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Accumulation of experience in a vast number of cases: Enactivism as a fit framework for the study of spatial reasoning in mathematics education

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Accumulation of experience in a vast number of cases:

Enactivism as a fit framework for the study of spatial reasoning in mathematics education

Steven Khan, Krista Francis, Brent Davis

Abstract

As we witness a push towards studying spatial reasoning as a principal component of mathematical competency and instruction in the 21st century, we argue that enactivism, with its strong and explicit foci on the coupling of organism and environment, action-as-cognition, and sensory motor coordination provides an inclusive, expansive, apt, and fit framework. We illustrate the fit of Enactivism as a theory of learning with data from an ongoing research project involving teachers and elementary-aged children's engagement in the design and assembly, of motorised robots. We offer that, spatial reasoning, with its considerations of physical context, the dynamics of a body moving through space, sensorimotor coordination, and cognition appears different from other conceptual competencies in mathematics. Specifically, we argue that learner engagements with diverse types of informationally 'dense' visuo-spatial interfaces (eg. blueprints, programming icons, blocks, maps etc.) as in the research study, affords some of the necessary experiences with/in a vast number of cases described by Varela et al. (1991) that enable the development of other mathematical competencies.

> [U]nlike the world of chessplaying, movement among objects is not a space that can be said to end neatly at some point…successfully directed movement…depends upon acquired motor-skills and the continuous use of common sense or background know-how. … Such commonsense knowledge is difficult, perhaps impossible, to package into explicit, propositional knowledge – “knowledge *that*”…since it is largely a matter of readiness to hand or “knowledge *how*” based on the accumulation of experience in a vast number of cases. (Varela, Thompson & Rosch, 1991, 147–148)

Introduction

Awareness of the importance of spatial reasoning to mathematics education is increasing. In North America, the NCTM intends to increase spatial reasoning in the early years standards matching the focus on number (Gojak, 2012). Canadian curricula are likely to follow. Contemplation is needed to determine what spatial skills are, how they might be envisioned in educational settings and the characteristics of tasks that support their robust development.

We open by inviting readers to consider a typical psychological measure used to assess one aspect of spatial reasoning – visual rotation tasks (Figure 1). The final position of a two-dimensional L-shape must be recognised from a series of similar gnomons that are related by rotation and/or reflection. Such tasks are believed to isolate and measure a singular dimension of spatial reasoning. The measure is also intended to be diagnostic of spatial-rotational abilities and can be framed propositionally, i.e. IF agent correctly matches gnomons THEN capable of rotational spatial reasoning. The task boundary is well defined.

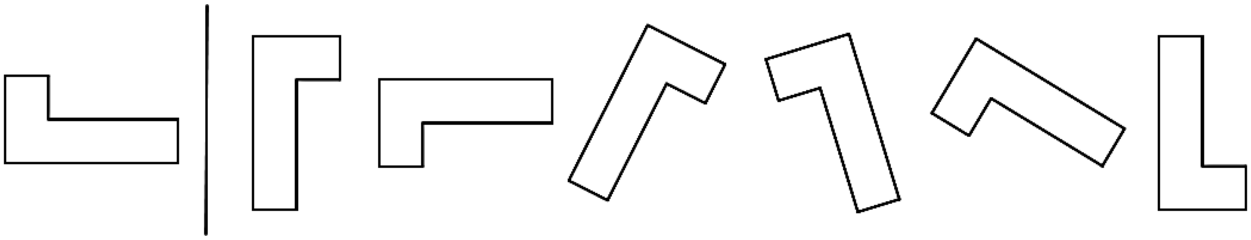

Figure 1: Rotation Instrument (Kayhan, 2005)

Consider the difference in engagement illustrated by Eric, a 9-year-old participant in a robotics summer camp (see Video 1). Beginning at 1:08, he looks at the L that is already partially attached; he flips that one a few times to see its orientation; he moves it down to the middle rod, puts that in and then he attaches the L-shape accurately (almost immediately). This action/fitting/assembling involves more 'real world' complexity than the visual rotation task in that it involves Eric moving among the (2D) visual representation in the instruction booklet/guide, selecting the appropriate (3D) element from a diverse/myriad collection of shapes (>80 different ones), then returning to the 2D representation, all while manipulating the 3D element in order to figure out how to attach it to the developing robot appropriately.

In his coordinations of the Lego pieces through his manipulations, the boundary of the task, while constrained, is not as clearly defined – it shifts as the robot develops. The 'task' in a sense can be seen as diagnostic in that he can either accomplish the task(s) or not. We claim that the task(s) are also developmental in that he can and is *learning* how to accomplish the task(s). Unlike the psychological rotation task feedback is provided by the system in that the piece fits or it does not. Eric can make as many attempts as necessary until the piece fits. The dominance of the psychologically influenced task in education over educative tasks is still very much at play today in curriculum structure and content. Unlike the abstract diagnostic psychological task, the educative dimension of the task(s) emerges from the interplay among physical context, the dynamics of a human body moving through space with other non-human bodies, cognitions and the coordination of these.

The two scenarios described above illustrate what we believe we do in education based on that type of psychology that abstracts away from the complexity of the phenomena, the relationship of the phenomena with the body, and what we observe children as being capable of doing. Below we address the question, "What are some necessary features to which a theory of human learning must deliberately and specifically attend in order to make sense of the type of learning/engagement that we construe is taking place in scenario 2?" Before presenting our response, we present an early concern by John Dewey about the influence of psychology on education, in particular the metaphor that was chosen to organize psychologists' thinking about learning.

There's Something about Dewey…

> What shall we term that which is not sensation-followed-by-idea-followed-by-movement, but which is primary; which is, as it were, the psychical organism of which sensation, idea and movement are the chief organs? (Dewey, 1896, p. 358)

In *The Reflex Arc Concept in Psychology*, Dewey (1896) claimed that the metaphorical image of the reflex-arc arising from neurology was transposed into and satisfied a demand for an organizing principle for psychology. He argued that the metaphor of the reflex-arc and its attendant principles of stimulus-response did not unseat previously held dualistic conceptions in which sensation and idea, or body and soul were construed as separate, but rather repeated them. He proposed instead the concept of "sensori-motor coordination,"[1] which unites, "sensory stimulus, central connections and motor responses…as divisions of labor, functioning factors, within the single concrete whole now designated the reflex arc" (p. 358). Dewey was elaborating the well-established relationship between seeing and learning by identifying both seeing and learning as instances of sensorimotor coordination. The metaphorical image of the reflex-arc underpins schools of thought in 20th-century psychology, most notably behaviourism. It has led to observational protocols of stimulus and responses, uni-dimensional measurement, and "rigid distinctions between sensations, thoughts and acts" (p. 358), but it has ignored or obfuscated such considerations as feedback and subjectivity.

Our opening contrast of tasks exemplifies the focus of Dewey's critique – that is, a narrow emphasis on diagnostic measures rather than a broad conception of action that includes the organism itself and its activities. In short, Dewey drew attention to the lack of a psychological framework of learning that is attentive to what he called sensori-motor coordination. Following Cummins (2013), what is necessary for the study of perception *and* action is a relational approach that is attentive to the limitations of "dualist mediated epistemologies" (p. 178) – that is, theories of knowing in which human experience is not *a priori* separated from the world.

We do not intend to suggest that Dewey's pragmatist philosophy is an earlier form of enactivism, but that the type of sensitivities and sensibilities that have come to be

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[1] We use the current spelling – sensorimotor – which in our opinion also serves to signal the juxtaposition of sensory and motor coordination.

associated with enactivist approaches are also found in his work. While we would argue that enactivism aligns with pragmatism in powerful ways, that is not our purpose here.

Enactivism: A fit framework

Enactivism is viewed as a "relatively young paradigm" (Villalobos, 2013, p. 159). Despite increasing attention in philosophy, cognitive science, and education, the "Theory of Enaction" has not yet managed to achieve significant traction in mathematics education outside of a few established social and professional researcher networks.

In this section, we address the question, 'What is enactivism?' by asking 'To what does it attend?' and 'How does it attend to it?' By way of initial, brief response, enactivism is (1) a theory of engagement (2) that is simultaneously attentive to the coupling of organisms and their environments, action as cognition, and sensorimotor coordination. (3) It involves a methodological eclecticism that is concerned with inter-agent dynamics that include feedback from the system and the organism's responses. We work from the position of Varela et al. (1991) that the enactivist approach comprises two principles, viz. that, "(1) perception consists in perceptually guided action and (2) cognitive structures emerge from the recurrent sensorimotor patterns that enable action to be perceptually guided" (p. 173).

Enactivist theories of human learning attend explicitly and deliberately to action, feedback, and discernment. Enactivism emphasises the bodily basis of meaning, distinguishing it from most accounts of constructivism – which, while not denying the body as ground and mediator of meaning, have not focused so intensely on the physicality of knowing and being. Rather, constructivisms have tended to be more concerned with conceptual understanding and propositional knowledge (Begg, 2013; Davis, 1996) – an emphasis that perhaps inadvertently "[d]evelops rigid distinctions between sensations, thoughts and acts" (Dewey, 1896, p. 358).

As is frequently noted, radical constructivism is a theory of how people assemble ideas, not a theory of how teachers might direct the assembling of ideas. It is thus relatively silent teaching practices, such as grading or distinguishing student interpretations as right or wrong – noting only that while learning may be dependent on such teaching acts, it is certainly not determined by them. In contrast, enactivism is attentive to the many feedback structures in a greater-than-the-individual-learner system, and this quality prompts us to regard enactivism as a much more educatively minded theory. More descriptively, following Begg (2013), enactivism should not be thought of as "creating dichotomies between non-cognitive and cognitive or between experiential and academic, but as ensuring that complementary ways of knowing are all given attention and credit" (p. 93). For us, this quality positions enactivism as a particularly useful frame for contrasting the two scenarios presented above – that is, the paper-based rotation task and the physically engaged robot building. Only in the second scenario does Eric receive feedback from the system on his progress with L-shapes.

In recent work, Hutto (2013) argued that enactivism, with its starting assumption that mental life can be understood as embodied activity, is a good candidate for "defining and demarcating [psychology's] subject matter" (p. 174) – that is, in his terms, for "unifying psychology." Traditional perspectives, he argued delimit psychological explanations to ones that rely on inner representational states. He noted that enactivism, in its original formulation by Varela et al., attended explicitly to organisms' varied engagements with contexts "not only of the biological kind but also of sociocultural varieties" (p. 177). The mental-rotation task illustrated in Figure 1 could be interpreted in this way, as merely a manipulation of an inner representation. Enactivism draws attention to the fact that similar neural circuits in the sensory-motor cortex are engaged across three seemingly distinct events: performing the actual physical rotation oneself, imagining the mental rotation, and observing another perform the rotation (Bergen, 2012) although the subjective experience would likely be different. This takes us to an enactivist re-framing of spatial reasoning.

How Enactivism Might Frame Spatial Reasoning

Much debate exists within and between communities of researchers on the precise definitions and subdivisions of spatial abilities and spatial cognition, along with their relationship to visualization, to experiences with problem solving in spatial contexts, and to the curricular form of geometry. Drawing on Tepylo's (2013) literature review, definitions of spatial reasoning skills generally include:

- visualizing part-whole objects (e.g., imagining how to put them together) and mental rotation of part-whole objects (i.e., imagining how two-dimensional and three-dimensional objects appear when rotated);
- locating objects, recognizing shapes, their relations to each other, and their paths of motion (Newcombe, 2010);
- manipulating spatially presented information, which may involve multisteps but not multiple solution strategies; rotating a two or three dimensional figure rapidly and accurately (Linn & Petersen, 1985);
- thinking about objects in three dimensions and being able to draw conclusions about the object with limited information (Barnett, 2013).

These definitions may appear divergent, but they share some key assumptions. For example, they all cast spatial reasoning as a sequential process of perceiving a separate-from-actor object in the environment, encoding particular features of that object (e.g., orientation), thinking about those features to generate motor actions and/or recognitions (e.g., of orientation or similarity). In this sense, spatial reasoning would be analogous to popular understandings of mental mathematics as, in Proulx's (2013) terms, "solving of mathematical tasks without paper and pencil or other computational/material aids" (p. 317).

Within an enactivist frame, the implicit separation of sensorimotor action from cognitive process is likely inappropriate. For instance, young children's fine motor coordination and spatial reasoning have been identified as key to mathematics learning and ability. In a longitudinal study that followed 213 three and four year-olds through to the end of kindergarten, the ability to redraw designs or shapes was a predictor of the ability to solve mathematical problems (Grissmer, Grimm, Aiyer, Murrah, & Steele, 2010). Carlson, Rowe and Curby (2013) found that the fine motor skills associated with visual spatial abilities was a predictor of mathematical problem solving for children aged 5 through18. In an intervening pilot study, Grissmer (as cited in Sparks, 2013) studied kindergartners and first graders who played games that required them to copy designs and shapes, cut and paste construction paper to make chains, and build models with clay or Lego for seven months 4 days a week. At the end of the study, the children made significant improvements to their mathematics skills. These studies point to the connections of sensorimotor skills, spatial reasoning and mathematics ability.

As Cummins (2013) argued, “the reciprocity of perception and action is obscured in a perception-then-cognition-then-action framework” (p. 183). Put differently, there is a tendency to describe away most of the phenomenon of spatial reasoning in formal definitions. We aim to recover some of the original complexity and dynamics by attending to the presence of the knower and the materiality of the knower in the spatial reasoning. The acts of isolating such aspect of spatial reasoning as rotation, orientation, and scale, and of divorcing the knower from the context of knowing diminishes the complexity of the construct of spatial reasoning.

We thus find it more productive to describe spatial reasoning as the constrained co-occurrence of sensory flux (sensation), recognition/discrimination (perception), and situated movement of a body. As might be illustrated in video clips of learner engagement (Video 1 and Video 2), this tripartite set constitutes an act of spatial reasoning, rather than either invisible cognitions or actual movements. This is the enactivist shift – in that, following Dewey (1896), “both sensation and movement lie inside, not outside, the act” (p. 359).

Analysis: From sensorimotor coordination to sensory-motor control

Figure 2: L-shaped (Video 1 Link)

In the video-linked episode , Eric was looking for an L-shaped piece to attach to his robot-in-progress. (See Figure 2: the piece appears in two places – and in two ways – on the page of instructions.) As we aim to illustrate, this engagement is an instance of perceptually guided action that arises from perception involving visual, tactile, and sensorimotor stimuli. Before delving into our analysis, we emphasize that we make no claims about cognitive structures (which we cannot see), but we do talk about the recurrent sensorimotor patterns that *enable* action.

The required L-shaped piece was in a section of the kit to Eric's right, buried beneath another shape, and differently oriented to both illustrations in the instruction booklet. As Eric sought to find it, his gaze and hand acted in concert as he reached into the orange container (see 1'15" of Video 1). Echoing Dewey (1986), the acts of seeing and reaching were bound together. The eyes, the fingers, the wrist and the arms all worked in unison to both see and grasp the L-shape. The seeing and reaching were part of a grander coordination of recognizing and selecting the unique piece among an assortment of more than 80 distinct shapes.

When Eric's eyes, fingers, hands, wrist, and arm coordinated in the attempt to attach the L-shaped piece, there was a new whole constituted in the cycle "which makes it impossible to say which started first in the exchange of stimuli and responses" (Merleau-Ponty, 1963, as cited in Cummins, 2013, p. 183). As Eric tried to fit the L-shaped piece to the robot his perception was that it didn't fit, which led him to manipulate the object and try again (see 1'28" of Video 1). He continued to try many times, until his perception was that it did fit. The perception (arising from sensorimotor information) guided (and arises from) the cognition which then guided the action. As noted earlier, the task serves both diagnostic and developmental purposes.

Structural Coupling & Uncoupling: The role/space of the social from an enactivist perspective

In this section we illustrate the enactivist notion of structural coupling and point to an important but often overlooked dimension to the phenomenon in a social system (in contrast to a physical system), viz. uncoupling. The notion of structural coupling derives from a biological perspective of an organism and environments co-adaption or evolution to each other. The mutual interaction of the organism and the environment causes changes and transformations in both.

We argue that, in an educational setting, learning is dependent on both socio-cognitive coupling *and* uncoupling. The former serves as a trigger or a perturbation and the latter provides opportunities for pursuing personal interest/focus necessary for individual learning. Varela et al. (1991) drew on Darwin's notion of evolution to describe structural coupling of the co-adaptation of an organism and its environment, noting that the ability of an organism to un-couple from its environment is also important for the organism's survival. Being too tightly coupled to a specific environment may lead to extinction if the environment changes in even minor ways (e.g., should water levels drop substantially, a fish has fewer options for survival than an amphibian). From an educational perspective, the combined socio-cognitive coupling and uncoupling can provide opportunities for learning that enable the organism to adapt learning to other environments.

Figure 3: Coupling (Link to Video 2 clip)

Figure 4: Uncoupling (Link to Video 2 clip)

An event of socio-cognitive coupling-and-uncoupling is presented through the video links above (Figures 3 & 4). In these clips, Declan is working on his own robot, Christopher approaches to find clarification of a problem he is having with robot construction (see 0'40" of Video 2). Declan observes, analyses Christopher's current status, points, describes where the error is, and identifies the stage where the error occurred. From and enactivist perspective, this coupling of Declan and Chris triggers a number of processes.

Applying our definition above of spatial reasoning, as the constrained co-occurrence of sensory flux (sensation), recognition/discrimination (perception), and movement, Figure 3 (and the linked Video 2 clip) shows a coupling of two children whose object of focus – the shared basis constraining the coupling – is one of their robots. The sensory flux in this coupling involves speaking, pointing, looking, touching, and holding. The recognitions of what is not right and when help is needed are the prompts for coupling. We claim that spatial reasoning is occurring as a part of the coupling in this moment. Spatial reasoning in this instance can be viewed in terms of either individual cognitive process or as interpersonal social process – or, in more educationally productive terms, as a socio-cognitive process.

On this count, we find dual-process theory (Kahneman, 2011) a useful supplement to enactivism. Briefly, dual-process theorists posit that individual knowing arises in the coactivity of two quasi-distinct knowing systems, the Automatic (System 1) and the Reflective (System 2). System 1 is quick and intuitive, rooted in memory and rehearsed experience, more given to analogy than to logic, and usually accurate in its reads and responses. System 2 is slow and deliberate, based in conscious though and concerned with novelty or perceived incoherences. It tends to be much more logical and, at the same time, much more prone to misreadings and unfitting responses.

Most of the time, the Reflective System 2 defers to the Automatic System 1. It is only when a threshold of unfamiliarity or confusion is met that System 2 is triggered into action.

We see both systems in play for both actors in the clips above. To discern the coupled and uncoupling of Declan's and Chris's Systems 1, we find it helpful to mute the sound and focus on the fluid choreography of their mutually specifying actions. These actions are smooth, precisely timed, exquisitely coordinated, and astonishingly free of excess motion. Such are the hallmarks of automatized action – which, to our observation, are appropriately characterized as embodied or enacted knowings.

Of course, we must be careful not to understate the roles and couplings of the actors' Systems 2 in this episode. After all, the event was triggered by Chris's conscious recognition of a difficulty. That is, in terms of dual-process theory, Chris encountered an instance of insufficient and/or inadequately integrated experiences to evoke a routinized response in a novel situation. Lacking that, another of his automatized responses appears: he calls on a likely-to-know and proximate other. Once System 2 is oriented to this course of action, System 1 appears to take over again, as suggested by the fluidity of the actions and articulations.

Declan's response is similarly interesting. Its immediacy indicates that the solution he offers to Chris was drawn from his repertoire of rehearsed actions. But more interesting to us is the seamless sequence of his couplings and uncouplings, starting with a cognitive uncoupling from his own work, a socio-cognitive coupling with Chris, a socio-cognitive uncoupling from Chris, and a cognitive recoupling with the original task. These subevents occurred in just seconds, at a speed that exceeds the capability of System 2. They were embodied.

Turning back to Chris, the instant of social uncoupling afforded him an opportunity to process what Declan disclosed (see 0'56" of Video 2). He then returns to Declan to explain what the problem was (see 1’05” of Video 2). The temporary uncoupling provides the time and space for individual reflection, which within a social context that supports accumulating experiences in a vast number of cases, serves as an occasion for sharing learning.

To be clear, we are still talking about spatial reasoning here. Our point is that spatial reasoning competence is not a solitary achievement, but one that arises in the main amid such socio-cognitive couplings and uncouplings. Of course, the same might also be said of other mathematical competencies. However, the particular advantage of the topic of spatial reasoning is that understandings are typically much more available to observation. We can, literally, see Chris’ and Declan’s understandings in their actions.

Moreover, and somewhat provocatively, when we watch an accelerated version of the more complete recording of the extended engagement from which this episode is extracted, we observe a distinct pattern of structured interactivity in which agents pull together and move apart in a rhythmic pulse as they structurally couple and uncouple. In our enactivist framing, we would be curious about those moments of coming together and those moving apart, that pulse of complexity, as occasions for accumulating experience in a vast number of cases for individuals and for the collective.

Implications

> To understand mentality, however complex and sophisticated it may be, it is necessary to start by appreciating how living beings dynamically interact with their environments, both shaping and being shaped by those encounters; ultimately there is no prospect of understanding minds without reference to ongoing interactions between organisms and their environments. (Hutto, 2013, p. 176)

We note that constructivisms, as adopted and adapted within the field of mathematics education in the 20th and early-21st centuries, have been mainly concerned with conceptual understanding of numerical and algebraic concepts. While some enactivist-aligned contributions have highlighted the importance of taking into account the body in efforts to make sense of these areas of mathematical competence (eg. Lakoff & Núñez, 2000), it is telling that relatively little of the research into arithmetic and algebraic competence delves deeply into the bodily basis of meaning.

In contrast, with the emergent recognition that spatial reasoning is a core element of mathematics competence, it is apparent that theories of learning that are principally focused on the evolutions of personal conceptual coherence are inadequate. Spatial reasoning is much more obviously and directly anchored to one's experiences, situatedness, and intentions – in brief, one's enactments. Returning to our title, what counts as sufficiently vast for individual learners will not be the same for others.

When we consider this realization alongside the longstanding tensions between knower-centered constructivisms and socio-cultural accounts of learning and knowing, we are even more compelled toward an enactivist frame. Intricate dances of cognitive and social coupling and uncoupling surpass perspectives that privilege one or another domain of (inter)action.

Our enactivist framing above as the constrained co-occurrence of sensory flux (sensation), recognition/discrimination (perception), and movement of a body, presents one opportunity for the field to re-consider both the phenomenon of interest and the way of studying it. However, we conclude that the phenomenon for which the concept/signifier 'spatial reasoning' is used as a descriptor is a complex one. Enactivist perspectives we believe offer fit frameworks for interpreting and investigating what it means to weave one's embodied and knowing self through the world.

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