Minimum-Energy Cooperative Routing in Wireless Networks with Channel Variations

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Abstract

This paper considers the problem of finding minimum-energy cooperative routes in a wireless network with variable wireless channels. We assume that each node in the network is equipped with a single omnidirectional antenna and, motivated by the large body of physical layer research indicating its *potential* utility, that multiple nodes are able to coordinate their transmissions at the physical layer in order to take advantage of spatial diversity. Such coordination, however, is intrinsically intertwined with routing decisions, thus motivating the work. We first formulate the energy cost of forming a cooperative link between two nodes based on a two-stage transmission strategy assuming that only *statistical knowledge* about channels is available. Utilizing the link cost formulation, we show that optimal *static* routes in a network can be computed by running Dijkstra's algorithm over an extended network graph created by cooperative links. However, due to the variability of wireless channels, we argue that many-to-one cooperation model in static routing is suboptimal. Hence, we develop an *opportunistic* routing algorithm based on many-to-many cooperation, and show that optimal routes in a network can be computed by a stochastic version of the Bellman-Ford algorithm. We use static and opportunistic optimal algorithms as baselines to develop heuristic link selection algorithms that are energy efficient while being computationally simpler than the optimal algorithms. We simulate our algorithms and show that while optimal cooperation and link selection can reduce energy consumption by almost *an order of magnitude* compared to non-cooperative approaches, our simple heuristics perform reasonably well achieving similar energy savings while being computationally efficient as well.

Index Terms

Cooperative communication, minimum energy routing, variable wireless channels.

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I. Introduction

Energy efficient communication is a fundamental problem in wireless networks. First, in a wireless network, excessive transmission energy leads to increased interference in the network resulting in decreased network throughput. Second, in wireless ad hoc and sensor networks, which are typically battery powered, inefficient use of energy causes rapid depletion of batteries resulting in a disconnected network.

Over the past several years, this problem has been studied extensively at different layers of the protocol stack, notably at the network (*e.g.*, [1]) and physical layer (*e.g.*, [2]). At the physical layer, in particular, it has been shown that using multiple antennas at the transmitter or receiver achieves considerable transmission energy savings compared to a single antenna thanks to the spatial diversity inherent in wireless networks [3]. But, in some cases, the use of multiple antennas on a transmitter or receiver may be impractical (*e.g.*, due to small size of sensors) or too costly (*e.g.*, due to costly analog circuitry). In such situations, it has been recently shown that, by allowing cooperation among spatially distributed single-antenna nodes, the so-called *cooperative communication* can achieve significant energy gains comparable to those achieved by multi-antenna systems [4].

However, although there has been extensive work at the physical layer demonstrating the utility of cooperative communications under metrics such as bit error probability or outage probability, there have been very few works that consider how to incorporate such links into practical networks [5]–[11]. This is a critical shortcoming, since cooperative communication inherently disrupts the normal separation of routing from the physical layer specification. In this work, we formulate energy optimal cooperative routing as a joint optimization of *cooperative link formation* at the physical layer and *route selection* at the network layer. Our objective is to characterize the energy gain of cooperative communication in a network with *variable* wireless channels. This obviously has direct application in practical wireless networks, but also provides a perspective for the research field on the utility of cooperative communication when measured with network-level metrics.

Wireless channels are inherently variable and fluctuate over time due to noise, shadowing, fading, *etc*. In cooperative communication, a critical issue is the availability of channel state information at the transmitters. If *instantaneous* channel information *including channel phase* is available, then transmitters can cooperatively beamform to a receiver to minimize transmission energy [12]. Using such channel information, optimal link formation and selection can be formulated *deterministically* to compute minimum-energy routes in a network as studied in [6], [8]–[11]. Whereas there have been recent examples of cooperative beamforming [13], the synchronization requirements for such are onerous in a mobile ad hoc network. Moreover, collecting instantaneous channel information at every transmission epoch is challenging when channels fluctuate rapidly, for example due to mobility. Hence, some recent work has instead explored cooperative communication assuming that no channel information is available [5]. When no channel information is available, cooperative links are formed by allocating equal transmission power to cooperative transmitters, effectively reducing the cooperative routing problem to finding the shortest path in a network. While being simpler from the implementation perspective, the cooperative routes computed using this approach are less energy efficient compared to the routes computed by optimal link formation when channel information is available.

Although instantaneous channel information is difficult to obtain in practice, some *partial* information (*e.g.*, probability distribution of the channel fading process) is usually available. Thus, in this work, rather than taking one of the above extreme approaches (*i.e.*, instantaneous channel information or nothing), we assume that the *distribution* of the channel variations is known. Throughout, we will assume that the channel variation is due to multipath fading, although this is not necessary for the algorithm specification. Given the distribution of the channel variation, optimal cooperative link formation is essentially a *stochastic* optimization problem, the objective being minimization of the transmission power with respect to the known fading distribution. We solve this optimization problem for the case of Rayleigh fading (which is widely used in literature [14] and serves as the worst-case over a broad class of fading distributions), although our analysis can be generalized to other classes of fading models as well.

In our previous work [7], we studied diversity-based cooperative routing with a quite different approach. In particular, in [7], the transmitting set is continuously grown (i.e. nodes are added but never removed) until the receiver is able to obtain the message. However, such a cooperation model is considerably restrictive in multiflow scenarios, because, as the routing progresses, almost the entire network will cooperate to transmit the same message to the same destination, which severely penalizes network throughput [15]. In the present work, we consider a general two-stage cooperation model (see Section II) that does not suffer from this problem while being more amenable to implementation. As we show, however, the analysis of the optimal cooperative link formation and selection is significantly more challenging in this model.

We first develop an optimal *static* routing algorithm by applying Dijkstra's algorithm to an extended graph of the network formed by cooperative links. However, due to the variability of wireless channels, we argue that the unicast-based many-to-one cooperation model in static routing is suboptimal. To address this issue, we then develop an *opportunistic* routing algorithm based on anycast many-to-many cooperation, and show that optimal routes in a network can be computed using a stochastic version of the Bellman-Ford algorithm. The optimal routing algorithms we develop are centralized and have exponential computational complexity. Nevertheless, they provide useful upper bounds in characterizing the energy gain that can be obtained from cooperation in fading environments. Moreover, we use optimal algorithms as a baseline to evaluate the performance of (computationally simpler) heuristic algorithms developed in this work and elsewhere [5].

Our main contributions in this paper can be summarized as follows:

1) We formulate energy cost of forming a cooperative link between two nodes in a fading environment subject to a constraint on link reliability.
2) Utilizing our link cost formulation, we develop static as well as opportunistic routing algorithms to find minimum-energy routes in a network.
3) We develop several heuristic routing algorithms in order to mitigate the complexity of the optimal algorithms, and evaluate their performance using simulations.

The rest of this paper is organized as follows. In Section II, we describe our system model. Section III presents our optimal power allocation and routing formulation. Opportunistic cooperative routing is presented in Section IV. Section V presents several heuristic algorithms. Simulation results are presented in Section VI, and Section VII concludes the paper.

II. System Model

We consider a wireless network consisting of a set of nodes distributed randomly in an area, where each node has a single omnidirectional antenna. We assume that each node can adjust its transmission power and that multiple nodes can coordinate their transmissions at the physical layer to form a cooperative link. As no beamforming is performed, only rough packet synchronization is required [16]. We denote the set of the nodes in the network by $\mathcal{N}$, and assume that there are $N = |\mathcal{N}|$ nodes in the network.

A. Channel Model

Consider a transmitting node t_i, and a receiving node r_j. Let x_i and y_j denote the transmitted and received signals at nodes t_i and r_j, respectively. Without loss of generality, we assume that x_i has unit power and that transmitter t_i is able to control its power p_i in arbitrarily small steps up to some limit $\mathrm{P}_{\max}$. Let η_j denote the noise and other interferences received at r_j, where η_j is assumed to be additive white Gaussian with power density P_η. The received signal at receiver r_j is expressed as follows

$$y_j = \sqrt{\frac{p_i}{d_{ij}^{\alpha}}} h_{ij}\, x_i + \eta_j, \tag{1}$$

where, d_{ij} is the distance between the transmitting and the receiving nodes t_i and r_j, α is the path-loss exponent, h_{ij} is the complex channel gain between t_i and r_j modeled as $h_{ij} = |h_{ij}|e^{\jmath\theta_{ij}}$, where $|h_{ij}|$ is the channel gain magnitude and θ_{ij} is the phase. We assume a non line-of-sight (LOS) environment, implying that $|h_{ij}|$ has a Rayleigh distribution with unit variance, *i.e.*, $\mathbb{E}\left[|h_{ij}|^2\right] = 1$.

Let γ_{ij} denote the Signal-to-Noise-Ratio (SNR) at receiver r_j due to transmitter t_i transmitting with power p_i. It is obtained that

$$\gamma_{ij} = \frac{1}{\mathrm{P}_\eta} \frac{p_i}{d_{ij}^{\alpha}} |h_{ij}|^2 \,. \tag{2}$$

Since $|h_{ij}|$ is Rayleigh distributed with unit variance, $|h_{ij}|^2$ is exponentially distributed with mean 1. Consequently, γ_{ij} is exponentially distributed with decay rate $\mu_{ij} = \mathrm{P}_\eta \frac{d_{ij}^{\alpha}}{p_i}$.

B. Cooperation Model

We consider a two-stage cooperation model to send a message from a transmitter t_k to a receiver r_k as follows:

- **Stage 1:** t_k broadcasts the message to its neighborhood with some transmission power P_b.
- **Stage 2:** Every node t_i $(i \neq k)$ that has successfully decoded the message will join t_k to form a cooperative transmitting set $T_k = \{t_1, \ldots, t_m\}$. Transmitting set T_k cooperatively transmits the message to r_k using some power allocation vector $\mathbf{p} = (p_1, \ldots, p_m)$.

We use the notation $\langle T_k, r_k \rangle$ to denote the cooperative link between transmitting set T_k and receiver r_k. Using the channel model (1), the total received power at r_k is then given by $p_k = \sum_{t_i \in T_k} \left(\frac{|h_{ij}|^2}{d_{ij}^{\alpha}}\right) p_i$.

Let γ_k denote the total SNR at receiver r_k due to m cooperative transmitters in T_k. We have $\gamma_k = \sum_{i=1}^{m} \gamma_{ik}$, which is the summation of m independent and exponentially distributed random variables γ_{ik} (as derived in (2)). Let $F_{\gamma_k}(y)$ denote the cumulative distribution function of γ_k. The summation of independent and exponentially distributed random variables can be modeled as a Hypoexponential random variable. Therefore, $F_{\gamma_k}(y)$ can be expressed as

$$F_{\gamma_k}(y) = 1 - \alpha e^{y\mathbf{\Theta}_k}\mathbf{1}, \tag{3}$$

where

$$\mathbf{\Theta}_k = \begin{bmatrix} -\mu_{1k} & \mu_{1k} & 0 & \ldots & 0 & 0 \\ 0 & -\mu_{2k} & \mu_{2k} & \ddots & 0 & 0 \\ \vdots & \ddots & \ddots & \ddots & \ddots & \vdots \\ 0 & 0 & \ldots & 0 & -\mu_{(m-1)k} & \mu_{(m-1)k} \\ 0 & 0 & \ldots & 0 & 0 & -\mu_{mk} \end{bmatrix},$$

and $\alpha = [1, 0, \ldots, 0]$. Also, $\mathbf{1}$ is a column vector of ones of size m, and $e^{\mathbf{A}}$ denotes the matrix exponential of matrix $\mathbf{A}$.

Let β denote the minimum SNR required at receiver r_k to decode the transmitted message at some desired rate λ, *i.e.*, $\beta = 2^{\lambda} - 1$. Due to fading, the cooperative link $\langle T_k, r_k \rangle$ may not be able to sustain the rate λ, resulting in *outage*. Let $\mathcal{S}(T_k, \mathbf{p}, r_k)$ denote the probability that link $\langle T_k, r_k \rangle$ is not in outage for power allocation vector $\mathbf{p}$, *i.e.*, the transmission is successful. We obtain that:

$$\mathcal{S}(T_k, \mathbf{p}, r_k) = \mathbb{P}\left\{\gamma_k \geq \beta\right\} = \alpha e^{\beta\mathbf{\Theta}_k}\mathbf{1}\,. \tag{4}$$

C. Routing Model

A K-hop cooperative route ℓ is a sequence of K links $\{\ell_1, \ldots, \ell_K\}$, where each link $\ell_k = \langle t_k, r_k \rangle$ is formed between a transmitting node t_k and a receiving node r_k, using the two-stage cooperative transmission at the physical layer. The sequence of links ℓ_k connects a source 's' to a destination 'd' in a loop-free path. Our objective is to find a path that minimizes end-to-end transmission power to reach the destination.

Definition 1 (**Link Cost**). *The cost of link $\ell_k = \langle t_k, r_k \rangle$ denoted by $\mathcal{C}(t_k, r_k)$ is defined as the minimum expected transmission power to deliver a message from t_k to r_k using the two-stage cooperative transmission subject to rate λ and outage probability p_ϵ.*

Then, the problem of energy efficient routing can be formulated as follows

$$\min_{\ell \in \mathcal{L}} \sum_{\ell_k \in \ell} \mathcal{C}(t_k, r_k), \tag{5}$$

where $\mathcal{L}$ denotes the set of all possible paths in the network (any loop-free sequence of nodes from the source to the destination is a potential path in this model).

III. Optimal Cooperative Routing

A. Link Cost Formulation

Consider link $\langle t_k, r_k \rangle$ formed between nodes t_k and r_k using the two-stage cooperative transmission. Let T_k denote the set of cooperative nodes in Stage 2 of the transmission strategy. Let $\mathbf{p}$ denote the power allocation vector to form the cooperative link $\langle T_k, r_k \rangle$. The expected cost of cooperative link $\langle T_k, r_k \rangle$ denoted by $\mathcal{C}(T_k, r_k)$ is given by the following optimization problem:

$$\mathcal{C}(T_k, r_k) = \min_{\mathbf{p} \in \mathcal{P}} \frac{\sum_{t_i \in T_k} \mathbf{p}_i}{\mathcal{S}(T_k, \mathbf{p}, r_k)}, \tag{6}$$

where $\mathcal{P}$ denotes the set of all *feasible* power allocation vectors $\mathbf{p}$, where $\mathbf{p}_i \leq \mathrm{P}_{\max}$ is the power allocated to transmitter $t_i \in T_k$.

The main benefit of cooperative communication is in fading environments where diversity can be used to combat fading. The applications that benefit from cooperative communication typically have a stringent requirement in terms of link reliability (*i.e.*, outage). The link cost formulation in (6), however, does not provide any specific target outage probability, and hence no limit on the number of retransmissions and consequently the link delay. To address this issue, we modify optimization problem (6) to include a constraint on target outage probability as follows. Let p_ϵ denote the target per-link outage probability that can be tolerated. Then, the cooperative link cost $\mathcal{C}(T_k, r_k)$ is the solution to the following constrained optimization problem:

$$\begin{aligned} \mathcal{C}(T_k, r_k) = &\min_{\mathbf{p}\in\mathcal{P}} \sum_{t_i \in T_k} \mathbf{p}_i \\ &\texttt{s.t.} \;\; \mathcal{S}(T_k, \mathbf{p}, r_k) \geq 1 - p_\epsilon \,. \end{aligned} \tag{7}$$

To this end, the total transmission cost to form link $\langle t_k, r_k \rangle$ is the summation of transmission powers in Stage 1 and 2. That is

$$\text{Total Power to form } \langle t_k, r_k \rangle = P_b + \mathcal{C}(T_k, r_k).$$

Two comments are due regarding the above expression:

1) The total required power is highly dependent on the broadcasting power P_b used in Stage 1. By increasing P_b, a larger cooperative set T_k is formed. It can be shown that as the cooperative set gets larger, the transmission power required to form a cooperative link (*i.e.*, $\mathcal{C}(T_k, r_k)$) decreases [7]. Our goal is to find the optimal value of P_b that minimizes the total transmission power.
2) If instantaneous fading coefficients are available (*e.g.*, a non-fading environment) then for any given P_b the corresponding cooperative set T_k can be deterministically specified (*i.e.*, with probability 1, it can be decided whether a node has received the message or not). Consequently, the minimum link cost corresponding to optimal P_b can be computed as presented in [9]. However, in a fading environment, where nodes might be in outage, a more complicated formulation is required to enumerate over all possible memberships for T_k.

Let T denote an arbitrary subset of $\mathcal{N} - \{t_k\}$. Let $\mathcal{S}(t_k, P_b, T)$ denote the probability that every node $t_i \in T$ successfully receives a message broadcast by t_k with broadcasting power P_b, and that every other node in the network (except t_k) is in outage. We obtain that

$$\mathcal{S}(t_k, P_b, T) = \prod_{t_i \in T} \mathcal{S}(t_k, P_b, t_i) \prod_{t_j \notin T} \left(1 - \mathcal{S}(t_k, P_b, t_j)\right). \tag{8}$$

Using this expression, the link cost $\mathcal{C}(t_k, r_k)$ can be expressed as the following optimization problem:

$$\begin{aligned} \mathcal{C}(t_k, r_k) = \min_{P_b \leq \mathrm{P}_{\max}} \Big\{ & P_b + \big(1 - \mathcal{S}(t_k, P_b, r_k)\big) \\ & \times \sum_{T \subseteq \mathcal{N} - \{t_k\}} \mathcal{S}(t_k, P_b, T) \cdot \mathcal{C}(T \cup \{t_k\}, r_k) \Big\}. \end{aligned} \tag{9}$$

Note that cooperative transmission is necessary only if receiver r_k fails to receive the message in Stage 1. This is reflected by the term $\big(1 - \mathcal{S}(t_k, P_b, r_k)\big)$ in (9).

B. Minimum Cost Route Selection

We can now model our network as a weighted graph $G = (\mathcal{N}, E, \mathcal{C})$, where $\mathcal{N}$ is the set of nodes in the network, E is the set of all possible edges between the nodes, *i.e.*, $E = \{(t_k, r_k) \,|\, t_k, r_k \in \mathcal{N}\}$, and $\mathcal{C} = \{\mathcal{C}(t_k, r_k) \,|\, (t_k, r_k) \in E\}$ is the set of link costs defined over the edges. The problem of energy efficient routing can now be formulated as the *shortest path* problem on graph G. Using Dijkstra's algorithm, the minimum energy path between a source and a destination can be computed in $O(N \log N)$ if the link costs $\mathcal{C}$ are known. Although the link costs $\mathcal{C}$ are computed once (and can be computed off-line), computing the cost of a link involves enumerating exponential number of cooperative sets T (see (9)). To mitigate this problem, one approach is to reduce the search space for T as discussed in the next subsection.

C. Restricted Cooperation

The idea is that the nodes that are far away from the transmitter have little chance to receive the message successfully. Hence, they can be ignored when searching for the optimal cooperative set T as their inclusion only marginally improves the link cost. Specifically, for a given broadcast power P_b, we restrict the search space to those nodes for which the probability of successfully receiving the message is at least $1 - p_\epsilon$. Let $\widetilde{\mathcal{N}}$ denote the set of such nodes. That is

$$\widetilde{\mathcal{N}} = \{n_j \in \mathcal{N} \,|\, \mathcal{S}(t_k, P_b, n_j) \geq 1 - p_\epsilon\},$$

which essentially defines a disk around the transmitter t_k with the radius $d(P_b)$ given by

$$d(P_b) = \Big(-\frac{\beta}{P_b} \ln\left(1 - p_\epsilon\right)\Big)^{1/\alpha}.$$

Although this restriction does not change the asymptotic order of the routing complexity, it is highly effective in finite networks that are of interest in this paper.

IV. Opportunistic Cooperative Routing

The analysis presented in Section III assumes that a *static* routing algorithm is employed in the network. That is, a route (which is essentially a set of intermediate relays) is computed *a priori* and all messages will be transmitted over the same route. At each intermediate relay, a unicast cooperative link is constructed between a set of transmitters and a specific receiver in a many-to-one manner. When channels are variable (which is typically the case in wireless networks), it has been shown that static routing may not be efficient as it unicasts a message to a pre-determined relay that may currently have a bad channel. To mitigate this problem, the broadcast nature of wireless channels can be explored to determine the best intermediate relay *opportunistically* after broadcasting a message. To implement this strategy, an opportunistic routing algorithm anycasts messages at intermediate nodes (in a many-to-many manner) and selects the next relay from the set of nodes that have received the message successfully. Similar to [17], our algorithm is essentially a stochastic version of the Bellman-Ford routing algorithm.

In our opportunistic routing, cooperative transmitters anycast a message to a *set* of potential receivers. Any node in this set that receives successfully may be used as the next relay. We refer to this set as the *candidate relay set*. Let $R(t_k)$ or simply R_k denote the candidate relay set for transmitter t_k.

Definition 2 (Opportunistic Route). *Because of anycasting, messages reach the destination through potentially different routes. An opportunistic route is the union of all possible routes between a source and a destination created by a choice of candidate relays at each intermediate node.*

A. Anycast Link Cost

Consider a transmitter t_k and its corresponding candidate relay set R_k (to be specified later). In Stage 1, t_k broadcasts a message with some power P_b. Nodes that successfully receive the message join t_k to form a cooperative transmitting set T_k. In Stage 2, T_k cooperatively anycast the message to the candidate relay set R_k.

Definition 3 (Anycast Link Cost). *The anycast cost of link $\ell_k = \langle t_k, R_k \rangle$ denoted by $\mathcal{C}(t_k, R_k)$ is defined as the minimum expected transmission power to deliver a message from t_k to any node in R_k using the two-stage cooperative transmission subject to rate λ and outage probability p_ϵ.*

Let $\mathcal{C}(T_k, R_k)$ denote the minimum power required for cooperative anycast from T_k to R_k.

Then, $\mathcal{C}(T_k, R_k)$ is given by the following optimization problem:

$$\begin{aligned} \mathcal{C}(T_k, R_k) = & \min_{\mathbf{p}\in\mathcal{P}} \sum_{t_i \in T_k} \mathbf{p}_i \\ & \texttt{s.t.} \;\; \mathcal{A}(T_k, \mathbf{p}, R_k) \geq 1 - p_\epsilon, \end{aligned} \tag{10}$$

where $\mathcal{A}(T_k, \mathbf{p}, R_k)$ denotes the probability that at least one node in set R_k successfully receives the message and is expressed as

$$\mathcal{A}(T_k, \mathbf{p}, R_k) = 1 - \prod_{r_j \in R_k} \left(1 - \mathcal{S}(T_k, \mathbf{p}, r_j)\right). \tag{11}$$

Using (10), the anycast link cost $\mathcal{C}(t_k, R_k)$ is given by the following optimization problem over broadcasting power P_b:

$$\begin{aligned} \mathcal{C}(t_k, R_k) = \min_{P_b \leq \mathrm{P}_{\max}} & \Big\{ P_b + \big(1 - \mathcal{A}(t_k, P_b, R_k)\big) \times \\ & \sum_{T \subseteq \mathcal{N} - \{t_k\}} \mathcal{S}(t_k, P_b, T) \cdot \mathcal{C}(T \cup \{t_k\}, R_k) \Big\}, \end{aligned} \tag{12}$$

where $\mathcal{S}(t_k, P_b, T)$ is given by (8), and $\mathcal{A}(t_k, P_b, R_k)$ can be computed from (11) by substituting $T_k = \{t_k\}$.

B. Cost of a Trajectory

A trajectory ℓ in an opportunistic route Υ is a possible path that a message can traverse to reach the destination. Hence, a trajectory is a sequence of nodes $\ell = (s, t_1, t_2, \ldots, t_K, d)$ connecting a source node s to the destination d. Each of the nodes in the sequence anycasts to its candidate relay set defined in the opportunistic route Υ. Consequently, the cost of trajectory ℓ in the opportunistic route Υ denoted by $\mathcal{C}(\ell|\Upsilon)$ is the sum of the anycast link costs of the nodes in ℓ, which is expressed as follows

$$\mathcal{C}(\ell|\Upsilon) = \mathcal{C}(s, R_s) + \mathcal{C}(t_1, R_1) + \cdots + \mathcal{C}(t_K, R_K). \tag{13}$$

Assuming each trajectory ℓ in Υ is used with probability $\mathbb{P}\{\ell\}$, the expected cost for the opportunistic route Υ is given by

$$\mathcal{C}(\Upsilon) = \sum_{\ell \in \Upsilon} \mathbb{P}\{\ell\} \, \mathcal{C}(\ell \,|\, \Upsilon). \tag{14}$$

C. Optimal Candidate Relay Set

In opportunistic routing, the candidate relay set that minimizes the expected cost to the destination is chosen as the anycast destination. Thus, we need to compute the expected cost of delivering a message to the destination from a given relay set in order to find the best relay set.

Definition 4 (Remaining Path Cost). *The remaining path cost $\mathcal{R}(t_k, R_k)$ with respect to opportunistic route Υ is the expected remaining cost to reach the destination if the candidate relay set R_k is chosen by t_k.*

$\mathcal{R}(t_k, R_k)$ is calculated as the weighted sum of costs from each node in $R_k = \{r_1, \ldots, r_n\}$ to the destination. Let D_j denote the cost to reach the destination from node r_j in R_k. In case $D_j = D$ for every $r_j \in R_k$, then the remaining cost is simply $\mathcal{R}(t_k, R_k) = D$. Next, consider the case where D_j's are not all equal. Without loss of generality, assume that $D_1 < D_2 < \cdots < D_n$. Assuming a cooperative transmitting set T_k, the expected remaining path cost for candidate relay set R_k denoted by $\mathcal{R}(T_k, R_k)$ is expressed as

$$\mathcal{R}(T_k, R_k) = \frac{1}{\mathcal{A}(T_k, \mathbf{p}, R_k)} \Big(\mathcal{S}_1 D_1 + \sum_{j=2}^{n} \mathcal{S}_j D_j \prod_{i=1}^{j-1} (1 - \mathcal{S}_i) \Big),$$

where, $\mathcal{S}_j = \mathcal{S}(T_k, \mathbf{p}, r_j)$, and $\mathbf{p}$ is obtained by solving the optimization problem in (10). To find the expected remaining cost, we average over all possible cooperative sets T_k that can be formed by transmitter t_k. Hence, the expected remaining cost from the candidate set R_k is given by

$$\mathcal{R}(t_k, R_k) = \sum_{T \subseteq \mathcal{N} - \{t_k\}} \mathcal{S}(t_k, P_b, T) \cdot \mathcal{R}(T \cup \{t_k\}, R_k), \tag{15}$$

where P_b is obtained by solving the optimization problem in (12). Consequently, the cost of the opportunistic routing from node t_k to the destination denoted by D_k is expressed as

$$D_k = \min_{R \subseteq \mathcal{N}} [\mathcal{C}(t_k, R) + \mathcal{R}(t_k, R)] \,. \tag{16}$$

The above equation gives an iterative representation of the minimum expected cost from a node to the destination similar to the familiar Bellman-Ford algorithm. At the h-th iteration, each node t_k updates D_k^h, its cost estimate to the destination. An estimate of the remaining path cost $\mathcal{R}^h(t_k, R)$ is also computed using (15). In the next iteration the estimated cost is updated for each node (except the destination d) as follows

$$D_k^{h+1} = \min_{R \subseteq \mathcal{N}} [\mathcal{C}(t_k, R) + \mathcal{R}^h(t_k, R)], \qquad \text{for } t_k \neq d\,. \tag{17}$$

The initial conditions for the iterative algorithm are $D_k^0 = \infty$ for all $t_k \neq d$, and $D_d^h = 0$ for all h. The algorithm terminates when $D_k^h = D_k^{h-1}$ for all t_k.

V. Heuristic Cooperative Routing

As discussed earlier, the optimal routing algorithms developed in Sections III and IV are centralized and have exponential computational complexity. In this section, we use those optimal algorithms as a baseline and modify them using several heuristics in order to develop routing algorithms that are computationally simpler yet achieve considerable energy efficiency (as shown in our simulations).

A. Probabilistic Cooperation (PC)

Consider a two-stage transmission from the transmitting node t_k to a receiving node r_k. When computing the optimal broadcasting power P_b, instead of enumerating $O(2^N)$ potential cooperative transmitting sets, we let the cooperative set include all nodes in the network, each with a certain *grade* of membership. We define the grade of membership $w(t_i)$ for node t_i as the probability of successfully decoding the broadcast message. For broadcating power P_b, the grade of membership can be expressed as

$$w(t_i) = e^{-\frac{d_{ki}^{\alpha}}{P_b}\beta}, \qquad \text{for all } t_i \in \mathcal{N} \text{ and } t_i \neq t_k, r_k\,. \tag{18}$$

Note that, in our model, the transmitting node t_k always participates in the cooperative transmission. Therefore, we have $w(t_k) = 1$. On the other hand, the cooperative transmission is performed only if the intended receiver fails to decode the broadcast message, implying that $w(r_k) = 0$.

In the Stage 2 of the cooperative transmission, the signal transmitted by node t_i is scaled by the membership grade of t_i, yielding the following expression for the received signal at r_k

$$y_k = \sum_{t_i \in T_k} w_i \sqrt{\frac{p_i}{d_{ik}^{\alpha}}} h_{ik}\, x_i + \eta_k, \tag{19}$$

where $w_i = w(t_i)$. Thus, γ_{ik}, the SNR at the receiver due to transmitter t_i, is given by

$$\gamma_{ik} = \frac{1}{\mathrm{P}_\eta} w_i^2 \frac{p_i}{d_{ik}^\alpha} |h_{ik}|^2 \,. \tag{20}$$

Similarly, the total SNR at r_k is expressed as $\gamma_k = \sum_{t_i \in T} \gamma_{ik}$, which is the summation of $N-1$ independent and exponentially distributed random variables with parameters $\lambda_{ik} = 1/\mathbb{E}\left[\gamma_{ik}\right]$.

The success probability and expected cost of the cooperative transmission denoted by $\mathcal{S}(T_k, \mathbf{p}, r_k)$ and $\mathcal{C}(T_k, r_k)$, respectively, can now be calculated using (4) and (7). This leads to the following expression for the link cost between t_k and r_k

$$\mathcal{C}(t_k, r_k) = \min_{P_b \leq \mathrm{P}_{\max}} \left\{ P_b + \big(1 - \mathcal{S}(t_k, P_b, r_k)\big)\mathcal{C}(T_k, r_k) \right\} . \tag{21}$$

Clearly, this heuristic algorithm has polynomial computational complexity in the network size.

B. Equal Power Allocation (EP)

In our model, to form a cooperative link, optimal power allocation is performed as expressed in (7). A simpler, albeit suboptimal, approach is to allocate equal power to every node in the cooperative set [5]. In this subsection, we modify our model to incorporate equal power allocation in our routing algorithm.

1) Computing Success Probability: Consider a cooperative link between nodes t_k and r_k. We assume that the cooperative transmitting set T consists of M nodes that are almost equally distant from the receiving node r_k. To estimate the distance of the nodes in T from the receiver, we compute the average distance of the nodes to the receiver with respect to the membership grades w_i's. This results in the following relation

$$d_{Tk} = \frac{1}{\sum_{t_i \in T} w_i} \sum_{t_i \in T} w_i d_{ik} \,. \tag{22}$$

Following the discussion in Section II, the total SNR at receiver r_k due to all nodes in the cooperative transmitting set is now expressed as

$$\gamma_k = \frac{1}{\mathrm{P}_\eta} \frac{p}{d_{Tk}^\alpha} \sum_{t_i \in T} |h_{ik}|^2, \tag{23}$$

which is the summation of M independent and exponentially distributed random variables with parameter $\mu_k = (\mathrm{P}_\eta d_{Tk}^\alpha)/p$. Hence, γ_k follows an Erlang distribution with parameters M and μ_k.

Consequently, we obtain the following expression for the success probability of the cooperative transmission under equal power allocation vector $\mathbf{p} = (p, \ldots, p)$:

$$\mathcal{S}(T, \mathbf{p}, r_k) = e^{-\mu_k \beta} \sum_{n=0}^{M-1} (\mu_k \beta)^n / n! \,. \tag{24}$$

2) Cooperative Link Cost: Assuming that nodes are uniformly distributed over the plane with density σ, the number of cooperative nodes M can be approximated as follows.

$$M \approx 1 + 2\pi\sigma \int_0^\infty r e^{-\frac{r^\alpha}{P_b}\beta} dr, \tag{25}$$

where one is added because the broadcasting node also takes part in the cooperative transmission. For the special case of $\alpha = 2$, we obtain that

$$M \approx 1 + \frac{\sigma\pi}{\beta} P_b \,. \tag{26}$$

Next, the minimum expected cost for cooperative transmission between T and r_k is obtained as follows

$$\begin{aligned} \mathcal{C}(T, r_k) = & \min_{p \le \mathrm{P_{max}}} \; Mp \\ & \texttt{s.t.} \;\; \mathcal{S}(T, \mathbf{p}, r_k) \ge 1 - p_\epsilon, \end{aligned} \tag{27}$$

which results in the following optimization problem for the link cost between t_k and r_k:

$$\mathcal{C}(t_k, r_k) = \min_{P_b \le \mathrm{P_{max}}} \left\{ P_b + \mathcal{C}(T, r_k) \right\}. \tag{28}$$

3) Cooperative Power Allocation: Using (28), a suitable broadcasting power P_b can be computed. Then, node t_k broadcasts its message using the computed P_b. After the broadcast, the cooperative set T is formed and equal power p is allocated to nodes in T for cooperative transmission. We consider two approaches for allocating power to cooperative transmitters.

- **EP-H1:** Once T is known, the optimal power p can be computed using a technique similar to that of Section III. Note that in this case, the power allocation vector is of the form $\mathbf{p} = (p, \ldots, p)$.
- **EP-H2:** Alternatively, we can simply use the value of p that is pre-computed in the optimization problem (28). In this case, a completely distributed routing algorithm can be designed assuming a certain spacial distribution for node locations (*e.g.*, uniform distribution over the plane).

Discussion: Recall that γ_k is an Erlang random variable with parameters μ_k and M. For simplicity of notation, we drop the index k in the following derivation. We are interested in approximating the success probability (24) in order to derive approximate closed-form expressions for P_b and p for the equal power allocation heuristic. Our derivation is based on Chernoff bounds for a non-negative random variable expressed as follows

$$\mathbb{P}\{X < a\} \leq \inf_{\theta<0} e^{-\theta a} M_X(\theta), \tag{29}$$

where, $M_X(\theta) = \mathbb{E}\left[e^{\theta X}\right]$ denotes the moment generating fuction of the random variable X. Applying the Chernoff bound for Erlang random variables [18], we obtain that

$$\mathbb{P}\{\gamma < \beta\} \leq e^{-(\beta\mu - M)} \left(\frac{\beta\mu}{M}\right)^M \approx \left(\frac{\beta\mu}{M}\right)^M. \tag{30}$$

To meet the outage probability p_ϵ, the following condition must be satisfied

$$\mathbb{P}\{\gamma < \beta\} \approx \left(\frac{\beta\mu}{M}\right)^M \leq p_\epsilon, \tag{31}$$

which yields the following result for optimal transmission power p:

$$p \geq \frac{\mathrm{P}_\eta \beta d^\alpha}{M \sqrt[M]{p_\epsilon}} \geq \frac{\mathrm{P}_\eta \beta d^\alpha}{M}, \tag{32}$$

where we have simply used the distance between t_k and r_k denoted by d to approximate d_{Tk}. As can be seen, p is inversely proportional to the number of cooperative transmitters. That is, as the set of cooperative transmitters become larger, p becomes smaller. Next, for the case of $\alpha = 2$, we obtain that

$$p \approx \left(\frac{\mathrm{P}_\eta \beta^2 d^2}{\pi\sigma}\right) P_b^{-1}, \tag{33}$$

which indicates that the optimal value of p decreases by increasing P_b. By substituting the above expression in (28), we obtain that

$$P_b = \min\left\{\sqrt{\frac{\mathrm{P}_\eta}{\pi\sigma}}\beta d, \mathrm{P}_{\max}\right\}. \tag{34}$$

Using the above expressions for p and P_b, a completely distributed cooperative routing algorithm can be designed (we leave this topic for future research).

VI. Performance Evaluation

In the following subsections, we present our simulation results and compare the performance of different algorithms in terms of energy consumption.

In addition to cooperative routing algorithms, we simulate the *optimal non-cooperative routing (ONCR)* algorithm as a benchmark to measure energy savings achieved by cooperative routing. ONCR is basically the least-cost non-cooperative route computed using Dijkstra's algorithm. In simulating the routing algorithms, if the cooperative transmission in Stage 2 fails to deliver the message (due to outage), we implement retransmissions until the message is successfully delivered to the next hop.

A. Simulation Parameters

We simulate a wireless network, in which nodes are deployed uniformly at random. Nodes are distributed in a square of area 5×5, and node density is set to $\sigma = 2$, *i.e.*, there are $N = 50$ nodes in the network. We choose two nodes s and d located at the lower left and the upper right corners of the network, respectively, and find cooperative and non-cooperative routes from s to d. We then compute the total amount of energy consumed on each route using different routing algorithms. For simulation purposes, we take path-loss exponent $\alpha = 2$, noise power $\mathrm{P}_\eta = 1$ and SNR threshold $\beta = 0.65$, unless otherwise specified. The numbers reported are obtained by averaging over multiple simulation runs with different seeds. The max node power $\mathrm{P}_{\max}$ is set in such a way that the network is connected without cooperation (the absolute value of $\mathrm{P}_{\max}$ does not affect the results).

B. Effect of Broadcast Power on Link Cost

In general, the link cost is a non-monotonic function of P_b. To demonstrate this in our simulations and find the optimal value of P_b, we choose a pair of transmitting and receiving nodes that are far apart in a randomly generated network of size 10×10 (total of 200 nodes in the network) and compute link cost between them for various broadcasting powers. We set $p_\epsilon = 0.2$ in this experiment.

To show the full extend of the trade-off between P_b and link cost, in this experiment, we do not consider any limit on the transmission power (*i.e.*, $\mathrm{P}_{\max} = \infty$), hence nodes are able to use arbitrary power levels. Fig. 1 illustrates the total transmission power (*i.e.*, link cost) for

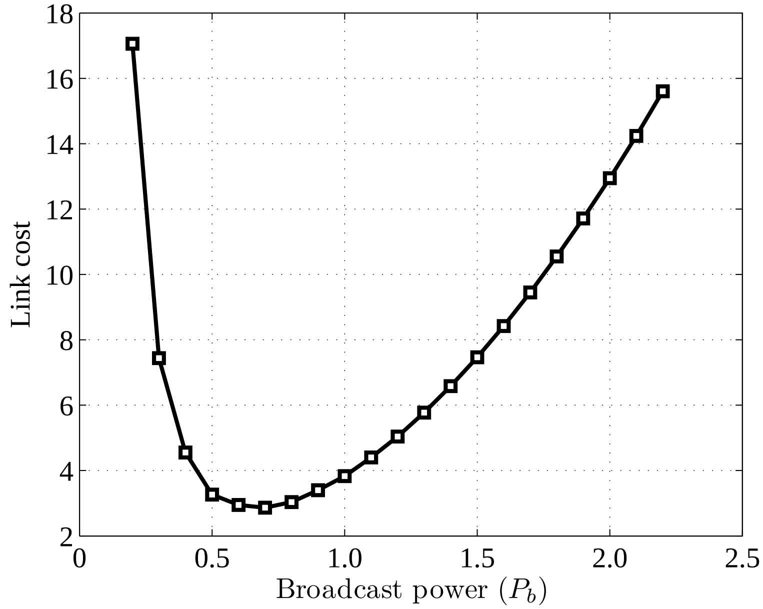

Fig. 1. Link cost as a function of broadcasting power P_b.

different values of the broadcasting power. Clearly, there is an optimal P_b that minimizes the total transmission power. This optimal value can be computed by solving optimization problem (9).

C. Performance of Optimal and Heuristic Algorithms

In this subsection, we investigate the performance of our optimal and heuristic routing algorithms in terms of energy savings achieved compared to the optimal non-cooperative routing algorithm (ONCR).

In the implementation of the routing algorithms, we consider two cases. In the first case, we assume that there is a requirement on link reliability (corresponding to some link delay) given in terms of the success probability. In this case, we use our constrained optimization relations (*e.g.*, (7)) to calculate the link cost. In the second case, we assume that there is no hard requirement on link reliability (hence, no delay target) and use our unconstrained optimization relations (*e.g.*, (6)) to compute the link cost. The link cost computed by constrained optimization relations is subject to a given outage probability p_ϵ. If a transmission fails then retransmissions are required. Thus, the average cost of successfully delivering a message over link $\langle t_k, r_k \rangle$ denoted by $\mathcal{C}_s(t_k, r_k)$ is given by $\mathcal{C}_s(t_k, r_k) = \mathcal{C}(t_k, r_k)/(1 - p_\epsilon)$, with the corresponding delay of $(1 - p_\epsilon)^{-1}$.

The simulation results for the case of unconstrained routing (*i.e.*, no target link reliability)

TABLE I

ENERGY EFFICIENCY OF UNCONSTRAINED ROUTING.

Algorithm	Min Cost	Success Prob.
ONCR	2.83	0.38
Static	2.74	0.43
Opportunistic	2.15	0.54
EP-H1	3.05	0.53
EP-H2	4.32	0.51
PC	2.75	0.48

are summarized in Table I. The table shows the minimum achieved energy costs along with corresponding success probabilities that resulted in the minimum energy cost for different routing algorithms. As can be seen, without any constraint on link reliability, the optimal success probability that results in minimum energy cost might be considerably low. Hence, delivering a message may require multiple rounds of transmissions causing extensive end-to-end delay. Furthermore, the energy savings in this regime compared to ONCR are negligible (and even negative in case of heuristics). Recall that this was expected as cooperative diversity is most effective in networks where retransmissions are not effective (*e.g.*, due to tight delay constraints and/or low mobility, where the fading does not change appreciably between transmissions). Thus, in the remainder of this section, we ignore unconstrained routing and focus instead on constrained routing algorithms.

1) Performance of Optimal Routing Algorithms: Due to the computational complexity of the optimal algorithms, it is challenging to simulate these algorithms in large networks. Instead, in this experiment, we simulated a fairly small network with 12 nodes distributed randomly on a square area of dimension 2×2. Energy cost of the two optimal algorithms is shown in Fig. 2. Interestingly, even on such a small network, the static cooperative algorithm performs about 30% better than the non-cooperative algorithm for larger values of success probability. The improvements in energy saving are even more significant with the opportunistic algorithm, consuming about 65% less transmission energy compared to the non-cooperative algorithm. As we show in Subsection VI-D, the energy savings will dramatically increase for higher success probabilities.

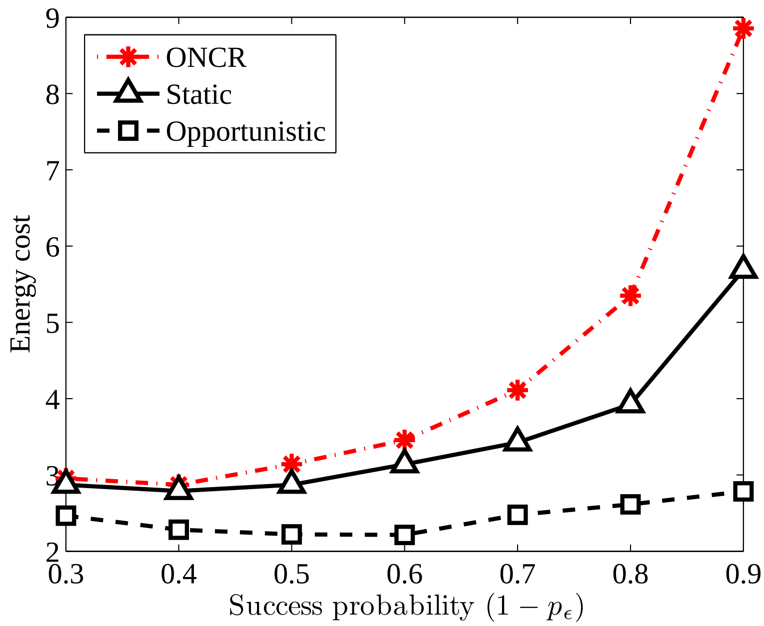

Fig. 2. Cost of optimal routing as a function of success probability.

2) Performance of Heuristic Routing Algorithms: Fig. 3 illustrates the energy cost of the heuristic algorithms described in Section V. As shown in the figure, the heuristic routing cooperative algorithms achieves considerable energy savings for higher values of success probability. In particular, PC achieves energy savings of about 10% and 300% for low and high success probabilities. Similarly, equal power allocation heuristics, namely EP-H1 and EP-H2, perform considerably better than the non-cooperative algorithm for higher success probabilities, which is the appropriate region of operation for cooperative communication.

D. Effect of Network Parameters

The results presented in Fig. 3 clearly show the superiority of our probabilistic cooperation heuristic (PC). We note that PC has polynomial computational complexity in the size of the network, making it a viable candidate for implementation in a network. Thus, in this subsection, we focus on PC and study the effect of various network parameters on its performance.

1) Effect of Path-Loss: The effect of *path-loss* exponent (α) on energy cost of PC is presented in Fig. 4. Although path-loss affects the energy cost across different success probabilities, the overall performance behavior does not change with respect to α.

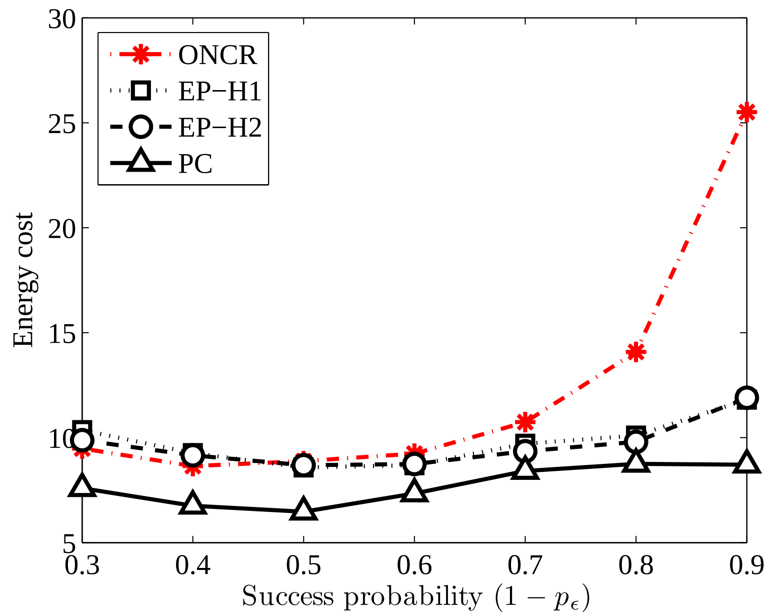

Fig. 3. Cost of heuristic routing as a function of success probability.

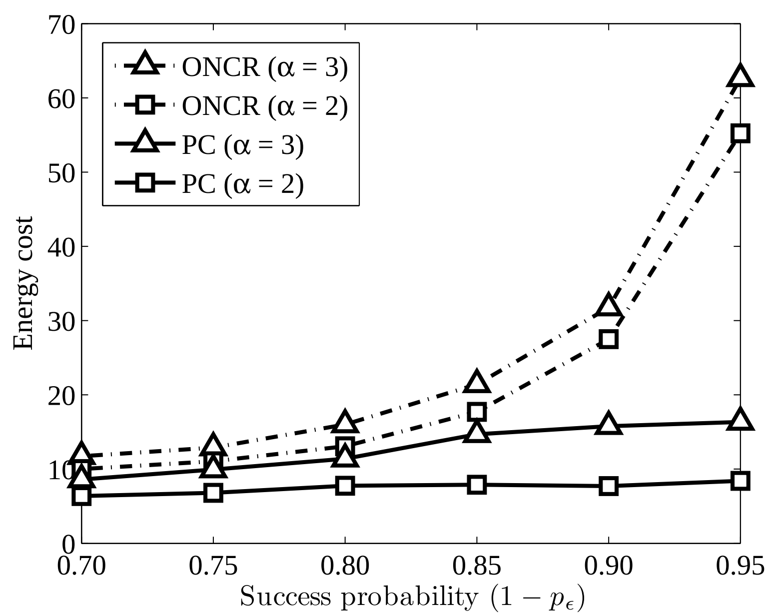

Fig. 4. Effect of path-loss exponent (α).

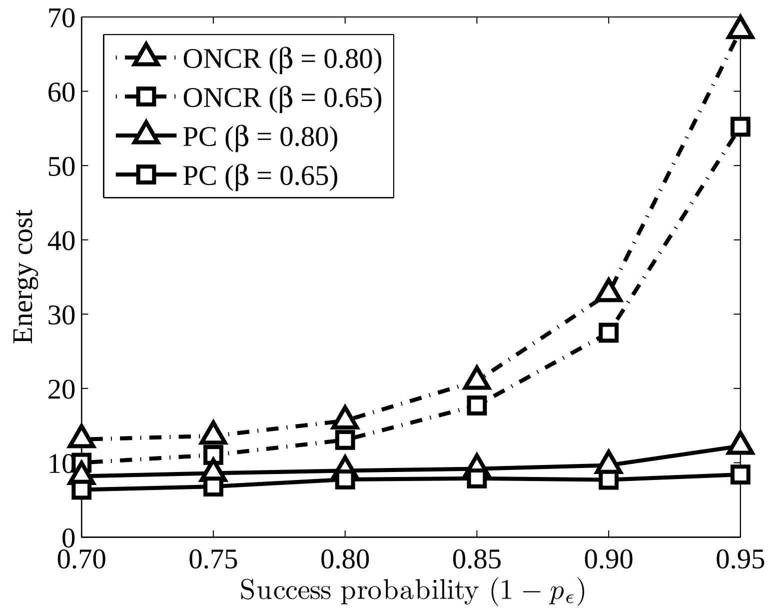

Fig. 5. Effect of target SNR (β).

2) Effect of SNR Threshold: We run the simulations with different values for β, corresponding to different link throughputs. Results from the simulations are shown in Fig. 5.We observe that the results under varying SNR thresholds remain consistent with the results presented in Fig. 3.

3) Effect of Node Density: Fig. 6 shows the impact of node density on the performance of PC. All other parameters remain the same as in Fig. 3. Again, we observe a consistent performance similar to what was observed in previous experiments.

E. Effect of Cooperation on Path Length

Although our focus in this paper is on minimizing energy, it would be interesting to see how cooperation affects path length as it has direct consequences for the network throughput. We set $p_\epsilon = 0.2$ in this experiment.

In this experiment, we consider a large network with 200 nodes uniformly distributed on a square of size 10×10. Fig. 7 compares the number of hops required to reach the destination from the source (which are located in opposite corners) using different routing algorithms. As shown by in figure, not only our proposed algorithms achieve considerable energy savings compared to non-cooperative routing, but also they form longer-range links resulting in fewer hops to reach

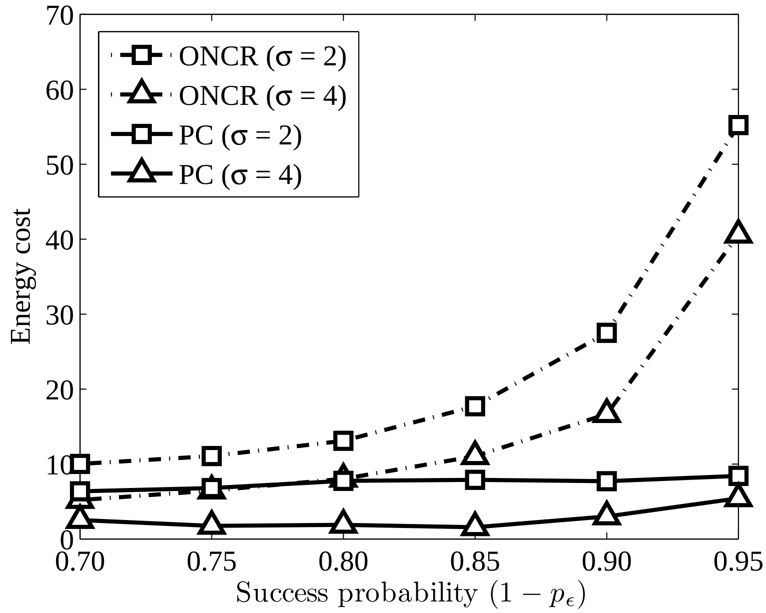

Fig. 6. Effect of node density (σ).

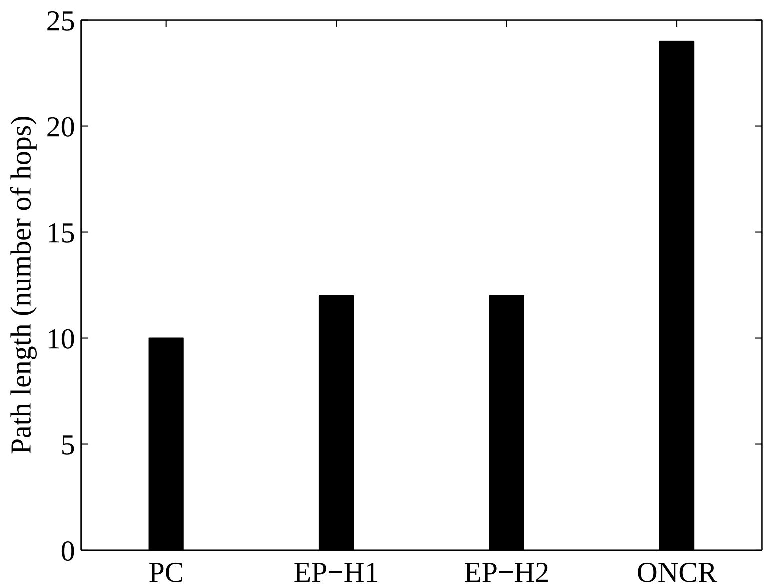

Fig. 7. Path length (*i.e.*, hop count) of different routing algorithms.

the destination. We suspect the reduced hop count of cooperative routing algorithms has positive implications for the network throughput achieved under cooperative routing.

VII. Conclusion

Cooperation among single-antenna nodes in wireless networks has been widely studied as a promising method to improve physical layer metrics. Such cooperation obviates the standard model on which routing algorithms are built, yet there has been little attention paid to understanding how to perform routing when cooperation is employed, particularly in the most pertinent case where partial channel information is available to the network.

Here we have formulated the minimum energy cooperative routing problem with partial channel information, and provided both optimal and heuristic algorithms. We have also simulated our algorithms in random wireless networks and studied their performance with respect to various network parameters. Our simulations show that while optimal cooperation and link selection can reduce energy consumption by almost an order of magnitude compared to non-cooperative approaches, our simple heuristics perform reasonably well achieving similar energy savings while being computationally efficient as well.

The most pressing future work is the extension of our formulation to the consideration of multiple flows in the network. Two directions are of interest. In the first, the algorithms developed here and extensions can be employed in a multi-flow scenario and the network throughput compared to that of traditional non-cooperative approaches. In the second, new algorithms can be developed that directly take into account the coupling between different flows. It is apparent that the complexity of optimal algorithms that jointly consider multiple flows will likely be prohibitive, so heuristic versions will need to be considered. The result will help to answer the crucial question of whether and to what extent physical layer cooperation has utility when measured with network-level metrics in practical networks.

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