by the top chord, the tensile longitudinal reinforcement is represented by the bottom chord, the transverse shear reinforcement acts as tensile vertical members and the concrete between inclined cracks corresponds to compressive diagonal members with an assumed 45° inclination angle. Consequently, all the applied loads were replaced by point loads acting at the truss nodes. He also pointed out that the stirrups carry tensile vertical forces, not horizontal shear. His truss model neglected any contribution of the concrete in tension. He proposed an equation for the design of stirrups based on this truss analogy. However, his viewpoints were vastly ignored in professional circles. Two schools of thought developed at the turn of the last century regarding the shear strength and failure of reinforced concrete members: One was the proponent of horizontal shear stress (i.e., pure shear failure), while the other one was the proponent of the diagonal tension (i.e., failure occurs due to tension along diagonal planes).

Withey (1907,1908) introduced Ritter’s truss model to the American literature and showed that it gave conservative results when compared with test results especially
for lightly reinforced members. To improve the correlation between the truss model and the experimental data, he suggested adding an empirical concrete contribution to the shear reinforcement contribution. This model with additional concrete contribution term was called the "Modified 45° truss model", which was the basis for the shear design provisions in the current ACI code.

Mörsch (1909) resolved the debate on whether shear failures were pure shear phenomenon or tensile phenomenon as he pointed out that when pure shear exists, a tensile stress of equal magnitude must also exist on a 45° plane. Therefore, since concrete is a weak material in tension, failure will occur due to tensile stresses along diagonal planes. He also pointed out that using stresses acting on diagonal planes is difficult to establish because of the uncertainties in determining the diagonal tension strength of a material. Therefore, an acceptable design procedure in shear for concrete structures was based on the assumption that a shear failure at the critical section occurs on an uncracked vertical plane when the applied shear stress at the section, $v$, exceeds the ultimate shear strength of concrete, $v_u$. He introduced the concept of shear strength, $v_w$, as a nominal measure of diagonal tension strength. He also supported Ritter's viewpoints by pointing out that stirrups contribute to shear resistance of reinforced concrete members by resisting tensile stresses not shear stresses. However, Mörsch believed that the nominal shear strength was a property of the concrete; accordingly, he related it to one variable, namely, the concrete compressive strength.

Talbot (1909) confirmed Withey's findings, and disputed the concept that nominal shear strength depends only on the compressive concrete strength. He also thought that diagonal tension is caused by horizontal stresses due to bending as well as vertical shear stresses. Based on the results of 106 reinforced concrete beams without web reinforcement, he demonstrated that the nominal shear strength depends not only on the concrete compressive strength but also on the amount of longitudinal reinforcement, the length of the beam, and the depth (i.e., stiffness) of the beam. However, Talbot's important findings were not reflected in design equations or expressed in a mathematical model and were forgotten until much later.
In the early 1910s, design specifications for shear in the United States restricted the allowable shear strength due to service loads to $0.02 f'_c$ with maximum values either stated or implied. Accordingly, the shear strength was assumed to be a function of the concrete compressive strength, $f'_c$, only. Numerous tests were done during the First World War, which demonstrated that the use of the concrete compressive strength as the sole measure of the nominal shear strength was unduly conservative.

**From 1920-1950**

The ACI Standard Specification No. 23 (1920) permitted allowable service shear stress of $0.025 f'_e$, but not exceeding 0.41 MPa, for beams without web reinforcement and without "mechanical anchorage" of the longitudinal reinforcement. However, with anchored longitudinal reinforcement, with 180° hooks or with plates rigidly connected to the bars, the allowable shear stress in service was $0.03 f'_c$, but not exceeding 0.62 MPa. For beams with transverse shear reinforcement, the allowable nominal shear stress at service was $0.06 f'_c$ with a maximum of 1.24 MPa or $0.12 f'_c$ with a maximum of 2.48 MPa without or with mechanical anchorage of the longitudinal reinforcement, respectively. This shear stress was intended to prevent diagonal web-crushing prior to yielding of the shear reinforcement.

In 1951, the distinction between mechanically anchored and non-anchored longitudinal reinforcement was omitted and replaced by the requirement to anchor the longitudinal bars, smooth or deformed ones. Therefore, the maximum allowable shear stress on the concrete beams in service was $0.03 f'_c$ or $0.12 f'_c$, for beams without or with web reinforcement, respectively. In addition, the ACI 318-51 stipulated that web shear reinforcement must be provided if the service shear stress exceeded $0.03 f'_c$. Moreover, the shear reinforcement was calculated on the basis of the 45° truss analogy to carry the difference between the total shear stress and the shear assumed to be carried by concrete.

In the late 1940s, Moretto adopted an empirical equation for the prediction of the nominal shear strength that included two parameters, namely, the concrete compressive strength and the longitudinal tensile reinforcement ratio.
From 1950 to present

Research and publications on the subject of shear and diagonal tension intensified as a result of a few structural failures, e.g., the August 1955 shear failure of beams in the warehouse at Wilkins Air Force Depot in Shelby, Ohio. This collapse was caused by the shear failure of 36” deep beams, which at the failure location, did not contain stirrups and had only 0.45 percent of longitudinal reinforcement. However, many research efforts were conducted to reveal the main parameters that caused this type of brittle failure at low service shear stresses. The research showed that shear in reinforced concrete members is a complex phenomenon involving more than one variable. This brought to the surface the forgotten findings of Talbot (1909).

In the early 1950s, Ferguson turned attention to the fact that the beam shear strength varies with the shear span-to-depth ratio, $a/d$. In addition, Clark represented Talbot’s notions into a mathematical model that reflected the effects of $a/d$, the longitudinal reinforcement ratio, and the compressive strength on the nominal shear strength of concrete.

The shear span-to-depth ratio, $a/d$, was immediately recognized as an important variable, but the problem was that this quantity is valid only for beams with concentrated loads. However, the research done at the University of Illinois in 1950 showed that $a/d$ relates the horizontal flexural stress to the diagonal tension stress and accordingly, it could be replaced by the ratio $M/Vd$ for any other cases of loading.

In the period from 1960s to 1980s, the pioneering work of Ritter and Mörsch received new impetus and there was an intensive research effort aimed at improving their design models. The early notion for calculating the shear strength by adding both concrete and steel contributions was adopted by most of the design codes. Moreover, two truss models were the basis of the design codes’ procedures. The first approach was based on Withey’s modified 45° truss model. The second approach used “Variable Angle Truss” that was brought after research had shown that the angle of inclination of the compressive concrete struts is not necessarily 45°. This truss with variable angles of diagonal struts provided more realistic basis for shear design.
In 1978, Loov proposed an equation to calculate the shear stress, \( v \), based on shear friction theory. Through the last 23 years, he developed this theory and gave an easy notional equation for shear design as will be discussed in Section 3.2.8 of Chapter 3.

Schlaich et al. (1987) extended the truss model for beams in the form of Strut-and-Tie model, which is relevant for designing and checking regions where the distribution of strains is significantly nonlinear along the depth. Schlaich et al. introduced two types of regions in their model: namely, the D and B regions where D stands for discontinuity or disturbed and B stands for beam or Bernoulli.

The work of Collins with Mitchell (1980) and with Vecchio (1982, 1986) resulted in what they called the modified compression field theory (MCFT). Their theory includes calculations for the average tensile stress in diagonally cracked concrete. They assumed that the concrete tensile stress is zero at crack positions and reaches its maximum value halfway between cracks. This theory was adopted as a design procedure in the CSA A23.3-94 General method and in the AASHTO-LRFD Bridge Design Specifications. The assumptions and derivation of this theory are presented in Section 3.3.1 of Chapter 3.

1.3 Double – head studs, alternative reinforcement solutions

Conventional stirrups and ties are widely used in many structural members like columns, walls, slabs and beams for concrete confinement and for resisting shearing forces. In most of the design codes, the stirrups and ties should have hooks with bends of at least 135° unless the concrete cover is restrained against spalling, in which cases 90° bends are permitted. In addition, a heavier longitudinal bar lodged inside each bend is necessary to enhance the anchorage.

Ghani and Dilger (1998) showed that the conventional stirrups and ties could not develop the full yield strength of the stirrup or tie material in the leg adjacent to the hook. That is because the high compressive stress developed on the concrete inside the hook (≈ 0.30 \( f_y \)) causes crushing of the concrete and slippage before the tensile stress in the bar reaches its yield strength. They also showed that straight studs with an enlarged head at each end with an area 9 to 10 times the cross-sectional area of the bar stem, could develop the full yield strength of the bar’s stem material immediately behind the anchor,
without the need of the development length mandatory when the anchorage is with conventional hooks. In addition, double-head studs do not need the bends and the associated longitudinal bars required for conventional stirrups, as the anchorage of the studs is achieved by bearing beneath the heads. Accordingly, three main advantages could be drawn from using double-head studs. First, double-head studs increase the concrete contribution to the shear strength of the structural members. Second, they increase the shear reinforcement contribution by attaining the full yield strength of bars behind the heads. Finally, they permit easier installation procedure and prevent congestion that may result when using conventional stirrups.

Dilger and Ghali (1997) tested a number of columns reinforced transversely with cross ties and double-head studs. They concluded that slippage, which accompanies the usage of conventional cross ties with 90° or 180° hooks would cause a loss of the force carried by these stirrups, especially with small leg lengths. Therefore, the conventional cross ties could not develop their full yield strength when the failure load in the columns were reached in spite of the enhanced anchorage achieved by the longitudinal bars lodged inside the hooks. On the other hand, tests revealed the superior behavior of the double-head studs as they reached higher strain values than the yield strain of the bars’ material. Moreover, because their anchorage is achieved by enlarged head at each end, there was no need to provide special longitudinal bars to enhance the anchorage. Columns reinforced transversely with double-head studs showed more ductile behavior than those reinforced with conventional cross ties.

In 1992, Ghali and Hammill compared the behavior of two full-size slab-column connection specimens, the first was reinforced with stud shear reinforcement (SSR) (Fig. 1-1) and the second with conventional closed stirrups detailed in accordance with the ACI 318-89 provisions. They showed the higher strength and the increased ductility of the slab reinforced with SSR in terms of higher deflection and higher load reached before failure. In addition, they showed the beneficial use of SSR as shear reinforcement in flat slabs. Using SSR increases the effective depth of flexural reinforcement, saves time and effort of labor during placement of the reinforcement cages in the congested reinforcement zone near columns. As a result of the extensive research work on using
SSR as shear reinforcements at slab-column connections, the Canadian Standard, CSA A23.3-94, allows higher upper limit to the nominal shear force, $V_n$, and larger spacing between studs; at the same time higher value of the concrete nominal shear force, $V_c$, is permitted in design.

![Stud Shear Reinforcement used in flat slab-column connections](image)

**Fig. 1-1 – Stud Shear Reinforcement used in flat slab-column connections**

**1.4 Objectives**

The major objective of the research project presented in this thesis is to examine the feasibility of the use of double-head studs as replacement of the single-leg hooked stirrups. To achieve the goal of this research, a series of concrete beams having I-shape cross-sections has been conducted. The work involves:

1- Presenting an original experimental program on simply supported I-beams reinforced transversely with single-leg hooked stirrups in one half of their length and with double-head studs in the other half.
2- Comparing the strength and ductility of the two shear reinforcement systems.
3- Evaluating the shear strength equations presented in the literature survey.

**1.5 Scope**

Concrete beams having I-shape cross-sections studied in this thesis are reinforced with non-prestressed steel bars. The thesis consists of five chapters and an appendix.

Chapter 2 presents a literature review of the shear theory in reinforced concrete beams, the parameters believed to affect their shear strength and the different types of cracks in shear tests.

Chapter 3 discusses provisions of several codes and a number of research equations for calculating the shear strength of reinforced concrete beams with
conventional stirrups. These equations are used in Chapter 4 to calculate the shear strength of the I-beams tested in the present research program.

Chapter 4 describes details of the experimental program, the basis upon which the dimensions and reinforcement of the tested I-beams are chosen and presents the test results. Discussion of the test results and their implications are presented at the end of the chapter in terms of the shear strength, shear cracks, and ductility by comparing the two systems of shear reinforcement.

Chapter 5 includes a summary of the main findings from the research presented here and some recommendations for future research in the field of use double-head studs as shear reinforcement in beams.

Appendix A shows example applications of the shear strength equations, discussed in Chapter 3. The calculations are based on the actual material properties of the concrete and steel for one of the tested beams.

Appendix B shows an example of the iterative method applied for calculating the shear strength and the critical shear failure plane using the friction model (Loov 2001).
CHAPTER 2
LITERATURE REVIEW

2.1 Introduction

This chapter is aimed to explore shear failure phenomenon, mechanisms of shear forces in reinforced concrete beams and the parameters affecting shear strength of beams. Different types of shear cracks exist and are related to various parameters. These cracks and early-investigated models explaining forces and factors affecting their appearance are reviewed.

2.2 Basic theory

Consider the loaded simply supported beam shown in Fig. 2-1. The internal forces at any section are: bending moment, \( M \), and shearing force, \( V \). Figure 2-2 shows Mohr’s circle for stresses at a small element represented by a square in the tension zone in Fig. 2-1. The circle gives the principal tensile and compressive stresses, \( f_1 \) and \( f_2 \) respectively, and their orientations before cracking. When \( f_1 \) exceeds the concrete tensile strength, cracking occurs perpendicular to its direction.

![Figure 2-1 - Simple beam and internal forces acting on a portion of its span](image)

Fig. 2-1 – Simple beam and internal forces acting on a portion of its span

The principal stresses differ in magnitude and direction from point to point on the beam. The lines defining the orientations of the principal stresses at any point on the beam are called “stress trajectories”.

\[
M \begin{aligned} V \\ M+\Delta M \end{aligned} V+\Delta V
\]

\[
\Delta \quad \Delta
\]
Similar to the principal stresses, the stress trajectories are of two types: tensile and compressive trajectories, which are orthogonal to each other. Since the strength of concrete is low in tension, cracks are always normal to the tensile trajectories, i.e., follow the shape of the compressive trajectories.

Figure 2-3 shows the orthogonal stress trajectories in a beam with two point loads, where compressive and tensile trajectories are represented by solid and dashed lines, respectively.
**Beam action and arch action**

Figure 2-4 shows a cracked beam and the forces acting on a concrete segment lying between two shear cracks. From equilibrium of internal forces,

\[ T = \frac{M}{jd}, \quad V = \frac{dM}{dx} = \frac{d(T \cdot jd)}{dx} \] \[ \text{[2-1]} \]

The expression of \( V \) can be expanded as

\[ V = \frac{dT}{dx} \cdot jd + \frac{d(jd)}{dx} \cdot T \] \[ \text{[2-2]} \]

where \( T \) is the tensile force in the longitudinal reinforcement; and \( jd \) is the flexural internal lever arm (Fig. 2-4).

The first and second terms in Eqn. 2-2 represent beam and arch actions, respectively. Investigations have shown that the extent to which each action contributes to the shear resistance primarily depends on the shear span-to-depth ratio, \( a/d \). Consequently, most of the shear strength equations are separated in two categories: the first for long slender beams with \( a/d \geq 2.5 \) where the beam action governs, while the other for short deep beams with \( a/d < 2.5 \) where the arch action governs.

**2.2.1 Beam action**

The beam action governs the behavior of beams having long shear spans \((a/d \geq 2.5)\). The first term in Eqn. 2-2, \( jd \cdot dT/dx \), represents the beam action, where the internal tensile force acts on a constant lever arm, \( jd \). On any horizontal plane
between the compression zone and the reinforcement, the shear flow is equal to \(dT/dx\). This shear flow develops through bond transfer along the tensile reinforcement.

In 1964, Kani considered that the concrete portions between inclined shear cracks act as cantilevers, loaded with bond forces developing along the tensile reinforcement (Fig. 2-5). Since these cantilevers support the bond forces in the longitudinal reinforcement, he assumes that their strength governs the beam action behavior. Failure of the teeth takes place at their bases connected to the compression zone of the beam. After failure of the concrete teeth, beams having small \(a/d\) ratios transform into tied arch mechanisms, which permit further increase in the beam shear strength. In beams having medium to high \(a/d\) ratios, the concrete teeth capacity was found to be higher than the tied arch capacity; thus, sudden failure occurs after the teeth break away.

Kani expressed the moment directly under the loading point, which causes the failure of the concrete teeth as

\[
M_{cr} = \frac{M_{flex}}{(a/d)_{TR}} \cdot \frac{a}{d} \tag{2-3}
\]

where \(M_{flex}\) is the flexural moment capacity of the beam cross-section; and \((a/d)_{TR}\) is the value of the shear span-to-depth ratio at which a transition from diagonal shear failure to flexural failure takes place (Figs 2-6 and 7). Kani proposed Eqn. 2-4 for calculating the value of \((a/d)_{TR}\) for under-reinforced sections.

\[
\left(\frac{a}{d}\right)_{TR} = 6\rho_i \frac{f_y}{f_t} \frac{l_{cr}}{\Delta x} \tag{2-4}
\]
where $p_i$ is the longitudinal tensile reinforcement ratio that equals to $A_s/(bd)$; $A_s$ and $f_y$ are the cross-sectional area and the yield strength of the tensile reinforcement; $f'_t$ is the concrete tensile strength; $l_{cr}$ is the effective height of cantilever; and $\Delta x$ is the spacing between cracks, i.e., the depth of the concrete cantilever base (Fig. 2-5).

![Diagram showing the relation between beam flexural capacity and $a/d$](image)

Fig. 2-6 – Relation between beam flexural capacity and $a/d$ (Kani 1964)

### 2.2.2 Arch action

Arch action governs the behavior of beams having short shear spans ($a/d < 2.5$) where it is represented by the second term of Eqn. 2-2, $T \cdot d(jd)/dx$. When this action governs, the beam behaves as an arch with a tie in which the compression force reaches to the supports in inclined concrete struts and the main steel transmits the tensile force through the shear spans to the supports. The compressive force in a concrete strut changes its position along the beam shear span from the zone beneath the loading point to the supports. This change in the compressive force position along the shear span is expressed by the change of the internal lever arm in Eqn. 2-2, $d(jd)/dx$.

Kani (1964) studied the capacity of the arching mechanism, and showed that the transformation from beam to arch action takes place after failure of the concrete teeth forming between the inclined shear cracks. It was also shown that in beams having short shear spans, a high reserve shear strength exists beyond the cracking strength due to the remaining arch capacity. While for slender beams, failure suddenly takes place at the onset of diagonal cracking. Kani introduced Eqn. 2-5 to calculate the moment capacity of the remaining tied arch – representing the failure moment for the range of $1.0 \leq a/d \leq (a/d)_{\text{min}}$ in Fig. 2-7 – in terms of $M_{flex}$ of the beam cross section.
The coefficient 0.9 in the denominator of Eqn. 2-5 was proposed to account for the favorable effect of the biaxial compressive stress at the loading point section.

Equating the two moments expressed for the remaining arch capacity (Eqn. 2-5) and at the failure of the concrete teeth (Eqn. 2-3) gives the \((a/d)_{\text{min}}\) value at which beam behavior transforms from arch to beam action (Fig. 2-7).

\[
\frac{a}{d}_{\text{min}} = \sqrt{\frac{(a/d)_{\text{TR}}}{0.9}}
\]

Fenwick and Paulay (1968) explained the higher reserve shear capacity associated with arching mechanism in beams having low \(a/d\) ratios in terms of the thrust force inclination between the load and support. They demonstrated that the steeper the line of thrust between the load and support, the larger the area resisting the compressive force would be. Thus, higher shear capacity could be sustained after the breakdown of the beam action.

Swamy et al. (1970) studied the arch action mechanism on beams without shear reinforcement. They emphasized the importance of "an end-to-end break of bond" for the tensile reinforcement as a condition for the arch action development. Their experimental results for beams with smooth and adequately anchored tensile bars showed that "a prestressing effect" is produced by the tensile force constantly developed over the
whole length of the beam. Under this prestressing effect, the web is subjected to compressive stresses, which enhance the shear resistance of beams with smooth bars. Some of the tested beams with smooth bars attained their flexural capacities.

Generally at ultimate the shear resistance in any reinforced concrete beam is a combination of both beam and arch actions since the full bond force in the longitudinal reinforcement is typically not achieved due to bond slippage and cracking.

2.3 Shear transfer components

Research in the last century identified qualitatively four shear resisting components in beams (Fig. 2-8). Shear transfers across a beam by: the shear in the uncracked compression zone, $V_{cz}$, the vertical component of the aggregate interlock, $V_{ay}$, the dowel action of the tensile reinforcement, $V_d$, and the tensile strength developed in the shear reinforcement, $V_s$. However, since quantifying each one of the first three components is difficult, codes and research approaches lumped them in one term identified as the concrete contribution to the shear resistance, $V_c$. Moreover, there is an interaction between the strengths provided by these components. For example, increasing the shear reinforcement ratio in a reinforced concrete beam results in a better confinement for the concrete that improves $V_{cz}$. In addition, the shear reinforcement limits widening of the shear as well as the longitudinal splitting cracks along the tensile bars, so that respectively the shear components $V_{ay}$ and $V_d$ are improved.

Fig. 2-8 – Shear transfer components in a reinforced concrete beam
Although the contribution of $V_s$ to the shear strength of a beam cannot be greater than the nominal yield capacity – defined by the stirrup yield stress multiplied by the sum of its branches’ area – its effect on the concrete contribution can be included in the $V_c$ term. This approach has been adopted by a number of researchers as will be discussed in Chapter 3.

2.3.1 Shear strength provided by concrete, $V_c$

The CSA A23.3-94 code uses $v_c = 0.2 \lambda \phi_c \sqrt{f'_c}$ to express the concrete contribution in resisting punching shear when stirrups are used in the slab-column connections. However, it uses higher concrete stress equal to $0.3 \lambda \phi_c \sqrt{f'_c}$, to express the confinement positive effect on the concrete carrying capacity when mechanically anchored studs are used as punching shear reinforcement in these regions. In beams, as given in Clause 11 of the CSA A23.3-94 code, the concrete contribution to the shear strength is considered constant for all beams with or without shear reinforcement. Thus, the code design approach neglects the improved confining effect of the mechanically anchored shear reinforcement. The three different resisting components composing $V_c$ are discussed in the following sections.

2.3.1.1 Shear force transferred by uncracked concrete, $V_{ci}$

Prior to cracking of a beam, shearing forces are resisted by the inclined principal compressive and tensile stresses referred to as the stress trajectories (Fig. 2-3). However, after cracking, these trajectories modify their positions and orientations so that they exist only in the uncracked compression zone of the section. Thus, this resisting component commences at early load phases prior to cracking and continues after cracking. Although the compressive confining force increases directly with loading, its depth and shear resistance decrease extensively prior to failure. Brittle flexural or shear failure takes place in beams respectively when the compressive or shear strength of this zone is exceeded.
2.3.1.2 Shear force transferred by aggregate interlock, $V_{ay}$

It is also known as the shear interface mechanism. The protruding aggregates from the slip surfaces form a jagged uneven surface that provides resistance against slippage. This resistance component depends on a number of parameters including: the concrete strength, the crack width, the amount of shear reinforcement and the size and percentage of aggregates in the concrete mixture.

In normal-strength concrete beams, the crushing strength of the aggregates is relatively higher than the concrete compressive strength so that the shear cracks skirt across the aggregates. Thus, the resulting slip surface has higher jaggedness that improves $V_{ay}$. On the other hand, smooth interface surfaces result in high-strength concrete beams because cracks cross through aggregates; thus, $V_{ay}$ is reduced in high-strength concrete members. In general, the aggregate interlock resistance decreases near the ultimate shear load due to the excessive widening of the shear cracks.

2.3.1.3 Shear force transferred by dowel action, $V_d$

The dowelling action of the tensile reinforcement starts when the shear crack crosses these bars. Relative slippage between interface surfaces of an inclined crack activates the wedging action of the bar deformations. A number of parameters affects the shear force resisted by this component such as the concrete strength, the geometry of the beam cross-section, the amount of shear reinforcement provided and the amount and distribution of the tensile reinforcement in the cross section.

Normally, the dowel shear resistance is insignificant in transversely unreinforced beams as it is limited by the stiffness of the concrete cover directly beneath the tensile bars. Thus, the net width of the beam at the reinforcement level and the concrete cover control $V_d$ in beams without shear reinforcement. Many investigators (Zsutty 1971, and Collins et al. 1999) demonstrated the favorable effect of distributing the tensile bars in more than one layer on the shear capacity of beams. Moreover, Fenwick and Paulay (1968) studied the effect of the bar positioning in the concrete at the time of casting and the associated phenomenon of water gain or sedimentation under the bars on the dowel action contribution. Experiments showed more reduction in the dowelling force of the top
cast dowels than of the bottom cast ones. As the concrete depth beneath the bar increases, a thicker soft and spongy layer of entrapped air and water forms, which reduces the elastic response of the concrete when the dowel is displaced into this weak zone.

In 1967, Bresler and MacGregor discussed that when the inclination of a shear crack increases, the dowel force increases until it reaches a critical value at which a splitting crack along the longitudinal reinforcement occurs. When splitting along the longitudinal bars takes place, $V_d$ cannot be maintained unless adequate transverse shear reinforcement exists which provides partial arrest of this split cracking. Thus, the shear reinforcement improves $V_d$ as it prevents the breakout and the splitting failure of the concrete around the tensile bars.

2.3.2 Shear force transferred by shear reinforcement, $V_s$

Prior to shear cracking, strain in the shear reinforcement is equal to that of the adjacent concrete and since the concrete cracks at a very small strain, the stress in the shear reinforcement prior to inclined cracking is found to be in the range of 20 to 40 MPa. Consequently, shear reinforcement comes into play after formation of major inclined cracks. However, it has been customary to view $V_s$ in concrete beams as a part of a truss mechanism, i.e., neglecting the concrete resistance components.

The joint ASCE-ACI committee 426 in 1973 demonstrated the beneficial effect of shear reinforcement on the concrete shear transfer components. It showed that in addition to the shear supported by the shear reinforcement itself, it maintains both $V_{ay}$ and $V_d$ as it respectively limits the widening of inclined cracks and supports the longitudinal bars to prevent splitting cracks. Pendyala and Mendis (2000) assumed that $V_c$ is limited by the value at which inclined cracking occurs; thus, $V_s$ extensively increases when increasing the shearing force beyond this level as shown in their assumed model in Fig. 2-9. However, since $V_s$ is limited by the nominal yield strength ($A_v f_{yu}$) of the shear reinforcement bars, after yielding of these bars the compression zone of the beam cross section tries to support the excessive load until its compressive strength is reached when failure takes place.
In spite of the simplicity of the previous model, illustrated in Fig. 2-9, it assumes that $V_c$ is constant beyond the inclined cracking load until failure. This assumption contradicts logic because after the shear cracking takes place, the shear supported by the various concrete components changes as mentioned above.

### 2.4 Parameters influencing shear strength of reinforced beams

#### 2.4.1 Cross-sectional dimensions of beams (size effect)

Based on early investigations (Kani 1967, Shioya 1989, Sarsam and Al-Musawi 1992, and Collins et al. 1993), the ACI committee 445 on shear and torsion in 1998 explained that size effect is a main reason for shear strength reduction in transversely unreinforced large beams due to the formation of wide diagonal cracks. However, experiments conducted by Sarsam and Al-Musawi (1992), Collins and Kuchma (1999), Forsch (2000) on large-scale beams having minimum shear reinforcement demonstrated that the size has insignificant effect on shear strength of transversely reinforced beams.

In the experimental research presented in this thesis, a reduction in the cracking and the ultimate shear stress has been observed with the increase in beam height (Chapter 4). This observed reduction in shear stress could be attributed to the size effect.
Placas and Regan (1971) investigated the effect of changing the flange width, \( b_f \), in T-beams on their shear strength. In their tests, T-beams having \( b_f / b_w \geq 2 \) had 20\% higher shear strength compared to rectangular beams having the same web width, \( b_w \). Accordingly, it was assumed that a portion of the flange immediately adjacent to the web participates in resisting the shear stresses. Zsutty (1972) proposed this portion of the flange area to be equal to \( 2h_f^2 \) with \( h_f \) being the thickness of the flange; while Loov (2000) used \( h_{f2}^2 \), which is the area of the largest two 45° triangles that can fit within the junction of the flange with the web.

### 2.4.2 Concrete compressive strength, \( f'_c \)

Although concrete strength has been the first parameter identified to affect shear strength of beams, there is still a lot of debate about its extent and how to express it mathematically in shear strength equations. Since 1950s, it was assumed that the shear transferred through concrete is a function of \( \sqrt{f'_c} \), which was adopted by some codes like the ACI code, the Canadian code and the New Zealand code. However, in 1999, Rebeiz showed that in beams without shear reinforcement, there was a weak correlation between \( \sqrt{f'_c} \) and the experimental values of cracking and ultimate shear stresses for both normal and high-strength concrete beams. Thus, he concluded that \( \sqrt{f'_c} \) was inadequate as a sole predictor of either cracking or ultimate shear stress. He used the technique of dimensional analysis to develop equations for both the cracking and ultimate shear stresses. He introduced a shape adjustment term, \( A_d \) to account for the increase in shear strength for beams having \( a/d < 2.5 \) and gave the following equations:

\[
\begin{align*}
V_c &= \frac{V}{b_w d} = 0.4 + \sqrt{f'_c \rho_i (d/a) \left[ 2.7 - 0.4A_d \right]} \\
V_u &= \frac{V}{b_w d} = 0.4 + \sqrt{f'_c \rho_i (d/a) \left[ 10 - 3A_d \right]} \\
A_d &= \begin{cases} 
\frac{a}{d} & \text{for } 1.0 < a/d < 2.5 \\
2.5 & \text{for } 2.5 \leq a/d
\end{cases}
\end{align*}
\]  

[2-7]
where $v_c$, $v_u$ and $f'_c$ are in MPa. Equation 2-7 showed good correlation with the experimental data for a range of $f'_c$ up to 104 MPa.

Clark (1951) found that the shear strength varies linearly with $f'_c$ multiplied by a factor representing the slenderness effect of beams. He used the available experimental data to introduce an empirical expression (Eqn. 2-8) for calculating the shear stress of beams failing due to diagonal tension.

$$v_u = 7000 \rho_l + 0.12 f'_c \frac{d}{a} + 2500 \sqrt{\rho_t}$$  \hspace{1cm} [2-8]

where $\rho_l$ is the longitudinal reinforcement ratio, equal to $A_s/bd$; $A_s$ (in$^2$) is the longitudinal reinforcement area; $b$ and $d$ are the beam width and depth, respectively in inches; $\rho_t$ is the transverse shear reinforcement ratio equal to $A_v/(b_w s)$; $A_v$ (in$^2$) is the cross-sectional area of all the shear reinforcement branches within a spacing $s$ measured parallel to the beam axis; $v_u$ and $f'_c$ are in psi.

Zsutty (1971) introduced $\sqrt[3]{f'_c}$ in an empirical shear strength equation, which was considered a significant improvement so that the British code adopted this trend in the shear design provisions. In 1984, Mphone and Frantz conducted an experimental investigation to study the effect of varying $f'_c$ (from 21 to 103 MPa) on the shear strength of transversely unreinforced concrete beams having $a/d = 1.5, 2.5$ and $3.6$ and $\rho_l = 3.36\%$. The results demonstrated that increasing $f'_c$ increases the cracking shear stress while reduces the difference between cracking and ultimate stresses, i.e., reduces the reserve shear stress over the cracking stress. Moreover, they introduced an empirical equation for the ultimate stress of beams with $a/d = 3.6$ in which $\sqrt[3]{f'_c}$ was introduced to represent the concrete strength effect on the shear stress.

$$v_u = 10.10 \sqrt[3]{f'_c} + 71 \ (f'_c \text{ in psi})$$  \hspace{1cm} [2-9]

It was also concluded that the effect of $f'_c$ on shear capacity becomes more significant as the $a/d$ ratio decreases.
Experiments conducted by Pendyala and Mendis (2000) on high-strength concrete beams ($f'_c = 100$ MPa) without shear reinforcement showed that there is an extensive increase in their shear strength when compared with identical beams made of normal-strength concretes.

Many investigators (Johnson and Ramirez 1989, Roller and Russell 1990, and Angelakos et al. 2001) studied the effect of $f'_c$ on the shear capacity of transversely reinforced concrete beams. They all emphasized that the minimum ratio of shear reinforcement should be related to $f'_c$. Johnson and Ramirez (1989) showed that shear crack widths are relatively larger in high-strength concrete beams because of the high stresses associated with the increase in diagonal tension cracking load. Moreover, smooth crack surfaces in high-strength concrete beams result in a redistribution of the internal forces, which may turn the shear reinforcement to be the weakest link in the load-carrying mechanism. In 2001, Angelakos et al. investigated the shear strength of lightly reinforced large beams with and without shear reinforcement. Results of the beam series without shear reinforcement showed that changing $f'_c$ from 21 to 80 MPa has insignificant effect on the ultimate shear capacity of the beams. It was also found that for the same $f'_c$ range, the ratio of the shear strength of beams having $\rho_{t,\min}$ (in accordance with the ACI code) to that of transversely unreinforced beams ranged from 1.54 to 2.44. Although high-strength concrete beams with shear reinforcement exhibited higher ductility than those without, they showed an abrupt drop in their carrying capacity after reaching the peak load value. Therefore, Angelakos et al. suggested a revision for $\rho_{t,\min}$ for high-strength concrete beams in the ACI code so that they could reach the relatively higher ductility ratios attained by low-strength concrete beams.

Collins and Kuchma (1999) showed that in beams made with high-strength concretes, diagonal cracks pass through the aggregates resulting in smooth crack surfaces, i.e., a lower contribution of aggregate interlock component to the shear resistance. Moreover, experiments showed that the shear strength reduction in large beams due to size effect is higher in beams made with high-strength concretes than in normal-strength concrete beams.
2.4.3 Ratio and distribution of longitudinal reinforcement (dowel action)

Many investigators (Bresler and Scordelis 1963, Zsutty 1971, Mphonde 1989, Sarsam and Al-Musawi 1992, and Collins and Kuchma 1999) studied the effect of longitudinal reinforcement distribution in beams with and without shear reinforcement on their shear strengths. It was found that beams reinforced with small diameter bars in more than one row result in higher shear strengths than beams having the same $\rho_l$ but with one row of larger diameter bars. Regan’s (2000) experimental research on transversely unreinforced beams showed that increasing $\rho_l$ by increasing the number of bars in one row of the same beam width, $b_w$, reduces the shear strength. Thus, he concluded that this decrease in shear strength is caused by the reduction of the net breadth of concrete at the longitudinal reinforcement level supporting the dowel rather than by the variation of the stiffness of the dowels. Regan also reported the results of an investigation on the effects of the flexural stiffness of the main reinforcement, and the concrete cover thickness to the main bars on the shear strength of transversely unreinforced beams. Experiments by Regan on different reinforcement types including smooth bars, deformed bars, prestressing strands and Bi-steel units, showed that the flexural stiffness of the main reinforcement bars has no apparent effect on the shear strength of beams. Thus, the dowel action component, $V_d$, has a small contribution to the shear strength when the main bars are not transversely supported by means of shear reinforcement. Furthermore, results of three identical beams differed only in the cover thickness showed an extensive reduction in shear strengths of beams having large concrete covers. This reduction was attributed to the increased spacing between flexural cracks in the case of large covers, which consecutively increased the crack widths and reduced the interface shear across their surfaces.

2.4.4 Transverse shear reinforcement ratio (stirrup effectiveness)

Kani (1969) studied three different types of shear reinforcements: bent-up bars, inclined and vertical stirrups, to investigate the most efficient type in supporting the internal concrete arches. Experimental results showed that bent-up bars and inclined stirrups are more efficient in supporting his assumed concrete internal arches (the shown
arches in Figs 2-10a, b and c) than vertical stirrups. Bent-up bars and inclined stirrups have good anchorages in the outermost supported arch (the shaded parts of the beams in Fig. 2-10) that enable transfer of the internal arch reactions. Most of the tested beams reinforced with bent-up bars and inclined stirrups attained their flexural capacities. On the other hand, it was shown that the anchorage quality of vertical stirrups differs from one location to another along the shear span. Vertical stirrups nearest to the loading point have poor anchorage while others farther from this point have better anchorage as shown in Fig. 2-10c.

Thus, it is to be noted here that double-head studs can offer better anchorage than stirrups to efficiently support the assumed internal arches.

Fig. 2-10 – Investigated three types of shear reinforcement (Kani 1969)

Many investigators (Haddadin et al. 1971, and Johnson and Ramirez 1989) studied the effect of $\rho_t$ on the behavior of transversely reinforced concrete beams. Experiments showed that the higher the $\rho_t$, the more and the closer the diagonal cracks appear in the tested shear spans. Experiments of Haddadin et al. (1971) on beams subjected to a combination of shear and axial loads (compression or tension) showed that the stirrup effectiveness is greater in beams subjected to axial compressive loads than in those subjected to axial tensile loads. Diagonal cracks in beams subjected to compressive loads are flatter (they intersect more stirrups) than those subjected to tensile loads. The
shear reinforcement ratio also affected the beam cracking as two types of failure cracks were reported in this investigation. The first appeared in beams having low ratios of $\rho_t$ where failure took place after yielding of shear reinforcement and was denoted as diagonal tension cracks. The second type of cracks appeared in beams having higher ratios of $\rho_t$ where failure was characterized by crushing of the concrete in the compression zone and was called shear-compression. It was also noted that in beams having $a/d > 2.5$ and high $\rho_t$ ratios, the diagonal-tension cracks were more steeply inclined to the longitudinal axis of the beam.

Fig. 2-11 – Truss systems of concrete members (Johnson and Ramirez 1989)

Johnson and Ramirez (1989) discussed the relation between the concrete strength and the minimum shear reinforcement ratio in concrete beams. The strut-and-tie models shown in Fig. 2-11 were used to illustrate their discussion. They demonstrated that both truss systems in Fig. 2-11 were in equilibrium as long as their members and nodes were capable of transferring the internal forces to the supports. However, increasing the concrete strength allowed for more redistribution of the internal forces as well as for strengthening the concrete struts and nodes. Consequently, if the stirrups were inadequate
to sustain the higher share of forces resulting from the redistribution process, they might be the weakest link in the truss system; thus, failure would take place after their yielding. Therefore, they argued that if the truss systems in Figs 2-11a and b represent beams having $\rho_{t,\text{min}}$ and those having higher ratios than $\rho_{t,\text{min}}$, respectively, the later system allows for more redistribution of forces than the former one, as indicated by the more concrete struts in this truss, and consecutively higher shear capacity.

Zsutty (1971) discussed that the presence of shear reinforcement in beams substantially improves both the dowel action and the aggregate interlock contributions to the shear strength as the shear reinforcement respectively prevents longitudinal splitting along the tensile bars and eliminates diagonal crack propagation. In addition, Zsutty discussed that these criteria exist at some intermediate phase of loading while they diminish after yielding of the shear reinforcement.

Shear reinforcement in wide beams was studied by Hsiung and Frantz (1985) and Anderson and Ramirez (1989). They demonstrated the importance of anchoring and distributing shear reinforcement across the width of wide beams. Moreover, Hsiung and Frantz (1985) reported two potential problems related to the use of single U-shaped stirrups in wide beams. The first was the reduction of the dowel action contribution and the splitting across beam webs due to the unsupported interior longitudinal bars. The second was the relatively wide cracks at the interior portion of the web that reduces the aggregate interlock contribution to the shear strength. However, experimental results showed that wide beams with transversely distributed stirrup branches across the web were slightly better than those with outer stirrup legs only. It was also observed that shortly before failure, crack widths increased with higher rates in the beams with outer stirrup legs than in those with distributed legs across the web.

Anderson and Ramirez (1989) used the truss model to emphasize on the importance of anchoring the shear reinforcement in beams. They explained that since the shear reinforcement forms the vertical ties of the truss system where the tensile forces are equilibrated at the joints, the shear reinforcement requires "full-strength mobilization throughout their entire heights". It was also discussed that the stirrup hooks anchored in the compression zone of a beam benefit from the compressive pressure and result in
better distribution of the bearing stresses in the hook interior especially when hooked
around relatively large bars. On the other hand, they discussed that hooks in the flexural
tensile zone are “questionable” because of the tensile cracking and the reduced
confinement in this zone of the beam section. Furthermore, it was demonstrated that
adequate stirrup spacing in the longitudinal as well as in the transverse (across the width
of wide beams) directions, prevents both the concentration of diagonal compressive
stresses at the truss nodes and the overloading of the outer truss nodes, respectively.

2.4.5 Shear span-to-depth ratio, $a/d$

Shear behavior of beams with respect to their shear span-to-depth ratios was
extensively studied by many researchers (Kani 1964, Zsutty 1968, Haddadin et al. 1971,
and Sarsam and Al-Musawi 1992). Kani (1964) classified shear behavior of beams into
three main bands with respect to the $a/d$ ratio (Fig. 2-7). First, beams having small shear
spans ($a/d < 2.5$) most likely behave as tied arches. Second, for beams having
intermediate shear spans $[(a/d)_{\text{min}} < a/d < (a/d)_{TR}]$, a brittle failure occurs after failure
of the concrete teeth, forming between the inclined shear cracks. Third band extends
beyond the transition point, $(a/d)_{TR}$, where beams attain their flexural capacities
resulting in ductile failures.

Zsutty (1968) analyzed early experimental data of reinforced concrete beams
without shear reinforcement. From the analysis he concluded that a change in the beam
behavior, from arch action at low ratios of $a/d$ to beam action at higher $a/d$ ratios,
exists at some intermediate ratio of $a/d$ that was found to be approximately equal to 2.5,
which agreed with Kani’s findings (see lowest point of the graph in Fig. 2-7).

High shear capacities of beams having small shear spans were experimentally
proved by a number of researchers (Clark 1951, Kani 1964, Mphonde and Frantz 1984,
(1984) discussed that reducing the $a/d$ ratio in transversely unreinforced beams
increases the reserve shear stress attained beyond the shear cracking stress. Crack
patterns appeared in their investigated beams showed a dominant flexural behavior in
slender beams (high \(a/d\) ratios) where many vertical cracks appeared within the shear spans, while showed a tied arch behavior in shorter beams where inclined cracks developed between load and support blocks with much less vertical cracks. Although in slender beams inclined cracks develop gradually and much slower than in shorter beams, their failures are sudden after the formation of a major inclined crack. However, in deep beams where the arch action governs, there is a significant reserve of strength after the crack pattern is fully developed.

2.4.6 Bond characteristics of flexural reinforcement

Effect of bond quality between concrete and the longitudinal reinforcement on the shear strength was observed for the first time in 1961 at the Technical University of Stuttgart when beams with "poor bond characteristics for the tensile reinforcement" attained higher shear capacities than typical beams with deformed bars. Since then, many investigators (Kani 1964, Swamy et al. 1970, Collins and Kuchma 1999, Kim and White 1999, Ma et al. 2000, and Regan 2000) studied this effect on the shear capacity of beams.

The importance of anchoring the flexural reinforcement specially smooth bars at the beam-ends in order to avoid premature shear-bond failure was demonstrated by Swamy et al. (1970) and Ma et al. (2000). Swamy et al. (1970) explained that high tensile stresses exist at the anchorage zones where these stresses have to be adequately transferred from the reinforcement bars to the concrete.

Collins and Kuchma (1999) conducted a number of shear tests on transversely unreinforced beams having the same \(\rho_l\) ratio but with three different arrangements of flexural reinforcement: epoxy-coated reinforcement, an array of three layers of small-diameter bars, and a bundle of large-diameter bars. Results showed that the highest shear strength was attained by beams with the array of three layers of small-diameter bars and that the beams with epoxy-coated bars had higher shear strengths than identical beams with deformed bars.

Kim and White (1999) related the shear strength reduction in beams with bonded tensile bars to the spread of the horizontal splitting cracks along them. Figure 2-12 shows their assumed model in which they argued that for a beam with deformed tensile
reinforcement, the tensile force $T$ is transferred to the concrete through the shear bond forces on the shear surface (e-f). Moreover, they discussed that the element (aefd) in Fig. 2-12 is analogous to a simple pullout test, which shows a concentration of shear bond stresses near the loaded end (point f). The distribution of the shear stresses on planes (e-f) and (h-g) are shown in the figure. This argument implies that after formation of the flexural crack (df), bond causes a magnification of the shear stress in the zone above the reinforcement and adjacent to the outermost flexural crack (the hatched zone shown in the figure). This shear stress magnification is thought to lead to horizontal splitting cracks parallel to the reinforcement initiating from the shown critical zone. However, in beams with smooth bars, the bond-induced shear stress is smaller; thus, the vulnerability to shear strength reduction due to the splitting cracks is reduced. It is argued that extensive horizontal splitting crack produces severe modification of the internal stress system; this results in an extensive increase of the stress at the shear crack apex and causes the shear crack apex to be unstable, which causes brittle diagonal tension failure.

Fig. 2-12 – Shear stress concentration causing longitudinal splitting cracks (Kim and White 1999)

2.4.7 Level of axial force on member

Haddadin et al. (1971) conducted a test series on T-beams with shear reinforcement and supported by prestressed bolts passing through the web width. The tested beams were subjected to combinations of either shear and axial loads (tension or compression) or to shear load only. The experiments showed that in beams with axial compressive forces, the onset of shear cracking load was delayed and cracks tended to be
at smaller angle with the beam axis. Both the growth and widening of the cracks were slower than beams without axial loads; thus, it was concluded that beams with axial compressive forces are stiffer than those without. The reverse case was observed in beams with axial tensile forces. To fit the experimental data, Eqs 2-10 and 11 were proposed for the ultimate shear stress at different shear reinforcement nominal strengths, $\rho_i f_{sv}$.

$$v_u = v_c (ACI318-63) + 1.75 \rho_i f_{sv}$$

for $50 \text{ psi} < \rho_i f_{sv} \leq 0.06 f'_{c} \left[ \frac{b}{b_w} \sqrt{\frac{Vd}{M}} \right]$  \[2-10\]

and

$$v_u = v_c (ACI318-63) + 0.5 \rho_i f_{sv} + 0.075 f'_{c} \left[ \frac{b}{b_w} \sqrt{\frac{Vd}{M}} \right]$$

for $0.06 f'_{c} \left[ \frac{b}{b_w} \sqrt{\frac{Vd}{M}} \right] < \rho_i f_{sv} \leq 630 \text{ psi}$  \[2-11\]

where $v_c (ACI 318-63)$ is

$$v_c = 1.9 \sqrt{f'_{c}} + 2500 \frac{\rho_i Vd}{M} \leq 3.5 \left[ f'_{c} \left( 1 + \frac{0.002N}{A_e} \right) \right]$$  \[2-12\]

$$M' = M - N \left( \frac{4h - d}{8} \right)$$  \[2-13\]

where $V$ is the shearing force in lbs; $M$ is the bending moment in lb-in; $N$ is the axial load in lbs (positive when compressive); $h$ is the total height of beam cross-section in inches; and $v_u$, $v_c$, $f'_{c}$ and $f_{sv}$ are in psi. Moreover, it was suggested that if the axial tension exceeded $4 \sqrt{f'_{c}}$ (with $f'_{c}$ in psi), the concrete stress, $v_c$, should be taken equal to zero in Eqs 2-10 and 11.

### 2.4.8 Beam loading and supporting mechanisms

Early investigations showed that this parameter is more significant in beams with low $a/d$ ratios than in slender beams with higher $a/d$ ratios. Experimental investigations conducted by Taylor (1966), Kani (1964), Haddadin et al. (1971) and Kim
and White (1999) demonstrated that beams with conventional loading and supporting blocks at top and bottom surfaces of beams, respectively have higher strengths over those supported against the side faces of their webs. This phenomenon was attributed to the arch action behavior that forms between the conventional loading and supporting plates.

In 1999, Kim and White investigated beams with eliminated reactions by supporting them on corbels, projected from the upper portion of the beams’ side faces. Elimination of the support reaction confinement led to extensive splitting cracks along the tensile bars with premature failure at greatly reduced load capacity.

2.5 Types of failure cracks in shear tests

Many investigators discussed the different shear failure cracks, however, this introduction is mainly based on the report by Bresler and MacGregor (1967) in which they identified two types of shear cracks: primary and secondary. The primary cracks were classified in two types: web-shear cracks which appear before the formation of any flexural cracks in their vicinity, and flexural-shear cracks, which extend from previously developed flexural cracks. Secondary cracks appear due to the splitting forces along the tensile bars. They showed that the type and distribution of shear cracks depend upon: the cross-sectional shape, $f_c$, $\rho$, $\rho_v$, $f_y$, $f_{yv}$, the detailing of anchorage of the longitudinal tensile reinforcement and the type of loading. In the following sections, the frequently appeared cracks in shear tests are reviewed according to early investigators’ efforts.

2.5.1 Shear-compression failure

Many investigators (Moody et al. 1954, Bresler and Scordelis 1967, Bresler and MacGregor 1967, Broms 1969, and Kim and White 1999) showed that this type of failure occurs due to crushing of the compressed concrete zone after the formation of a major diagonal tension crack in transversely unreinforced beams having short shear spans. From a strut-and-tie model point of view, they described this failure type accompanying concrete crushing in the beam compression zone as failure of an upper node in the compression chord.
In 1999, Kim and White discussed that the horizontal cracking intensity along the tensile bars in beams having high $a/d$ ratios affects the severity and speed of diagonal tension cracking which controls the shear-compression failure. However, in beams with short shear spans, since the critical section – at which diagonal cracks initiate – is relatively close to the support reaction confinement, the splitting cracks are retarded resulting in higher reserve strength than in beams having higher $a/d$ ratios. Furthermore, normal stresses at the apex of inclined shear cracks in beams having small $a/d$ ratios are more stable; thus, additional loads are needed for crack extension and failure.

2.5.2 Shear-tension failure

It is also known as "bond-shear" failure. Early investigations observed that this failure is preceded by severe horizontal cracking at the tensile reinforcement level. Investigators (Moody et al. 1954 and Swamy et al. 1970) and the ASCE-ACI committee 426 (1973) referred this failure mode to the slippage along the tensile reinforcement, caused by inadequate anchorage of the bars at the beam-ends. The ASCE-ACI committee 426 (1973) discussed that when the tensile bars begin to slip, their "wedging action" accelerates the extension of splitting crack, which may result in an anchorage failure. Other researchers (Bresler and Scordelis 1963, Bresler and MacGregor 1967, and Kim and White 1999) argued that this failure mode occurs in transversely unreinforced beams due to lack of confinement for the horizontal splitting cracks, extending along these bars. The horizontal cracking was attributed either to transverse shearing force acting on the tensile bars or to the extension of shear cracking along the tensile reinforcement bars. Experiments demonstrated that horizontal cracking reduces the dowel action shear contribution and causes spalling of the concrete cover below the longitudinal bars, which leads to premature failures at greatly reduced capacities. Therefore, it was concluded that adequate shear reinforcement supports the longitudinal bars, increases their dowel action contribution to shear resistance and retards the extension of horizontal splitting cracks.

Kim and White (1999) studied the effect of confining the horizontal splitting cracks on the shear strength of transversely unreinforced beams. They compared the shear strength of simply supported beams without shear reinforcement with typical ones
having an external prestressed clamp at a section of distance equal to the beam depth, \( d \), from the support. Results demonstrated that confining the horizontal splitting cracks increases the reserve shear strength from which they concluded that the horizontal splitting cracks along the tensile bars is a prerequisite for the shear-tension failure.

### 2.5.3 Diagonal tension failure

In 1954, Moody et al. conducted a number of shear tests on beams having a range of \( a/d \) from 1.5 to 3.4 with and without shear reinforcement and with different anchorage conditions for the tensile bars (straight or hooked bars). Since all the tested beams failed with the diagonal tension mode, it was concluded that the presence of either shear reinforcement or anchorage of the tensile reinforcement has no significance on the failure mode. However, it was noted that for beams having long shear spans (\( a/d > 2.5 \)), failure occurred at the onset of diagonal cracking load while for beams with shorter shear spans there was a reserve strength beyond this load level. Investigators (Moody et al. 1954, and Bresler and Scordelis 1963) referred the reserve shear strength over the diagonal cracking load in beams having short shear spans to the redistribution of stresses, occurring after the formation of the diagonal cracking. The increase in compressive and shearing stresses at the compression zone above the diagonal crack and the development of transverse shear and local bending in the longitudinal reinforcement were mentioned to be the main results of the stress redistribution. Moreover, they argued that the redistribution of stresses creates a stable condition that permits further increase in load over the diagonal cracking.

Fig. 2-13 – Effect of horizontal cracking on diagonal tension crack propagation (Kim and White 1999)
In beams having \( a/d > 2.5 \), a brittle failure occurs after the formation of a major diagonal crack. Researchers (Bresler and Scordelis 1963, and Kim and White 1999) referred this brittle failure to the extensive horizontal splitting cracks occurring along the tensile reinforcement near the supports of slender beams. It was discussed that stresses at the inclined crack apex increase extensively after the propagation of horizontal splitting crack (Fig. 2-13). The increase of the normal stresses at the apex of diagonal crack results in an unstable shear crack that extends rapidly within the compressive zone with insignificant increase in load. Therefore, it was concluded that arresting the propagation of both the inclined shear cracks and the horizontal splitting cracks are main roles of the shear reinforcement in beams.

2.5.4 Bearing compression failure

It is characterized by concrete crushing at the supports that can be referred to as nodal failure – due to compressive stresses exceeding the concrete strength – from the strut-and-tie model point of view. The ASCE-ACI committee 426 (1973) on Shear and Diagonal Tension discussed that this failure mode might occur in beams having short shear spans after the formation of an inclined shear crack joining the load and support blocks or in other words after transformation from beam to a tied arch action. To prevent such failure, Clause 11.5 in the CSA A23.3-94 code limits the compressive stress in the nodal zones according to the degree of confinement at these regions.

2.5.5 Web-crushing failure

Shear failure that is caused by web-crushing was reported in investigations by Bresler and MacGregor (1967), ASCE-ACI committee 426 on Shear and Diagonal Tension (1973), and Rangan (1991). It was shown that it is one of the possible failure modes in deep beams \( (a/d < 1) \) or in narrow-webbed beams (I- or T-beams) where the concrete struts, isolated by diagonal cracks, are subjected to high diagonal compressive stresses that exceed the concrete strength.

The ASCE-ACI committee 426 (1973) explained that shear stresses are much higher relative to the flexural stresses in narrow-webbed I-beams than they are in rectangular beams. Therefore, inclined shear cracks initiate in the web of the former
beams, due to the principal tensile stresses, rather than extend from preceding flexural cracks in the latter beams. These inclined shear cracks in beams with limited web widths, referred to as "web-shear" cracks, have inclination angles close to 45° due to the dominance of shear stresses within their webs. In addition, the ASCE-ACI committee 426 (1973) and Rangan (1991) discussed that web-crushing failure might precede yielding of shear reinforcement; thus, the diagonal compressive stresses and consecutively the shearing stresses should be limited to avoid such failure. Results of experiments conducted by Rangan (1991) showed the applicability of Nielsen's formula that was based on the plasticity theory (Eqn. 2-14) in limiting web-crushing failure in reinforced and prestressed concrete I-beams.

\[
\begin{align*}
V_{\text{max}} &= 0.5 f'_c b_w d_v \\
f'_c &= f'_c \left(0.7 - \frac{f'_c}{200}\right)
\end{align*}
\]  

[2-14]

where \(d_v\) is the internal flexural arm in meters that can be taken equal to \((0.9 d)\) for design purposes; and \(f'_c\) and \(V_{\text{max}}\) are in MPa and MN, respectively. Similar to limiting the \(\rho_i\) ratio in beam flexural design so that yielding of the tensile bars precedes crushing of concrete, moderate \(\rho_i\) ratios in beams would give the same effect as yielding of the shear reinforcement might precede crushing of the compressed web.

**2.5.6 Arch–rib cracks**

This failure mode was studied by a number of investigators (Bresler and Scordelis 1963, Bresler and MacGregor 1967, Fenwick and Paulay 1968, Broms 1969, and Swamy et al. 1970). It was attributed to the eccentricity of the thrust line in beams without or with shear reinforcement as illustrated in Figs 2-14a and b, respectively. However, in transversely reinforced beams, shear reinforcement causes more curvature of the thrust line than in beams without shear reinforcement (compare Fig. 2-14b with a). It is shown in Fig. 2-14c that these flexural cracks on the compression surface of beams are similar to those that appear in a concrete prism due to an eccentric compressive load. Fenwick and Paulay (1968) argued that this failure mode would occur in beams having relatively high \(a/d\) ratios.
In 1969, Broms introduced the model illustrated in Fig. 2-15 for the forces affecting diagonal shear cracking of rectangular beams. It was shown that due to the internal rotation resulting from splitting cracks along the tensile reinforcement, the inclination of the strut above the diagonal tension crack increases (comparing the inclination of the force \( R_a \) in Figs 2-15a and b). Consequently, the thrust line, joining the force \( R_a \) and the support, moves closer to the diagonal crack at some section within the shear span. Thus, in slender beams, this eccentric force causes tensile stresses at the uppermost face of beam.
Swamy et al. (1970) conducted a number of shear tests on rectangular and T-beams without shear reinforcement having a range of $a/d$ ratio between 1 and 7. "Vertical tension cracks emanating from the compression face" were observed in the beams having $1 < a/d < 4$ at loads close to failure. Since these cracks were observed just before failure, they were considered as evidence of the transformation from beam to arch action in those beams. On the other hand, in beams having longer shear spans ($4 < a/d$), compressive strains at the upper surface were found to increase continually with increasing the load up to failure. Since the flexural-tensile cracks observed in beams having $1 < a/d < 4$ result from local tensile failure and instability of the compression zone, it was concluded that these cracks are failure phenomenon that appear due to local stress conditions.
CHAPTER 3
SHEAR STRENGTH PREDICTION EQUATIONS

3.1 Introduction

In this chapter a number of selected researcher efforts in expressing the shear strength mathematically is presented in 3.2. In 3.3, a number of shear provisions in some of the major design codes are discussed. The shear strength equations presented here are calibrated in Chapter 4.

3.2 Research approaches

3.2.1 Zsutty's (1968, 1971) equation

A large record of early test data for beams without shear reinforcement, covering a wide range of beam variables \((a/d, \rho_l\) and \(f'_c\)), and lack of an approach that integrates these variables in an accurate shear strength equation at that time, inspired Zsutty to develop a rational shear strength equation for beams. A combination of dimensional and regression analyses was used to develop this equation and to distinguish between the shear capacity of arch action in short beams and beam action in slender beams.

Dimensional analysis technique was used to weigh each one of the main variables affecting the cracking strength, \(V_{cr}\). The cracking shear stress, \(v_{cr}\), was expressed as
\[
v_{cr} = V_{cr}/(b_o d) = k (f'_c)^{b_1} (\rho_l)^{b_2} (d/a)^{b_3}
\]  
[3-1]

where the constants \(b_1, b_2\) and \(b_3\) were found from the analysis of 151 beams – having \(a/d > 2.5\) – to be equal to \(\frac{1}{3}\). Thus, the cracking shear stress, \(v_{cr}\), for long slender beams \((a/d > 2.5\) ) without shear reinforcement was put in the form
\[
v_{cr} = k (f'_c \rho_l d/a)^{\frac{1}{3}}
\]  
[3-2]

Regression analysis of different laboratories’ data was used to determine the constant value \(k\). The analysis reflected the laboratory differences in identifying the test value of \(V_{cr}\). According to the individual laboratory, \(V_{cr}\) might be either: the load at which the diagonal crack crosses the beam’s neutral axis, the load at which a break in slope of
the strain gauge reading versus load curves occurs, or the load at a sudden diagonal
tension failure. Analysis of slender beams – having \( a/d > 2.5 \) – without shear
reinforcement showed that test data corresponding to the first definition of \( V_{cr} \) gave the
lowest coefficient of variation and \( k_{cr} = 59 \) (with \( v_{cr} \) and \( f_c' \) in psi). When analyzing the
data from all sources with recorded diagonal tension or sudden failure, a value of \( k_u = 61.2 \) (with \( v_u \) and \( f_c' \) in psi) was obtained for the ultimate shear stress. Therefore,
substituting both \( k_{cr} \) and \( k_u \) values into Eqn. 3-2 gives the cracking and ultimate shear
stresses, respectively.

\[
\begin{align*}
\nu_{cr} &= V_{cr}/(b_v d) = 59 \left( f_c' \rho_i d/a \right)^{1/3} \quad (\text{with } \nu_{cr} \text{ and } f_c' \text{ in psi}) \\
\nu_{cr} &= 2.14 \left( f_c' \rho_i d/a \right)^{1/3} \quad (\text{with } \nu_{cr} \text{ and } f_c' \text{ in MPa})
\end{align*}
\]

and

\[
\begin{align*}
\nu_u &= V_u/(b_v d) = 61.2 \left( f_c' \rho_i d/a \right)^{1/3} \quad (\text{with } \nu_u \text{ and } f_c' \text{ in psi}) \\
\nu_u &= 2.22 \left( f_c' \rho_i d/a \right)^{1/3} \quad (\text{with } \nu_u \text{ and } f_c' \text{ in MPa})
\end{align*}
\]

In 1971, Zsutty conducted a regression analysis on short beams (\( a/d < 2.5 \)) with
and without shear reinforcement and with top and bottom plates for load and support
conditions, respectively. Analysis of 108 short beam tests resulted in a preliminary
equation (Eqn. 3-5) for the central behavior of the data.

\[
\bar{\nu}_u = 167 \left( f_c' \right)^{0.26} \left( \rho_i \right)^{0.47} \left( d/a \right)^{1.30} \quad (\text{with } \bar{\nu}_u \text{ and } f_c' \text{ in psi})
\]

Inspired from the exponent values of the main parameters in Eqn. 3-5 and noting the
higher shear strength corresponding to beams having short shear spans (\( a/d < 2.5 \)),
Zsutty concluded that the simplest mathematical model that could represent this strength
increase was to multiply Eqn. 3-4 by a linear arch action factor equal to \([2.5/(a/d)]\).

\[
\begin{align*}
\nu_u &= 60 \left( f_c' \rho_i d/a \right)^{1/3} \left( \frac{2.5}{a/d} \right) = 150 \left( f_c' \rho_i \right)^{1/3} \left( d/a \right)^{4/3} \quad (\nu_u \text{ and } f_c' \text{ in psi})
\end{align*}
\]

In addition, Zsutty discussed that in transversely reinforced beams, their ultimate
shear capacity can be represented by adding the cracking shear strength for beams
without shear reinforcement (Eqn. 3-3 or 6) to the nominal yield capacity of the shear
reinforcement, $\rho_t, f_{yv}$. Thus, he introduced Eqn. 3-7 for the ultimate shear capacity of slender ($a/d > 2.5$) and short beams ($a/d < 2.5$), respectively.

$$v_{u-Zsutty} = \begin{cases} 60 \left( f'c \rho_t \frac{d}{a} \right)^{1/3} + \rho_t f_{yv} & \text{for } a/d > 2.5 \\ 150 \left( f'c \rho_t \right)^{1/3} \left( d/a \right)^{4/3} + \rho_t f_{yv} & \text{for } a/d < 2.5 \end{cases}$$  \hspace{1cm} \text{(psi)}  \hspace{1cm} [3-7]

In 1984, from their experimental results of slender beams without shear reinforcement, Mphonde and Frantz concluded that Eqn. 3-4 gave close prediction values to the test ultimate shear stress. However, a reduction in safety factor, measured by means of $\left( \frac{\nu_{test}}{\nu_{Zsutty}} \right)$ ratio, was observed for high-strength concrete beams having $f'c > 12,000$ psi.

### 3.2.2 Research of Haddadin et al. (1971)

In 1971, Haddadin et al. studied the variables affecting the shear strength of simply supported and restrained T-beams having shear reinforcement when subjected to axial loads (tension or compression). The variable parameters in their study were: the nominal yield strength of shear reinforcement, $\rho_t, f_{yv}$, the concrete strength, $f'c$, the axial load applied to beams (either tensile, compressive or no axial load), and the $a/d$ ratio. Experimental results were used to verify the shear strength equations adopted by ASCE-ACI Committee 326 and ACI 318-63 code in which the ultimate shear stress was expressed as

$$\nu_u = V_u / (b_u d) = K \rho_t, f_{yv} + v_c$$  \hspace{1cm} [3-8]

where $K = \left( \sin \alpha + \cos \alpha \right) \sin \alpha$; $\alpha$ is the inclination angle of the shear reinforcement with respect to the longitudinal axis of beam; $f_{yv}$ is the yield strength of shear reinforcement; and $v_c$ is the concrete shear stress expressed by Eqn. 2-12.

$$v_c = 1.9 \sqrt{f'c} + 2500 \frac{\rho_t V d}{M'} \leq 3.5 \sqrt{f'c} \left( 1 + \frac{0.002 N}{A_g} \right)$$  \hspace{1cm} [2-12]

The term $(K \rho_t f_{yv})$ in Eqn. 3-8, representing for the shear reinforcement contribution, was based on the pin-jointed 45° truss model that assumes yielding of the shear reinforcement.
within a distance $d/2$ from each side of the center of the shear crack. Haddadin et al. concluded that this approach is reasonable and does not need to be changed either for beams with or without axial loads.

From the experimental observations, some notes concerning the development of flexural and shear cracks in the tested T-beams having a mid-point concentrated load can be summarized as follows:

1- Flexural cracks form first at sections of maximum moment. With the load increased, the flexural cracks bend over toward the loading point forming flexural-shear cracks.

2- At higher loads, diagonal tension cracks form at the level of the section neutral axis and with further increase in load, more flexural and diagonal tension cracks appear within the shear span.

3- The number of diagonal shear cracks within the shear span increases and the spacing between them decreases as the shear reinforcement ratio, $\rho_s$, increases.

4- In their investigation, two modes of shear failure were identified, which depend on the shear reinforcement nominal strength, $\rho_t f_{yv}$: diagonal tension and shear-compression. The former and latter modes of failure occur in beams having small and high amounts of $\rho_t f_{yv}$, respectively.

5- A changeover from diagonal tension to shear-compression failure mode was found to depend on the variables $\rho_t f_{yv}$, $f'_c$ and $a/d$ ratio. Figure 3-1 illustrates the extent of these three variables on determining the failure mode. The failure mode would change from diagonal tension to shear-compression at lower values of $\rho_t f_{yv}$ as the $a/d$ ratio increases and the concrete strength decreases (Figs 3-1a and b, respectively).

6- It was found from the experimental data that two straight lines with different slopes (1.75 and 0.5) intersecting at the changeover point represent the relation between the ultimate shear stress, $v_u$, and the shear reinforcement nominal strength, $\rho_t f_{yv}$. These two straight-line equations are given in Eqns 2-10 and 11.
It was shown that $f'_c$ has a greater effect on shear strength when the failure mode is the shear-compression rather than when it is the diagonal tension mode. Since the concrete tensile strength is approximately proportional to $\sqrt{f'_c}$, it will decrease with a slower rate than the compressive strength, $f'_c$, itself. Thus, when $f'_c$ decreases, the shear strength calculated by Eqn. 2-11 will decrease more rapidly than that calculated by Eqn. 2-10.

![Graph showing the effect of variation of $a/d$ and $f'_c$ on relation between $\rho_l f_{sv}$ and $v_u$](image)

Fig. 3-1 – Effect of variation of $a/d$ and $f'_c$ on relation between $\rho_l f_{sv}$ and $v_u$ (Haddadin et al. 1971)

### 3.2.3 Equation of Bazant and Kim (1984)

Fracture mechanics studies on diagonal shear failure in concrete members showed that the nominal shear stress of beams without shear reinforcement should be proportional to $1/\sqrt{(1+\lambda/\lambda_o)}$. The factor $\lambda$ is defined as the relative structural size factor that equals to $d/d_a$ where $d_a$ is the maximum aggregate size in the concrete mixture; and $\lambda_o$ is a constant value. Bazant and Kim discussed that the majority of the tested beams in laboratories are of small size for which the shear failure stress is nearly constant and equal to the tensile strength of concrete, $f'_t$, while for larger beams, shear failure stress decreases extensively with increasing the size. Figure 3-2 illustrates the relation between the nominal shear stress, $\sigma_N$, and the size logarithms. It was discussed that in linear elastic
fracture mechanics, $\sigma_N$, is proportional to $1/\sqrt{d}$; thus, the plot of $(\log \sigma_N)$ versus $(\log d)$ is a straight line with slope $-\frac{1}{2}$. Combining the strength criterion (proportionality between $\nu$ and $f_t$), which applies for small beams with the size effect can be expressed as

$$\sigma_N = \frac{f_t^{\prime}}{\sqrt{(1 + \lambda/\lambda_o)}} \quad [3-9]$$

For small size beams, the factor $\lambda/\lambda_o$ is negligibly small compared to unity; thus, Eqn. 3-9 yields $\sigma_N = f_t^{\prime}$, which is a straight line representing the strength criterion (Fig. 3-2). On the other hand, for large size beams having large values of $\lambda/\lambda_o$, Eqn. 3-9 results $\sigma_N = f_t^{\prime}(\lambda/\lambda_o)^{-1/2}$ that is represented by a straight line with a slope of $-\frac{1}{2}$. Therefore, Eqn. 3-9 is a gradual transition from the strength criterion of small beams to the linear elastic fracture mechanics of large beams.

Fig. 3-2 – Size effect on nominal failure stress (Bazant and Kim 1984)

Bazant and Kim developed a shear strength equation (Eqn. 3-10) that takes into account the contribution extent of beam and arch actions to the shear strength.

$$V_{r-Bazant & Kim} = b_w d \left(0.83 \xi \sqrt{\rho_t + \rho_1 f_t} \right) \quad [3-10]$$

where $b_w$ and $d$ are the beam width and depth, respectively in meters; $\xi$ is a dimensionless size effect factor that equals to $\left(l/\sqrt{1+d/25d_o}\right)$; $d_o$ is the maximum aggregate size in meters; $\chi$ is a factor that accounts for the concrete strength and steel ratio, expressed as
\[ \chi = \sqrt{f_c} + 250\sqrt{\rho_l (d/a)^5} \] \[ \rho_l \] is the longitudinal reinforcement ratio; \[ f_c \] is the compressive concrete strength in MPa; \[ a \] is the shear span in meters; \[ \rho_t \] is the transverse shear reinforcement ratio; and \[ f_{yw} \] is the yield strength of shear reinforcement in MPa. It is to be noted in Eqn. 3-10 that Bazant and Kim considered the size effect through the factor \( \zeta \).

### 3.2.4 Mphonde’s (1989) equation

Experimental investigation conducted by Mphonde (1989) demonstrated that although theoretically the shear reinforcement contribution cannot be greater than its nominal yield strength, \( \rho_t f_{yw} \), it improves some of the concrete resisting components like the aggregate interlock and the dowel action. Therefore, he introduced Eqn. 3-11 as a general form of the shear strength in which \( \beta \) is a factor that reflects this increase in the concrete resisting components in beams with shear reinforcement.

\[ v_n = \beta C + \rho_t f_{yw} \] \[ 3-11 \]

However, since the increase in the concrete resisting components is due to the beneficial effect of stirrups, Mphonde proposed Eqn. 3-12 – similar to Eqn. 3-8 in the ACI 318-63 code – in which \( K \geq 1.0 \) was referred to as the “stirrup effectiveness factor” related to the stirrup contribution.

\[ v_n = C + K \rho_t f_{yw} \] \[ 3-12 \]

where \( C \) in Eqns 3-17 and 18 is the base concrete-shear contribution that can be taken equal to the inclined cracking shear stress.

Mphonde conducted a number of shear tests on beams with shear reinforcement from which the following two expressions were found to match the test results.

\[ v_n = (1.52 \sqrt{f_c} + 135) + \rho_t f_{yw} \] \[ 3-13 \]

\[ v_n = (1.51 \sqrt{f_c} + 90) + 1.60 \rho_t f_{yw} \] \[ 3-14 \]

where \( v_n \), \( f_c \) and \( f_{yw} \) in both equations are in psi. The terms in parentheses in Eqns 3-19 and 20 represent the concrete ultimate strength, \( \beta C \), and the inclined cracking strength, \( C \), of beams without shear reinforcement, respectively.
Since Eqn. 3-14, which uses the inclined cracking strength and \( K = 1.60 \) gives closer estimates of the test data than Eqn. 3-13 that uses a simple superposition of the concrete ultimate strength and \( \rho_l f_{yv} \), Mphonde concluded that using the shear reinforcement effectiveness equation (Eqn. 3-14) is appropriate and economic for design.

### 3.2.5 Equation of Sarsam and Al-Musawi (1992)

In an investigation on the applicability of the normal-strength concrete shear design provisions for high-strength concrete beams, Sarsam and Al-Musawi (1992) conducted a number of shear tests on high-strength concrete beams with shear reinforcement and having moderate slenderness ratios \( (a/d) = 2.5 \) and 4). From their study, they proposed a shear strength equation (Eqn. 3-15) that was concluded to have appropriate safety margins in high-strength concrete beams.

\[
V_{r,Sarsam\ et\ al.} = 0.85 \left[ 1.8 \left( f'_c \rho_l V_w d / M_u \right)^{0.38} b_w d + A_v f_{yv} d / s \right]
\]  

where \( V \) is in MN; \( b_w, d \) and \( s \) are in meters; \( M_u \) is in MN·m; \( A_v \) is in \( m^2 \); and \( f'_c \) and \( f_{yv} \) are in MPa. Compared to Zsutty’s equation (Eqn. 3-3), Eqn. 3-15 used a different power for the concrete strength, \( f'_c \), which is 0.38 and a lower constant factor for the concrete contribution, which is 1.8 instead of the 2.14 in Eqn. 3-3 to get more conservative design equation. However, test results showed that Eqn. 3-15 gives a lower coefficient of variation than Zsutty’s strength equation (Eqn. 3-7).

### 3.2.6 Equation of Russo and Puleri (1997)

Russo and Puleri (1997) developed a mechanical model that takes into account the effectiveness of shear reinforcement on the concrete shear resistance and its influence on the failure mode. They discussed that this model – based on the shear reinforcement effectiveness – is more rational than the conventional design procedure, which uses the simple superposition of the shear reinforcement and concrete capacities. This design approach was adopted by the ACI 318-63 code provisions (Eqn. 3-8) and also by Mphonde (1989) (Eqn. 3-14). Based on Eqn. 3-10, developed in 1984 by Bazant and Kim, they introduced a shear strength expression (Eqn. 3-16) as
where $b_w$ and $d$ are the beam width and depth, respectively in meters; $\xi$ is a dimensionless size effect factor that equals to $(1/\sqrt{1 + d/25d_a})$; $d_a$ is the maximum aggregate size in meters; $\chi$ is a factor that accounts for the concrete strength and steel ratio, expressed as $\chi = \sqrt{f'_c + 250\gamma_f(d/a)^4}$; $\rho_l$ is the longitudinal reinforcement ratio; $f'_c$ is the compressive concrete strength in MPa; $a$ is the shear span in meters; $\psi$ is the stirrup effectiveness factor that equals to $1.67\sqrt{f'_c/\chi}$; $\rho_t$ is the transverse shear reinforcement ratio; and $f_{yw}$ is the yield strength of shear reinforcement in MPa.

### 3.2.7 Equation of Kim et al. (1998, 1999)

Kim et al. developed a new approach for calculating the shear strength of reinforced concrete beams by quantifying the beam and arch action contributions. Based on their experimental results, an approach was based for determining the internal moment arm height, which then, by using the general shear equation (Eqn. 2-2), led to a shear strength equation (Eqn. 3-17).

$$V_{r-Kim\text{ et al.}} = 0.2(1 - \sqrt{\rho_l})(\frac{d}{a})^{0.6} \left[ \sqrt{f'_c + 1020\rho_l^{0.9} \left( \frac{d}{a} \right)^{0.6}} \right] b_w d + A_v f_{yw} d/s$$  \hspace{1cm} [3-17]

where $\rho_l$ is the longitudinal reinforcement ratio; $a$ is the shear span in meters; $b_w$ and $d$ are the beam width and depth, respectively in meters; $f'_c$ is the compressive concrete strength in MPa; $A_v$ and $f_{yw}$ are the cross-sectional area in m$^2$ and the yield strength in MPa of all the shear reinforcement branches within a spacing $s$ (in meters) measured parallel to the beam axis, respectively; and $r$ is a function of $a/d$ and $\rho_l$ ratios that was determined from regression analysis as

$$r = k(d/a)^n (\rho_l)^{n_2} \leq 1.0 \text{ where,}$$

$$k = 1.0; \ n_1 = 0.6; \ n_2 = -0.1 \text{ for beams without stirrups}$$

$$k = 0.6; \ n_1 = 1.4; \ n_2 = -0.2 \text{ for beams with stirrups}$$  \hspace{1cm} [3-18]
3.2.8 Shear friction theory, Loov (2001)

Loov developed a shear strength equation to supplement the empirical simplified equation of the CSA A23.3-94. Although he used the simple superposition principle of the concrete and steel contributions (Eqn. 3-19), he introduced rational principles for assessing these contributions based on the beam dimensions and stirrup spacing.

\[ V_r = V_c + V_s \]  \hspace{1cm} [3-19]

The stirrup contribution was based on the number of stirrups crossed by a potential shear crack, \( m \). When the shear crack intercepts a well-anchored stirrup, yield develops and produces a force \( V_{s1} \) in that stirrup.

\[ V_{s1} = A_v f_{yw} \]  \hspace{1cm} [3-20]
\[ V_s = m V_{s1} \]  \hspace{1cm} [3-21]

Loov showed that when the tensile reinforcement is adequately anchored at the beam ends, so that the force developed in these bars, \( F \), is larger than a value \( F_o \) (given by Eqn. 3-22), the concrete shear strength, \( V_c \), can be expressed by Eqn. 3-23.

\[ F_o = V_{45} \left( 2 + \tan^2 \theta \right) \]  \hspace{1cm} [3-22]
\[ V_c = V_{45} \tan \theta \]  \hspace{1cm} [3-23]

where \( V_{45} \) is the concrete shear strength when the potential shear failure plane is at 45°; and \( \theta \) is the inclination angle of the critical shear failure plane with respect to the beam axis. The term \( V_{45} \) in Eqn. 3-23 was introduced as

\[ V_{45} = \beta_v \lambda \phi_c \sqrt{f'_c} A_w \]  \hspace{1cm} [3-24]

where \( A_w \) is the effective web area participating in resisting the shear force, which equals to \( b_w h \) in case of a rectangular beam or to \( b_w h + h_f^2 \) for a T-beam, with \( h_f^2 \) being the area of the largest two 45° triangles that can fit within the junction of the flange with the web.

The calibration coefficient \( \beta_v \) in Eqn. 3-24 is a constant value that has been taken equal to 0.36 as the best fit for beams with \( f'_c = 30 \) MPa and heights from 300 to 500 mm. For beams with different concrete strengths other than 30 MPa and heights more than 500 mm, \( \beta_v \) is equal to 0.36 multiplied by two factors: \( (30/f'_c)^{0.25} \) and \( (500/h)^{0.25} \).
The inclination angle of the critical shear plane, $\theta$, was found to be influenced by: the effective height of shear reinforcement bars, $d_{ev}$, their spacing, $s$, and the number of spaces crossed by this plane, $m + 1$. The slope of the shear plane can be expressed as

$$\tan \theta = \frac{d_{ev}}{[(m + 1)s]}$$  \[3-25\]

in which the effective height of stirrups, $d_{ev}$, is

$$d_{ev} = h - c_t - c_b - 2d_b$$  \[3-26\]

with $c_t$ and $c_b$ are the clear covers to the stirrups at top and bottom of the beam, respectively; and $d_b$ is the nominal diameter of the stirrup bar.

Loov (2000) proved that the stirrups support less than or equal to half the shearing force, i.e., $V_s \leq V_f/2$. However, if in the extreme case $V_s$ exceeded $V_{45}d_{ev}/s$, the shear failure plane would bypass all the stirrups so that the entire shear would be supported by the concrete only. Therefore, to determine the number of stirrups, which would be crossed by the failure plane at a given shear level, $V_f$ is

$$m = \frac{0.5V_f}{V_{sl}} \downarrow \text{(to be rounded down)}$$  \[3-27\]

The following expression gives the maximum spacing that insures that the failure plane be steep enough, so that the concrete can carry the shear not supported by the stirrups, $V_c = V_f - V_s$:

$$s \leq \frac{V_{45}d_{ev}}{(m-1)(V_f - V_s)}$$  \[3-28\]

By setting $V_s$ equal to $V_c$ from Eqns 3-21, 23 and 25, we get a quadratic equation for $m$:

$$m = \frac{1}{2}\left[\sqrt{1 + \frac{4V_{45}d_{ev}}{V_{sl}s} - 1}\right] \downarrow \text{(to be rounded down)}$$  \[3-29\]

It may be noted that $V_{45}d_{ev}/s$, represents the concrete shear strength for a failure plane bypassing all the stirrups for which $\tan \theta = d_{ev}/s$. Equation 3-29 is for use when $V_c$ is to be calculated for a beam for which the dimensions and $f_c'$ are known, while Eqns 3-27 and 28 are for use in design of the shear reinforcement.
3.2.9 Equation of Lee et al. (2000)

Based on equilibrium consideration of a truss model in the vertical direction, Lee and Watanabe (2000) proposed the shear strength equation

\[ V_{r-Lee et al.} = b_w j d \rho_t f_{ws} \cot \theta \]  

where \( f_{ws} \) is the shear reinforcement stress at the shear strength load level; \( jd \) is the internal flexural arm, taken equal to \( (0.9 d) \); and \( \theta \) is the inclination angle of the concrete principal compressive stress with respect to the longitudinal beam axis. It was demonstrated that the shear reinforcement stress, \( f_{ws} \), is influenced by the governing failure mode. Two main shear failure modes were identified for transversely reinforced beams: tension failure (STF), and compression failure (SCF). Experimental data showed that the former failure mode is likely to occur in beams having small amounts of shear reinforcement, \( \rho_t f_{yw} \), while the latter mode takes place in beams with higher values of \( \rho_t f_{yw} \). Consequently, \( f_{ws} \) may be larger (considering strain hardening of reinforcement) or smaller than the yield stress, \( f_{yw} \), when failure mode is STF or SCF, respectively.

The involved calculations in this model can be executed in three steps:

1- Classification of the shear failure mode:

Experimental data from 92 reinforced concrete beams varying in \( \rho_t f_{yw} \) and \( f'_c \) were analyzed to investigate the modes of shear failure, from which the limits given in Eqn. 3-31 were concluded.

\[
\begin{align*}
2.8 & < f_{yw} \sqrt{\rho_t / f'_c} < 11 \quad \text{for STF mode} \\
11 & < f_{yw} \sqrt{\rho_t / f'_c} \quad \text{for SCF mode}
\end{align*}
\]  

[3-31]

2- Determination of the shear reinforcement stress, \( f_{ws} \):

a- SCF mode:

Since failure occurs by concrete crushing before yielding of shear reinforcement in beams having large amounts of shear reinforcement, it is reasonable to assume that \( f_{ws} \approx f_{yw} \). Moreover, the shear reinforcement stress for this mode is denoted as \( f_{wsc} \) and is expressed as

\[ f_{wsc} = 11 \sqrt{f'_c / \rho_t} \quad \text{(with } f_{wsc}, f'_c \text{ and } f_{yw} \text{ in MPa)} \]  

[3-32]
This equation shows that as $\rho_t$ increases or $f'_c$ decreases, the shear reinforcement stress, $f_{wcs}$, decreases. If the value of $f_{wcs}$ from Eqn. 3-32 is larger than $f_{yw}$, it will be taken equal to $f_{yw}$ and be substituted in Eqn. 3-30 for the $f_{ws}$ term.

b- STF mode:

The shear reinforcement stress $f_{wst}$ replaces $f_{ws}$ in Eqn. 3-30 when STF mode governs and its value $f_{wst} \geq f_{yw}$ if the strain hardening of the shear reinforcement is considered. Lee and Watanabe expressed this stress as

$$f_{wst} = f_{yw} \left(1 + 0.02 \frac{f_{wcs} - f_{yw}}{f'_c} \right) \geq f_{yw}$$  \[3-33\]

The term $f_{wcs}$ in Eqn. 3-33 is the shear reinforcement stress, given by Eqn. 3-32, assuming the shear reinforcement does not yield in the STF mode (Fig. 3-3). The difference $(f_{wcs} - f_{yw})$ serves as a fictitious stress indicating the potential of strain hardening.

![Fig. 3-3 – Steel stress in the STF mode considering strain hardening (Lee and Watanabe 2000)](image)

3- Inclination angle of the concrete principle compressive stresses, $\theta$:

The value of $\theta$ was expressed in Eqn. 3-34 by curve fitting as a function of both $\rho_t$, $f_{ws}$, and $f'_c$.  

\[ \theta = 1.2 \sqrt{\rho, f_{ws}/f_c'} \text{ where,} \\
f_{ws} = f_{wcs} \text{ for SCF mode} \\
f_{ws} = f_{yw} \text{ for STF mode} \]  

[3-34]

3.3 Shear strength equations in codes
3.3.1 CSA A23.3-94 general method – Clause 11.4

The general shear design method is based on the approach called \textit{"Modified Compression Field Theory"} (MCFT) developed by Collins and Vecchio in 1986. Although this theory was developed to predict the load-deformation response of membrane elements subjected to in-plane shear and normal stresses, it was extended in 1990s to include shear design of prestressed and non-prestressed concrete beams with and without shear reinforcement. However, the concrete panels tested in 1986 by Collins and Vecchio provided better understanding for the stress-strain characteristics of cracked concrete. The cracked concrete subjected to tensile strains in a direction normal to the compression was found to be softer and weaker under compression than concrete in a standard cylinder test. In addition, the MCFT takes into account the transformation of tensile stresses from concrete at the crack locations to the reinforcing steel, which is referred to as the \textit{"tension stiffening"} effect. These tensile stresses in cracked concrete were totally ignored in the original compression field theory developed by Collins and Mitchell in 1980.

In order to simplify the complex mechanism of shear transfer in cracked elements, the following assumptions were introduced:

1- Stresses and strains considered are assumed to be average values when taken over areas or distances large enough to include several cracks.

2- Tensile stresses in concrete between cracks vary from zero at the crack surfaces to a maximum value midway between cracks due to the tension stiffening mechanism between concrete and the steel reinforcement.

3- Principal strain directions coincide with the principal stress directions under monotonic loading conditions.
Figure 3-4a show a cracked concrete panel due to shear loading, while Figs 3-4b and c show the average stress state between two shear cracks and the local stresses at a crack, respectively. Since the applied load is constant, these two states of stresses are equivalent. It can be shown that the average tensile stresses in concrete $f_i$ (Fig. 3-4b) between two shear cracks are replaced by increased steel stresses and by an interface shear stress, $v_{ci}$, at the crack surface as shown in Fig. 3-4c.

Experiments showed that the principal compressive stress in concrete, $f_2$, is proportional not only to the principal compressive strain, $\varepsilon_2$, but also to the co-existing principal tensile strain, $\varepsilon_1$.

$$f_2 = f_{2\text{max}} \left[ \frac{2\varepsilon_2}{\varepsilon_c} - \left( \frac{\varepsilon_2}{\varepsilon_c} \right)^2 \right]$$

$$f_{2\text{max}} = \frac{f_c'}{0.8 + 170\varepsilon_1} \leq f_c'$$

where $\varepsilon_c'$ is the strain in a concrete cylinder at the peak stress, $f_c'$; and $f_2$, $f_{2\text{max}}$ and $f_c'$ are in MPa.
While it was shown that the relationship between the average principal tensile stress and the average principal tensile strain is nearly linear prior to cracking, $f_1 = E_c \varepsilon_1$, where $E_c \approx 2 f_{c'}/\varepsilon_{c'}$ is the concrete modulus of elasticity in MPa. After cracking, $f_1$ decreases as $\varepsilon_1$ increases according to the following proposed relation (Eqn. 3-36).

$$f_1 = \frac{f_{cr}}{1 + \sqrt{500\varepsilon_1}}$$

(with $f_{cr}$, $f_{c'}$, and $E_c$ in MPa) \[3-36\]

$$f_{cr} = 0.33 \sqrt{f_c}$$

where $f_{cr}$ is the concrete cracking stress. It has been demonstrated that Eqn. 3-36 applies only for small values of $\varepsilon_1$, however, for larger values, cracks become wider and the $f_1$ value will be controlled by yielding of the reinforcement at the crack and by the ability of transmitting the shear stresses, $v_{ci}$, across the cracked interface. The shearing force transmitted across the crack interface is a function of the crack width, $w$, and the coarse aggregate size, $d_a$, as given by Eqn. 3-37.

$$v_{ci} = \frac{0.18 \sqrt{f_{c'}}}{0.3 + \frac{24w}{d_a} + 16}$$

(with $v_{ci}$ and $f_{c'}$ in MPa, and $w$ and $d_a$ in mm) \[3-37\]

The average crack width, $w$, is taken as the product of the principal tensile strain, $\varepsilon_1$, and the crack spacing, $s_\theta$.

$$w = \varepsilon_1 \cdot s_\theta$$

$$s_\theta = \frac{1}{\left(\sin \theta + \cos \theta \right)}$$

(with $w$, $s_\theta$, $s_{mx}$ and $s_{my}$ in mm) \[3-38\]

where $s_{mx}$ and $s_{my}$ are the average spacing of cracks in the two orthogonal directions $x$ and $y$, respectively.

Moreover, when shear stress transmitted by aggregate interlock, $v_{ci}$, exceeds the value given by Eqn. 3-37, slippage between concrete parts and yield of shear reinforcement occur. In such cases, the average tensile stress, $f_1$, should be reduced until the shear stress, $v_{ci}$, reduces to the value of Eqn. 3-37, then, $f_1 = v_{ci} \tan \theta$ \[3-39\]
Regarding the calculation of the average principal strains in cracked concrete, the MCFT assumptions and procedure can be summarized as follows:

1- It was assumed that the shear stress is uniform over the effective web area of the beam, $b_wd_v$.

![Mohr's circle for average concrete strains and stresses](image)

Fig. 3-5 — Mohr’s circles for average concrete strains and stresses

2- From Mohr’s circle for strain (Fig. 3-5a), it was shown that the average principal tensile strain can be expressed as

$$
\varepsilon_1 = \varepsilon_x + (\varepsilon_x - \varepsilon_2) \cot^2 \theta \tag{3-40}
$$

where $\varepsilon_x$ is the concrete longitudinal (in $x$-direction) strain that can be simplified and equated to the longitudinal reinforcement strain given by Eqn. 11-22 in the CSA A23.3-94 code (Eqn. 3-41 in this thesis) for non-prestressed beams without axial loads.

$$
\varepsilon_x = \frac{M_f/d_v + 0.5V_f \cot \theta}{E_sA_s} \tag{3-41}
$$

in which $E_s$ is the longitudinal reinforcement modulus of elasticity.

Equations 3-40 and 41 imply that the flatter the inclination angle of cracks, $\theta$, the higher the $\varepsilon_1$ and $\varepsilon_x$ values are.
3- The average principal compressive strain is calculated from Eqn. 3-35 after substituting $\varepsilon_c = -0.002$.

$$\varepsilon_2 = -0.002 \left(1 - \sqrt{1 - \frac{f_2}{f_{2\text{max}}}}\right) \tag{3-42}$$

\[\text{Fig. 3-6 – Tension in web reinforcement of diagonally cracked beam}\]

(Collins and Mitchell 1991)

4- Figure 3-6 shows the vertical equilibrium achieved by the shear reinforcement.

$$A_v f_{yv} = \left(f_2 \sin^2 \theta - f_1 \cos^2 \theta\right) b_v s \tag{3-43}$$

$$\text{At the same time, from Mohr’s circle for stresses (Fig. 3-5b), it can be shown that}$$

$$f_1 + f_2 = v \left(\tan \theta + \cot \theta\right) \tag{3-44}$$

$$\text{where } v = \frac{V}{(b_w j d)} \text{ is the shear stress.}$$

$$\text{From Eqns 3-43 and 44, a shear strength equation was developed and expressed as}$$

$$V_r = V_c + V_s = f_1 \cdot b_w d_v \cot \theta + A_v f_{yv} \frac{d_v \cot \theta}{s} \tag{3-45}$$

$$\text{Substituting the term } f_1 \cot \theta = \beta \sqrt{f_c} \text{ in Eqn. 3-45 and using Eqns 3-36, 37 and 39 for the average and the upper limit of the principal tensile stress, } f_1, \text{ yields an expression that gives the average and the upper limit values of the factor } \beta.$$

$$\beta = \frac{0.33 \cot \theta}{1 + \sqrt{500 \varepsilon_1}} \leq \frac{0.18}{0.3 + \frac{24w}{d_a + 16}} \tag{3-46}$$
Then, substituting Eqns 3-35, 42 and 44 into Eqn. 3-40 results an expression for the average principal tensile strain, \( \varepsilon_1 \).

\[
\varepsilon_1 = \varepsilon_x + \left[ \varepsilon_x + 0.002 \left( 1 - \sqrt{1 - \frac{V}{f_c'} (\tan \theta + \cot \theta)(0.8 + 170 \varepsilon_1)} \right) \right] \cot^2 \theta \quad [3-47]
\]

The CSA A23.3-94 code expressed Eqn. 3-45 in the following form for both concrete and reinforcement contributions in transversely reinforced beams

\[
V_{rs} = V_{cg} + V_{sg} = 1.3 \lambda \phi_c \beta \sqrt{f_c'} \cdot b_w d_v + \phi_f A_f f_y \cdot d_v \cdot \cot \theta / s \quad [3-48]
\]

where the coefficient 1.3 is a correction factor that accounts for the low \( \phi_c = 0.6 \) in the Canadian code. Values of \( \beta \) and \( \theta \) are given in Table 11-1 and Figure 11-1 in the code as functions of both the normalized shear stress, \( \sqrt{f_c'} \), and the longitudinal strain, \( \varepsilon_x \). They also have been based on an assumed crack spacing, \( s_\theta = 305 \text{ mm} \) and maximum aggregate size \( d_a = 19 \text{ mm} \). The value of \( \theta \) in this case has been solved, so that \( f_2 \leq f_{2\text{max}} \) \( \text{and the strain in the shear reinforcement, } \varepsilon_n \), is at least equal to the yield strain (assumed to be 0.002). Within the possible range of \( \theta \) values, the one that corresponds to the minimum \( \rho_t \) was chosen in Table 11-1 of the code. Once the \( \theta \) value is determined, the value of \( \varepsilon_1 \) can be computed using Eqn. 3-47 and then the \( \beta \) value is calculated by Eqn. 3-46.

Using the limit design approach, the ultimate design shear, \( V_u \), should be less than the nominal shear strength of the beam, \( V_n \), multiplied by the appropriate material reduction factor, \( \phi \), i.e., \( V_u \leq \phi V_n \). Therefore, knowing the \( V_u \) value with the beam section and concrete properties, the shear reinforcement is proportioned according to the relation, \( V_s \geq V_f - V_c \).

Because any shear failure plane has a horizontal projected length of \( (d_v \cot \theta) \) along the beam axis, the values of \( V_f \) and \( M_f \) to be used in its analysis should be equal to the values at the middle of its length. Thus, the first section to be considered is \( (0.5 \ d_v \cot \theta) \) from the support face, which for the sake of simplicity may be taken as \( d_v \).

Figure 3-7 illustrates the effect of an inclined shear crack at a support on the longitudinal reinforcement. Although the moment at centerline of the support is zero, some tensile forces result from the shear crack inclination. Neglecting the small moment produced by
the crack interface shearing forces, the CSA A23.3-94, Clause 11.4 introduced Eqn. 3-49 for the calculation of this anchorage tensile force.

\[
T = (V_f - 0.5 V_s) \cot \theta \tag{3-49}
\]

Fig. 3-7 – Effect of shear on longitudinal reinforcement force

In beams without shear reinforcement, it was discussed that using \( s_\theta = 305 \text{ mm} \), which has been assumed for beams with shear reinforcement is not conservative because of the relatively large spacing between shear cracks in such cases. Moreover, the ability of cracked concrete beams without shear reinforcement to transfer shear across the inclined crack interface is governed by the width of the diagonal cracks, \( w = \varepsilon_1 s_\theta \). Consequently, Fig. N.11.4.4b in CSA A23.3-94, which is presented in Fig. 3-8 here, illustrates the assumptions made concerning the spacing of diagonal cracks in transversely unreinforced beams. Fig. 3-8b shows the benefit of using crack control reinforcement in reducing the diagonal crack spacing. Table 11-2 and Fig. 11-2 in CSA A23.3-94 code give the \( \beta \) and \( \theta \) values as functions of the crack spacing, \( s_z \), and the longitudinal strain, \( \varepsilon_x \). These values were derived assuming that \( d_a = 19 \text{ mm} \). However, the MCFT allows the use of an equivalent crack spacing, \( s_{ze} \), instead of \( s_z \), for determining the values of \( \beta \) and \( \theta \), when using aggregate sizes other than 19 mm.

\[
s_{ze} = \frac{35 s_z}{d_a + 16} \quad \text{(with } s_{ze}, s_z \text{ and } d_a \text{ in mm)} \tag{3-50}
\]
For transversely unreinforced beams made of high-strength concretes, diagonal shear cracks pass through aggregates resulting in relatively smooth crack interfaces. Therefore, it seems appropriate to use \( d_a = 0 \) in Eqn. 3-50.

In 2000, Vecchio reported that the MCFT showed good correlation with experimental results of beams reinforced transversely with \( \rho_t \geq 0.1\% \), while its accuracy was reduced in beams without shear reinforcement. Vecchio proposed two additional limits to the original constitutive relations: the first is related to the concrete tensile stresses, and the second to the crack widths. Regarding the first limit, Vecchio discussed that post-cracking tensile stresses in unreinforced or lightly reinforced concrete elements like webs of shear-critical beams result from the combination of: the tension stiffening, the aggregate interlock, and the tension softening mechanisms. A conservative value of \( 0.10 f'_t \) was proposed as a lower limit of these residual tensile stresses. Concerning the second limit on crack widths, it was discussed that shear slip along wide crack surfaces may lead to divergence between the directions of principal strains and those of the principal stresses. Moreover, an overestimation of the reorientation of the stress...
trajectories may result. Vecchio showed that the principal compressive stress, $f_2$, diminishes when the crack width exceeds 2 mm, for which he proposed Eqn. 3-51 for the reduction in $f_2$ value.

$$f_2 = \begin{cases} 
\frac{f_2}{\text{Eqn. (3-35)}} \left( \frac{5 - w}{3} \right) & \text{for } 2 \text{ mm} \leq w \leq 5 \text{ mm} \\
0 & \text{for } w > 5 \text{ mm}
\end{cases}$$  \[3-51\]

### 3.3.2 CSA A23.3-94 simplified method – Clause 11.3

The simplified method in CSA A23.3-94 is permitted for shear design in flexural members, which are not subjected to axial loads. This method requires the nominal shear resistance, $V_r$, that is composed of the concrete and shear reinforcement contributions, to be larger than the maximum factored shearing force, $V_f$. From Eqns 11-6 and 11 in the code, the resistance, $V_r$, for members having more than the minimum shear reinforcement or $d < 300$ mm is expressed in the form of Eqn. 3-52 as

$$V_r = 0.2 \lambda f_c \sqrt{f'_c} b_w d + \phi_s A_v f_{sv} d/s$$  \[3-52\]

with $V_r$ in MN, $f'_c$ and $f_{sv}$ in MPa, $A_v$ in m$^2$, and $b_w$, $d$ and $s$ in meters. However, when comparing the predictions of Eqn. 3-52 with experimental results, the CSA A23.3-94 stipulates using 0.167 instead of the 0.2 coefficient in this equation.

$$V_r = 0.167 \sqrt{f'_c} b_w d + A_v f_{sv} d/s$$  \[3-52a\]

Equation 11-5 in the CSA A23.3-94 code gives an upper limit for the shear reinforcement contribution, which is $V_s < 0.8 \lambda f_c \sqrt{f'_c} b_w d$, to guard against excessive crack widths. Consequently, the upper limit of the shear resistance is $V_r < 0.967 \lambda f_c \sqrt{f'_c} b_w d$. For members with effective depths greater than 300 mm and with transverse reinforcement less than required by Eqn. 11-1 in the CSA A23.3-94, the concrete contribution to shear resistance is given by Eqn. 11-7 in the code (given as Eqn. 3-53 in this thesis).

$$V_c = \left( \frac{260}{1000 + d} \right) \lambda f_c \sqrt{f'_c} b_w d \geq 0.1 \lambda f_c \sqrt{f'_c} b_w d$$  \[3-53\]

with $V_c$ in MN, $f'_c$ in MPa, and $b_w$ and $d$ in meters.
The minimum amount of shear reinforcement in the CSA A23.3-94 code is taken the greater of two values given in Clauses 11.2.8.4 and 11.2.11 as:

1- Clause 11.2.8.4 gives Eqn. 11-1 (Eqn. 3-54 in this thesis), which calculates the minimum area of shear reinforcement as

\[ A_{v_{\text{min}}} = 0.06 \sqrt{f'_c} \frac{b_w s}{f_{yw}} \]  

with \( A_{v_{\text{min}}} \) in \( \text{mm}^2 \), \( f'_c \) and \( f_{yw} \) in \( \text{MPa} \), and \( b_w \) and \( s \) in \( \text{mm} \).

2- Clause 11.2.11 gives the maximum spacing between shear reinforcement related to the shear demand at a section expressed as the factored shear stress, \( V_f/(b_w d) \), for non-prestressed beams.

\[ s_{\text{max}} = 600 \text{ mm or } 0.7 \ d \quad \text{if} \quad V_f / [10^{-6}(b_w d)] < 0.1 \phi_c f'_c \quad \text{(Low demand)} \]

\[ s_{\text{max}} = 300 \text{ mm or } 0.35 \ d \quad \text{if} \quad V_f / [10^{-6}(b_w d)] \geq 0.1 \phi_c f'_c \quad \text{(High demand)} \]

with \( b_w \) and \( d \) in \( \text{mm} \), \( V_f \) in \( \text{MN} \), and \( f'_c \) in \( \text{MPa} \).

In 1992, Sarsam and Al-Musawi concluded from their experimental investigation that this simplified method for shear design is conservative although it ignores the effect of both \( \rho_i \) and \( a/d \) ratios on the shear resistance of beams.

### 3.3.3 ACI 318-99

In 1963, Bresler and Scordelis suggested Eqn. 11-3 in the ACI 318-99 code (Eqn. 3-55 in this thesis) for calculation of concrete shear strength for non-prestressed members subjected to shear and flexure only. It was demonstrated that this equation is a close approximation for the inclined cracking strength and is satisfactory for design of beams with “ordinary proportions”, which means \( \rho_i V_d/M < 0.01 \).

\[ V_c = \begin{cases} 
2 \sqrt{f'_c} b_w d & \text{\( f'_c \) in psi, \( b_w \) and \( d \) in inches, and \( V_c \) in lb} \\
\sqrt{f'_c} / 6 b_w d & \text{\( f'_c \) in MPa, \( b_w \) and \( d \) in meters, and \( V_c \) in MN} 
\end{cases} \]  

[3-55]

Equation 3-55 was originally proposed as a replacement of the more detailed equation, Eqn. 11-5 in ACI 318-99 (or Eqn. 3-56 in this thesis). The latter equation that was based
on the collaborative work of Viest (1959) with others, accounts for more parameters, which are believed to influence the shear strength such as, $f_c$, $\rho_i$ and $a/d$.

$$V_c = \begin{cases} 
1.9\sqrt{f_c} + 2500\rho_i \frac{V_u d}{M_u} b_w d & (f_c \text{ in psi, } V \text{ in lb, } b_w \text{ and } d \text{ in inches}) \\
0.16\sqrt{f_c} + 17 \rho_i \frac{V_u d}{M_u} b_w d & (f_c \text{ in MPa, } V \text{ in MN, } b_w \text{ and } d \text{ in meters})
\end{cases} \tag{3-56}$$

The current ACI code shear provisions use the limit design approach for cross-sections design, $V_u \leq \phi V_n$, where $V_n$ is composed of two additive contributions, $V_c$ and $V_s$. In addition, $V_c$ may be calculated by one of the previous two equations (Eqn. 3-55 or 56), while $V_s$ is based on the traditional 45° truss model expressed in Eqn. 3-57 for vertical shear reinforcement.

$$V_s = \frac{A_v f_{vy} d}{s} < 8\sqrt{f_c} b_w d \tag{3-57}$$

with $f_c$ and $f_{vy}$ in psi, $A_v$ in in$^2$, $b_w$, $d$ and $s$ in inches, and $V_s$ in lb. The ACI 318-99 specified that the minimum area of shear reinforcement for prestressed and non-prestressed members should not be less than $A_{v_{\text{min}}}$ determined from Eqn. 3-58.

$$A_{v_{\text{min}}} = 50 b_w s / f_{vy} \tag{3-58}$$

with $f_{vy}$ in psi, $A_{v_{\text{min}}}$ in in$^2$, and $b_w$ and $s$ in inches.

The ACI shear provisions have been subjected to extensive research work to study their applicability for designing beams with different material properties, reinforcement details, shear reinforcement amounts and loading configurations. Most of the investigations conducted in the last 30 years, examined the ACI code's adequacy for representing the size effect, $\rho_i$, $\rho_i$ and $f_c$ and its applicability to present safe design factors.

Regarding the size effect on shear strength, investigators (Mphonde and Frantz 1984, Collins and Kuchma 1999, and Angelakos et al. 2000) showed that Eqn. 3-55 is conservative and thus applicable only for small size beams with or without shear reinforcement; but its safety factor decreases in large beams without shear reinforcement. Moreover, Collins and Kuchma (1999) suggested Eqn. 3-59, which accounts for the
decrease in the shear associated with increasing the size of beams having shear reinforcement less than the minimum amount, to replace Eqn. 3-55 of the ACI 318-99 code.

\[
V_c = \begin{cases} 
\frac{115}{50 + s_{ze}} \sqrt{f_c' b_w d} & (f_c' \text{ in psi, } b_w, d \text{ and } s_{ze} \text{ in inches, and } V_c \text{ in lb}) \\
\frac{245}{1275 + s_{ze}} \sqrt{f_c' b_w d} & (f_c' \text{ in MPa, } b_w, d \text{ and } s_{ze} \text{ in meters, and } V_c \text{ in MN})
\end{cases} \quad [3-59]
\]

where \(s_{ze}\) is the parameter that accounts for the longitudinal crack spacing, \(s_z\), given by Eqn. 3-50. In 1984, Bazant and Kim discussed that Eqn. 3-56 does not account for the size effect. Therefore, it is applicable for small size beams where the strength criterion governs (Fig. 3-2).

Concerning the shear reinforcement ratio, \(\rho_t\), Collins and Kuchma (1999) showed that Eqn. 3-55 is appropriate for beams having shear reinforcement more than the minimum amount. However, they proposed Eqn. 3-60, which has been adopted by the AASHTO-LRFD code, to replace the current ACI equation for the minimum shear reinforcement (Eqn. 3-58 of the ACI 318-99 code).

\[
A_{v_{\min}} = \begin{cases} 
\sqrt{f_c'} b_w s & (f_c' \text{ and } f_{yv} \text{ in psi, } b_w \text{ and } s \text{ in inches, and } A_{v_{\min}} \text{ in in}^2) \\
\frac{\sqrt{f_c'}}{12 f_{yv}} b_w s & (f_c' \text{ and } f_{yv} \text{ in MPa, } b_w \text{ and } s \text{ in mm, and } A_{v_{\min}} \text{ in mm}^2)
\end{cases} \quad [3-60]
\]

Moreover, the work done by Zsutty (1968), Haddadin et al. (1971) and Johnson and Ramirez (1989) demonstrated that Eqn. 3-56 is more conservative in beams having high amounts of shear reinforcement, while it has poor correlation with test results of beams without shear reinforcement.

Investigations conducted by Zsutty (1968), Mphonde (1989) and Angelakos et al. (2000) demonstrated that Eqn. 3-56 underestimates the effect of \(\rho_t\) on the shear strength. Zsutty (1968) referred this to the evaluation of the coefficients included in Eqn. 3-56 (the 1.9 and 2500 coefficients) by empirical fit to data representing two different types of beam behavior: beam and arch actions. In 1984, Mphonde and Frantz suggested that the shear provisions in the ACI code should reflect a reduced shear capacity for transversely
unreinforced beams having $\rho_l$ less than a critical value. However, since the shear strength and the applied moment at a shear failure increase due to the increase of $f'_c$, they argued that this critical value of $\rho_l$ should be proportioned to $f'_c$.

Researchers (Zsutty 1968, Haddadin et al. 1971, Mphonde and Frantz 1984, Johnson and Ramirez 1989, Roller and Russell 1990, and Angelakos et al. 2000) showed that Eqn. 3-56 overestimates the benefit of increasing $f'_c$. However, Johnson and Ramirez (1989) and Roller and Russell (1990) concluded that limiting $\sqrt{f'_c}$ to 100 psi, that was proposed by the ACI committee 318, would limit the concrete contribution to the shear strength for beams having $f'_c >10,000$ psi. In addition, Roller and Russell suggested that when $f'_c >10,000$ psi, the minimum nominal $v_s$ provided should be greater than $0.01f'_c$ but not exceeding 150 psi.

3.3.4 British code BS 8110: 1985

The shear strength in the British code, BS 8110:1985, is given by

$$V_{r-BS} = 0.79(100\rho_l)^{1/3}(f'_c/20)^{1/3}(400/d)^{1/4}b_w d + 0.87 A_v f_y, d/s$$  \[3-61\]

with $f'_c$ and $f_y$ in MPa, $b_w$, $d$ and $s$ in meters, $A_v$ in $m^2$, and $V_r$ in MN. The coefficients $1/1.25$ and 0.87 included in the concrete and reinforcement contributions are partial material safety factors for the concrete and reinforcement, respectively.

3.3.5 New Zealand code NZS 3101: 1982

The shear strength in the New Zealand code, NZS 3101:1982, is given as

$$V_{r-NZS} = (0.07 + 10\rho_l)\sqrt{f'_c} b_w d + A_v f_y, d/s$$  \[3-62\]

with $f'_c$ and $f_y$ in MPa, $b_w$, $d$ and $s$ in meters, $A_v$ in $m^2$, and $V_r$ in MN.

3.3.6 EuroCode 2: 1991

The shear strength in EuroCode 2 (1991) is governed by the minimum of two values: $V_{RD2}$ which is the upper limit of the shear strength intended to prevent web-
crushing failures, and $V_{RD3}$ that is equal to the summation of the concrete and reinforcement contributions.

The former value, $V_{RD2}$, is expressed as

$$V_{RD2} = 0.5v f_{CD} b_w d_v (1 + \cot \alpha) \quad [3-63]$$

where $v$ is the "effectiveness factor" taken equal to $(0.7 - f_c' / 200) > 0.5$ (with $f_c'$ in MPa); $f_{CD}$ is the design concrete strength in MPa taken equal to $f_c' / 1.5$; $\alpha$ is the angle between the shear reinforcement and the beam axis; $b_w$ and $d_v$ are the web width and the moment lever arm in meters, respectively; and thus $V_{RD2}$ is in MN.

While the latter value, $V_{RD3}$, is given as

$$V_{RD3} = V_{WD} + V_{CD} \quad [3-64]$$

where $V_{WD}$ is the shear reinforcement contribution in MN taken equal to $(A_v f_{yl} d_v / s)$ with $A_v$ in m$^2$, $d_v$ and $s$ in meters and $f_{yl}$ in MPa, and $V_{CD}$ is the concrete contribution in MN expressed by Eqn. 3-65.

$$V_{CD} = 1.5 \tau_{RD} k (1.2 + 40 \rho_1) b_w d_v \quad \text{where,}$$

$$\tau_{RD} = \begin{cases} 
0.01 f_c' + 0.06 & \text{for } f_c' \leq 20 \text{ MPa} \\
0.008 f_c' + 0.1 & \text{for } f_c' > 20 \text{ MPa} 
\end{cases}$$

$$k = 1.6 - \left[ d / (1.0 \text{ m}) \right] > 1.0$$

$$\rho_1 = A_v / A_{cross-section}$$

with $\tau_{RD}$ and $f_c'$ in MPa, $b_w$, $d_v$ and $k$ in meters, and $A_v$ and $A_{cross-section}$ in m$^2$.

### 3.3.7 AASHTO-LRFD code: 2000 update

Similar to the General method of the CSA A23.3-94, the shear design provisions in the AASHTO-LRFD (2000 update) code were based on the MCFT. The shear strength of a reinforced concrete section is expressed as

$$V_{r-AASHTO} = 0.083 \beta \sqrt{f_c'} b_w d_v + A_v f_{yl} d_v \cot \theta / s \quad [3-66]$$

where the $\beta$ and $\theta$ values are determined from the MCFT and are given in tables as functions of the normalized shear stress, $v/f_c'$, and the average longitudinal strain over
the depth of the web, \( \varepsilon_x \) (Fig. 3-9), \( f_c' \) and \( f_{yw} \) in MPa, \( b_w, d_v \) and \( s \) in meters, \( A_s \) in m\(^2\), and thus \( V_r \) in MN.

The values of \( \beta \) and \( \theta \) in the MCFT derivation were based on the longitudinal strain that was taken in the CSA A23.3-94 General method equal to the strain in the longitudinal reinforcement. However, in the AASHTO-LRFD code since the concrete strain, \( \varepsilon_c \), on the compression side is quite small in comparison to the longitudinal reinforcement strain, \( \varepsilon_t \), it is adequate to take \( \varepsilon_x = 0.5 \varepsilon_t \). The longitudinal reinforcement strain, \( \varepsilon_t \), is calculated by Eqn. 3-41 of the CSA A23.3-94 code.

**3.3.8 Norwegian Standard NS 3473E: 1992**

Shear strength in the Norwegian Standard, NS 3473E: 1992, is given as

\[
V_{r-NS} = V_{cd} + V_{sd}
\]

where \( V_{cd} \) and \( V_{sd} \) are the concrete and reinforcement contributions in MN, expressed in Eqns 3-68 and 69, respectively.

\[
V_{cd} = 0.33 \left( f_t' + \frac{k_A A_s}{\gamma_c b_w d} \right) b_w d k_v \leq 0.66 f_t' b_w d k_v
\]

in which \( k_A = 100 \) MPa; \( \gamma_c \) is the material coefficient for concrete; \( k_v = 1.5 - \left[ d/(1.0 \text{ m}) \right] \) or 1.0 for members without or with shear reinforcement, respectively; \( f_t' \) is the concrete tensile strength in MPa; \( A_s \) is in m\(^2\); and \( b_w \) and \( d \) are in meters.
\[ V_{sd} = \frac{A_v f_{yw}}{s} z (1 + \cot \alpha) \sin \alpha \]  

[3-69]

where \( z \) is the internal moment arm of the cross-section in meters, taken equal to 0.9 \( d \) for sections having a compression zone; \( \alpha \) is the angle between the shear reinforcement bars and the longitudinal axis of beam; \( A_v \) is in \( m^2 \); \( f_{yw} \) is in MPa; and \( s \) is in meters.

### 3.3.9 Australian code AS 3600: 1994

The shear strength expression in the Australian code, AS 3600: 1994 (Eqn. 3-70) is valid for normal-strength concretes (\( f'_c \leq 50 \) MPa).

\[ V_{r\text{-AS}} = V_{uc} + V_{us} = \beta_1 \beta_2 \beta_3 \frac{A_s f'_c}{b_w d} b_w d + \frac{A_v f_{yw} d_v (\cot \theta_v + \cot \alpha) \sin \alpha}{s} \]  

[3-70]

where \( \beta_1 \) is a factor that accounts for the depth of the section in mm and is given as \( \beta_1 = 1.1 (1.6 - d/1000) \geq 1.1 \); \( \beta_2 \) is a factor accounting for the axial forces in the member, and is equal to 1 for members without axial loads; \( \beta_3 \) is a factor, which accounts for the presence of a large concentrated load near a support and equals to \( 2d/a \); \( \theta_v \) is the inclination angle of the diagonal compressive stresses with respect to the beam longitudinal axis; \( f'_c \) and \( f_{yw} \) are in MPa; \( b_w, d, d_v \) and \( s \) are in meters; \( A_s \) and \( A_v \) are in \( m^2 \); and thus \( V_r, V_{uc} \) and \( V_{us} \) are in MN.
CHAPTER 4
EXPERIMENTS ON I-BEAMS REINFORCED TRANSVERSELY WITH SINGLE-LEG HOOKED STIRRUPS AND DOUBLE-HEAD STUDS

4.1 Introduction
The experiments conducted in this research and presented in this part of the thesis are concerned primarily with exploring the feasibility of use of studs as replacement of the single-leg hooked stirrups in thin webbed I-beams. Six reinforced concrete beams have been tested to compare the shear behavior of the different reinforcement types in terms of shear strength and ductility. Half the length of each beam has been reinforced transversely with studs, the other half with single-leg hooked stirrups. The test specimens, their dimensions, reinforcement and the test variables are presented first.

Test results in terms of the load-deflection and the load-shear reinforcement strains are plotted. Experimental shear strengths have been used in calibrating shear strength equations provided by researchers and by some selected codes. The advantage of using double-head studs as shear reinforcement is emphasized in the analysis of the test results.

4.2 Description of experimental program
4.2.1 Test specimens
Elevation and cross-sections of the tested I-beams are shown in Fig. 4-1. The span between the centers of supports is 3.2 m, while the overall length of the beams is 4.0 m. The width of the upper and the lower flanges is equal to 0.3 m for all the tested beams. Height of the beams is the main variable in this research, where it is varied from 300 to 600 mm. The longitudinal reinforcement is provided with 180° hooks at both ends in accordance with CSA A23.3-94, Clause A12.2 provisions. Closely spaced regular stirrups are placed in the end blocks. Each beam is reinforced transversely over one half of its length with single-leg hooked stirrups with 90° and 135° bends at the ends. The other half
of each beam is reinforced with double-head studs (Fig. 4-1). The studs are made of straight bars, cut form the same stock used for fabricating the single-leg hooked stirrups, welded to circular heads at each end. The diameter of the stirrups and studs in all the tested beams is 8 mm.

![Elevation and sections of the test I-beams](image)

Fig. 4-1 – Elevation and sections of the test I-beams, $P$ is a single load applied in two different positions.

Each test specimen is labeled according to its height and shear reinforcement spacing (Table 4-1). For example, “400IB-250” indicates that, the height of specimen = 400 mm; and the spacing of shear reinforcement = 250 mm. The tested shear span, $a = 1350$ mm, is the part between the left-hand support and the concentrated load in Fig. 4-1. While this part is tested, the remaining part of the span (1.85 m) is reinforced by external vertical Dywidag bars (Fig. 4-2). After failure of the shear span, the prestressing bars are moved to strengthen the failing part and same beam is reused to test the untested part. The position of the concentrated load is changed, so that the shear span, $a$, is kept constant in all tests. The letter R in the designation of specimens indicates that the part with double-head studs is tested first. Absence of the letter R in the label of the specimen indicates that the part with stirrups is tested first.
Table 4-1 – Test variables

<table>
<thead>
<tr>
<th>Beam</th>
<th>$d$</th>
<th>$\rho_I$</th>
<th>$\frac{\rho_I}{\rho_b}$</th>
<th>$a/d$</th>
<th>$\rho_t$</th>
<th>$\rho_t f_{yy}$</th>
<th>$f_c$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>mm</td>
<td>%</td>
<td></td>
<td></td>
<td>%</td>
<td></td>
<td></td>
</tr>
<tr>
<td>300IB-185</td>
<td>264</td>
<td>7.56</td>
<td>1.08</td>
<td>5.11</td>
<td>0.27</td>
<td>1.65</td>
<td>26.8</td>
</tr>
<tr>
<td>400IB-250</td>
<td>364</td>
<td>5.49</td>
<td>0.94</td>
<td>3.70</td>
<td>0.20</td>
<td>1.22</td>
<td>27.0</td>
</tr>
<tr>
<td>600IB-390</td>
<td>564</td>
<td>3.54</td>
<td>0.72</td>
<td>2.39</td>
<td>0.13</td>
<td>0.78</td>
<td>28.2</td>
</tr>
<tr>
<td>400IB-250R</td>
<td>364</td>
<td>5.49</td>
<td>0.89</td>
<td>3.70</td>
<td>0.20</td>
<td>1.22</td>
<td>28.0</td>
</tr>
<tr>
<td>500IB-320R</td>
<td>464</td>
<td>4.31</td>
<td>0.92</td>
<td>2.91</td>
<td>0.16</td>
<td>0.96</td>
<td>23.9</td>
</tr>
<tr>
<td>600IB-390R</td>
<td>564</td>
<td>3.54</td>
<td>0.74</td>
<td>2.39</td>
<td>0.13</td>
<td>0.78</td>
<td>27.3</td>
</tr>
</tbody>
</table>

4.2.2 Proportioning the specimens

All the tested beams are proportioned to fail in shear. For this goal, precautions have been taken to guard against flexural and bearing-compression failure. The maximum shear strength of the investigated beams, $V_r$, has been based on a target concrete strength of 25 MPa and is calculated using the shear friction Eqn. 3-19 (See Loov, Section 3.2.8). This equation has been selected because it has been shown in published work to be accurate. The weight of the beams, varying between 6.4 and 9.3 kN, is ignored in the calculation of the shear strength. Thus, the bending at the loading point, $M$ is:

$$M = 1.35 V_r$$

[4-1]

and the shear force in the tested shear span

$$V_r = P \left( 3.2 - 1.35 \right) / 3.2 = 0.58 P$$

[4-1a]

where $P$ (kN) is the value of the applied load.

In the calculations for $V_r$ and $M_{flex}$ given in Table 4-2, the material resistance factors specified by code provisions are taken equal to unity. Four #25 bars are used as longitudinal tensile reinforcement and three #10 bars as upper longitudinal reinforcement in each of the tested beams. The anchorage of the tensile reinforcement, by 180° hooks, is sufficient to prevent slippage. Table 4-2 gives the values of $V_r$, $M$, $M_{flex}$ and $M_{flex} / M$ for each specimen. $M_{flex}$ is the ultimate flexural strength of the section below $P$. The ratio $M_{flex} / M$ being greater than unity indicates that failure will be by shear rather than by flexure.
Fig. 4-2 – Test setup
Steel plates - Steel cylinder

Details of bearing block

Cross frame girder (Hatched)

Actuator ram

Load cell

Spherical seat

Spacer (Loading block)

External prestressing rods

Steel pedestal

Plywood

Hollow box steel section

Fig. 4-2 – Test setup, continued
Appendix A includes the calculation of $V_r$ and $M_{flex}$ for a beam of height 400 mm. The values calculated in the appendix differ slightly from the values $V_r = 121$ kN and $M_{flex} = 272$ kN-m given in Table 4-2, because of the difference between the actual value of the concrete strength and the value of 25 MPa used in Table 4-2. The spacing, $s_{code}$, allowed by CSA A23.3-94 and the spacing, $s$, adopted in the tested beams are given in Table 4-3.

Table 4-2 – Calculated values of the ratio $M_{flex}/M$

<table>
<thead>
<tr>
<th>Beam height</th>
<th>$V_r$</th>
<th>$M_{flex}$</th>
<th>$M_{flex}/M$</th>
</tr>
</thead>
<tbody>
<tr>
<td>mm</td>
<td>kN</td>
<td>kN-m</td>
<td></td>
</tr>
<tr>
<td>300</td>
<td>110</td>
<td>149</td>
<td>169</td>
</tr>
<tr>
<td>400</td>
<td>121</td>
<td>163</td>
<td>272</td>
</tr>
<tr>
<td>500</td>
<td>127</td>
<td>171</td>
<td>362</td>
</tr>
<tr>
<td>600</td>
<td>133</td>
<td>180</td>
<td>452</td>
</tr>
</tbody>
</table>

Table 4-3 – Spacing of transverse shear reinforcement

<table>
<thead>
<tr>
<th>Beam height</th>
<th>Depth, $d$</th>
<th>$s_{code}$ *</th>
<th>$s$</th>
</tr>
</thead>
<tbody>
<tr>
<td>mm</td>
<td>mm</td>
<td>(Cl. 11.2.11)</td>
<td>mm</td>
</tr>
<tr>
<td>300</td>
<td>264</td>
<td>185</td>
<td>185</td>
</tr>
<tr>
<td>400</td>
<td>364</td>
<td>255</td>
<td>250</td>
</tr>
<tr>
<td>500</td>
<td>464</td>
<td>325</td>
<td>320</td>
</tr>
<tr>
<td>600</td>
<td>564</td>
<td>395</td>
<td>390</td>
</tr>
</tbody>
</table>

* Value of $s_{code}$ is the maximum spacing allowed for low demand shear, taken equal to 0.7$d$.

4.2.3 Test variables

The main variables in this experimental research are the type of shear reinforcement (single-leg hooked stirrups or double-head studs) and the beam height. The height of beams has been varied between 300 and 600 mm in increments of 100 mm. The nominal shear reinforcement strength expressed by the quantity $\rho_t f_{te}$ is different from one beam to another; where $\rho_t = A_s / (b_w s)$. Keeping the width of flanges, $b_{fl}$, the width of web, $b_w$, the amount of flexural-tensile reinforcement, $A_s$, and the distance of the applied
load from support, \( a \), as constants, results in variations in the values of the longitudinal 
reinforcement ratio, \( \rho_i = \frac{A_i}{(b_w d)} \) and the shear span-to-depth ratio, \( a/d \). Another 
unavoidable variable is the difference between the target and the actual concrete 
compressive strength, \( f'_c \). However, this difference is less than \( \pm 15 \) percent of the target 
concrete strength of 25 MPa. Table 4-1 lists the test variables for each one of the tested 
beams.

Figure 4-3 shows details of the two different types of shear reinforcement. The single-leg 
hooked stirrups are detailed in accordance with the CSA A23.3-94. The anchor head of 
the studs has diameter and thickness of 25 mm and 6 mm, respectively. This diameter 
provides a ratio of about 10 for the area of the head to that of the bar stem. If the tension 
in the stud reaches yielding (\( f_{sv} = 608 \) MPa), and the bond between the bar stem and 
concrete is ignored, the average bearing stress on concrete beneath the head will be close 
to 70 MPa. This stress does not cause crushing of concrete.
4.2.4 Materials

4.2.4.1 Concrete

Two different concrete mixes have been used so that beams made of the second mix could be tested within a week from their casting day. The two concrete mixes have been composed of normal type No. 10 cement and coarse and fine aggregates with nominal maximum sizes of 14 and 5 mm, respectively. Table 4-4 lists the two mix proportions used for the test specimens. Table 4-5 lists the actual concrete compressive strength, \( f'_c \), on testing days of each beam based on the average of at least three 100 mm \( \times \) 200 mm control cylinders.

Table 4-4 – Design of concrete mixes No. 1 and 2

<table>
<thead>
<tr>
<th>Component</th>
<th>Mix No. 1</th>
<th></th>
<th>Mix No. 2</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Mass (kg)</td>
<td>Ratio by mass</td>
<td>Mass (kg)</td>
<td>Ratio by mass</td>
</tr>
<tr>
<td>Cement</td>
<td>270</td>
<td>1.00</td>
<td>300</td>
<td>1.00</td>
</tr>
<tr>
<td>Water</td>
<td>189</td>
<td>0.70</td>
<td>174</td>
<td>0.58</td>
</tr>
<tr>
<td>Fine aggregate</td>
<td>995</td>
<td>3.69</td>
<td>870</td>
<td>2.90</td>
</tr>
<tr>
<td>Coarse aggregate</td>
<td>950</td>
<td>3.52</td>
<td>1085</td>
<td>3.62</td>
</tr>
</tbody>
</table>

Table 4-5 – Concrete strength and mix type of test specimens

<table>
<thead>
<tr>
<th>Beam</th>
<th>Mix design #</th>
<th>( f'_c ) (Phase 1)</th>
<th>( f'_c ) (Phase 2)</th>
<th>Age at test day</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>MPa</td>
<td>MPa</td>
<td>Days</td>
</tr>
<tr>
<td>300IB-185</td>
<td>1</td>
<td>27</td>
<td>27</td>
<td>69</td>
</tr>
<tr>
<td>400IB-250</td>
<td>1</td>
<td>27</td>
<td>27</td>
<td>68</td>
</tr>
<tr>
<td>600IB-390</td>
<td>1</td>
<td>28</td>
<td>28</td>
<td>58</td>
</tr>
<tr>
<td>400IB-250R</td>
<td>2</td>
<td>28</td>
<td>29</td>
<td>6</td>
</tr>
<tr>
<td>500IB-320R</td>
<td>2</td>
<td>24</td>
<td>25</td>
<td>7</td>
</tr>
<tr>
<td>600IB-390R</td>
<td>2</td>
<td>27</td>
<td>28</td>
<td>6</td>
</tr>
</tbody>
</table>

4.2.4.2 Flexural and transverse shear reinforcements

The longitudinal reinforcement of each test specimen consists of 4 bars #25 at the bottom and 3 bars #10 at the top. The bars have been delivered to the structural laboratory at the University of Calgary in a single shipment. A number of bar specimens
have been randomly selected and subjected to tensile tests to determine their stress-strain characteristics. The steel used for both types of transverse shear reinforcement has been deformed bars with nominal diameter of 8 mm. Pieces of appropriate lengths have been cut and shipped to Decon, Brampton, Ontario where anchor heads are welded. Figures 4-4 through 6 show typical stress-strain curves for #25, #10 and the 8 mm diameter bars, respectively. Table 4-6 lists the values of $\varepsilon_y$, $f_y$, $f_u$ and $E_s$; where $\varepsilon_y$ is the yield strain; $f_y$ and $f_u$ are the yield and ultimate stresses, respectively; and $E_s$ is the modulus of elasticity. For the 8 mm diameter bars $\varepsilon_y$ and $f_y$ are the strain and stress corresponding to a proof strain of 0.2%.

<table>
<thead>
<tr>
<th>Bar size</th>
<th>$\varepsilon_y \times 10^6$</th>
<th>$f_y$</th>
<th>$f_u$</th>
<th>$E_s$</th>
</tr>
</thead>
<tbody>
<tr>
<td>#25</td>
<td>2280</td>
<td>451</td>
<td>678</td>
<td>197.8</td>
</tr>
<tr>
<td>#10</td>
<td>2220</td>
<td>437</td>
<td>617</td>
<td>196.8</td>
</tr>
<tr>
<td>8 mm diameter</td>
<td>4884</td>
<td>608</td>
<td>645</td>
<td>219.1</td>
</tr>
</tbody>
</table>

4.2.5 Casting of test specimens

4.2.5.1 Formwork

An existing steel form having U-shape has been used in casting the beams. Its dimensions are 4.15 m in length and 0.30 m in both width and depth. For beams with heights more than 300 mm, additional wooden sides have been used. Figures 4-7a and b show the forms used in casting specimens 300IB-185 and 600IB-390, respectively. All the form surfaces have been sprayed with oil to facilitate their removal after hardening of the concrete. Four short threaded rods have been placed at the appropriate positions of the loading points (250 mm from both sides of the midspan). These threaded rods have been used to fasten the loading block to the tested beams. Figure 4-8 shows the rods before casting a typical beam.
Fig. 4-4 – Typical stress-strain curve of #25 deformed bars

Fig. 4-5 – Typical stress-strain curve of #10 deformed bars
Fig. 4-6 – Average stress-strain curve of 8 mm deformed bars

(a)- Steel forms used for 300IB-185  (b)- Form work used for 600IB-390

Fig. 4-7 – Forms used for casting the I-beam specimens
Fig. 4-8 – Threaded rods used to fix the loading block to a typical beam

4.2.5.2 Placement of steel cages

Figure 4-9 shows the steel cage in the two halves of one of the tested beams. The first picture shows the single-leg hooked stirrups, while the other shows the double-head studs.

Two Styrofoam blocks, cut from sheets of 2" thickness, have been used in shaping the webs of the I-beams. The two Styrofoam blocks have been held 100 mm apart by ¼" threaded rods (Fig. 4-10). Lifting lugs of high strength cables have been provided at both ends for transporting the steel cages and the finished beams before and after the casting, respectively.
4.2.5.3 Concrete casting

Each beam has been cast in one phase with two identical batches using one of the concrete mixes shown in Table 4-4. All batching has been done by mass. Six control concrete cylinders 100 mm x 200 mm have been cast from each batch and vibrated on a shaking table. Concrete has been placed in the forms with the aid of an electric internal vibrator. The beam surface has then been finished with a steel trowel.
One day after casting, the beam and the control cylinders have been removed from their forms and molds, respectively. Then the Styrofoam blocks, forming the webs, have been removed and the specimens cured.

4.2.5.4 Concrete curing

Curing of the beams and the control cylinders has begun one day after casting. They have been covered with wet burlap and plastic sheeting for 7 or 3 days for beams made of the first or second concrete mix, respectively. Then, the specimens have been allowed to dry in the laboratory till the testing date at age of at least 58 or 6 days for the beams of the first or second concrete mix, respectively.

4.2.6 Test setup

After concrete curing, specimens have been painted in white to facilitate crack observation. Vertical lines have been marked on the webs at shear reinforcement locations. Figures 4-2 and 11 illustrate the different components of the test setup. These components are a portal steel frame, an actuator ram attached to a load cell and a spherical seat, two steel pedestals anchored to the laboratory floor and the data scan modules connected to a personal computer.

The frame girder has been sustained at a constant level for all the tests; thus, steel blocks with different heights have been used as spacers to accommodate the differences in beam heights. These loading blocks have been fixed to the test specimens by the threaded rods embedded in the upper flanges (Fig. 4-8). A concentrated load has been applied monotonically at a distance of 1.35 m from one support. Hydraulic MTS (material testing system) connected to a 500 kN capacity actuator ram has been used. The load has been monitored by a 330 kN capacity OMEGADYNE load cell.

During the testing of a shear span of length 1.35 m, the remaining length of the beam (1.85 m), between the concentrated load and one support, is strengthened by external prestressing Dywidag bars of \( \frac{3}{8} \)" diameter passing through HSS sections (64 mm × 6 mm), at top and bottom beam faces (Fig. 4-12a). These external prestressing rods have been placed midway between each two of the internal shear reinforcement bars within the untested 1.85 m length of the beam.
4.2.7 Instrumentation

The magnitude of the applied load, $P$, has been given by the load cell in voltage readings, which have then been converted to load values. The load magnitude has been monitored by the data scan modules. Two LVDTs (Linear Variable Differential Transformers) have been used to measure the deflection at the point load position (one at the north and the other at the south side of the beam) as shown in Figs 4-12b and 13. No strain gauges have been used to measure the axial strain in the longitudinal tensile reinforcement, or in the concrete anywhere. However, strain gauges have been mounted on the mid-height points of every transverse shear reinforcement bar within the shear spans of both beams’ halves. Locations and numbers of the strain gauges in the tested beams are shown in Fig. 4-14.
Fig. 4-13 - LVDTs used to monitor the beam deflection
Readings of the applied load, $P$, the deflection, and the strain gauges have been continually recorded and monitored on a personal computer through the data scan modules. During testing, concrete cracks have been marked after each load increment with the corresponding load value indicated on the marked cracks.

Fig. 4-14 – Shear reinforcement spacing and strain gauges' layout in tested beams
4.2.8 Test procedure

Each beam is tested in two consecutive phases. During any phase, the load $P$ is applied in a load-controlled mode up to the service load level in constant load intervals then unloaded to a value of 10 kN. The service load level is assumed equal to half the predicted failure load. The lower load of the unloading cycle (10 kN) has been chosen to get a strong clear signal from the load cell. The loading-unloading cycles are repeated three times for each specimen. After reaching the load value of 10 kN at the end of the third cycle, the specimen is loaded to a value of 80% of the predicted strength in a load-controlled mode. At this load level, a displacement-controlled load application has been adopted to slow down the failure process and to get the descending branch of the load-deflection curve beyond the ultimate shear strength point. At the end of each load interval in the load-control mode, cracks have been traced and marked. However, in the displacement-controlled load application, load has been continually applied until the end of the test. After failure of the first shear span, the external prestressing rods are used in strengthening and repairing the failed half of the beam and the same loading procedure has been applied for the second phase of test.

4.3 Test results

As mentioned earlier, each beam has been tested in two phases. The results of the two phases are presented in this section.

4.3.1 Modes of failure

All beams have failed in shear in the two test phases. Diagonal cracks and yielding of the intercepted shear reinforcement have occurred in all phases, except in the second phase of specimen 600IB-390. In this phase, failure has occurred by shear-compression before yielding of the shear reinforcement.

Table 4-7 lists the values of $P_u$ and $V_u$ in each phase of testing as well as $P_{cr}$ in phase 1; where $P_u$ is the highest recorded value of the concentrated load applied at 1.35 and 1.85 m from the supports; $V_u$ is the value of the corresponding shearing force in the tested shear span. $P_{cr}$ is the value of the load at the start of shear cracking.
### Table 4-7 – Loads and modes of failure of I-beam specimens

<table>
<thead>
<tr>
<th>Beam</th>
<th>$P_{cr}$ (kN)</th>
<th>$P_u$ (kN)</th>
<th>$V_u$ (kN) in shear span reinforced with</th>
<th>Identification of the shear span tested in phase 1</th>
<th>Mode of failure for phase</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Phase 1</td>
<td>for phase 1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>KN 1 2</td>
<td>Stirrups</td>
<td>Studs</td>
<td>Stirrups</td>
<td>DT DT</td>
</tr>
<tr>
<td>1 300IB-185</td>
<td>80 165 174</td>
<td>95</td>
<td>101</td>
<td>Stirrups</td>
<td>DT DT</td>
</tr>
<tr>
<td>2 500IB-320R</td>
<td>100 208 191</td>
<td>110</td>
<td>120</td>
<td>Studs</td>
<td>DT DT</td>
</tr>
<tr>
<td>3 400IB-250</td>
<td>80 185 160</td>
<td>107</td>
<td>93</td>
<td>Stirrups</td>
<td>DT DT</td>
</tr>
<tr>
<td>4 400IB-250R</td>
<td>90 199 176</td>
<td>102</td>
<td>115</td>
<td>Studs</td>
<td>DT DT</td>
</tr>
<tr>
<td>5 600IB-390</td>
<td>90 230 220</td>
<td>133</td>
<td>127</td>
<td>Stirrups</td>
<td>DT SC</td>
</tr>
<tr>
<td>6 600IB-390R</td>
<td>115 258 218</td>
<td>126</td>
<td>149</td>
<td>Studs</td>
<td>DT DT</td>
</tr>
</tbody>
</table>

DT = diagonal crack and tension yielding of transverse reinforcement; SC = shear-compression.

### 4.3.2 Crack patterns

During the first phase of testing the beams, flexural cracks have occurred on the bottom surfaces of the beams near the loading point. As the load has increased, these cracks have propagated vertically upwards and new cracks have formed between the supports and the loading point in the tested shear span, as well as in the strengthened part of the beams. During this initial development of flexural cracks, no shear cracks have formed and thus, the strain readings in the shear reinforcement bars are practically equal to zero.

The shear cracks in the webs of the 300 to 500 mm beams are closely spaced and parallel to each other. In the 600 mm beams, the shear cracks have extended from flexural cracks. The shear cracks in the webs have inclinations ranging from 45° to 60° with the beam axis. As the load has increased, the shear cracks in the webs have extended with their original inclination. At approximately 70% of the failure load, these cracks have extended horizontally along the upper and lower web-flange connections, causing splitting between the web and the flanges. Shortly prior to failure, in the displacement-control load application mode, the upper splitting cracks near the loading point have extended in the upper flange towards the loading block; new flatter cracks, crossing the previously developed web-shear cracks, have occurred. Figure 4-15 illustrates the development of the shear cracks at two load levels for beam 400IB-250R. Figures 4-15a and b correspond to 45% of ultimate load; the former and the latter pictures are taken before and after load cycling, respectively. The picture in Fig. 4-15c is taken after failure.
Fig. 4-15 – Development of shear cracks in 400IB-250R

Shortly prior to failure of the 600 mm beams, an inclined crack has extended from the support to the loading block. Figures 4-16a and b show the shear cracks in beam 600IB-390R at 45% of ultimate load; the former and the latter pictures are taken before and after load cycling, respectively. Figures 4-16c and d show pictures taken at 72% of ultimate load and after failure, respectively.

In all the tested beams, loads beyond 80% of ultimate have caused extensive increase in the deflection and crack widths. Table 4-8 lists the measured inclination angles of failure shear cracks and those calculated by provisions of codes and proposed by researchers. This table is only for the shear spans tested in the first phase.

In all the beams, except the beams of 600 mm height, cracks have appeared on the compression flanges shortly before failure. These may be explained by Fig. 4-17. The dashed lines in the figure represent assumed struts and ties. Failure of the two shear reinforcements caused by their yielding is associated with local loss of equilibrium and
outward movement of the compression flange relative to the web as obvious in the picture.

Fig. 4-16 – Development of shear cracks in 600IB-390R

Table 4-8 – Inclination angles of shear cracks in tested beams

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>300IB-185</td>
<td>30</td>
<td>37</td>
<td>35</td>
<td>35</td>
<td>27</td>
</tr>
<tr>
<td>400IB-250</td>
<td>25</td>
<td>36</td>
<td>34</td>
<td>26</td>
<td>25</td>
</tr>
<tr>
<td>600IB-390</td>
<td>35</td>
<td>36</td>
<td>34</td>
<td>26</td>
<td>21</td>
</tr>
<tr>
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<td>36</td>
<td>34</td>
<td>26</td>
<td>21</td>
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* Example computation for $\theta$ using the shear friction model for beam 400IB-250R is given in Appendix B.

During the second phase of testing of all beams, some of the shear cracks formed during the first phase of testing, either have extended with their original inclination or
have adjusted their inclination by new flatter cracks. Figures 4-18a, b and c show pictures of shear spans after failure in second phase of testing for three beams. The transverse reinforcements in these shear spans are studs. The dashed lines in the figures encircle cracks developed during the first phase of testing. Figure 4-19 is similar to Fig. 4-18 except that the transverse reinforcements in the former are stirrups.

![Figure 4-17](image.png)

**Fig. 4-17 – Upper flange cracks in an extensively cracked beam**

### 4.3.3 Beam deflections

The load-deflection curves in phases 1 and 2 of testing are plotted for each beam in Figs 4-20 through 23. The deflection is measured below the loading point (Fig. 4-12b). The points indicated by a solid square and triangles correspond to the deflection $\Delta_{cr}$ at the start of shear cracking in the first phase and the deflection $\Delta_{yy}$ at which the yielding started in a stirrup or a stud, respectively.

In all specimens, the slope of the load-deflection graph at zero load level is steeper in the first phase. As would be expected, the smaller slope corresponds to the stiffness of a beam cracked prior to the start of the second phase. Table 4-9 lists the deflection values of $\Delta_{cr}$ (for first phase tests), $\Delta_{yy}$ and $\Delta_u$ at the highest recorded load $P_u$ for both phases and the corresponding load values, $P_{cr}$, $P_{yy}$ and $P_u$, respectively.
Fig. 4-18 – Crack patterns after failure: second phase testing

Cracks in dashed rectangles have developed during the first phase of testing
Fig. 4-19 – Crack patterns after failure: second phase testing
Cracks in dashed rectangles have developed during the first phase of testing
Fig. 4-20 – Load-deflection curves for beam 300IB-185

Fig. 4-21 – Load-deflection curves for 400 mm I-beams
Fig. 4-22 – Load-deflection curves for beam 500IB-320R

Fig. 4-23 – Load-deflection curves for 600 mm I-beams
Table 4-9 – Load and deflection values for the tested beams

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<th>First yield of web reinforcement</th>
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<td>mm</td>
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Fig. 4-24 – Strains in the transverse reinforcement of 300IB-185

Fig. 4-25 – Strains in transverse reinforcement of 500IB-320R
Fig. 4-26 – Strains in transverse reinforcement for beams of height 400 mm

Fig. 4-27 – Strains in transverse reinforcement for beams of height 600 mm
4.3.4 Strains in the shear reinforcement

As shown in Fig. 4-14, the strain gauges on the shear reinforcement are located at mid-height of the beam; thus, the strain readings are not necessarily the maximum strain in the stirrups or the studs. The maximum is normally at the location of the shear cracks. Figures 4-24 and 25 show graphs for $P$ versus the strain readings in the shear reinforcement for beams with heights 300 and 500 mm, respectively. The plots are for the stirrups or studs, which have exhibited high strain readings during the test. Figures 4-26 and 27 show similar graphs for beams of heights 400 and 600 mm, respectively.

4.4 Discussion of test results

Based on the results presented in the previous section, some shear strength equations selected from a number of codes and some research investigations will be calibrated in this section. The accuracy of the calibrated shear strength equations will be measured by the mean value of the ratio $V_u/V_r$. The different shear crack patterns, flexural cracks on the compression flange and their implications will be discussed in 4.4.2. The shear ductility of half-beams reinforced with either type of shear reinforcement (single-leg hooked stirrups or double-head studs) are compared by means of the displacement ductility factors.

4.4.1 Shear strength

The order in which the shear spans are tested has affected the values of the ultimate shearing force, $V_u$. In the first phase of testing, a pattern of cracks develops in the tested shear span as well as in the part strengthened with prestressed external bars. A shear span tested in the second phase is weakened by the pre-cracking developed in the first phase. Lines 3 and 4 in Table 4-7 are for two identical beams, differing only in the order in which the shear spans are tested. The values of $V_u$ for the shear spans reinforced with stirrups are 107 and 102 kN, when the shear span is tested first and second, respectively. For the same beams, the values of $V_u$ for the shear spans reinforced with studs are 115 and 93 kN in first and second phases of testing, respectively. Similar pattern of results can be seen by comparing $V_u$ values for the identical beams reported on lines 5 and 6. It
can be seen that a shear span tested first has a value of $V_u$ that exceeds the value of $V_u$ when identical span is tested second by 5 to 24 percent.

In all beams, the shear spans reinforced with studs have higher values of $V_u$ than those with stirrups. In a beam of height 400 mm, use of studs instead of stirrups increases $V_u$ by 8 percent (107 to 115 kN, lines 3 and 4 of Table 4-7, first-phase tests). In the beams of heights 600 and 500 mm, the corresponding increase is 12 and 9 percent, respectively (133 to 149 kN, lines 5 and 6 of Table 4-7, first-phase tests and 110 to 120 kN, line 2). In the beam of height 300 mm, the increase in $V_u$ due to the use of studs is 6 percent, although in this beam the shear span with studs is tested in the second phase (line 1 of Table 4-7).

In calibrating the shear strength equations discussed in Chapter 3, the resistance factors for the different materials (concrete and steel) and the included load factors are taken equal to unity. Furthermore, the measured properties for the concrete and the steel are used. Table 4-10 lists the shear strength values, $V_r$, calculated by the shear equations adopted from early investigations or from selected code approaches. In Table 4-11, the results of the tested beams are represented as three groups. The shear spans tested first with stirrups are presented in group 1. The shear spans tested first with studs are presented in group 2. Group 3 combines groups 1 and 2. Figures 4-28 and 30 compare test results with the shear strength, $V_r$, using the equations provided by codes and by researchers, respectively. Here, only the shear spans tested during the first phase are considered. The plotted ($V_u/V_r$) values for the beams of heights 400 and 600 mm are separated in two groups: the values plotted on the left-hand side of the group are for shear spans with stirrups; while, those on the right-hand side are for shear spans with studs.

Table 4-12 lists the $V_s$ values based on the simple shear provisions in the CSA A23.3-94 and the ACI 318-99 codes, and the concrete shear strength, $V_c = V_u - V_s$, for the tested beams. Furthermore, the normalized concrete and ultimate stresses, $v_c/\sqrt{f'_c}$ and $v_u/\sqrt{f'_c}$, respectively are listed and plotted versus the beam height in Fig. 4-29 for the tested beams. The upper limits for the ultimate normalized stress for both the CSA A23.3-94 and the ACI 318-99 codes are also given in Fig. 4-29.
Fig. 4-28 – Comparison between test results and equations of codes

Fig. 4-29 – Variation of normalized stresses with beam height
Table 4-10 – Shear strength values (in kN) calculated by equations of codes

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* Calculations for the shear strength $V_r$ for beam 400IB-250R are given in Appendix A.
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* Calculations for the shear strength $V_r$ for beam 400IB-250R are given in Appendix A.
Table 4-11 – Values of $V_u/V_r$ for comparison of test results and equations of codes

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Table 4-11 – Values of $V_u / V_r$ for comparison of test results and equations proposed by researchers, continued

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<td>1.09</td>
<td>1.06</td>
<td>1.03</td>
<td>1.28</td>
<td>0.53</td>
<td>0.83</td>
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</tr>
<tr>
<td></td>
<td>Mean</td>
<td>1.00</td>
<td>1.04</td>
<td>1.06</td>
<td>1.04</td>
<td>1.24</td>
<td>0.57</td>
<td>0.83</td>
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<tr>
<td></td>
<td>COV (%)</td>
<td>9.85</td>
<td>3.83</td>
<td>3.51</td>
<td>4.57</td>
<td>3.77</td>
<td>5.17</td>
<td>3.66</td>
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<td>Group 3</td>
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<td>Mean</td>
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<td>0.99</td>
<td>1.05</td>
<td>1.03</td>
<td>1.21</td>
<td>0.50</td>
<td>0.83</td>
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<tr>
<td></td>
<td>COV (%)</td>
<td>9.35</td>
<td>6.09</td>
<td>5.77</td>
<td>6.31</td>
<td>4.51</td>
<td>5.10</td>
<td>7.19</td>
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</table>
Table 4-12 – Normalized shear stresses of the tested beams

<table>
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<tr>
<th>Beam</th>
<th>$V_s$ (kN)</th>
<th>$V_u$ (kN)</th>
<th>$V_c$ (kN)</th>
<th>$v_c/\sqrt{f'_c}$</th>
<th>$v_u/\sqrt{f'_c}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 300IB-185</td>
<td>44</td>
<td>95</td>
<td>51</td>
<td>0.375</td>
<td>0.694</td>
</tr>
<tr>
<td>2 500IB-320R</td>
<td>44</td>
<td>120</td>
<td>76</td>
<td>0.333</td>
<td>0.529</td>
</tr>
<tr>
<td>3 400IB-250</td>
<td>45</td>
<td>107</td>
<td>62</td>
<td>0.330</td>
<td>0.565</td>
</tr>
<tr>
<td>4 400IB-250R</td>
<td>45</td>
<td>115</td>
<td>70</td>
<td>0.365</td>
<td>0.596</td>
</tr>
<tr>
<td>5 600IB-390</td>
<td>44</td>
<td>133</td>
<td>89</td>
<td>0.296</td>
<td>0.444</td>
</tr>
<tr>
<td>6 600IB-390R</td>
<td>44</td>
<td>149</td>
<td>105</td>
<td>0.355</td>
<td>0.505</td>
</tr>
</tbody>
</table>

Many early investigations have been conducted on beams without shear reinforcement from which concrete strength equations have been introduced. As discussed early, the shear reinforcement presence has an insignificant effect on the cracking shear strength; thus, it seems appropriate to evaluate the accuracy of these equations as shown in Tables 4-13 and 14. Figures 4-31 and 32 compare the cracking strength observed from tests, $V_{c-test}$, with the concrete shear strength, $V_c$, using the equations provided by codes and by researchers, respectively. Here, only the shear spans tested during the first phase are considered. In a similar way to Figs. 4-28 and 30, the plotted $\left(\frac{V_{c-test}}{V_{c-eqn}}\right)$ values for beams of heights 400 and 600 mm are separated in two groups in Figs 4-31 and 32.

From the above tables and figures, the following discussion can be drawn:

**Shear provisions in some investigated codes:**

1- In general, all the studied code provisions except the NZS 3101 (1982) are conservative. The AS 3600 (1994) gives two unconservative values when compared with the experimental results of beams 400IB-250 and 500IB-320R.

2- Although both the general method of the CSA A23.3-94 and the shear provisions of the AASHTO-LRFD are based on the MCFT, the former is more conservative than the latter. The overall mean values of $V_u/V_r$ are 1.33 compared with 1.18, respectively; where $V_u$ and $V_r$ are the test and the calculated values, respectively.
Fig. 4-30 – Comparison between test results and equations proposed by researchers

Fig. 4-31 – Comparison between test results and code provisions for $V_c$
For some of the tested beams, the iterative procedure given in the AASHTO-LRFD (2000 update) for calculation of shear strength, $V_r$, does not converge. The calculated strength in consecutive iteration continues to fluctuate between two different values and does not converge to a specific value for $V_r$. An example will be shown in A.2.7. of Appendix A.

The results of both the simplified and general approaches in the CSA A23.3-94 code (Clauses 11.3 and 4, respectively) are close to each other.

The more detailed equation in the ACI 318-99 code (Eqn. 3-56) is 11% more accurate than the simplified one (Eqn. 3-55).

The Australian code, AS 3600 gives the nearest prediction of the experimental values $V_u$ if one considers the overall mean.

**Shear stress provided by concrete:**

The way in which the concrete shear strength, $V_c$, has been calculated in Table 4-12 ($V_c = V_u - V_s$) may be criticized since the shear strength provided by the shear reinforcement cannot be assured to reach the yield strength ($V_s = A_v f_{ys} d/s$).
Moreover, the shear reinforcement bars crossed by a shear crack are not necessarily equal to \( d/s \). The writer has no alternative method to determine \( V_c \).

8- The simplified strength approaches in the ACI 318-99 (Eqns 11-3 and 15) and the CSA A23.3-94 (Eqn. 11-4) codes are similar and conservative for design purposes although they did not account for \( a/d \) or \( \rho_l \) on the shear strength. However, the upper limit for the shear stress in the former approach is more appropriate than the unduly high value given by the CSA A23.3-94 code (Fig. 4-29).

9- The test series conducted in this investigation has shown that in the presence of studs, the concrete contribution to the shear strength is 11 to 20% higher than that exhibited with stirrups. This may be attributed to the slip-free anchorage achieved by the studs. However, no change for the codes is proposed here.

10- The experimental results have shown that both the cracking and the ultimate shear stresses are decreasing with the increase in the beam height, which confirms the findings of Bazant and Kim (1984), discussed in 3.2.3.

**Anchorage of stirrup legs with 90° bends:**

11- Both the ACI 318-99 and CSA A23.3-94 codes stipulate the necessity of anchoring the stirrup ends with 135° bends unless the adjacent concrete is protected against spalling in which case 90° bends are permitted. In addition, the tensile stress reaches yield in a stirrup leg only at or in the vicinity of shear crack. Thus, unless a crack intersects the leg of a stirrup close to the anchorage, there is no demand for the anchorage to develop the full-yield strength. In the tested beams where stirrups are anchored with 90° bends at the bottom ends (Fig. 4-3), opening of the bends and spalling of the concrete cover has not occurred as reported by Dilger and Ghali (1997) in their columns subjected to axial force. This may be attributed to the fact that in the tests conducted here the demand of the full-yield strength on the stirrup legs has not occurred close to the bend.
Table 4-13 – Concrete shear strength, $V_c$, values (in kN) calculated by equations of codes

<table>
<thead>
<tr>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
<th>(5)</th>
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<th>(8)</th>
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<td>Beam</td>
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<td>BS 8110</td>
<td>NZS 3101</td>
<td>Euro Code 2</td>
<td>AASHTO LRFD</td>
<td>NS 3473E</td>
<td>AS 3600</td>
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<td>Clause 11.4</td>
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<td>Eqn. 3-56</td>
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<td>Eqn. 3-65</td>
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<td>500IB-320R</td>
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<td>114</td>
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<td>40</td>
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<td>600IB-390R</td>
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<td>49</td>
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<td>125</td>
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<td>55</td>
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Table 4-13 – Concrete shear strength, $V_c$, values (in kN) calculated by equations proposed by researchers, continued

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<tr>
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<td>Eqn. 3-3 or 4</td>
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<td>175</td>
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<td>54</td>
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<td>101</td>
<td>97</td>
<td>73</td>
<td>239</td>
<td>73</td>
<td>86</td>
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<td>47</td>
<td>175</td>
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<td>54</td>
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<td>85</td>
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Table 4-14 – Values of $V_{c-test} / V_{c-eqn}$ for comparison between test results and equations of codes

<table>
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<tr>
<th>Beam</th>
<th>CSA A23.3-94</th>
<th>ACI 318-99</th>
<th>BS 8110</th>
<th>NZS 3101</th>
<th>Euro Code 2</th>
<th>AASHTO-LRFD</th>
<th>NS 3473E</th>
<th>AS 3600</th>
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<td>Clause 11.4</td>
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<td>Eqn. 3-56</td>
<td>Eqn. 3-61</td>
<td>Eqn. 3-62</td>
<td>Eqn. 3-65</td>
<td>Eqn. 3-66</td>
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<td>0.81</td>
<td>0.39</td>
<td>0.97</td>
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<td>0.74</td>
<td>0.41</td>
<td>0.90</td>
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<td>0.89</td>
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<td>1.60</td>
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<td>1.53</td>
<td>1.20</td>
<td>0.95</td>
<td>0.51</td>
<td>1.18</td>
<td>1.41</td>
</tr>
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<td>600IB-390R</td>
<td>1.36</td>
<td>1.44</td>
<td>1.35</td>
<td>1.08</td>
<td>0.96</td>
<td>0.53</td>
<td>1.17</td>
<td>1.20</td>
</tr>
<tr>
<td>Mean</td>
<td>1.50</td>
<td>1.61</td>
<td>1.50</td>
<td>1.20</td>
<td>0.88</td>
<td>0.45</td>
<td>1.07</td>
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Table 4-14 – Values of $V_{c-test} / V_{c-eqn}$ for comparison between test results and research approaches, continued

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<th>(15)</th>
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<th>(17)</th>
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<td></td>
<td></td>
<td></td>
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<tr>
<td></td>
<td>Eqn. 3-23</td>
<td>Eqn. 3-16</td>
<td>Eqn. 3-3 or 4</td>
<td>Eqn. 3-15</td>
<td>Eqn. 3-17</td>
<td>Eqn. 3-13</td>
<td>Eqn. 2-7</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Stirrups</th>
<th>Studs</th>
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</thead>
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<td>0.58</td>
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<tr>
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<td>0.51</td>
</tr>
<tr>
<td>600IB-390</td>
<td>0.71</td>
</tr>
<tr>
<td>400IB-250R</td>
<td>0.56</td>
</tr>
<tr>
<td>500IB-320R</td>
<td>0.88</td>
</tr>
<tr>
<td>600IB-390R</td>
<td>0.91</td>
</tr>
</tbody>
</table>

| Mean | 0.69  | 0.77  | 0.79  | 1.02  | 0.28  | 1.02  | 0.88  |
| COV (%) | 24.90 | 23.16 | 22.96 | 21.48 | 13.69 | 21.39 | 22.10 |
Shear strength equations proposed by researchers:

12- Shear strength equations introduced by Zsutty (1968), Bazant and Kim (1984), and Sarsam and Al-Musawi (1992) give conservative values for all the tested beams.

13- The shear friction model (Loov 2001) is the only approach that accounts for the effective height of the shear reinforcement bars in calculating the critical shear plane. For that, a similar equation to that used for stirrups (Eqn. 3-26) has been assumed here for the calculation of the studs’ effective height.

\[ d_{ev} = h - c_i - c_b - 2t_h \]  

where \(t_h\) is the thickness of the steel circular head welded to the bar segment at both ends that is equal to 6 mm (Fig. 4-3). It is to be noted here that in beams having the same height, and clear covers for the shear reinforcement at top and bottom surfaces, those reinforced with studs have higher \(d_{ev}\) than those reinforced with single-leg hooked stirrups. The non-conservative shear strengths predicted by the shear friction model (Loov 2001) may be attributed to: the high value of \(\beta_v\) coefficient (0.36), and the high estimated value of the effective height of the stirrups, \(d_{ev}\), involved in its formulation. However, if the approach presented by Loov (2000) in which \(\beta_v = 0.3\) and \(d_{ev} = h - c_i - c_b - 4d_s\) (for stirrups) is adopted, the mean value of \(V_u/V_{R-Loov}\) will be 1.08, 1.12 and 1.09 with coefficients of variation 3.97, 7.91 and 6.50% respectively for the shear spans reinforced with stirrups, studs and for all the shear spans tested in the first phase. Therefore, with such modifications in \(\beta_v\) and in the effective height of the stirrups, the shear friction model has the best correlation with the test results reported in this thesis. In addition, the shear friction model is more conservative for the half-beams reinforced with studs than those with stirrups, which can be attributed to: the \(\beta_v\) coefficient that has been originally calibrated on tests of beams reinforced with stirrups, and to the estimated effective height of the studs, which may be unduly overestimated by Eqn. 4-2.
14- It is obvious that applying the current shear provisions of the studied codes or of the researchers efforts to beams reinforced with studs rather than stirrups offers safer and more conservative design. This can be attributed to the slip-free anchorage provided by the enlarged heads at both ends of the studs and consecutively to the longer effective height of the stud stem that has beneficial effects on both the shear reinforcement and the concrete contributions.

**Concrete shear strength in different codes and equations proposed by researchers:**
15- From Table 4-14 and Figs 4-31 and 32, it can be seen that the value of $(V_{c-test}/V_{c-eqn})$ corresponding to the shear spans reinforced with studs are higher than those with stirrups. This can be attributed to the slip-free anchorage in the former than in the latter shear spans.
16- The NS 3473E (1992) and the EuroCode 2 approaches for the concrete shear strength show the best correlation with the cracking shear strengths of the tested beams.

**4.4.2 Shear crack patterns**

The implications of the observed crack patterns in the tested beams (presented in 4.3.2) can be summarized as follows:

**General comments for all tested shear spans:**
1- The strain readings of the shear reinforcements within the tested shear spans are practically equal to zero prior to diagonal shear cracking. However, they increase extensively at the positions where a shear crack intersects them. Within the strengthened part of the beams, the strains of the shear reinforcement are negligible even at failure, which indicates the effectiveness of the external prestressing bars in strengthening these untested shear spans. However, the prestressing bars have not prevented completely the occurrence of shear cracks.
2- Shortly before failure, cracks inclined at a relatively small angle with the beam axis occur. In the deeper beams the cracks create a compression member joining the support to the load, indicating a clear arch action.
Slender shear spans (beams of heights from 300 to 500 mm):
3- Cracks of the beams of height 300 to 500 mm indicate the dominance of the beam action in which the beam acts as a truss. The concrete parts between the parallel shear cracks act as diagonal struts, the tensile reinforcement and the compression concrete zone along the shear span represent the lower and upper chords, respectively while the shear reinforcement bars behave as tensile web members.

4.4.3 Shear ductility
Table 4-15 gives values of shear ductility parameters that are defined as:

$$\mu_1 = \frac{\Delta_u}{\Delta_{yv}} ; \mu_2 = \frac{\Delta_{u80}}{\Delta_{yideal}}$$

where the $\Delta$'s are four deflection values defined in a typical load-deflection graph in Fig. 4-33; $\Delta_u$ is the deflection at the highest recorded load $P_u$; $\Delta_{yv}$ is the deflection at which yielding started in a stirrup or a stud; $\Delta_{u80}$ is the deflection after the strength drops to 80 percent of $P_u$; and $\Delta_{yideal}$ is defined by a graphical construction adopted from Paulay and Priestly (1992) as the deflection at yield in an idealized bilinear $P-\Delta$ graph.

![Idealized curve](image)

Fig. 4-33 – Definition of parameters used for shear ductility calculations
Table 4-15 – Shear ductility parameters of the shear spans tested in the first phase

<table>
<thead>
<tr>
<th>Beam designation</th>
<th>Shear ductility parameter</th>
<th>Shear ductility parameter</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\mu_1$ (Stirrups)</td>
<td>$\mu_1$ (Studs)</td>
</tr>
<tr>
<td>1 300IB-185</td>
<td>1.6</td>
<td>-</td>
</tr>
<tr>
<td>2 500IB-320R</td>
<td>-</td>
<td>1.8</td>
</tr>
<tr>
<td>3 400IB-250</td>
<td>2.1</td>
<td>-</td>
</tr>
<tr>
<td>4 400IB-250R</td>
<td>-</td>
<td>2.2</td>
</tr>
<tr>
<td>5 600IB-390</td>
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<td>-</td>
</tr>
<tr>
<td>6 600IB-390R</td>
<td>-</td>
<td>2.0</td>
</tr>
</tbody>
</table>

* Test terminated at $\Delta_{87}$; this value is used in lieu of $\Delta_{80}$.

The phase in which a shear span is tested also affects the shear ductility. Thus, comparison of the shear ductility parameters for the shear spans reinforced with stirrups and studs can be done only for tests where both types of shear reinforcement are tested first in beams with same heights. The shear ductility parameters $\mu_1$ and $\mu_2$ can be compared for the beams of height 400 mm (line 3 with 4, Table 4-15) and for the beams of height 600 mm (lines 5 with 6). It can be seen that provision of studs instead of stirrups increases both $\mu_1$ and $\mu_2$ by 5 to 11 percent.

For beams 300IB and 500IB, the values of $\mu_1$ reported on lines 1 and 2 of Table 4-15 are 1.6 and 1.8, respectively. No other values of $\mu_1$ are given for these beams because their testing has not been repeated with reversed order of phases, as done for the beams of heights 400 and 600 mm. The tests for 300IB and 500IB have ended shortly after reaching $P_u$, to avoid damaging the beam before it can be tested in phase 2. Thus, the descending branch of the $P$-$\Delta$ graph and $\Delta_{u80}$ are not available and no $\mu_2$ values are reported in Table 4-15.

The first testing phase of the beam with height 400 mm (400IB-250) has been terminated (arbitrarily) at a load level equal to 87 percent of $P_u$. Thus, $\Delta_{87}$ is used to replace $\Delta_{u80}$ in calculation of $\mu_2$ reported in Table 4-15.
CHAPTER 5
CONCLUSIONS AND RECOMMENDATIONS

5.1 Introduction

Conclusions of the study presented in this thesis are given in this chapter. In addition, recommendations for future research are addressed.

5.2 Conclusions

The main results of the present investigation can be briefly summarized as follows:

1- Double-head studs can be used as substitute for stirrups in I- or T-beams, with advantage of ease of installation. They also offer improvements in strength and ductility (respectively, 6 to 12 percent and 5 to 11 percent in the tested beams). Although the concrete contribution to the shear strength is higher by 11 to 20 percent when the studs are used, no change to the code provisions is proposed here.

2- The test series conducted in this investigation confirms the known fact that there is a decrease in both the cracking and the ultimate shear stresses with the increase in beam height.

3- The following equations are recommended for calculating the shear strength in design (based on the shear friction model, Loov 2000):

\[ V_r = V_c + V_s \]

\[ V_c = V_{45} \cdot \tan \theta = \beta_v \lambda \phi_c \sqrt{f_c} A_w \cdot \tan \theta \]

Take \( \lambda \) and \( \phi_c = 1 \) when the equation is used for calibration of experimental data.

\[ V_s = V_{st} \cdot m = \phi_s A_v f_{yv} \cdot m \]

Take \( \phi_s = 1 \) for calibration of experimental data.

\[ d_{ev} = h - (c_l + c_b) - 4d_b \] (for stirrups); \[ d_{ev} = h - (c_l + c_b) - 2t_h \] (for studs)

where

\[ \beta_v = 0.3 \]
\( \lambda \) = a factor that accounts for the concrete density

\( \phi_c \) and \( \phi_s \) = concrete and steel resistance factors that equal to 0.6 and 0.85, respectively

\( f'_c \) = concrete uniaxial compressive strength

\( f_{yw} \) = yield strength of transverse shear reinforcements

\( A_w \) = effective web area participating in resisting the shear force, which equals to \( b_w h \) in case of a rectangular beam or \( b_w h + h_f^2 \) for a T-beam with \( h_f \) being the height of the largest 45° triangle that can fit within the flange

\( A_v \) = area of all shear reinforcement branches within a spacing \( s \)

\( b_w \) and \( h \) = beam web width and height, respectively

\( \theta \) = inclination angle for the failure shear crack

\( m \) = number of shear reinforcements crossed by the critical shear plane that equals to \( \frac{1}{2} \left[ \sqrt{1 + \frac{4V_{s1} d_{ev}}{V_{s1} s}} - 1 \right] \) (to be rounded down)

\( d_{ev} \) = Effective height of the shear reinforcements

\( c_t \) and \( c_b \) = clear covers at the top and bottom of the section, respectively

\( d_b \) = nominal diameter of the shear reinforcements

\( t_h \) = Thickness of steel circular head at both ends of a stud

5.3 Recommendations for future research

1- The use of one beam, for testing in two phases, to compare the behavior of shear spans having different shear reinforcement types complicates the comparison and thus is not recommended for future research. This is because the shear span tested in the second phase is weakened by the crack pattern developed in the first phase.

2- Precast girders are a common product for constructions of simply supported or continuous bridges. Girders are placed first and a concrete deck is then poured. Continuity between girders is then achieved by continuous prestressing. It is
proposed here to use precast girders with thin webs and external prestressing. The double-head studs, placed at the middle plane of the web, will act as shear reinforcement and as shear connectors for the composite action between the web and the deck. This proposal, published recently by others, may require experimental verifications to highlight its advantages and to discover any difficulties that may occur in practice.
REFERENCES


APPENDIX A
ESTIMATES OF FAILURE LOADS

A.1 Introduction

In proportioning the test beams the ultimate flexural and shear strengths have been estimated. The amount of the flexural and shear reinforcements is chosen such that the anticipated failure made by shear rather than by bending. For all test beams, the estimates have been based on an assumed value of the concrete strength, $f'_c$. This appendix shows as example, the calculations done for beam 400IB-250R, measured values of the concrete and steel properties are used. These are: $f'_c = 28$ MPa; $f_y = 451$ MPa; and $E_s = 197.8$ GPa for the # 25 bottom longitudinal bars; $f_y = 437$ MPa and $E_s = 196.8$ GPa for the # 10 top longitudinal bars; and $f_{yv} = 608$ MPa for the 8 mm nominal diameter shear reinforcement bars.

![Fig. A-1 - Dimensions and reinforcement of beam with height 400 mm](image)

The beam dimensions and reinforcement areas are given in Fig. A-1. Using CSA A23.3-94 provisions the balanced steel ratio is calculated to be $\rho_b = 6.15\%$ and $A_{sb} = 2245$ mm$^2$. This is greater than the provided cross-sectional area of the bottom reinforcement, $A_s = 2000$ mm$^2$. Thus, at flexural failure the bottom reinforcement yields
and the strain $\varepsilon_{cu} = 0.0035$ at the extreme compression failure. The ultimate moment is calculated to be $M_{flex} = 277$ kN-m.

The values of the shear strength $V_r$ have been calculated using different methods. The details of the calculations are given below for beam 400IB-250R. The greatest value of $V_r$ is 133 kN determined by Lee et al. (2000). The corresponding bending moment, immediately below the concentrated load is 1.35 $V_r = 180$ kN-m. This calculation presented below also serves as example computation of the values $V_r$ listed in Table 4-10.

A.2 Shear strength values using different equations

A.2.1 CSA A23.3-94 Clause 11.3, simplified method

Clause 11.3 in the CSA A23.3-94 code requires that this simplified method for calculating shear strength of reinforced beams be applied for members with no axial forces. Moreover, Clause 11.3.5.1 permits the application of Eqn. 11-6 for the calculation of $V_c$ only if the shear reinforcement ratio provided in beam is higher than the minimum (calculated by Eqn. 11-1 in the code). Since the minimum shear reinforcement area, calculated from Eqn. 11-1 of the code, is $A_{v\min} = 13$ mm$^2$ and the actually used area (of 8 mm diameter bars) is $A_v = 50$ mm$^2$, the first term of Eqn. 3-52a is applied for calculating $V_c$ as follows:

$$V_c = 0.167 \lambda f_c \sqrt{f_w} b_w d = 32 \text{ kN}$$

Using the simple equation based on the pin-jointed truss model adopted by the code to calculate the shear reinforcement strength, $V_s$, results

$$V_s = \frac{\phi_s A_v f_{yw} d}{s} = 45 \text{ kN}$$

The shear resistance is calculated by a simple superposition of concrete and shear reinforcement strengths (Eqn. 3-52a in this thesis).

$$V_r = V_c + V_s = 32 + 45 = 77 \text{ kN}$$

The code also gives Eqn. 11-5 to calculate the maximum allowable shear resistance that intends to insure yielding of shear reinforcement prior to web-crushing failure.

$$V_{r-max} = 0.967 \lambda f_c \sqrt{f_w} b_w d = 186 \text{ kN}$$
Accordingly, the failure is governed by diagonal tension rather than by web-crushing, thus;

\[ V_{r-CSA, simplified} = 77 \text{ kN} \]

**A.2.2 CSA A23.3-94 Clause 11.4, general method**

**First iteration**

Since this method depends on the values of the longitudinal strain, \( \varepsilon_x \), and the normalized shear stress, \( \nu_f / (\lambda \phi_c f'_c) \), an assumed value for \( V_f \) has to be chosen. Therefore, as a first assumption \( V_f = 77 \text{ kN} \), taken from the above simplified approach. Consequently, the corresponding moment at a section at \( (d_v \cot \theta/2 = 229 \text{ mm}) \) from the applied load is \( M_f = 77 \times 1.12 = 86 \text{ kN-m} \).

Using the code Eqn. 11-22 (Eqn. 3-41 in this thesis)

\[
\varepsilon_s = \frac{M_f}{d_v} + 0.5 V_f \cot \theta = \frac{E_s A_s}{E_s A_s}
\]

with \( d_v = 0.9 \ d; d = 364 \text{ mm}; A_s = 2000 \text{ mm}^2; E_s = 197.8 \text{ GPa} \) and a trial value \( \cot \theta = 1.4 \) gives

\[
\varepsilon_s = \frac{86 \times 10^3/(0.9 \times 0.364)}{197.8 \times 10^9 \times 2000 \times 10^{-6}} = 800 \times 10^{-6}
\]

\[
\nu_f = \frac{V_f}{b_w d} = 2.1 \text{ MPa} \quad \Rightarrow \quad \frac{V_f}{\lambda \phi_c f'_c} = 0.076
\]

Entering \( \varepsilon_s \) and \( \nu_f / (\lambda \phi_c f'_c) \) in Fig. 11-1 of the code gives an improved \( \theta \) value equal to 36° and \( \beta = 0.174 \).

The values of \( \theta \) and \( \beta \) are substituted into Eqn. 3-47 giving \( V_{rg} \)

\[
V_r \equiv V_{rg} = \lambda \phi_c \beta \sqrt{f'_c b_w d_v} + \frac{\phi_s A_v f_{ys} d_v \cot \theta}{s}
\]

\[
V_r = 30 + 55 = 85 \text{ kN}
\]

**Second iteration**

As a better approximation for the new iteration, the values of \( V_f = 85 \text{ kN} \) and \( \theta = 36^\circ \) (from the first iteration) will be used in the same procedure, which results in \( V_r = 85 \text{ kN} \), as assumed. Thus, \( V_{r-CSA, general} = 85 \text{ kN} \).
A.2.3 ACI 318R-99, equations (3-55 and 57)

Similar to the Canadian simplified method, the ACI 318-99 simple approach permits the superposition of the concrete and steel contributions for calculating the shear strength. The concrete contribution is calculated from Eqn. 3-55, if the beam is with ordinary proportions \((\rho, Vd/M \approx \rho, d/a < 0.01)\). Since the beam cross-sectional value for \(\rho, d/a = 0.0068 < 0.01\), Eqn. 3-55 applies.

\[
V_{c,\text{ACI}} = \sqrt{f'_c b_w d / 6} = 32 \text{ kN}
\]

While, the steel contribution is calculated by Eqn. 3-57.

\[
V_{s,\text{ACI}} = \frac{A_s f_y d}{s} = 45 \text{ kN}
\]

However, the ACI 318-99 gives an upper limit for the shear reinforcement contribution to insure yielding prior to the concrete web-crushing.

\[
V_{s,\text{max}} = 0.66f'_c b_w d = 128 \text{ kN} > V_s = 45 \text{ kN}
\]

\[
V_{r,\text{ACI}} = V_c + V_s = 77 \text{ kN}
\]

A.2.4 British code, BS 8110: 1985

The concrete shear strength in this code is calculated by the first term of Eqn. 3-61 in 3.3.4. Similar to the ACI 318-99 approach for the reinforcement contribution, it is calculated from the pin-jointed truss model \((V_s = 45 \text{ kN})\).

\[
V_{c,\text{BS}} = 0.79\sqrt[3]{100 \rho_i} \sqrt[4]{\frac{f'_c}{20}} \frac{400}{d} b_w d
\]

\[\text{with } \rho_i = 5.49\%; \quad f'_c = 28 \text{ MPa}; \quad b_w = 100 \text{ mm}; \quad \text{and } d = 364 \text{ mm gives}
\]

\[
V_{c,\text{BS}} = 0.79\sqrt[3]{5.49 \times 3} \sqrt[4]{\frac{28}{20}} \times \frac{400}{364} \times 0.1 \times 0.364 \times 10^3 = 58 \text{ kN}
\]

\[
V_{r,\text{BS}} = V_c + V_s = 103 \text{ kN}
\]

A.2.5 New-Zealand code, NZS 3101: 1982

The shear strength \(V_r\) in this code is Eqn. 3-62:

\[
V_{r,\text{NZS}} = (0.07 + 10 \rho_i)\sqrt{f'_c b_w d + A_s f_y d / s}
\]
with $\rho_l = 5.49\%$; $f'_c = 28$ MPa; $b_w = 100$ mm; $d = 364$ mm; $A_v = 50$ mm$^2$; $f_{yw} = 608$ MPa; and $s = 250$ mm gives

$$V_{r,NZS} = (0.07 + 10 \times 0.0549)\sqrt{28 \times 0.1 \times 0.364 \times 10^3} + \frac{50 \times 608 \times 0.364 \times 10^{-3}}{0.25}$$

$$V_{r,NZS} = 119 + 45 = 164 \text{ kN}$$

A.2.6 EuroCode 2: 1991

The shear strength in this code is governed by the smaller of the shear value that causes web-crushing failure, and the conventional summation of concrete and shear reinforcement contributions. In the beam considered, the latter value is much smaller than the former. According to this code the equation for $V_r$ is written in the form:

$$V_{r,EC2} = 1.5 \tau_{RD} \left(1.2 + 40 \rho_l\right)b_w d + A_v f_{yw} d_v/s$$

In this application $b_w = 100$ mm; $d = 364$ mm; $\rho_l = 2.6\%$; $A_v = 50$ mm$^2$; $f_{yw} = 608$ MPa; $d_v = 0.9 d = 328$ mm; $s = 250$ mm. The code gives the following equations for $\tau_{RD}$ and $k$.

$$\tau_{RD} = 0.008 f'_c + 0.1; \text{ and } k = 1.6 - [d/(1.0 \text{ m})]$$

Substitution gives $\tau_{RD} = 0.324$ MPa; and $k = 1.24$.

$$V_{r,EC2} = 1.5 \times 0.324 \times 10^3 \times 1.24 \left(1.2 + 40 \times 0.026\right) \times 0.1 \times 0.364 + \frac{50 \times 608 \times 0.328 \times 10^{-3}}{0.25}$$

$$V_{r,EC2} = 49 + 40 = 89 \text{ kN}$$

A.2.7 AASHTO-LRFD: 2000 update

**First iteration**

The AASHTO-LRFD (2000 update) shear provisions are based on the MCFT, which includes an iterative procedure as the General method of the CSA A23.3-94. Thus, for the first iteration, an assumed value of $V_f = 85$ kN will be taken. Consequently, the moment at a section at $(d_v \cot \theta/2 = 229$ mm) from the applied load is $M_f = 85 \times 1.12 = 96$ kN-m.

As discussed in Section 3.3.7, the longitudinal strain, $\varepsilon_x$, in the AASHTO-LRFD code is taken equal to half that in the flexural tensile reinforcement (Fig. 3-9).
\[ \varepsilon_s = \frac{M_f}{d_v} + 0.5V_f \cot \theta \]

with \( d_v = 0.9 \, d; \, d = 364 \, \text{mm}; \, A_s = 2000 \, \text{mm}^2; \, E_s = 197.8 \, \text{GPa} \) and a trial value \( \cot \theta = 1.4 \) gives

\[ \varepsilon_s = \frac{(96 \times 10^3/0.328) + 0.5 \times 85 \times 10^3 \times 1.4}{2 \times 197.8 \times 10^9 \times 2000 \times 10^{-6}} = 445 \times 10^{-6} \]

\[ \frac{V_f}{f_c} = \frac{V_f}{b_w d f_c} = 0.093 \]

From the AASHTO-LRFD tables, \( \theta_{\text{AASHTO}} = 30.8^\circ \) and \( \beta_{\text{AASHTO}} = 2.50 \). Consequently, the shear strength \( V_r \) is calculated from Eqn. 3-66.

\[ V_r = 0.083f_c b_w d_v + \frac{A_s f_v d_v \cot \theta}{s} \]

\[ V_r = 36 + 67 = 103 \, \text{kN} \]

**Second iteration**

As a better approximation for the new iteration, the values of \( V_f = 103 \, \text{kN} \) and \( \theta = 30.8^\circ \) will be used in the same procedure, which results in \( V_r = 91 \, \text{kN} \).

**Third iteration**

As a better approximation for the new iteration, the values of \( V_f = 91 \, \text{kN} \) and \( \theta = 34.4^\circ \) will be used in the same procedure, which results in \( V_r = 103 \, \text{kN} \).

Because the shear strength, \( V_r \), is varying between the two values 91 and 103 kN, the shear strength will be taken the minimum, thus; \( V_{r,\text{AASHTO}} = 91 \, \text{kN} \).

A.2.8 Norwegian standards, NS 3473E: 1992

The Norwegian Standard calculates the shear strength of transversely reinforced beams by a simple superposition of concrete and steel contributions to the shear strength (Eqn. 3-67). The concrete contribution, \( V_{cd} \), is calculated by Eqn. 3-68.

\[ V_{cd} = 0.33 \left( f_r + \frac{k_A A_s}{\gamma_c b_w d} \right) b_w d k_v \]

with \( f_r = 1.8 \, \text{MPa}; \, k_A = 100 \, \text{MPa}; \, A_s = 2000 \, \text{mm}^2; \, \gamma_c = 1.0; \, b_w = 100 \, \text{mm}; \, d = 364 \, \text{mm}; \) and \( k_v = 1.0 \) for members with shear reinforcement.
\[ V_{cd} = 0.33 \left( \frac{1.8 \times 10^6 + \frac{100 \times 2000}{0.1 \times 0.364}}{0.1 \times 0.364 \times 10^{-3}} \right) \]
\[ V_{cd} = 88 \text{ kN} \]

However, the NS 3473E sets an upper limit for the concrete contribution.

\[ V_{cd-max} = 0.66 f'_{ck} b_w d_k = 42 \text{ kN} < V_{cd} \text{ (governs)} \]

The shear reinforcement strength is calculated by Eqn. 3-69 in which:

- \( A_v = 50 \text{ mm}^2 \)
- \( f_{yv} = 608 \text{ MPa} \)
- \( z = 0.9d \)
- \( s = 250 \text{ mm} \)
- \( \alpha = 90^\circ \) is the angle between the shear reinforcement and the beam axis.

\[ V_{sd} = \frac{A_v f_{yv}}{s} z (1 + \cot \alpha) \sin \alpha = 40 \text{ kN} \]  

\[ V_{r-NS} = V_{cd-max} + V_{sd} = 82 \text{ kN} \]

### A.2.9 Australian code, AS 3600: 1994

The concrete contribution to shear strength in AS 3600 is calculated by the first term in Eqn. 3-70.

\[
\beta_1 = 1.1 \left( 1.6 - \frac{d}{1000} \right) = 1.36 > 1.1 \\
\beta_2 = 1.0 \quad \text{for members with no axial force} \\
\beta_3 = 2 \frac{d}{a} = 0.54 \\
\]

The shear reinforcement contribution is calculated from the second term in Eqn. 3-70 in which: \( \theta_v = 30^\circ \) (the measured inclination angle of the critical shear cracks); and \( \alpha = 90^\circ \).

\[ V_{us} = \frac{A_v f_{yv}}{s} d_v \left( \cot \theta_v + \cot \alpha \right) \sin \alpha = 69 \text{ kN} \]

\[ V_{r-AS} = V_{uc} + V_{us} = 100 \text{ kN} \]

### A.2.10 Shear friction model, Loov (2001)

Since the shear span reinforced with double-head studs has been tested in the first testing phase for beam 400IB-250R, Eqn. 3-19 will be used in evaluating the shear strength for this shear span. Equation 4-2 is used to calculate the effective height of studs.

\[ d_{ev} = h - c_t - c_b - 2 t_h = 358 \text{ mm} \]
As shown in 3.2.8, the effective web area in resisting shear is shown in Fig. A-2 and can be calculated as:

\[ A_w = b_w h + (\text{parts of the upper and lower flanges}) = 63400 \text{ mm}^2 \]

![Fig. A-2 - Cross-sections at different beam halves and the effective web area](image)

In 2000, Loov showed that in beams with shear reinforcement, the shear reinforcement supports maximum half the applied shear. This notion led to Eqn. 3-29, which calculates the number of shear reinforcement bars crossed by the critical shear crack, \( m \). The parameter \( V_{45} \) represents the concrete shear strength along a shear crack with 45° and is calculated by Eqn. 3-24 as:

\[ V_{45} = \lambda \phi_c \beta_v \sqrt{f_c} A_w = 121 \text{ kN} \quad [3-24] \]

The yield strength of one shear reinforcement bar is denoted as \( V_{y1} \) and is equal to \( A_v f_{yy} = 31 \text{ kN} \). Then substituting the values of \( V_{45}, V_{y1} \) and \( d_{ev} \) in Eqn. 3-29 results,

\[ m = \frac{1}{2} \left( \sqrt{1 + \frac{4V_{45} d_{ev}}{V_{y1} s}} - 1 \right) = 1.93 \quad [3-29] \]

As discussed previously this value has to be rounded down to the nearest integer; thus, \( m = 1 \), so that the number of spaces crossed by the shear crack is \( m + 1 = 2 \), from which the inclination of the shear crack is

\[ \tan \theta = \frac{d_{ev}}{(m+1)s} = 0.716 \quad [3-25] \]

The concrete shear strength, \( V_c \), is calculated by Eqn. 3-23.
\[ V_c = V_{45} \tan \theta = 87 \text{ kN} \quad [3-23] \]

The shear reinforcement strength, \( V_s \), is calculated by Eqn. 3-21.
\[ V_s = \phi_s A_v f_{yr} m = 31 \text{ kN} \quad [3-21] \]

Two adjustment factors for beams having height and concrete strength other than 500 mm and 30 MPa should be applied to the \( V_c \) term.
\[
\alpha = \alpha_{\text{depth}} \cdot \alpha_{\text{strength}} = \left( \frac{500}{h} \right)^{0.25} \cdot \left( \frac{30}{f'_c} \right)^{0.25} = 1.08
\]

\( V_{r-\text{Loov}} = \alpha V_c + V_s = 124 \text{ kN} \)

However, using the detailed procedure, shown in Appendix B, results in \( V_r = 125 \text{ kN} \). In addition, if Loov (2000) is applied, a more conservative value would be obtained: \( V_{r-\text{Loov}} = 109 \text{ kN} \).

A.2.11 Equation of Russo and Puleri (1997)

Russo and Puleri introduced Eqn. 3-16 for calculating the shear strength of reinforced concrete beams. The size effect has been taken into account through the \( \zeta \) parameter, which is calculated as
\[
\zeta = \frac{1}{\sqrt{1 + d/(25d_a)}} = 0.7
\]

In addition, the \( \chi \) coefficient included in calculating the shear reinforcement effectiveness function, \( \psi \), is calculated as
\[
\chi = \sqrt{f'_c + 250 \sqrt{\rho_i} (d/a)} = 7.51
\]
\[
\psi = 1.67 \sqrt{f'_c / \chi} = 1.18
\]

The contributions of the concrete and the reinforcement to the shear strength are calculated from the first and the second terms of Eqn. 3-16 as:
\[
V_c = (0.83 \xi \chi \sqrt{\rho_i}) b_w d = 60 \text{ kN} \quad ; \quad V_s = (\psi \rho_i f_{yr}) b_w d = 53 \text{ kN}
\]

Thus, the ultimate shear strength of the beam is calculated from Eqn. 3-16.
\[
V_{r-\text{Russo \& Puleri}} = V_c + V_s = 113 \text{ kN}
\]
A.2.12 Zsutty’s (1968, 1971) equation

Since the slenderness ratio $a/d = 3.7$ is higher than 2.5, which has been specified as the upper limit for short deep beams, the concrete shear strength according to Zsutty’s model will be calculated by Eqn. 3-3; while the shear reinforcement strength is the same as the ACI 318-99 that is based on the pin-jointed truss model ($V_s = 45$ kN).

$$V_c = 2.14 \sqrt[3]{f'_c \rho_i d/a} b_w d = 59 \text{ kN} ; \quad V_s = A_s f_y d/s = 45 \text{ kN} \quad V_{r-Zsutty} = V_c + V_s = 104 \text{ kN}$$


They introduced Eqn. 3-10 for calculating the shear strength of transversely reinforced concrete beams. In this equation, the same parameters ($\chi$ and $\zeta$), calculated in A.2.11 (Russo and Puleri 1997) are used.

$$V_{r-Bazant \& Kim} = b_w d \left(0.83 \xi \chi^{3/4} + \rho_n f_{yv}\right) \quad [3-10]$$

$$V_{r-Bazant \& Kim} = 0.1 \times 0.364 \times 10^3 \left(0.83 \times 0.7 \times 7.51 \times 0.0549 + 0.002 \times 608\right) = 105 \text{ kN}$$

A.2.14 Equation of Sarsam and Al-Musawi (1992)

The first term between brackets in Eqn. 3-15 is used to calculate the concrete shear strength.

$$V_c = 1.8 \left(f'_c \rho_i \frac{V_u d}{M_u}\right)^{0.38} b_w d$$

with $\rho_i = 5.49\%$; $f'_c = 28$ MPa; $b_w = 100$ mm; $d = 364$ mm; and $(V_u d/M_u) = d/a = 0.27$ gives $V_c = 47$ kN.

The shear reinforcement is the same as in the ACI 318-99 code, $V_s = 45$ kN. Thus, the shear strength at the support is

$$V_{r-Sarsam \ et \ al.} = V_c + V_s = 92 \text{ kN}$$

A.2.15 Equation of Kim et al. (1999)

The first term of Eqn. 3-17 is used to calculate the concrete shear strength in which the exponent, $r$, is given by Eqn. 3-18 for transversely reinforced beams as
\[ r = 0.6(d/a)^{1.4} (\rho_i)^{0.2} = 0.171 \]  

\[ V_c = 0.2 \left( 1 - \sqrt{\rho_i} \right) (d/a) \left( \sqrt{f'_c} + 1020 \rho_i^{0.9} (d/a)^{0.6} \right) b_w d = 175 \text{kN} \]

The shear reinforcement strength is the same as that of the ACI 318-99 code, \( V_s = 45 \text{kN} \).

\[ V_{r-Kim\ et\ al.} = V_c + V_s = 220 \text{kN} \]

**A.2.16 Equation of Lee et al. (2000)**

The procedure introduced in 3.2.9 is applied in which \( \rho_t = A_v/(b_w s) = 0.2\% \); and \( jd = 0.9 d = 328 \text{mm} \). The procedure steps are applied as follows:

1. Classification of the shear failure mode:
   Since \( 2.8 < f_{yy} \sqrt{\rho_i} / f'_c = 5.2 < 11 \), the shear failure is a STF mode.

2. Determination of the shear reinforcement stress, \( f_{ws} \):
   Shear reinforcement stress is calculated by Eqn. 3-33 in which \( f_{wcs} \) is calculated from Eqn. 3-32.

   \[ f_{wcs} = 11 \sqrt{f'_c / \rho_i} = 1298 \text{MPa} \]  
   
   \[ f_{ws} = f_{wst} = f_{yy} \left( 1 + 0.02 \frac{f_{wcs} - f_{yy}}{f'_c} \right) = 908 \text{MPa} \]

3. Inclination angle of the concrete compressive stresses, \( \theta \):

   \[ \theta = 1.2 \sqrt{\rho_i} f_{yy} / f'_c = 24^\circ \]

Substituting the values of \( f_{ws} \) and \( \theta \) into Eqn. 3-30 results

\[ V_{r-Lee\ et\ al.} = b_w jd \rho_i f_{ws} \cot \theta = 133 \text{kN} \]
APPENDIX B

SHEAR FRICTION DETAILED PROCEDURE

A detailed procedure has been followed to determine the critical shear crack inclination angle, $\theta$, and the shear strength, $V_r$. The $\theta$ values, when applying this procedure for each one of the investigated beams, are given in Table 4-8. In this procedure, the inclination of potential cracks is changed ($\tan \theta$), and the number of shear reinforcement bars crossed by each potential crack, $m$, is thus known so that the shear strength for each one is calculated using the equations presented in Section 3.2.8.

Fig. B-1 – Detailed procedure for the determination of the critical shear plane

Figures B-1a and b show the pivoting point, respectively, on the studs and stirrups about which the shear crack inclinations are varied. Figure B-2 is a plot for the inclination angle
of the shear crack versus the shear strength corresponding to this potential crack, applied for the beam 400IB-250R.

From this procedure, the values of $V_r$ and $\theta$ are equal to 125 kN and 26°, respectively.