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USING VARIATION TO CRITIQUE AND ADAPT MATHEMATICAL TASKS

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We report on four key ideas we have found important in our work with teachers based on almost five years of research with the Math Minds Initiative. These ideas combine the Variation Theory of Learning with a strong focus on continuous assessment to inform the way teachers adapt task sequences offered in the resource used by project teachers. In doing so, we expect that teachers aim to better serve both struggling students and those who need extension as they develop coherent mathematical knowing. We elaborate on each one of these ideas, with examples from the Initiative in this paper.

Keywords: Variation Theory of Learning, continuous assessment, mastery learning

For almost five years, the Math Minds Initiative has centered on improving mathematics instruction at the elementary level in Calgary. Part of this work has focused on teacher professional learning, which is the focus of this paper. The initiative integrates research on formative assessment (Wiliam, 2011), intrinsic motivation (Pink 2011), mastery learning (Guskey, 2010) cognitive load (Clark, Kirschner, & Sweller, 2012), and variation theory (Marton, 2015), which we have integrated in a four-part framework to support the feedback we offer teachers (see Figure 1).

Here, what we call ribboning refers to an alternating pattern of tasks that draw attention to key ideas and assessment of each key idea. Ribboning draws on both variation theory and cognitive

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load theory to effectively direct attention to key ideas without overwhelming working memory. Clear patterns of variation among ideas that may be held together in working memory makes it more likely that connections between those ideas may be discerned. Formative assessment with careful monitoring of student feedback and teaching that is responsive to that feedback are essential to ensure that intended distinctions are in fact made before we ask students to extend or connect them. In this paper, we focus primarily on variation, but it is important to emphasize how variation supports and is supported by the other elements in the protocol.

Ribboning	Monitoring	Adapting	Connecting
Use structured variation to draw attention to potentially novel discernments necessary to a concept.	Ensure every student is able and obligated to provide feedback to the teacher in response to each ribboned query.	Revise/devise tasks, explanations, and other engagements to fit with demonstrated understandings.	Move between “part” and “whole” when ribboning to ensure that learners do not lose sight of the concept(s) under study.

Figure 1: Math Minds Principles (adapted from Davis, 2016)

In previous work, we have described some of the challenges teachers face in (a) supporting struggling students and (b) offering meaningful extensions (*cf*. Preciado-Babb, Aljarrah, Sabbaghan, Metz, Pinchbeck, & Davis, 2016). We have also highlighted some of the difficulties they experience when creating their own patterns of variation (Metz, Preciado-Babb, Sabbaghan, Pinchbeck, Aljarrah, & Davis, 2016). We have since asked teachers to consider the following questions as they examine the patterns of variation offered in the resource used by all project teachers: (a) What features of the concept are separated for attention? (b) Are the features we wish to draw attention to systematically varied? (c) Is variation set against a constant background? and (d) Are key ideas juxtaposed in a manner that highlights the desired pattern of variation?

Here, we highlight four ideas that have emerged as significant in our work with teachers and students around these questions. The first reiterates the importance of contrast: When *too many things* change or the *wrong things* change, students may not make intended distinctions. The second stresses that even tightly controlled variation may not be noticed if relevant items are not clearly *juxtaposed* and if students are not *invited* to notice what changes and what stays the same from example to example. In the third, we further distinguish between *parsing work into manageable chunks* and *using clear patterns of variation* to draw attention to key concepts within those chunks. Finally, in the fourth idea, we address concerns that tightly structured variation may allow students to merely extend patterns without understanding why they occur. In the following sections, we offer selected examples from our data set to exemplify each of these four ideas. Throughout, we maintain an emphasis on continuous assessment and adaptive response: None of the identified patterns should be taken as ideal or sufficient unto themselves. When tasks are organized such that each piece builds on understanding of the previous, assessing understanding at the end of a lesson or even at several intervals during a lesson is not enough.

ATTENDING TO DIFFERENCE: THE POWER OF CONTRAST

A subtle but powerful insight may be found in the distinction between systematic variation that varies the feature we want students to notice while holding other features constant (i.e., contrast) from systematic variation that varies everything else while holding the key idea constant (i.e., generalization). Marton (2015) noted that we tend intuitively toward the latter; i.e., when we want to help someone understand something, we offer many examples. But this only works if the idea has already been discerned and we need only to broaden perception of what it may encompass. It does not work well for introducing new ideas. Appreciating this distinction can shift how we adapt explanations and tasks to support struggling learners.

In presenting rounding at Grade 5, the resource used by the teacher first uses a number line to prompt attention to whether various numbers are closer to 0 or 10, then 10 or 100. It then offers a common procedure that involves checking the digit after the one being rounded to; if 5 or up, round up; if less than 5, round down. Practice includes a collection of numbers to round to 10, a different collection to round to 100, and another to round to 1000. This pattern was unproblematic for some.

One student, however, didn't make sense of this until asked to contrast *the same number* rounded to *different place values*, as in Figure 2. This student was particularly intrigued by the staircase pattern of zeroes that forms in the first set and by the fact that ten million rounded to zero. He was also interested in the third set, where all but the last case rounds to 2,000,000. What began as support for a struggling student went significantly beyond the original task of rounding to 1000.

	1	6	2	7	3	8	4
10							
100							
1000							
10,000							
100,000							
1,000,000							
10,000,000							

	1	9	6	9	7	9	8
10							
100							
1000							
10,000							
100,000							
1,000,000							
10,000,000							

	1	9	9	9	9	9	9
10							
100							
1000							
10,000							
100,000							
1,000,000							
10,000,000							

Figure 2: Using contrast to deepen understanding of rounding.

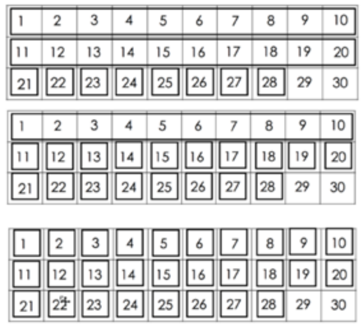
JUXTAPOSITION: THE IMPORTANCE OF *DIRECT* CONTRAST

Even when patterns of variation seem clear and teaching is responsive, however, there are times when patterns go unnoticed or unmarked. Marton (2015) did not directly address cognitive load (Clark, Kirschner, & Sweller, 2012). However, he stressed the importance of holding ideas simultaneously in consciousness, which requires attention to working memory as well as careful consideration of what it is that needs to be held together: "[T]he experience of sameness and difference is not only a function of what there is to be experienced but of what things are experienced simultaneously" (Marton, 2015, p. 66).

Effective juxtaposition supports simultaneous experience, and there are various tools and strategies that can be used to support this. Mathematical representations often juxtapose ideas in ways that allow particular relationships to be discerned. Consider the number line, the intersection of number lines that form the axes of a Cartesian plane, or the plotting of a unit circle on a mere 2 x 2 portion of that plane: Each allows new ways to experience mathematical relationships. We have also observed pedagogical strategies that support the clear and uncluttered juxtaposition of ideas in time and space.

In the rounding example offered earlier (and in other examples that follow), notice that the sequences were presented on grid paper. While this helped keep work neat and organized, it also did something more powerful: It made tracking variation easier by making it easier to see what changed and what stayed the same from example to example. It was still important to *ask* "What changes? What stays the same?" and to explore "Why?" in response to the student excited to see emerging patterns such as the staircase pattern of zeroes in the first set of rounding tasks.

The resource used by Math Minds teachers includes a series of slides for each lesson. The simple act of laying those slides side by side can help prompt attention to significant contrasts between subsequent examples. In the original lesson highlighted in Figure 3, each of the three images representing 28 and the T-Table were presented on different slides.



Tens	Ones
2	8
1	18
0	28

Figure 3: Representing 28 (Adapted from Mighton, Sabourin, & Klebanov, 2012, Slides 2-4).

After juxtaposing the images, the teacher asked “What is the *same* about the three charts on the smartboard?” Initial responses focused on the open 29 and 30 in each chart. Her next question prompted attention to difference: “Do they all look *exactly* the same way?” The students then noted the changing number of tens and ones blocks, which she recorded in the T-Table beside the images.

ATTENDING TO LIMITS ON WORKING MEMORY: PARSING WITHOUT FRAGMENTING

At times, demands on working memory (Clark, Kirschner, & Sweller, 2012) make it useful to separate parts of a more complex procedure such that they may be learned in manageable pieces and then put back together. While important, this alone does not draw attention to the mathematical ideas embedded in those steps. In the resource used by teachers in our study, long division at Grade 4 is parsed into steps designed to reduce load on working memory. Students first work with two-digit dividends and a one-digit divisor; the procedure is parsed into steps that are explained conceptually, then practiced for fluency before adding the next step. For some, this was sufficient. Others struggled to remember the steps; among those who did, some could not explain them. We then offered a practice sequence that systematically varied the divisor, as in Figure 4.

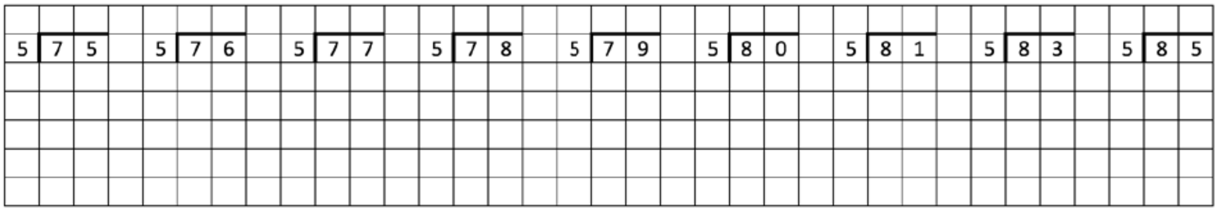


Figure 4: Systematic variation of practice for long division.

Because only one thing changed, students could now see (when prompted to look) how various parts of the algorithm changed (or not) each time the dividend went up by one (again, note the use of grid paper to allow easier tracking of changes from example to example); they could also see that the remainder went up by one each time. When asked why, they could more easily trace the cause of the changes. Further, because some of the requisite number facts were constant from question to question, less attention was diverted from understanding the algorithm. At 80/5, the remainder cycles back to zero, which prompted further insights. It became clearer to students that the quotient is counting *groups* (alternatively, it could have been seen as representing an amount *per group*), while the remainder is counting *singles*; i.e., there were important opportunities for noticing that wouldn't have been there with more random practice. Significantly, it didn't take long before the students using these patterns started *looking* for connections. Again, what began as a support for a struggling student soon exposed a pattern that prompted deeper insights into division than the original task sequence.

HIGHLIGHTING CONNECTIONS: BRIDGING BIG IDEAS

Tightly structured sequences in which only one thing changes sometimes create predictable sequences that allow students to predict answers based on observed patterns rather than understanding of those patterns. However, it is easy to trivialize what to students may be important insights foundational for deeper understanding.

A Grade 5 class was trying to find all the ways to make 45 cents with dimes and nickels. They had been instructed to start with zero dimes and work their way up to 4 dimes, each time figuring out how many nickels would be needed to make up the balance and recording answers in a T-Table. Two students struggled to find the balance of nickels needed. To support them, they were asked to make marks for nickels, counting by fives to know when they had 45 cents. When asked, "How many nickels?" they at first wanted to count by fives again and had to be reminded that each mark was a nickel; they then counted nine sticks. Next, they were instructed to circle two nickels and to think of this as trading two nickels for a dime. They counted the total again, this time starting with 10 for the dime and adding five for each nickel: 10, 15, 20, 25, 30, 35, 40, 45. When asked how many nickels remained, both counted the remaining sticks and wrote the total in their T-Tables. They were then asked to make another dime, count the money to ensure the total was still 45 cents, and to count the remaining nickels. While this may seem like rote repetition of a procedure, after three repetitions, one of the students excitedly noted, "The nickels are going down by two every time!" and related this to the circled nickels. For him, this was a big insight. The other was excited to discover that the leftover stick at the end couldn't be used to make a dime. The students needed help setting this up again for 75 cents, but they started using observed *patterns* in ways that they hadn't done before; i.e., they could see more clearly that every time they added a dime, they had to take away two nickels. They then worked with quarters and nickels, nickels and pennies, and dimes and pennies.

IMPLICATIONS AND NEXT STEPS

By using clear patterns of variation coupled with prompts to attend to that variation, bridges as appropriate when perceptual jumps are too big, and breaks in patterns that push understanding beyond predictable patterns, we have asked teachers to move beyond what the resource often describes (appropriately but insufficiently) in terms of scaffolding and attending to limits on working memory. Teachers in the initiative have relied heavily on the resource to define and sequence critical features for learning particular topics. However, they are assuming greater responsibility for what is set side by side in the moment-by-moment sequence of a lesson.

To date, much of our work with teachers has been on connections *within* lessons. Moving forward, we will further emphasize how careful attention to patterns of variation might support stronger connections over time—i.e., between lessons, units, and grades. Like scaffolding, review of material addressed in previous lessons is emphasized and supported by the resource. As with scaffolding, conceptualizing review in terms of using variation to carefully juxtapose the old and the new can support strong connections between key ideas. Contrast is most powerful when directly experienced and not reliant on memory of a lesson done last year, last week, or even earlier in a lesson. As doing so requires a broader awareness of what students are expected to understand over time, we will consider ways to support teachers who are often most familiar with a single grade or a small range of grades to look beyond the boundaries of their teaching assignments.

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