# A SELF-DIRECTED PROFESSIONAL DEVELOPMENT APPROACH TO TRANSFORMING TEACHERS' PRACTICE TO SUPPORT MATHEMATICAL THINKING

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*This paper reports on a self-directed, school-based, practice-based professional development experience aimed at helping elementary teachers to develop expertise in inquiry-based teaching to support mathematical thinking. The goal of the study reported here was to identify characteristics of the self-directed orientation of this professional development that supported the teachers' learning, the nature of the inquiry-based knowledge they constructed, and the impact on their teaching. In general, the approach was successful in helping the teachers to transform their teaching in more desirable ways that support students' development of mathematical thinking.*

*Este artículo reporta una experiencia de desarrollo profesional auto-dirigida, basada en la escuela y basada en la práctica orientada a ayudar a profesores de primaria a desarrollar competencia en la enseñanza basada en un modelo indagatorio para sustentar el pensamiento matemático. El objetivo del estudio que se reporta aquí fue identificar características de la orientación auto-dirigida de este desarrollo profesional que sustentó el aprendizaje de los profesores, la naturaleza de conocimiento construido basado en la indagatoria y el impacto en sus prácticas docentes. En general, esta aproximación tubo éxito en ayudar a los profesores a transformar sus enseñanzas en modos más deseables que sustentan el desarrollo del pensamiento matemático de los estudiantes.*

## INTRODUCTION

This paper reports on a study of a self-directed, school-based, practice-based professional development experience aimed at helping elementary teachers to develop expertise in inquiry-based teaching to support mathematical thinking. The focus is on identifying the theoretical constructs, the nature and characteristics of a self-directed orientation of a professional development that supported the teachers' learning, the nature of the inquiry-based knowledge they constructed, and the impact on their teaching.

Mathematical thinking has been considered in a variety of ways in the research literature (cf., Chapman, 2011a). However, the underlying conception is that it stresses the mental activities or methods used in learning/doing mathematics, e.g., it is associated with thinking, reasoning and coming to know a concept (Mason, Burton, & Stacey, 1982). Schoenfeld (1992) suggests that in order for students to think mathematically, they need to develop a predilection to analyze and understand, to perceive structure and structural relationship, to see how things fit together, and the ability to reason in extended chains of arguments. The NCTM (2000) process standards suggest ways of supporting mathematical thinking that include allowing students to: apply and adapt a variety of appropriate strategies to solve problems; select and use various types of reasoning and methods of proof; analyze and evaluate the mathematical thinking and strategies of others; understand how mathematical ideas

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interconnect; and select, apply, and translate among mathematical representations to solve problems. The NCTM (1991) standards also highlight the importance of worthwhile mathematical tasks and discourse in helping students to develop their mathematical thinking. Mason, Burton, and Stacey (1982) argued that an atmosphere of questioning, challenging and reflecting supports mathematical thinking while Watson and Mason (1998) discussed questioning as an important part of the teacher's ability to establish a classroom atmosphere conducive to the development of mathematical thinking. In general, as Chapman (2011a) noted, mathematical thinking can be supported through an inquiry-based pedagogy that allows students' questions and curiosities to drive curriculum, honours previous experience and knowledge, makes use of multiple ways of knowing, and allows for creation or adoption of new perspectives when exploring issues, content, and questions. In this approach, students are given the opportunity to direct their own investigations and find their own answers. In this study, then, understanding a professional development approach on inquiry-based teaching is important as a means to help teachers to support mathematical thinking.

## RESEARCH METHOD

Participants of the study were 14 practicing teachers with representation from grades 1 to 6 of the same school. They ranged from three to 20 years of teaching experience, with most being over 10 years. They formed a mathematics study group for their "professional growth plan" because they needed more help in this discipline. The study focused on the first year of the study group that met for one and a half to two hours every three weeks and on special school professional development days. The teachers' goal was to take on the role of "researcher" to develop their version of how to engage their students in inquiry-based learning of mathematics. Thus they engaged in inquiry as a basis of their learning in order to develop their own understanding and use of inquiry in the context of their practice. I was invited to join the group as an "expert-friend". My role was to provide support through a nonthreatening, non-authoritarian presence, by responding to their needs rather than imposing direction, and not deliberately influencing the process of events by dictating what they should do or how to do it.

Data collection focused on two aspects of the professional development: the way it evolved for the teachers and the way it impacted their learning and practice. Data sources included notes of the teachers' discussions about, for example, what to do; how, when, or where to do it; why to do it; and their planning, conducting, observing and evaluating of the research lessons. There were also individual and group interviews about their thinking and learning and classroom observations of their teaching. Two rounds of data analysis directly addressed the research questions. The first round of analysis focused on identifying the nature of the teacher's inquiry-based process of learning and the characteristics of the self-directed orientation that supported the teachers' learning. This was guided by theoretical constructs of teacher learning and inquiry. The second round of analysis focused on the nature of the inquiry-based knowledge the teachers constructed for teaching elementary mathematics and the impact on their teaching.

## FINDINGS

The findings reported here do not consider the unique ways in which each teacher developed and applied her learning about inquiry-based teaching. The focus is only on what was common in terms of their experience in the professional development, the knowledge constructed and impact on their teaching.

### Theoretical constructs underlying the self-directed professional development

Analysis of the self-directed learning perspective the teachers engaged in indicated that one way to interpret it theoretically is in terms of three key constructs: agency (Bruner, 1996), practical knowledge (Elbaz, 1983; Fensternmacher, 1994), and situated learning (Brown, Collins, & Duguid, 1989; Lave & Wenger, 1991). Chapman (2011b) provides more details of these constructs. Each is briefly described here in terms of how it emerged in the professional development.

*Agency*: The teachers engaged in an inquiry learning process based on their sense-making of it as opposed to adopting a predetermined, theoretical version, thus asserting their agency. They were instrumental in determining the inquiry process for both their learning and teaching. They made the experience about them and maintained autonomy of the process. They made decisions that gave them a sense of ownership to the goal and meaningfulness to the process.

*Practical knowledge*: The teachers' practical knowledge [PK] played a significant role in the self-directed process. Their preference was generally to build from and on their PK or through that of others (e.g., via videos) as opposed to through theory that seemed too abstract and disconnected to be a meaningful starting point. This use of PK was an important connection to self and way to empower self to make the learning through and about inquiry personal, real, relevant, important, and meaningful. The teachers accessed their PK through stories of experience, i.e., they shared, resonated in, and reflected on stories of past and present pedagogical experiences to facilitate their decision making, planning, and enacting of the inquiry-based learning and teaching processes they engaged in and created.

*Situated learning*: In the context of a learning community, situated learning was also central to the teachers' self-directed approach. Their learning was situated in the context of their individual and collective teaching. They created an *authentic environment* (Brown et al., 1989), i.e., the learning tasks paralleled real-world situations. The situatedness was also reflected in their decision to learn about something only as it became necessary to make progress in their inquiry and achieve their goal. For example, they initially sought only the knowledge they needed to interpret inquiry-based teaching and communication. They later consulted external sources and engaged in mathematical activities to expand their mathematics knowledge for teaching, but this was still linked to the particular situations involved.

### Self-directed inquiry cycle of the professional development

Analysis of the self-directed inquiry learning process the teachers engaged in was based on Dewey's (1938) perspective of inquiry. From this perspective, simply stated, the inquiry learning process begins with the learner encountering a puzzling situation that initiates the generation of questions. This leads the learner to become aware of aspects of the situation that point to suggestions of a solution. Reflective thinking on these aspects results in the formulation of hypotheses or possible lines of action which are tested in a concrete situation. Based on the knowledge resulting, guided by reflective thinking, a new puzzling situation could emerge and the process begins again. Table 1 shows the components of this inquiry process used to make sense of the teachers' self-directed inquiry process. It is the first of several cycles the teachers went through to attend to new questions that emerged at the end of each cycle.

| Component | Description |
|---|---|
| Pose questions/ problems | Overarching puzzling situation for the teachers was how to transform their teaching to an inquiry-based perspective. This led to the question of what topic to study to achieve their goal. |
| Create hypotheses/ conjectures/predictions | Hypothesized that the official curriculum would be a good starting point to identify a topic. |
| Investigate/ explore | Investigated the theoretical section of the curriculum, which they had not read before. |
| Create new knowledge | Created new knowledge focused on the mathematical processes emphasized in the curriculum (i.e., problem solving, connections, communication, reasoning, visualization, estimation). |
| Communicate, evaluate, reflect | Discussed their understandings of the processes that meant the most to them. After critically examining and evaluating these processes in relation to their teaching, they concluded that "communication" in an inquiry-teaching context was the key process that they wanted to study as a starting point. |
| New inquiry | New question on how to learn about inquiry-based teaching focusing on the communication process |

Table 1: Components of inquiry cycle 1

**Mathematical Pedagogical Knowledge constructed**

There were several interrelated aspects to the knowledge the teachers' constructed during the professional development over the year. However, only two aspects that relate to the primary aim of the professional development are considered here, that is, growth in knowledge of inquiry-based communication and inquiry-based teaching of elementary school mathematics. While the teachers held this knowledge with unique features relative to them individually, the focus here is on what they developed collectively during the professional development. The knowledge they constructed regarding inquiry-based teaching consisted of a model with ten components (Table 2) and key questions/prompts (e.g., what do you notice?) to guide the structure and enactment of an inquiry-based lesson. Their inquiry of "experimental lessons" led them to conclude that the components of the model can be sequenced in different ways, that is, the structure of the inquiry lesson is flexible in terms of which components of the model are used and how they are sequenced. Thus they decided to represent the model in the form of a jigsaw puzzle (Chapman, 2011b) and called it "The Jigsaw Teaching Model".

| |
|---|
| (1) Engage students in learning *goals* for conceptual understanding |
| (2) Engage students in sharing *prerequisite* knowledge/conceptions of concept being taught |
| (3) Engage students in making *predictions* of possible outcomes related to the concept |
| (4) Engage students in *free exploration* of the concept |
| (5) Engage students in *focused exploration* |
| (6) Engage students in *application* of concept |
| (7) Engage students in *comparison* of their learning |
| (8) Engage students in *evaluation* of their learning |
| (9) Engage students in *reflection* of their learning |
| (10) Engage students in *extension* of concept to other situations or related concepts |

Table 2: Components of the Jigsaw Teaching Model

Table 3 is an example of one of the lessons the teachers used to test and develop their model. This is their actual plan of this grade 1 lesson on two properties of 3D objects.

| Inquiry Model | Activities |
|---|---|
| Free exploration | Talk/experiment/observe 11, 3D geometric objects [in groups] |
| Discussion/comparison | What did you notice? [whole class] |
| Prediction | Will shapes roll or slide? Record individually on worksheet with pictures of the eleven 3D objects and columns for rolls only, slides only, and rolls and slides |
| Focused exploration | Test prediction and record findings on same worksheet |
| Discussion/justification | Discuss solutions and support answers |
| Evaluation/reflection | Venn diagram to sort pictures of shapes and make general statements about "What I know about 3D shapes!" |
| Extension: application 1 | Think of self as a builder. Suppose I want to build a house on a mountain, what would I need to know about shapes? |
| Extension: application 2 | Look for things at home and around school that roll or slide. |

Table 3: Experimental lesson – Introduction to Geometric solids

The knowledge the teachers constructed about inquiry-based communication focuses on key questions to engage students in inquiry, e.g., questions that allowed them to see for themselves patterns, properties, or relationships of mathematics concepts embodied in a situation, problem, or object. They also focused on questions that allowed students to think about the mathematics or their thinking and share their thinking.

## CONCLUSION

This study offers an example of a self-directed professional development and illustrates its potential and factors that are important to frame it theoretically and to facilitate teachers' growth in inquiry-based teaching of mathematics. It highlights the potential effectiveness of teachers' inquiry of inquiry as a means of transforming their thinking and teaching in a way to support mathematical thinking. In particular, as discussed in Chapman (2011b), it suggests that there could be significant benefits from professional development to support elementary teachers' growth in inquiry-based teaching of mathematics if the professional development is based on the following factors: inquiry of inquiry-based teaching from the teachers' perspective; agency, practical knowledge, and situated learning; development of personalized models or guidelines; understanding of inquiry-oriented questioning; a common pedagogical problem; and relevant and accessible mathematics topics.

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