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UNIVERSITY OF CALGARY

Novel approaches towards non-cryogenic quantum repeaters

by

Jiawei Ji

A THESIS

SUBMITTED TO THE FACULTY OF GRADUATE STUDIES IN PARTIAL FULFILLMENT OF THE REQUIREMENTS FOR THE DEGREE OF DOCTOR OF PHILOSOPHY

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Abstract

The successful implementation of global quantum networks would have many applications such as secure communication, blind quantum computing, and private database queries, ultimately leading to a "quantum internet" of networked quantum processors. This will require photons for establishing long-distance connections. However, the inevitable losses in transmission and the fact that they cannot be compensated by amplification significantly limit the distance. Therefore, quantum repeaters have been proposed to solve this issue but this typically requires stationary quantum memories for storing and processing the quantum information. Currently, a vast majority of approaches to quantum networks need either vacuum equipment and optical trapping or cryogenic cooling, which makes scaling up such architectures very difficult. In this thesis, we explore two hardware platforms for realizing quantum repeaters that can operate without cryogenics: nitrogen-vacancy (NV) centers and optomechanics-based repeaters and hot hybrid alkali-noble gases-based repeaters. We show how entanglement generation and entanglement swapping can be achieved in both schemes. Moreover, we quantify the performance of these two proposed repeater architectures in terms of repeater rates and overall entanglement fidelities and make a comparison between them. We also discuss the experimental feasibility of these two schemes, demonstrating that both can be within reach of current technologies.

Preface

Below is the list of publications during my graduate studies, all of which are either published or submitted. They are presented in chronological order. In this thesis, I only include the work from papers 4 and 5. The copyright permissions are included in appendix A.

- J.-W. Ji and D. L. Feder. "Extending matchgates to universal quantum computation via the Hubbard model." Phys. Rev. A 100, 052324 (2019).
- S. C. Wein, J.-W. Ji, Y.-F. Wu, F. Kimiaee Asadi, R. Ghobadi, and C. Simon. "Analyzing photon-count heralded entanglement generation between solid-state spin qubits by decomposing the master-equation dynamics." Phys. Rev. A 102, 033701 (2020).
- C. Qian, Y.-D. Wu, J.-W. Ji, Y. Xiao, B. C. Sanders. "Multiple uncertainty relation for accelerated quantum information." Phys. Rev. D 102, 096009 (2020).
- J.-W. Ji, Y.-F. Wu, S. C. Wein, F. Kimiaee Asadi, R. Ghobadi, and C. Simon. "Proposal for room-temperature quantum repeaters with nitrogen-vacancy centers and optomechanics." Quantum 6, 669 (2022).
- F. Kimiaee Asadi, J.-W. Ji, and C. Simon. "Proposal for transduction between microwave and optical photons using ¹⁶⁷Er-doped yttrium orthosilicate." (Editor's suggestion) Phys. Rev. A 105, 062608 (2022).

- D. B. Higginbottom, F. Kimiaee Asadi, C. Chartrand, J.-W. Ji, L. Bergeron, M. L. W. Thewalt, C. Simon, and S. Simmons. "Memory and transduction prospects for silicon T center devices." PRX Quantum 4, 020308 (2023).
- 7. J.-W. Ji, F. Kimiaee Asadi, K. Heshami, and C. Simon. "Proposal for non-cryogenic quantum repeaters with hot hybrid alkali-noble gases." arXiv:2210.09504 (2022).

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List of Abbreviations

Abbreviation	Definition
U of C	University of Calgary
Qubit	Quantum bit
BS	Beam splitter
BSM	Bell-state measurement
LHV	Local hidden variable
BQC	Blind quantum computation
LEO	Low-Earth-orbit
RE	Rare-earth
QKD	Quantum key distribution
NV	Nitrogen-vacancy
NISQ	Noisy intermediate-scale quantum
ZPL	Zero-phonon line
PSB	Phonon sideband
RWA	Rotating wave approximation
QND	Quantum nondemolition
FWM	Four-wave mixing
ORCA	Off-resonant cascaded absorption
hBN	Hexagonal boron nitride
GPS	Global positioning system

SQL	Standard quantum limit
DLCZ	Duan-Lukin-Cirac-Zoller
AFC	Atomic-frequency comb
FT	Fourier transform
SEOP	Spin-exchange optical pumping

Epigraph

Discovery is the privilege of the child: the child who has no fear of being once again wrong, of looking like an idiot, of not being serious, of not doing things like everyone else.

- Alexander Grothendieck

Chapter 1

Introduction

Quantum communication promises to perform some communication tasks that are beyond the capability of classical communication, which requires optical photons to exchange quantum information (quantum states) between remote quantum processors. The successful implementation of quantum communication will bring many exciting applications such as secure communication [1], blind quantum computing [2], private database queries [3], and a quantum internet that connects quantum processors, including quantum computers, [4, 5, 6]. As the transmission of photons is adversely affected by loss in the channel, the distance of connecting quantum devices is significantly limited. In addition, unlike its classical counterparts, unknown quantum states cannot be cloned [7], thus eliminating the possibility of performing amplification to overcome the loss.

Therefore, quantum repeaters have been proposed to solve this issue [8]. The basic idea is that we divide a long distance into pieces of shorter distance (called elementary links), and we generate entanglement in these elementary links and then perform entanglement swapping to propagate the entanglement further. However, this typically requires quantum memories to process and store entanglement. So far, most approaches to realizing quantum repeaters are based on cryogenics (temperatures below 120 K) or optical trapping [9, 10, 11, 12], which significantly undermines the scalability of these architectures. Hence, this thesis aims to present the work done for proposing non-cryogenic quantum repeaters.

The thesis is organized as follows. In Chapter 1, I will introduce some underlying physics in quantum communication and quantum networks. In Chapter 2, I will focus on the details of quantum networks, including the potential applications. Then, I will move on to introduce physical systems that are the building blocks of realizing non-cryogenic quantum repeaters in Chapter 3. Chapters 4 and 5 present two papers that contain all the details of the proposals. Chapter 6 provides conclusions and an outlook.

1.1 Quantum bits (Qubits)

In the classical world, information is typically presented by classical bits which can take on either the state 0 or 1. For example, a classical bit can be physically implemented by one of two levels of DC voltage. However, in the quantum world, the basic unit representing information is called the quantum bit (qubit) which is the quantum analogue of the classical bit. It can be implemented in a two-level quantum system, which makes a striking difference from the classical implementation. In such a two-level quantum system, the state of a qubit can be much more general than just being 0 or 1, which can be described as follows:

$$\alpha \left| 0 \right\rangle + \beta \left| 1 \right\rangle, \tag{1.1}$$

where $|0\rangle$ and $|1\rangle$ are the basis states of a qubit and the basis set $\{|0\rangle, |1\rangle\}$ is referred to as the computational basis. According to Born's rule, the coefficients α and β represent the probability amplitudes so the probability of being in states $|0\rangle$ and $|1\rangle$ are $|\alpha|^2$ and $|\beta|^2$. Moreover, they must add up to be unity $|\alpha|^2 + |\beta|^2 = 1$. As α and β can be complex, the quantum state also contains the relative phase of α and β .

In fact, there are four parameters in α and β but as they are constrained to the unit probability, and the global phase is irrelevant, there only exist two free parameters. Hence,



Figure 1.1: The representation of a qubit on the Bloch sphere with the probability amplitude $\alpha = \cos \frac{\theta}{2}$ and $\beta = e^{i\psi} \sin \frac{\theta}{2}$. The North and South poles represent states $|0\rangle$ and $|1\rangle$ respectively.

we have

$$\alpha = \cos\frac{\theta}{2},\tag{1.2}$$

$$\beta = e^{i\psi} \sin\frac{\theta}{2},\tag{1.3}$$

where θ and ψ are real numbers, defining a point on a unit three-dimensional sphere called Bloch sphere as shown in Fig. (1.1) [13]. Now, a qubit can be written as:

$$\cos\frac{\theta}{2}\left|0\right\rangle + e^{i\psi}\sin\frac{\theta}{2}\left|1\right\rangle. \tag{1.4}$$

This is the Bloch representation of a qubit, which is often used to visualize the state of a single qubit. In contrast to qubits which can appear anywhere on the surface of this sphere, classical bits can only be in the North or South pole.

1.1.1 Multiple qubits

For classical bits, if there are two of them, we have four possible states: 00, 01, 10, and 11. Similarly, one can define the basis states for two qubits, which are $|00\rangle$, $|01\rangle$, $|10\rangle$, and $|11\rangle$. Like a single qubit, two qubits can be in a coherent superposition of these four basis states with probability amplitudes corresponding to each basis state. Thus, a general two-qubit state can be written as

$$|\psi\rangle = \alpha_1 |00\rangle + \alpha_2 |01\rangle + \alpha_3 |10\rangle + \alpha_4 |11\rangle, \qquad (1.5)$$

where α_1 , α_2 , α_3 , and α_4 are complex probability amplitudes. Now, we can generalize it to the case where there are *n* qubits. A *n*-qubit state has 2^n computational basis states, which are given by

$$|\psi\rangle = \sum_{x_1 x_2 \cdots x_n} \alpha_{x_1 x_2 \cdots x_n} |x_1 x_2 \cdots x_n\rangle, \qquad (1.6)$$

where x_i can take the value of either 0 or 1. To store such a quantum state in a classical computer can be very challenging as the number of amplitudes is gigantic even if n is just several hundred, and its size increases exponentially. This is one of the main reasons to develop quantum computers for processing quantum information.

1.1.2 Mixed states

So far, the quantum states of qubit(s) are represented as linear superpositions of normalized state vectors as in Eq. (1.4) and Eq. (1.6). These quantum states are referred to as pure states. However, sometimes some quantum states cannot be captured by this characterization as they are statistical ensembles of different quantum states, which are referred to as mixed states. Even when the whole quantum system is in a pure state, if we try to only look at a part of the system by tracing out the others, it becomes a mixed state. We typically use the symbol ρ to represent mixed states, which is referred to as density matrix. This representation is also an alternative way of expressing pure states. For example, Eq. (1.1) can also be written as:

$$\rho = (\alpha |0\rangle + \beta |1\rangle)(\alpha^* \langle 0| + \beta^* \langle 1|)$$

$$= |\alpha|^2 |0\rangle \langle 0| + \alpha\beta^* |0\rangle \langle 1| + \beta\alpha^* |1\rangle \langle 0| + |\beta|^2 |1\rangle \langle 1|.$$
(1.7)

For mixed states, they can only be written using density matrix, e.g. the state $\rho = \frac{1}{2} |0\rangle \langle 0| + \frac{1}{2} |1\rangle \langle 1|$. In general, a mixed state takes the following form:

$$\rho = \sum_{i} p_{i} |\psi_{i}\rangle \langle\psi_{i}|, \qquad (1.8)$$

where $\sum_{i} p_{i} = 1$ with p_{i} being the probability of occurrence. One of the key features of mixed states is that they satisfy $\text{Tr}(\rho^{2}) < 1$ whereas for pure states, we have $\text{Tr}(\rho^{2}) = 1$. The mixed states of a single qubit can also be visualized on the Bloch sphere, where the length of the corresponding state vector is less than 1, which means this vector is inside the Bloch sphere.

1.1.3 Fidelity of quantum states

In quantum mechanics, fidelity refers to the degree to which two quantum states are similar or identical. More specifically, it is a measure of the overlap between two quantum states. In general, the fidelity of two states ρ and σ (both are density matrices) is defined as follows [14]:

$$F(\rho,\sigma) = \left(\mathrm{tr}\sqrt{\sqrt{\rho}\sigma\sqrt{\rho}}\right)^2 = \left(\mathrm{tr}\sqrt{\rho\sigma}\right)^2. \tag{1.9}$$

The quantum state fidelity satisfies several properties. First of all, it is bounded, $0 \leq F(\rho, \sigma) \leq 1$. When $\rho = \sigma$, we have $F(\rho, \rho) = (\operatorname{tr}(\rho))^2 = 1$. Second, it is symmetric: $F(\rho, \sigma) = F(\sigma, \rho)$. This can be easily seen from the definition above as $\rho\sigma = \sigma\rho$. If the states are pure states, meaning that $\rho = |\psi\rangle \langle \psi|$ and $\sigma = |\phi\rangle \langle \phi|$, the fidelity becomes:

$$F(\rho,\sigma) = \left(\operatorname{tr}\sqrt{|\psi\rangle\langle\psi|\phi\rangle\langle\phi|}\right)^2 = \langle\psi|\phi\rangle\left(\operatorname{tr}\sqrt{|\psi\rangle\langle\phi|}\right)^2 = |\langle\psi|\phi\rangle|^2.$$
(1.10)

Throughout the thesis, we stick to this fidelity definition for quantum states in Eq. (1.9).

1.2 Quantum gates

In classical information processing, classical gates are used to form circuits for computational tasks. Among many types of classical gates, the NOR or NAND gates are universal, which means any Boolean function can be implemented only by either of these two types of logic gates. Likewise, there exist different types of quantum gates that can be used to execute arbitrary algorithms. In this section, I will introduce some elementary one and two-qubit gates.

1.2.1 Single-qubit gates

The only nontrivial classical one-bit gate is the NOT gate, which maps 0 to 1 and 1 to 0 (simply flipping the bit). Its quantum counterpart (the quantum NOT gate) must be able to map a general single-qubit state $\alpha |0\rangle + \beta |1\rangle$ to $\alpha |1\rangle + \beta |0\rangle$, that is flipping the basis states. In order to find this gate, let us first write this general quantum state as a vector:

$$\begin{bmatrix} \alpha \\ \beta \end{bmatrix}. \tag{1.11}$$

The quantum NOT gate is expected to satisfy the following equation:

$$X\begin{bmatrix} \alpha\\ \beta \end{bmatrix} = \begin{bmatrix} \beta\\ \alpha \end{bmatrix}.$$
 (1.12)

Now, after solving this equation, we obtain

$$X = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}. \tag{1.13}$$

This is the quantum NOT gate. It satisfies $XX^{\dagger} = X^{\dagger}X = I$ where X^{\dagger} is the conjugate transpose of X. This implies that the X gate is unitary, which can be attributed to the fact that the probability amplitudes of a general single-qubit state are constrained to the unity condition. This unitary property actually can be generalized to any quantum gates. In fact, the X gate is one of the Pauli matrices, which are the following:

$$I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \quad X = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad Y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad Z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}.$$
(1.14)

These Pauli matrices are of great importance and interest as they are not only unitary but also Hermitian (its conjugate transpose is itself). The exponentiation of Pauli matrices forms so-called rotation operators/matrices, which are themselves unitary. They play a significant role in quantum gates. They are given by

$$R_X(\theta) = e^{-iX\frac{\theta}{2}} = \cos(\frac{\theta}{2})I - i\sin(\frac{\theta}{2})X,$$
(1.15)

$$R_Y(\theta) = e^{-iY\frac{\theta}{2}} = \cos(\frac{\theta}{2})I - i\sin(\frac{\theta}{2})Y,$$
(1.16)

$$R_Z(\theta) = e^{-iZ\frac{\theta}{2}} = \cos(\frac{\theta}{2})I - i\sin(\frac{\theta}{2})Z,$$
(1.17)

where I is the identity matrix and θ is a real number. Now, it is possible to express any 2×2 unitary matrix in terms of two of these three rotation matrices:

$$U = e^{i\phi} R_X(\theta_1) R_Z(\theta_2) R_X(\theta_3), \qquad (1.18)$$

where ϕ is a global phase that has no effect in observation. θ_1 , θ_2 and θ_3 are real numbers, which are rotation angles. This general unitary matrix U is able to map a single-qubit state to any other single-qubit state, which is why it is also called an arbitrary single-qubit rotation. On the Bloch sphere shown in Fig. (1.1), any point on the surface can be reached by applying this unitary matrix. It simply rotates a unit vector from pointing to one spot to another.

It is worth talking about the rotation matrix $R_Y(\frac{\pi}{2})$ as it is very important in single-qubit gates. By plugging $\theta = \frac{\pi}{2}$ in $R_Y(\theta)$, we obtain

$$R_Y\left(\frac{\pi}{2}\right) = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1\\ 1 & -1 \end{pmatrix}.$$
 (1.19)

This gate is known as the *Hadamard* gate *H*. It maps the state $|0\rangle$ to $\frac{|0\rangle+|1\rangle}{\sqrt{2}}$ and the state $|1\rangle$ to $\frac{|0\rangle-|1\rangle}{\sqrt{2}}$. This can be visualized on the Bloch sphere: the *Hadamard* gate *H* rotates $|0\rangle$ by $\frac{\pi}{2}$ to the positive **x** axis and $|1\rangle$ by by $\frac{\pi}{2}$ to the negative **x** axis.

1.2.2 Two-qubit gates

Analogous to two-bit gates in classical computation, there are two-qubit gates that play an important role in quantum information processing as they are able to generate entanglement in qubits. I will discuss quantum entanglement in Sec. 1.3. One of the most important two-qubit gates is the CNOT gate (*controlled*-NOT), which is given by

$$CNOT = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}.$$
 (1.20)

This gate flips the second qubit only when the state of the first qubit is $|1\rangle$. It can also be called CX gate (*controlled*-X) as it performs the X operation on the second qubit conditioned

on the state of the first qubit. Now, if we apply this gate to two separate qubits, we obtain

$$CNOT[(\alpha_1 |0\rangle_1 + \beta_1 |1\rangle_1) \otimes (\alpha_2 |0\rangle_2 + \beta_2 |1\rangle_2)] = \alpha_1 \alpha_2 |00\rangle + \alpha_1 \beta_2 |01\rangle + \beta_1 \beta_2 |10\rangle + \beta_1 \alpha_2 |11\rangle$$
(1.21)

It is worth noticing that this outcome is a new two-qubit state that cannot be written as the tensor product of two single-qubit states as before for any possible values that $\alpha_1, \alpha_2, \beta_1$, and β_2 can take. We can use the following mathematical expression to describe this effect:

$$|\psi\rangle_1 \otimes |\psi\rangle_2 \xrightarrow{\text{CNOT}} |\psi'\rangle \neq |\psi'\rangle_1 \otimes |\psi'\rangle_2.$$
 (1.22)

Quantum states like this are called entangled states as opposed to the states called product states which can be decomposed into the product form of two single-qubit states. The CNOT gate serves as an entangler to generate the entanglement between two single-qubit states. Once entanglement is formed between two qubits, they no longer can be treated separately but should be regarded as a whole. Entanglement is an amazing implication of quantum mechanics, which cannot be explained by local-hidden variable theories according to Bell's theorem [15]. Thus, one has to accept non-locality in quantum entanglement (more discussion in the next section), which gives rise to various important applications in quantum information science such as quantum teleportation [16](more detail can be found in Sec. 1.6).

Another interesting two-qubit gate is called the CZ gate (controlled-Z gate), which is

$$CZ = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}.$$
 (1.23)

Conditioned on the state of the first qubit, only the state $|11\rangle$ is mapped to $-|11\rangle$, and the other states remain the same. The CZ gate is actually a member of a large family of gates

called *controlled phase* gates. When the phase is set to be π , we obtain the CZ gate. The CZ gate entangles two single-qubit states by changing the sign of $|11\rangle$), and it is closely related to the CNOT gate by the following transformation:

$$CZ = (I \otimes H)CNOT(I \otimes H), \qquad (1.24)$$

where and I is the *identity* gate, and H is the *Hadamard* gate. This transformation can be seen as changing the computational basis from $\{|+\rangle, |-\rangle\}$ to $\{|0\rangle, |1\rangle\}$ for the second qubit.

Another powerful two-qubit gate worth introducing is the SWAP gate. It can be written as:

$$SWAP = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}.$$
 (1.25)

It is called the SWAP gate simply because it swaps two qubits. This can be proved as follows:

$$SWAP[(\alpha_{1} | 0\rangle_{1} + \beta_{1} | 1\rangle_{1}) \otimes (\alpha_{2} | 0\rangle_{2} + \beta_{2} | 1\rangle_{2})]$$

$$= \alpha_{1}\alpha_{2} | 00\rangle + \alpha_{1}\beta_{2} | 10\rangle + \beta_{1}\alpha_{2} | 01\rangle + \beta_{1}\beta_{2} | 11\rangle. \qquad (1.26)$$

$$= (\alpha_{2} | 0\rangle_{2} + \beta_{2} | 1\rangle_{2}) \otimes (\alpha_{1} | 0\rangle_{1} + \beta_{1} | 1\rangle_{1})$$

The SWAP gate is equivalent to three consecutive CNOT gates but with the middle reversed:

$$SWAP = CNOT_{12}CNOT_{21}CNOT_{12}, \qquad (1.27)$$

where for the second CNOT gate, the first qubit is controlled by the second qubit, which is reversed.

1.3 Quantum entanglement and nonlocality

I briefly mentioned entanglement in the previous section when introducing two-qubit gates. In this section, I will touch more on it. Quantum entanglement can be described as a nonclassical correlation appearing in two or more particles that could be separated far from each other. When a group of particles is entangled, the quantum state of each particle cannot be treated independently of the others, i.e. they are a whole. Mathematically speaking, it is impossible to express the quantum state of the whole system as a tensor product of the quantum state of each subsystem or when the state of each subsystem is a mixed state, there is no way to write the whole quantum state as a convex combination of product states.

If we have two unentangled quantum systems, they can be described as:

$$|\psi\rangle_{1,2} = |\psi\rangle_1 \otimes |\psi\rangle_2. \tag{1.28}$$

This state sometimes is referred to as separable state. For two separable mixed states, they can be written as

$$\rho_{1,2} = \sum_{i} p_i \rho_1^{(i)} \otimes \rho_2^{(i)}, \tag{1.29}$$

where $\sum_{i} p_i = 1$ with p_i being the corresponding state probability. More generally, if there are *n* separable quantum systems, their state can be written as:

$$|\psi\rangle_{1,2,\cdots,n} = |\psi\rangle_1 \otimes \cdots \otimes |\psi\rangle_n \quad \text{(pure state)}$$

$$\rho_{1,2,\cdots,n} = \sum_i p_i \rho_1^i \otimes \cdots \otimes \rho_n^i \quad \text{(mixed state)}. \tag{1.30}$$

However, if we have two entangled particles, there is no way to write their state as Eq. (1.28) or Eq. (1.29), e.g. $1/\sqrt{2}(|00\rangle) + |11\rangle$). For this state, it is obvious to see the correlation/entanglement between two particles as two qubits are always in the same basis state. No matter how far these two qubits are separated, once we measure one qubit to be in $|0\rangle$ or $|1\rangle$, the other must be in state $|0\rangle$ or $|1\rangle$ as well.

The fact that the state collapse of two entangled particles is independent of the distance implies that this correlation is nonlocal. At first look, this seems to be at odds with special relativity but as the collapse of quantum states is totally random, there is no actual information exchange between two particles. The problem of nonlocality in quantum mechanics was first raised by Einstein, Podolsky, and Rosen in [17], which is now known as the EPR paradox. This led to the development of local hidden variable (LHV) theories, which argue that there are some local and hidden variables in reality (they are preassigned some values) that are not captured by quantum mechanics. However, Bell's inequality [15] and CHSH (Clauser-Horne-Shimony-Holt) inequality [18] were later proposed to reject LHV theories, which state that quantum correlations must violate these two inequalities. Subsequently, there have been a series of experiments to verify the violation of these two inequalities [19, 20, 21, 22]. Also, in Hardy's thought experiments, LHV theories fail to explain the outcomes [23].

Moreover, quantum entanglement is a powerful resource for performing numerous interesting tasks in quantum information processing. Especially in quantum communication/quantum networks, a great deal of effort is put to generate entanglement between remote quantum systems, which will be discussed in detail in Chapter 2.

1.4 Bell-state measurement

The Bell states are a set of maximally entangled two-qubit states [24], which are important resources used in quantum teleportation as discussed in more detail in Sec. 1.6. They are given by:

$$\begin{split} \left| \Phi^{+} \right\rangle &= \frac{1}{\sqrt{2}} (\left| 0 \right\rangle \otimes \left| 0 \right\rangle + \left| 1 \right\rangle \otimes \left| 1 \right\rangle), \\ \left| \Phi^{-} \right\rangle &= \frac{1}{\sqrt{2}} (\left| 0 \right\rangle \otimes \left| 0 \right\rangle - \left| 1 \right\rangle \otimes \left| 1 \right\rangle), \\ \left| \Psi^{+} \right\rangle &= \frac{1}{\sqrt{2}} (\left| 0 \right\rangle \otimes \left| 1 \right\rangle + \left| 1 \right\rangle \otimes \left| 0 \right\rangle), \\ \left| \Psi^{-} \right\rangle &= \frac{1}{\sqrt{2}} (\left| 0 \right\rangle \otimes \left| 1 \right\rangle - \left| 1 \right\rangle \otimes \left| 0 \right\rangle). \end{split}$$
(1.31)

There are various ways to generate the Bell states, one of which is to use a H gate and a CNOT gate. For example, if the initial two-qubit state $|0\rangle_1 \otimes |0\rangle_2$, it is then transformed as follows:

$$\begin{aligned} |0\rangle \otimes |0\rangle \xrightarrow{(H \otimes I) \text{CNOT}} |\Phi^+\rangle, \\ |1\rangle \otimes |0\rangle \xrightarrow{(H \otimes I) \text{CNOT}} |\Phi^-\rangle, \\ |0\rangle \otimes |1\rangle \xrightarrow{(H \otimes I) \text{CNOT}} |\Psi^+\rangle, \\ |1\rangle \otimes |1\rangle \xrightarrow{(H \otimes I) \text{CNOT}} |\Phi^-\rangle. \end{aligned}$$
(1.32)

The standard Bell-state measurement (BSM) is composed of a CNOT gate and a H gate acting on the first qubit, which projects a Bell state to its corresponding two-qubit basis state. Then, we need to perform measurements in the computational basis to obtain classical bits. This operation can be seen as the reverse of the operation for preparing the Bell states, i.e. Eq. (1.32) though without the measurements in the last step. Thus, the BSM outcomes are given above. Via the BSM, one can easily differentiate all four Bell states.

In quantum communication, as the information carriers are typically photons, the entangling gates are performed using linear optics. Photons have plenty of degrees of freedom such as frequency, polarization, spatial location, and arrival time (early or late). They all can be used to encode quantum information. Here we use Fock states to represent the number of photons in a mode so they are also called number states. $|0\rangle$ stands for the vacuum state, and $|1\rangle$ represents a mode containing one photon. They can be viewed as two basis states of a qubit. This type of encoding is often used in many quantum information processing tasks, particularly in quantum communication. The Bell states shown in Eq. (1.31) can now be seen as the Bell states of photons in two modes. In this case, we can use a beam splitter (BS) and two detectors to perform the BSM on these Fock states, the detection outcomes of which can be used to distinguish the input Bell states. A beam splitter either transmits the incoming photon or reflects it. For a generic BS, its transformation matrix can be written as:

$$U_{\rm BS} = \begin{pmatrix} \cos\theta & i\sin\theta\\ & \\ i\sin\theta & \cos\theta \end{pmatrix}.$$
 (1.33)

It satisfies the unitary condition that $U_{\rm BS}U_{\rm BS}^{\dagger} = U_{\rm BS}^{\dagger}U_{\rm BS} = I$. In the case of 50/50 BS $(\theta = \pi/2)$, it becomes

$$U_{\rm BS} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & i \\ i & 1 \end{pmatrix}. \tag{1.34}$$

Now, let us apply this 50/50 BS transformation to the photon Bell states, which results in the following:

$$\begin{split} \left| \Phi^{\pm} \right\rangle_{1,2} &\longrightarrow \frac{1}{\sqrt{2}} \left| 00 \right\rangle_{3,4} \mp \frac{i}{2} \left| 20 \right\rangle_{3,4} \pm \frac{i}{2} \left| 02 \right\rangle_{3,4}, \\ \left| \Psi^{+} \right\rangle_{1,2} &\longrightarrow \left| 10 \right\rangle_{3,4}, \\ \left| \Psi^{-} \right\rangle_{1,2} &\longrightarrow -i \left| 01 \right\rangle_{3,4}, \end{split}$$
(1.35)

where subscripts 1 and 2 stand for the input ports, and 3 and 4 stand for the output ports of the beam splitter. Thus, measuring the photon states at the output ports allows us to distinguish these states. However, we cannot differentiate the states $|\Phi^{\pm}\rangle$ by measuring the outputs as they only differ by a phase but they can be distinguished from $|\Psi^{\pm}\rangle$. Hence, the efficiency of performing the BSM using a BS and detectors is only 50%. This setup is commonly used in entanglement generation and entanglement swapping in quantum networks which will be discussed in detail in Chapter 2.

1.5 No-cloning theorem

The no-cloning theorem states that it is impossible to create perfect copies of an arbitrary unknown quantum state [7], which is in huge contrast to its classical counterparts where classical bits are often copied/cloned to counteract the loss in transmission and errors in computation. In order to see how the no-cloning theorem works, let us first consider two quantum systems A and B where the quantum state of A is given by $|\psi\rangle_A$ and the quantum state of B is given by $|0\rangle_B$. The state $|\psi\rangle_A$ can be completely arbitrary. Now, the initial state of these two systems is given by:

$$\left|\psi\right\rangle_{A}\otimes\left|0\right\rangle_{B},\tag{1.36}$$

which is a product state. Our goal is to apply a quantum cloning machine (characterized by a unitary U) to this initial state to make a copy of $|\psi\rangle$ of A to B:

$$U(|\psi\rangle_A \otimes |0\rangle_B) = |\psi\rangle_A \otimes |\psi\rangle_B.$$
(1.37)

Now, since this cloning machine is able to make a copy of any quantum state of system A, we can randomly select another quantum state $|\phi\rangle_A$ such that we have:

$$U(|\phi\rangle_A \otimes |0\rangle_B) = |\phi\rangle_A \otimes |\phi\rangle_B.$$
(1.38)

Then, we obtain the following equation:

$$(\langle \phi |_A \otimes \langle 0 |_B) U^{\dagger} U(|\psi\rangle_A \otimes |0\rangle_B) = (\langle \phi |_A \otimes \langle \phi |_B) (|\psi\rangle_A \otimes |\psi\rangle_B).$$
(1.39)

As U is unitary and $|0\rangle$ is normalized, the above equation can be simplified as:

$$\langle \phi | \psi \rangle = (\langle \phi | \psi \rangle)^2. \tag{1.40}$$



Figure 1.2: An example of how to teleport an unknown qubit (denoted as C) from Alice to Bob using the shared Bell state $|\Phi^+\rangle_{A,B}$. By performing Bell-state measurement and sharing the outcomes with Bob, Alice is able to teleport the qubit. Here, we assume that there is no decoherence in shared entanglement.

The solutions to this equation are trivial: $\langle \phi | \psi \rangle = 0$ or 1. This means either they are orthogonal or they are the same up to a phase. However, this cannot be the case for two arbitrary states. Therefore, a general quantum cloning machine does not exist. It is worth noticing that even though one cannot clone unknown quantum states perfectly, it is still possible to make imperfect copies with a high fidelity of 5/6 [25].

The immediate implication of the no-cloning theorem is that unknown quantum signals cannot be eavesdropped on in transmission without being noticed. This enables quantum-safe communication and development of quantum key distribution protocols [26, 1]. However, this also has drawbacks as the no-cloning theorem prevents us from amplifying quantum signals during transmission. This makes long-distance key distribution tasks and other connecting remote quantum devices very difficult to achieve. Hence, quantum repeaters have been proposed to solve this issue [8], which is also motivated by the application of quantum teleportation as discussed in the next section.

1.6 Quantum teleportation

Suppose Alice wants to send a qubit to Bob who is spatially separated from her, Alice can do so via direct transmission. Surprisingly, there is an alternative way to achieve this without directly sending the qubit. This is known as quantum teleportation, which uses entanglement as resources to transfer qubits [16]. As shown in Fig. 1.2, Alice and Bob share the Bell state $|\Phi^+\rangle_{A,B}$, and she also holds an unknown qubit $|\psi\rangle_C$. At this point, the quantum state of these three particles is given by:

$$|\psi\rangle_C \otimes \left|\Phi^+\right\rangle_{A,B} = (\alpha \left|0\right\rangle_C + \beta \left|1\right\rangle_C) \otimes \frac{1}{\sqrt{2}} (\left|0\right\rangle_A \left|0\right\rangle_B + \left|1\right\rangle_A \left|1\right\rangle_B). \tag{1.41}$$

Now, we notice that two-qubit basis states can be written in terms of the Bell states:

$$\begin{aligned} |0\rangle \otimes |0\rangle &= \frac{1}{\sqrt{2}} (|\Phi^+\rangle + |\Phi^-\rangle), \\ |0\rangle \otimes |1\rangle &= \frac{1}{\sqrt{2}} (|\Psi^+\rangle + |\Psi^-\rangle), \\ |1\rangle \otimes |0\rangle &= \frac{1}{\sqrt{2}} (|\Psi^+\rangle - |\Psi^-\rangle), \\ |1\rangle \otimes |1\rangle &= \frac{1}{\sqrt{2}} (|\Phi^+\rangle - |\Phi^-\rangle). \end{aligned}$$
(1.42)

We then can rewrite Eq. (1.41) using the above equations as

$$\begin{split} |\psi\rangle_{C} \otimes \left|\Phi^{+}\right\rangle_{A,B} &= \frac{1}{2} \Big(\left|\Phi^{+}\right\rangle_{C,A} \left(\alpha \left|0\right\rangle_{B} + \beta \left|1\right\rangle_{B}\right) + \left|\Phi^{-}\right\rangle_{C,A} \left(\alpha \left|0\right\rangle_{B} - \beta \left|1\right\rangle_{B}\right) \\ &+ \left|\Psi^{+}\right\rangle_{C,A} \left(\alpha \left|1\right\rangle_{B} + \beta \left|0\right\rangle_{B}\right) + \left|\Psi^{-}\right\rangle_{C,A} \left(\alpha \left|1\right\rangle_{B} - \beta \left|0\right\rangle_{B}\right) \Big). \end{split}$$
(1.43)

Up to this point, Alice and Bob have not done anything. We just rearranged their joint quantum state to Eq. (1.43) where it shows that Bob now holds a qubit but its state is yet to be determined by measurements on Alice's side. Now, Alice needs to perform BSM from her side, and depending on the measurement outcomes, Eq. (1.43) will collapse into the corresponding state, and Alice needs to inform Bob of her outcomes to correct the qubit

state. For example, if Alice gets the bit-string 10, then Bob needs to perform the Z operation on the qubit. Moreover, if Alice obtains the bit-string 01, she must inform Bob to perform the X operation. This also ensures no superluminal communications in quantum teleportation as classical communication is imperative. In this way, Alice can send a qubit to Bob by just performing BSM and communicating classically to Bob. This is the basic idea of quantum teleportation.

Chapter 2

Introduction to quantum networks

In this chapter, I will introduce the basic elements of quantum repeaters and the potential applications of quantum networks.

2.1 Quantum repeaters

Similar to classical networks where distant classical devices are connected to exchange information, we wish to connect distant quantum devices enabled by quantum networks. One can imagine that this may be achieved by directly sending quantum information over the channel, but the attenuation in the channel is significant, which extremely limits the transmission distance. For telecom band signals (wavelengths: 1530-1565 nm), the loss in the optical fiber is characterized by $\exp(-L/L_{att})$, where $L_{att} = 22$ km is the attenuation distance, which defines the distance when the amplitude drops to 1/e, and L is the total transmission distance. This is mainly due to the absorption in the fiber, and in addition, there are other types of loss in the channel including scattering loss, dispersion loss, bending loss, and connector loss. However, the absorption loss is the most significant, and the exponential decay is a direct consequence of the Beer-Lambert law, which makes the chance of successful transmission over a long distance very slim. As a result, the average time for successfully receiving a photon (referred to as the average waiting time) becomes very long even with a high-repetition-rate


Figure 2.1: The illustration of a quantum repeater. The total distance is L, and it is divided into four elementary links with the length of L_0 . The entanglement distribution is achieved in three steps. First, we generate entanglement in elementary links, and upon the success heralded by single-photon detection, we then perform BSM to swap the entanglement between adjacent elementary links (indicated in blue dashed circles), and we again perform entanglement swapping to further propagate the entanglement. The red nodes indicate quantum memories which are typically required in repeaters.

single-photon source. For example, with the best available source with a repetition rate of 10 GHz, the average waiting time is expected to be 10^{10} s for a distance of over 1000 km. The same challenge also exists in classical communication, but this issue can be solved using amplification which directly amplifies the signal to compensate for the loss.

However, the same technique cannot apply to quantum signals due to the no-cloning theorem discussed in Sec. 1.5. Thus, quantum repeaters have been proposed to solve this problem [8], which is centered on the idea of quantum teleportation that allows us to use entanglement to transfer quantum information. In such quantum repeaters as shown in Fig. 2.1, a long distance L is divided into small pieces of equal length $L_0 = L/N$ with N being the total number of subdivisions. We first need to generate entanglement in these elementary links, the success of which is announced by the single-photon heralding in detectors, and then we perform entanglement swapping (blue dashed circle) between two adjacent links to propagate the entanglement to a longer distance. The entanglement swapping is based on BSM, which projects two Bell pairs to a single Bell pair with a 50% efficiency. As these steps are probabilistic, quantum memories are required to store the entanglement (labeled as red dots) which is discussed in detail in the next section. Moreover, entanglement purification [27] can be used to improve the fidelity of distributed entanglement, which is actually a part of the repeater protocol in [8] but this boosts the fidelities at the cost of lower distribution rates.

2.2 Quantum memory

In most repeater protocols [28, 12, 11], quantum memory is an essential part of a quantum repeater, which allows us to store and process quantum information. Still, there are some memory-less repeater protocols that use quantum error correction codes to encode signals and correct errors that happen during the direct transmission [29, 30, 31, 32]. In spite of the much higher rates that these memory-less protocols could potentially yield, they require a lot more resources such as the near-deterministic preparation of a many-photon cluster state and many high-fidelity two-qubit gates, which is currently very challenging to achieve. In this thesis, I will only talk about quantum repeaters that need quantum memory, and the memory-less repeaters are beyond the scope of this thesis.

Quantum memories typically operate at cryogenic temperatures but some can operate at room temperature, which can either be atomic ensembles or single emitters. Some of the wellstudied and leading quantum memory platforms include rare-earth (RE) ions [12, 11], alkali atoms [33, 34] and nitrogen-vacancy (NV) centers in diamond [35, 36, 37, 38], the last two platforms of which operate at room temperature with incredibly long coherence times. More detailed information about these two types of systems can be found in Chapter 3. The two most important figures of merit to quantify the performance of quantum memory are storage time and efficiency [28]. The storage time measures how long quantum entanglement can remain coherent in the memory, which limits the entanglement distribution distance. The efficiency measures how much incoming signal can be successfully stored in the memory, which is particularly important in ensemble-based memories as the entanglement swapping is always probabilistic in the atomic ensemble-based repeater protocols. Thus, the efficiency directly affects the repeater rates. For atomic ensemble memories, the bandwidth is also a metric to consider as it directly tells us how short the signal pulse can be and thus the maximum repetition rate of the protocol.

2.3 Figures of merit of quantum repeaters

There are two main figures of merit for quantum repeaters: rates and fidelities. The rates characterize the average time of distributing the final entangled state, and the fidelities characterize the quality of the final entangled state. As mentioned before, direct transmission suffers from an exponential loss in the channel. The rates of quantum repeaters must beat those of direct transmission, particularly at longer distances. However, realizing such repeaters involves a few challenges, which include having efficient detectors, efficient and long-lived quantum memories, and efficient single-photon sources or entangled photonpair sources. For different repeater protocols, the rates vary but in general, the repeaters equipped with single-emitter quantum memories have higher rates than the repeaters with atomic ensemble memories, which is presented in more detail in the next two sections.

The repeater fidelities are limited by various factors including imperfect photon sources, imperfect entanglement swapping, noise photons in the signal retrieval, dark counts in detectors, and memory decoherence. Moreover, the fidelities can be boosted by entanglement purification [27] but at the cost of lower rates. These limiting factors play slightly different roles in the repeaters with single-emitter memories and atomic ensemble memories (more details can be found in the next two sections).

2.4 Single-emitter-based quantum repeaters

The single-emitter-based quantum repeaters use single-emitter systems as quantum memories. Similar to atoms and ions, these quantum emitters have addressable electron spin and optical levels to serve as spin-photon interfaces. They are often embedded in different host materials and the spin and optical properties of the emitters depend not only on the host material but also on their compositions and structures. So far, there are numerous quantum emitters that have been actively investigated including carbon-based emitters (nitrogenvacancy (NV) centers and silicon-vacancy (SiV) centers), silicon-based emitters (T centers and G centers), rare-earth-ion-doped crystals, and semiconductor quantum dots. Among them, silicon color centers are of particular interest due to their scalability. The host materials and single emitters also possess rich isotopes (e.g. ¹³C, ¹⁴N and ¹⁵N in NV centers), which provide nuclear spins nearby to utilize. These nuclear spins have ultra-long coherence times, thus offering the possibility of storing quantum states via hyperfine interaction with the electron spins. Moreover, the high-fidelity quantum transfer between the electron spins and the nuclear spins nearbles near-deterministic entanglement swapping, which could boost repeater rates [39].

Despite the above-mentioned attractive properties of single emitters, they are subject to various decoherence sources such as a nuclear spin bath and lattice phonons [40]. Also, lattice phonons can broaden the photon emission line, which makes the use of the emitters at non-cryogenic temperatures challenging. But at the same time, these nuclear spins can also be used to correct errors [41].

2.4.1 The Barrett-Kok scheme

One of the most widely used entanglement generation schemes in solid-state platforms is the Barrett-Kok scheme [42]. It was utilized to demonstrate the first loophole-free Bell inequality violation [22]. In order to generate entanglement between two solid-state qubits (single emitters), the most straightforward way is to create spin-photon entangled states. As illustrated in Fig. 2.2a, a spin has two states $|\uparrow\rangle$ and $|\downarrow\rangle$, and $|\uparrow\rangle$ can be optically excited to the excited state $|e\rangle$. The excited state only decays back to $|\uparrow\rangle$ with a single photon emitted. Thus, if we first put the spin in a superposition state as $1/\sqrt{2}(|\uparrow\rangle + |\downarrow\rangle)$ and excite $|\uparrow\rangle$ to the excited state $|e\rangle$ using an optical π pulse, then after the decay, we generate a spin-photon



Figure 2.2: This figure is directly taken from [43]. (a) An illustration of an L-type quantum system. There is a ground state doublet system $|\uparrow\rangle$ and $|\downarrow\rangle$ with one optically excited state $|e\rangle$ that decays back to its initial ground state $|\uparrow\rangle$. (b) An entanglement generation diagram. Two fields from each quantum system interfere at a central beam splitter (BS) with two single-photon detectors D_1 and D_2 .

entangled state:

$$\frac{1}{\sqrt{2}}(\left|\uparrow\right\rangle\left|1\right\rangle+\left|\downarrow\right\rangle\left|0\right\rangle),\tag{2.1}$$

where $|1\rangle$ and $|0\rangle$ represent a single photon state and vacuum state. Now, if we create two such entangled states and interfere the emitted photons from the systems as shown in figure 2.2b, the which-path information is erased and a single detection event will herald one of the Bell states $|\psi^{\pm}\rangle = (|\uparrow\downarrow\rangle \pm |\downarrow\uparrow\rangle)/\sqrt{2}$ as discussed in Sec. 1.4. However, not all single clicks in detectors lead to the desired entangled states as the event where two photons with each emitted from a single emitter can also contribute to spurious clicks when one photon gets lost in transmission. This results in infidelities in the final state.

Barrett and Kok proposed to solve this problem by performing the second round of singlephoton detection after flipping the spin state for both systems by applying X operation and re-exciting the state $|\uparrow\rangle$ to $|e\rangle$ using an optical π pulse [42]. In this way, the single-photon detection event can eliminate the components due to spurious clicks in detectors. This scheme is known as the Barrett-Kok scheme. Using two rounds of single-photon detection, one can generate entanglement between two distant spin qubits with a fidelity that is robust against photon loss and detector loss. However, there are two main experimental imperfections that still reduce the fidelity: decoherence of spin qubits and dark counts in detectors. At cryogenic temperatures, superconducting nanowire detectors can offer ultralow dark count rates, thus significantly reducing this effect. The effect of decoherence of spin qubits depends on the specific physical platforms we use. The detailed study on this can be found in [43].

2.5 Atomic ensemble-based quantum repeaters

The very first realistic quantum repeater scheme was proposed by Duan, Lukin, Cirac, and Zoller [9] which is widely known as the DLCZ scheme. Ever since then, atomic ensemblebased quantum repeaters have been under active investigation. The atomic ensembles are utilized as quantum memories and offer the collective enhancement of interaction with single photons. In the DCLZ protocol, the signal is stored as a collective excitation in the spin state, and the retrieval of the signal can be achieved by reversing the process, which emits the photon in a well-defined direction [9]. However, this protocol has a few limitations including a strong trade-off between rate and high fidelity due to multiphoton emission, phase fluctuations in the channel, inefficient entanglement generation, and non-telecom photon emission [12]. Thus, numerous improvements have been proposed to overcome the issues in the DLCZ scheme, which include the single-photon protocol [44], the two-photon detection protocol [45], and the atomic-frequency comb (AFC)-based protocol [46]. All of these protocols greatly improve the rates and robustness but their physical implementations could require different platforms such as rare-earth-ion doped crystals for the AFC memory. Moreover, the decoherence of ensemble-based repeaters only affects the distribution rate, not the fidelity as the vacuum components are filtered out by post-selection.

2.5.1 The DLCZ protocol

The basic idea of the DLCZ protocol uses spontaneous Raman emission to create two ensemble-photon entangled states that are far away from each other, as well as a central BS to interfere the two fields to create entanglement between two ensemble memories. The entanglement swapping is then achieved by reading out the collective excitation in the memories and interfering the retrieved photons using a BS. As illustrated in Fig. 2.3, the system has a huge number of atoms that have three levels: the excited state $|e\rangle$ and two ground states $|g\rangle$ and $|s\rangle$. In the write process, all the atoms are first prepared in $|g\rangle$, and then a laser pulse (write) excites the $|g\rangle - |e\rangle$ transition with a detuning Δ . The excited state decays to the spin state $|s\rangle$ by emitting a single photon. This occurs with a small probability due to the small excitation amplitude that depends on the laser intensity [12]. The emitted photon is called the Stokes photon. As the probability of emitting one photon is small, the probability of emitting two photons or more is even smaller. The created excitation in the state $|s\rangle$ is called the collective atomic excitation as the process moves one atom from $|g\rangle$ to $|s\rangle$ from all the atoms, thus generating a multi-atom superposition state. This process creates a two-mode entangled state, which is given by [12]:

$$(1 - \frac{1}{2}(\chi t)^2) |0\rangle_p |0\rangle_s - i\chi t |1\rangle_p |1\rangle_s - (\chi t)^2 |2\rangle_p |2\rangle_s + O((\chi t)^3),$$
(2.2)

where t stands for time, and χ characterizes the laser intensity and the subscripts s and p stand for the photon and the collective spin with $|0\rangle_s = |gg\cdots g\rangle$ and $|1\rangle_s = 1/\sqrt{N}(|sg\cdots g\rangle + |gs\cdots g\rangle + |gg\cdots s\rangle$ for single excitation in the ensemble. $|2\rangle_s$ then represents two excitations. This equation shows that the probability of creating multiple pairs of excitations is non-zero although it can be made quite small by choosing χt to be small. This is the main limiting factor of the DLCZ protocol as multiple photon errors degrade the repeater fidelity and there is a strong trade-off between the repeater rates and the fidelities.

With a small χt , one creates two ensemble-photon entangled states in two distant loca-



Figure 2.3: The illustration of the level scheme in the DLCZ protocol. The system consists of three levels: the collective ground state $|g\rangle$, the collective spin state $|s\rangle$, and the excited state $|e\rangle$. In the write process, atoms are prepared in $|g\rangle$, and a laser pulse off-resonantly excites the $|g\rangle - |e\rangle$ transition with a detuning Δ , which emits a Stokes photon with a small probability. In the read process, a read pulse excites the $|s\rangle - |e\rangle$ transition, converting the single atomic excitation back to $|e\rangle$. This collectively emits an anti-Stokes photon in a well-defined direction.

tions A and B. The Stokes photons interfere at a central BS and the single-photon detection event projects the systems into an entangled state between two distant quantum memories as shown in Fig.2.4(a). The created entangled state is given by [12]:

$$|\psi_{ab}\rangle = \frac{1}{\sqrt{2}} (|0\rangle_a |1\rangle_b + |1\rangle_a |0\rangle_b e^{i\theta_{ab}}), \qquad (2.3)$$

where θ_{ab} is the relative phase between the modes a and b due to transmission in the fiber and pump lasers. After we establish two links A-B and C-D, we can perform entanglement swapping to create entanglement in A and D. This is accomplished using the "read process" shown in Fig. 2.3. A read pulse converts the atomic excitation to the excited state $|e\rangle$ followed by the emission of an anti-Stokes photon in a well-defined direction. The anti-Stokes photons then are sent to a central BS, and the single photon detection heralds the successful swapping, thus creating entanglement between A and D as illustrated in Fig. 2.4(b).

As mentioned earlier, the DLCZ protocol has several limitations [12]. The strong trade-



Figure 2.4: (a) Entanglement generation of two remote ensembles at A and B in the DLCZ protocol. The ensembles (QM) emit Stokes photons with a small probability and the photons travel through optical fibers (dashed line) to a central BS where a single photon detection either in detector d or \tilde{d} projects the systems to an entangled state. (b) Entanglement swapping of two links A-B and C-D. The anti-Stokes photons are read out and interfere at a central BS for B and C. Again, the single photon detection heralds the storage of a single excitation in either A or D, thus creating an entangled state between the memories at A and D.

off between its efficiency and fidelity significantly restricts its performance. As the emission probability of Stokes photon must be kept low in order to reduce multi-photon errors, the entanglement generation efficiency is thus very low. This drawback can be eliminated using the single-photon protocol which is presented in the next section and is adopted in the paper presented in Chapter 5. Moreover, as can be seen in Eq. (2.3), the relative phase must be stable in the channel during transmission. This could be realized using Sagnactype configurations or using a two-photon detection scheme [45]. Besides, telecom photons are ideal for long-distance communications but so far, none of the atomic ensembles offer photon emission in telecom bandwidths. Thus, we either need frequency conversion or use photon-pair sources [46] to overcome this issue.

2.5.2 The single-photon protocol

The single-photon protocol was proposed by Sangouard et al. in 2007 to overcome the limitation imposed by multiphoton emission in the DLCZ protocol [44]. It utilizes single-photon sources and atomic ensembles to create ensemble-photon entangled states for entanglement generation and entanglement swapping is achieved by heralding retried signals, similar to the DLCZ protocol. However, the memory protocol can be pretty flexible depending on the properties of ensembles species. The entanglement generation process is illustrated in Fig. 2.5(a). For the node at A, a single photon emitted from the source after a beam splitter can be described as $(\alpha a_1^{\dagger} + \beta a_2^{\dagger}) |0\rangle$ where α , β are reflection and transmission amplitudes of a beam splitter, and they satisfy the relation $|\alpha|^2 + |\beta|^2 = 1$. a_1^{\dagger} and a_2^{\dagger} are the creation operators for the reflected photon and transmitted photon respectively. The same is true for the node at B where the state of a single photon after a beam splitter is $(\alpha b_1^{\dagger} + \beta b_2^{\dagger}) |0\rangle$. Thus, the joint state is given by:

$$\left[\alpha^{2}a_{1}^{\dagger}b_{1}^{\dagger} + \alpha\beta(a_{1}^{\dagger}b_{2}^{\dagger} + a_{2}^{\dagger}b_{1}^{\dagger}) + \beta^{2}a_{2}^{\dagger}b_{2}^{\dagger}\right]|0\rangle.$$
(2.4)



Figure 2.5: (a) Entanglement generation of two remote ensembles at A and B in the singlephoton protocol. The single-photon source emits a photon that either transmits through the beam splitter or gets reflected to enter the quantum memory (QM). The transmitted photon travels through optical fibers (dashed line) to a central BS where a single photon detection either in detector d or \tilde{d} projects the systems to an entangled state. (b) Entanglement swapping of two links A-B and C-D. The stored photons are read out and interfere at a central BS for B and C. The single photon detection heralds the storage of a single photon in either A or D, thus creating an entangled state between the memories at A and D.

The first term in this state is the case where both single photons are reflected to be stored in quantum memories, ideally yielding no heralding in detectors. The second and third terms are the main contributions to single photon heralding where a_1^{\dagger} and b_1^{\dagger} are to be stored in quantum memories. The last term could also lead to the single-photon detection event when one of the two photons gets lost in the transmission, thus creating vacuum components. After the detection of a single photon in one of the detectors, we obtain [44]:

$$\alpha^{2} \left| \psi_{ab} \right\rangle \left\langle \phi_{ab} \right| + \beta^{2} \left| 0 \right\rangle \left\langle 0 \right|, \qquad (2.5)$$

where $|\psi_{ab}\rangle = (1/\sqrt{2})(a_1^{\dagger} + b_1^{\dagger})|0\rangle$ that is the entangled state shared between the two remote memories at A and B and the vacuum state $|0\rangle$ represents an empty memory.

Once we successfully establish two links A-B and C-D, we can perform entanglement swapping to propagate the entanglement between A and D. As illustrated in Fig. 2.5(b), stored photons in memories can be retrieved and sent to a central BS through optical fibers. Again, the single-photon detection heralds the entanglement between A and D, thus completing the swapping process. In practice, there are no ideal single-photon sources but one can use atomic ensembles to well approximate single-photon sources. Although this protocol could still suffer from multiphoton errors due to imperfect single-photon sources, the tradeoff between the repeater fidelity and rate is not as significant as that in the DLCZ protocol. In addition, the entanglement generation efficiency is considerably enhanced compared to the DLCZ protocol [12].

2.6 Applications

2.6.1 Secure communications

One of the most important applications of a quantum network is quantum-enabled secure communication. In classical communication, suppose we wish to share a random secret key between two parties that can be far away from each other, in reality, this secret key can be eavesdropped by a third party which could intercept the communication channel and make copies of the shared key. However, this is not the case in quantum communication where photons are used to encode the information, and thanks to the no-cloning theorem, it is impossible for the eavesdropper to perfectly clone the shared quantum key. This method was first proposed by Charles Bennett and Gilles Brassard in 1984, which is now known as BB84 protocol [26]. In their protocol, photons are prepared in two non-orthogonal polarization bases (e.g. rectilinear and diagonal), and they are transmitted from Alice to Bob. Then, Bob needs to measure these photons in the basis of his choosing, and they need to classically communicate to compare the outcomes. At last, they only keep the same measurement outcomes to establish the key.

As the eavesdropper cannot perfectly clone the quantum states, he/she has to make a guess about what basis each photon is in, but the success probability of doing so scales as $(1/2)^n$ with *n* being the length of the bit string of the transmitted key, which becomes very small when *n* is large. Moreover, any wrong guess would lead to disagreements on the measurement outcomes shared between Alice and Bob, which would enable them to detect the eavesdropper. Notably, there is also the entanglement-based quantum key distribution protocol [47]. The quantum key distribution (QKD) protocols are secure against eavesdropping based on the principles of quantum mechanics, which offers security beyond classical communication. For long distances, quantum repeaters are needed to overcome the loss in direct transmission to distribute keys, and for longer distances over 1000 km, quantum satellites are required to perform QKD between continents. To date, there has been plenty of experimental demonstrations of QKD [48, 49, 50, 51, 52], which has achieved over 300 km distance on the ground and about 7600 km on satellite.

2.6.2 Blind quantum computing

In classical computation, despite that personal computer has become readily accessible, the need for the client to delegate a computational task to the server to perform calculations is never abated, which is often in the form of cloud computing. This need can also be found in quantum computation where ideally the classical client delegates a certain quantum computational task to the server who has a universal quantum computer to perform this task while keeping the structure of the task hidden. This is known as blind quantum computation (BQC) [53]. However, so far the security of BQC is restricted to two settings: a semi-classical client with restricted quantum capabilities and a single quantum server [54, 55] or a classical client without any quantum resources and multiple non-communicating quantum servers [56]. In the first setting, quantum communications are needed to send information between the client and the server using photons. In the second setting, the client only classical communicate with the multiple servers who share entanglement with each other. Thus, quantum networks are imperative to achieve BQC. Moreover, the client also should be able to verify the returned results from the server(s) to confirm that the calculations have been done correctly. So far, a few experimental demonstrations of BQC have been successfully shown [2, 57, 58] although they are only performed in a short distance. For long-distance BQC, we need quantum networks integrated with quantum repeaters, which are currently under intensive development.

2.6.3 Distributed quantum computing

Scaling up a quantum computer has been a major roadblock to overcome in order to use it for solving complex problems both in science and industry. So far, we have entered the era of having noisy intermediate-scale quantum (NISQ) devices [59], which have been demonstrated to offer computational advantages over classical computers in solving specific problems [60, 61, 62, 63] with less than 100 qubits. However, we need many more qubits to take it further to run many quantum algorithms to solve classically-intractable computational problems such as Shor's algorithm [64], Grover's algorithm [65] and quantum algorithm for linear systems of equations [66]. One promising approach to this problem is to have smaller quantum processors and connect them by quantum and classical networks. This method has been used in classical computing to distribute a heavy computational task to many processors. The same idea can apply to quantum computing as well but it is more complicated. The main striking difference is that besides classical communication, we also need quantum networks for networking distant quantum processors in order to perform nonlocal CNOT gates [67]. This serves as another important motivation for developing quantum networks.

2.6.4 Quantum clock synchronization

The optical atomic clocks are so far the most precise tools for measuring frequency and time [68]. Having access to a set of synchronized clocks distributed in geographically separated locations could have a significant impact on various current technologies such as the global positioning system (GPS), traffic control, and long baseline interferometry [69], and could also lead to new applications such as probing the properties of dark matter [70] and spacetime variation of fundamental constants [71]. For independent quantum clocks, there are two main methods for achieving synchronization: the Einstein protocol combined with time transfer links [72] and quantum versions of Eddington's protocol [73], both of which are inspired by their classical counterparts. However, the accuracy and precision of measuring independent quantum systems are limited by the standard quantum limit (SQL) where the error goes inversely proportional to \sqrt{N} with N being the number of clocks. But if these clocks are entangled, then measurements can beat the SQL to achieve the Heisenberg limit where the error goes inversely proportional to N. Thus, this leads to a third way of clock synchronization: entanglement-based quantum clock synchronization, originally proposed by Jozsa and others [74]. As this method utilizes non-local resources to achieve synchronization instead of exchanging light or matter between parties, it is more robust. This two-party protocol has been generalized to multi-party protocol [75] with experimental demonstrations for both [76, 77].

Both the entanglement-based protocols and quantum Eddington's protocols have an outstanding challenge that a common phase reference is required for all parties to control and stabilize the phase that each local clock qubit picks up during precession. For quantum Eddington schemes, this issue can be solved by introducing a two-way exchange of clock qubits between parties [78] but it is challenging to do so for a long distance (even intercontinental distance) as space-based quantum communication for distributing entanglement is likely needed. This potential solution in principles takes advantage of entanglement as well. Moreover, it has been shown that by using quantum purification, the quantum entanglement-based scheme, i.e. in [74] can overcome the common phase reference issue [79]. Also, by introducing a center clock that connects all other local clocks in a quantum network of clocks, one can interrogate the collective phase contributed by all parties, stabilize it and use it to correct each clock [80].

2.6.5 The quantum internet

The ultimate application of quantum networks is a global quantum internet that is able to connect remote quantum computers and quantum information processors [4, 5, 6], which could even allow computations that outperform the most powerful monolithic quantum computers in the future. This likely requires quantum satellites to reach intercontinental connections. There have been a few successful demonstrations of quantum experiments based on low-Earth-orbit (LEO) satellites [51, 81, 82, 52, 83]. In order to have longer service and wider coverage, geostationary satellites can be used, or we can combine LEO satellite links with the terrestrial quantum repeaters [84] which also offers a solution to avoid significant photon loss in the channel between the ground stations and the cities [5], or we can equip LEO satellites with quantum memories [85]. Moreover, as the first-generation quantum computers will likely be based on superconducting qubits, a global quantum network should be able to interface optical photons with microwave photons. This requires the development of quantum transducers, which can interconvert microwave photons and optical photons (ideally telecom photons for long-distance connections) into each other [86]. There has been a great deal of effort in using different quantum systems to design transducers, which include atomic ensembles [87, 88, 89, 90, 91], electro-optomechanical systems [92, 93], and electrooptical systems [94, 95]. Among them, rare-earth (RE) ions and T centers [96] are promising as they offer level structures with addressable microwave transitions and optical transitions in telecom bands. In particular, ¹⁶⁷ Er ions have a hyperfine splitting of 5 GHz even at zero magnetic fields, which can be used to design transducers for superconducting qubits while avoiding disturbing them [91]. Also, solid-state systems are attractive in scalability and could be potentially integrated with superconducting systems. The details of ¹⁶⁷ Er ions-based and T center-based transducers can be found in [91, 90] which are not included in the thesis.

Chapter 3

Room-temperature quantum systems

In this chapter, I will introduce the main features of quantum systems that can operate at ambient conditions, which serve as important ingredients in our proposed room-temperature quantum repeaters [97, 98]. The details of these two proposals can be found in Chapter 4 and Chapter 5.

3.1 Nitrogen-vacancy centers

The nitrogen-vacancy (NV) center is a naturally occurring point defect in diamond, which is composed of a nitrogen atom and a vacancy as shown in Fig. 3.1(A). The vacancy can be neutral (NV⁰) or has an electron (NV⁻). The energy structure of the negatively charged NV center (NV⁻) is shown in Fig. 3.1(B) where the ground state is a triplet state containing two degenerate states $|m_s = \pm 1\rangle$ and the state $|m_s = 0\rangle$ along the z axis. The states $|m_s = \pm 1\rangle$ can be separated by applying an external magnetic field, one of which can be used with $|m_s = 0\rangle$ as an electron qubit. The excited states have a large phonon sideband (PSB) at room temperature but at cryogenic temperatures, the states $|m_s = \pm 1\rangle$ in the excited state ³E can be resolved. The ground states $|m_s = \pm 1\rangle$ can be excited to their excited states with a green laser at 532 nm but the luminescence spectrum ranges from 638 nm to 800 nm due to the PSB. In addition, there are singlet states ¹A₁ and ¹E, which allows for non-



Figure 3.1: (A) Schematic of the NV center in diamond, which is directly taken from [99]. The NV center is a point defect composed of a nitrogen atom (green) and a vacancy (V). When the vacancy has an electron, it is negatively charged, which is often labeled as NV⁻. The grey dots represent ¹²C with a natural abundance of around 98%, and the yellow dot is ¹³C isotope with an abundance of around 1.1%. (B) The energy structure of NV⁻ at room temperature. The ground state is a triplet state with two degenerate states $|m_s = \pm 1\rangle$ which can be resolved by applying an external magnetic field. At ambient conditions, the excited states and the zero-phonon-line (ZPL) emission (638 nm) are significantly broadened. Also, the ZPL only takes up 2% of the emission spectrum. There also exist meta-stable single states ¹A₁ and ¹E, which allows for non-radiative decay from the excited states to the ground states and the emission of near-infrared photons.

radiative decay from the excited states to the ground states and the emission of near-infrared photons. The non-radiative transitions are spin-dependent, which enables the excited states $|m_s = \pm 1\rangle$ to decay to the ground state $|m_s = 0\rangle$ without preserving the spin. Thus, this property is used to achieve spin polarization for initializing the system. Moreover, there are two types of carbon atoms ¹² C and ¹³ C, and the isotopes ¹³ C and ¹⁵ N have a nuclear spin of I = 1/2, and ¹⁴ N has a nuclear spin of I = 1. They can interact with the electron spin in the NV center via hyperfine coupling [100, 101, 35]. The strength of the coupling can vary from a few MHz to more than 100 MHz, depending on the position [102, 103, 35]. The electron-nuclear spin Hamiltonian is given by

$$H_{e,n} = \Delta_0 S_z^2 + \mu_e B S_z + \mu_n B I_z + A S_z I_z, \qquad (3.1)$$

with the zero-field splitting $\Delta_0=2.87$ GHz, the electronic spin gyromagnetic ratio $\mu_e = -2.8$ MHz/Gauss, the nuclear spin gyromagnetic ratio $\mu_n = 1.07$ kHz/Gauss. S_z and I_z are spin operators in the z direction for the electron spin and nuclear spin respectively. The external magnetic field B is applied along the symmetry axis of the NV. The $C_n NOT_e$, where n is the nuclear spin which is the control qubit and e is the electron spin which is the target qubit gate can be implemented by a Ramsey sequence on the electron spin at room temperature.

The spin properties of NV centers are excellent even at room temperature, which has millisecond-long electron spin coherence times [35, 104, 105, 106]. Moreover, nuclear spins in diamond have even longer coherence times at room temperature than the electron spins (exceeding a second [35]), which can be used as quantum memories. The mapping between the nuclear spin and the electron spin can be achieved via hyperfine coupling, which consists of the CNOT gates and readout of the electron spin states. The gate operations have been demonstrated to achieve a high fidelity of 99.92% at room temperature [107]. The readout of the nuclear spin can only be done through the electron spin as the nuclear spin is optically inaccessible. There are various methods for the electron spin state readout at room temperature such as spin-to-charge conversion [108] and photoelectrical imaging [109] to achieve a high-fidelity readout.

Despite the outstanding spin properties of NV center at room temperature, the zerophonon line (ZPL) where the emission and absorption wavelengths are not phonon-assisted only takes up 2% at room temperature, and the ZPL itself is considerably broadened, which makes it challenging to use NV centers for the spin-photon interface at ambient conditions [110]. Thus, if we can bypass the phonon-induced broadening in the ZPL, we will be able to fully utilize them to build quantum networks that operate at room temperature. We will see that this can be achieved by using optomechanics, and the details can be found in Chapter 4. Furthermore, due to the coupling with a nuclear spin bath (numerous nuclear spins surrounding the electron spin), the coherence time of NV electron spin is adversely affected. The dynamical decoupling technique is typically used to mitigate this effect where the electron spin can be decoupled from the nuclear spin bath via a sequence of pulses that flip the spin, enabling the millisecond-long coherence time at room temperature [105, 111]. Furthermore, one can use a microwave source to dress the triplet spin states, which produces the dressed states that are robust against the nuclear spin bath while providing high sensitivity to the external magnetic field [112]. Both the dynamical decoupling and microwave dressing are used in Chapter 4.

3.2 Optomechanics

A generic optomechanical system comprises a laser-driven optical cavity and a mechanical oscillator as shown in Fig. 3.2. Due to radiation pressure, the end mirror vibrates, which shifts the cavity resonance frequency by the amount determined by the mechanical amplitude. The total Hamiltonian of the optomechanical system is given by [113, 114]:

$$\hat{H}_0 = \hbar\omega_{\rm cav}(x)\hat{a}^{\dagger}\hat{a} + \hbar\Omega_m\hat{b}^{\dagger}\hat{b} + \hbar\alpha_{\rm in}(\hat{a}e^{\omega_L t} + \hat{a}^{\dagger}e^{-i\omega_L t}), \qquad (3.2)$$

Mechanical oscillator



Figure 3.2: Schematic of a generic optomechanical system. The optical cavity is driven by a laser, and the end mirror is attached to an oscillator. The cavity mode \hat{a} couples to the mechanical mode \hat{x} by the radiation pressure from the laser.

where $\omega_{\text{cav}}(x)$ is the cavity resonance frequency that is modulated by the mechanical amplitude x and Ω_m is the mechanical frequency, and ω_L is the driving laser frequency. α_{in} is the drive strength, and \hat{a} and \hat{b} are the cavity photon and mechanical phonon annihilation operators respectively. $\omega_{\text{cav}}(x)$ can be well approximated as $\omega_{\text{cav}} - Gx$ with $G = -\partial \omega_{\text{cav}}/\partial x$ being the optical frequency shift displacement. We also have $\hat{x} = x_{\text{ZPF}}(\hat{b} + \hat{b}^{\dagger})$ with x_{ZPF} being the zero-point fluctuation of the oscillator. Thus, the total Hamiltonian can be written as

$$\hbar\omega_{\rm cav}\hat{a}^{\dagger}\hat{a} + \hbar\Omega_m\hat{b}^{\dagger}\hat{b} - \hbar g_0\hat{a}^{\dagger}\hat{a}(\hat{b} + \hat{b}^{\dagger}) + \hbar\alpha_{\rm in}(\hat{a}e^{i\omega_L t} + \hat{a}^{\dagger}e^{-i\omega_L t}), \tag{3.3}$$

where $g_0 = Gx_{\text{ZPF}}$ is the vacuum optomechanical coupling strength. It is convenient to work with a time-independent Hamiltonian so we need to rotate the above Hamiltonian at the laser frequency ω_L by applying the unitary $\hat{U} = \exp(i\omega_L \hat{a}^{\dagger} \hat{a} t)$ [113], which gives the transformed Hamiltonian:

$$-\hbar\Delta\hat{a}^{\dagger}\hat{a} + \hbar\Omega_{m}\hat{b}^{\dagger}\hat{b} - \hbar g_{0}\hat{a}^{\dagger}\hat{a}(\hat{b} + \hat{b}^{\dagger}) + \hbar\alpha_{\rm in}(\hat{a} + \hat{a}^{\dagger}), \qquad (3.4)$$

where $\Delta = \omega_L - \omega_{\text{cav}}$.

Now, we introduce the "linearized" approximation of this Hamiltonian. Driven by a strong laser, the dynamics of the cavity field reach a steady state, and the cavity mode can be written as $\hat{a} = \alpha + \delta \hat{a}$ where α is the mean-field amplitude and $\delta \hat{a}$ is the quantum fluctuations. This is also true for the mechanical mode \hat{b} , which reaches a steady state and can be written as $\hat{b} = \beta + \delta \hat{b}$ where β is the average mechanical amplitude and $\delta \hat{b}$ is the corresponding quantum fluctuations. By substituting these into Eq. (3.4), choosing α and β properly, and omitting the terms that are independent of $\delta \hat{a}$ and $\delta \hat{b}$, we end up obtaining [114]:

$$-\hbar(\Delta + \frac{2g_0^2\alpha^2}{\Omega_m})\delta\hat{a}^{\dagger}\delta\hat{a} + \hbar\Omega_m\delta\hat{b}^{\dagger}\delta\hat{b} - \hbar g_0\alpha(\delta\hat{a} + \delta\hat{a}^{\dagger})(\delta\hat{b} + \delta\hat{b}^{\dagger}) - \hbar g_0\delta\hat{a}^{\dagger}\delta\hat{a}(\delta\hat{b} + \delta\hat{b}^{\dagger}).$$
(3.5)

Furthermore, the last term can also be omitted as it is not enhanced by α compared to the third term. Now, if we choose a new $\Delta = \Delta + 2g_0^2 \alpha^2 / \Omega_m$, we obtain

$$-\hbar\Delta\delta\hat{a}^{\dagger}\delta\hat{a} + \hbar\Omega_{m}\delta\hat{b}^{\dagger}\delta\hat{b} - \hbar g(\delta\hat{a} + \delta\hat{a}^{\dagger})(\delta\hat{b} + \delta\hat{b}^{\dagger}), \qquad (3.6)$$

where $g = g_0 \alpha$ is often referred to as "the optomechanical coupling strength", which depends on the laser strength. Depending on the detuning Δ , there are three different regimes that the optomechanical system can be in. When $\Delta \approx -\Omega_m$, the system is in the so-called red-detuned regime. In this situation, the interaction term in Eq. (3.6) can be written as $-\hbar g(\delta \hat{a}^{\dagger} \delta \hat{b} + \delta \hat{a} \delta \hat{b}^{\dagger})$ using rotating-wave approximation (RWA) [113]. This allows for quantum state exchange between the two modes, thus making the cooling of the oscillator to the ground state possible, even from temperature [115]. When $\Delta \approx \Omega_m$, the system is in the blue-detuned regime. The interaction now can be written as $-\hbar g(\delta \hat{a}^{\dagger} \delta \hat{b}^{\dagger} + \delta \hat{a} \delta \hat{b})$ using RWA, which allows for efficiently entangling the two modes [113]. The last regime is when $\Delta = 0$, the interaction term remains. In this regime, we are able to perform quantum nondemolition (QND) measurement on the optical amplitude $\delta \hat{a} + \delta \hat{a}^{\dagger}$ or on the mechanical displacement $\delta \hat{b} + \delta \hat{b}^{\dagger}$ as either of them commutes with the Hamilton in Eq. (3.6).

So far, we have not considered any dissipations in the system but in most cases, we are dealing with open quantum systems which inevitably suffer from decoherence. It is common to use quantum Langevin equations to describe the dynamics of open quantum systems. Based on Eq. (3.6), the linearized quantum Langevin equations are given by

$$\delta \dot{\hat{a}} = (i\Delta - \frac{\kappa}{2})\delta \hat{a} + ig(\delta \hat{b} + \delta \hat{b}^{\dagger}) + \sqrt{\kappa} \hat{a}_{\rm in}$$
(3.7)

$$\dot{\delta b} = -(i\Omega_m + \frac{\Gamma_m}{2})\delta \hat{b} + ig(\delta \hat{a} + \delta \hat{a}^{\dagger}) + \sqrt{\Gamma_m}\hat{b}_{\rm in}, \qquad (3.8)$$

where κ is the cavity decay rate and $\Gamma_m = \Omega_m/Q_m$ is the mechanical damping rate with Q_m being the quality factor. \hat{a}_{in} and \hat{b}_{in} are the vacuum noise operator and the thermal noise operator for the cavity mode and the mechanical mode respectively. Here, we ignore the thermal noise in the cavity as for optical fields, the thermal occupation is very close to zero even at room temperature [113]. However, this is not the case for the oscillator as it operates at MHz or even lower frequencies, which makes the thermal occupation very large at room temperature. This is why the setup is typically cooled to cryogenic temperatures. In order to make the setup work even at room temperature, we need to have an oscillator with an ultra-high quality factor. Amazingly, levitated particles feature quality factors up to 10⁸ at room temperature [116], and patterning a photonic crystal on the ultra-thin, high-stress SiN membranes enables quality factors up to 10⁸ at room temperature as well [115]. Moreover, soft clamping with strain engineering on SiN membranes has allowed room-temperature quality factors to approach 10⁹ [117, 118].

3.3 Hot alkali gases

The alkali metals are group 1 chemical elements, which consist of Li, Na, K, Rb, Cs, and Fr. The energy structure of alkali atoms has many addressable excited states and two ground



Figure 3.3: Schematic of a general Λ -type memory interacting with a control field Ω and a signal field S. The control field can also couple the $|g\rangle - |e\rangle$ transition which generates the anti-Stokes field A during the retrieval process. As there are four fields present in the system, it is also called four-wave mixing (FWM).

states which are separated by hyperfine splitting. One can select a specific excited state and together with two ground states, this three-level system can be used as quantum memories at both laser-cooled [119, 120] and room temperatures [34, 33, 121]. Unfortunately, none of the alkali gases has the telecom wavelength, so frequency conversion is needed for long-distance communication in optical fibres. At room temperature and above, a buffer gas is typically used to extend the storage time but still due to the atomic collisions between the hot alkali gas and the buffer gas, and the collisions in the hot alkali gas itself, the storage time is limited to tens of microseconds [122, 28]. This detrimental effect may be reduced by the motional averaging method [123, 124], which enables millisecond-long coherence time but is still insufficient for long-distance quantum networks [12]. Also, using a decoherence-free subspace of spin states [33] can extend the storage time to a second but this has not been achieved at the single-photon level. Interestingly, noble gases can be used to interface with alkali gases to enable hours-long storage time, which will be introduced in the next section.

The most frequently used hot alkali gases in quantum memories are Rb and Cs as these two types of gases have ground state splittings that are on the order of a few GHz, which makes the effect of four-wave mixing (FWM) less severe than using other types of hot alkali atoms such as Potassium but FWM is still a significant roadblock towards using hot alkali gases for quantum networks [125, 126]. As shown in Fig. 3.3, FWM arises in Λ -type quantum memories where a strong control field not only couples the transition from the excited state $|e\rangle$ to the spin state $|s\rangle$ but also couples the transition from the ground state $|g\rangle$ to the excited state $|e\rangle$, thus generating the anti-Stokes field when we retrieve the signal [127]. In some literature, the spin state $|s\rangle$ is chosen to be the higher of the two ground states so the off-resonant Raman scattering produces Stokes field but here the state $|s\rangle$ is chosen to be the lower one. The effect of FWM of electromagnetically induced transparency (EIT) memory in free space can be computed using the so-called FWM strength parameter $x = d\gamma_e/\Delta_{gs}$ where d is the optical depth of the ensemble and γ_e is the decoherence rate of the excited state and Δ_{gs} is the splitting between $|g\rangle$ and $|s\rangle$. When $x \ll 1$, the fidelity is well approximated as

$$F_{\rm FWM} \approx \exp(-\sqrt{3}d \frac{|\Omega|^2}{\Delta_{gs}^2})$$
 (3.9)

for single-photon signals with a frequency width much narrower than the transmission width [126]. In principle, this formula can apply to other types of adiabatic quantum memories both in free space and in the cavity. However, in the cavity, one needs to replace the optical depth d with the cooperativity C that measures the strength of the interaction between the cavity field and the ensemble [128]. At low temperatures, it is readily easy to achieve $x \ll 1$ even with high optical depths (more than 100) but at room temperature, as γ_e is largely increased due to the collision with buffer gas, the Doppler broadening, and self-broadening, it is very difficult to achieve $x \ll 1$ so FWM becomes quite significant.

Therefore, there has been some effort in solving this problem, which includes blocking FWM channels by polarization selection rules [129], Raman absorption-enabled suppression [130], cavity engineering [131], by means of coherent destructive interference of FWM [132], and noise-free memory protocol based on two-photon off-resonant cascaded absorption (ORCA) [121]. The idea of using a cavity to suppress FWM is based on tuning the cavity

to be in anti-resonance with the anti-Stokes field and in resonance with the signal. The advantage of using a ring cavity to suppress FWM noise compared to other solutions is that it offers enhanced light storage and retrieval efficiency while only introducing a cavity. It has been experimentally verified, reporting a noise floor of around 1.5×10^{-2} photons per pulse in a Raman-type hot vapor memory [133]. This method is adopted in our second proposal paper presented in Chapter 5.

3.4 Noble-gas spins

The noble gases are group 18 chemical elements, which are He, Ne, Ar, Kr, and Xe. As they have very low chemical reactions, they are often called inert gases. The odd isotopes of noble gases (e.g. ³He and ¹²⁹Xe) possess non-zero nuclear spins, which are isolated from the environment by electronic shells. Thus, they maintain hours-long coherence time even at room temperature [134]. However, these nuclear spins states are optically inaccessible, thus making it difficult to utilize them in quantum information processing, especially quantum communication. A quantum interface between noble-gas spins and alkali atoms has been proposed based on weak spin-exchange collisions [135], which enables the coherent coupling between these two ensembles of atoms. The effective coupling rate is given by:

$$J = \zeta \sqrt{(2I+1)p_a p_b n_a n_b/4},$$
(3.10)

where ζ is the local average interaction strength of an alkali-noble atom pair in a single collision. p_a and p_b are the polarization degrees of alkali and noble gases, and n_a and n_b are the densities of alkali and noble gases in the cell. I is the nuclear spin of an alkali atom. Thus, this interaction strength J is the effective interaction strength in multiple collisions with each collision averaging over all alkali-noble atom pairs in the ensembles. Given the fixed polarization degrees and pressures, larger values of J can be obtained by choosing alkali-noble gas mixtures that have larger values of ζ . Moreover, we have $\zeta = \langle \sigma v \phi \rangle / q$ where σ is the spin-exchange cross section and v is the relative thermal velocity and ϕ is the mutual precession angle and q is the so-called Larmor slowing down factor determined by the degree of polarization [135]. It is clear that atom pairs that have larger spin-exchange cross sections are preferred to have strong interactions. Thus, Rb-Xe and Cs-Xe are typically preferred. However, the decoherence rates of alkali and noble gases also depend on various parameters including cross sections, which are given by

$$\gamma_a = n_b (k_{\rm SE} + \sigma_{\rm SR} v) + n_a \sigma_{\rm SD} v_a / 2, \qquad (3.11)$$

$$\gamma_b = k_{\rm SE} n_a + T_b^{-1},\tag{3.12}$$

where $k_{\rm SE} = 1/4 \langle \sigma v \phi^2 \rangle$ is the binary spin-exchange rate coefficient and $\sigma_{\rm SR}$ and $\sigma_{\rm SD}$ are the collisional spin-rotating cross section and spin-destruction cross section (alkali atoms) respectively. v_a is the mean-field atomic velocity of alkali atoms. T_b is the coherence time of noble-gas atoms when there are no alkali atoms, which is limited by the fluctuations and inhomogeneity of the magnetic field [134]. Thus, the hybrid species K-He is preferred in this case to minimize the decoherence. In general, we have $\gamma_b \ll \gamma_a$. Overall, one needs to take both factors into consideration to determine what mixture is more favorable.

Due to the effect of spatial diffusion, there could be many spatial modes for the alkali and noble gases. For simplicity, here we assume that they are single uniform modes, which is a good approximation in the so-called light-dominated regime [136] and this approximation is adopted in our work as well as presented in Chapter 5. Now, the dynamics of the alkali and noble-gas spins can be well described by the following Heisenberg equations of motion [135]:

$$\partial_t \hat{a} = -(i\omega_a + \gamma_a) - iJ\hat{b} + \hat{F}_a, \qquad (3.13)$$

$$\partial_t \hat{b} = -(i\omega_b + \gamma_b) - iJ\hat{a} + \hat{F}_b, \qquad (3.14)$$

where ω_a and ω_b are the Larmor frequencies of the alkali and noble-gas atoms respectively,

and \hat{F}_a and \hat{F}_b are the corresponding Langevin noise operators for the two types of atoms. The coherent exchange interaction between alkali and noble gases enables us to store signals (photons) as a collective spin excitation in noble gas through alkali atoms, and the storage time can be significantly enhanced, which has been experimentally demonstrated with the coherence time of a minute [137] and an hour [138]. However, due to the fact that the exchange interactions are generally pretty weak, the coupling strength so far has only been demonstrated to be 78 ± 8 Hz [138] but this can be increased to 1000 Hz with higher densities (by increasing pressures) and higher polarization degrees for both alkali and noble gases [135]. Moreover, in high-pressure configuration, radiation trapping becomes severe, which makes the polarization of alkali atoms difficult. Therefore, we need to use extra buffer gas such as N_2 to mitigate this effect, but this further broadens the linewidth of the excited state of alkali atoms. When selecting the species of the alkali and noble gases, 39 K- 3 He mixture has been preferred as in high-pressure configuration, the decoherence rate of the collective spin state in Potassium is much smaller than that in Caesium and Rubidium (More details are discussed in Chapter 5). Therefore, this opens the door for using hybrid alkali-noble gases to build quantum networks that can operate without cryogenics.

Chapter 4

Proposal for room-temperature quantum repeaters with NV centers and optomechanics

4.1 Preface

The successful implementation of global quantum networks would have many applications from secure communication, blind quantum computing, and private database queries to a "quantum internet" of networked quantum computers and other quantum devices. Here we propose a quantum repeater architecture that operates at ambient conditions, which is based on spin-optomechanical interfaces and nuclear spins in diamond with the latter serving as long-lived quantum memories.

This work was done in collaboration with a few co-authors. My main contributions to this work include proposing the architecture scheme, quantifying the entanglement generation fidelity and efficiency, and computing the repeater fidelities. I also drafted the first manuscript.

Proposal for room-temperature quantum repeaters with nitrogen-vacancy centers and optomechanics

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Abstract

We propose a quantum repeater architecture that can operate under ambient conditions. Our proposal builds on recent progress towards non-cryogenic spin-photon interfaces based on nitrogen-vacancy centers, which have excellent spin coherence times even at room temperature, and optomechanics, which allows to avoid phonon-related decoherence and also allows the emitted photons to be in the telecom band. We apply the photon number decomposition method to quantify the fidelity and the efficiency of entanglement established between two remote electron spins. We describe how the entanglement can be stored in nuclear spins and extended to long distances via quasi-deterministic entanglement swapping operations involving the electron and nuclear spins. We furthermore propose schemes to achieve high-fidelity readout of the spin states at room temperature using the spin-optomechanics interface. Our work shows that long-distance quantum networks made of solid-state components that operate at room temperature are within reach of current technological capabilities.

4.2 Introduction

The successful implementation of global quantum networks would have many applications such as secure communication [1], blind quantum computing [2], private database queries [3], ultimately leading to a "quantum internet" [4, 5, 6] of networked quantum computers and other quantum devices. This will require photons for establishing long-distance connections, as well as stationary qubits for storing and processing the quantum information. In particular, since quantum information cannot be amplified, quantum repeaters are likely to be required [12, 139, 5]. Most current approaches to such quantum networks require either vacuum equipment and optical trapping or cryogenic cooling [9, 10, 11, 140, 36, 12, 141, 142, 143], which adds significantly to the difficulty of scaling up such architectures. There is notable recent work towards quantum networks with room-temperature atomic ensembles [123, 33, 144, 145, 124], but it is also of interest to investigate solid-state approaches, which might ultimately be the most advantageous in terms of scalability.

Nitrogen-vacancy (NV) centers have millisecond-long electron spin coherence times even at room temperature [35, 104, 105, 106], making them excellent candidates for being stationary qubits in quantum networks [140, 36, 22]. So far, NV-based room-temperature quantum information processors were proposed based on the spin-chain model where the interactions between electron spin qubits are mediated by the nuclear spin chain [101] or based on the strongly interacting fluorine nuclear spins [146]. It is intriguing to ask whether photonic links can be implemented for NV centers at room temperature. Unfortunately, the phononinduced broadening of optical transition poses a serious challenge to using NV centers in generating spin-photon entanglement at room temperature [110]. An alternative approach to overcome this problem could be using quantum optomechanics [147], where the effective spin-photon coupling is mediated by an ultra-low loss mechanical resonator [117, 148] to bypass the direct spin-photon interface. It was shown theoretically that this approach allows the emission of highly indistinguishable photons [149] at room temperature, which suggests that high-fidelity entanglement creation should be possible as well. Further, this interface allows the freedom of choosing the wavelength of emitted photons. Thus, one could have the emission at telecom band, which is ideal for connecting distant NV centers through optical fibers.

Nuclear spins in diamond have even longer coherence time at room temperature than the electron spins, exceeding a second [35]. Therefore, these nuclear spins can be used as quantum memory to store the entanglement both at ambient conditions [150], similar to what is being done at cryogenic temperatures [37]. Electron and nuclear spin qubits can be coupled via hyperfine interactions [100, 101, 35].

Based on the above line of thought, we here propose a room-temperature quantum repeater architecture based on NV centers and optomechanics. In our proposal the entanglement between two distant NV electron spins is established via photons following the Barrett-Kok scheme [42, 38, 22]. We apply the photon number decomposition method [43] to quantify and analyze the entanglement generation efficiency and fidelity. Mapping of the electron spin entanglement onto nuclear spins is achieved via performing CNOT gates and electron spin readout through the spin-optomechanics interface. Finally, entanglement swapping is done using the same gate operations assisted by the readout of electron spin and nuclear spin states. The quasi-deterministic gate operations allow us to distribute the entanglement in the nesting-level free manner which outperforms other conventional nested repeater protocols.

This paper is organized as follows. In Sec. 2, we introduce the quantum repeater architecture, including the spin-optomechanics interface, as well as entanglement generation, entanglement storage in nuclear spins, and entanglement swapping. The NV electron spin readout at room temperature is discussed in Sec. 3. Sec. 4 discusses the repeater rate and fidelity. Sec. 5 gives more details in implementation. We conclude and provide an outlook in Sec. 6.



Figure 4.1: (a) Room-temperature quantum repeater architecture. Here, we just show four nodes and three links to demonstrate the basic logic of the quantum repeater protocol, which proceeds in four steps. Step 1 is to generate the entanglement between two remote NV electron spins using the spin-optomechanics interface. Step 2 is memory mapping, which stores the entanglement between two electron spins into the entanglement between two nuclear spins. Step 3 is the same as step 1 for generating the entanglement between two remote NV electron spins. Step 4 is to perform the entanglement swapping that establishes the entanglement only between the first and the last nuclear spins. (b) Schematic of the spin-optomechanics interface with membrane-in-the-middle design. The optomechanical system consists of a SiN membrane oscillator placed inside the high-finesse cavity. A magnetic tip is attached to this membrane. An NV center in bulk diamond is placed near the tip, such that the oscillator is coupled to the dressed ground states of the NV center. A single telecom photon is produced via the mechanically mediated interaction between the control laser and the dressed NV center, while the cooling laser is on to keep the membrane oscillator near its ground state.

4.3 Quantum repeater architecture

The diagram in Fig. 4.1(a) illustrates the basic steps and components for building a roomtemperature quantum repeater architecture based on spin-optomechanics systems. A typical quantum repeater features two basic ingredients: the entanglement generation between two remote memories, and the entanglement swapping between two local memories to propagate it further [4, 5]. Here, our physical systems also have these two components, and they can operate at room temperature. One crucial component of our proposal is the spinoptomechanics interface which was first proposed by R. Ghobadi et.al. [149]. Moreover, our proposal features two kinds of qubits: the NV electron spins serve as communication qubits, and the nuclear spins serve as memory qubits for storing the entanglement because they have long coherence time even at room temperature [35, 150]. At cryogenic temperature, experimental realizations of such diamond-based nuclear-spin memories have already been demonstrated [37, 150].

This section is dedicated to the basic structure and components of our proposed architecture. We start with the introduction to the spin-optomechanics interface [149], and then quantify the efficiency and fidelity of entanglement generation between two remote nodes based on the recently developed photon number decomposition method [43]. Then we discuss entanglement storage and swapping under ambient conditions. The application of the spin-optomechanics interface for the electron spin state readout at room temperature, which serves as a crucial ingredient in the proposed architecture, is discussed in the next section.

4.3.1 Spin-optomechanics interface

The schematic of the spin-optomechanics interface is shown in Fig. 4.1(b). There are three main components in the system: the NV electron spin, the mechanical oscillator (SiN membrane), and the high-finesse optical cavity. The NV electron spin is coupled to the mechanical oscillator via a magnetic tip that is attached to the oscillator, which requires the magnetic

field gradient to produce the strong spin-mechanics coupling rate λ [149]. The red-detuned control laser is used to induce the optomechanical coupling rate g. The NV electron spin must be tuned to be resonant with the red-detuned control laser so that a single spin excitation would be converted to a single photon emitted at the cavity frequency via the mechanical oscillator. However, when the control laser is red-detuned from the cavity, it also starts to cool the mechanical oscillator via the phonon sideband. This converts phonons to single photons at the cavity frequency as well, which causes a thermal noise that degrades the quality of the single photon from the NV electron spin. In order to reduce this noise, we detune the control laser far from the phonon sideband ω_m . Since the control laser is detuned far from the phonon sideband, it is ineffective at cooling the mechanical oscillator. Hence, we introduce a different laser on resonance with the mechanical oscillator to efficiently cool it [149].

The triplet NV electron spin state $\{|0\rangle, |-1\rangle, |+1\rangle\}$ is under the dressing of a microwave source [149], which form a three-level dressed spin states $\{|0\rangle, |D\rangle, |B\rangle\}$ that are noiseprotected from the nuclear-spin bath [151]. Only the bright state $|B\rangle = (|+1\rangle + |-1\rangle)/\sqrt{2}$ and the dark state $|D\rangle = (|+1\rangle - |-1\rangle)/\sqrt{2}$ couple to the mechanical oscillator with the rate λ . The states $|+1\rangle$ and $|-1\rangle$ are two of the triplet ground states of the NV center. The transition frequency between $|B\rangle$ and $|D\rangle$ is ω_q , which is tuned to be the same as the control laser via controlling the Rabi frequency of the microwave dressing source. The detuning δ between the red-detuned control laser ω_q and the phonon sideband ω_m is $\delta = \omega_m - \omega_q$. The level diagram of this spin-optomechanics system is shown in Fig. 4.2(a).

Then, the system Hamiltonian is given by $(\hbar = 1)$

$$\hat{H} = \omega_{\rm q}(\hat{\sigma}_+ \hat{\sigma}_- + \hat{a}^{\dagger} \hat{a}) + \omega_{\rm m}(\hat{b}^{\dagger} \hat{b} + \hat{c}^{\dagger} \hat{c}) + \hat{H}_{\rm I}, \qquad (4.1)$$

where $\hat{\sigma}_{-} = |D\rangle \langle B|$ is the lowering operator for the dressed NV spin states, and \hat{a} and \hat{c} are the control cavity mode and cooling cavity mode respectively, and \hat{b} is the oscillator mode.


Figure 4.2: (a) The level diagram illustrates the coupling between the excited dressed NV electron spin state and the mechanical phonon with the rate λ , and the coupling between the mechanical phonon and the cavity photon with the rate g. Coupled states are denoted as $|\text{spin}, \text{mechanics}, \text{cavity}\rangle$. A single photon is generated via the indirect coupling between the spin and cavity mode through the oscillator and is then released by the cavity at the rate κ , leaving the whole system in $|D00\rangle$. The dressed spin state has a dephasing rate γ_s^* , and the mechanical oscillator is dissipatively driven by the environment with the rate $\gamma_m n_{\text{th}}$. (b) The schematic of the four-level spin-cavity system after the adiabatic elimination of oscillator mode. The effective coupling strength between the cavity and the NV spin is $\lambda g/\delta$. This effective spin-cavity system has five effective decoherence rates: the pure spin dephasing rate γ_s^* , the mechanically-induced thermal decay and excitation rates γ_1 and γ_2 for the spin, and the effective decay rate κ_1 and mechanically-induced thermal excitation rate κ_2 for the cavity mode.

 $\hat{H}_{\rm I}$ stands for the interaction term, and it takes the following form:

$$\hat{H}_{\rm I} = (\lambda \hat{\sigma}_- + g\hat{a} + g_{\rm c}\hat{c})(\hat{b}^{\dagger} + \hat{b}) + \text{H.c.}, \qquad (4.2)$$

where λ is the spin-mechanics coupling strength, g is the control optomechanical coupling rate, and g_c is the cooling optomechanical coupling rate.

Under the condition that $\delta \gg \{\lambda, g\}$ and $\kappa_c < \omega_m$ (the sideband-resolved regime), and the cooling mode significantly reduces the thermal noise from the mechanical oscillator, making it near the ground state [149], it is valid to adiabatically eliminate the δ -detuned mechanical phonon mode to achieve the effective coupling between the dressed spin state and a cavity photon [149, 152]. The cooling mode can also be ignored as it cools the mechanical oscillator, converting phonons to photons that are emitted at a different frequency than the desired single photon from the NV spin. The effective coupling rate is $\lambda g/\delta$ as indicated by the blue arrow in Fig. 4.2(b). After adiabatic elimination and rotating-wave approximation $(\delta \ll \omega_q, \omega_m)$, the simplified Hamiltonian is given by [149]

$$\hat{H}_{\text{eff}} = \frac{g^2}{\delta} \hat{a}^{\dagger} \hat{a} + \frac{\lambda^2}{\delta} \hat{\sigma}_+ \hat{\sigma}_- + \Omega(\hat{a}^{\dagger} \hat{\sigma}_- + \hat{a} \hat{\sigma}_+), \qquad (4.3)$$

where $\Omega = \lambda g/\delta$ is the effective coupling strength between the cavity photon and NV bright state. Although this system is a three-level system containing two coupled ground states of NV spin $\{|D\rangle, |B\rangle\}$ and the cavity mode, it is convenient to include the uncoupled ground state $|0\rangle$ in the system for the later analysis. From now, we call this system a four-level system. Then, the corresponding effective master equation is given in Sec. 4.8.2:

$$\dot{\hat{\rho}} = -i[\hat{H}_{\text{eff}},\hat{\rho}] + \kappa_1 \mathcal{D}[\hat{a}]\hat{\rho} + \gamma_s^* \mathcal{D}[\hat{\sigma}_z]\hat{\rho} + \gamma_1 \mathcal{D}[\hat{\sigma}_-]\hat{\rho} + \gamma_2 \mathcal{D}[\hat{\sigma}_+]\hat{\rho} + \kappa_2 \mathcal{D}[\hat{a}^{\dagger}]\hat{\rho},$$
(4.4)

where $\kappa_1 = \kappa + g^2 \gamma_m (n_{\rm th} + 1)/\delta^2$ is the effective cavity decay rate with original cavity decay rate κ , and $\kappa_2 = g^2 n_{\rm th} \gamma_m / \delta^2$ is the mechanically-induced thermal excitation rate for the cavity photon with the oscillator damping rate γ_m , and the average phonon number $n_{\rm th}$ determined by the environment temperature is given by $n_{\rm th} = 1/(e^{\hbar\omega_m/k_BT} - 1)$, and γ_s^* is the pure spin dephasing rate, and $\gamma_1 = \lambda^2 \gamma_m (n_{\rm th} + 1)/\delta^2$, $\gamma_2 = \lambda^2 n_{\rm th} \gamma_m / \delta^2$ are the mechanicallyinduced thermal decay and excitation rates for the NV spin state, respectively. Here $\mathcal{D}[\hat{A}]\hat{\rho} =$ $\hat{A}\hat{\rho}\hat{A}^{\dagger} - \hat{A}^{\dagger}\hat{A}\hat{\rho}/2 - \hat{\rho}\hat{A}^{\dagger}\hat{A}/2$. The inherent NV spin flip-flop rate is ignored because it is much smaller than the pure spin dephasing rate γ_s^* even at ambient temperature [104].

4.3.2 Entanglement generation

Step 1 in Fig. 4.1 is to generate entanglement between two remote NV electron spins at room temperature. This can be achieved using the protocol described in Sec. 4.3.1. Photons with high indistinguishability, brightness and purity can be produced using this spinoptomechanics interface at room temperature [149]. Each of the two spin-optomechanical interfaces can be modeled as described in the previous section.

If the initial state of the NV center is prepared as $(|B\rangle + |0\rangle)/\sqrt{2}$, a single photon would be released from $|B0\rangle$ at the cavity frequency via the effective coupling between $|B0\rangle$ and $|D1\rangle$. Therefore, a spin-photon entangled state $(|D1\rangle + |00\rangle)/\sqrt{2}$ is created. Then, after interfering the photonic modes from each interface at a beam splitter, detection of a single photon projects the two spins into an entangled state. Here, we propose to use the spin-time bin protocol (the Barrett-Kok scheme) to generate the entanglement between two distant nodes, which is much more robust against some important errors such as photon loss, detector loss and cavity parameters mismatch compared the single-photon detection scheme [38, 42]. In this protocol, two rounds of single-photon detection are required. After the first round, we flip the spin states $|D\rangle$, $|0\rangle$ of both systems and re-excite $|D\rangle$ to $|B\rangle$. The detection of two consecutive single photons (one at each round), will then project the joint state of the quantum systems onto a Bell state. Depending on which detectors click in these two rounds, we obtain two Bell states $|\psi_{\pm}\rangle = (|D0\rangle \pm |0D\rangle)/\sqrt{2}$ with a 50% total probability.

Due to the existing mechanically-induced cavity emission at room temperature, the initial state of the cavity is not perfectly the vacuum state. A more precise initial state can be obtained by solving the steady state of cavity mode with only the optomechanical coupling g turned on. Then, the initial state of the cavity is given in Sec. 4.8.4:

$$\rho_{ic} = \frac{\kappa_1 - 2\kappa_2}{\kappa_1 - \kappa_2} \left| 0 \right\rangle \left\langle 0 \right| + \frac{\kappa_2}{\kappa_1 - \kappa_2} \left| 1 \right\rangle \left\langle 1 \right|, \qquad (4.5)$$

where $\kappa_1 \gg \kappa_2$. The mechanically-induced initial thermal occupation $\kappa_2/(\kappa_1 - \kappa_2)$ is quite small, which is estimated to be around 0.1% using the parameters in Fig. 4.3. Since this thermal occupation is so small, and it does not affect the quantum system dynamics significantly, we can treat its contributions classically by modeling it as dark counts to simplify the calculations [43]. This dark count rate is given by $\mathcal{D}_{\rm th} = \kappa_1 \kappa_2/(\kappa_1 - \kappa_2)$. Therefore, we start with the initial state of the system: $\hat{\rho}(t_0) = |\psi(t_0)\rangle \langle \psi(t_0)|$ where $|\psi(t_0)\rangle = (1/2)(|0,0\rangle + |B,0\rangle)^{\otimes 2}$.



Figure 4.3: Entanglement generation fidelity $F_{\rm gen}$ and efficiency $\eta_{\rm gen}/\eta_t^2$ for a single link as a function of protocol time $t_{\rm f}$. The mechanically-induced initial thermal noise in the cavity is modeled as dynamical dark counts as described in the text, while the detector dark count rate is set to 10 Hz [153]. The detection time window for each time bin $T_{\rm d}$ is set to be equal to half the total detection time window: $t_{\rm f} = 2T_{\rm d}$. Due to the loss in the channel, it is difficult to see the efficiency curve so it is divided by the factor $\eta_t^2 = \exp(-L_0/L_{\rm att})$, where $L_0 = 100$ km is the length of the link, and $L_{\rm att} = 22$ km is fiber attenuation distance of telecom photons. The peak value of the fidelity curve $F_{\rm gen}$ is around 97%. All parameters are chosen to be the same for both spin-optomechanics systems and similar to those in Ref. [149], where the parameters are optimized for achieving high indistinguishability and single-photon purity: $\lambda = g = 2\pi \times 100$ kHz, $\delta = 2\pi \times 1$ MHz, $Q_m = 3 \times 10^9$, $\kappa_1 = 2\Omega = 2\pi \times 20$ kHz, $\gamma_s^* = 0.01\kappa_1$ [104], and $\gamma_1 = \gamma_2 = 1.0 \times 10^{-3}\kappa_1$.

Under this approximation, the mechanically-induced thermal excitation rate in the cavity mode can be set to 0 in Eq. (4.4), i.e., $\kappa_2 = 0$. In this way, the total number of quantum states to simulate is reduced.

Now, in order to quantify the entanglement fidelity and efficiency, we follow the photon number decomposition method developed in [43] to compute the time dynamics. The basic idea of this method is to decompose the master equation dynamics into evolution conditioned on single photon detection, which can be done by rewriting the master equation of the whole system (in this case two distant spin-optomechanical systems) as follows:

$$\dot{\rho} = \mathcal{L}_0 \hat{\rho}(t) + \sum_{i=1}^2 \mathcal{S}_i \hat{\rho}(t), \qquad (4.6)$$

where $\mathcal{L}_0 = \mathcal{L} - \sum_{i=1}^2 S_i$ with \mathcal{L} being the Liouville superoperator that contains all the dynamics of this composite system, and $S_i\hat{\rho} = \hat{d}_i\hat{\rho}\hat{d}_i^{\dagger}$ is the collapse superoperator of the source field \hat{d}_i at the *i*th single-photon detector [43]. As can be seen, at a given detection time window t_f if there is no photon detected, then the system evolves only subject to \mathcal{L}_0 , but if there is a photon detected during this time window, then we apply the collapse superoperator to the system. Moreover, as the final state of the system depends on the detected photon count, we would obtain a set of different states, which we call conditional states.

In the Barrett-Kok scheme, there are four possible detected photon counts: $\{\mathbf{n}_l, \mathbf{n}_e\} = \{(1,0), (1,0)\}, \{(1,0), (0,1)\}, \{(0,1), (1,0)\}, \text{ and } \{(0,1), (0,1)\}\)$ where \mathbf{n}_l and \mathbf{n}_e stand for the photon count in the early and late detection time window, and each can take two possible outcomes (1,0), (0,1) which correspond to the click in the left detector and the right detector as shown in Fig. 4.1. Thus, the entanglement generation efficiency and the entanglement generation fidelity can be defined in the following way:

$$\eta_{\text{gen}} = \text{Tr}[\hat{\rho}(t_f)] = \sum_{\mathbf{n}} \text{Tr}[\hat{\rho}_{\mathbf{n}}(t_f)]$$

$$F_{\text{gen}} = \frac{1}{4} \sum_{\mathbf{n}} \frac{\langle \psi_{\pm} | \ \hat{\rho}_{\mathbf{n}}(t_f) | \psi_{\pm} \rangle}{\text{Tr}[\hat{\rho}_{\mathbf{n}}(t_f)]},$$
(4.7)

where **n** stands for the detected photon count as mentioned above. We use $|\psi_+\rangle$, when $\mathbf{n} = \{(1,0), (1,0)\}, \{(0,1), (0,1)\}$, otherwise we use $|\psi_-\rangle$. Further, due to dark counts (both from detectors and the initial thermal occupation as mentioned above), zero or single-photon conditioned states would give spurious photon counts. This imperfection is also taken into account when estimating the entanglement generation fidelity and efficiency, which is discussed in more detail in [43].

Fig. 4.3 shows the entanglement generation fidelity F_{gen} and efficiency curves $\eta_{\text{gen}}/\eta_t^2$ for the effective spin-cavity system described by Eq. (4.4) over the total detection time window t_{f} for a link of 100 km. T_{d} is the detection time window for each time bin, which is set to be half the total detection time window $t_{\rm f}$. The loss in the channel degrades the entanglement efficiency in proportion to the square of the transmission rate, i.e., $\eta_t^2 = \exp(-L_0/L_{\rm att})$, which makes the efficiency curve difficult to see, so it is divided by this factor. We assume a dark count rate of 10 Hz, which is predicted to be achievable for photons in the telecom band using up-conversion single photon detectors (USPDs) in the free-running regime [153] (which do not require cryogenic cooling). After taking the loss in the channel into account, this detector dark count rate is comparable to the rate $\mathcal{D}_{\rm th} \sim 100$ Hz. This type of detector is also predicted to have low afterpulsing probability [153], making afterpulsing negligible in estimating entanglement fidelity and efficiency. For the detection efficiency, we consider 45% [153], which is later used in the readout fidelity estimates and the repeater rates calculations.

Fig. 4.3 shows that the efficiency degrades gradually after it reaches the maximum due to the thermal-induced flip-flop effect between the bright and dark states. Under the influence of the flip-flop effect, both systems continue to emit photons, resulting in the probability of detecting only two photons to vanish when the detection time $t_{\rm f}$ goes to infinity. Likewise, the fidelity decreases after it reaches the maximum, and it starts with fairly low values due to the small signal-to-noise ratio in the beginning. If we choose to terminate the measurement at a proper time as $\kappa_1 t_{\rm f} \sim 10$, then the fidelity is approaching 97% at room temperature.

One can obtain approximate analytical expressions for the entanglement fidelity and efficiency by following the methods developed in [154, 155]. In the incoherent regime ($2\Omega \leq \kappa + 2\gamma_s^* + 2\Gamma_{\rm th}$), we can model this four-level system as a three-level system with the effective emission rate by adiabatically eliminating the spin-photon coherence [149]:

$$R = \frac{4\Omega^2}{(\kappa + 2\gamma_s^* + 2\Gamma_{\rm th})},\tag{4.8}$$

where $\Gamma_{\rm th} = \lambda g n_{\rm th} \gamma_m / \delta^2$ is the thermal-induced noise. By applying the photon number decomposition method to this spin-optomechanics system [43], we get the entanglement

generation efficiency in the Barrett-Kok scheme:

$$\eta_{\rm BK} = \frac{\eta_t^2}{2} (1 - e^{\frac{-t_f}{2}R})^2, \tag{4.9}$$

where R is the effective emission rate for each system, and η_t is the transmission rate in the channel. This is proportional to the product of the two total emission intensities from the two emitters. However, for the room-temperature case where the cavity starts with a small thermal occupation, a more precise expression of the efficiency is given by taking the dark counts into consideration as discussed in [43].

The entanglement generation fidelity $F_{\rm BK}$ is then given by [43]

$$F_{\rm BK} = \frac{1}{2} \left(1 + \frac{1}{2\eta_{\rm BK}} |\tilde{C}(t_{\rm f})|^2 \right), \tag{4.10}$$

where $\tilde{C}(t_{\rm f})$ takes the following form

$$\tilde{C}(t_{\rm f}) = \frac{\eta_t R}{R_{\rm tot}} \left(1 - e^{-\frac{1}{2}t_{\rm f}R_{\rm tot}} \right), \qquad (4.11)$$

where $R_{\text{tot}} = R + 2\gamma_s^*$ is the spectral width of the emitted photons for both systems. This fidelity equation is the upper bound for the cryogenic temperature case when there is only optical dephasing. For the room-temperature case, one needs to take into account the mechanically-induced thermal contribution in the cavity and the mechanically-induced spin flip-flop effect, which makes the precise analytical fidelity expression very difficult to obtain.

4.3.3 Entanglement mapping

After the successful entanglement generation, we need to store the entanglement between two remote NV electron spins in nuclear spins via performing memory swapping between an electron spin and a nuclear spin at both ends of the link as indicated by two yellow arrows in Fig. 4.1. This operation is achieved through performing a $C_n NOT_e$ gate between the electron and nuclear spins plus the measurement of the state of the electron spin.

Assuming that $|\psi^+\rangle$ is obtained in step 1, since quantum systems are in the dressed basis $\{|B\rangle, |D\rangle, |0\rangle\}$, we need to bring them back to the original basis $\{|+1\rangle, |-1\rangle, |0\rangle\}$ by turning off the microwave source adiabatically. Then, $|D\rangle$ returns to $|-1\rangle$ and $|0\rangle$ remains the same. Here, we denote $\{|-1\rangle, |0\rangle\}$ as $\{|\uparrow_e\rangle, |\downarrow_e\rangle\}$ for the electron spin. Then, we prepare the nuclear spin in the superposition of the spin-up and spin-down states by applying a $\pi/2$ RF pulse to the nuclear spin that is initially polarized to the spin-down state via the combination of optical, microwave, and RF fields as discussed in [156]. There are several options for nuclear spins in diamond such as ¹⁴N [157] and ¹⁵N [158]. Here, we use ¹³C as the nuclear spin in an isotopically purified sample, which has the nuclear spin I = 1/2 [104, 35, 159]. The state is then given by

$$\frac{1}{\sqrt{2}}(|\Downarrow_n\rangle + |\Uparrow_n\rangle) \otimes \frac{1}{\sqrt{2}}(|\downarrow_e\uparrow_e\rangle + |\uparrow_e\downarrow_e\rangle) \otimes \frac{1}{\sqrt{2}}(|\Downarrow_n\rangle + |\Uparrow_n\rangle), \tag{4.12}$$

where $|\Downarrow_n\rangle$ and $|\Uparrow_n\rangle$ correspond to $m_I = -1/2$ and $m_I = +1/2$ individually. Now, a $C_n \text{NOT}_e$ gate can be performed between the electron and nuclear spins using the hyperfine interaction between them. Fig. 4.4 shows the hyperfine structure for performing two-qubit gates between the electron spin and the nuclear spin and one-qubit gates on each of them individually.

The electron-nuclear spin Hamiltonian is given by

$$H_{e,n} = \Delta_0 S_z^2 + \mu_e B S_z + \mu_n B I_z + A S_z I_z, \tag{4.13}$$

with the zero-field splitting $\Delta_0=2.87$ GHz, the electronic spin gyromagnetic ratio $\mu_e = -2.8$ MHz/Gauss, the nuclear spin gyromagnetic ratio $\mu_n = 1.07$ kHz/Gauss, the external magnetic field *B* is applied along the symmetry axis of the NV, and the hyperfine coupling *A* ranges from tens of kHz to 100 MHz for a ¹³C nuclear spin [102, 103, 35]. The C_nNOT_e gate can be implemented by a Ramsey sequence on the electron spin at room temperature, where the free precession time is chosen to be $t = \pi/A$ with the magnetic field of several hundred



Figure 4.4: The NV center with a ¹³C can be modeled as a four-level system. Nuclear spin sublevels $|\uparrow_n\rangle$ and $|\downarrow_n\rangle$ are addressed by RF radiation with Rabi frequency Ω_{RF} . The electronic spin sublevels are driven via a microwave field Ω_{MW} but when the electron spin is $|\downarrow_e\rangle$, the microwave field has relative detuning given by hyperfine interaction A.

Gauss [35, 157, 156]. The efficient realization of the CNOT gate with fidelity of 99.2% at ambient conditions has been demonstrated using composite pulses and an optimized control method [160] as well as the dynamical decoupling technique [161, 162, 111]. The dynamical decoupling technique is also important in the entanglement generation where the electron spin can be decoupled from the nuclear spin bath to have millisecond-long coherence time at room temperature [105, 111]. However, in our entanglement generation step the NV electron spin is in dressed states under a far-detuned microwave source, which itself is already robust against the nuclear-bath-induced noise [151, 112].

Two $C_n NOT_e$ gates on both ends of the link lead to a four-qubit entangled state. So the projective measurement in the Z basis on the state of the electron spin is required to complete the entanglement storage, which projects this four-qubit entangled state to an entangled state of the nuclear spins. Typically, fluorescence detection can be used to determine the state of the electron spin after the projective measurement at a low temperature of around 4K with good fidelity [163], which enables the cryogenic-temperature entanglement storage in nuclear spins [37, 159]. Unfortunately, at room temperature, the intensity of electronic spin-up and spin-down states only differ by roughly a factor of 2 due to the fact that the phonon-induced broadening greatly diminishes the resolution of these two Zeeman states [157]. Thus, the past decade has seen a great deal of experimental efforts put into solving this problem [156, 108, 164, 109]. In Sec. 4.4, we propose two electron spin readout schemes based on the spin-optomechanics system.

4.3.4 Entanglement swapping

After mapping the entanglement to the nuclear spins, the electron spins are free and we can use them again to generate entanglement between the electron spins i and i + 1. This is done in step 3 as illustrated in Fig. 4.1. Then, the entanglement swapping is achieved as follows: a $C_n NOT_e$ gate at each endpoint of this link is applied, giving us an entangled state of these six spins. Via performing measurements on the electron spin in the Z basis, one ends up obtaining an entangled state of four nuclear spins. Depending on the measurement outcomes, one gets different entangled states. Here, we assume that we get the following four-qubit entangled state:

$$\frac{1}{\sqrt{2}}(|\Uparrow_n\Uparrow_n\Uparrow_n\Uparrow_n\rangle + |\Downarrow_n\Downarrow_n\Downarrow_n\Downarrow_n\rangle).$$
(4.14)

In order to complete the entanglement swapping, i.e. to only entangle nuclear spins i - 1and i + 2, one still needs to disentangle two nuclear spins i and i + 1 in between. This can be done by measuring them in the X basis but unfortunately, one cannot optically read out the nuclear spin directly. However, it turns out that the nearby electron spins can be used to indirectly read out the nuclear spin state [157, 165]. The basic idea is as follows: first, a Hadamard gate is performed on the nuclear spins i and i + 1 individually by applying a $\pi/2$ RF pulse to make $|\Uparrow_n\rangle \rightarrow 1/\sqrt{2}(|\Downarrow_n\rangle + |\Uparrow_n\rangle)$ and $|\Downarrow_n\rangle \rightarrow 1/\sqrt{2}(|\Downarrow_n\rangle - |\Uparrow_n\rangle)$.

Second, the electron spin nearby is initialized to $|\uparrow_e\rangle$, and we again perform a $C_n NOT_e$ gate, mapping the nuclear spin state to the electron spin state. Therefore, the readout of the nuclear spin could be achieved by performing the measurements in the Z basis on the electron spin, followed by the readout of the measurement outcome which is discussed in detail in Sec. 4.4. The post-measurement state is given by

$$\begin{cases} \frac{1}{\sqrt{2}} (|\Uparrow_n \Uparrow_n \rangle - |\Downarrow_n \Downarrow_n \rangle) & \uparrow_e \downarrow_e \text{ or } \downarrow_e \uparrow_e \\ \frac{1}{\sqrt{2}} (|\Uparrow_n \Uparrow_n \rangle + |\Downarrow_n \Downarrow_n \rangle) & \uparrow_e \uparrow_e \text{ or } \downarrow_e \downarrow_e, \end{cases}$$
(4.15)

where the final state depends on the outcomes of the electron spins readout. Therefore, nuclear spins i-1 and i+2 are entangled as indicated by the long red wavy line in Fig. 4.1(b). As we can see, the entanglement swapping process is in fact equivalent to the entanglement mapping process plus the readout of two nuclear spins.

4.4 The electron spin readout

Applying previously proposed readout methods to our system is quite challenging since they require extra techniques and apparatus such as using nuclear spin ancillae, spin-tocharge conversion [108] and photoelectrical imaging [109] to achieve a high-fidelity readout of electron spin at room temperature. Hence, we propose to read out the electron spin state at room temperature using the spin-optomechanics interface. In this section, two intensity-based readout schemes are proposed to distinguish the electron spin state at room temperature.

4.4.1 Readout scheme using periodic driving pulses

In the readout scenario, the aim is to distinguish the states $|0\rangle$ or $|D\rangle$. The intuitive idea is to perform a π pulse on the transition between $|B\rangle$ and $|D\rangle$, which will excite the state $|D\rangle$ to $|B\rangle$ while keeping the state $|0\rangle$ unchanged. Then the state $|B\rangle$ will decay back to $|D\rangle$ according to the process described in Fig. 4.2(a) and will emit a single photon. By measuring a single photon, we can determine that the state is initially in the state $|D\rangle$ or $|0\rangle$. However, measuring a single photon may not be the optimal way to distinguish these two spin states due to the photon loss in the channel and the dark counts in detectors.



Figure 4.5: (a) Periodic driving pulse scheme. The driving pulse is applied once the NV spin occupation is nearly 0, meaning that the spin state decays from $|B\rangle$ to $|D\rangle$. (b) Continuous driving pulse scheme. The NV spin and the cavity mode will reach a non-zero equilibrium state. The red solid curve and the purple dot-dashed curve represent cavity photon number and NV bright state ($|B\rangle$) population respectively. The gray shaded area corresponds to the time window for the readout operation, and the hatched area in (a) corresponds to the detection time window in one pulse cycle. The parameters used in (a) and (b) are the same as the ones in the entanglement generation section.

Therefore, we provide two extended readout schemes, the periodic driving scheme, and the continuous driving scheme to achieve the high-fidelity readout of NV electron spin states.

In the periodic driving scheme, periodic pulses are used to drive a cycling transition between the states $|B\rangle$ and $|D\rangle$. Assuming a perfect MW π pulse is applied to the state $|D\rangle$, it is excited to the state $|B\rangle$ and then returns to the state $|D\rangle$ with a single photon emitted. Then we repeat this process. In the adiabatic elimination regime, the total Hamiltonian is given by

$$\hat{H}_1 = \hat{H}_{\text{eff}} + g_d [\hat{\sigma}_+ \exp(-i\omega_q t) + \hat{\sigma}_- \exp(i\omega_q t)] f(t), \qquad (4.16)$$

where \hat{H}_{eff} is given by Eq. (4.3), and g_d is the coupling strength for the driving pulse, and f(t)is a periodic delta function with the form $\delta(t - nT_p)$ and the period T_p is the inverse of the decay rate R. The simulation results are shown in Fig. 4.5 (a). The solid red and dot-dashed purple curves are the cavity photon population and the NV spin population respectively when the NV spin is initially in the state $|D\rangle$, and the dashed red and purple lines are the cases where the initial NV spin state is $|0\rangle$. We can define the brightness (intensity) as the average number of emitted photons: $\beta_i = \kappa \int_{t_0}^{t_0+T} dt \langle \hat{a}^{\dagger}(t) \hat{a}(t) \rangle_i$ with i = D or 0 representing the initial NV spin states in $|D\rangle$ and $|0\rangle$ respectively, where $\langle \hat{a}^{\dagger}(t)\hat{a}(t)\rangle_i$ is the corresponding average cavity photon number. A single photon is emitted within a period shown as the gray shade in Fig. 4.5(a).

To estimate the readout fidelity, we consider the measurement being repeated N times and each measurement is independent. Thus, the number of photons detected within the total measurement time NT_p can be described by a binomial distribution, and the probability of detecting n photons is $P_{N,n,p} = \binom{n}{N}p^n(1-p)^{N-n}$, where $p_i = \eta\beta_i$ is the probability of detecting a single photon within the detection time window, and η is the total efficiency with which an emitted photon can be detected. One can plot $P_{N,n,p}$ corresponding to β_D and β_0 and find the intersection point (details can be found in Sec. 4.8.5). The intersection point is the threshold that decides the measurement result: if the number of photons detected is more than the threshold, the photons are most likely coming from the emitter and therefore the NV spin state is decided to be $|D\rangle$; if the number of photons are highly possible from the threshold, the NV state is assumed to be $|0\rangle$ because these photons are highly possible from the thermal noise.



Figure 4.6: The relation between the readout infidelity (1 - F) and the total readout time with the parameters used in Fig. 4.5. For the periodic driving scheme (plotted as purple squares), $\beta_D = 0.929$, $\beta_0 = 0.034$, and the driving period is T = 0.02 ms; for the continuous driving scheme (plotted as red triangles), $\langle a^{\dagger}(t)a(t)\rangle_D = 0.202$ and $\langle a^{\dagger}(t)a(t)\rangle_0 = 0.014$. The solid, dashed, and dash-dotted lines correspond to the total detection efficiency $\eta = 0.05$, 0.1, and 0.5, respectively [153, 166, 167]. The time axis is the total readout time NT_p , where N is the total pulse number in the periodic driving scheme. The discontinuity of the first derivative shown on the curves is due to the change in the threshold (because the threshold is always an integer).

4.4.2 Readout scheme using continuous driving pulses

The continuous driving scheme employs a continuous-wave (CW) laser to drive the bright and the dark spin states. Similarly, the Hamiltonian in this case is given by

$$\hat{H}_1 = \hat{H}_{\text{eff}} + g_d(\hat{\sigma}_+ \exp(-i\omega_q t) + \hat{\sigma}_- \exp(i\omega_q t)).$$
(4.17)

Under this Hamiltonian, the cavity mode will eventually reach a non-zero equilibrium state as shown in Fig. 4.5(b). To give the calculation of the readout fidelity, we assume that the detection is a Poisson process, where the probability of detecting n photons is given by $P(n, \lambda) = \lambda^n e^{-\lambda}/n!$, where λ is the average photon counts within total detection time T_0 , given by $\lambda_i = \eta \kappa \int_{t_0}^{t_0+T_0} dt \langle \hat{a}^{\dagger}(t) \hat{a}(t) \rangle_i$ with i = D or 0 corresponding to the initial states $|D\rangle$ or $|0\rangle$ respectively. Similarly to the treatment in the periodic driving scheme, the intersection point of these two plots of the probability distribution functions gives the threshold and the detailed discussion can be found in Sec. 4.8.5.

Instead of showing the readout fidelity, here we show the readout infidelity (1-F) of these two schemes in Fig. 4.6 for a clearer demonstration of how well our readout schemes work. The dark count rate is taken to be 10 Hz in detectors [153], which is negligible because the average number of dark counts within ms time period is on the order of 10^{-3} , much smaller than the average number of emitted photons during the whole readout process. Also, the afterpulsing probability can be efficiently suppressed to be lower than 1% [153], which makes it negligible as well. Comparing these two schemes, the continuous driving scheme requires more time to have the same infidelity due to the lower signal-to-noise ratio in the present parameter regime than the periodic driving scheme. To achieve the high-fidelity readout (> 99%), the readout time is typically in the ms timescale for both of our schemes with detectors that have pretty poor efficiencies. However, a high-fidelity readout can be achieved in a shorter timescale if we use higher-efficiency detectors, which are however challenging to realize for telecom wavelength photons [166, 168] at non-cryogenic temperatures. In comparison to other proposed methods [156, 108, 169, 164, 109, 170], which also demonstrate a high-fidelity readout of the electron spin in NV centers in ms timescale, these two readout schemes appear to predict comparable performance, without having to add extra elements to our setup. Thus, in our proposal for building a room-temperature quantum network, these spin-optomechanics system-based readout schemes serve as more natural and friendly candidates than other room-temperature readout methods.

4.5 Entanglement generation rates and overall fidelities

We use a "two-round" repeater protocol. During the first round, the entanglement is generated between electron spins in every other elementary link and then is mapped to corresponding nuclear spins, which also sets those electron spins free. For the remaining links, the entanglement is generated in the second round, followed by the entanglement swapping that distributes entanglement between the first and last nuclear spins. Although entanglement generation between the electron spins is probabilistic, the failure of such an attempt does not disturb the entanglement stored in the nuclear spins if the dynamical decoupling is being applied during the entanglement generation [171, 162, 111, 172]. This means that the second round of the entanglement generation process can be repeated many times until success while not affecting the stored entanglement. However, this is true only when the decoherence of nuclear spins is negligible, which is discussed in more detail below. Hence, our two-round repeater protocol makes the widely-used nested repeater structure no longer necessary [11, 10, 12].

Considering an even number of links m, each with length L_0 , the total entanglement distribution time is given by

$$\langle T \rangle_L = 2f(m/2)\frac{L}{cmp_0} + T_{\rm mp} + T_{\rm sw},$$
(4.18)

where f(m/2) is the factor of the average number of attempts required to successfully establish entanglement in all m/2 links, and p_0 is the entanglement generation probability, and L is the total distance, and $c = 2 \times 10^8 \text{ ms}^{-1}$ is the speed of light in optical fiber, and $T_{\rm mp}$, $T_{\rm sw}$ are the total entanglement mapping time and the total entanglement swapping time respectively. Both of these times are made up of CNOT gate time plus the measurement time as discussed in Sec. 4.3.3 and Sec. 4.3.4. The numerical results shown in Sec. 4.8.6 show that $f(x) = 0.64 \log_2(x) + 0.83$ is a good approximation, and one can recover the well-known 3/2 factor by setting x = 2. In contrast to the nested repeater approach [11], where the average entanglement distribution time has a linear dependence on the number of links, we here have a logarithmic dependence. Intuitively, the scaling improvement of the two-round protocol comes from the fact that there is no hierarchy of the entanglement swapping process, where higher-level swapping can only start under the condition of the success of the lower level. Therefore, the main thing left for us is to successfully generate the entanglement simultaneously for these links, which is calculated to have logarithmic dependence on m/2. This scheme could significantly enhance the entanglement distribution rate for a quantum network with much more links, e.g., networked quantum computing [173]. Fig. 4.7(a) shows the repeater rates as the function of distance for four different numbers of links and direct transmission. With 45% detection efficiency, our protocol yields 10 Hz with 8 links at 800 km. This rate is comparable to cryogenic schemes, such as the rare-earth ion-based scheme [11] and the microwave cat qubit-based scheme [10], and it outperforms the well-known DLCZ protocol for laser-cooling based systems [12], which gives less than 1 Hz rate at 800 km. However, if the detection efficiency is significantly lower, e.g. 10% [167], multiplexing would be needed with about 15 multiplexed channels to achieve similar rates.

The whole repeater protocol consists of three parts described in Sec. 4.3. However, instead of taking the fidelity of each part into consideration, here we consider the overall fidelity as

$$F_{\text{tot}} = \left(F_{\text{gen}}\right)^m \times \left(F_{\text{mp}}\right)^m \times \left(F_{\text{nro}}\right)^{m-1},\tag{4.19}$$

where F_{gen} is the fidelity of entanglement generation given in Fig. 4.3, which needs to be established over m elementary links. F_{mp} is the fidelity of an entanglement mapping operation as described in Sec. 4.3.3, and F_{nro} is the readout fidelity of the nuclear spin. This overall fidelity equation is only valid in the high-fidelity regime. The fidelity of entanglement swapping includes the fidelity of entanglement mapping plus the readout of two nuclear spins. Therefore, in total we need to generate entanglement for m links and perform m times entanglement mapping to obtain a chain of nuclear spins followed by the readout of m - 1nuclear spins to achieve the final entangled state between the first and the last nuclear spins. The nuclear spin readout can be achieved by mapping its state to the electron spin and applying the readout methods discussed in Sec. 4.4. Fig. 4.7(b) shows the overall fidelities with respect to the total distance for this quantum network with the detection efficiency of 45%. At 800 km, the overall fidelities are still fairly high, except for the case of 4 links where



Figure 4.7: (a) Repeater rates for different numbers of links and direct transmission (dashed line). $p_0 = 0.5p^2\eta_d^2\eta_t^2$ is the success probability of entanglement generation for the Barrett-Kok scheme with the success probability of emitting a single photon p = 0.9, the transmission loss $\eta_t = \exp\{(-L_0/2L_{\text{att}})\}$ where $L_0 = L/m$ is the length of each elementary link, and the detector efficiency $\eta_d = 0.45$ [153]. (b) Fidelity plots with respect to the total distance with a detection efficiency of 45%. The CNOT gate fidelity is taken to be 99.2% [160]. The electron spin readout fidelity is taken to be 99.9% based on Fig. 4.6. At 800 km, the overall fidelity for four links drops below 60%, which is due to detector dark counts.

the overall fidelity drops below 60% due to the comparatively large effect of detector dark counts when the transmission loss for the comparatively long elementary links is taken into account.

For an eight-link repeater with 45% detection efficiency, the rate is far above 10 Hz at the cross-over point (around 450 km) as shown in Fig. 4.7(a), on which time scale it is well within the coherence time of nuclear spins which can be longer than a second [35] so the decoherence is negligible in this case. This is also true for the four-link, six-link, and ten-link cases. Thus, Fig. 4.7(b) is a valid approximation of overall fidelities in this regime. For the repeaters with much lower detection efficiencies, e.g. 10%, the rates are significantly lower so the decoherence of nuclear spins would seriously degrade the final fidelities. In this case, we can use multiplexing to enhance the rates (about 15 multiplexed channels are needed), which will make the decoherence of nuclear spins negligible.

In addition, our eight-link repeater yields a final fidelity of around 74% at the cross-over point (around 450 km) with 45% detection efficiency and the six-link repeater yields around 80% final fidelity at the cross-over point (around 470 km) with 45% detection efficiency. These fidelities are comparable to the DLCZ protocol for laser-cooling-based systems with 75% for eight links [12], and cryogenic schemes such as the rare-earth ion-based scheme with around 80% for eight links and the microwave cat qubit-based approach with around 60% for eight links [10]. The overall entanglement fidelity could be further improved using entanglement purification protocols [27, 174, 159], which would make this quantum network architecture fault-tolerant.

4.6 Implementation

The spin-optomechanics setup proposed in Ref. [149] is mainly composed of a high-Q cavity patterning with a SiN membrane of ultrahigh Qf (quality×frequency) product, where a small magnetic tip is attached. This hybrid device allows a single NV electron spin to be effectively coupled to photons inside the cavity, emitting a single photon with high purity and indistinguishability at room temperature. However, due to the design where the SiN membrane serves as a part of the optical cavity, the cavity finesse is limited to the order of 10⁴. The other key requirement for this system to work well is the low decay rate, $\kappa \sim 10^4$ Hz in the optical cavity. These two key factors constrain the length of the cavity to be around 0.6m [149]. Here, we propose a new design for this spin-optomechanics interface that uses the membrane-in-the-middle geometry to greatly reduce the cavity length. With this membrane-in-the-middle design, one could significantly reduce the cavity length using a high-finesse cavity, since the finesse scales as $F = \pi c/L_c \kappa$, where κ is the cavity damping rate. As previously estimated, the cavity length is around $L_c = 60$ cm with finesse F = 12000. With the new design, it might be possible to reduce this to around $L_c = 0.6$ cm, if a finesse of order 10^6 can be achieved, see e.g. Ref. [175].

The spin-optomechanics interface shown in Fig. 4.1(b) illustrates our envisioned spinoptomechanical transducer. A SiN membrane is placed between the node and the antinode of the cavity modes (of both the cooling mode and the control mode) such that the optomechanical coupling is still linear and not quadratic like many other membrane-in-themiddle experiments [176, 177, 113]. The membrane-in-the-middle design allows us to use a membrane with a thickness much smaller than the light wavelength, which reduces the potential optical losses such as absorption and scattering due to the significantly smaller overlap between the membrane and the optical field [176]. Similar to the previous proposal, a red-detuned control laser is used to drive the cavity for single photon extraction, which is set to be equal to the transition energy between dressed spin states ω_q . The other reddetuned laser with detuning equal to the phonon sideband ω_m is used to cool the oscillator from room temperature, which is also possible to achieve in this proposed device [148, 178].

Moreover, the spin-mechanics coupling is achieved by a magnetic tip that is attached to the SiN membrane at the bottom, and a NV center in bulk diamond is placed nearby as shown in Fig. 4.1(a). The required strong spin-mechanics coupling ($\lambda \sim 10^5$ Hz) can be realized by a magnetic field gradient of 10^7 T/m with a SiN membrane of ~pg effective mass [149]. This SiN membrane also needs to have ultra-low damping rate γ_m , which is discussed in [117, 149]. As the magnetic tip is attached to the SiN membrane, the quality factor of the membrane may be degraded. This could be compensated by further improving the initial quality factor of the membrane without the tip, which is possible to implement as the limit of the quality factor still has been not reached. With the combination of the methods in [117] and [148], one can get quality factors as high as 10^{10} , which gives some room to improve our current Q factor ~ 10^9 .

4.7 Conclusions and outlook

We presented a room-temperature quantum network architecture based on NV centers in diamond and a spin-optomechanical interface. We showed that high-fidelity entanglement between electron spins can be generated between two distant nodes under realistic conditions. Nuclear spins associated with the NV centers can be utilized as quantum memories. We showed that the spin-optomechanical interface also offers the possibility to read out electron spins at room temperature with high fidelity on ms timescales. Furthermore, we proposed an entanglement distribution protocol where the average distribution time shows logarithmic scaling with the number of links as opposed to linear scaling in conventional nested protocols. A membrane-in-the-middle design may allow to reduce the dimensions of the spin-optomechanics interface to the sub-cm range, thus improving its potential for integration and scalability.

We have here focused on room-temperature quantum repeaters as a medium-term goal, but the proposed approach also holds promise for the implementation of distributed quantum computing [179, 173], extending photonic approaches to quantum information processing in diamond [180, 181] beyond cryogenic temperatures. Nuclear spins in diamond offer the possibility to implement quantum error correction codes [159, 182, 183, 30], which, when integrated into our present approach, may enable fault-tolerant quantum communication and quantum computation under ambient conditions.

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4.8 Supplementary material

4.8.1 S1: System Hamiltonian and dissipation

The spin-optomechanics system has the following Hamiltonian $(\hbar = 1)$:

$$\hat{H} = \omega_q \hat{\sigma}_+ \hat{\sigma}_- + \omega_q \hat{a}^{\dagger} \hat{a} + \omega_m \hat{b}^{\dagger} \hat{b} + \lambda (\hat{b}^{\dagger} \hat{\sigma}_- + \hat{b} \hat{\sigma}_+) + g(\hat{b}^{\dagger} \hat{a} + \hat{b} \hat{a}^{\dagger}),$$
(4.20)

where we already performed rotating wave approximations and the cooling laser mode \hat{c} is ignored as it just cools the mechanical oscillator to be close to the ground state, converting phonons to photons that are emitted at a different frequency than the desired single photon from the NV spin. In order to perform the adiabatic elimination, we need to convert the Hamiltonian into the natural picture [184] by entering the rotating frame using the following transformation:

$$\hat{H}_1 = \hat{U}\hat{H}\hat{U}^\dagger - i\hat{U}\hat{U}^\dagger, \qquad (4.21)$$

where $\hat{U} = e^{i\omega_q(\hat{a}^{\dagger}\hat{a} + \hat{\sigma}_+ \hat{\sigma}_- + \hat{b}^{\dagger}\hat{b})t}$. Then, one obtains the Hamiltonian

$$\hat{H}_{1} = \delta \hat{b}^{\dagger} \hat{b} + \lambda (\hat{b}^{\dagger} \hat{\sigma}_{-} + \hat{b} \hat{\sigma}_{+}) + g(\hat{b}^{\dagger} \hat{a} + \hat{b} \hat{a}^{\dagger}), \qquad (4.22)$$

where $\delta = \omega_m - \omega_q$ is the detuning between the control laser (and the dressed NV spin) and the phonon sideband. Taking dissipation into consideration, the master equation is given by

$$\dot{\hat{\rho}} = -i[\hat{H}_1, \hat{\rho}] + \kappa \mathcal{D}[\hat{a}]\hat{\rho} + \gamma_s^* \mathcal{D}[\hat{\sigma}_z]\hat{\rho} + n_{\rm th}\gamma_m \mathcal{D}[\hat{b}^{\dagger}]\hat{\rho} + (n_{\rm th} + 1)\gamma_m \mathcal{D}[\hat{b}]\hat{\rho},$$
(4.23)

where the intrinsic NV spin flip-flop rate is ignored because it is much smaller than the spin dephasing rate γ_s^* in an isotopically purified diamond [104].

Then, the corresponding Heisenberg-Langevin equations are given by

$$\dot{\hat{a}} = -\frac{\kappa}{2}\hat{a} - ig\hat{b} + \sqrt{\kappa}\hat{a}_{\rm in}$$
$$\dot{\hat{b}} = -i\delta\hat{b} - \frac{\gamma_m}{2}\hat{b} - ig\hat{a} - i\lambda\hat{\sigma}_- + \sqrt{\gamma_m}\hat{F}_b(t)$$
$$\dot{\hat{\sigma}}_- = -2\gamma_s^*\hat{\sigma}_- + i\lambda\hat{\sigma}_z\hat{b},$$
(4.24)

where $\hat{F}_b(t)$ and \hat{a}_{in} are the input noise operators that satisfy:

$$\langle \hat{F}_{b}^{\dagger}(t)\hat{F}_{b}(t')\rangle = n_{\rm th}\delta(t-t'),$$

$$\langle \hat{a}_{\rm in}(t)\hat{a}_{\rm in}^{\dagger}(t')\rangle = \delta(t-t'),$$

$$\langle \hat{a}_{\rm in}^{\dagger}(t)\hat{a}_{\rm in}(t')\rangle = 0.$$

(4.25)

Here the operator \hat{a}_{in} is manually included for completeness as the cavity is driven by the control laser.

4.8.2 S2: Adiabatic elimination

When $\delta \gg \lambda, g$, one can adiabatically eliminate the oscillator either by following the method [185] to obtain the Heisenberg-Langevin equations for cavity mode \hat{a} and NV spin $\hat{\sigma}_{-}$ after the elimination of \hat{b} or by setting $\dot{\hat{b}} = 0$, and obtaining \hat{b} in terms of \hat{a} and $\hat{\sigma}_{-}$. Here, we follow the second way to obtain

$$\hat{b} = \frac{ig\hat{a} + i\lambda\hat{\sigma}_{-} - \sqrt{\gamma_m}\hat{F}_b(t)}{-i\delta - \gamma_m/2}.$$
(4.26)

Under the conditions $\delta \gg \gamma_m/2$ and $\gamma_m \ll 1$, which are true in this system, this can be well approximated as

$$\hat{b} \approx \frac{g\hat{a} + \lambda\hat{\sigma}_{-}}{-\delta},$$
(4.27)

where we ignore decay-related terms and only keep coherent parts. Now, substituting this in the Hamiltonian (Eq. (4.22)), we obtain the effective Hamiltonian after the adiabatic elimination

$$\hat{H}_{\text{eff}} = \frac{\lambda^2}{\delta} \hat{\sigma}_+ \hat{\sigma}_- + \frac{g^2}{\delta} \hat{a}^{\dagger} \hat{a} + \Omega(\hat{a}^{\dagger} \hat{\sigma}_- + \hat{a} \hat{\sigma}_+), \qquad (4.28)$$

where $\Omega = \lambda g/\delta$ is the effective interaction between the cavity mode and the NV electron spin. In order to get the effective master equation, we also need to compute the decoherence terms related to the oscillator mode \hat{b} . Using Eq. (4.27), the thermal relaxation Lindbladian $(n_{\rm th} + 1)\gamma_m \mathcal{D}[\hat{b}]\hat{\rho}$ can be rewritten as

$$(n_{\rm th}+1)\gamma_m \Big[(\frac{g\hat{a}+\lambda\hat{\sigma}_-}{-\delta})\hat{\rho}(\frac{g\hat{a}^{\dagger}+\lambda\hat{\sigma}_+}{-\delta}) - (\frac{g\hat{a}^{\dagger}+\lambda\hat{\sigma}_+}{-\delta})(\frac{g\hat{a}+\lambda\hat{\sigma}_-}{-\delta})\hat{\rho}/2 - \hat{\rho}(\frac{g\hat{a}^{\dagger}+\lambda\hat{\sigma}_+}{-\delta})(\frac{g\hat{a}+\lambda\hat{\sigma}_-}{-\delta})/2 \Big]$$

$$\approx (n_{\rm th}+1)\gamma_m \Big[\frac{g^2}{\delta^2}\hat{a}\hat{\rho}\hat{a}^{\dagger} + \frac{\lambda^2}{\delta^2}\hat{\sigma}_-\hat{\rho}\hat{\sigma}_+ - \frac{g^2}{\delta^2}\hat{a}^{\dagger}\hat{a}\hat{\rho}/2 - \frac{\lambda^2}{\delta^2}\hat{\sigma}_+\hat{\sigma}_-\hat{\rho}/2 - \frac{g^2}{\delta^2}\hat{\rho}\hat{a}^{\dagger}\hat{a}/2 - \frac{\lambda^2}{\delta^2}\hat{\rho}\hat{\sigma}_+\hat{\sigma}_-/2 \Big]$$

$$= \frac{g^2}{\delta^2}(n_{\rm th}+1)\gamma_m \mathcal{D}[\hat{a}]\hat{\rho} + \frac{\lambda^2}{\delta^2}(n_{\rm th}+1)\gamma_m \mathcal{D}[\hat{\sigma}_-]\hat{\rho}, \qquad (4.29)$$

where the off-diagonal terms correspond to the incoherent interaction between the cavity mode and the spin and the thermal-induced cross-decoherence between these two modes, which can be ignored if $\delta \gg n_{\rm th} \gamma_m$. This is satisfied in our system even at ambient conditions. The same is true for the thermal excitation Lindbladian $n_{\rm th} \gamma_m \mathcal{D}[\hat{b}^{\dagger}]\hat{\rho}$, which can be written as

$$n_{\rm th}\gamma_m \mathcal{D}[\hat{b}]\hat{\rho} \approx \frac{g^2}{\delta^2} n_{\rm th}\gamma_m \mathcal{D}[\hat{a}^{\dagger}]\hat{\rho} + \frac{\lambda^2}{\delta^2} n_{\rm th}\gamma_m \mathcal{D}[\hat{\sigma}_+]\hat{\rho}.$$
(4.30)

Therefore, the effective master equation is given by

$$\dot{\hat{\rho}} = -i[\hat{H}_{\text{eff}}, \hat{\rho}] + \kappa_1 \mathcal{D}[\hat{a}]\hat{\rho} + \kappa_2 \mathcal{D}[\hat{a}^{\dagger}]\hat{\rho} + \gamma_s^* \mathcal{D}[\hat{\sigma}_z]\hat{\rho} + \gamma_1 \mathcal{D}[\hat{\sigma}_-]\hat{\rho} + \gamma_2 \mathcal{D}[\hat{\sigma}_+]\hat{\rho},$$
(4.31)

where $\kappa_1 = \kappa + g^2 \gamma_m (n_{\rm th} + 1)/\delta^2$ is the effective cavity decay rate, and $\kappa_2 = g^2 n_{\rm th} \gamma_m/\delta^2$, $\gamma_1 = \lambda^2 \gamma_m (n_{\rm th} + 1)/\delta^2$, and $\gamma_2 = \lambda^2 n_{\rm th} \gamma_m/\delta^2$ are the mechanically-induced thermal excitation rate for the cavity mode, and the mechanically-induced thermal flip-flop rates for the spin respectively.

4.8.3 S3: Effective emission rate

Under the condition $\lambda = g$, the effective Hamiltonian shown in Eq. (4.28) can be rewritten in the rotating frame of the spin frequency λ^2/δ

$$\hat{H}_{\rm int} = \Omega(\hat{a}^{\dagger}\hat{\sigma}_{-} + \hat{a}\hat{\sigma}_{+}). \tag{4.32}$$

Together with the effective master equation shown in Eq. (4.31), we obtain a set of optical Bloch equations for the cavity photon population, NV spin population and the coherence between them as

$$\frac{d\langle \hat{a}^{\dagger}\hat{a}\rangle}{dt} = i\Omega(\langle \hat{a}\hat{\sigma}_{+}\rangle - \langle \hat{a}^{\dagger}\hat{\sigma}_{-}\rangle) - (\kappa_{1} - \kappa_{2})\langle \hat{a}^{\dagger}\hat{a}\rangle + \kappa_{2},$$

$$\frac{d\langle \hat{\sigma}_{+}\hat{\sigma}_{-}\rangle}{dt} = i\Omega(\langle \hat{a}^{\dagger}\hat{\sigma}_{-}\rangle - \langle \hat{a}\hat{\sigma}_{+}\rangle) - (\gamma_{1} + \gamma_{2})\langle \hat{\sigma}_{+}\hat{\sigma}_{-}\rangle + \gamma_{2},$$

$$\frac{d\langle \hat{a}^{\dagger}\hat{\sigma}_{-}\rangle}{dt} = i\Omega(\langle \hat{a}^{\dagger}\hat{a}\hat{\sigma}_{z}\rangle + \langle \hat{\sigma}_{+}\hat{\sigma}_{-}\rangle) - \frac{\kappa_{1} - \kappa_{2}}{2}\langle \hat{a}^{\dagger}\hat{\sigma}_{-}\rangle - \frac{\gamma_{1} + \gamma_{2}}{2}\langle \hat{a}^{\dagger}\hat{\sigma}_{-}\rangle - 2\gamma_{s}^{*}\langle \hat{a}^{\dagger}\hat{\sigma}_{-}\rangle,$$

$$\frac{d\langle \hat{a}\hat{\sigma}_{+}\rangle}{dt} = -i\Omega(\langle \hat{a}^{\dagger}\hat{a}\hat{\sigma}_{z}\rangle + \langle \hat{\sigma}_{+}\hat{\sigma}_{-}\rangle) - \frac{\kappa_{1} - \kappa_{2}}{2}\langle \hat{a}\hat{\sigma}_{+}\rangle - \frac{\gamma_{1} + \gamma_{2}}{2}\langle \hat{a}\hat{\sigma}_{+}\rangle - 2\gamma_{s}^{*}\langle \hat{a}\hat{\sigma}_{+}\rangle.$$
(4.33)

Since we are mainly interested in the single-photon regime, the term $\langle \hat{a}^{\dagger} \hat{a} \hat{\sigma}_z \rangle$ can be simplified as $-\langle \hat{a}^{\dagger} \hat{a} \rangle$. Hence, these optical Bloch equations can be rewritten as

$$\frac{d\langle \hat{a}^{\dagger}\hat{a}\rangle}{dt} = i\Omega(\langle \hat{a}\hat{\sigma}_{+}\rangle - \langle \hat{a}^{\dagger}\hat{\sigma}_{-}\rangle) - (\kappa_{1} - \kappa_{2})\langle \hat{a}^{\dagger}\hat{a}\rangle + \kappa_{2},$$

$$\frac{d\langle \hat{\sigma}_{+}\hat{\sigma}_{-}\rangle}{dt} = i\Omega(\langle \hat{a}^{\dagger}\hat{\sigma}_{-}\rangle - \langle \hat{a}\hat{\sigma}_{+}\rangle) - (\gamma_{1} + \gamma_{2})\langle \hat{\sigma}_{+}\hat{\sigma}_{-}\rangle + \gamma_{2},$$

$$\frac{d\langle \hat{a}^{\dagger}\hat{\sigma}_{-}\rangle}{dt} = i\Omega(\langle \hat{\sigma}_{+}\hat{\sigma}_{-}\rangle - \langle \hat{a}^{\dagger}\hat{a}\rangle) - \frac{\kappa_{1} - \kappa_{2}}{2}\langle \hat{a}^{\dagger}\hat{\sigma}_{-}\rangle - \frac{\gamma_{1} + \gamma_{2}}{2}\langle \hat{a}^{\dagger}\hat{\sigma}_{-}\rangle - 2\gamma_{s}^{*}\langle \hat{a}^{\dagger}\hat{\sigma}_{-}\rangle,$$

$$\frac{d\langle \hat{a}\hat{\sigma}_{+}\rangle}{dt} = -i\Omega(\langle \hat{\sigma}_{+}\hat{\sigma}_{-}\rangle - \langle \hat{a}^{\dagger}\hat{a}\rangle) - \frac{\kappa_{1} - \kappa_{2}}{2}\langle \hat{a}\hat{\sigma}_{+}\rangle - \frac{\gamma_{1} + \gamma_{2}}{2}\langle \hat{a}\hat{\sigma}_{+}\rangle - 2\gamma_{s}^{*}\langle \hat{a}\hat{\sigma}_{+}\rangle.$$
(4.34)

In the incoherent regime, the cross terms that are responsible for the Rabi oscillation, i.e., $\langle \hat{a}^{\dagger}\hat{\sigma}_{-}\rangle$ and $\langle \hat{a}\hat{\sigma}_{+}\rangle$, can be eliminated [186], resulting in

$$\frac{d\langle \hat{a}^{\dagger} \hat{a} \rangle}{dt} = -(R + \kappa_1 - \kappa_2) \langle \hat{a}^{\dagger} \hat{a} \rangle + R \langle \hat{\sigma}_+ \hat{\sigma}_- \rangle + \kappa_2,$$

$$\frac{d\langle \hat{\sigma}_+ \hat{\sigma}_- \rangle}{dt} = -(R + \gamma_1 + \gamma_2) \langle \hat{\sigma}_+ \hat{\sigma}_- \rangle + R \langle \hat{a}^{\dagger} \hat{a} \rangle + \gamma_2,$$
(4.35)

where R is the effective decay rate which describes the population transfer between the cavity photon and the NV spin, and it is given by

$$R = \frac{4\Omega^2}{\kappa_1 - \kappa_2 + \gamma_1 + \gamma_2 + 2\gamma_s^*}.$$
(4.36)

Moreover, given that at room temperature $n_{\rm th} \gg 1$, the effective decay rate R can be written in a more compact form

$$R = \frac{4\Omega^2}{\kappa + 2\gamma_s^* + 2\Gamma_{\rm th}},\tag{4.37}$$

where $\Gamma_{\rm th} = \lambda^2 n_{\rm th} \gamma_m / \delta^2 = \lambda g n_{\rm th} \gamma_m / \delta^2$ is the thermal noise for the NV electron spin.

4.8.4 S4: Initial state of the cavity

The initial state can be obtained by solving the steady state of cavity mode with only the optomechanical coupling g turned on. Thus, we set $\Omega = 0$, and we obtain the following equation:

$$\frac{d\langle \hat{a}^{\dagger}\hat{a}\rangle}{dt} = -(\kappa_1 - \kappa_2)\langle \hat{a}^{\dagger}\hat{a}\rangle + \kappa_2 = 0.$$
(4.38)

Solving this equation, we get the average occupation number of the cavity mode: $\bar{n}_c = \langle \hat{a}^{\dagger} \hat{a} \rangle = \frac{\kappa_2}{\kappa_1 - \kappa_2}$. As this occupation is very small $\bar{n}_c \approx 10^{-3}$, it is valid to truncate the Hilbert space up to $|1\rangle$. Hence, the initial state of the cavity is given by:

$$\rho_{ic} = \frac{\kappa_1 - 2\kappa_2}{\kappa_1 - \kappa_2} \left| 0 \right\rangle \left\langle 0 \right| + \frac{\kappa_2}{\kappa_1 - \kappa_2} \left| 1 \right\rangle \left\langle 1 \right|.$$
(4.39)



Figure 4.8: Photon-counting histogram for pulsed driving scheme (a) and the continuous driving scheme (b) with a total readout time of 2 ms. The threshold is determined by the intersection of the two plots.

4.8.5 S5: Photon counting statistics

Our goal is to distinguish spin states $|D\rangle$ and $|0\rangle$. Let us denote the conditional probabilities of measurement outcome \pm given that the initial state of system is $|i\rangle$, with $i \in \{D, 0\}$, as $P(\pm|i) = p_i^{\pm}$. The total probability of outcome \pm is then given by $p^{\pm} = p_D p_D^+ + p_0 p_0^+$ where p_i is the total probability of the system being in state i. Then the conditional fidelity is defined as the conditional probability P(D|+) (P(0|-)) of having state D (0) given outcome + (-). This is given by Bayes' theorem: $F^+ \equiv P(D|+) = p_D p_D^+ / p^+$ and $F^- \equiv P(0|-) = p_0 p_0^- / p^-$. We can then define the total fidelity as the weighted average $F = (p^+F^+ + p^-F^-)/p_\eta$ where $p_\eta = p^+ + p^-$ is the total probability of having a measurement outcome. In the case that $p_D = p_0 = 1/2$ and $p_\eta = 1$, the fidelity reduces to the average of the conditional probabilities $F = (p_D^+ + p_0^-)/2$.

The most widely-used approach for spin readout is to use a cycling transition, which involves the emission and detection of a large number of photons. The photon-counting histogram shows the probability distribution of the number of photons detected and has two traces: one for photons emitted from the emitter and the other for the thermal noise contribution (non-zero cavity photon number when the spin state is at $|0\rangle$). The cross-over point of the two traces corresponds to the photon number threshold, above which we can be confident that the photons come from the emitter, thus determining that the spin state is $|D\rangle$; otherwise, the spin state is $|0\rangle$, meaning that the photons most likely come from the thermal noise.

Here we show the photon-counting histogram for the pulsed driving scheme and the continuous driving scheme in Fig. 4.8. For the pulsed driving scheme, the photon-counting histogram is described by a binomial distribution $P_{N,n,p} = {n \choose N} p^n (1-p)^{N-n}$, where $p = \eta\beta$, η is the total efficiency that an emitted photon can be detected, and β is the brightness of the cavity photon. For the parameters used in Fig. 4.5, $\beta = 0.929$ and $\beta = 0.034$ for the initial spin states $|D\rangle$ and $|0\rangle$ respectively. We plot the photon-counting histogram in Fig. 4.8(a) for a total pulse number of 100 (so the corresponding total readout time is 2 ms). The blue solid line and the yellow solid line show the probability distribution with respect to the detected photon number when the spin is in state $|D\rangle$ and $|0\rangle$, respectively. The threshold is thus determined by the corresponding number of photons at the intersection of the two lines, and it is $n_t = 9$ in this case. The readout fidelity is given by

$$F = \frac{1}{2} \left(\sum_{n < n_t} P_{N,n,p_2} + \sum_{n \ge n_t} P_{N,n,p_1} \right).$$
(4.40)

Then the estimated fidelity is 0.99999.

For the continuous driving scheme, we plot the photon-counting histogram for the corresponding Poisson distribution, shown in Fig. 4.8(b). In this case, the probability distribution of detecting n photons is $P(n, \lambda) = \lambda^n e^{-\lambda}/n!$, where λ is the average number of photons detected and is proportional to the readout time. For the parameters we used in Fig. 4.6, $\lambda_D/\lambda_0 = 14.43$, where λ_D and λ_0 are for the case of spin state $|D\rangle$ and $|0\rangle$, respectively. This gives two probability distributions that intersect at a photon number of 4. This means that the threshold is 4, and the readout fidelity is 0.997 using Eq. (4.40).

4.8.6 S6: f(x) **Derivation**

Here we provide a derivation of f(x) used in Sec. 4.5. For x elementary links, we define the average number of attempts required to independently generate entanglement in all xlinks as $n_{max,x} = f(x)/p_0$, where p_0 is the entanglement generation probability. For a single link, the probability of a successful entanglement generation with n attempts is given by $P(n) = p_0(1 - p_0)^{n-1}$. Thus the joint probability of successful entanglement generation for all x links with attempts $n_1, n_2, ..., n_x$ is

$$P_{j}(n_{1}, n_{2}, ..., n_{x}) = \prod_{k=1}^{x} P(n_{k})$$

$$= p_{0}^{x} (1 - p_{0})^{\sum_{k=1}^{x} n_{k} - x}.$$
(4.41)

The probability distribution function (PDF) of $n_{max,x}$ is

$$P(n_{max,x}) = \sum_{k=1}^{x} P_j(n_k = n_{max,x}, n_{\neq k} < n_{max,x}) + \sum_{k=1,l=2}^{l,x} P_j(n_{k,l} = n_{max,x}, n_{\neq k\neq l} < n_{max,x}) + \dots + P_j(n_1 = n_2 = \dots = n_x = n_{max,x}).$$

$$(4.42)$$

However, it is difficult to calculate $n_{max,x}$ from Eq. (4.42). To simplify the problem, we assume $x = 2^k$. The PDF of $n_{max,x} = n_{max,2^k}$ can be calculated iteratively by separating 2^k links into two groups of sublinks with each having 2^{k-1} sublinks. $n_{max1,2^{k-1}}$ and $n_{max2,2^{k-1}}$ denote the number of attempts for these two sublinks respectively. Then the probability distribution of $n_{max,x}$ can be expressed as

Table 4.1: Numerical results of f(x)

х	2	2^2	2^3	2^{4}	2^{5}
f(x)	1.5	2.08	2.72	3.38	4.05

$$P_{j}(n_{max1,2^{k-1}}, n_{max2,2^{k-1}}) = P(n_{max1,2^{k-1}}) * P(n_{max2,2^{k-1}}),$$

$$P(n_{max,2^{k}}) = P_{j}(n_{max1,2^{k-1}} = n_{max,2^{k}}, n_{max2,2^{k-1}} < n_{max,2^{k}})$$

$$+ P_{j}(n_{max1,2^{k-1}} < n_{max,2^{k}}, n_{max1,2^{k}} = n_{max,2^{k}}.$$

$$(4.43)$$

The simplest case is k = 1,

$$P_{j}(n_{1}, n_{2}) = P(n_{1})P(n_{2}),$$

$$P(n_{max,2}) = P_{j}(n_{1} = n_{max,2}, n_{2} < n_{max,2})$$

$$+ P_{j}(n_{1} < n_{max,2}, n_{2} = n_{max,2}).$$
(4.44)

We numerically calculate f(n) with respect to k = 1 to 5, shown in Tab. 4.1.

One can check that the function $f(2^k)$ almost linearly increases with k, and the regression result gives

$$f(2^k) = 0.64k + 0.83. \tag{4.45}$$

Therefore, we obtain the following empirical expression for f(x) by replacing 2^k with x and k with $\log_2(x)$ in Eq. (4.45).

$$f(x) = 0.64 \log_2(x) + 0.83. \tag{4.46}$$

Chapter 5

Proposal for non-cryogenic quantum repeaters with hot hybrid alkali-noble gases

5.1 Preface

The successful implementation of global quantum networks would have many applications from secure communication, blind quantum computing, and private database queries to a "quantum internet" of networked quantum computers and other quantum devices. Here we propose a non-cryogenic quantum repeater architecture, which builds on a cell of hot alkali atoms and noble-gas spins which offer ultra-long storage times. The cell is placed inside a ring cavity which allows for the strong suppression of FWM.

This work was done in collaboration with a few co-authors. My main contributions to this work include proposing the architecture scheme, quantifying the entanglement generation fidelity and efficiency, and computing the repeater rates and the fidelities. I also wrote the first draft of the manuscript.

Proposal for non-cryogenic quantum repeaters with hot hybrid alkali-noble gases

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Abstract

We propose a quantum repeater architecture that can operate without cryogenics. Each node in our architecture builds on a cell of hot alkali atoms and noble-gas spins which offer an hours-long storage time. Such a cell of hybrid gases is placed in a ring cavity, which allows us to suppress the detrimental four-wave mixing (FWM) noise in the system. We investigate the protocol based on a single-photon source made of an ensemble of the same hot alkali atoms. A single photon emitted from the source is either stored in the memory or transmitted to the central station to be detected. We quantify the fidelity and success probability of generating entanglement between two remote ensembles of noble-gas spins by taking into account finite memory efficiency, channel loss, and dark counts in detectors. We describe how the entanglement can be extended to long distances via entanglement swapping operations by retrieving the stored signal. Moreover, we quantify the performance of this proposed repeater architecture in terms of repeater rates and overall entanglement fidelities and compare it to another recently proposed non-cryogenic quantum repeater architecture based on nitrogen-vacancy (NV) centers and optomechanical spin-photon interfaces. As the system requires a relatively simple setup, it is easier to perform multiplexing, which enables achieving rates comparable to the rates of repeaters with NV centers and optomechanics, while the overall entanglement fidelities of the present scheme are higher than the fidelities of the previous scheme. Our work shows that a scalable long-distance quantum network made of hot hybrid atomic gases is within reach of current technological capabilities.

5.2 Introduction

The realization of global quantum networks would bring many fascinating applications to the world, which include secure communication [1], blind quantum computing [2], private database queries [3], and eventually, a quantum internet that connects quantum computers and other quantum information processing devices [4, 5, 6]. In such a quantum network, photons are used as information carriers for establishing long-distance connections, but they are adversely affected by transmission loss, which significantly limits the distance of connecting remote locations. Unlike its classical counterparts, photon loss cannot be compensated by amplification as unknown quantum states cannot be perfectly cloned according to the no-cloning theorem [7]. Therefore, quantum repeaters have been proposed to solve this issue but this requires stationary quantum memories for storing and processing the quantum information [12, 139, 5]. Currently, a vast majority of approaches to quantum networks need either vacuum equipment and optical trapping or cryogenic cooling [9, 10, 11, 140, 36, 12, 141, 142, 143], which makes scaling up such architectures very difficult. However, there have been some efforts in proposing quantum networks that operate at room temperature based on solid-state systems [149, 39] but they require complex setups and have high demands in designing the hardware for realizing the spin-photon interface. On the other hand, hot alkali vapors have been actively investigated as quantum memories for the application of quantum networks [123, 145, 187, 188, 124], and as they require relatively simple setups, it is easier to scale, which even offers a great potential for being deployed in space [85].

In spite of the appealing features of hot alkali vapor, there are a few challenges in the system. The main roadblock towards using this system for quantum networks is four-wave mixing (FWM) noise as it is quite significant and ubiquitous in Λ -type hot atomic ensembles, posing serious challenges to the single-photon level applications [125, 126]. Proposed solutions to this issue include blocking FWM channels by polarization selection rules [129], Raman absorption-enabled suppression in a mixed hot vapor [130], cavity engineering [131], and by means of coherent destructive interference of FWM [132]. The advantage of using a cavity to suppress FWM noise compared to other solutions is that it offers enhanced light storage and retrieval efficiency while only introducing a cavity. It has been experimentally verified, reporting a noise floor of around 1.5×10^{-2} photons per pulse in a Raman-type hot vapor memory [133]. Another significant challenge in the Λ -type hot atomic ensembles is short storage time in the collective spin state, which is mainly affected by the atomic collisions between the hot vapor and the buffer gas and the collisions in the hot vapor itself. Due to this detrimental effect, the storage time in hot ensembles is limited to a microsecond [189], thus restricting its application in quantum networks. However, there has been some work towards reducing this detrimental effect either by the motional averaging method [123, 124] or by using a decoherence-free subspace of spin states [33] with the spin coherence time extended to a second but even second-long coherence time may not be sufficient for longdistance quantum networks [12]. It is worth noting that minute-long spin relaxation time in hot alkali atoms has already been achieved [190] but no light storage was demonstrated in this experiment.

Rare isotopes of noble gas have non-zero nuclear spins, which are isolated from the environment by electronic shells. Thus, they maintain hours-long coherence time even at room temperature [134]. They can be accessed either via the collisions with metastable helium atoms or via the collisions with alkali atoms [134]. A quantum interface between noble-gas spins and alkali atoms has been proposed based on weak spin-exchange collisions [135]. Using this interface, the storage time can be significantly enhanced, which has been experimentally demonstrated with the coherence time of a minute [137] and an hour [138].

In this work, we propose a quantum repeater architecture without cryogenics, which is based on hot alkali vapor and noble-gas nuclear spins. In our proposal, we adopt the cavity engineering method to suppress FWM noise when the input gets stored as a collective spin excitation in hot vapor via the off-resonant Raman protocol [131], and then it is mapped to noble-gas spins via weak spin-exchange collisions [135, 136]. We consider the singlephoton-based protocol [12] where single-photon sources and quantum memories are used for entanglement generation and swapping. We propose to use the same hot alkali atomic ensembles for single-photon sources. We quantify and analyze the entanglement generation efficiency and fidelity between two remote ensembles of noble-gas spins. Then, we show how entanglement swapping can be done to extend the entanglement to longer distances. Finally, we compute the repeater rates and overall fidelities and compare them to quantum repeaters with NV centers and optomechanics.

This paper is organized as follows. In Sec. 5.3, we introduce the hybrid system of hot vapor and noble-gas spins in a ring cavity. The single-photon protocol is presented in Sec. 5.4. Sec. 5.5 discusses the repeater rates and fidelities. Sec. 5.6 gives more details on system implementation. We conclude and provide an outlook in Sec. 5.7.

5.3 Hybrid atomic gas system

As shown in Fig. 5.2(a), the hybrid atomic gas system is composed of a ring cavity and a cell of two hot atomic gases: alkali atoms and noble-gas atoms. This cell placed inside the cavity is driven by the control field (red) and the Stokes field (green). The ring cavity consists of two



Figure 5.1: The level diagram of a hybrid system with hot vapor and noble gases. The $|g\rangle - |e\rangle$ transition is coupled by the input signal (Stokes field) with the strength proportional to $\sqrt{n_a}$ where n_a is the number density of alkali atoms in the cell. The control field is coupled to the $|e\rangle - |s\rangle$ transition with a time-dependent Rabi frequency $\Omega(t)$. Both fields are detuned from $|e\rangle$ by Δ_s , and the control field can also couple the $|g\rangle - |e\rangle$ transition with the detuning Δ_a , which generates the anti-Stokes field A. The collective spin state for noble-gas atoms is denoted by $|k\rangle$. The $|s\rangle - |k\rangle$ transition is coupled to each other via the spin-exchange collision with a constant strength J when these two states are in resonance with other, i.e. $\delta_k = 0$. The collective excited state, alkali spin state, and noble-gas spin state decohere at the rates of γ_e , γ_s , and γ_k respectively. Typically, $\gamma_k \ll \gamma_s$ as noble-gas spins have extremely low decoherence rates.
fully reflective mirrors and a mirror that serves as an input-output coupler with amplitude reflectivity r. The coherent interaction between the noble-gas spins and alkali atoms is achieved by spin-exchange collisions [135]. As shown in Fig.5.1, an ensemble of alkali atoms is modeled as a Λ -type system with a collective ground state $|g\rangle$, a collective spin state $|s\rangle$ and an excited state $|e\rangle$. Each noble-gas atom is modeled as a spin-1/2 system with up and down states $|\uparrow\rangle$, $|\downarrow\rangle$. Here, we denote the collective noble-gas spin state as $|k\rangle$. The input signal (Stokes field) S couples the $|g\rangle - |e\rangle$ transition with the strength proportional to the density of alkali atoms n_a , and the control field couples to the $|e\rangle - |s\rangle$ transition with the Rabi frequency $\Omega(t)$. Both fields are detuned from $|e\rangle$ by Δ_s . The control field can also couple the $|g\rangle - |e\rangle$ transition with the detuning of Δ_a , which generates the anti-Stokes field A (FWM noise) because here all alkali atoms are prepared in one of the ground states that has higher energy [131]. Due to the effect of spatial diffusion, there could be many spatial modes for the alkali and noble gases. However, in the light-dominated regime where the power broadening in the alkali atoms due to the control beam dominates over diffusion in the alkali atoms, the collective spin mode of the alkali gas and the collective spin mode of the noble gas can be well approximated as single uniform modes by engineering the spatial profile of the control field [136]. This condition is satisfied in this work, which is discussed in detail in Sec. 5.6.

We need to polarize both the alkali and noble gases along the vertical axis. The former can be done using standard optical pumping, and the latter can be done using spin-exchange optical pumping (SEOP). The SEOP takes around 10 hours to complete, but this preparation can last up to 100 hours. The collective alkali spin state $|s\rangle$ can be coupled to the collective noble-gas spin state $|k\rangle$ via weak spin-exchange collision with the strength J. The coupling strength J is given by $J = \zeta \sqrt{(2I+1)p_a p_b n_a n_b/4}$ where ζ is the local average interaction strength of an alkali-noble atom pair in a single collision, and p_a and p_b are the polarization degrees of alkali and noble gases, and n_a and n_b are the densities of alkali and noble gases in the cell. I is the nuclear spin of an alkali atom. Thus, J is the effective interaction



Figure 5.2: (a) Schematic of the setup of hybrid quantum memories. A cell that contains alkali atoms (red dots) and noble-gas atoms (blue dots) is placed inside a ring cavity where the green signal and red control field interact with alkali atoms directly. The FWM noise is the generated blue anti-Stokes field during the storage and retrieval processes, which can be largely suppressed via tuning the cavity. The interface between alkali atoms and noble-gas spins is based on spin-exchange collisions. (b) Schematic of a two-link repeater with singlephoton sources [44] as an example. There are two elementary links with four nodes and each one has a single-photon source which is a hot alkali atomic ensemble, a beam splitter, and a hot alkali-noble gases hybrid system serving as a quantum memory. The single-photon source emits a photon that either transmits through the beam splitter or gets reflected to enter the quantum memory. Two steps are required to establish the entanglement between nodes A and D. The first step is to generate the entanglement between A and B, C and D. The second step is to perform the entanglement swapping between B and C to distribute the entanglement to A and D. (c) Schematic of post-selection. This is the same as in the DLCZ protocol [9]. The entanglement is established both in links A-D and A'-D', and the stored photons are retrieved to be detected, which allows us to rotate the measurement basis by adjusting the transmission coefficients and phases of the beam splitters [12].

strength in multiple collisions with each collision averaging over all alkali-noble atom pairs in the ensembles [135]. δ_k is the detuning between these two states, which can be tuned by applying a magnetic field along the vertical axis. This detuning can be used to decouple these two species of atoms [135, 138]. γ_e , γ_s , and γ_k are the decoherence rates for the collective excited state, spin state, and noble-gas spin state respectively. Moreover, we have $\gamma_k \ll \gamma_s$ as noble-gas spins have an extremely low decoherence rate.

The Maxwell-Bloch equations of this hybrid system with the excited state $|e\rangle$ being

adiabatically eliminated take the form [131, 136]

$$(c\partial_{z} + \partial_{t})S = ic\sqrt{\frac{d\gamma_{e}}{L_{c}}}\frac{\Omega}{\Gamma_{s}}B - \kappa_{s}S,$$

$$(c\partial_{z} + \partial_{t})A = ic\sqrt{\frac{d\gamma_{e}}{L_{c}}}\frac{\Omega}{\Gamma_{a}}B^{\dagger} - \kappa_{a}A,$$

$$\partial_{t}B = -i\sqrt{\frac{d\gamma_{e}}{L_{c}}}\frac{\Omega^{*}}{\Gamma_{s}}S + i\sqrt{\frac{d\gamma_{e}}{L_{c}}}\frac{\Omega}{\Gamma_{a}}A^{\dagger} - (\frac{1}{\Gamma_{s}} + \frac{1}{\Gamma_{a}^{*}})|\Omega|^{2}B - \gamma_{s}B - iJK,$$

$$\partial_{t}K = -(\gamma_{k} + i\delta_{k})K - iJB,$$
(5.1)

where S, A, B, and K are the annihilation operators for the signal field, anti-Stokes field, bosonic collective spin wave, and collective noble-gas spin wave (we use the same notation as in [135]). $\Gamma_{s,a} = \gamma_e - i\Delta_{s,a}$ is the complex detuning of the signal and anti-Stokes fields. $d \propto g^2 p_a N_a / \gamma_e$ is the optical depth where N_a is the total number of alkali atoms inside the cell, and q is the average coupling strength between the Stokes/anti-Stokes fields and the alkali atoms, which is given by $g = \sqrt{\frac{1}{N_a} \sum_{i=1}^{N_a} |g_i(r_i)|^2}$. This approximation is valid when the number of excitations is much smaller than N_a [191], which is the case here. L_c is the length of a roundtrip in the cavity, and c is the speed of light. The coordinate z indicates the direction along the optical path inside the cavity. Moreover, the bosonic operators B and K take the form as $B(z,t) = e^{i\omega_{gs}(t-z/c)} \sum_{j \in [z,z+\delta z]} |g\rangle_j \langle s| / (\delta z \sqrt{p_a n_a})$ and K = $e^{i\delta_k(t-z/c)}\sum_{i\in[z,z+\delta z]}|\psi\rangle_i\langle \uparrow |/(\delta z\sqrt{p_b n_b})$. As mentioned before, the spin-exchange coupling rate J is proportional to the densities of the two atomic gases n_a and n_b , i.e. $J \propto \sqrt{n_a n_b} [135]$. Thus, by increasing the pressure, one can increase this interaction strength. $\kappa_s = c d\gamma_e / (L_c \Gamma_s)$ and $\kappa_a = cd\gamma_e/(L_c\Gamma_a^+)$ stand for the decay rates of the Stokes field and anti-Stokes field in the ring cavity where $\Gamma_a^+ = \gamma_e - i(\Delta_a + \delta_s)$ with δ_s being the splitting between the states $|g\rangle$ and $|s\rangle$. Strictly speaking, Eq. (5.1) should also have the Langevin noise operators. However, for both the signal and the anti-Stokes field, the noise is vacuum which is zero in normal ordering [128]. The first two equations in the above set of equations describe the dynamics of the signal and anti-Stokes field inside the cavity, and the third equation describes the dynamics of the density of the collective spin state of the alkali atoms, which couples not only to the signal and anti-Stokes field but also to the noble-gas spins. The last equation describes the dynamics of the density of the noble-gas spin state.

Now, we have the boundary condition where the intra-cavity fields S_0 and A_0 (z = 0) can be related to the input fields S_{in} , A_{in} and the fields at $z = L_c$: S_{L_c} and A_{L_c} by the input-output coupler. Thus, we obtain the following relations [131]:

$$S_{0} = r e^{ik_{s}L_{c}} S_{L_{c}} + t_{r}S_{\text{in}},$$

$$A_{0} = r e^{ik_{a}L_{c}} A_{L_{c}} + t_{r}A_{\text{in}},$$
(5.2)

where $t_r = \sqrt{1 - r^2}$ is the transmission coefficient of the coupler, and k_s and k_a are the wavevectors of the signal and the anti-Stokes fields respectively. Moreover, S_{L_c} and A_{L_c} can be directly related to S_0 and A_0 by Taylor expansion. To the first-order approximation, we have the following:

$$S_{L_c} \approx e^{-k_s \frac{L_c}{c}} (S_0 + iL_c \sqrt{\frac{d\gamma_e}{L_c}} \frac{\Omega}{\Gamma_s} B_0 - \frac{L_c}{c} \partial_t S_0),$$

$$A_{L_c} \approx e^{-k_a \frac{L_c}{c}} (A_0 + iL_c \sqrt{\frac{d\gamma_e}{L_c}} \frac{\Omega}{\Gamma_a} B_0^{\dagger} - \frac{L_c}{c} \partial_t A_0),$$
(5.3)

where B_0 is the collective alkali spin operator for z = 0. Combining these two sets of relations, one can obtain the following Maxwell-Bloch equations:

$$\begin{aligned} \partial_t s &= -\tilde{\kappa}_s s + i\sqrt{\frac{d\gamma_e}{\tau}} \frac{\Omega}{\Gamma_s} b + e^{-i\phi_s} \frac{t_r}{\mu_s \sqrt{\tau}} S_{\rm in}, \\ \partial_t a &= -\tilde{\kappa}_a a + i\sqrt{\frac{d\gamma_e}{\tau}} \frac{\Omega}{\Gamma_a} a + e^{-i\phi_a} \frac{t_r}{\mu_a \sqrt{\tau}} A_{\rm in}, \\ \partial_t b &= -\gamma_s b + i\sqrt{\frac{d\gamma_e}{\tau}} (-\frac{\Omega^*}{\Gamma_s} s + \frac{\Omega}{\Gamma_a} a) - (\frac{1}{\Gamma_s} + \frac{1}{\Gamma_a^*}) |\Omega|^2 b - iJk, \\ \partial_t k &= -(\gamma_k + i\delta_k) k - iJb, \end{aligned}$$
(5.4)

where $\tau = L_c/c$ is the cavity roundtrip time. $s = \sqrt{\tau}S_0, a = \sqrt{\tau}A_0, b = \sqrt{L_c}B_0$, and $k = \sqrt{L_c}K_0$ (K_0 is obtained by setting z = 0 in K) are the intra-cavity amplitudes for

the signal, anti-Stokes field, collective alkali spin state, and collective noble-gas spin state respectively. $\tilde{\kappa}_{s,a}$ is the resonant and anti-resonant decay rates for the signal and anti-Stokes field. They are given by [131]:

$$\frac{1}{\tilde{\kappa}_{s,a}} = \tau \frac{\mu_{s,a} e^{i\phi_{s,a}}}{1 - \mu_{s,a} e^{i\phi_{s,a}}},\tag{5.5}$$

where $\phi_{s,a} = k_{s,a}L_c - \text{Im}\{\kappa_{s,a}\}\tau$ is the accumulated phases in the cavity roundtrip by the signal and anti-Stokes fields, and $\mu_{s,a} = re^{-\operatorname{Re}\{\kappa_{s,a}\}\tau}$ is the cavity roundtrip amplitude transmission for the fields.

Eq. (5.4) can be solved in the bad-cavity regime where the signal/anti-Stokes field evolved at a rate much slower than the corresponding decay rate, i.e. $|\tilde{\kappa}_{s,a}| \gg |\sqrt{d\gamma_e/\tau}\Omega/\Gamma_{s,a}|$ [128]. In this limit, we can set $\partial_t a \approx 0$ and $\partial_t s \approx 0$. Moreover, as we can decouple the alkali and noble gases by applying a large magnetic field, we can break the storage into two steps: first consider the storage in the alkali atoms in the presence of the anti-Stokes field and then consider the transfer from the alkali to the noble gas. This sequential storage is optimal when the signal pulse duration T satisfies $T \ll 1/\gamma_s$ [136], which is adopted in this work. We discuss how this sequential storage is achieved in detail and the optimal storage efficiency in Sec. 5.4.1. The main noise present in the system is FWM, and in order to achieve the maximum suppression of this noise, we need to tune the ring cavity to be in resonance with the signal and to be in anti-resonance with the anti-Stokes field, which means $\phi_s = 0$ and $\phi_a = \pi$. This is crucial in the first step of storage and retrieval, which is discussed in more detail in Sec. 5.4.2. Other sources of noise in hot vapor systems include collision-induced fluorescence noise, the Doppler broadening, and inhomogeneous broadening for the $|g\rangle - |e\rangle$ transition. However, in this system, we ignore these effects as it has been demonstrated that fluorescence noise is negligible for the off-resonant scheme with a short pulse input [192], and this is also true for the Doppler broadening and inhomogeneous broadening with the detuning Δ_s much larger than their bandwidth, which is discussed in Sec. 5.6.

5.4 The single-photon repeater

Here, we focus on the single-photon-based protocol [44] for entanglement generation and entanglement swapping where each node consists of a beam splitter (BS), a single-photon source (SPS) and a hybrid quantum system as depicted in Fig. 5.2(b), where we just show a two-link repeater as an example. As the noble-gas spins offer ultralong coherence time at room temperature, they will be used as the memory for storing the signal. There are two steps to establish the entanglement between two remote locations, and Fig. 5.2(a) shows how to achieve this between nodes A and D by cutting this distance into small pieces of equal length. In Fig. 5.2(b), it is cut into two equal pieces: A-B and C-D but it can be more general to have more links. In this example, we need to first establish the entanglement between A and B, and C and D, which is called entanglement generation, and then we perform the entanglement swapping between two local memories B and C to distribute the entanglement to A and D, i.e. only entangling A and D. Moreover, as a single excitation in the noble-gas spins is shared between A and D, it is difficult to perform measurements in other bases than the basis $\{|k\rangle, |0\rangle\}$. In order to relax this, we can introduce another entangled link A' - D'where nodes A' and D' are in the same locations as A and D respectively [44], which is depicted in Fig. 5.2(c). In this way, we can use two beam splitters and two detectors in each location to read out the stored photons, which allows measurements in an arbitrary basis by choosing the transmission coefficients and phases. This step is known as postselection. In this section, we show how entanglement generation, entanglement swapping, and post-selection can be achieved in our hybrid system, and we also quantify the established entanglement generation fidelity and efficiency in the elementary link.

5.4.1 Entanglement generation

Before we characterize how the entanglement generation can be done, we would like to first talk about how signal storage can be achieved and give optimal storage efficiency. Our goal

is to store the signal in the quantum memory as a collective excitation in the noble-gas spins, and this process can be divided into two steps: storing the signal in the collective spin excitation of alkali atoms and transferring this excitation to the collective excitation in noble-gas spins. This sequential storage is optimal when the signal pulse duration T satisfies $T \ll 1/\gamma_s$ [136]. In order to execute the first step, we make the detuning δ_k between $|s\rangle$ and $|k\rangle$ large enough such that $\delta_k \gg J$, and when this condition is satisfied, the states $|s\rangle$ and $|k\rangle$ are decoupled from each other [135]. Then, this process is simply described by the first three equations in Eq. (5.4) with J = 0. Given that the maximum suppression of noise is achieved by tuning the ring cavity to be in resonance with the signal and to be in anti-resonance with the anti-Stokes field ($\phi_s = 0$ and $\phi_a = \pi$), it has been shown that the optimal storage efficiency in the first step is $\eta_1 = 1 - \sqrt{d\gamma_e}/(\sqrt{2}\Delta_s)$ in the strong coupling regime (more details can be in Sec. 5.4.2) and the far-detuned regime $(\Delta_s \gg \gamma_e)$ without mode mismatch in the cavity [131]. This efficiency could be achieved when using lossless optical components. The requirements for all the related parameters can be realized experimentally, which are discussed in Sec. 5.6. It is worth noting that this optimal efficiency depends on the signal detuning Δ_s as opposed to the previous result in [128] where the optimal efficiency was found to be detuning-independent in the absence of FWM. The second step is to transfer the signal stored in the alkali atoms to the noble-gas spins. Thus, we need to turn off the control field $\Omega(t)$ and tune $|k\rangle$ on resonance with $|s\rangle$ to make them interact, which can be done using an external magnetic field [138]. The efficiency of this transfer is maximized when the transfer time is set to be $\pi/(2J)$ and it is in the strong coherent coupling regime, i.e. $J \gg \gamma_s \gg \gamma_k$ [136]. Then, we obtain the optimal transfer efficiency $\eta_2 = \exp(-\frac{\pi(\gamma_s + \gamma_k)}{2J})$, which gives us the total storage efficiency:

$$\eta_s = \eta_1 \eta_2 = \left(1 - \frac{\sqrt{d\gamma_e}}{\sqrt{2\Delta_s}}\right) \exp\left(-\frac{\pi(\gamma_s + \gamma_k)}{2J}\right).$$
(5.6)

Now, we shall see how entanglement can be established in an elementary link. There

are two links illustrated in Fig. 5.2(b), and here we focus on the first link for describing how the entanglement generation is achieved. In this link, for the left node, a single photon emitted from the source after a beam splitter can be described as $(\alpha a_1^{\dagger} + \beta a_2^{\dagger}) |0\rangle$ where α , β are reflection and transmission amplitudes of a beam splitter, and they satisfy the relation $|\alpha|^2 + |\beta|^2 = 1$. The same is true for the right node where the state of a single photon after a beam splitter is $(\alpha b_1^{\dagger} + \beta b_2^{\dagger}) |0\rangle$. Thus, the joint state is given by:

$$\left[\alpha^{2}a_{1}^{\dagger}b_{1}^{\dagger} + \alpha\beta(a_{1}^{\dagger}b_{2}^{\dagger} + a_{2}^{\dagger}b_{1}^{\dagger}) + \beta^{2}a_{2}^{\dagger}b_{2}^{\dagger}\right]|0\rangle.$$
(5.7)

The first term in this state is the case where both single photons are reflected to be stored in quantum memories, ideally yielding no heralding in detectors. However, the detector dark counts could potentially lead to spurious clicks, thus causing infidelity in the desired entangled state. This probability is given by $\epsilon_0(1-\epsilon_0)\alpha^4$ where ϵ_0 is the probability of having no dark counts in detectors. Here, we take it into account, but later on, we will see that its effect can be negligible if we choose the detector and detection window time properly. The second and third terms are the main contributions to single photon heralding where a_1^{\dagger} and b_1^{\dagger} are to be stored in quantum memories. We use noble-gas nuclear spins as quantum memories where the storage of a single photon is achieved in two steps as described above. As the finite storage efficiency, η_s could create vacuum components, we take it into consideration in this work. The probability of having this contribution is given by $\epsilon_0 \alpha^2 \beta^2 \eta_t \eta_c \eta_d \eta_s$ where η_t, η_d, η_c are the transmission, detection, and frequency conversion efficiencies. The last term could also lead to the single-photon detection event when one of the two photons gets lost in the transmission, thus creating vacuum components as well. As discussed in Sec. 5.4.2, although the hindsight from post-selection tells us that the vacuum components can be eliminated, which seems to have no effect on overall fidelity, it still could decrease the overall repeater rates. This probability is given by $\epsilon_0 \beta^4 \eta_t \eta_c \eta_d (1 - \eta_t \eta_c)$. Moreover, we assume that the probability that the single-photon source emits a photon is p_1 , which depends on the source we use.

Here, we choose to use the same hot alkali gas as a single-photon source, which can be charged with a single excitation via the FWM process used in the DLCZ protocol [9], and this atomic excitation can then be reverted to emit a single photon. A few experimental works have been reported for using hot rubidium atoms to generate bright and indistinguishable photons [193, 194]. In this way, we do not need to perform frequency conversion to match with the alkali gas we use in the system, but the frequency conversion is needed for longdistance communication, i.e. for a_2 and b_2 . Using atomic ensembles to generate single photons could lead to multi-photon errors thus degrading the repeater fidelities. This is discussed in detail in Sec. 5.5. We envision using the reverse-proton exchange (RPE) PPLN waveguide technique to convert a single photon emitted from the source to a telecom photon, which can operate at room temperature with a conversion efficiency of 23% for the 863 nm signal [195] but it is promising to apply it to the signal of different wavelengths. Moreover, by choosing the proper waveguide mode filter and fibre type, one can greatly improve this conversion efficiency to 60% [195], and we use a higher value of 80% in Sec. 5.5 for rates calculations. Also, we assume that the relative phase in two optical fibres remains stable. Practically, this requirement can be achieved by actively stabilizing the lengths of fibre [12]. or through the use of self-compensating Sagnac-type configurations [196].

After taking all these effects into account, the entanglement generation fidelity and efficiency of the state created by detecting a single photon in one of the detectors are given by:

$$F_{\rm gen} = \frac{\alpha^2 \beta^2 \eta_t \eta_c \eta_d \eta_s}{\beta^2 \eta_t \eta_c \eta_d + (1 - \epsilon_0) \alpha^4 - \beta^4 \eta_t^2 \eta_c^2 \eta_d},\tag{5.8}$$

$$\eta_{\rm gen} = 2p_1(\epsilon_0\beta^2\eta_t\eta_c\eta_d + \epsilon_0(1-\epsilon_0)\alpha^4 - \epsilon_0\beta^4\eta_t^2\eta_c^2\eta_d), \tag{5.9}$$

where $\epsilon_0 = \exp(-\lambda T_d)$ with λ being the dark count rate, and T_d is the detection window time which is set to be the time duration of the signal, that is $T_d = T$. η_t is the function of the length of an elementary link L_0 , which takes the following form: $\eta_t = \exp(-L_0/2L_{\text{att}})$ with $L_{\text{att}} = 22$ km being the attenuation length for telecom photons. The factor of 2 in the efficiency expression comes from the fact that the detectors are symmetric, and the heralding in either of them contributes to the efficiency. We envision using silicon single-photon avalanche diodes (Si SPADs) [197, 198] and frequency conversion to detect telecom photons. Si SPADs combined with a monolithic integrated circuit of active quenching and active reset (AQAR) can enable detection efficiency as high as 75% with dark count rates below 100 Hz at 785 nm [197]. This type of detector can operate at non-cryogenic temperatures which only require a thermoelectric cooler. The parameters are taken to be $\alpha^2 = 0.84$, $\beta^2 = 0.16$, $\eta_d = 0.6$, $\eta_c = 0.8$, $T_d \sim 12.5$ ns (the signal bandwidth is around 80 MHz, which is compatible with the hot vapor bandwidth as discussed in Sec. 5.6). In this regime, the term $(1 - \epsilon_0)\alpha^4$ is a few orders of magnitude smaller than $\beta^2 \eta_t \eta_c \eta_d$ so Eq. (5.9) can be approximately written as $F_{\text{gen}} \approx \alpha^2 \eta_s$, and $\eta_{\text{gen}} \approx 2p_1 \beta^2 \eta_t \eta_c \eta_d$. Moreover, we can now write the entangled state for each elementary link as

$$\alpha^2 \eta_s \left| \psi_{ab} \right\rangle \left\langle \psi_{ab} \right| + \left[\alpha^2 (1 - \eta_s) + \beta^2 \right] \left| 0 \right\rangle \left\langle 0 \right|, \qquad (5.10)$$

where $|\psi_{ab}\rangle = \frac{1}{\sqrt{2}}(|k_a\rangle |0_b\rangle + |0_a\rangle |k_b\rangle)$. The storage inefficiency $1 - \eta_s$ increases the vacuum component proportion, and therefore it decreases the repeater rates. The required input pulse is short as it satisfies the condition $T \ll 1/\gamma_s$, which is also the requirement for the optimal signal storage in noble-gas spins using the sequential scheme [136]. Moreover, when we have two elementary links, there is some waiting time for both links to establish entanglement, and as noble-gas spins offer ultralong coherence time, the decoherence that happened during the waiting time is ignored.

5.4.2 Entanglement swapping

After we successfully establish the entanglement in two adjacent elementary links as shown in Fig. 5.2(b), we then need to perform entanglement swapping to propagate the entanglement between A and D. This can be done by recalling the single photon stored in either quantum memories B or C that are in the same location, and the heralding at one of the beam splitters informs us of the success in the swapping process, leading to the entangled state shared between A and D. At this level, it is well known that the swapping probability takes the following form [12]:

$$P_1 = \frac{p_1 F_{\text{gen}} \eta}{2} (2 - p_1 F_{\text{gen}} \eta), \qquad (5.11)$$

where $\eta = \eta_d \eta_r$ is the product of the detection efficiency and the retrieval efficiency. Here, the retrieval process happens in two phases as well. First, we map the excitation in noblegas spins to the excitation in hot vapor via the spin-exchange interaction by turning on the magnetic field for the amount of time of $\pi/(2J)$ [135, 136]. Second, we need to read out the signal from the collective spin state of the hot vapor. In this process, we need to turn on the control field $\Omega(t)$ and decouple the hot vapor from the noble gas by applying an external magnetic field to detune $|s\rangle$ from $|k\rangle$. The efficiency of retrieving the signal from hot vapor is the same as η_1 , and it only holds under the condition that the decoherence of $|s\rangle$ is negligible during this process, which is true as the decoherence happens on the time scale much slower than that of memory interactions [136, 131]. Thus, the overall retrieval efficiency is $\eta_r = \eta_s$, which is given in Eq. (5.6).

Now, putting all together, we can further simplify Eq. (5.11) as $P_1 = p_1 \alpha^2 \eta_{\text{tot}} (1 - \frac{1}{2}p_1 \alpha^2 \eta_{\text{tot}})$, where $\eta_{\text{tot}} = \eta_s \eta_r \eta_d$. If we have more than two elementary links, the entanglement swapping is nested, which requires higher levels of swapping. This leads to a more general expression for the success probability of entanglement swapping at the *i*th level [12]:

$$P_{i} = \frac{p_{1}\alpha^{2}\eta_{\text{tot}}}{2} \frac{\left[2^{i} - (2^{i} - 1)p_{1}\alpha^{2}\eta_{\text{tot}}\right]}{\left[2^{i-1} - (2^{i-1} - 1)p_{1}\alpha^{2}\eta_{\text{tot}}\right]^{2}}.$$
(5.12)

After the entanglement swapping, a single excitation in noble-gas spins is shared between two remote locations (in Fig. 5.2(b), it is between A and D). As mentioned before, we need to perform post-selection by reading out the stored photons in each location, which allows us to generate an effective state $1/\sqrt{2}(|k_A k_{D'}\rangle + |k_{A'} k_D\rangle)$. Here, the dark counts are negligible because of the short detection time T_d as mentioned in Sec. 5.4.1. Then, the success probability of performing this projection is given by [12]:

$$P_{ps} = \frac{(p_1 \alpha^2 \eta_{\text{tot}})}{2} \frac{1}{[2^i - (2^i - 1)p_1 \alpha^2 \eta_{\text{tot}}]^2}.$$
(5.13)

This post-selection step enables us to eliminate the vacuum components in Eq. (5.10) as it is impossible to detect a single photon on each side if both links are vacuum. Hence, the overall fidelity is not affected by the vacuum components in Eq. (5.10) but as mentioned they have a significant impact on repeater rates.

In the retrieval process, FWM noise can be strongly suppressed by choosing $\phi_s = 0$ (on resonance) and $\phi_a = \pi$ (anti-resonance). In the strong coupling regime, this noise can be quantified by calculating the $g_{\rm re}^{(2)}$ function of the retrieved signal, which is equal to $2|x|^2\zeta_1|\Gamma_s|^2/|\Gamma_a|^2$ [131] when there is no mode mismatching, and the input signal contains one photon. x is the FWM noise suppression factor, which is given by

$$x \approx \frac{1 - \mu_s}{2\mu_s} = \frac{1 - re^{-d(\frac{\gamma_e}{\Delta_s})^2}}{2re^{-d(\frac{\gamma_e}{\Delta_s})^2}}.$$
 (5.14)

 $\zeta_1 \gg 1$ is the dimensionless coupling strength between both the signal and anti-Stokes field and the alkali gas, which in this case is given by:

$$\zeta_1 \approx 2 \left| \frac{\sqrt{C_s \gamma_e W}}{\Gamma_s} \right|^2, \tag{5.15}$$

where $C_s = d\mu_s/(1-\mu_s) \approx d/2x$, and $W = \int_0^{T_c} |\Omega(t)|^2 dt$ stands for the integrated Rabi



Figure 5.3: (a) Repeater rates as a function of total distance L with $F_{\text{targ}} = 0.9$ for hot hybrid gases-based scheme (referred to as scheme 1), and repeater rates for NV centers and optomechanics-based scheme (referred to as scheme 2) [39] with corresponding fidelities shown in (b). Here, we plot 4-link case (B), and 8-link case (A) for scheme 2, and we also plot 4-link case (D), multiplexed 4-link case (C), and 8-link case (E) for scheme 1 with the efficiency $\eta_s = \eta_r = 0.9$, and 4-link case (G) for scheme 1 with efficiency $\eta_s = \eta_r = 0.8$. The choice of the storage/retrieval efficiency is justified in Sec. 5.6. The direct transmission (F) is plotted with a single-photon source of 10 GHz. For C, it is multiplexed by a factor of 100. In general, the rates of scheme 2 are much higher than the rates of scheme 1. All these repeaters outperform direct transmission. We assume $\eta_c = 0.8$, $\eta_d = 0.6$, $\alpha^2 = 0.84$, $\beta^2 = 0.16$, $t_{\text{trans}} = 1.5$ ms for all cases in scheme 1. We also use $t_{\text{ch}} = 0.048$ ms for D and G, and 1.03 ms for E. The emission probability for single-photon source p_1 is assumed to be 0.9 for all repeaters in both schemes. (b) Repeater fidelities as a function of total distance L for schemes 1 and 2. A and B are 8-link and 4-link cases in scheme 2 [39]. C stands for a 100-multiplexed 4-link repeater in scheme 1 with $F_{\text{targ}} = 90\%$ and $F_{\text{re}} = 98.6\%$. As a multiplexed 8-link repeater in scheme 1 has fidelities very close to C, it is not shown here. In general, scheme 1 yields much higher fidelities than scheme 2, and they are independent of the total distance.

frequency with T_c being the control pulse duration. Then, the readout fidelity is given by

$$F_{\rm re} = \frac{1}{1 + {\rm SNR}^{-1}},\tag{5.16}$$

where $\text{SNR}^{-1} = g_{\text{re}}^{(2)}/2$ is the signal-to-noise ratio [131]. Here, we ignore the infidelity that comes from the detector's dark counts as the detection window time is assumed to be around 12.5 ns. Using the parameters discussed in Sec. 5.6, it is possible to have a readout fidelity as high as 98.6%.

5.5 Repeater rates and overall fidelities

In our system as the storage and retrieval time are mainly limited by how fast we can transfer the coherence from hot vapor to noble-gas spins via the spin-exchange collisions, and these times are given by $t_{\text{trans}} = \pi/2J$, which is around 1.5 ms based on the parameters in Sec. 5.6. This transfer time is on the same order as the two-way communication time L_0/c for L_0 ranging from 50 km to 100 km with $c = 2 \times 10^8$ m/s, which makes the total length of an 8-link repeater ranging 400 km to 800 km. Furthermore, the average charging time t_{ch} in the ensemble also needs to be taken into account as it is comparable to t_{trans} both in a four-link repeater and an eight-link repeater with the final target fidelity $F_{targ} = 0.9$ as discussed later in this Section. Now, taking η_{gen} , P_i , and P_{ps} into the standard entanglement distribution time for the single-photon protocol [12] plus the extra time spent for retrieving the signal and charging the ensemble, we obtain

$$T_{\rm tot} = \frac{3^{n+1}}{2} \left(\frac{L_0}{c} + t_{\rm trans} + t_{\rm ch}\right) \frac{\prod_{i=1}^n \left(2^i - (2^i - 1)p_1 \alpha^2 \eta_{\rm tot}\right)}{\eta_t \eta_c \eta_d p_1^{n+3} \beta^2 \alpha^{2n+4} \eta_{\rm tot}^{n+2}}.$$
(5.17)

where n indicates the number of nesting levels, and the number of links associated with it is 2^n . Thus, the total length of a repeater is $L = 2^n L_0$. In Fig. 5.3(a), we plot 4-link case (D), multiplexed 4-link case (C), and 8-link case (E) for hot hybrid gases-based scheme (referred to as scheme 1) with respect to total distance when $\eta_s = \eta_r = 0.9$ and $F_{\text{targ}} = 0.9$ as discussed below, and we plot 4-link case (G) for scheme 1 with $\eta_s = \eta_r = 0.8$. The choice of the storage/retrieval efficiency is discussed in detail in Sec. 5.6. Also, we plot 4-link case (B), and 8-link case (A) for NV centers and optomechanics-based scheme (referred to as scheme 2) [39] for comparison. The direct transmission (F) is plotted with a source of 10 GHz. For C, it is multiplexed by a factor of 100, which can be implemented spatially [199, 12] or spectrally [11] as discussed in Sec. 5.6. All these repeaters outperform direct transmission at some point but in general, the rates of scheme 2 are much higher than the rates of scheme 1. The lower rates for scheme 1 are due to the fact that the single-photon protocol is nested as the entanglement swapping and post-selection are probabilistic as opposed to the non-nested scheme used in [39], and the interface between alkali atoms and noble-gas spins is also quite slow, which further degrades the repeater rates. The other factors that limit the repeater rates in this proposal are detection efficiency η_d and frequency conversion efficiency η_c , which could be improved to further enhance the rates. We expect an order of magnitude increase in rates when we increase η_d from 0.6 to 0.9. The slow interface between hot vapor and noblegas spins also plays a role in reducing the rates, but the room for improving the speed of this interface is limited as it is based on weak spin-exchange interactions [135], which means J cannot be too large. It is worth noticing that it is much easier to perform multiplexing in hot hybrid gases-based repeaters than NV centers and optomechanics-based repeaters because the latter requires much more complex setups than the former [39]. Moreover, there is a trade-off between the target fidelity F_{targ} and repeater rates as F_{targ} determines t_{ch} . However, the improvement in rates is not significant when we set a lower target fidelity.

The infidelities in our repeaters mainly come from multiphoton emissions of the singlephoton source and FWM noise in the entanglement swapping and post-selection. The effect of FWM noise in the signal readout has been estimated in Sec. 5.4.2 based on the parameters discussed in Sec. 5.6, which gives us a high readout fidelity of 98.6%. In addition, the decoherence of the noble-gas spins does not affect the final fidelities as we perform postselection in end to filter out the vacuum components. Now, the overall fidelity is given by

$$F_{\rm tot} = F_{\rm targ} \times (F_{\rm re})^{n+2}, \tag{5.18}$$

where n+2 is the number of performing readouts. F_{targ} is a target fidelity of repeaters, which we choose to be 90% for all repeaters in scheme 1 with different nesting levels. This fidelity is determined by errors due to multiphoton emission in the ensemble-based single-photon source [12]. The probability of having a two-photon contribution is given by $p_2 = 2p(1-\eta_{st})p_1$ where p is the probability of emitting the Stokes photon when charging the ensemble, and η_{st} is the efficiency of detecting a Stokes photon, assumed to be 0.75 using a silicon single-photon detector [197, 198]. In order to make p_2 small enough to have $F_{\text{targ}} = 0.9$, we need to make p sufficiently small. It can be shown that when we have a four-link repeater, the maximum value that p_2 can take is 0.00093 [12], which leads to p = 0.0021. This emission probability results in a charging time given by $t_{\rm ch} = 1/(Rp) = 0.048$ ms with the repetition rate R = 10MHz. If we have an eight-link repeater with $F_{\text{targ}} = 0.9$, we obtain $p = 9.73 \times 10^{-5}$, which leads to $t_{\rm ch} = 1.03$ ms. Assuming the readout fidelity for both swapping and post-selection is 98.6%, the overall fidelities of a 4-link and 8-link repeaters in scheme 1 are estimated to be 85.1% and 83.87%. In Fig. 5.3(b), we plot the overall fidelities as a function of total distance L for scheme 1 and scheme 2. A and B are 8-link and 4-link repeaters in scheme 2 which decrease as total distance increases due to thermal noise present in the system which are treated as dark counts [39]. C is the multiplexed 4-link repeater in scheme 1. which is independent of the total distance. In general, scheme 1 yields fidelities that are significantly higher than the fidelities in scheme 2, which is mainly due to the fact that the accumulated infidelities induced by vacuum components are eliminated in the end by postselection. Overall, these two schemes have their own advantages and disadvantages. Scheme 1 is much slower than scheme 2 but has much higher fidelities, and scheme 1 requires much less complex setups than scheme 2 which also facilitates multiplexing. Moreover, it is possible to boost the fidelities using entanglement purification (200), but this comes at the cost of further reducing the rates. A quantitative discussion of repeaters including purification goes beyond the scope of the present work.

5.6 Implementation

Here, we consider ³⁹K atoms as the hot vapor and ³He atoms as the noble-gas spins in our system, where the optical depth d of the hot vapor is assumed to be 100, which can be achieved in the high-density hot vapors. The linewidth of the excited state $2\gamma_e$ is taken to be 27 GHz for broadened D_1 line due to collisions with buffer gas, which is much smaller than the assumed detuning $\Delta_s = 2700$ GHz so it makes the system in the far-off resonant regime [131]. Moreover, such a large detuning Δ_s also makes the Doppler broadening negligible, around 1 GHz at 230° C. But this often comes at the cost of reducing the efficiency due to large detuning so we also need to make sure the system is still in the strong coupling regime in the far off-resonant regime, which then requires a strong control pulse as discussed below in this section. In fact, it has been shown that at high enough optical depth (or high enough cooperativity), the effect of Doppler broadening or any inhomogeneous broadening is negligible [201]. So far, the experimentally achieved value of J is around 78 Hz [138] but if we further increase the pressure to increase the gas densities, it is possible to have J = 1000 Hz [135]. In this condition, for γ_s and γ_k , they are estimated to be 17.5 Hz and 2.8×10^{-6} Hz respectively, dominated by intra-gas and inter-gas collisional spin-rotation couplings [135, 137, 138]. However, one also needs to take the diffusion-induced effect into account. For the single uniform mode of the alkali atoms, the diffusion-induced decay rate is given by $D_a \pi^2/R^2$ where D_a is the diffusion coefficient of the alkali atoms, and R is the radius of the spherical cell. For a cell with $R \sim 0.15$ cm, in high buffer-gas pressure configuration $(D_a = 0.054 \text{cm}^2/\text{s})$, the diffusion-induced decay rate is estimated to be around 28 Hz. Thus, the actual decay rate of $^{39}\,\mathrm{K}$ atoms γ_s is the sum of the original rate and the diffusioninduced rate, which becomes 45.5 Hz. For the single uniform mode of the noble-gas atoms, the spatial diffusion does not affect its own decay rate, thus leaving γ_k unchanged. To verify that we are in the light-dominated regime, the power broadening due to the control beam is $\operatorname{Re}(|\Omega|^2(1/\Gamma_s+1/\Gamma_a^*))$, which is around 3700 Hz, thus much larger than the diffusion-induced decay rate 28 Hz. Overall, this yields a storage efficiency of around 90%. In real experiments, this efficiency could be lower due to possible mode mismatch in the cavity, so we also use a lower value of 80% in calculating repeater rates and fidelities in Fig. 5.3.

Moreover, $\Delta_a = \Delta_s + \delta_s$ where δ_s is the splitting between the states $|g\rangle$ and $|s\rangle$, which is around 0.46 GHz in $^{39}\,\mathrm{K}$ vapor. The dimensionless coupling strength ζ_1 is taken to be 10 to ensure we are in the strong coupling regime, which could be achieved by using a square pulse as the control field with the Rabi frequency $\Omega \sim 2\pi \times 1$ GHz and the duration $T_c \sim 50$ ns. The power of the control laser can be related to the Rabi frequency as $P = (\hbar \Omega/d_i)^2 c \epsilon_0 \pi R_w^2/2$ where d_i is the dipole moment of the D₁ line, and ϵ_0 is the vacuum permittivity, and R_w is the waist width of the control beam. Then, given the waist is on the same order as the radius of the cell, \sim 0.15 cm, the required power of the control laser is estimated to be around 2 W, and the energy is 100 nJ. In general, the larger the signal detuning Δ_s is, the more difficult it is to achieve the strong coupling regime as we need a stronger and longer control pulse, which can be seen from Eq. (5.15). In the strong coupling regime, the noise suppression factor x is given in Eq. (5.14), and when the storage and retrieval efficiencies are optimized, the reflectivity r is given by $r = (1 - \sqrt{1 - \alpha_s^2})/\alpha_s$ with $\alpha_s \approx \exp\{-d(\gamma_e/\Delta_s)^2\}$, which is estimated to be 93.2%. Thus, we obtain the signal readout fidelity $F_{\rm re} \sim 98.6\%$. The cavity linewidth κ_c is linked to r and the hyperfine splitting δ_s as $\kappa_c = 8\delta_s(1-r)/r$, which is estimated to be 0.27 GHz. Moreover, in the bad cavity regime, the bandwidth δ_B of this hybrid quantum memory is upper bounded by the cavity linewidth as $0.3\kappa_c$ [131], which gives $\delta_B \sim 80$ MHz. The size of the ring cavity is given by the length of roundtrip $L = \pi c/(2\delta_s) = 160$ mm. As for the time-bandwidth product, this hybrid quantum memory yields an unprecedented value of 2.8×10^{13} which is mainly attributed to the hours-long storage time in the noble gas and the large bandwidth of the hot vapor. The multiplexing can be implemented either spatially or spectrally. For spatial multiplexing, we envision having many hybrid memories in each node [199]. The spectral multiplexing also requires many hybrid memories in one node but the emitted photons need to be converted to different frequencies fed into a common channel [11, 202]. This can be accomplished using frequency translation which can be noise-free using waveguide electro-optic modulators [203]. The feeding to a common channel can be achieved by a tunable ring resonator filter that enables MHz-level resonance linewidths [204].

5.7 Conclusions and outlook

We presented a quantum network architecture based on hot hybrid alkali-noble gases that can operate without cryogenics. We showed that under realistic conditions, high-fidelity entanglement can be distributed over long distances thanks to the ultra-long coherence time of noble-gas spins. We showed that the rates of our proposed quantum repeaters can outperform direct transmission, and with realistic multiplexing, the rates can be greatly enhanced, close to the corresponding rates of NV centers and optomechanics-based repeaters. Furthermore, compared to the complex setup in room-temperature repeaters based on NV centers and optomechanics, this hybrid gas system only requires a moderate-finesse ring cavity, an external magnetic field, and optical pumping equipment. This significantly reduces the complexity of the system while offering a great potential to be scalable. We hope that this work could further stimulate the development of high-efficiency silicon single-photon detectors and even room-temperature detectors that offer both high detection efficiencies and low dark count rates for telecom photons.

We here have focused on hot atomic gas-based quantum repeaters on the ground, but this compact hybrid quantum system also offers a good potential for being used as memory in space [85], which could unlock the possibility of establishing a truly global quantum network [205, 206, 84] that goes beyond the limit of terrestrial quantum repeaters, and such a global quantum network could enable ultra-long distance quantum teleportation, quantum entanglement and applications in fundamental physics tests [85].

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Chapter 6

Conclusion and outlook

The realization of quantum networks will bring numerous applications that are beyond the capability of classical networks such as secure communication [1], blind quantum computation [2], quantum clock synchronization [80, 77], enhanced imaging of distant optical telescopes [207], and ultimately a global quantum internet that connects a broad range of quantum devices on a global scale. Quantum repeaters are indispensable components of quantum networks in the long-distance transmission of quantum information, thus having drawn substantial attention in the past two decades, and how to realize them still poses great challenges. Among the numerous challenges, the need for cryogenics and optical trapping greatly compromises the scalability of most proposed quantum repeaters. Thus, the main goal of this thesis is to advance the development of quantum networks by proposing two novel quantum repeater architectures that can operate without cryogenics and optical trapping.

The first direction we pursued is to utilize NV centers' excellent electron and nuclear spin properties at room temperature as communication qubits (generate spin-photon entanglement) and quantum memories. However, the zero-phonon emission line is considerably broadened at room temperature, thus preventing us from directly generating spin-photon entanglement. Inspired by recent progress in the development of room-temperature spinoptomechanical interface [149] which bypasses the phonon-broadening issue using optomechanics, we proposed a room-temperature quantum repeater architecture that builds on NV centers, high-Q cavities, and ultra-high Q membranes. We quantified the entanglement generation fidelity and efficiency using the so-called photon-number decomposition method. We also proposed to use the spin-optomechanical interface to read out electron spins at room temperature with high fidelity on ms timescales. Furthermore, we proposed an entanglement distribution protocol where the average distribution time shows logarithmic scaling with the number of links as opposed to linear scaling in conventional nested protocols, and the rates of the repeaters beat direct transmission.

The second approach to realizing non-cryogenic quantum repeaters is based on hot alkali and noble gases. Our approach builds on the cavity engineering method to suppress the detrimental four-wave mixing, in which a cell of hybrid gases is placed. The long-lived noble-gas spins are used as quantum memories, which coherently interact with alkali atoms via weak spin-exchange collisions. We investigated the single-photon protocol and quantified the entanglement generation fidelity and efficiency by taking into account the finite memory efficiency, channel loss, and dark counts in detectors. We also estimated the repeater rates and fidelities and compared them with the rates and the fidelities of the NV centers and optomechanics-based repeaters. Although the rates of this approach are generally lower than the rates of the previous proposal, it is promising to perform multiplexing to achieve comparable rates as it requires a relatively simple setup. Moreover, the overall entanglement fidelities of the previous scheme are higher than the fidelities of the previous scheme.

The NV centers and optomechanics-based approach holds promise for distributed quantum computing at room temperature [179, 173] where a cell containing a few electron and nuclear qubits can be entangled with other cells to perform computational tasks. Moreover, nuclear spins in diamond also offer the possibility to perform quantum error correction codes even at ambient conditions [159, 182, 208], which could enable fault-tolerant quantum communication [183, 41] and quantum computation. For hot hybrid alkali-noble gases, we only investigated the possibility of using them for building terrestrial repeaters but this system also offers the possibility of being deployed in space as it is relatively simple, which could unlock the potential of establishing a truly global quantum network [205, 5] that will bring many new exciting applications. At the same time, there is another interesting room-temperature quantum system: two-dimensional hexagonal boron nitride (hBN), which contains a color center that exhibits ultra-bright emission and Fourier transform (FT) limited lines at room temperature [209]. This system may serve as a spin-photon interface for long-distance quantum communication at room temperature without extra components and complex engineering in the future but it is still under investigation.

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