Technical Note on "An improved algorithm for solving a multi-period facility location problem"

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Abstract

This note develops a pruning rule for the multi-period facility location algorithm published previously in this journal. This rule can be used to reduce the number of candidate single period location configurations to be examined. An important feature of the proposed rule is that very little additional computation is required in order to apply it. The rule is illustrated using one of the examples in the previously published paper. Finally an experiment is also conducted to examine the effectiveness of the proposed rule under different conditions.

Keywords: facility location, multi-period, dynamic programming, pruning rule

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1. Introduction

This note develops a pruning rule for the multi-period facility location problem (MPFLP) algorithm developed by Hormozi and Khumawala (1996) in this journal. This problem can be modelled as a mathematical programming problem which is difficult to solve for practical sizes. Thus Hormozi and Khumawala introduce procedures to reduce the number of single period solutions that need to be examined through the use of improved bounding and solution procedures. They showed that their improvement is effective in providing better solutions for the MPFLP. Their procedure involves the use of dynamic programming (DP) to determine the final solution. In this paper we extend their work by proposing a new pruning rule.

The MPFLP DP has the property that each stage is a period and each arc connecting different states in a stage consists of two costs – a material flow cost, and a location configuration rearrangement cost. By considering each period independently (which is possible in DP) and separating the two costs we show in this paper that some configurations that have inferior material flow costs within a period can be eliminated from consideration in DP. This occurs if the maximum advantage these configurations have in rearrangement costs does not offset the disadvantage in their high material flow cost. This reduces the size of the DP and makes it more effective. The proposed rule is also computationally efficient. Thus it may be useful in solving practical sized MPFLPs

In Section 2 the rule is proposed and then proved. Implementation within the Hormozi and Khumawala (HK) algorithm is discussed in Section 3. Section 4 illustrates the proposed rule using an example from their paper. Finally in Section 5, an experiment is conducted to provide an indication of the effectiveness of the rule under different conditions. Wherever possible, the notation used in this paper is consistent with that of Hormozi and Khumawala's earlier work.

2. Pruning Rule

In the multi-period (MP) solution process, in any period t the $r(t)^{th}$ best single period (SP) location configuration $K_1[V_{t,r(t)}]$ with material flow cost $V_{t,r(t)}$ will not replace the best SP location configuration $K_1[V_{t,1}]$ with material flow cost $V_{t,1}$ if

$$\text{Max}_{\forall\, i,j} \{(C^{t}_{i1} + C^{t+1}_{1j}) - (C^{t}_{ir(t)} + C^{t+1}_{r(t)j})\} < (V_{t,r(t)} - V_{t,1}) \qquad (1)$$

where

$C^{t}_{ir(t)}$: configuration rearrangement cost from any SP configuration $K_1[V_{t-1,i}]$ in period (t-1) to an SP configuration $K_1[V_{t,r(t)}]$ in period t.

Proof

Let $MPFLP_1$ represent the cost for a T-period solution containing $K_1[V_{t,1}]$. Then

$$MPFLP_1 = V_{1,k} + C^2_{km} + V_{2,m} + .. + V_{(t-1),i} + C^t_{i1} + V_{t,1} + C^{t+1}_{1j} + V_{(t+1),j} + \ldots + V_{T,u}$$

where k, m, i, 1, j and u represent SP location configurations.

Now, consider $MPFLP_2$ which differs from $MPFLP_1$ only in that $K_1[V_{t,r(t)}]$ replaces $K_1[V_{t,1}]$.

$$MPFLP_2 = V_{1,k} + C^2_{km} + V_{2,m} + .. + V_{(t-1),i} + C^t_{i\,r(t)} + V_{t,\,r(t)} + C^{t+1}_{r(t)j} + V_{(t+1),j} + \ldots + V_{T,u}$$

and

$$MPFLP_2 - MPFLP_1 = (C^t_{i\,r(t)} + V_{t,\,r(t)} + C^{t+1}_{r(t)j}) - (C^t_{i1} + V_{t,1} + C^{t+1}_{1j})$$

Separating the material flow and rearrangement costs we get

$$MPFLP_2 - MPFLP_1 = (V_{t,\,r(t)} - V_{t,1}) - \{(C^t_{i1} + C^{t+1}_{1j}) - (C^t_{i\,r(t)} + C^{t+1}_{r(t)j})\} \qquad (2)$$

$$= DIF_{Material\_Flow\_Cost} - DIF_{Configuration\_Rearrangement\_Cost}$$

By definition of rank ordering where $V_{t,1} \le V_{t,2} \ldots \le V_{t,\,r(t)} \le \ldots V_{t,\,R(t)}$, $DIF_{Material\_Flow\_Cost}$ is always positive. Thus for equation (2) to be negative, i.e., for $MPFLP_2$ to be a better solution than $MPFLP_1$,

$$DIF_{Configuration\_Rearrangement\_Cost} > DIF_{Material\_Flow\_Cost} \qquad (3)$$

Based on equation (2), we know

$$\text{Max}(\text{DIF}_{\text{Configuration\_Rearrangement\_Cost}}) = \text{Max}_{\forall\, i,j}\ \{(C^{t}_{i1} + C^{t+1}_{1j}) - (C^{t}_{i\, r(t)} + C^{t+1}_{r(t)j})\}$$

Therefore, if

$\text{Max}(\text{DIF}_{\text{Configuration\_Rearrangement\_Cost}}) < \text{DIF}_{\text{Material\_Flow\_Cost}}$

i.e. $\text{Max}_{\forall\, i,j}\ \{(C^{t}_{i1} + C^{t+1}_{1j}) - (C^{t}_{i\, r(t)} + C^{t+1}_{r(t)j})\} < \{ V_{t,\, r(t)} - V_{t,1}\}$

equation (3) will never be satisfied for any i, j and $K_1[V_{t,r(t)}]$ will not replace $K_1[V_{t,1}]$. So $K_1[V_{t,r(t)}]$ can be eliminated from further consideration.

In the first period there is no period (t-1). Thus equation (1) reduces to $\text{Max}_{\forall\, j}\ (C^{2}_{1j} - C^{2}_{r(t)j}) > (V_{1r(t)} - V_{1,1})$. Similarly for the last period (T), there is no (t+1) and equation (1) reduces to $\text{Max}_{\forall\, i}\ (C^{T}_{i1} - C^{T}_{ir(t)}) > (V_{T,r(t)} - V_{T,1})$. Note that a feasible MPFLP solution is not required in order to apply the pruning rule. A rank ordering of the SP configurations in a period is sufficient.

3. Implementation within the Hormozi and Khumawala algorithm:

The proposed pruning process can be implemented in two places. It can be implemented after Step 3b of the algorithm, or if the algorithm proceeds to Step 4, it can also be repeated after 4a. For clarity the HK algorithm along with the implementation of the proposed rule (shown in bold) is briefly described below.

Step 1: MP delta and omega rules are used to decide the open/closed status of facilities. In practice the status can be decided only for some facilities, so Step 2 is initiated.

Step 2: The two best SP solutions for each period are determined to generate MPFLP upper and lower bounds. UB – LB = DIF

Step 3: R(t) =2T rank ordered solutions are generated so long as $(V_{t,2T} - V_{t,1}) <$ DIF.

a) If delta and omega procedures can be applied go to 3b, otherwise to 3b'

b) Initiate delta and omega procedures to reduce further reduce state space

b') Use proposed pruning rule to reduce state space further if possible.

c) Update UB by using DP to determine a feasible (and possibly optimal) solution with the updated SP solutions and the appropriate open/closing costs.

Step 4: Update DIF = UB – LB

a) If $(V_{t, R(t)} - V_{t,1}) >$ DIF $\forall$ t, UB is the optimal solution. Otherwise add SP solutions $\forall$ t till $(V_{t, R(t)} - V_{t,1}) >$ DIF

b) Use proposed pruning rule to reduce state space further if possible.

Solve DP to obtain optimal solution.

Step 3 and 4a involves the rank ordering of SP location configurations, i.e., determining $V_{tr(t)}$ $\forall$ t, r(t). In step 3c and in Step 4, the MPFLP solution is obtained using DP with the SP solutions and the appropriate open/closing costs. The appropriate opening and closing costs are given by $C^{t}_{i\ r(t)}$ $\forall$ i, r(t). These costs, which are generated by the original HK algorithm are also used in the proposed algorithm and thus no additional computation is required in the cost generation stage for the proposed pruning rule.

The only additional computation once $V_{tR(t)}$, $C^{t}_{i\ r(t)}$ and $C^{t+1}_{\ r(t)j}$ $\forall$ i, j ,r(t) are obtained is as follows:

For each r(t) up to R(t), compute

1. $\forall$ i,j Max $\{(C^{t}_{i1} + C^{t+1}_{1j}) - (C^{t}_{i\ r(t)} + C^{t+1}_{\ r(t)j})\}$
2. $V_{t,\ r(t)} - V_{t,1}$
3. If $Max_{\forall\ i,j}\ \{(C^{t}_{i1} + C^{t+1}_{1j}) - (C^{t}_{i\ r(t)} + C^{t+1}_{\ r(t)j})\} < (V_{t,\ r(t)} - V_{t,1})$, remove $K_1[V_{t,r(t)}]$ from further consideration and go to next r(t). Else go to next r(t)

In tests with 12 facilities and R(t) = 400 (which is the maximum that Hormozi and Khumawala used in their tests), the proposed rule was processed for all 400 configurations within each period in less than 0.4 CPU seconds using Excel VBA on a Pentium III 700 MHz computer. Thus the proposed pruning rule is computationally quite efficient.

4. Illustration

The rule is now illustrated using periods 4 and 5 from the Sweeney and Tatham (1976) example shown in Table 1 of the Hormozi and Khumawala paper. Table 1 shows the rearrangement costs of changing from each SP location configuration in Period 4 to period 5's best configuration ABE ($K_1[V_{5,1}]$), and to period 5's seventh best configuration ABD ($K_1[V_{5,7}]$) , given Sweeney and Tatham's facility opening cost of $200 and closing cost of $100.

For example, if BCE was the chosen configuration in period 4 and ABE was the chosen configuration in period 5, B and E would remain open, while C is closed incurring a closing cost of $100 and A is opened incurring a cost of $200. Thus the total rearrangement cost in changing from BCE to ABE is $300 as shown in Table 1.

If ABD was the chosen configuration in period 5, B would remain open, while C and E would be closed for a cost of $100 each. Facilities A and D would be opened at cost of $200 each. Thus the total rearrangement cost in changing from BCE to ABD is $600. As shown in the $(C^5_{j1} - C^5_{j7})$ column in the BCE row, the rearrangement costs for ABD is $300 greater than that for ABE i.e., $(C^5_{21} - C^5_{27}) = -300$.

From Table 1, it is seen that regardless of the SP configuration in period 4 with associated rearrangement costs (C^5_{11}, C^5_{21}, .. C^5_{81} when changing to ABE in period 5, and C^5_{17}, C^5_{21}, .. C^5_{87} when changing to ABD in period 5) the rearrangement costs for changing to ABE is never more than $300 greater than when changing to ABD (this value occurs twice, when changing from period 4 configurations ABD and BCD). Thus $\text{Max}_{\forall\ i}\ (C^5_{i1} - C^5_{ir(t)}) = 300$. On the other hand, from Table 1 of Hormozi and Khumawala the configuration operation cost of ABD is $367 more than that of ABE in period 5 i.e., $V_{5,7} - V_{5,1} = 367$. So this means that regardless of the SP configuration in period 4, one would never change to ABD in period 5 when compared to ABE, since the maximum potential saving in rearrangement cost $\{\text{Max}_{\forall\ i}\ (C^5_{i1} - C^5_{ir(t)})\}$ is $67 less than the increase in operating cost $(V_{5,7} - V_{5,1})$. Thus ABD can be eliminated from consideration in Step 3c.

If the closing cost were reduced to \$50, and the opening cost were reduced to \$100, every SP configuration in period 5 can be eliminated when compared to ABE ($K_1[V_{5,1}]$) as $V_{5r(t)} - V_{51} > \mathrm{Max}_{\forall j}\ (C_{51} - C_{5r(t)})$ for every r(t). This will result in a significant reduction in the state space and the resulting DP computation time in Step 3c.

Period 4 SP Config-uration Number i	Period 4 Config-uration $K_1[V_{4,i}]$	Rearrangement costs for configurations ABE and ABD in Period 5 given each i in Period 4		
		ABE (best configuration) $K_1[V_{5,1}]$	ABD (seventh best configuration) ($K_1[V_{5,7}]$)	Difference in rearrangement costs ($C^5_{i1} - C^5_{i7}$)
1	ABE	0	300	-300
2	BCE	300	600	-300
3	ABDE	100	100	0
4	ABCE	100	400	-300
5	ABD	300	0	**300**
6	ABC	300	300	0
7	BCDE	400	400	0
8	BCD	600	300	**300**

Table 1: Difference in rearrangement costs

5. Empirical Testing

In order to examine the general effectiveness of the rule, an experiment was set up. Two factors that determine its effectiveness are the relative value of the rearrangement cost and the relative value of ($V_{t, r(t)} - V_{t,1}$). The computer generated problems consisted of 12 facilities with 400 unique rank ordered random SP solutions. It was also assumed that periods t-1, t , and t+1 all had these same 400 SP solutions. From this data $C^{t}_{i\ r(t)}$ $\forall$ i, t, r(t) could be determined. The material flow costs were also randomly generated. $V_{t,1}$ was set equal to 100000. $V_{t,2}$, $V_{t,3}$ etc were generated using the normal distribution where $V_{t,(i+1)} = V_{t,(i)} + \omega$ where $\omega \sim N(\mu_1,\sigma_1)$. This was done in order to ensure that the $V_{t,r(t)}$ increased by a controlled amount for increasing r(t). Three different values of μ_1, 0.05%, 0.075% and 0.1% of $V_{t,1}$ respectively were used. This meant that the average difference between two consecutively rank ordered solutions for μ_1 values of 0.05%, 0.075%, and 0.1% were 50, 75 and 100 respectively. The standard deviation, σ_1 was 0.01% of $V_{t,1}$. If the generated increase between two consecutively ranked configurations was negative it was set to zero, since the definition of rank ordering implies that the configurations are to be in order of increasing cost.

The twelve location opening and closing costs were also generated from a normal distribution $\{N(\mu_2,\sigma_2)\}$ where μ_2 represented the average location closing/opening cost. In order to control the relative magnitude of the location closing/opening costs, three different relative values of μ_2 , 0.5% 1.0% and 1.5% of $V_{t,1}$ respectively were used. Thus the three absolute values of μ_2 were 500, 1000 and 1500 . The standard deviation, σ_2 was set as 20% of μ_2.

The values in Table 2, which show the percentage of the 400 location configurations that could be eliminated represent the averages of ten replications under each of the nine conditions. Multiple replications were done to get more reliable values. Each replication was different from the previous one in that the material flow costs for the 400 configurations were regenerated using the method described earlier. The variance in the percentage of locations eliminated was minimal and so are not shown. Table 2 also shows the average $(V_{t,R(t)} - V_{t,1})$ as a percentage of $V_{t,1}$ for different μ_1. So when $\mu_1 = 0.05\%$, the 400th ranked configuration (R(t)) was about 22% higher in cost than the best configuration.

Average Location Closing/Opening cost (% of $V_{t,1}$)	Average Increase in Material Flow Costs Between Two Consecutive Configurations (% of $V_{t,1}$)		
	0.05%	**0.075%**	**0.1%**
0.5%	46.2%	64.1%	73.7%
1.0%	0%	27.7%	45.8%
1.5%	0%	0%	17.3%
Average $(V_{t,R(t)} - V_{t,1})$ as a percentage of $V_{t,1}$	22%	35%	49%

Table 2: Percentage of configurations eliminated

It is seen that the pruning rule is more effective when the rearrangement costs are low relative to the material flow cost or the increase in the material flow cost between rank ordered solutions is high. When the increase in the material flow cost between rank ordered solutions is high, as r(t) increases there will be more poor SP solutions in the rank ordered list. This is indicated by the fact that in the last row of table 2, when μ_1 = 0.05%, the 400th ranked configuration is only 22% higher in cost than the best configuration, whereas when μ_1 = 0.1% the 400th ranked configuration is almost 50% higher in cost. In addition to this if the shifting costs are low, the maximum configuration rearrangement cost will also be low. Thus $\mathrm{Max}_{\forall\, i,j} \{(C^{t}_{i1} + C^{t+1}_{1j}) - (C^{t}_{i\, r(t)} + C^{t+1}_{r(t)j})\} < (V_{t,\, r(t)} - V_{t,1})$ for more r(t) and more SP solutions can be eliminated from the DP. On average over 73% of the 400 SP solutions were eliminated when the increase in the material flow cost was at its highest (0.1%) and the location closing/opening cost was at its lowest (0.5%). This will lead to a significant decrease in the computation time required for the DP solution. The opposite is true for high rearrangement costs and low increase in the material flow cost between rank ordered solutions, where no SP solution could be eliminated. Note that even in this case, using the pruning rule would not be disadvantageous due to its short computation time.

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