Fast Digital Erosion/Dilation of Micropore Images

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I. Introduction

Computer assisted analysis of the pore complex of reservoir rocks is required due to the complexity of the images involved and the time consuming nature of performing the same analysis manually. A key technique for extracting relevant data from pore complex images is ***erosion-dilation*** [1,2], wherein layers of pixels are removed from the raster image representation of each pore. This has the general effect of smoothing out irregularities in the pore perimeter. Layers of pixels are then added back to the outer edges of the pores (dilation), restoring the pores to an approximation of their original size and shape but without the surface irregularities.

Erosion-dilation is usually performed in cycles [3]. First, an image would have one layer of pixels eroded and then would be dilated by one pixel. This removes one-pixel irregularities. Then the image would be eroded by two layers and dilated by two pixels to remove two-pixel irregularities. This process can be repeated until the erosion process removes all pixels belonging to a given pore. At this point, dilation will have no effect, since at least one pixel must remain to provide a 'seed' for the dilation process.

For large images, which includes any practical image used for analysis, this is a time consuming process. Each erosion step requires that the entire image be examined; for an image of size N rows by N columns, the time needed is a function of $2N^2$, assuming that erosion is performed in place and a second copy is not used. Similarly, each dilation step requires on the order of $2N^2$ time. To erode two layers of pixels requires twice this amount of time, and the same for the corresponding dilation step. If all erosions and dilations from 1 to k pixels are needed, the total time needed is a function of $2k(k-1)N^2$.

If sufficient computer memory space is available the time needed for erosions can be shortened. The eroded image can be saved in a second image, dilated, then the saved image can be eroded by one more layer. If this is done then the time needed is a function of $k(k+3)N^2$. For typical applications N has a value of 512 or more; the time needed is therefore excessive for most microprocessor systems and for many larger time-shared computers as well. However, it is possible to reduce the time needed by a large factor without affecting the quality of the results.

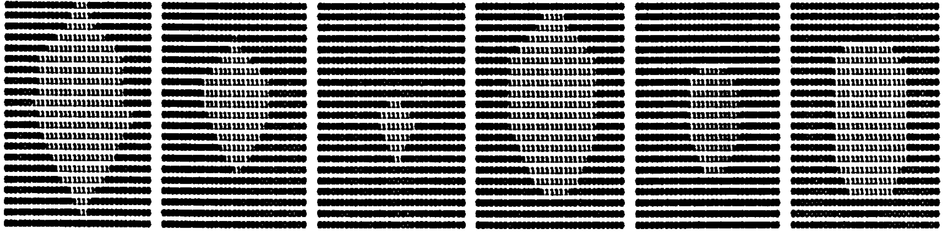
1a. 1b. 1c. 1d. 1e. 1f.

Figure 1 - Sample Results from Standard Erosion-Dilation

II. Computing Erosion and Dilation

Figure 1a shows a small portion of a pore-complex image to be eroded. Assume that all pixels in the image have a value of 0 or 255, where a 255 value represents a pixel that belongs to a pore and a 0 value represents rock. The standard in-place erosion method can be described as follows:

1) Scan the image for a pixel with a value of 255.
2) If one of the eight immediate neighbors of this pixel has a value of zero, then *mark* this pixel. This is done by changing its value to, say, 128.
3) Continue from step 1 until all pixels have been examined.
4) Scan the image again, this time looking for any pixel that has been *marked.*
5) Change the value of this marked pixel to zero, thereby removing it.
6) repeat from step 4 until all *marked* pixels have been set to zero.

Figures 1b and 1c represent erosions of Figure 1a by two pixels and four pixels respectively using this method. The value of 255 permits the erosion of regions smaller than 512 by 512 pixels, an allows the pixel values to be stored in a single memory byte. The standard method used for computing in-place dilations is similar, and with the same assumptions for pixel values as before can be described as:

1) Scan the image for a pixel with a value of zero.
2) If one of the eight immediate neighbors of this pixel has a value of 255, then *mark* this pixel.
3) Continue from step 1 until all pixels have been examined.
4) Scan the image again, this time looking for any pixel that has been *marked.*
5) Change the value of this marked pixel to 255, thereby adding a new pixel.
6) repeat from step 4 until all *marked* pixels have been set to 255.

Figures 1d and 1e show the result of dilating Figures 1b and 1c, respectively, by two pixels. Figure 1f shows the result of dilating Figure 1c by four pixels. The desired effect of removing small irregularities has been achieved at some expense of computer time.

III. A Faster Erosion-Dilation Method

The fast erosion method is based on the construction of a distance map of the pore image, where the value of each pixel belonging to a pore is replaced by a new value representing the distance of that pixel from the boundary of the pore. Thus, pixels on the pore boundary would be given a value of 1, since they are 1 pixel away from the boundary. Pixels that are 2 pixels away from the boundary would be given the value 2, and so on. The result has the appearance of a contour map, where the contours represent the distance from the edge of the pore.

The resulting image contains enough information to perform an erosion by any number of pixels in just one pass through the image; in other words, all possible erosions have been encoded into a single image. This *globally eroded* image can be produced in just two passes through the original image. The method is:

1) Starting at the upper left of the image, scan along successive rows, from left to right, until a pixel with a value of 255 is found. This becomes the current pixel.
2) Look at all 8 possible neighbors of the current pixel, and let V be the *minimum* value found in these pixels.
3) Set the value of the current pixel to V+1.
4) Continue from step 1 until all pixels have been examined.
5) Now starting at the lower right of the image, scan along successive rows from right to left moving UP the image, looking for a pixel whose value is non-zero. This becomes the current pixel.
6) Look at all 8 neighbors of the current pixel, and let V be the minimum value found in these pixels.
7) If V has a value less than that of the current pixel, set the value of the current pixel to V+1.

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2a. Global Erosion of 1a

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2b. 2a Dilated by 2 Pixels

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2c. 2a Dilated by 4 Pixels

Figure 2 - Results of Global Erosion-Dilation

8) Repeat from step 5 until all pixels have been examined.

Steps 1 through 4 represent the first phase of the global erosion, and steps 5 through 8 represent the second phase. After the first phase is complete the pixel values represent the minimum distance, expressed as numbers of pixels, to the nearest upper, upper left, left, and upper right boundaries. The second phase must be performed by scanning in the opposite direction in order to collect the distances in the other four directions in one pass.

The resulting image has all possible erosions encoded within the pixel values. Figure 2a shows the result of globally eroding the pore image of Figure 1a. It would normally not be necessary to actually perform an erosion by removing pixels, since the image will be immediately dilated again. If the actual eroded image is desired a simple thresholding operation will do the job. To remove k layers of pixels from a globally eroded image, simply set all pixels that have a value less than of equal to k to zero, and set all other pixels to 255.

The process of *global dilation* must be applied only to an image that has been globally eroded. The result is an image that has been eroded and then dilated by k pixels. The method is:

1) Scan the image for a pixel with a value greater than zero that is also *unmarked*. Let this pixel be the current pixel.

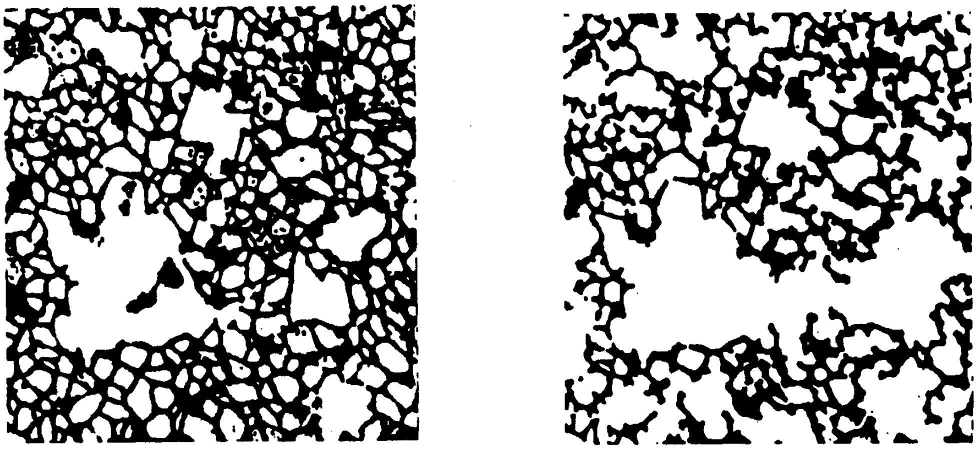
2) If the value of the current pixel is less than $k+1$ then set the current pixel to zero; if the value of the current pixel is greater than $k+1$ then *mark* the current pixel. In either case, skip to step 4. If the current pixel has a value equal to $k+1$ go to step 3.

3) The current pixel must have a value of $k+1$. *Mark* the current pixel, and also *mark* all pixels within k pixels of the current pixel.

4) Repeat from step 1 until all pixels have been examined.

The marked pixels in the image that results belong to the dilated pore; all other pixels are background. Figures 2b and 2c show the result of globally dilating the image of Figure 2a by 2 and 4 pixels respectively. Notice that there is no difference between these dilated images and those of Figures 1d and 1f.

If further dilations are needed, the original globally eroded image may be saved and dilated by a different number of pixels or the globally dilated image can be unmarked and re-dilated. Depending upon the method chosen for marking pixels, the globally dilated image may need to be thresholded before it is displayed.



3a - Sample 1 3b - Sample 2

Figure 3 - Test Images Used For Execution Timings

IV. Results

All of the computer programs for performing erosions and dilations were written in the C language on a VAX 11/780 running UNIX. Conversion to FORTRAN or another language would provide no difficulty. The code for performing global erosions and dilations was actually a little shorter than the code for the traditional method: 70 as opposed to 76 lines. Both methods can be coded and debugged in just a few hours.

Both methods were applied to the micropore images seen in Figure 3 which are both of size 443 by 440 pixels. As well, a modified version of the standard method in which eroded images are saved for later erosion steps was timed, and is referred to as the Standard II method. All erosions and dilations from 1 to 12 pixels were requested. The computer time needed for each method was:

Method		CPU Time (Seconds)			
	Image	Erode	Dilate	Total	Real Time (Min)
Standard:	3a	1242.10	730.76	1972.86	182.03
	3b	1067.61	709.77	1777.38	85.0
Standard II:	3a	191.15	724.31	915.46	46.77
	3b	188.99	754.60	943.59	65.98
Global:	3a	120.24	102.69	222.93	29.02
	3b	90.40	95.04	185.44	17.60

The values for real time would depend on the load on the computer at the time, and are included only as a rough estimate.

V. Conclusions

The method of global erosion-dilation is no more difficult to implement than is the standard technique, but yields a significant saving of computer time during execution.

It turns out that any normally eroded image can be globally dilated using a method very similar to the one described here. The time performance is still worse than for global erosion-dilation, but is better than for Standard II.

VI. Acknowledgements

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VII. References

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