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Understanding Math 10 Students’ Experiences of Struggle to Learn Mathematics

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Understanding Math 10 Students’ Experiences of Struggle to Learn Mathematics

by

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Abstract

Students have much to tell educators about their experiences of learning mathematics. Their experiences include struggles that may either be productive as they persevere to understand or destructive as they feel that learning is frustrating and painful. The purpose of this study was to gain an understanding and new insights into how students struggle to learn math, the different ways that students struggle, and how this experience shapes how they respond to future encounters of difficulty, as well, as their attitude towards math. Six Math 10 students were interviewed using semi-structured questions. Students’ lived experiences were explored using interpretive methodology. Students spoke about their experiences of confusion, disconnection, encountering hidden rules, not understanding why, freezing on a test, and having to learn concepts that they found were irrelevant to their lives. These experiences may provide new insights and understandings of how teachers may better teach students.
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Chapter 1: My experience teaching mathematics

Throughout the eight years of teaching junior and senior high math, I encountered many students in my classes who struggled in mathematics. Typically, these students would tell me that they were stuck on a question, or were unclear about a concept. I noticed various ways that students worked to overcome these struggles. Some students looked through their notes or textbook, while others asked a peer or me for help. In most cases, they would be able to find an answer to their question, and move forward to complete the task. But others would not. These students continued to struggle and became very frustrated. Sometimes, after class, some would come to a help session with me for individual help. During this time, we would review earlier concepts, and sometimes, just giving them a few cues would help. However, sometimes during these sessions, students would sit down and tell me they just did not understand any of the math. They seemed extremely unhappy; some even had tears in their eyes. It appeared to me that their experiences in math caused them unnecessary suffering.

It was through these individual help sessions that the nature of student struggles became clearer to me. Every student I worked with presented their problems with the hope that I would help them overcome their difficulties. As we talked and worked through questions, they repeatedly demonstrated their understanding of mathematics and how concepts related to each other. However, their reflection and problem solving successes were not always enough to convince them that they understood and could continue. The students I worked with could demonstrate their abilities to me, but they were unable to translate that experience into future success in math. Why could I teach them a math concept, but not instil the confidence to work through their next challenge in math? Why did they still not believe in their ability to be successful at mathematics? Why were they not able to understand the concepts? What was
missing? Were my observations not reflective of their experience (Rodgers, 2006)? I am curious about this phenomenon. As such, it frames the basic questions for my study: how do students struggle in mathematics, and how do their struggles influence their attitudes toward mathematics?

**Struggle**

It is interesting to look at the word “struggle” itself. It is derived from the German word *straucheln*, which means to stumble (OED online, 2017). I can picture myself stumbling after my foot bumps against an exposed tree root on the trail. I might trip and possibly fall, but I would likely get up and continue to travel along the pathway toward my destination. In learning math, when I stumble, I may stop, review, modify the work, or ask for help, but I continue to solve the problem. The reason I continue is because I enjoy this journey. I enjoy mathematics. My curiosity motivates me to find solutions and to find connections between the concepts. I am always in awe when I find a link and realize the interconnectedness of mathematics. I have struggled and continue to struggle in understanding math; however, I believe that it is part of the process and not a reflection of my inability.

Mathematicians also struggle to find solutions to problems. For example, finding a proof to Fermat’s last theorem took 350 years (Singh, 1997). In his book, Fermat’s Last Theorem, Singh (1997) tells the story of the mathematician, Andrew Wiles, who spent eight years working on this unproven statement. At a conference in the company of fellow awestruck mathematicians, he wrote out his proof. Everyone was astounded. Unfortunately, days after his proof had been written out for all to critique, an error was found. He too “stumbled”. However, with the help of another mathematician, Wiles was able to find another theorem to support his
earlier proof and finally prove the theorem that had long evaded him and many other mathematicians. Why did Wiles continue despite frustration and possible embarrassment of his initial failure? Perhaps he did not see the erred proof as a failure. Perhaps the belief that a solution to Fermat’s last theorem existed, compelled Wiles to continue to search for a solution.

Mathematicians speak about their work in mathematics without using the term struggle. G.H. Hardy believes mathematics to be a creative art, in which the mathematician makes patterns of the ideas (Hardy, 1967). W.S. Anglin and Andrew Wiles compare their experiences to a journey in which one may get lost or stumble in the darkness (Singh, 1997). In the documentary, The Proof, by Lynch (1997), Wiles said

Perhaps I could best describe my experience of doing mathematics in terms of entering a dark mansion. One goes into the first room, and it's dark, completely dark. One stumbles around bumping into the furniture, and gradually, you learn where each piece of furniture is, and finally, after six months or so, you find the light switch. You turn it on, and suddenly, it's all illuminated. You can see exactly where you were. (0:04)

Interestingly, Wiles uses the word stumble, the derivation of struggle, to describe his lengthy mathematical experience in which he is searching for understanding. He describes an exploration of his surroundings in search of familiarity. Singh (1997) describes Wiles’ attempt to find a proof, as a battle. Alternately, I view each problem as a puzzle that can be solved by looking for patterns. These views of mathematics show a willingness to create, discover, engage, and persevere with the belief that all problems are solvable.

The students that I interviewed did not share the same curiosity and passion about math as the mathematicians; however, they too struggled. Five students in this study said that
mathematics is a system of steps that must be understood and followed. One student used
the word linear to describe this process of following steps and another student said that she simply
must find the correct formula to solve a question. They described their struggles to understand
math with feelings of confusion, frustration, and dread. Yet one students said that she enjoyed
working with numbers and found it satisfying to get the correct answer. Students’ experiences of
struggling may be different from those of mathematicians; however, their stories both describe a
perseverance to move through these struggles.

Warshauer (2015) investigated the types of struggles that students faced and the ways in
which teachers responded. In her study, Warshauer (2015) discusses a “productive struggle” as a
mathematical challenge that a student faces which falls within their reasonable capability. She
believes productive struggle to be a natural part of learning mathematics with understanding. Her
research shows that during this struggle, a student “muddles through” (Brownwell & Sims, 1946
as cited in Warshauer, 2015), “grapples” (Warshauer, 2015), and “reaches an impasse” (Van
Lehn et al., 2003) to construct deep understanding (Hiebert & Grouws, 2007). This raises the
question: as students struggle to learn math, are they experiencing a productive struggle or one
that causes them to suffer unnecessarily, or what might be called a “destructive struggle”?

As an educator, I have a commitment to help students learn math. As a mathematician
and a lifelong student of the subject, I want students to enjoy learning and solving math. I want
students to be enthusiastic and curious during their math class and take risks without the fear of
making mistakes. I have always found math interesting and exciting. Its rich history, curious
puzzles, and many connections are enjoyable to learn and to teach. My hope is to provide a
learning environment so that every student can have a positive and enjoyable math experience in
which they want to persevere when faced with a struggle.
The Purpose of My Study

The purpose of my study was to understand the experiences of grade 10 students who had difficulties learning mathematics, both those who struggled productively, and those for whom struggle was a destructive force. By “destructive force” I mean the struggle that causes students to find learning and working at math to be painful enough to give up trying to understand, and possibly even hate math and quit. The six students that I interviewed all shared experiences of productive struggles and two students spoke of experiencing Math 10 as a destructive struggle. As I listened to their experiences, I heard students reiterate their need to understand the concepts and procedures. They did not simply want to memorize what to do, they wanted to know why. On their journeys to achieve understanding, the students in this study shared with me what it felt like to experience confusion and disconnection, encounter hidden rules and freezing on a test, and learn math topics that seemed to have no relevance to their lives. Their experiences have provided me with insight into what allows some students to persist and feel successful, while others to want give up.
Chapter 2: What the literature says about struggling in math

To understand students’ productive and unproductive struggle in math, I examined the literature on what both students and mathematicians say about their mathematics experiences, and their perceptions of mathematics. As my study was shaped by student voice, I have primarily included qualitative literature as well as some quantitative literature of students’ experiences and perceptions in their struggles with mathematics. Based on what they revealed about their encounters, I have categorized these studies into three areas: 1) mathematics is boring, 2) mathematics is difficult, and 3) mathematics is enjoyable.

Mathematics is Boring

A 2003 study by Nardi and Stewart investigated the reasons why some students are disengaged and bored during their math classes. In their study, they observed and interviewed seventy 13 and 14 year-old students in England and found five characteristics that these students used to describe math: tedious, isolated, rote learning, elitist, and depersonalized. The students’ lack of enjoyment or appreciation of math classes led to feelings of resigned acceptance. These students took math because it was required, but they found it to be dull and boring. They considered concepts to be meaningless, irrelevant, unimaginative, and tedious. Some of the real-life problems or projects teachers used were viewed by students as unrealistic situations in which they would never find themselves. For example, students were asked to design a table which had to have a specified pattern of squares. Students felt these types of problems were boring, unpleasant, and irrelevant to their lives.

There is evidence that students feel that teaching methods that include collaboration and provide more meaningful understandings, can improve their engagement in mathematics (Attard,
The traditional method of textbook style teaching which involves rote memorization and task completion has been shown to isolate students (Nardi & Stewart, 2003) from their peers. The students in Nardi and Stewart’s (2003) study preferred working with others to working in isolation because they believe that their peers give them clear, straightforward explanations. In addition, they believe that their conversations about mathematics with their peers allow for a better understanding because students must explain and negotiate their understanding with each other. Students in this study suggest that in situations where they are stuck and can no longer proceed with a question, it may be easier to ask a peer than wait for the teacher to assist. Students in my study echoed the need to sit with their peers, because they quickly and easily could ask for help when they were stuck on a question. Working in groups enables students to ask each other to clarify or offer a new perspective. This collaboration, according to Nardi and Stewart (2003) leads to a more efficient understanding.

Students in the Nardi and Stewart (2003) study also found the repetition from rule and cue following to be frustrating and annoying. The authors wondered if the rigidity of memorizing and following exact procedures, although considered efficient, impaired a deeper understanding and enjoyment of mathematics. For example, when presenting an abstract form of a mathematical concept, such as a formula, the authors suggested that teachers should show students how the algorithm works. They believe that simply providing a formula without presenting the mathematical ideas to explain why it works, “mystifies” the learner. Similar to students in my study, students want learning opportunities to understand why and how formulae, concepts, and procedures work.

In a longitudinal study over three years, Boaler (1998) conducted both quantitative and qualitative research methods in the form of questionnaires, interviews, assessments, and
observations of students from two schools with differing instructional practices. At Phoenix Park school, teachers designed open ended math projects for students to work on, and at Amber Hill school traditional methods were used to teach math. Boaler and Selling (2017) described the learning at Amber Hill to be passive engagement. In a passive engagement setting, the math class begins with the teacher introducing a concept, and the remainder of the time available for students to practice questions from their textbooks. Boaler (1998) found that most students from this school described math as boring because the questions were repetitive and they simply had to follow a procedure to get the right answer. Students from this school reported that the key to being successful in math was to memorize a formula or method.

The ways in which teaching methods can influence students’ understanding of mathematical concepts was examined in a study by Pesek and Kirshner (2000). Students in six grade 5 classes were divided into two groups and taught a perimeter and area unit. One group was given rule and cue instruction, also defined by Skemp (1978) as instrumental learning, in which students are taught the rules without developing the understanding of why they exist. For a period of five days, students in this group were presented with the formulas for the area and perimeter of squares, rectangles, triangles, and parallelograms. The instructor explained and demonstrated the operations and connected them to the proper shape and finally assigned problems for practice. The construction of the formulas and how they related to the shapes was not discussed. After this series of lessons, over a course of three days, both groups were instructed relationally. Relational instruction as defined by Skemp (1978) provides students with an understanding of what they are doing and why they are doing it. Understanding is gained through reflection, explanations, reasoning, making connections, and communication. While working in groups, students in this study used concrete and personal objects to construct their
own methods to calculate area and perimeter. The instructor never provided specific strategies for efficient calculations. Pesek and Kirshner (2000) found that students who had only received relational instruction performed better on a post test and retention test (given two weeks after the post test) than students who initially received instrumental instruction for five days and then relational instruction for five days. Six students from each group were interviewed after the lessons and all agreed that they enjoyed the relational instruction. However, the interviews showed that students who received both types of instruction demonstrated confusion around when to use a certain formula and had troubles explaining the formulas. Yet, students who only received relational instruction could provide partial explanations of the formulas. The authors concluded that “initial rote learning of a concept can create interference to later meaningful learning” (p. 537). They noticed that students who received only relational instruction within a shorter period of time (three days) could conceptually and flexibly construct solutions, whereas students from the mixed instructional group, who received eight days of instruction, referred to their formulas and fixed procedures. The authors noted the significance of this observation, because the general opinion of teachers is that procedural or rote learning is more efficient.

Rote learning or rule following is considered an easy and quick way to teach the concepts required for standardized tests (Pesek & Kirshner, 2000; Nardi & Stewart, 2003). However, Nardi and Stewart (2003) observed that this learning experience is aimed at helping students be successful on tests, rather than focussing on an individual meaningful understanding. Furthermore, testing stratifies students into two groups: those who succeed and those who do not. Those who did not succeed expressed alienation from their mathematical experience. Their struggles in mathematics led to anxiety and nervousness on tests. Students testified that their lack of success risked exposing their weaknesses, and worsened their perception of their own
intellectual ability. This painful experience of learning mathematics was believed to further disengage students.

There is research to support the important role that a teacher holds in engaging students and making math relevant and interesting (Attard, 2011; Boaler 2008; Nardi & Stewart, 2003). A longitudinal study conducted by Attard (2013) in Australia followed 20 grade six students over three years to determine their views on mathematics teaching and learning. In semi-structured interviews, students with high levels of engagement were asked to discuss qualities of good lessons and good teachers. Similar to a student in my study, these students preferred “tasks that required active learning, elements of choice, challenge and the option of self-directed activities” (Attard, 2013, p.575). When students were engaged in learning they said that they felt empowered, flexible, and creative (Attard, 2011). They believed that their teacher was the most important influence in their mathematical engagement and that the teaching methods used, as well as the relationship developed with students, improved their enjoyment of mathematics. A student in my study also spoke of the need for teachers to provide creative learning situations to accommodate different learning styles. Attard’s research (2011, 2013) showed that a good teacher, one who engages students, is aware of both students’ needs and abilities, and will differentiate a task when necessary to suit the individual. The students in this study also agreed that this type of teacher is passionate and enthusiastic about mathematics, and explains things well.

Attard (2011, 2013), Brown et al. (2008), and Masingila (2002) found that connecting in-school mathematics to out-of-school mathematics improved students’ perceptions of mathematics. Masingila (2002) had 20 middle school students keep a daily log detailing the mathematics they used outside the classroom. Her findings supported the idea “that one’s
perception of how one uses mathematics is a function of one’s view of mathematics” (Masingila, 2002, p. 37). In other words, meaningful connections of mathematics to an individual’s life lead to engaged students.

In contrast, incorporating real life situations was viewed by a group of grade 9 students in the United Kingdom as unimaginative, personally irrelevant, and unoriginal (Nardi & Stewart, 2003). As mentioned earlier, these students found the real-life situations intended to be representations of reality, such as constructing a table or building a fence, to be contrived, nonsensical, and boring (Boaler, 2016). Nardi and Stewart (2003) propose that it might be important for teachers to select appropriate and relevant situations for dynamic open-ended mathematical problem solving and to learn transferable skills that are based in the students’ lives. Boaler (2016) recommends an appropriate situation might involve students using modeling to “apply the mathematics that they know to solve problems arising in everyday life, society, and the workplace” (p. 195) or proposing an interesting question which might arouse a natural curiosity to critically and creative think, reason, and engage.

**Mathematics is Difficult**

Brown et al. (2008) explored why students stopped studying A-level mathematics after age 16, by means of a four-page questionnaire which surveyed 1997 sixteen-year-old students from 17 schools in England. The primary reason identified by students was that they believed that the math would be too difficult. In the free response section of the questionnaire, students indicated that this perceived difficulty originated from information from older students, families, and their teachers. The researchers believed that these external messages had negative consequences on student identities and thereby on their choice not to take higher level courses.
Students who obtained a B on their compulsory exam were receiving messages from teachers that they had not done well, even though the majority of colleges and schools accepted a grade of B. Brown et al. (2008) concluded that a set of strategies to address the attitudes of 16-year-old students, their teachers, and older students is necessary to improve student participation in higher level mathematics. The researchers also recommended that schools break down mathematical structural and personal barriers. Structural barriers are defined as curriculum hierarchies, tiered classes, and assessments which lead to students feeling that they are unqualified, when undeniably they would be accepted into higher level mathematics courses. According to the authors, breaking down personal barriers, such as a student’s perceived inability in mathematics can lead to an increase in their confidence to learn math. These strategies have also been supported by other research (Boaler, 1997; Nardi & Stewart, 2003). However, Brown et al. (2008) caution against protecting students from the challenge of mathematics in that it may result in boredom rather than positive attitudes and engagement. They believe that it is not possible nor desirable to take the “challenging sting” (p. 16) out of mathematics because it is a natural part of learning and life.

In the free response section of the questionnaire, Brown et al. (2008) noticed that students expressed stress, anger, and pain about their mathematical experience. One student said “I enjoy it when I get it right, but I didn’t choose it because I hate it when I get it wrong and get frustrated” (p. 10). Perhaps this student meant that he/she does not enjoy math because of the frustration that arises when their answer is wrong. Another student simply said “I hate mathematics and I would rather die” (p. 10). These researchers believe that these emotionally charged responses may be from students that have frequently been “completely stuck and unable to proceed with a problem” (p. 10), what I have termed earlier, a destructive struggle. However,
Brown et al. (2008) also suggest that a dislike of math may also stem from other reasons, such as how mathematics is taught or its relevance to their lives.

Nardi and Stewart (2003) found that students perceived intelligent people to be successful in mathematics. Those students weak in the subject say that they risk exposing their perceived weak intelligence. This belief was supported by the overwhelmingly negative responses of students’ self-images of their mathematical ability. One of the students in the study described the top math students “as ‘frightening’ because ‘they just seem so clever’” (p. 359). The researchers note that this same student is a top English student. Nardi and Stewart (2003) wonder why students perceive mathematical ability to be linked to an individual’s intelligence.

An alternate approach to overcoming student difficulty is to simply accept students’ belief that mathematics is difficult, and that encounters of struggle are a normal response to learning mathematics (Pendlington, 2006; Boaler, 2016). Over 11 weeks, Pendlington (2006) worked in a separate classroom with six of the lowest achieving 10-year-olds. Using video and interviews, her intent was to improve both student’s self-esteem and their ideas about learning mathematics. These students shared their past difficulties: lack of understanding, lack of time to finish their work, and puzzlement over their failures. Pendlington (2006) realized that they associated their struggles with failure because they made negative comparisons. She believes that struggles are not problems to be avoided, but are “a normal part of learning” (p. 7). I concur, learning mathematics is challenging and math problems can be difficult. Pendlington (2006) advises that it is the “giving up” that should be avoided. Pendlington (2006) used affective strategies such as warning students that the task that they would be working on would be difficult and praising their persistence. She successfully brought the concept of struggle into the mathematics classroom culture to encourage students to remain engaged. She made connections
with a real life hero known to students, and his struggles to succeed. She demonstrated that, at least in her case, acknowledging students’ struggles can be the first step in helping students to persist and create an understanding that struggling is not shameful.

**Mathematics is Enjoyable**

While mathematics frustrates some, others students seem to enjoy it and build careers around it. In this section, I will explore what students, mathematicians, and scholars of math have to say about their experiences.

There is little research on students’ experiences of enjoying mathematics. A study by Darragh (2015) interviewed 22 students four times over the span of grade 8, 9 and 10 to understand their experiences in math and what they say makes a good math student. Parents and teachers were also interviewed to gain a better portrayal of the participant. Grade 8 students defined students who were good at math to be those who did well on tests, finished their work quickly, just knew what to do, could explain the concepts, and help others. As this group of students moved to secondary schools, grade 9, their responses changed. To them, successful students were the first to respond to a teacher’s question and those students who always asked questions. One student in particular, Estelle, spoke about her enjoyment of mathematics during the grade 8 interview. She said that she liked learning cool new things and how everything in math links together. She shared how she liked to “play” with numbers. “Just making little pictures out of numbers and writing down random numbers and just seeing what different equations I can make of those numbers … it’s just really fun” (p. 97). However, once Estelle began secondary school, Darragh (2015) found her to be somewhat disenchanted with math. During their interviews, Estelle described math as memorizing and writing rules. “We just learn
the proper rules and the proper procedure of working stuff out” (p. 97). As such, Estelle portrayed a person to be good at math if they could understand all the rules and proper procedures. She claimed that although she tries to reach understanding, she is not always successful. In grade 8, Estelle designated math to be her favourite subject, but by grade 10, it had fallen to a third place tie with science. What may have caused her excitement to wane?

In the book *Loving and Hating Mathematics* (2011), Hersh and John-Steiner portray the delights and challenges encountered by both mathematicians and those who have a studied math. When asked about their experiences in math, mathematicians and scholars express their fascination with numbers, the excitement of getting a result, or the astonishment over the relationship that mathematics has with other subject areas. They also share their experiences of frustration and insecurity.

Curiosity about the numbers themselves and observations of number patterns are part of a common story that is told by mathematicians. In *Loving and Hating Mathematics* (2011), neurologist Oliver Sacks recounts his experiences and his love of numbers. As a child, he was quick with numbers and liked that they were absolute and certain. During the World War II bombing of London, he was sent away from his family, and while in solitude, he mused with numbers, in particular, prime numbers. He loved them because they are indivisible and form the building blocks of other numbers. He wondered whether a pattern to finding other prime numbers exists, and whether they go on forever. He spent hours searching for new prime numbers, factoring numbers, and memorizing them. Numbers offered him hours of immersed solitary play during a time of confusion. A similar experience with numbers was told by physicist Eugene Wigner. At the age of 11, Wigner was diagnosed with tuberculosis and spent a difficult time in a sanatorium. To get through this period he played with geometry, challenging
himself to construct triangles given the lengths of the three altitudes. It took him months of focused determination to solve this problem. Later when he returned to school, Wigner was introduced to the Rule of Fifth Powers. He describes himself as enchanted when he noticed “that the fifth power of any one-digit number ends in that same number” (as cited in Hersh et al., 2011, p. 11). Both individuals experienced feelings of enchantment and fascination with the patterns that exist among numbers.

It was the connectivity and real life applications of algebra and other areas of math or science that amazed mathematicians Steven Strogatz and Israel Moiseevich Gelfand. Strogatz recalls his high school physics lab in which he compared the length of a pendulum string with the time taken for the pendulum to complete a swing. Once he had plotted this data on a graph, he was surprised to see it formed a parabola. The same parabola that he had constructed in his math class and the same shape that water formed as it comes out of a fountain. It was at this moment that he observed a connection between the order of the universe and mathematics. This moment was an epiphany for him, that he says he never really recovered from. Gelfand also describes the surprise he felt, when at age 15 he received a math book showing a formula which revealed that algebra and geometry were united in mathematics and physics. There is an appeal to math because it is a complex subject which embodies many areas such as algebra, geometry, probability, as well as nature.

Some of the mathematicians examined in the book say that their enjoyment of math stems from its nature: that it is trusted, real, sequenced, practiced and clear. Mathematician Terence Tao said that he likes “situations where there are very clear rules of what to do” (Chang, 2007, p. 2). Oliver Sacks likes numbers because “they were solid, invariant, they stood unmoved in a chaotic world” (Sacks, 2001, p. 26). Mathematician Julia Robinson expressed her like for natural
numbers (1, 2,) because “they are the one real thing” (Reid, 1996. p. 3). The ability to trust in the certainty of the patterns of numbers provides these mathematicians a sense of security and appreciation.

In a study of mathematicians by William Gustin (1985), he found that mathematicians are both gripped by excitement and fear. Feelings of excitement and satisfaction arise after a concentrated effort that has led to a new result. However, some of Gustin’s participants said that they felt insecure in their ability to create something new or were fearful that their efforts may lead them to a dead end. Despite the occasional failure, it “is a feeling of awe” and privilege to add “a couple of things to the field” that drives a mathematician (Gustin, 1985, p.329).

Each of these mathematicians has found enjoyment in learning or doing mathematics. Patterns amongst numbers sparked their curiosity, the connectivity of mathematics to different subject areas intrigued them, and they have felt pure joy over solving a problem.
Chapter 3: The need, approach, and conduct of the study

There is much quantitative literature on students’ experiences in their math classes, but little qualitative literature about what students have to say about their struggles in mathematics. However, there seems to be a common belief that many students do struggle. This study began with my interest in understanding students’ lived experience of struggle, because I felt I was not able to help students who continued to struggle. I could not understand why they struggled even after I had helped them individually. There seemed to be some part of their experience that I was missing. My goal was to listen to students about what they had to say about their experience so I might discover a new understanding of what caused students to struggle, how they struggle and how I could better help them.

van Manen (1990, 2014) ascertains that reflecting on lived experience can provide new insights and understandings this experience. van Manen (2014) observes that as individuals recount an experience, details that were not previously apparent may be revealed.

“Experience as we live it from moment to moment, is always more complex, more nuanced, more richly layered than we can fathom, and meanings emerging from reflecting on lived experience are always ambiguous, enigmatic, and ultimately unfathomable.” (p. 42)

In talking to students about their everyday experience of struggle, I heard details which led me to a richer understanding of their struggle. Details that I had not previously considered contained descriptions of behaviors, expressions, actions, intentions as well as the experience itself. As each student recollected experiences, I became thoughtfully aware of seemingly insignificant
details that offered a deeper understanding of what this experience was like for the student and how I could use this information to help future students with struggles (van Manen, 1990).

**Research Questions**

This study attempted to generate understandings of students’ mathematical experiences relating to the following research questions:

1. How do students experience struggling in math?
   a. What are there differences in the way they struggle?
   b. How are some ways productive: experienced as a challenge where students want to persevere?
   c. How are some ways unproductive: experienced as painful and destructive?
   d. How does a student’s experience of struggling shape how they respond to subsequent encounters of difficulty?

2. How does the way that students struggle influence their attitudes toward math?

**Methodology**

This study used an interpretive methodology to answer the research questions and gain new understandings of what it means to struggle in math. Interpretive methodology is used to uncover the meaning of students’ lived experience. Since I was curious about the broader scope of how students struggle in math, I used an in depth interview to gain insight into the bigger picture of both their struggle and how they understand and create meaning out of their struggles. “The purpose of studying other people in depth is not to measure, predict, or classify them. Our purpose is to understand, more extensively or more deeply, other people’s experiences of some aspects of their lives (Josselson, 2013, p. viii).
German philosopher, Hans-Georg Gadamer (1900-2002) believed that dialogue and conversation are essential tools to create new understandings of a phenomenon such as struggling (Binding & Tapp, 2008). In Binding and Tapp’s (2008) exploration of Gadamer’s notion of genuine conversation, they suggest that it is within the questioning and answering that participants may reveal something new about the topic, broadening its scope to create new understandings for each participant. Within this study, I was not able to discover the understandings that the students may have gained through our conversations. However, the conversations revealed new and further insights about my understanding of how and why students experience struggling.

The combination of my own experiences as a teacher and my review of the literature helped form my initial understanding of how and why students struggle in math. Further exploration into student experience has helped me deepen my understanding of what it means to struggle, why students respond the way they do, what effect that has, and how they understand their experience (Rubin & Rubin, 1995). I chose interpretive methodology to provide students with the opportunity to freely discuss their mathematics experience, so that I may see, and understand how the students live through it, an approach confirmed by Smith, Flowers & Larkin (2009). Often educators infer either from evaluations or observations, “looking in from the outside” and do not get an accurate interpretation of student experience (Flutter & Rudduck, 2004). Indeed, eliciting students’ stories about their experiences has provided me with rich insights on the experience of struggle in math.

I conducted my interviews guided by hermeneutic principles to gain a better understanding of struggle. These principles are authentic dialogue and the movement from the part to the whole. Authentic dialogue requires actively listening to the other person with the
notion that the other person is right. Josselson (2013) describes that it requires an openness to listen what the other person is saying and asking questions to make meaning of how others understand their experience. Words cannot be assumed to carry the same meaning for both participants. Through this questioning and answering, the participants move from gaining understanding of parts of the topic to gaining a better understanding of the entire topic. Josselson says that this holistic view “must attend to the intricate relationship of the whole and the parts, recognizing that they are mutually constitutive” (p.6, Josselson, 2013). Each piece of reading or conversation contributes to a greater understanding of the topic. Hermeneutics as discussed by Moules, McCaffrey, Field, and Laing (2015) is the practice of interpreting the meaning of lived experience. Understanding lived experience requires being in that person’s life world or experience and truly understanding what that situation is like for the individual (Moules et al., 2015). The hope in hermeneutics is that both the participant and the researcher are transformed through an authentic dialogue (Moules et al., 2015). In Gadamer’s hermeneutic practice, a conversation takes place in which each person is open to the other’s perspective and attentive to what is said (Moules et al., 2015; Gadamer, 2007). In my interviews, this dialogue explored the student’s experience with the intent that both I and the student interviewed might expand our personal understandings of struggle. I had opportunities to hear and gain insights into students’ past and present experiences of how they struggle. The possibility arises that through this dialogue, both the participant and researcher are influenced by each other to see, think about, and act differently in the future (Moules et al., 2015; Gadamer, 2007). Although I was unable to hear whether students experienced such a transformation, I certainly gained new understandings that have influenced the way I approach mathematics teaching. This transformation of self is an
outcome of hermeneutics which takes “what is assumed and unquestioned and look(s) at it as something new and exotic” (p. 75, Moules, et. al., 2015).

I was challenged to view struggling in math with fresh eyes. In order to learn from the student’s experience of math, it was imperative that I listen attentively and without judgement about their experience, and humbly view that what they say, think, and feel. By humble, I mean that I invited the students to have an impact on the way I think about student struggle in math and believed what they had to say to be true.

The nature of the interviews

I endeavoured to engage students in a meaningful conversation about their experiences within semi-structured interviews. By semi-structured, I mean an interview in which the interviewer enters the topic with an open-ended question, and then follows these with questions specific to the participant’s story to guide the discussion (Rubin & Rubin, 1995). During the forty-minute interview I followed my opening question with four questions which enabled me to understand their experience from different perspectives. I noticed that sometimes students answered these questions even before I asked them. I will discuss these questions in more detail later. In contrast to structured interviews, in which specific questions are posed, I chose this approach because it allowed for movement of the conversation to form around the student’s particular experience. I thought that using an unstructured interview method, in which the interviewer opens with a question but allows the participant to lead the discussion, might lose focus of the topic. The semi-structured interview seemed to provide an atmosphere of empathy, curiosity, and respect that enabled students to speak easily and freely about their experience of struggle, while remaining close to the topic.
This kind of interview requires the researcher to employ good listening skills. Josselson (2013) urges the interviewer to actively listen, as if from inside the participant’s perspective and wonder about how this experience was for this person. It necessitates paying attention to what students have to say, learning from them and valuing what they have to say. In doing so, any follow up questions that I asked were used to explore and provide a better understanding of the meaning of the experience of struggling. “Good listening means exposing ourselves to the unknown; it involves giving up our usual frameworks and immersing ourselves, intellectually and affectively, in the viewpoints and experiences of the Other (Josselson, 2013, p. 80). I found it humbling to listen to students’ interpretation of their experience, the Other. These stories called attention to the need for me to recognize that each student has a unique history which I cannot possibly know. Only through dialogue was I able to hear about their different ways of learning and understanding. This kind of interview gave me insight into the complexity of human experience and the impact these experiences have on students.

Davis (n.d.) also recognises the active nature of listening that is required for individuals to gain new insight and deeper understanding. During a classroom discussion, he observed students leaning into each other, reaching out, focussing, and attending to each word and action. He regards listening as “something that we enter into, something that we are, emerging from our occupation with others and with their meanings“ (Davis, n.d., p. 4). While listening to students about their experiences, I entered into their world, with the belief that each student had something new to teach me about how they struggle. I found it challenging to remember to inquire about their meanings, rather than assume we shared a common understanding. Although this open and active stance of listening was difficult for me to maintain throughout the entire
interview, I noticed that when I listened, I learned much from these students and found myself wanting to hear more.

Smith et al. (2009) encourage the researcher to take time to non-judgementally listen for matters or concerns that “we did not even anticipate needing to know” (p.59). Listening well means listening to the content, the choice of words, the important omissions, as well as non-verbal clues (Josselson, 2013; Rubin & Rubin, 1995). I found listening to the audio recordings while reading over the transcripts gave me a deeper understanding of what students were saying. Their intonations, pauses, and laughs provided depth to their words.

Another reason for using interviews was to have students use their own words or labels to describe their experience. My choice of words describes my experiences and understandings. I tried to allow participants to use their own words to describe what it was like to struggle in math and not make too many assumptions about their experience. Students used words such as difficulty, discouraged, confusion, handle it, does not click with me, as well as struggle. During the interviews, I tried to remain open minded by asking students to elaborate or explain their choice of words and clarify the meaning of the words they chose so that I might better understand their life world. However, there were times when I read over the transcript and realized that I had not asked them to explain what they meant. I had lapsed into assuming that we shared common meanings.

Interviews are an invitation to a new perspective of a phenomenon, learning mathematics, that I have already experienced (Josselson, 2013, Moules et al., 2015). My experiences as a teacher are different than those of my students and it is these differences that I want to understand better. Rodgers (2006) reminds teachers that interviews with students are important
because they may reveal that what students actually experienced may be completely different than what was intended by the teacher. As a teacher, I have seen students struggle. Through observations and questions, I have tried to find suitable solutions for students to work through their difficulties, yet my interpretation of their struggle did not enable me help them. My goal in the interviews was to listen attentively and actively as students elaborated on their experience, in the hope that I might gain a better understanding of what it is for them to struggle. I endeavoured to give students the opportunity to speak openly and freely about their experience to facilitate a more valid interpretation of this experience.

Providing an atmosphere in which students feel comfortable to share their world requires openness to what is said, trust, and empathy (Josselson, 2013). I made every effort to minimize the power imbalance during the interviews to obtain meaningful and valid data (Williams, 2007). Although part of my introduction included that I was a math teacher, I told students that they had much to teach me and that I was interested in hearing what they had to say. We were on a first name basis and sat in chairs next to each other to create an equal setting. During the interviews, I carefully listened to students and was curious about their stories and interested in what they had to share. I did not interrupt them or suddenly change the subject. My intention was to demonstrate acceptance and understanding of what students said, so that I could establish an authentic dialogue in which I could learn from these students (Cook-Sather, 2006, Josselson, 2013). I hope that by being receptive, respectful and open-minded I was able to provide the conditions for a constructive and honest discussion.
The conduct of the interviews

For my study, I interviewed six Math 10 students about their experiences with math. I chose this group for two reasons. It was in grade 10 that I first struggled in mathematics. I have always wondered if it was just me or if there were other reasons. Secondly, I chose these students because they are in a transitory year as they begin high school. Research (Attard 2010; Darragh, 2015) has shown that this transition affects students both positively and negatively.

From my own experience, I began grade 10 in a city near my home town, far away from the familiarity of the school that I had attended since kindergarten. My new school had three stories and large bright classrooms with teachers that seemed to have higher expectations than my previous teachers. I had always been a great math student; however, that year, I could not seem to get the same honours test scores that had come so easily in the past. My Math 10 teacher was able to make math interesting and fun; however, it seemed that I did not have the same math background as my peers. I thought at the time, that other students knew more tricks and could quickly understand the math concepts being taught. I found that year to be frustrating and began to doubt my abilities as a strong math student. I do not remember what changed for me the next year, but my math understanding and scores improved so that I chose to study mathematics in University.

In my conversations with teachers and administrators, I hear that it is often grade 10 students that need extra help in math. This may be because many grade 10 students experience a transformation from primary to secondary schooling that may include changes in the physical structure of the learning environment, teaching practices, learning practices, and expectations.
(Attard, 2010). These changes combined with learning new courses can be challenging for students.

Prior to interviewing students, I needed approval from both The Conjoint Faculties Research Ethics Board and Rocky View Schools. After receiving approval from both boards, I approached the principal of a Calgary area grade nine to twelve school. The principal was interested in my research and after reviewing the consent forms and approval forms, I was given permission to visit the two Math 10 classes to ask for student volunteers.

I visited the classes on four occasions. The first was to introduce myself, the research, and distribute a letter to the parents which outlined the study and student/parent consent forms. The consent forms identified the purpose of the study, informed students and parents of a one-hour interview, as well as a possible 45 minute follow up interview, outlined the risks and benefits to participating, identified that information would be kept anonymous, that parts of the interview would be used in my thesis, and that students were free to discontinue participation at any time. After receiving communication from two students, I returned to the Math 10 classrooms to ask if students had any questions, needed consent forms, and to appeal for more male and female participants. Following these two classroom visits, I interviewed five female participants. In hopes of conducting the study with a minimum of 6 students, I returned again to both classes to ask for more participants and mentioned that I had not yet had any responses from boys. During one of these visits, a male student approached me saying that he would like to participate. After securing six participants, I did not return to the classroom to ask for more participants. At this point it was the beginning of May and I needed to complete the interviews and follow up interviews before the beginning of June when students would be preparing for their final exams.
Grade 10 students who consented and whose parents also gave consent were individually interviewed about their experiences. I conducted the interviews in either the school’s conference room or in a classroom during the lunch hour. Students met me in the available room and we talked while they ate their lunch.

As for the interviews, after going over the consent forms, I turned on the audio recorder, and introduced myself as a teacher who is interested in understanding students’ experiences in math class, so that I may better help future students that I teach. I asked students whether they had any questions for me and reminded them that I would preserve their anonymity, and that they could stop the interview at any time or discontinue their participation. I checked in with students to make sure that we could use the entire lunch time for the interview and then began. All six students participated in the interviews.

I opened five of the six interviews with the questions:

Everyone has different experiences in their math class, which is why I really want to learn about how you experienced it. Can you describe your math experience to me?

In asking the question like this, my intent was to provide each student the opportunity to tell their own story, in their own words. I wanted to hear what they experienced, what they heard, felt, and saw, so that I might be able to experience, as close as possible, what they had experienced. I wanted to understand what it felt like for each student to experience math in grade 10. The sixth interview did not begin with the opening question because the student immediately began to tell me about his math experiences as soon as I met with him in the hallway. Once we sat down in the classroom, I went over the consent form and asked if he had any questions. Once I began the
after the audio recording, I said “I want to know about your experiences. Let’s go back to what you were talking about.”

After posing the initial question, I used probing questions, such as “Please tell me more about that” or questions that clarified meaning. I also asked which specific math units went well for them and which, if any, did not go so well. In each interview, I was able to talk about specific concepts, tasks, tools, and methods to delve deeper into their experience. These questions allowed me to hear specific details about mathematical content (such as Factoring, Trigonometry, etc.), allowing me a clearer perspective of how this experience impacted them.

Josselson (2013) encourages researchers to have a list of auxiliary questions on hand which “comprehensively includes what you want to know about the experience” (p.51). These questions are not to be used during the interview, but instead to be reviewed at the end to make sure that some areas were not omitted. If part of the participant’s experience was not explored, these open ended questions would be useful to ask. During the interviews, I found myself using these questions, as they helped me understand students’ struggles from different perspectives. Some questions that I used are:

1) Can you tell me about a unit that went well for you?

This question allowed me to hear what it meant for students to be successful in math. It gave me insights into their understanding of concepts and processes, because they could describe their understanding. All students experienced a successful unit. The units they described were factoring, trigonometry, and linear relations. Two students commented on liking the visual aspect of trigonometry and graphing units. Students also spoke about test writing and their emotions about attending class and learning.
2) Please, can you tell me about time that you found a math question or concept that did not make sense to you?

This question enabled me to discuss specific concepts and uncover how students struggled within the unit. The difficult units included factoring, linear equations, and radicals. Students whose daily math experience did not describe any struggle or difficulties told me about a specific topic that they had difficulties with. The responses from students also allowed them to use their own word that reflects this experience. I gained a new perspective of their struggles as I could compare theses descriptions to those of their successes.

3) Do you see math as useful? Do you see math as being part of your life?

These questions provided me insight into students’ broader view of math and whether or not it has an impact in their lives. Students shared how mathematics impacted their present lives as well as their futures. It also opened up their views on the relevance of mathematics.

4) How does this experience help you to work through your next encounter with learning a “difficult” math concept?

I asked this question after students had spoken about their experience with struggle, but I replaced the word “difficult” with the word the student had used earlier. I found it challenging to wait for students to use their own word. There was only one interview in which I came to this question and they had not yet used their own word to describe struggling. In that conversation, I chose to use the word difficult. My intent was to hear about the magnitude of the impact that struggling had and whether it is a lasting impact. Students discussed the feelings of their experience as well as how they overcame their struggles. I noticed that students persevered despite an experience of frustration. From my teacher perspective of wanting to help students
move through these times of difficulties, I was curious about these experiences and asked them to expand upon them. As they shared their individual stories, I realized that their reactions to struggling did not come from a single experience, but from many different experiences both in and out of school.

5) What advice would you offer future grade 10 students in approaching their math learning?

I asked this question to five of the six students. This question provided me with an opportunity to hear what students believed to be the challenges of Math 10. They seemed excited about answering this question. I noticed that their personal experiences contributed to their advice. These responses allowed me further insight into their personal experiences with math and I asked students about how these suggestions had impacted their learning.

6) Can you describe your perfect math class?

This question gave me a perspective of their ideal math environment. Students spoke about the impact that the classroom space, the teacher, teaching style, and their peers has on their learning.

These questions were not always asked verbatim because I tried to incorporate them into the flow and context of our conversation. In some cases, I did not ask the question because the answers were conveyed independently within the students’ dialogue. The answers to all these questions provided me with a broader, richer, and deeper understanding of how they experienced Math 10.

Four interviews took place in the school’s conference room with windows and a long table and large comfortable chairs. I felt it was a relaxed space to conduct interviews.
Unfortunately, it was not available for two of my interviews. Instead we used a nearby classroom which had no windows and was filled wall to wall with desks. This classroom did not belong to a teacher so the walls were mostly devoid of posters, pictures, or student work. I felt that this classroom was not as inviting as the conference room. However, I rearranged the desks and chairs so that both the student and I were sitting in chairs without desks between us. I had a desk beside the student so that they could use it for their lunch. I tried to create a space that was inviting and comfortable. I noticed that one of the interviews in this classroom to be more question and answer based. Although I tried to have the student expand her answers, the student may not have been as comfortable in this space. The other student that I interviewed in this classroom did not seem to be affected by the surroundings as he openly shared his experiences. Neither of these students participated in follow up interviews. The first student contacted me a month later to apologize for not checking that email address and the second student did not reply to my email. I am not certain if they did not participate in the follow up interview because their initial experience was not what they expected.

During my first interview I felt that the questions were too circular and did not include enough breadth. I reflected on the interview with my advisor and while sharing some of the statements, I realized that I needed to ask more clarifying questions. I had assumed that we had shared meanings for a language that we have in common, yet began to realize it was not so. I was reminded to delve deeper so that I might understand this student’s experience from her perspective. In the follow up interview with this student, I had the opportunity for clarification. During subsequent interviews I tried to stay aware of assumed share meanings and have students elaborate on what their choice of words meant to them.
During the follow up interviews, students said they enjoyed participating in the interviews. One student said she wanted to participate because she thought it would be a good experience to be part of a study in case she wants to conduct research in the future. Another student said that she wanted the chance to convey her thoughts about the importance of passionate teachers who want to help their students.

I concluded each interview by asking the students if they had any questions to ask me or whether they had any concerns. As suggested by Josselson (2013) and Moules et al. (2015), I extended my sincere gratitude for their willingness to share their experiences and let them know that they have been helpful to my work.

I audio recorded and transcribed each interview. I checked the transcriptions for accuracy. I plan to store the transcripts for three years on a secure flash drive with a password that is only accessible by my supervisor and by me. The audio recordings will be erased once I complete my thesis. I labeled the interviews numerically and assigned pseudonyms. Initially I listened to the interviews, making notes of observations and recollections from the interview. Once the interviews were transcribed, I read through the text numerous times, to comment on what was said, how it was said, questions that I may have had as well as my reflections, as recommended by Smith et al. (2009). These notes helped me open up new meanings to the struggles students have in their math classes. After reading over my notes and transcripts I met again with four students for a follow up interview in which I asked questions to clarify meaning. I again asked students if they had any questions or concerns and thanked them for sharing their experiences.
Interpreting the Interviews

According to Smith et al. (2009), analysis of students’ lived experience requires a continual movement of flexible thinking, expansion, reduction, revision, and innovation. Similar to the hermeneutic circle, it requires continual movement from the part to the whole and then back again in search of new understandings or perspectives. The circular motion is created as the understanding of the whole requires an understanding of the parts, which then provides greater meaning of the parts and so on (Josselson, 2013). The hermeneutic circle provided me with new meaning of student struggle and was made up of these individual parts: my experiences of students’ struggles, my struggles, the literature that I read, the conversations that I had with my advisor and others, and the interviews with students. Each part influenced the other. As I moved from one to the other, and reflected upon what was said, the hermeneutic circle widened and my understanding of struggle expanded. As I continue to read and discuss this topic, I find that I gain new insights and meaning about students’ experiences of struggle.

To interpret the interviews, I began by focussing on the transcripts and audio recordings. While listening to and reading each interview, I began to identify quotes that resonated most with me. As suggested by Smith et al. (2009), I recorded words of understanding and concern that these six grade 10 students shared with me about their mathematics experiences, as well as, descriptive comments about their struggles and successes in math, specific words they used, as well as their emotions. I formed a list of meaningful quotes and categorized the quotes into seventeen broad categories. As I reread the quotes, I found that some categories overlapped while others revealed how students overcame their struggles.
As I revisit the quotes which spoke to student’s struggles, I noticed that ten themes emerged: creativity, checking in, homework, unit tests, relevance, resiliency, visualization, understanding, helpful strategies, and individualizing education. I began by writing on each theme using the quotes which I felt helped me gain a new perspective of the theme. It was after writing about each of these themes, and with the help of my advisor, that I realized I was having difficulties separating the students’ struggles from the strategies that helped. As a teacher, I struggled to put myself into their shoes. I found myself continually looking for solutions and writing about what helped these students, rather than listening to how they experienced math. With much guidance, I endeavoured to understand and write about their struggles. Again I listened to their voices on the audio recording while reading the transcripts. I tried to listen actively and thoughtfully as they told me what their math experience meant to them.

As I focussed on these six students’ experiences, the themes surrounding their struggles converged on students’ demand for understanding and meaningfulness. They spoke about feeling confused and bewildered about the concepts they were learning, the rules they needed to memorize, and the reasons to follow a procedure. One student talked about her experience of freezing on a test. All students maintained that they were learning Math 10 so that they may graduate, not because it was meaningful. As I began to incorporate the literature, I found myself learning about the importance of relational understanding for mathematical learning. Skemp’s (1978) article on relational understanding really resonated with me, because it gave me a new perspective on how students learn. He believes that mathematics can be learned if a student is familiar with the pathways and reference points that make up the map of mathematics. This article together with the stories that the six students shared, continued to help me gain a deeper understanding of their experience with mathematics.
The chapter on the strategies that students said helped them to learn math, allowed me to share the insights I learned from students. Students spoke about their need for finding and making connections and that visuals of any kind have a huge impact on their learning. They spoke about their resiliency, which the literature supports to be a vital part of learning. Students talked about importance of support from teachers, peers, and their families. I concluded this section with what the literature suggests can help students. Alberta Education has already made changes to its diploma exam and is presently working to revamp the Kindergarten to grade 12 curricula to better meet the needs of present day and future students.

In the last chapter, I share my reflections and probing questions that teachers may ask themselves, which have emerged from the interviews with these six students. I found myself surprised and humbled to realize that I did not know or understand that which seems clearly obvious now. I feel fortunate to have learned so much from these interviews and as I move forward in teaching and in life, I am committed to stay open to new understandings that will come my way.
Chapter 4: The students speak: understanding their mathematical experiences

Students in this study spoke to me about their experiences in their math classes. They talked about times when they struggled, what caused them to struggle, and how they overcame or moved through their struggles. In this chapter, I aim to describe their struggles and offer critical commentary from current research. In order to understand what they have experienced, I wanted to hear what they had to say about their struggles, and how it makes them feel. As a teacher, I found it difficult and at times saddening to listen to these students speak. Some of the experiences in their classrooms were similar to those that students I taught might have experienced. When I listened back to the interviews, I noticed that when they spoke about their difficulties, I quickly moved on to asking them about what strategies helped them. Perhaps, I wanted them to feel success and joy, rather than failure and pain. I was challenged by my advisor to revisit the interviews and again listen to their struggle, to truly hear and understand what it is like to see and feel their experience. I was challenged to step out of my educator and parent shoes, and into the shoes of a grade ten math student. I have tried to do so genuinely, and to take care in making assumptions about the intentions that their words meant to describe.

A brief introduction of the inspirational students in this study

I would like to begin by describing the six brave and inspirational grade ten students who took time from their demanding schedules to share their personal experiences and thoughts about math. They attended the same Calgary area high school and were taking the Math 10 course at the time of the interviews. The interviews took place during the last part of the second semester. During this semester, the school offered two Math 10 classes taught by the same teacher. The students that I interviewed were enrolled in one of the two classes.
Naia. Naia talked about working hard to achieve high marks so that she would be successful in school. She finds it challenging at times to juggle coursework and participate in school sport. Although managing these responsibilities can lead to late nights of studying, she described this stress as motivating. She maintains that she works better under pressure. She also spoke about enjoying the time she spends with her friends, and her belief that she is more successful when she has friends in her class. Naia finds that her friends help to make math fun and are supportive when she has difficulties or needs help to focus. Math is one of her favourite subjects. She describes it as a solitary game, whereby she personally challenges herself to use the math she has learned to solve new problems. She wins the game if she has successfully solved the problem. At home, Naia’s older siblings help her when needed. Finishing her homework and studying for a test are very important to her, and she will go to great lengths to do so, even at the expense of a few hours of sleep. Naia’s advice to future grade tens is to work hard, try their best, ask lots of questions, and rather than making “snarky” comments about math’s relevance, just learn it.

Valentina. Valentina is a busy grade ten student who has her sights on going to university when she graduates. Outside of school, she plays competitive volleyball and regularly babysits. To manage these three responsibilities, she has a number of apps on her phone and a large calendar on her wall at home that provide her with a visual reminder of the events occurring each day, week, and month. She said that unlike an agenda which she often forgets in her locker, her phone is readily available to input volleyball tournaments, babysitting, assignments, and tests. The apps send her reminders about upcoming events so that she can better manage her time. Although math is not her favourite subject, she is enjoying it more than she did in grade eight and nine. She believes that in previous years she was more off task
because she was not as serious about school. However, now that she has set a goal to go to university, she is working harder and wants to do well on tests. Valentina’s strategy in school is to repeatedly practice until she is confident to move on. She recommends that future math ten students keep up with their homework, because a new topic is learned every two classes, and it is easy to fall behind.

**Kyla.** Kyla spoke about the impact that her experiences as an artist and those of climbing have contributed to her learning. As an artist, she uses her talents to help her focus and organize the material she learns in her courses. Since the age of six, she has climbed competitively and recreationally, both indoor and outdoor. Her climbing experiences have positively impacted the way she views challenges in her life and at school. She believes that if she falls, she will climb the wall again; she will not let the wall defeat her. Kyla has transferred this mindset to the classroom, where similarly, she works through her struggles. Math is not her favourite subject. In the past she struggled with it, because she could not understand it. This year, with the help of a tutor, she discovered that she sometimes inadvertently switched numbers around or missed important information within a question. With a few changes in how she approaches her learning, she has managed to improve her understanding and says that she does not mind math so much. Kyla’s advice to future math ten students, who are faced with a complicated concept, is to take a step back and find the individual, easily manageable pieces that make up the whole.

**Danielle.** Danielle shared with me her enjoyment of math when she understands it, but when she is struggling, she gets frustrated and does not like math. She enjoys that math can be broken into steps to find one answer. With repetition and a knowledge of the basics (knowing her times tables), she says that she can learn the steps to solve problems so that she will score well on tests. She described her approach to learning math as robotic, and observed that when she is
struggling she simply needs to locate the right formula. However, this too can be a challenge. Her ideal learning environment includes working in groups and having the teacher go over a lesson on the board. Similar to other students in this study, she considers herself an auditory and visual learner. Her understanding improves when the teacher combines the two learning styles by writing out the information on the board and explaining how to get the answer. Her motive for participating in this study was to help teachers better support students’ learning. Her advice to future math ten students is to complete the homework, read through examples in the textbook, and most importantly, just keep practicing every night.

**Marin.** Marin is a grade ten student taking three core courses (social studies, math, and science) in the same semester. She finds it a challenge to keep up with learning the material and getting good marks in these classes. When she does struggle, she has strategies to help her, such as getting help from her sister, peers, or YouTube videos. In math class, she prefers to be presented a lesson by the teacher at an unhurried pace and then have time to work on assignments. Although math is not her favourite class, her favourite type of homework questions are word problems, because she finds them more interesting than numbers alone. Marin’s recommendation to future math ten students is to stay focused and keep up with the homework.

**Derek.** Derek spoke with conviction and passion about why he despises and dreads math class. He explained that his approach to learning is from a creative mindset, which he defined as open ended learning through construction. Derek likes to draw and he draws all the time, including while he is learning in class. The back of the consent form that he returned to me was completely covered in a pencil drawing of an individual wearing a top hat and suit, whose head is a giant eyeball. He detailed the picture through the use of shading. His love for drawing is obvious. Unfortunately, most of his math experiences have not allowed him to incorporate his
artistic skills into learning. He views math as boring, random, and linear, comprised primarily of assignments, quizzes, and tests that exclude creative freedom. He wishes there was more overlap between math and art. He found the two units in math ten that had visual features, trigonometry and graphing, to be a little more interesting. He wishes math was not just about numbers, but could include a more visual aspect.

**The significant aspects of student experience**

When these students spoke about their difficulties or struggles in math, I noticed that their experiences revealed times of confusion about both understanding the concepts and the meaningfulness of math. Each student in this study spoke about their struggle to understand the mathematical concepts and wondered about the relevance of what they were learning. In this section, I will share what they had to say about the kinds of experiences that caused them to struggle. I will discuss students’ experiences of wanting to memorize steps, wanting to know how concepts are related, how to solve a problem, and why certain processes are chosen to solve a question. These experiences are important because students told me that understanding the math led them to feel confident and enjoy math. I will conclude this section with the struggle that students face to learn a mathematics curriculum that they find irrelevant.

1. **Experiencing confusion.** Marin spoke to me about the confusion that arises from memorizing rules and laws, specifically in the exponents unit.

   I wasn’t really good, with remembering all the exponent laws and like when you flip it or when you do this negative or that stuff. I was like all jumbled around with what you have to do for those.
Marin feels that she must memorize the exponent laws in order to solve a question. She has memorized the process of flipping, but cannot remember which situation requires it. Flipping refers to the process of finding the reciprocal of a term with a negative exponent, so that it can be rewritten as a positive exponent. Research (Boaler, 2016; Schoenfeld, 1988; Skemp, 1978) shows that although memorizing is a method that may lead to solving a question correctly, it does not create deep, meaningful understanding. Further, if students forget “the rules”, they cannot solve a problem. This may be why Marin uses the word jumbled to describe the mess of numbers and variables. The rich learning experience to understand the processes is missing. Marin continues to describe this experience with exponents.

It just, took me a while to like understand like why this happened or which way you, flipped it and why this can’t be like this… it just kind of confused me altogether like what it needed you to do, for like the question… if I just reviewed it a bit more I’d probably understand it.

The arbitrary term “flipping” does not make sense to Marin, it is only a word to invoke a memory of a process and ultimately denies her a connection that may lead her to understanding. In her quest to learn, Marin wants to transition from memorizing the exponent laws to understanding them. By asking why something happens or why it does not happen, Marin is in search of relational understanding (Skemp, 1978) or conceptual understanding (Hiebert & Grouws, 2007). Skemp (1978) suggests students will find concepts easier to remember if they learn what to do and why to do it, because this type of learning allows students to find links among related concepts. The simplistic exponent rules originate from earlier learned mathematical concepts of multiplying and dividing. Building on these earlier concepts may provide the connections students need to understand.
Marin also shares her need to practice more problems, because in her experience, repetition helps her to understand. Similarly, students in other studies (Boaler, 1998; Mason, 2003; Nardi & Steward, 2003) also believe that repetition and task completion is needed to understand. However, repeatedly practicing a procedure may lead to memorization, which has the potential to be forgotten. These studies showed that linking understanding with practice may provide students with alternate perspectives of a concept leading to perhaps a deeper understanding. Marin’s experience of math as a rote learning activity strips away possible rich understanding and enjoyment (Boaler, 1998, 2016; Nardi & Steward, 2003; Schoenfeld, 1988).

Marin describes her enjoyment of math hinging on how well she grasped the concepts. I wasn’t really looking forward to the class because it was like, math and it’s not really my favourite subject. And plus it was also like, um, like a concept that I didn’t really understand, so every time we went in it was like oh no, why?

Anticipating an encounter of confusion and frustration creates feelings of fear and anxiousness for Marin. She does not want to go to math class if she believes that she will be exasperated. Marin’s frustrations and confusion with these math concepts and her anxiety to attend math class may be a caused by her lack of understanding of how the concepts relate to each other.

2. Feeling disconnected. Marin talked to me about why math was more difficult to understand than other courses.

Most of the concepts I’ll understand, but it’s just sometimes, um, when they’re explaining I don’t get how this goes with this. Or like how, why you’re supposed to like, multiply this. Or just like formulas a lot of the time, I don’t get how they like make sense with, to what you’re trying to do.
Marin sits in class trying to make sense of what is presented; however, the concept is unrelated to what she already knows. She has no reference points from which to orientate herself (Skemp, 1978). To Marin, even the processes and formulas used to solve a problem are random and hazy. She watches the teacher present the material in steps, but the procedure or formula do not offer clarity. Marin is confused and unable to take hold of something familiar; subsequently, the information presented by her teacher appears arbitrary and jumbled.

Derek too discussed his frustration with trying to figure out how to learn math.

I think the reason why I don’t like it so much is because most of it doesn’t work for me. I’m trying pretty desperately to make some connection.

His loathing of math comes from math concepts that do not build on each other. He knows that to learn he needs to make connections, but he is unable to do so.

Skemp (1978) suggests that students require a mental map of reference points and pathways, so that if they get lost, they can use these points to navigate a way to a solution. Simply memorizing exponent rules or formulas is not meaningful because memorization does not create connections. Connecting exponents to previously learned concepts such as multiplication and division may create the pathways to understanding. Students who have a map of concepts comprising of connected pathways can build on previous concepts as they learn new ones. This map provides students with the links to understand why the rules work.

Another viewpoint of this disconnection is that Marin and Derek might be struggling to blend new concepts with those that they have “met-before” (Tall, 2013). Tall (2013) defines met-befores as ideas and concepts that arise from previous experiences. For example, in elementary school, students are taught to place the larger number first when subtracting, because a smaller
number can only be subtracted from a larger number. In junior high, when students encounter negative numbers, this rule is no longer true and subtraction can cause confusion. The met-befores can be problematic and cause conflict, as in Marin’s frustration with connecting exponent rules to what she already knows about orders of operation (Tall, 2013). Both viewpoints can be considered to better understand Marin’s disjointed learning experience of exponents and Derek’s frustration to learn math.

In a study by Martin and Towers (2016), students’ understanding of a new mathematical concept was fostered by connecting previously learned knowledge or “folding back” to current knowledge. Through a discussion of their understanding of the previous concept, students journeyed the path towards understanding the new connected concept. This process requires bringing to surface any underlying misconceptions and misunderstandings and “working on and with them” (p.96). Martin and Towers (2016) believe that folding back is a cognitive action that can allow students to address their problematic met-befores to gain a connected, meaningful understanding. Perhaps Derek and Marin’s discussion of their understanding of previously learned concepts, complete with discussions misunderstandings, may provide the groundwork necessary to create a meaningful understanding of a new concept.

3. **Not knowing hidden rules.** Derek described his frustration with what he views as an arbitrary nature of mathematical procedures.

But I feel like the more and more we get into the curriculum the more linear it gets, the more grayer it gets, and, just the less tools I have, and (raising his voice) more random stuff. More random like, like, I don’t like when teachers say “oh this I solve it just, just cause that’s how math works”. And that’s what makes me mad. (he says sharply) Is why?
Why you do you do, oh and this times two, because it’s always like that, that’s the rule. And I’m saying whose rule is this? (he says with frustration and anger) I don’t get any of them. I don’t get any of these unwritten rules, I don’t like them, I don’t want to do them, I want to have my own rules! And you know solve something that can be solvable by some, by anyone. You don’t have to know these little rules to solve a formula. (angrily) It just feels like, the more and more these little rules get implemented, (bitterly) the more and more I know I will never ever use that. (angrily) I will never remember that and (raising his voice) nor will I ever, ever care for that. (pause) But I need it, (quietly) you know. That’s the thing.

Derek cannot make sense of the material presented, and is both furious and frustrated. He has no recollection of having learned the processes. The teacher’s assumption that he should know these rules irritates Derek because he has never seen them before. The rules are an unwritten secret code to which Derek is not privy, yet he must find a way to make some sense of them. Skemp (1978) defines these rules without reasons as instrumental understanding. He concludes that learning a rule and how to use it is a kind of understanding that is accepted by both teachers and students because, it is simple and students can quickly get the right answer. However, students such as Derek do not care about the arbitrary rules, they want to understand relationally. Perhaps making the rules meaningful by investigating and unpacking why they exist may create a deliberate framework.

4. I didn’t understand why. Some of the students talked about their struggle to find meaning in the procedures. They said that although they knew to implement a process, they wanted to know the reason why it was chosen.
Kyla was one such student who spoke to me about her experience of trying to understand the procedures in the unit on factoring. Factoring is a skill that is learned in grade 10 and applied within various units in grades 11 and 12. Similar to adding, subtracting, multiplying, and dividing, it is a skill which is used over and over again. There are different ways to factor depending on the expression that is given. This unit includes few applications as students must learn the skill before they can apply it. Here is what Kyla shared with me about her experience with the factoring unit.

I didn’t pick up on it as fast, and I absolutely dreaded going to class because I knew I was going to have trouble. Factoring is mostly equations, so because there was no, no visual part of it, no graphs, no diagrams, no visuals, no pictures or anything like that, I struggled in factoring a lot because it’s all just numbers and variables and coefficients and all of that … I mostly understood the steps, but I didn’t understand the concepts behind them. I didn’t understand why we doing what we were doing.

The numbers, variables, and coefficients are individual pieces without connections or meaning. Kyla cannot make sense of how they exist or interact. Without any visual representation or points of reference, Kyla feels lost amongst the numbers, variables, and coefficients. The meaningfulness of these parts creates feelings of fear and anxiety. She says that she does not want to endure another math class in which she cannot understand what is going on. Kyla lacks the mathematical understanding which could be shaped from the rich networks among concepts and processes.

Hiebert and Grouws (2007) compare skill efficiency of mastering procedures to conceptual understanding which provides mental connections among the facts, processes, and
ideas. In this sense, Kyla describes her experience of learning to factor as a skill to be mastered. She speaks about the individual parts, but has not understood how these parts connect to the whole. Perhaps if she had learned where the parts of the expression to be factored originated from, she would not need to memorize the steps. Skemp (1978) reminds teachers that learning conceptually is more difficult because there is more to learn; however, he believes that once learned, the result is long lasting.

Kyla’s frustration comes from not understanding what to do. She does not even want to be in the math classroom because she cannot make the connections that others can. She feels despair, hopelessness, and she blames herself.

All my friends were doing better than me, they were understanding the concepts. So I was really struggling. Um, I felt quite depressed with myself, um, quite deflated and defeated. Ultimately, I was quite, um, upset with myself because it was, it was my fault that I didn’t understand the concepts for some reason.

Kyla sees everyone else succeeding and moving forward. She too wants to be part of this group which progresses with understanding; however, she does not feel as if she can move forward. She is stuck, wondering why she cannot understand the next step, why she cannot make the transition to the new concept. As she compares herself to others, she feels badly about herself. She questions why she cannot understand and begins to blame herself. She believes that she is at fault. Without any hope of moving forward, she feels depressed and defeated.

In our interview, Kyla also spoke about the difficulties she encounters when a teacher takes what she calls a short cut.
Kyla: I tend to take whatever question I can and, I work through it visually. Some kids can do it in their head, and just think about it and solve the question. But for me, I need to be able to see it. And when I can see it, I can work through, the steps. I can manipulate the equations as long as I can actually see it and I don’t have to do it in my head.

Researcher: Can you give me an example of that?

Kyla: Um, an example, solving, like for example, the unit we’re doing right now is Functions of Linear Relations and what not. So when you’re given a question of take, -3x – 2y + 12 = 0 and write it out in slope-point form instead of general form, there are some people, like my math teacher who can just say, “that’s the answer” because he can do it in his head. And then there is people like me, where I have to write it out and I have to write out each individual step. Like I have to write out ok I have to take 12 away from this side and this side and I have to write out each and every single step on a piece of paper, so I can understand what I am doing.

Her struggle arises when she does not understand how the teacher came to the next step. If the steps are not all written out she is lost. The teacher may know the next step is to subtract twelve from both sides, do the calculation in their head, and write out the answer as the next step. But a student cannot read their teacher’s mind. Kyla does not know when a step was skipped, she simply assumes that the next step followed from the previous step. Tall (2013) differentiates the teacher’s complex understanding with the student’s novice understanding. He believes that over time, the mathematics teacher has repeatedly created links among the richly connected concepts and thinking processes, which then compress to be simpler and more efficient. Tall (2013) defines this understanding as compressed knowledge. This flexible understanding strengthens
conceptual links, so that lengthy operations can be compressed, into what Kyla calls short cuts. However, Tall (2013) believes that without this rich experience, students such as Kyla can become frustrated when they do not understand how the processes are demonstrated. Kyla is only seeing this concept for the first time and does not have the experience with numbers that a mathematics teacher may have.

5. **Being frozen.** Over the years of teaching, I have often heard from students about their experience of having studied for an exam, but not knowing how to proceed when writing the exam. One of the students in this study, Danielle described her experience of “freezing” during her factoring test.

Danielle: I had like a break down, and I came to (my teacher) and I was like, I’ve never froze in a test like that before. And like how do I do those questions? How did, like why did I freeze like that?

Researcher: So you said you froze, what does that mean?

Danielle: Like when I came to the question, I didn’t know where to begin. Like I didn’t know how to solve, I didn’t even know where to start for the question. And I like, I’ve never felt that way. Cause normally like, you have like, they’ll give you some context, and they’ll be like or they’ll say like find the slope or something. But this one said just said solve it. So like, ok I don’t know what to do. (she says this slowly with uncertain laughter and desperation) Like I had no idea.

Researcher: How did that make you feel?

Danielle: Um. Like I did not want to do the test anymore.
The question on her test makes a simple request: solve. The request is meaningless to Danielle. It gives her no clue about what to do or how to proceed. She is frozen, unable to move. She describes feelings of confusion, panic, and despair. She compares herself to others sitting around her that are succeeding and moving ahead at a faster pace. She feels as though she has been left behind. This debilitating experience frightens her, and she wonders how this experience came about and what went wrong. Boaler’s (1998) research on grade nine to eleven students’ test writing experiences showed similar results. The students in Boaler’s (1998) study also reported not having any cues that could help them figure out how to answer the question on a test. Similar to Danielle’s experience, Boaler (1998) observed that these students were unable to interpret the demands of unfamiliar questions because their mathematical knowledge came from practicing rules and procedures from the textbook.

We did a lot of practice on it. Like he made us like really. Which is weird because my first test that I did for Factoring, I got the test, and I did all the homework, but like the test was like, he stepped it up a notch. Because like when I do like a question I do it all by steps and like you can’t do that because each question is different. But um I got the test and I did not know where to begin with the question, like I did not know what to do. I did not do very good on that test.

Danielle expects to do well on a test for which she has completed the homework assigned by the teacher. Similarly, Boaler (1998) observed students in her study who did not know how to proceed, searching for cues to trigger their memories. These students had not gained meaningful knowledge which would enable them to understand an unfamiliar question and choose an appropriate procedure. Having completed the homework, Danielle assumes that she has a map to guide her; however, she does not know how to proceed. Danielle has not created familiar
pathways which connect the concepts and processes. She wonders if the questions are harder than the ones from her homework. Perhaps the textbook questions did not provide Danielle the flexibility to interpret a new mathematical situation (Boaler, 1998; Schoenfeld, 1988).

Boaler (2014) examined the impact that timed mathematics tests can have on students, causing them to feel panicked, worried, and fearful (also called test anxiety). She found a study by Bielock (2006) which investigated individuals’ response to stressful testing situations. This study showed that the pressure of the test can block an individual’s working memory. Further investigations by Bielock (2006) showed that the individuals who are most likely to fail in these stressful situations are the ones who “in the absence of pressure, have the greatest capacity for success” (p.342). In my study, Danielle said that this experience of freezing on the exam had only happened once during the Math 10 course, and might therefore not be related to test anxiety.

Danielle continues to describe this frustrating experience as having a question thrown at her.

I was like why is this like, every question like he is just going to throw something at me on the test and I’m not going to know like what’s going to come … I do like all my homework and all the questions in the textbook are basically the same, and then you come to a test and it’s like “this looks different, like I don’t know what to do” (she says this slowly and with confusion) … When I do all my homework, I feel confident about what’s going on in math and then he like throws that question at me, and I think that there’s so much more that I don’t know … A test is basing off of your knowledge. And if that’s not a part of your knowledge, then how do you figure it out?
Although she has prepared to the best of her ability, the instrumental understanding that Danielle has gleaned is useless to her if she cannot apply the procedures that she has learned on a test.

6. **Experiencing math as irrelevant.** I asked students about whether they saw math as useful and relevant to their futures. I heard varying responses ranging from a definite yes to a definite no. Some of these students said that although they worked hard to learn the math, they often wondered when they will ever use it. Part of their struggles in math are understanding the purpose of learning concepts that they may never use again.

Danielle was one such student who said that the math she learns in school will not be useful to her in the future.

Why do I need to know like factoring? Why do I need to know what x and y is of a graph? (laughing) But I think if we are all learning it, then if someone like brings it up to talk about it, you kind of know what you’re doing which is good. But other than that, I sometimes I just don’t know what the meaning of math is? (laughing)

Danielle’s reason to learn math is that she may help a classmate. That is the extent of her use for math. Factoring and graphing are not realistic skills that she sees herself using in the future. Danielle’s opinion is partially influenced by the time she went with her dad to his work at an accounting firm for “take your kid to work day”. There she spoke with her dad’s co-workers about the math they do.

They said like, like, when they even do multiplication, they just take out a calculator and like do their like math like it’s basic math. It’s not like anything like what’s y, or how to solve for slope. Like that’s stuff that they never really use. And I don’t how you would use that in like a job. Only if I was a like a teacher.
Similar to students in Nardi & Steward’s (2003) study, Danielle cannot see herself using math again. She said that the only job that she can see herself using these concepts again was if she was a math teacher. She learns the material, because it is mandatory.

I just do whatever is given to us. I don’t really like, I don’t know, cause you got to do it at the end of the day, it’s in the curriculum.

Danielle works through assignments to learn the concepts because she has no other choice, not because she can use the skills and tools later in life. Similarly, Derek, a student who despises math, attends class because it is a requirement.

But I need it, you know, that’s the thing. I need it. I come to, you know, I come to odds with that. I guess I need it. I don’t know, people say you need it.

Derek attends class and goes through the motions of learning, solely, because math is a requirement by Alberta Education to graduate. Derek is trapped by this constraint to be in an environment that he finds tedious and unfulfilling. Similarly, students in other studies (Andersson, Valero, & Meaney, 2015; Attard, 2013; Nardi & Steward, 2003) also reported resistance and disengagement, because the mathematics they learn has no relevance to their present or future.

The contents of school mathematics curricula, including Alberta’s math curriculum is questioned by researchers (Brown et al., 2008; Davis, 2014; Mason, 2003; Nardi & Steward, 2003; Wright, 2012) for its meaningfulness and relevance to our present world. Davis (2014) considers the archaic, impractical concepts and skills students must learn in math:
Programs of study have thus become not-always coherent mixes of topics drawn from ancient traditions, competencies imagined to constitute a necessary skill set for citizen of the modern (read: industrially based, consumption-driven) world, necessary preparations for postsecondary study, and ragtag collections of other topics that have been seen to add some pragmatic value at one time or another (p. 34).

Similar to students in this study, Davis (2014) questions the useless collection of concepts in mathematics. Davis (2014) challenges a reconsideration of the curriculum to include students engaging in mathematics that is “image rich, example dense, and rife with mathematical connections” (p. 17).
Chapter 5: Students’ strategies for learning

The interviews which enabled students to voice their struggles in mathematics also shed light on conditions and strategies that they use to work through these difficulties. Students in this study were excited to share what worked for them. In this chapter I will outline their recommendations together with supporting literature. In the first section I will describe their experiences of making connections, using visuals, and getting support. In the second section, I will share their experiences of resiliency when they confronted with struggling in math.

The Journey towards Understanding

According to students in this study, understanding mathematics and its processes is an integral part of learning. Marin explained what the importance of understanding meant to her.

Once I understand like a concept, I’m pretty good at it. But it’s just me getting to understanding it, that takes a little longer. Just cause like, some they’re, I like really click with, but then there’s others where it just takes me forever to understand what I’m supposed to do.

Marin’s quote represents the shared belief among students in this study that understanding math is a necessity. Memorizing the steps is not enough. These students said they want to know why something happens and why it does not happen. They want to know what they are doing and why they are doing it. This kind of understanding is called relational (Skemp, 1978) or conceptual (Hiebert & Grouws, 2007). Skemp (1978) contends that once students know what to do and why to do it, they can begin to see how concepts relate to each other and in doing so gain a deeper more meaningful understanding. Schoenfeld (1988) believes that understanding the connections among facts and procedures enables students to apply their knowledge flexibly and
meaningfully. Lampert (2001) asserts that students have acquired knowledge and skills if they can communicate their ideas through words, diagrams or manipulatives to convince others. In doing so, students gain the flexibility to interpret, negotiate, reason, and ultimately “understand” (Boaler, 1997).

The path that each student takes to gain understanding is different. Students shared with me the need to make connections, the importance of visual aids, and the need for support from their peers, families, and teachers. In this section, I will outline these strategies that guided them towards understanding.

1. Making connections. The need for understanding was expressed by students as making the connections which contribute to their comprehension of how and why concepts and processes in math work. Kyla says she feels confident in math when:

I can do that, I know how to do that, I know why we’re doing it.

Vergnaud (1983), Lampert (2001), and Hiebert & Carpenter (1992) recommend facilitating student understanding by connecting what students already know to a broader based theme. For example, Vergnaud (1983) proposed a field of multiplicative structures which would include “multiplication, division, fraction, ratio, rational number, linear and non-linear function, dimensional analysis and vector space” (p. 127). Since relationship already exists between these structures, introducing them together allows students to make beneficial links or pathways to construct a mathematical map. For example, the division of fractions could be taught by first introducing “the big ideas” (Lampert, 2001) of multiplication and division. In doing so, students link the new concept of division of fractions to concepts of multiplication and division that they have “met before” (Tall, 2013). With this experience, students can navigate similarities and
differences among concepts. The concepts of multiplication and division are linked to a larger network which includes addition and subtraction. Flexibly connecting knowledge over this vast network of mathematical concepts enables students to gain a richer understanding of mathematics (Duckworth, 2006; Hiebert & Carpenter, 1992; Lampert, 2001).

Naia and two more students realized that missing a step or a component can cause them to struggle.

As soon as you miss one step and you don’t see what happens there, it’s like this brand new kind of concept and you just you don’t know how you got there or like how to follow after that.

In these situations, Naia would ask for help from a friend, the teacher, or a family member. She also recommended taking a break and returning to the question with “fresh eyes”.

In her experience of teaching grade 5 math, Lampert (2001) also observed gaps in students’ understanding as they communicated their ideas with one another or to the class. To identify and close these gaps, she suggested that within an encouraging and supportive classroom, students should explain their thinking and reason why their answers made sense. As students explain their understanding through reflection and negotiation, they strengthen and further link it to a larger network (Lampert, 2001).

2. Visual aides. Some of the students in this study referred to themselves as visual learners. They spoke about their need for a visual representation of the information in a diagram, picture, or graph to help them understand what is being presented. When these visuals were absent, students found that they struggled or lacked focus to learn. Kyla told me about her experience.
I tend to work better with diagrams and with pictures… I need to be able to see it. And when I can see it, I can work through the steps. I can manipulate the equations, as long as I can actually see it and I don’t have to do it in my head.

As a visual learner, Kyla is aware of how the pictures, diagrams, and steps enrich her comprehension. Research by Andersson et al. (2015), Boaler (2016), and Lampert (2001) has shown that incorporating visual representations such as physical objects and drawings can be incredibly powerful for students, because they lead to higher levels of engagement and deeper understanding.

Derek spoke of a similar experience of needing a visual representation. For Derek any visual representation has the potential to spark interest in learning.

I’m a very visual learner and I’m a very like auditory. So if I, if I write something down and I copy some notes from the textbook and there’s, I can draw a little picture and that will, it’ll be like some, I don’t know, some flow chart of some sort with a doodle on the side, and then that would help me like just click with that, with that topic, definitely. I feel like that, that’s always worked for me.

Without a picture, Derek finds math boring and dull. There is nothing to entice his learning, nothing to make it meaningful. He sees numbers to which he cannot relate. However, any visual representation, even a doodle offers an interesting perspective to engage him. There is some research (Andrade, 2010; Tadayon & Afhami, 2016) which shows that students who doodle perform better educationally than those who do not. When I asked Derek about what the doodle on the side of his notes represented, this is what he told me.
I always doodle, there’s never a time that I’m not doodling. It’s just like my little doodle. It helps me focus. It definitely helps me focus and if I’m studying, I got to be drawing and if I’m working, I got to be drawing. It’s like I can’t shut that part of my mind off and then go to a whole different area and say “hey I’m just going to lock down and do this.” I don’t know, that’s, that’s one of the bigger dilemmas I’ve always faced.

For Derek, learning and doodling go hand in hand. Without doodling or picture making, he is unable to engage with math, his mind cannot make a connection to the numbers alone. A study by Tadayon and Afhami (2016) suggests that creatively thinking about and producing a visual representation of a problem provides opportunities to make connections between what is known to that which is being learned. These links enable students to remember relationships (Boaler, 2016). Derek’s need for creativity would be fulfilled if he was challenged to think about mathematics visually or create a visual representation of the math that he is learning.

Using various colours to highlight different concepts or procedures is another type of visual aide that one student said helped her gain understanding. Valentina spoke to me about how colours can orientate her understanding. She said that she found it confusing when the math is presented in one colour.

So all these numbers, if I like look away and then look back, sometimes they all turn into like a big mess.

Valentina sees only a jumbled mess of numbers in front of her. They make no sense to her. There is no organization and no clarity. She cannot understand where the numbers came from or how they flow. She simply sees a muddled mess. Breaking up a concept or process into parts using colors helps Valentina guide her learning and create clear pathways.
Colour coding is another visual aide that students can use to make sense of the concepts they are taught (Boaler 2016). The subtle differences among solutions can be easily seen when diagrams, pictures, or several colours are used, allowing students to more readily make connections needed for understanding.

3. Getting support. When students in this study had difficulties, they relied on help, encouragement, and collaboration from their teacher, peers, and family. In our interview, Derek shared his struggle to understand math the same way as his classmates. He said,

If (a teacher) would just listen how I approach the formula and he would, I would go through it, and he would point out you know once I’m done, or once I hit some roadblock or whatever, he would just say this is what you did, um this is also what you did, here are the ways to fix it and does this work for you?

In working through his struggles, Derek implores his teachers to listen to how he understands a concept, so that the teacher may use this understanding to teach Derek. Coles (2002) identified this kind of listening as transformative listening. Similar to Gadamer’s (2007) stance of listening, Coles’ (2002) transformative listening requires openly listening to the participant with the view that the other may be right. Rather than assuming where Derek has gone wrong, it requires a willingness by the teacher to hold valid Derek’s understanding and be open to change the way the teacher thinks He expects his teacher, who is considered the expert, to individualize his learning by telling him what he has done wrong and how to fix it. If Derek has misunderstood the concept or is missing a component, he wants input and guidance to get back on track. He is willing to learn math, he just feels that he approaches his learning differently and can be helped
if someone would take the time to connect with him. Similar to the students in the Nardi and Stewart (2003) study, Derek feels alienated in the absence of such individualization.

Derek told me that the learning he experienced by his friend’s help could transpire in class, if the teacher was aware of what he already knew. He says:

Figuring out where you are in, at that like you know on the spectrum, on understanding – not understanding. And I mean, the way most classes go is a teacher banters to the class and four people respond. And I don’t know if they’re not getting the memo, that obviously the whole class isn’t getting it, but um the people that are on the sidelines, the people that aren’t answering, it’s quite obvious that they don’t understand it. And those teachers should be checking on them. Not necessarily, you know, do you have any questions?

Derek is not included in the group of students who gain understanding. He is frustrated that he is simply a bystander watching the learning take place, but not learning himself. Students in other studies (Murray, 2011; Nardi and Stewart, 2003; Warshauer, 2015) also preferred teachers who responded to individual needs. Derek does not want to be asked whether he has any questions. He views such questioning as vague and impersonal. As he previously outlined, he prefers the teacher to actively listen and individually work through a question with him. As described by Josselson (2013), actively listening requires the listener to take time to non-judgementally listen and wonder how this experience is for this student.

If a teacher went up to me and said okay let’s do this question and then the teacher literally finds out the mistakes I’m making, says, you know fixes those mistakes, I’ve learned the whole unit. I’ve learned the whole chapter just from one little encounter …
And that’s all I ask, is just like just one question. And I mean I’m probably at fault for not approaching the teachers and saying let’s do this one. But I’m just afraid for the repercussions I guess.

Rather than appeal to the class as a whole, Derek simply wants the teacher, who has the experience and knowledge, to check in with him, to have a dialogue in which Derek will feel he is heard and can get help to understand. Josselson (2013) believes that teachers sometimes listen with the idea of how to fix the problem, rather than just openly listening. Derek is not comfortable to approach his teachers for help, yet in our interview he appeals for help to understand. Derek’s math experience would be greatly improved if his teacher periodically approached him, actively listened to how and what he understood.

Every student in this study spoke about the effectiveness of working together with someone, either a peer, teacher, parent, or tutor when a difficulty arose. Working alone causes frustration and despair, as Valentina describes:

When you have someone with you and you’re not kind of like alone, trying to and I kind of like get frustrated sometimes if I can’t talk through it. … Me reading it doesn’t fully kinda click in my head and I have to get some sort of verbal way to understand it.

Alone, Valentina has no one to speak to, to bounce ideas off of or to clarify her understanding. She remains in a state of confusion because there is no other way to understand or approach the question. She has no one with whom she can ask questions or navigate a path towards understanding. Similar to other studies (Attard, 2013; Beilock, 2010; Boaler, 2016; Mitchell, 1993; Nardi & Stewart, 2003; Whicker, Bol, & Nunnery, 1997) students found that collaboration with their peers allowed them to learn other ways to think, thereby strengthening their
understanding and making their learning environment more enjoyable. Studies (Beilock, 2010; Lampert, 2001; Mitchell, 1993; Nardi & Stewart, 2003; Whicker et al., 1997) also found that students gained a better understanding because they have to negotiate their understanding as they explain what their knowledge to others. A support network of peers, teachers, families, or tutors foster greater understanding because this network promotes thinking about, interpreting, negotiating the math that has been learned.

**Persevering in the Search to Understand**

During this study I was continually surprised by students’ experiences of resiliency and perseverance when faced with frustrations and failures. These students believe that they can overcome obstacles in their learning and that they have the tools and methods available to help them. Duckworth (2016) defines the combination of perseverance and passion as grit. Although I did not hear about the passion behind their perseverance, I certainly heard from students that they expected their efforts to lead to improvement (Duckworth, 2016).

One such student was Naia. She spoke to me about the times when she was “incredibly confused”.

I just have like this subconscious thing where just like you know if you got through those questions, like where you were unbelievably confused, then like you can get through more. Like you obviously, if you did it once, you can do it again. So, like you just keep going.

Naia’s inner drive to persevere stems from her past experiences. She believes that if she has triumphed in the past, she will triumph again. There is no question in her mind that she cannot succeed. Her previous experiences have shown her that difficulty in math is a natural part of
learning math. It is not a matter of whether or not she will be faced with difficulty, it is a matter of when. When she does confront confusion, she will work through it. Naia’s perseverance can be defined as either a productive struggle (Warshauer, 2015) or constructive struggle (Hiebert & Grouws, 2007). Struggling in this way may also create learning opportunities for a deeper understanding. Research (Hiebert & Grouws, 2007; Skemp, 1978; Warshauer, 2015) has shown that students who embrace struggling as a natural part of learning math are able to make important connections or re-examine what they have learned before.

Similarly, other students in the study spoke to me about their experiences to overcome their struggles.

Danielle: From that point where I froze, and I didn’t know what to do, I just wanted to improve.

Kyla: I’m not going to let this test mark bring me down, and I can, I can, I can get over this.

Valentina: Like the first test was kind of, oh ok I gotta, I gotta focus now again. That didn’t go well.

Students used the words “keep going”, “improve”, “get over this” and “focus again” to describe the motivation to move through a difficult learning experience and propel themselves towards understanding. Dewey (1910) also believed that struggling was essential to generate a deeper understanding because it presented students with an opportunity for reflective thinking. Reflective thinking, Dewey (1910), says “involves willingness to endure a condition of mental unrest and disturbance (p. 13).”
Kyla spoke about her experience of having a unit test mark bring her average down from 85% to 65%. I was curious about what it was that changed her perception from feeling deflated and defeated to believing that she could succeed. She said:

It was really my mindset. I got a low test mark or unit test mark on my Factoring Unit Test. I got 52% and kind of really snapped me out of my “oh I can’t do this, I’m not good enough”. It really snapped me out of that mindset, instead ok, if you work hard enough you can you can do this. Right? So it was really that one trigger moment that kind of flipped everything around.

Although Kyla describes being successful prior to her low test mark, it was this moment of defeat that changed her approach. This “trigger moment” was her turning point. She invested more time and energy into succeeding in her math course.

It was a lot of work to try and fix the mark that factoring had brought my average down to.

Kyla’s description of perseverance demonstrates that she believes that struggling is a natural part of her math experience, and she testifies that she is equipped to handle difficulties. She embarked to work through her struggles by firstly, practicing questions to better understand concepts and secondly, she enlisted the help of a tutor who helped her understand how she learns and identified some of the common errors she makes. Duckworth (2016) defines this practice of finding connections to gain understanding as deliberate practice. Duckworth (2016) believes that gritty individuals initially reflect on their experience of struggle and then choose an appropriate method to achieve their goal. This echoes Dewey’s (1910) belief that reflective thinking is
essentially to build supportive connections. Kyla’s reflection and grit is demonstrated by the combination of her courage to ask for help and her work ethic in pursuit of understanding.

Although learning mathematics can be considered by students to be difficult and frustrating (Nardi & Stewart, 2003), the process of struggling is viewed by some researchers (Boaler, 2016; Dewey, 1910; Hiebert & Grouws, 2007; Schoenfeld, 1988; Warshauer, 2014) to be both natural and necessary. Struggling which engages students to make sense of a problem (Hiebert & Grouws, 2007) may take time and effort (Schoenfeld, 1988), yet is comprehensible can be considered a productive struggle (Warshauer. 2014) The experiences of resiliency and perseverance which students in this study discussed, echo the findings by Warshauer (2014) that productively struggling is valuable process in learning with understanding and should be encouraged by teachers.

When students are struggling, Warshauer (2014) urges teachers to encourage continued effort by asking students questions such as what do you mean by that? or why does that happen? and support students to try. She believes that these questions offer students an opportunity for reflection because students must reason and justify their understanding during this conversation. Warshauer (2014) advises teachers to provide an environment in which students feel comfortable on a questions regardless of whether it is the right or wrong approach and are not rushed.

When confronted with difficulties, the students in this study persevered in their search for understanding. All the students in this study believed that making connections among concepts led to their understanding of what they are doing and why they are doing it. Using visual aides and getting support from teachers, peers, parents were strategies they said helped them achieve this desired understanding.
Chapter 6: What I learned from this study

Looking Back

In writing and researching the question of understanding students’ struggles in math, I have come to humbly realize that this study has given me the opportunity to reflect upon my own struggles to understand students’ learning experiences of mathematics. It has provided me a chance to consider my assumptions and actions. Prior to this study I assumed that students struggled because they did not have the confidence to proceed independently, even after I had worked through a question with them. I wondered if there was more that I could do in my teaching practice that would help struggling students. I had not considered that the teaching methods I had practiced at the time may not provide students with the tools to overcome their difficulties. Even during the study, as I listened to students speaking about their experiences, rather than hearing their struggles, I heard the strategies that helped them, possibly because these may become the practices that I could use to help future students. However, with the help of my advisor, I realized that I was struggling to hear students’ individual experiences of struggle in math. As I spent more time listening to their words and trying to see their experience as they experienced it, I began to better understand their meanings and perspectives of struggle. It was this transition from the practice of doing to the practice of being that was transformative for me.

Aoki (2004) suggests that teaching is not about “doing” rather, it is about “being”. He observes that the practice of teaching is often regarded as the skills and techniques necessary to help students learn the outcomes of a curriculum. I believe that I too was caught up in this practical view of teaching that focuses on teaching methods and techniques that I can use to transfer the mathematical knowledge to the students. Instead, Aoki (2004) offers the view that
teaching is about mindfully watching and being thoughtful. He encourages teachers to break from their teaching routines and reflect upon the essence of teaching, wherein teachers attune themselves to the care of their students. Interviewing students provided me the “break” that allowed me to move from being practical to experiencing what it was like to be a student in math class. I have had the opportunity to hear what it means to struggle in math from different perspectives than I had previously known. I have learned that if I want to help students who struggle, I will need to combine my practice of doing with the practice of being which will require listening to what they have to say about the meaning of their struggle. Only then can I really help them.

My journey towards understanding began with my concern for and curiosity about those students who I taught that found math frustrating and painful. I wanted to improve their mathematical journey so that they too might enjoy the subject that I regard as enjoyable. In the course of this study, I have learned that students have much to teach me about the practice of teaching mathematics.

Lampert (2001) describes the practice of teaching to be complex because it involves multiple relationships and collaborations between the teacher, students, and the content. In the past, I assumed that a student struggled because they did not believe in their ability to do math, despite being able to demonstrate their skills. I thought that as an educator, I should be able to teach all my students to learn math. This research has revealed that understanding a student’s struggles in mathematics is complex, it is not just about my relation with the content and students, but it is also about their relationship with me and the content. Together, students and I must collaborate to gain a deeper understanding of mathematics. I am grateful that I interviewed
students, because I could not have prepared a questionnaire with the right questions necessary to shed light on the multi-faceted experiences that these six students shared with me.

In this section I will discuss what I have learned from the process of my research, from students’ personal experiences in math, and from the literature. The interpretative method itself, offered me tools and insights that are useful in the practice of teaching. From students, I heard about how their experiences affected them, as well as their need for understanding and their determination to gain understanding. From the literature, I learned about the importance of relational understanding (Skemp, 1978), an awareness of compressed knowledge (Tall, 2013) and its impact on my teaching. Skemp (1978) describes relational understanding as a network of connected concepts and procedures which provides students an understanding of why and how a concept exists or a procedure is followed. Tall (2013) identifies that by observing similarities and differences and through practice, teachers develop a compressed knowledge of a concept or process. This knowledge is a simpler or more efficient understanding, that may lead to the use of short cuts. However, short cuts may not seem obvious to those who are learning the processes for the first time. The deep meaningful knowledge of mathematics which I have garnered over the years provides me with an understanding of concepts and processes that students who are learning for the first time simply do not have. I will conclude this section with what I learned from the literature and from the students in this study, about students’ struggles in mathematics, both those that are productive (Hiebert & Grouws, 2007; Warschauer, 2015) and those that are destructive. Warschauer (2015) and Hiebert and Grouws (2007) describe productive struggles as the necessary effort required by students to makes sense of concepts and procedures, whereas destructive struggles are the efforts which bring about frustration and pain. The students in this study experienced both types of struggle and their experiences revealed that although struggling
productively is an accepted part of learning, it has also been experienced destructively and caused students pain.

**Interpretive Methodology**

Interpretive methodology is used to gain an understanding of the meaning of lived experience (Moules et al, 2015; van Manen, 2014). Rubin and Rubin (1995) assert that through in-depth interviews with students, it is possible to gain insight into what is happening in their world, why they respond the way they do, and how they make sense of their experience. A semi structured interview can provide a deeper understanding of what students mean, because it provides opportunities for students to elaborate, clarify, and discuss their experience at length (Rubin & Rubin, 1995). During these interviews, I was able to hear from students the meanings they made about struggling. Through a one-on-one dialogue with students, I had an opportunity to gain new understandings of students, myself, and mathematics (Moules et al., 2015).

Interpreting students’ meanings so that I could gain new understandings about their lived experiences meant that I had to stay open minded to what students had to say, with the stance that what they say is true. Interpretive methodology has taught me that by openly and actively listening to students, I can learn something that is beyond my pre-existing assumptions so that I might provide a better learning experience (Josselson, 2013).

From the interview process I learned to ask questions that invited students to explore their educational experiences and to actively listen to their responses. Josselson (2013) believes that asking good questions that invite students to elaborate on their experience requires the interviewer to be empathetic and stay connected. These questions can be followed up with probing questions
that allow for students to provide more detail. I have learned that if I want to know more about how to help students learn, I need to engage them in a dialogue and then openly and actively listen.

Listening for the sake of gaining new insights about students’ struggles rather than thinking about how to change things may allow for new ideas and impressions (Josselson, 2013). I learned that I must actively listen to students about their experience, about the meanings they make about their experience, and how they view the challenges of mathematic so that I might gain valuable insights into how to better teach them. I learned not to assume that we share the same meanings of words, rather, it is necessary to ask students to explain their meanings.

Interpretive methodology has given me the tools to gain important insights into student learning by asking students about their experiences and attentively listen to their responses so that their experiences may challenge my pre-existing assumptions.

What I learned from Students about Understanding

Although I interviewed only six students, I gained important insights and details that I had not previously considered. Reliving their stories of math shed light on other ways that students struggle to learn math.

I was surprised to learn that every student in this study talked about their struggle to understand why they had to choose a certain method and how that method works. The students in this study told me that not knowing why or how made them feel confused, lost, worried, bewildered, and frustrated. It made learning math painful and they would dread going to class. From them I learned that taking time to explain why a concept or procedure exists, and how to use it is necessary for learning because these explanations can create understandings.
I was also surprised that students in this study continually said that providing visual aides, opportunities to investigate, and incorporating creativity would help them learn. Although I knew that these pieces were important and should be incorporated into my teaching, I did not truly understand their significance or widespread requirement for learning. Visual aides such as a diagram, picture, or objects can have a tremendous effect on helping students understand mathematical concepts (Boaler, 2016). Although I knew that visual aides could be effective, I did not consider how important it was to incorporate them into each new concept taught. Not all students learn the same way, but providing visual learners with visual aides throughout the entire course can lead to understanding and ultimately, a better learning experience.

One of my participants, Danielle, mentioned her enjoyment of investigating mathematical concepts. She said that after her investigation, she understood the concept better than she thinks she would have otherwise. The process of discovering how and why a concept works provided a more meaningful learning experience for her. Perhaps providing future students that I teach with opportunities to investigate how and why concepts work might allow them to understand and enjoy mathematics.

Another student, Derek, talked about his experiences of math as linear and lacking of creative freedom. Felder and Silverman (1988) propose that while he is being taught sequentially, he may be a global learner who would benefit from creativity exercises. Rather than learning the concepts “in a logically ordered progression” (p. 679), the authors believe that he should first be presented with the goal of the lesson, establishing “the context and relevance of the subject matter and relate it to the students’ experience” (p. 679). The authors recommend allowing creative learners to think divergently to generate and devise their own solutions.
Similar to the teachers in other studies (Aljughaiman & Mower-Reynolds, 2005), I had not previously considered incorporating creativity into mathematics. However, Lev-Zamir and Leikin (2011) believe that teaching creatively and for creativity can deepen learning. Mann (2006) broadens the learning scope to include enjoyment. “Encouraging mathematical creativity in addition to computational fluency is essential for children to have a productive and enjoyable journey while developing a deep conceptual understanding of mathematics” (p. 240). These authors affirm Derek’s request to incorporate creativity into his math classes.

Research (Aljughaiman & Mower-Reynolds, 2005; Atkas, 2015; Lev-Zamir & Leikin, 2011; Mann, 2006; Pehkonen, 1997) has shown that there is no agreed upon definition of creativity in teaching. In a study conducted by Aljughaiman and Mower-Reynolds (2005), 36 elementary teachers were asked about their conceptions and definitions of creativity. The top three responses indicated that teachers believe creativity involves original ideas, an aesthetic product, and intelligence. The authors were surprised that divergent thinking was only indicated in 14.7% of the responses. They concluded that teachers have different conceptions of creativity in the classroom. Mann’s (2006) review of the literature on creativity also found a variety of definitions. However, he believes that the essence of teaching mathematics creatively is to encourage children “to reach beyond the familiar and probe deeper into the relationships and structures of a problem” (p. 253). Mann’s definition echoes the connection that creativity has to conceptual understanding. Both Mann (2006) and Pehkonen (1997) believe that the key to teaching creatively is solving problems that require exploration. Both authors challenge teachers to reflect on their teaching practices and search for curricular materials that develop creativity without returning to heuristic problem solving strategies (Mann, 2006; Pehkonen, 1997).
Derek’s experience of a lack of creativity and the literature’s support of the importance of incorporating creativity into teaching made me reflect on how I could integrate creativity into my teaching. Approaching teaching with this new stance of creativity requires consideration of both the curriculum and my teaching practices. I will need to find meaningful opportunities for students to problem solve so that they elaborate on their knowledge, rather than reproduce it. However, I think that problems which enable flexible and innovative thinking in conjunction with dialogue may provide students with a deep and meaningful understanding of the concepts they are learning.

I was awestruck by the efforts students in this study made towards gaining understanding. Their strategies of using visual aides, sourcing help from teachers, family, peers, and the internet, and their perseverance to continue despite struggling amazed me. The clear message from them was that despite their failures, they each believed that they could succeed. I have learned to appreciate the different ways students work towards achieving their successes.

What I learned from the Literature about Understanding

In researching what students were telling me about the importance of understanding in math, I was surprised to find so many studies concerned with the need to teach math with understanding. Hiebert and Carpenter (1992) compared the goal of teaching students to learn by understanding, with finding the Holy Grail. They said that although educators agree that understanding is necessary to learning, it has been difficult to design learning that successfully encourages understanding. One reason it might be difficult to teach for understanding is because such diverse learners exist in our schools. Every student in my study had different learning needs: visual, auditory, creative. Two students said they would prefer a more creative learning
approach which may include investigations or projects, while four of the students said they
preferred being taught the material and then practicing it.

Skemp’s (1978) article on relational understanding provided me great insights into why
understanding is so important to students. Skemp (1978) describes instrumental knowledge as
rules without reasons, and believes that it is a common method of teaching. His examples of
presenting students with a formula for the circumference of a circle or the method of multiplying
fractions in which it is assumed students know to multiply the numerators together and then
multiply the denominators together to find a new fraction, resonates with me. I reflected on the
many times that I assumed that my high school students knew the reasons for these rules, and
that in some cases, I too, only knew the rule, but not the reasoning behind it. I realize that the
explanations of the processes could easily be investigated, providing students with the much
needed why. In addition to answering why the rule works, I believe that the explanations too can
be often be interesting and illuminating. In the future, I aim to arouse curiosity and explore the
reasons behind the rules.

Skemp (1978) also compared learning mathematics to drawing a mental map. The
concepts in math are related and can be better understood if they are introduced as such. By
teaching the concepts in a connected way, a mental map with joined pathways can be formed.
These connections give students the knowledge of what to do and why. Skemp (1978) believes
that if a student with relational knowledge gets lost, he or she may have a variety of choices of
connections or pathways to navigate a solution. Although I am embarrassed to admit that I did
not always teach for understanding, I am grateful for this “aha moment”. I will challenge myself
to understand the connections that already exist among concepts so that I may help students
understand the reasons behind what they are doing and why.
Martin and Towers’ (2016) study offered me practical suggestions on how to create these connections. They recommend beginning a lesson with existing understandings that students have about a concept and exploring possible misunderstandings. Muddling through these misunderstandings and misconceptions may enable students to strengthen their understanding. Taking time to fold back and discuss these problematic met-befores could provide students further insights and provide the groundwork to build a connecting concept. The authors suggest continually moving back and forth between old and new to strengthen or thicken understanding. I look forward to opening up these conversations with students and discover some misunderstandings or even different ways of understanding which I had not previously considered.

Tall’s (2013) discussion of compressed knowledge was another perspective I had not previously considered, yet in hindsight was evident. Through the years of recognizing similarities and differences among concepts, I have created a mental map of how concepts are connected. My experiences navigating this mental map have allowed me to understand and create short cuts among concepts. For example, when rearranging an equation to solve for a variable, I might not show the steps of how I isolated the variable, because the steps seem simple and obvious to me. However, the students in my class may be learning this material for the first time, or have little experience or understanding with these concepts (Tall, 2013). When I take a short cut that I assume is evident, I may confuse students. What I think may be simple and efficient, may be confusing and random (words that students used to describe the times they did not understand what the teacher was doing) to students that are learning these concepts for the first time. I have learned that I need to teach a concept as though I am seeing it for the first time,
and I need to stay in touch with students as to how this experience is for them, so that I do not assume they understand.

**Relevance**

Not all the students in this study believed that the concepts they were learning in Math 10 would be relevant or useful to their lives. Their insights make me question why I teach mathematics. I struggle with the belief that mathematics should be relevant and agree that the specific topics that I teach are not part of a student’s daily experience. What does being relevant mean? Does it mean that it is useful in a student’s day to day life? Or does it mean that the processes such as pattern making, observations, exploration, inventing, crafting arguments, and analysis, which are learned within a mathematical unit, are useful (Lockhart, 2009)? Specific mathematical units such as trigonometry, exponents, and linear equations are not experienced by individuals every day; however, the processes needed for making a decision are a part of our daily experience. Two students in my study, Naia and Kyla, both agreed that although the specific concepts that they learn are not important, mathematics provides opportunities to learn how to problem solve, which they believe is a skill they will always need.

In his book A Mathematician’s Lament, Paul Lockhart (2009) argues that mathematics is about “playing with patterns, noticing things, making conjectures, searching for examples and counterexamples, being inspired to invent and explore, crafting arguments and analyzing them, and raising new questions” (p. 118). It is about reasoning and deep meaningful understanding. He says that although it is not vitally important, doing math should be an adventurous and enjoyable experience. Although I agree with Lockhart that mathematical concepts can be taught in a way
that sparks curiosity and wonder, I will also search for meaningful and relevant connections that the students in my study need to gain understanding.

Researching and writing this thesis has provided me an opportunity to think differently about my previously held assumptions and practices. I have gained new perspectives on how students struggle to understand math, their belief that mathematics is not relevant and about the importance of listening and asking students about their learning.

**Productive and Destructive Struggles**

Research by Boaler (2016), Hiebert and Grouws (2007), Mann (2006), Schonefeld (1998) and Warshauer (2015) supports the importance of allowing students to struggle productively to develop conceptual understanding. Productively struggling engages students in their learning when students take risks and grapple with difficult problems whose solutions are within reach. Unlike destructive struggle which causes despair and needless frustration, I have learned that productive struggles are a natural part of learning mathematics that should be encouraged and supported in the classroom. Students in this study spoke about specific times that they struggled and shared both the situation of struggling and how they felt during this time of struggle. All the students experienced struggling in their classrooms when they learned a new concept or practiced it in their homework. One student spoke about struggling or freezing on an exam. Two students spoke of the feelings of getting back an exam that they had done poorly on.

Productive struggles should be viewed as a natural part of learning mathematics. In the documentary on Wiles’ proof of Fermat’s Last Theorem, Wiles remarked that “mathematicians just love a challenge” (Lynch, 1997, 7:19). This challenge is their productive struggle. They compare their work to taking a journey or stumbling in the darkness (Singh, 1997). They take
risks (Mann, 2006) and may spend many years to find a solution to a problem, as was the case for Andrew Wiles (Singh, 1997). In the documentary, Wiles spoke about his productive struggle to find this proof. He said he would scribble, doodle, find patterns, make calculations, try to make connections with previously known conceptual understandings, and sometimes just try to “find something new” (Lynch, 1997, 22:00). These are some of the same processes that the literature (Hiebert & Grouws, 2007; Mann, 2006; Skemp, 1978) suggests students undertake when learning math. I view these processes as enjoyment and aim to spark curiosity in future students that I meet.

Naia told me about a question that she could not understand and her struggle to find the solution. She said that after trying to solve it herself, she took a break. That evening she asked her sister for help, but could not make sense of her explanation. The following day when a classmate asked her about the question, she could finally make sense of it. This productive struggle required patience, time, support from her family, and perseverance. Naia described feeling incredibly confused, bewildered, and stressed as she struggled to understand. However, once she figured it out, she was incredibly happy. She said that she felt smart and confident to help other people. Schoenfeld (1988) recommends that teachers provide students the opportunity to engage with problems that cannot be simply solved in a few minutes require time and hard work. Grappling with mathematical ideas helps students make sense of the math and construct interpretations which may be more connected to what is already known (Hiebert & Grouws, 2007).

Conversely, destructive struggles create needless frustration and despair. Danielle experienced a destructive struggle. Danielle told me about a time when she froze on an exam. She said that she did not know how to solve any questions and did not want to take the test. She
was confused because she had done all her homework, yet she could not answer the questions. She believes that a test should reflect her knowledge. Daniele felt confident before writing the test because she had completed her homework and gained the knowledge that she needed to be successful on the test. However, she could not transfer her knowledge to the questions on the test. Research by Boaler (1998), Schoenfeld (1988), and Skemp (1978) indicate that a lack of deep meaningful conceptual understanding hinders flexible thinking. Danielle’s destructive struggle experience has strengthened my determination to ensure that I provide students with connected conceptual learning opportunities so that they may be better prepared to write a test.

I would like to prevent future students from experiencing destructive struggles. Although, I cannot make that guarantee, I have learned from both the students in this study and the literature that engaging students in a dialogue, in which I humbly and actively listen, as well as providing opportunities for students to make meaningful connections may enable them to struggle productively and gain conceptual understanding. These learning opportunities must also be provided in a supportive environment in which students are encouraged to take risk and embrace their mistakes as a natural part of learning and share these learning experiences with others (Boaler, 2016; Lampert, 2001).

**Answering the Research Questions: New Insights about Struggle**

The students and the literature have helped me gain new insights into answering the research questions of how students experience struggling in math and whether the way they struggle influences their attitudes towards math. Their experiences showed that students do indeed struggle differently, in the classroom learning or completing homework, and in writing an exam. However, their experiences of struggling do not always effect their attitudes towards math.
As their teachers introduced concepts, students spoke of their struggle with the presentation of the material. One student spoke about his struggle to learn when the concepts were presented in a linear fashion. Three students said that they struggled when they were not presented with visual representations of the concepts. Three students spoke about struggling when they missed a step within a procedure. One student indicated that missing a step sometimes happens when teachers skip a step that might seem obvious or too simple to include.

All the students in this study spoke about struggling when they could not understand the concepts. Three students indicated that the formulas did not make sense and seemed random. One student said that because math is about learning “how you are supposed to answer a question”, indicating that memorizing can be difficult. All six students said that they struggled to understand why and how to do what they are supposed to do.

While working on their homework, every student told me about their experiences of struggling to solve a problem without help. One student said that when she struggled in class, she would “ignore” her homework.

Students said that during the times they struggled, they felt bewildered, confused, worried, frustrated, discouraged, and stressed. Some students said that these feelings affected their interest in doing homework and going to class. Four students said that when they struggled to understand they dreaded going to class.

All six of the students talked about the strategies they used in times of struggling. Every student asked for help either from a peer, a family member, a tutor, or the teacher. Five students seemed to persevere through their struggles in math. Two students shared an experience of math that was unproductive. One student shared the pain and frustration of trying to learn concepts
that were uninspiring, meaningless, and irrelevant. Although two students said they liked math, the other five said they only learned math because it is a requirement to graduate.

In this study, the way students struggled did have an affect on their attitude towards math. When they destructively struggled to understand concepts, students said that they avoided doing homework, or dreaded going to class. However, when students talked about the productive struggle to persevere and understand, they were excited and proud of themselves. One student believed that struggling to understand is normal, that it takes time to understand. She advised students not to give up, because “they may not get it yet”, but she believes they will eventually understand. However, another student said he “absolutely despised” math. He felt that his destructive struggle to understand math stemmed from his experiences of learning a subject that was linear, random, and lacked creativity. The ways that students experienced learning math had both positive and negative impacts on their attitudes.

The students in this study shared their stories about the impact that struggling had on their mathematical learning experiences. Although their experiences were different, their stories have enabled me to view these experiences through their eyes and have impacted my understandings and practices of what it means to be a good mathematics teacher. As Aoki (2004) suggests, teaching is not simply about passing information to students, it is about mindfully watching students and being thoughtful in caring for them. I have learned that understanding my students is necessary to teach them and that a genuine understanding of students calls for dialogue. Only through open and active listening can I gain new insights into their experiences and perhaps together we can find better ways to teach and learn. Perhaps through this dialogue I might hear more about students’ need for understanding how and why, visual aides, creativity, and meaningful learning experiences. This study has provided me with profound insights into the
importance of not only how I want to teach, but how I want to conduct myself as attending to
students so that we might learn from each other.
Chapter 7: Reflecting on the study, and moving forward with (New) Understandings

Looking Ahead

In 2015, Alberta Education reduced the provincial diploma exam (a standardized exam that every grade twelve student must write in core subject areas) from a weighting of fifty percent of the final grade to thirty percent (the remaining seventy percent weighting is the teacher’s mark). One of the reasons for this change was the belief that the final mark should mainly be a reflection of a student’s year long efforts (Alberta Education, 2015). The following year, Alberta Education embarked on a six-year initiative to revamp six areas of study including mathematics to create an updated cohesive curriculum that is relevant to a changing world that “honour ways of knowing, facilitate learning, enable broad exploration, and deep understanding of subjects (Alberta Education, 2016a, [Video webcast], 2:12). Consultations on the development of this new curriculum will take place with “teachers, students, parents, superintendents, trustees, post-secondary institutions, employers, industry and apprenticeship bodies, and the broader community” (Alberta Education, 2016b, “What’s Next,” para.1). The goal is to design a curriculum which is relevant, current, and foster interdisciplinary learning (Alberta Education, 2016a, [Video webcast]).

On its website, Alberta Education has a series of seven mathematical processes which teachers are expected to utilize when teaching each grade’s program of studies or curriculum (Alberta Education, 2016c, “Support Documents”). These supporting documents give a description of the processes of communication, connections, mental math and estimation, problem solving, reasoning, technology, and visualization and how to support them within and
out of the classroom. The outcome of these processes is to enable students to learn, perform, and understand math, as well as foster life long learning (Alberta Education, 2016d).

The grade ten students that I interviewed told me that although their experiences of Math 10 are not meaningful, relevant, nor integrated into their present world, they know that they need this course to graduate and to continue onto post secondary education. They said that a common complaint among their peers was that they wonder why they need to learn these concepts. However, these students’ advice was to look past that complaint and learn the content. In their experience, successfully learning the Math 10 course requires an understanding of the concepts, the help of others, and in some cases, using visual representations. All six students talked about the importance of having support from others not only to ask questions, but to gain a deeper understanding of the concepts. Although the students’ experiences in this study concur with Alberta Education’s mathematical processes of communication, connections, problem solving, reasoning, and technology, they also provided insights into understanding and struggling.

Research by Boaler (2016), Lampert (2001), Mann (2006), Schoenfeld (1988), and Alberta Education (2016c) suggest that the math should be learned using problem solving to encourage student curiosity, exploration, and collaboration generate relational understanding. For those who are concerned that not all mathematical concepts can be included using this teaching style, Lockhart (2009) ponders, “if some issue never happens to come up in thirteen years of schooling, how interesting or important could it be” (p. 81)? This and other research (Boaler, 2016; Davis, 2014; Hiebert & Grouws, 2007; Skemp, 1978) echo the need for engaging, significant, and relevant mathematical learning experiences. Given the upcoming changes, consideration should be given to creating a curriculum that provides opportunities for student to persevere and engage in rich, deep, meaningful understandings.
Probing Questions

The sample size of this study is too small to draw definitive conclusions beyond this study. However, the six students in my study have raised penetrating questions that I think might be useful for mathematics teachers to reflect upon. Aoki (2004) encourages teachers to break from their daily routines and attune themselves to their students. Here are a few questions that may provide new insights and understandings as they have for me:

*How well do you know your students?*

Students have differing past experiences of learning mathematics which have shaped their attitude to learning. The students in this study have shown me that grade 10 students know a lot about what they need to be successful in their classes. Their voices can provide insights into how best they learn.

*Do you know which learning strategies will benefit each student: self-directed/inquiry/teacher-directed?*

Students in this study shared with me that their perfect mathematics learning environment included differing teaching styles.

*Do you know which ones need visual aids?*

I was surprised to hear that most of the students talked about their need for visual aides. Research (Lev-Zamir & Leikin, 2011; Mann, 2006; Pehkonen, 1997) supports visual aids to create meaningful learning connections.
Do you know when and which students like to work in groups or work alone?

Students in this study talked about their preference to work with others so that they could easily get help and that collaboration may create new ways of thinking about the concepts and procedures.

Do you know who will ask questions and who will not? Do you check in with your students beyond asking “are there any questions”?

One student in the study talked about his unwillingness to ask questions because he felt that the teacher would think it was a stupid question or that he would be scolded for missing a part of the lecture. However, this student said he wished his teacher would ask specific questions and listen openly to try to understand where he is coming from.

Do your students see the big picture of how the concepts are related and can be understood? Do they know how new concepts are related to previously learned concepts?

Students in this study said they want to understand how the procedures and rules came to exist, so that they do not need to memorize. Students want to gain deep meaningful knowledge of the concepts. Research (Skemp, 1978; Martin & Towers, 2016) demonstrates the need for relational understanding to learn mathematics and encouraging folding back to engage with previous “met-befores” to thicken their conceptual understanding.

Do they understand what to do and why to do it?

Not knowing what to do and why created confusion, frustration, and stress.
**Are students able to communicate their understanding through reflection and/or negotiation?**

Reflection or dialogue with peers, teachers, or family either through a conversation or writing in a journal can create an opportunity to negotiate their understandings in their own words.

**Do students understand the similarities and differences?**

Understanding and talking about similarities and differences further help students create deep, meaningful understanding.

**Do you know how they struggle, why they struggle, and who is struggling?**

Students struggle both productively and destructively in different ways to understand concepts and procedures, to answer problems, and in writing tests. There may be some common ways students struggle to understand a concept, but this study has shown me that unless I engage in dialogue with a student, I cannot assume I know how they struggle.

**Do your students know where to access support to help them understand?**

The experiences of the students in this study revealed that struggling is part of their learning. I believe that struggling productively should be taught as a natural part of the learning process. However, students require supports to help them through their difficulties. The students in this study showed tenacity and perseverance to gain understanding. They asked peers, teachers, parents, tutors, or checked the internet when they needed help. One students said they found these supports themselves, while another said that teachers had provided a list of supports. Although Math 10 students shared strategies for persevering when faced with challenges in learning math, educators can benefit students by offering alternate strategies, guides, and supports that students can access.
Do your students see relevance in the math they are learning?

Students in this study said they did not believe that the concepts they were learning would be useful to them, except to graduate. Reflecting on how to create more meaningful learning experiences can have a positive impact on students and their attitudes towards math.

These questions have provided an opportunity for me to reflect on ways to better understand students’ experiences in mathematics and thereby help create more meaningful, connected, and enjoyable mathematics learning.

Moving Forward with New Understandings

I am inspired and motivated first by the students’ shared experiences and then by the supporting literature to think differently about my teaching practice. The students’ lived experiences opened up new ways for me to understand how students learn and struggle. I better understand the need for making connections to past concepts and knowledge. Repeatedly hearing from these students that they want to know why and how they do things in math, makes me realize that it is very important to incorporate this understanding into their learning.

Moving forward, I am committed to not only asking myself the questions above, but incorporating what I have learned into my practice of teaching. I plan to improve my communication with students either using journals or interviews to try to better learn more about these students, how they view math, what they like and don’t like and what they need to be successful learners (visuals, opportunities for support). I want to incorporate more opportunities for collaboration, so that students have opportunities to negotiate their understanding. Situations that may allow students to deepen their understanding of concepts would include problem solving, open ended problems, and partner quizzes. I want to find relevant and interesting
problems which can be used to the curriculum. I will share my philosophy of the importance of making mistakes in learning, struggling productively, and embracing an ongoing perseverance and grit to learn.

Most importantly, I aim to create connections amongst concepts and procedures. I will link new knowledge to previously learned concepts, building a broader mathematical landscape with connecting pathways. Whenever possible, I will use visual aides and incorporate creativity in hopes of creating more engaging their understanding. Checking in with students will require me to actively listen and ask questions so as not to assume that we share common meanings. My intentions are to create a mathematical learning experience for my students where struggling productively is encouraged and conceptual understanding, is rich, meaningful, and enjoyable.
References


Appendix A

Letter to Parent or Guardians

Dear Parent or Guardian,

I am a graduate student at the University of Calgary in the Werklund School of Education. I am interested in understanding grade 10 students’ experiences in their math classrooms. As a mathematics teacher, I have a genuine interest to hear from students about their own experiences in math so that I may help future students that I teach. I am writing this letter to ask you for permission for your child to participate in the research which will form my graduate thesis. My advisor, James Colin Field will supervise this study which the Conjoint Faculties Research Ethics Board has approved.

Your child’s participation will involve a one hour interview in which she/he will describe their experience in their math class. This interview will only be audio recorded. Your child’s identity will remain anonymous with only myself and my supervisor having access to the audio recordings and transcripts. Your child may be asked to participate in a follow up interview to clarify or further discuss their responses.

No risks are anticipated in participating in this interview. The interview can be stopped at any time. Your child may withdraw from the study at any time without negative consequences and all data will be destroyed.

If you and your child are interested in participating in this study or if you have any questions, please send me an email.

Sincerely,
Stephanie Smith